NUCLEAR AND PARTICLE-PHYSICS ASPECTS OF CONDENSED-MATTER NANOSYSTEMS

NUCLEAR MANY-BODY METHODS IN CONDENSED-MATTER NANOSYSTEMS

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Computational Challenges in Nuclear and Many-Body Physics
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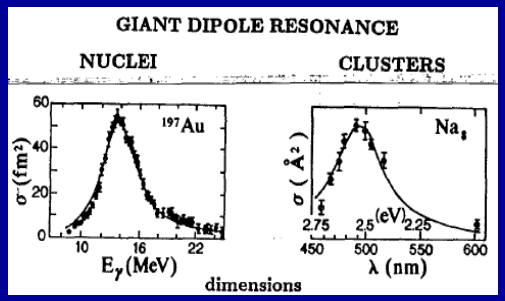
Three (among others) major nuclear analogies:

- *Surface plasmons/Giant resonances*
  (via matrix RPA/LDA) in metal clusters
  [see, e.g., Yannouleas, Broglia, Brack, Bortignon,
  PRL **63,** 255 (1989)]



- *Electronic shells/deformation/fission*
  (via Strutinsky/ Shell correction approach) in metal clusters
  [see, e.g., Yannouleas, Landman, Barnett, in "Metal Clusters",
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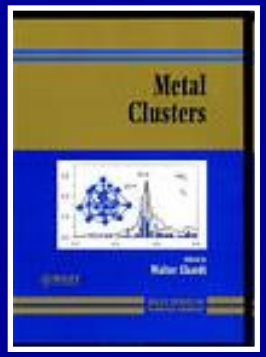
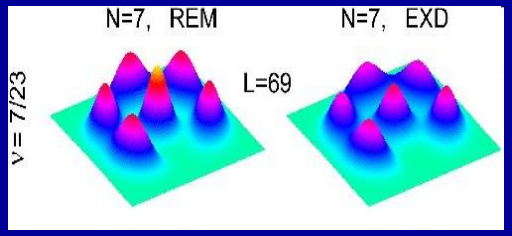


- *Strongly correlated states /* ***Repulsive interaction***
  (Quantum crystals/Wigner molecules/dissociation)
  in 2D semiconductor quantum dots and
  ultracold bosonic traps via
  symmetry breaking/symmetry restoration
  in conjunction with exact diagonalization (full CI)
  [see, e.g., Yannouleas, Landman,
  Rep. Prog. Phys. **70,** 2067 (2007)]



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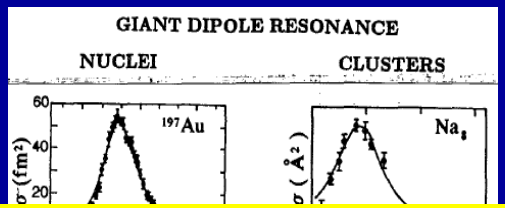
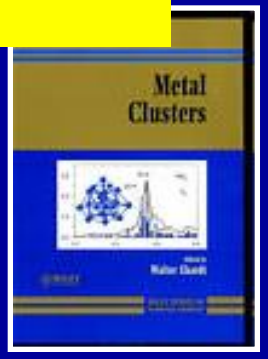


KS-DFT



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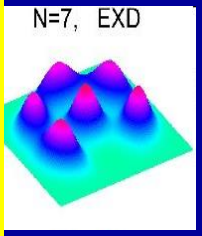
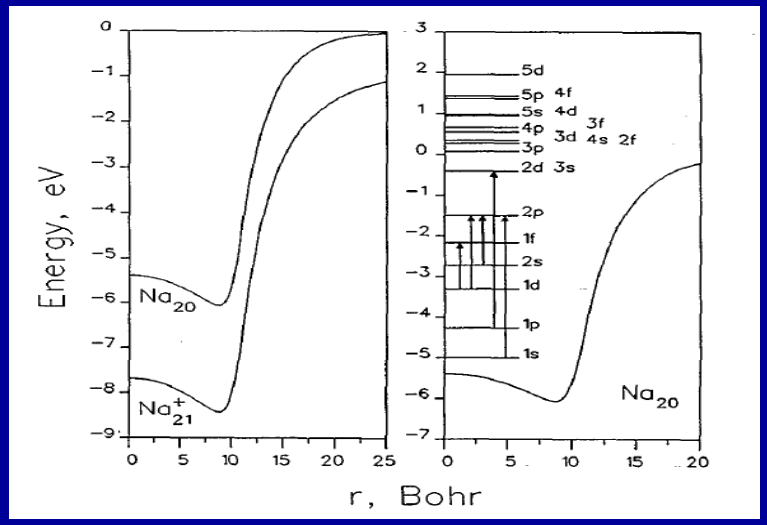
- *Strongly corre*
  (Quantum cry
  in 2D semicon
  ultracold bosc
  symmetry bre
  in conjunction
  [see, e.g., Yanno
  Rep. Prog. Phys.

