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Multi-Dimensionally Constrained Covariant Density Functional Theories: Formalism and Applications

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Introduction

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Normal nuclei

>1-, 2-, & 3-dim PES of ²⁴⁰Pu & fission barriers of actinides >Non-axial octupole (tetrahedral) shapes in N = 150 isotones >Superdeformed & hyperdeformed shapes in actinides

\square Λ hypernuclei

- Shape polarization effect of Λ
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- Summary & perspectives

Nuclear shapes

$$R(\theta,\varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta,\varphi) \right]$$



Courtesy of Bing-Nan Lu (吕炳楠)

Nonaxial quadrupole shape (β_{22} or γ)

J. Phys. G: Nucl. Part. Phys. 37 (2010) 064025

Meng_Zhang 2010_JPG37-064025



Figure 1. Left- and right-handed chiral systems for a triaxial odd-odd nucleus.

A static triaxial shape in atomic nuclei manifests itself by the wobbling motion & chiral doublet bands

Bohr & Mottelson 1975 Odegard ... 2001_PRL86-5866 Frauendorf_Meng1997_NPA617-131 Starosta ... 2001_PRL86-971

Octupole shape (β_{30})

ARTICLE

Gaffney... 2013_Nature497-199

doi:10.1038/nature12073

Studies of pear-shaped nuclei using accelerated radioactive beams

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Nonaxial (β_{22} or γ) & octupole (β_{30}) shapes in PES



Axial asymmetry plays important roles around the first barrier Reflection asymmetry plays important roles around the second barrier

Nonaxial (β_{22} or γ) & octupole (β_{30}) shapes in PES



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Covariant Density Functional Theory (CDFT)

$$\mathcal{L} = \bar{\psi}_{i} (i\partial - M) \psi_{i} + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - U(\sigma) - g_{\sigma} \bar{\psi}_{i} \sigma \psi_{i}$$

$$- \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_{\omega}^{2} \omega_{\mu} \omega^{\mu} - g_{\omega} \bar{\psi}_{i} \psi \psi_{i}$$

$$- \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{1}{2} m_{\rho}^{2} \vec{\rho}_{\mu} \vec{\rho}^{\mu} - g_{\rho} \bar{\psi}_{i} \vec{\rho} \vec{\tau} \psi_{i}$$
Serot_Walecka1986_ANP16-1
$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi}_{i} \frac{1 - \tau_{3}}{2} A \psi_{i},$$
Reinhard1989_RPP52-439
Ring1996_PPNP37-193
Vretenar_Afanasjev_Lalazissis_Ring2005_PR409-101
Meng_Toki_SGZ_Zhang_Long_Geng2006_PPNP57-470

$$(\boldsymbol{\alpha} \cdot \mathbf{p} + \boldsymbol{\beta}(\boldsymbol{M} + \boldsymbol{S}(\mathbf{r})) + \boldsymbol{V}(\mathbf{r}))\psi_{i} = \epsilon_{i}\psi_{i}$$

$$\left(-\nabla^{2} + m_{\sigma}^{2}\right)\sigma = -g_{\sigma}\rho_{S} - g_{2}\sigma^{2} - g_{3}\sigma^{3}$$

$$\left(-\nabla^{2} + m_{\omega}^{2}\right)\omega = g_{\omega}\rho_{V} - c_{3}\omega^{3}$$

$$\left(-\nabla^{2} + m_{\rho}^{2}\right)\rho = g_{\rho}\rho_{3}$$

$$-\nabla^{2}A = e\rho_{C}$$

MDC-CDFT ($\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}, \beta_{40}, ...$)

□ Axially deformed harmonic oscillator (ADHO) basis

$$\begin{bmatrix} -\frac{\hbar^2}{2M} \nabla^2 + V_B(z,\rho) \end{bmatrix} \Phi_{\alpha}(\boldsymbol{r}\sigma) = E_{\alpha} \Phi_{\alpha}(\boldsymbol{r}\sigma)$$

$$V_B(z,\rho) = \frac{1}{2} M(\omega_{\rho}^2 \rho^2 + \omega_z^2 z^2)$$

$$\Phi_{\alpha}(\boldsymbol{r}\sigma) = C_{\alpha} \phi_{n_z}(z) R_{n_{\rho}}^{m_l}(\rho) \frac{1}{\sqrt{2\pi}} e^{im_l \varphi} \chi_{s_z}(\sigma)$$

□ Fourier expansion for densities & potentials $f(\rho, \varphi, z) = f_0(\rho, z) \frac{1}{\sqrt{2\pi}} + \sum_{n=1}^{\infty} f_n(\rho, z) \frac{1}{\sqrt{\pi}} \cos(2n\varphi)$ $f = V \text{ or } \rho$

A modified linear constraint method

$$E' = E_{\rm RMF} + \sum_{\lambda\mu} \frac{1}{2} C_{\lambda\mu} Q_{\lambda\mu} \qquad C_{\lambda\mu}^{(n+1)} = C_{\lambda\mu}^{(n)} + k_{\lambda\mu} \left(\beta_{\lambda\mu}^{(n)} - \beta_{\lambda\mu} \right)$$

Lu_Zhao_SGZ 2014_PRC89-014323

MDC-CDFT (β_{20} , β_{22} , β_{30} , β_{32} , β_{40} , ...)

ph channel	Non-linear	Density-dependent
Meson exchange	NL3, NL3*, PK1,	DD-ME1, DD-ME2,
Point Coupling	PC-F1, PC-PK1,	DD-PC1,
	MDC-RMF	MDC-RHB
pp channel	BCS	Bogoliubov
Constant gap	\checkmark	
Constant strength	\checkmark	
Delta force	\checkmark	\checkmark
Separable force	\checkmark	\checkmark

Lu_Zhao_SGZ 2011_PRC84-014328 Zhao_Lu_Zhao_SGZ 2012_PRC86-057304 Lu_Zhao_SGZ 2012_PRC85-011301R Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

Δ (β₂, β₃, …)

Geng_Meng_Toki2007_ChinPhysLett24-1865

Δ (β₂, γ, …)

Meng_Peng_Zhang_SGZ2006_PRC73-037303

□ (β₂, ...)

Ring_Gambhir_Lalazissis1997_CPC105-77

Lu_Zhao_SGZ 2011_PRC84-014328 Lu_Zhao_SGZ 2012_PRC85-011301R Zhao_Lu_Zhao_SGZ 2012_PRC86-057304 Lu_Zhao_Zhao_SGZ 2014_PRC89-014323





Zhao_Lu_Zhao_SGZ 2012_PRC86-057304 Lu_Zhao_Zhao_SGZ 2014_PRC89-014323





Lu_Zhao_Zhao_SGZ 2012_1 RC80-037304



Lu_Zhao_SGZ 2012_PRC85-011301R Zhao_Lu_Zhao_SGZ 2012_PRC86-057304 Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

Potential energy surface: an example



Potential energy surface: an example



Courtesy of Bing-Nan Lu (吕炳楠)

²⁴⁰Pu: 1-dim. potential energy curve (β_{20})

- □ Triaxiality lowers inner barrier height by more than 2 MeV
- Octupole deformation lowers outer barrier dramatically
- □ Triaxiality lowers outer barrier height by about 1 MeV



Lu_Zhao_SGZ 2012_PRC85-011301R

²⁴⁰Pu: 2-dim. PES (β₂₀, β₃₀)



²⁴⁰Pu: 3-dim. PES ($\beta_{20}, \beta_{22}, \beta_{30}$)



Lu_Zhao_SGZ 2012_PRC85-011301R

B_f of actinide nuclei



- □ Influence of triaxiality
 - Inner fission barriers lowered by 1~2 MeV
 - ➢Outer fission barriers lowered by 0.5~1 MeV
- Problems
 - >²³⁰⁻²³²Th: out barriers primary
 - >²³⁸U: ?
 - ≻²⁴⁸Cm: two fission paths

Empirical values: RIPL-3 (NDS2010)

Lu_Zhao_SGZ 2012_PRC85-011301R

Skalski1991_PRC43-140 Hamamoto_Mottelson_Xie_Zhang1991_ZPD21-163 Li_Dudek1994_PRC49-1250R Takami_Yabana_Matsuo1998_PLB431-242



Dudek_Gozdz_Mazur_Molique_Rybak_Fornal 2010_JPG37-064032

PHYSICAL REVIEW C 77, 061305(R) (2008)

Nonaxial-octupole effect in superheavy nuclei

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The triaxial-octupole Y_{32} correlation in atomic nuclei has long been expected to exist but experimental evidence has not been clear. We find, in order to explain the very low-lying 2⁻ bands in the transfermium mass region, that this exotic effect may manifest itself in superheavy elements. Favorable conditions for producing triaxial-octupole correlations are shown to be present in the deformed single-particle spectrum, which is further supported by quantitative Reflection Asymmetric Shell Model calculations. It is predicted that the strong nonaxial-octupole effect may persist up to the element 108. Our result thus represents the first concrete example of spontaneous breaking of both axial and reflection symmetries in the heaviest nuclear systems.