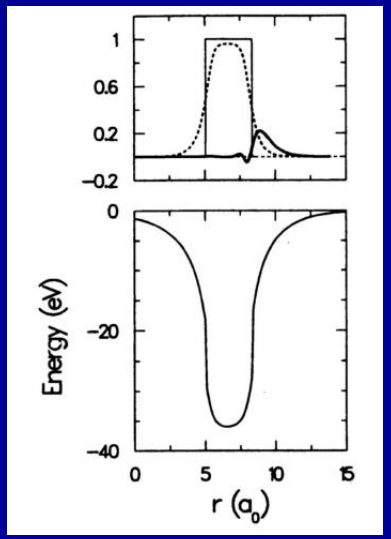
NO KS-DFT/ due to SIE, open problem of how to use multideterminants and restore symmetries



- *Electronic shells/ magic numbers/ deformation/ fission* in metal clusters
- *Surface plasmons/Giant resonances* in metal clusters

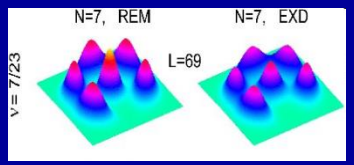
The physics of free nonrelativistic electrons confined in a central potential, like atomic nuclei
(conservation of symmetries/ independent particle model/ delocalized electrons)





- *Strongly correlated states (Quantum crystals/Wigner molecules/dissociation)* in 2D semiconductor quantum dots

No central potential/ electron localization (relative to each other) due to strong Coulomb repulsion/ mean-filed with broken symmetries



TWO VARIANTS OF SHELL CORRECTION METHOD (SCM) in condensed-matter nanosystems:

1) Fully microscopic (SCM-DFT) / **Orbital-free DFT**

Based on Extended Thomas Fermi (ETF)
sp densities and central potentials

Literature: **Y&L,** PRB **48**, 8376 (1993) (multiply anionic metal clusters)

Y&L, Ch. 7 in "Recent Advances in Orbital-Free Density Functional Theory," Y.A. Wang and T.A. Wesolowski Eds. (Word Scientific, Singapore, 2013)
(metal clusters, nanowires, fullerenes)

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2) Semiempirical (SE-SCM)

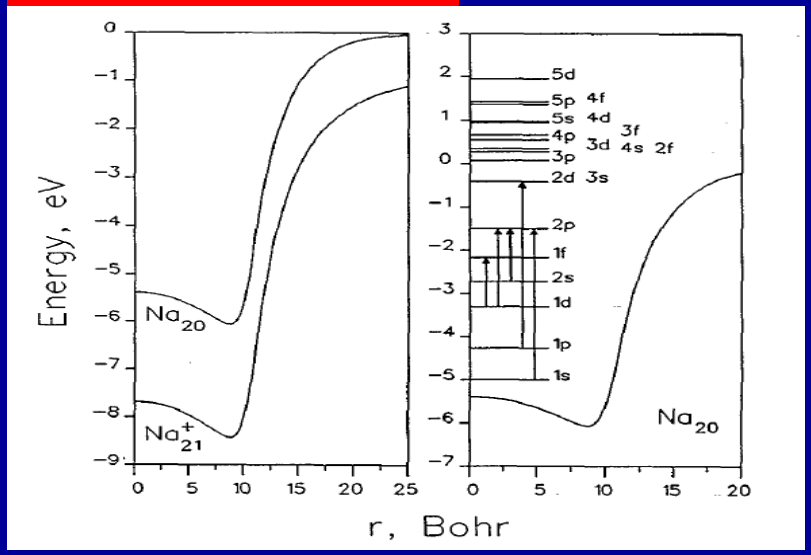
Based on a triaxial H.O. (Nilsson) central potential
+ liquid drop model for smooth variation