Zhao_Lu_Zhao_SGZ2012 PRC86-057304



□ Y_{32} correlations from near degeneracy of pair of orbitals with $\Delta I = \Delta j = 3 \& \Delta K = 2$

□ For ²⁴⁸Cf

 $\gg \pi 7/2[633] (1i_{13/2}) \& \pi 3/2[521] (2f_{7/2}) \\ \gg \nu 9/2[734] (1j_{15/2}) \& \nu 5/2[622] (2g_{9/2})$

Zhao_Lu_Zhao_SGZ2012 PRC86-057304

Moller1972_NPA192-529

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...
Blons_Mazur_Paya1975_PRL35-1749
...
Cwiok_Nazarewicz_Saladin_Plociennik_Johnson1994_PLB322-304
...
Csige et al. 2013_PRC87-044321
...
Kowal_Skalski2012_PRC85-061302R
Jochimowicz_Kowal_Skalski2013_PRC87-044308
Ichikawa_Moller_Sierk2013_PRC87-054326
McDonnell_Nazarewicz_Sheikh2013_PRC87-054327
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Zhao_Lu_Vretenar_Zhao_SGZ arXiv: 1404.5466 [nucl-th]

Shapes of Λ hypernuclei w/ MF models

- □ Non-relativistic mean field study of hypernuclei
 - >An axially deformed Skyrme Hartree-Fock (SHF) model
 - Similar shapes of core nuclei & the corresponding hypernuclei
 Zhou_Schulze_Sagawa_Wu_Zhao2007_PRC76-034312

□ However, a RMF study reveals

- In most cases the results are similar to the SHF calculations
 Several exceptions, e.g., ¹³_AC & ²⁹_ASi whose shapes change dramatically compared to their corresponding core nuclei Win_Hagino2008_PRC78-054311
- \blacksquare Different polarization effect of Λ in SHF & RMF

Schulze_Win_Hagino_Sagawa2010_PTP123-569

Triaxiality in hypernuclei w/ RMF model

	Skyrme HF	RMF
Spherical	Yes	Yes
Axially deformed	Yes	Yes
Triaxially deformed	Yes	???

Triaxially defromed RMF: With an additional Λ , no significant shape change occurs except that the PES becomes softer in the γ direction

Win_Hagino_Koike2011_PRC83-014301

What RMF models predict for trixiality in hypernuclei?

RMF model for Λ hypernuclei

 $\Box \text{ The Lagrangian density: } \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Lambda$ $\mathcal{L}_\Lambda = \bar{\psi}_\Lambda \left(i\gamma^\mu \partial_\mu - m_\Lambda - g_{\sigma\Lambda}\sigma - g_{\omega\Lambda}\gamma^\mu \omega_\mu \right) \psi_\Lambda$ $+ \frac{f_{\omega\Lambda\Lambda}}{4m_\Lambda} \bar{\psi}_\Lambda \sigma^{\mu\nu} \Omega_{\mu\nu} \psi_\Lambda,$

D The Dirac equation for Λ $[\vec{\alpha} \cdot \vec{p} + \beta (m_{\Lambda} + S_{\Lambda}) + V_{\Lambda} + T_{\Lambda}] \psi_{\Lambda i} = \epsilon_i \psi_{\Lambda i}$

$$T_{\Lambda} = -\frac{f_{\omega\Lambda\Lambda}}{2m_{\Lambda}}\beta\left(\vec{\alpha}\cdot\vec{p}\right)\omega$$

D Effective interactions

Parameter	m_{Λ}	$g_{\sigma \Lambda}$	ŚωΛ	$f_{\omega \wedge \wedge}$	N-N interaction
PK1-Y1	1115.6 MeV	$0.580g_{\sigma}$	$0.620g_{\omega}$	-gωλ	PK1
NLSH-A	1115.6 MeV	$0.621g_{\sigma}$	$0.667 g_{\omega}$	-g _{ωΛ}	NLSH

PK1-Y1: Song_Yao_Lü_Meng2010_IJMPE19-2538

Wang_Sang_Wang_Lü2014_Commu.Theor.Phys.60-479 NLSH-A: Win_Hagino2008_PRC78-054311

Carbon isotopes: w/ & w/o Λ



Potential energy surfaces of ${}^{12}C \& {}^{13}_{\Lambda}C$



Silicon isotopes: w/ & w/o Λ



Potential energy surfaces of ²⁸Si & ²⁹ $_{\Lambda}$ Si

Model dependence Parameter dependence



Potential energy surfaces of ²⁸Si & ²⁹ $_{\Lambda}$ Si











TABLE II. Calculated deformation parameter β_2 , the overlap I_{overlap} defined in Eq. (11), the binding energy E, and Λ separation energy S_{Λ} of some Λ hypernuclei. For comparison, the binding energy of the core nucleus E_{core} is also given. The energies are in MeV.