Y&L, PRB **51**, 1902 (1995) (deformed metal clusters)

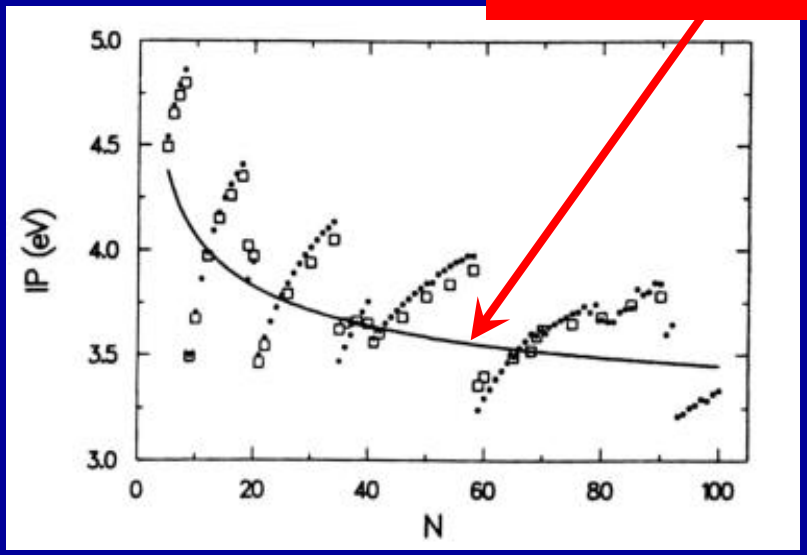
Used extensively in nuclear physics

SCM-DFT (based on ETF) ➡ KS-DFT

ETF potentials



ETF/ Smooth



Yannouleas & Landman,
PRB **48**, 8376 (1993)

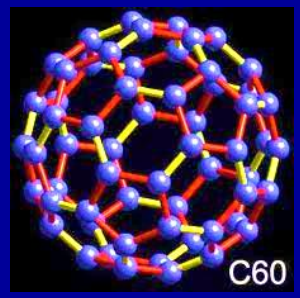
$$T_{sh} = \sum_{i=1}^{\text{occ}} \widetilde{\varepsilon}_i - \int \rho_{ETF}(\mathbf{r}) V_{ETF}(\mathbf{r}) d\mathbf{r},$$

Shell correction: Difference of two kinetic energy terms

$$\Delta E_{sh} = T_{sh} - T_{ETF}[\rho_{ETF}]$$

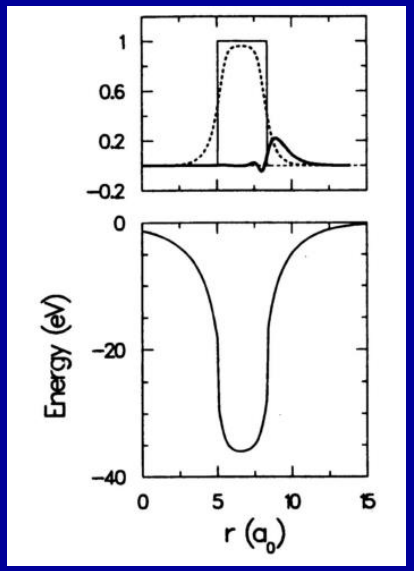
Applications of DFT-SCM: neutral fullerene C_{60}

Y&L, Chem. Phys. Lett. **217**, 175 (1994)



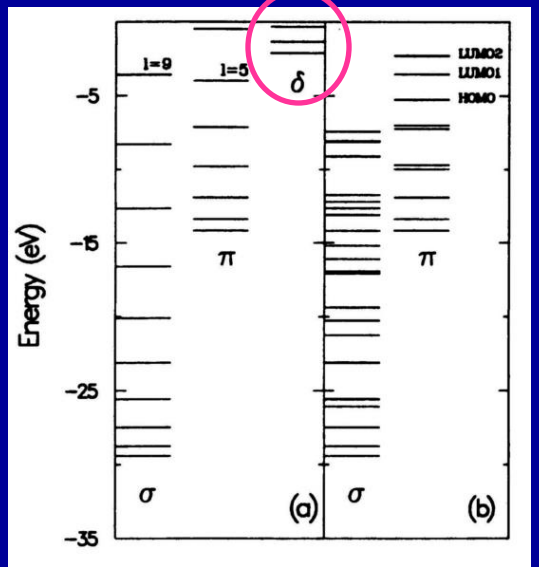
ETF density

ETF potential



Spherical

Icosahedral

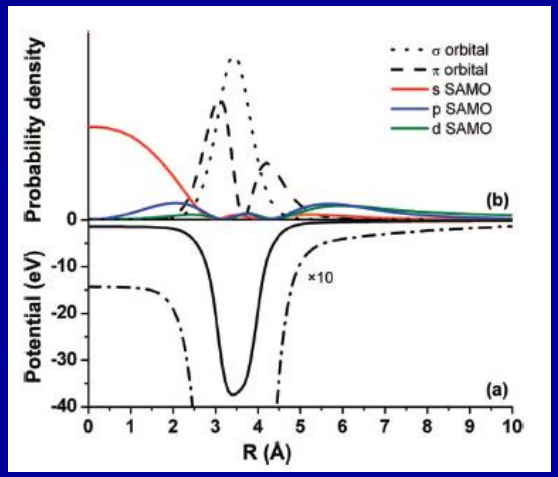


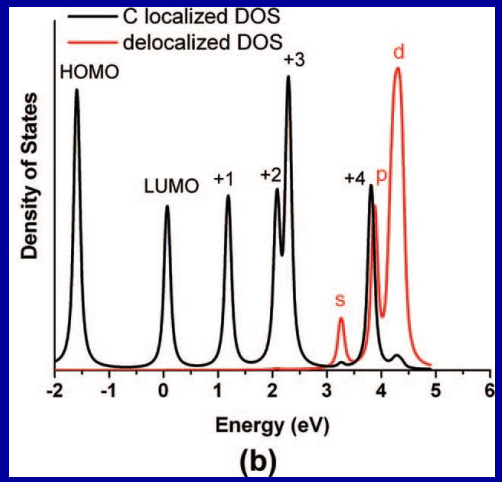
The Superatom States of Fullerenes and Their Hybridization into the Nearly Free Electron Bands of Fullerites

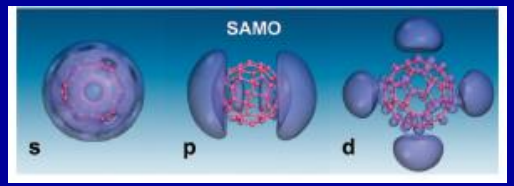
J. Zhao, M. Feng, J. Yang, H. Petek

ACS Nano **3**, 854 (2009)

LT-STM







SECOND PART

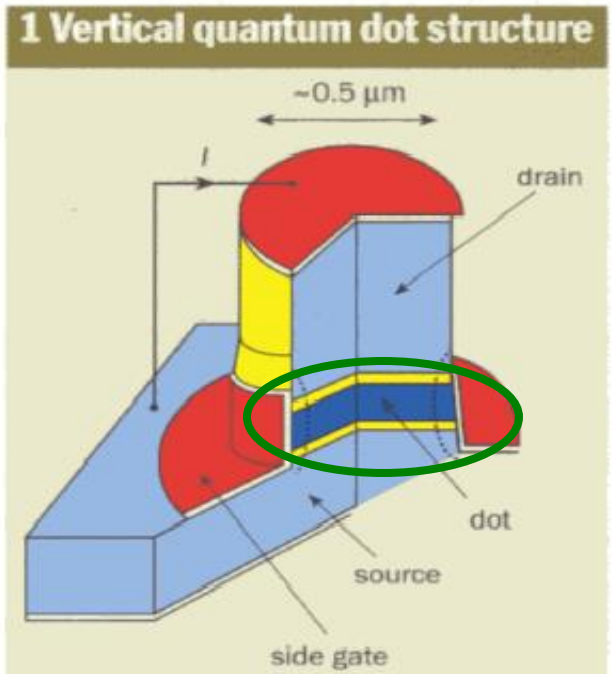
Strong correlations and symmetry breaking/restoration in 2D finite systems

Constantine Yannouleas and Uzi Landman

Phys. Rev. Lett. **82**, 5325 (1999);
Rep. Prog. Phys. **70**, 2067 (2007)

Collaborators:

Igor Romanovsky (ultracold bosons & graphene nanostructures)
Yuesong Li (electrons in QDs)
Ying Li (electrons in Quantum Dot Molecules)
Leslie O. Baksmaty (ultracold bosons & electrons in QDs)