	Nucleus	eta_2	Ioverlap	$E_{\rm tot}$	$E_{\rm core}$	S_{Λ}
	$^{37}_{\Lambda}$ Ar $^{37}_{\Lambda}$ Ar*	$-0.204 \\ 0.597$	0.1352 0.1370	-321.979 -315.194	-303.802 -296.670	18.177 18.524
	$^{39}_{\Lambda}$ Ar $^{39}_{\Lambda}$ Ar*	0.000 0.589	0.1360 0.1378	-344.896 -336.306	-326.455 -317.448	18.441 18.858
s	$^{41}_{\Lambda}$ Ar $^{41}_{\Lambda}$ Ar*	-0.117 0.491	0.1357 0.1378 0.1361	-361.398 -355.922 361.422	-342.613 -336.852 342.860	18.785 19.070 18.553
· _ · · · ·	$^{\text{A}}Ca^{41}$ Ca*	0.00	0.1301 0.1393	-301.422 -350.559	-342.809 -331.317	19.242
-0.2 0	$\frac{1}{10} \frac{1}{10} \frac$	0.26 0.97 0.00	0.1376 0.1243 0.1461	-285.095 -274.315 -506.665	-267.002 -257.951 -484.759	16.364 21.906
	$\int_{\Lambda}^{57} Ni^*$	0.40 0.22	0.1415 0.1438	-498.610 -534.565	-477.892 -512.924	20.718 21.641
	$^{61}_{\Lambda}$ Zn*	0.62	0.1415	-527.168	-506.238	20.930







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Isaka_Fukukawa_Kimura Hiyama_Sagawa_Yamamoto 2014_PRC89-024310

AMD: Opposite predictions !

TABLE I. Calculated excitation energy E_x in MeV, matter quadrupole deformation β and γ (deg), and rms radii (fm). The B_{Λ} defined by Eq. (14) is also listed in unit of MeV for $^{41}_{\Lambda}$ Ca.

	J^{π}	E_x (MeV)	β	γ (deg)	r _{rms} (fm)	B_{Λ}
$^{41}_{\Lambda}$ Ca	$1/2_{1}^{+}$	0.00	0.10	0	3.38	19.45
11	$1/2^{+}_{2}$	9.24	0.40	27	3.47	19.15
	$1/2_{3}^{+}$	11.41	0.55	13	3.58	18.01
⁴⁰ Ca	0_{1}^{+}	0.00	0.12	12	3.39	
	0^{+}_{2}	8.94	0.40	28	3.50	
	0_{3}^{-1}	9.97	0.60	17	3.63	

LETTER

Ebran_Khan_Niksic_Vretenar 2012_Nature487-341

doi:10.1038/nature11246



Summary

- Multidimensionally-constrained covariant density functional theories: (β₂₀, β₂₂, β₃₀, β₃₂, β₄₀, ...)
- PES & fission barriers of heavy normal nuclei
 - ➤Triaxiality lowering systematically 2nd barrier of actinides
 - >Non-axial octupole correlations in N=150 isotones
 - Shallow hyperdeformed minima in actinides
- □ Shape of hypernuclei
 - > Drastic shape changes in $({}^{12}C-{}^{13}_{\Lambda}C)$, $({}^{28}Si-{}^{29}_{\Lambda}Si)$, ...
 - >Localization effects in superdeformed 36,38,40 Ar & 40 Ca, leading to larger Λ separation energy than in ground state

Summary & perspectives

- Multidimensionally-constrained covariant density functional theories: (β₂₀, β₂₂, β₃₀, β₃₂, β₄₀, ...)
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 - >Localization effects in superdeformed 36,38,40 Ar & 40 Ca, leading to larger Λ separation energy than in ground state
- □ Systematic study of PES & B_f of superheavy nuclei □ Structure of S=-2 hypernuclei

Collaborators

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