Vertical QD (Delft)

Electrostatic confinement

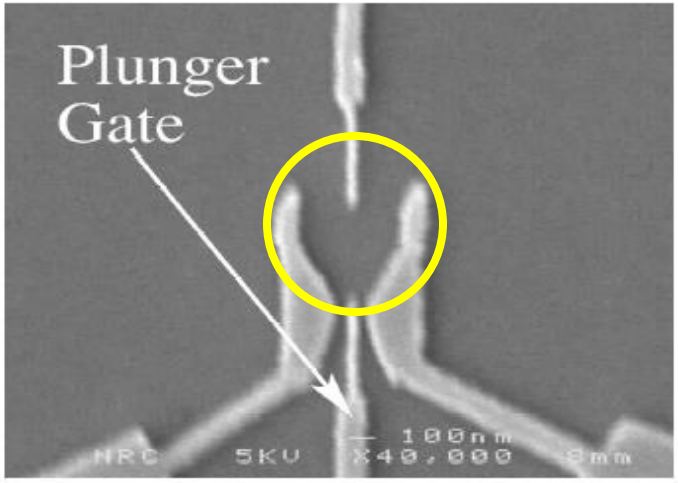
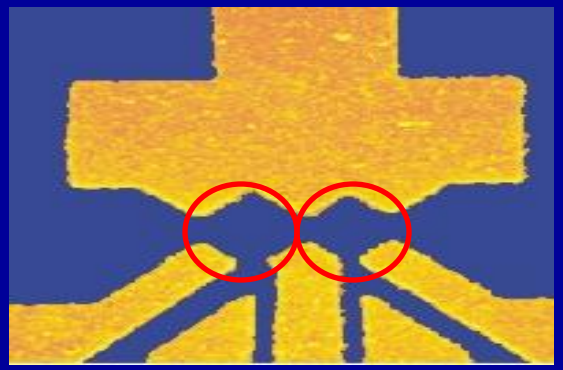


FIG. 1. SEM image of the gate geometry forming the quantum dot. This geometry enables a precisely known number of electrons ($N = 0,1,2,\ldots,50$) to be trapped (Ref. 13) and produces a quasiparabolic confinement potential. Sweeping the plunger-gate voltage tunes both the shape and the chemical potential of the quantum dot.

Lateral QD (Ottawa)

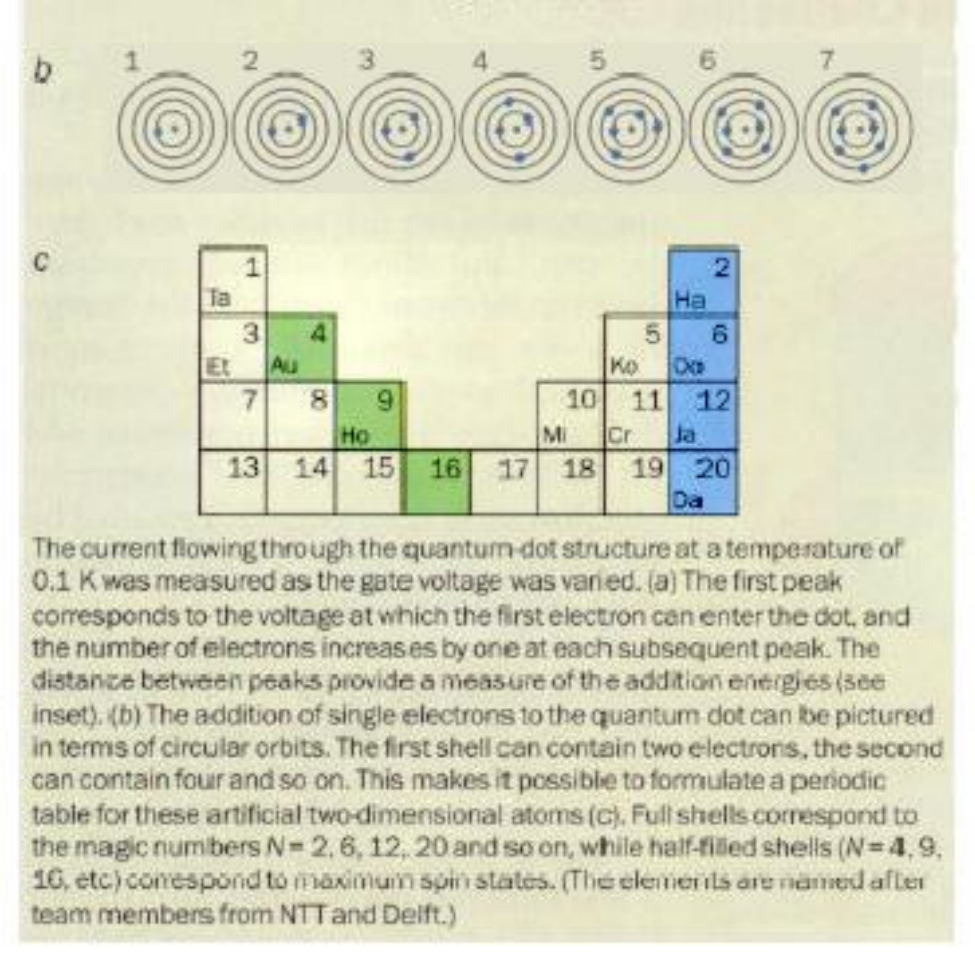


Lateral QD Molecule (Delft)

Central common confining potential?
Electronic Shells? (B=0; Circular QD)

2D Periodic Table?

4, 9, 16
Hund's Rule



The current flowing through the quantum-dot structure at a temperature of 0.1 K was measured as the gate voltage was varied. (a) The first peak corresponds to the voltage at which the first electron can enter the dot, and the number of electrons increases by one at each subsequent peak. The distance between peaks provide a measure of the addition energies (see inset). (b) The addition of single electrons to the quantum dot can be pictured in terms of circular orbits. The first shell can contain two electrons, the second can contain four and so on. This makes it possible to formulate a periodic table for these artificial two-dimensional atoms (c). Full shells correspond to the magic numbers $N = 2, 6, 12, 20$ and so on, while half-filled shells ($N = 4, 9, 16$, etc) correspond to maximum spin states. (The elements are named after team members from NTT and Delft.)

2, 6, 12, 20

Closed Shells

Kouwenhoven and Marcus, Physics World, June 1998

Wigner Crystals

DECEMBER 1, 1934 PHYSICAL REVIEW VOLUME 46

On the Interaction of Electrons in Metals

E. WIGNER, *Princeton University*

(Received October 15, 1934)

The energy of interaction between free electrons in an electron gas is considered. The interaction energy of electrons with parallel spin is known to be that of the space charges plus the exchange integrals, and these terms modify the shape of the wave functions but slightly. The interaction of the electrons with antiparallel spin, contains, fact that the electrons repell each other and try to keep as far apart as possible. The total energy of the system will be decreased through the corresponding modification of the wave function. In the present paper it is attempted to calculate this "correlation energy" by an approximation method which is, essentially, a development of the energy

… electrons repell each other and try to keep as far apart as possible. The total energy of the system will be decreased through the corresponding modification of the wave function.

… "correlation energy" …

"If the electrons had no kinetic energy, they settle in configurations which correspond to the absolute minima of the potential energy. These are close-packed lattice configurations, with energies very near to that of the body-centered lattice … "

WC: Classical Electron Crystals/ Mean Field/ Broken Symmetry

Our work: Quantum Crystals/ Beyond Mean Field/ Full Symmetry

Rotating Wigner molecule

Not a small piece of a classical
Wigner crystal (broken symmetries) ➡
Quantum crystallite (good symmetries)

B=0 ➡ Rigid rotor (L^2) and corrections (as in nuclear physics)

Large B -> ∞ (lowest Landau level - electrons) ➡

Superfloppy, supersolid: a solid flowing like a superfluid
(e.g., $aL + b/\sqrt{L}$)

Connection to FQHE

C.Y. & U.L., Physical Review B **69** (2004) 113306

HAMILTONIAN FOR CLEAN 2D QD'S AND QDM'S

$$\mathcal{H} = \sum_{i=1}^{N_e} H(i) + \sum_{i=1}^{N_e} \sum_{j>i} \frac{e^2}{\kappa r_{ij}}$$

$$H(i) = H_0(i) + H_B(i)$$

$$\frac{\vec{p}_i^{\,2}}{2m^*} + V(x_i, y_i)$$

External confinement

Parabolic, single QD

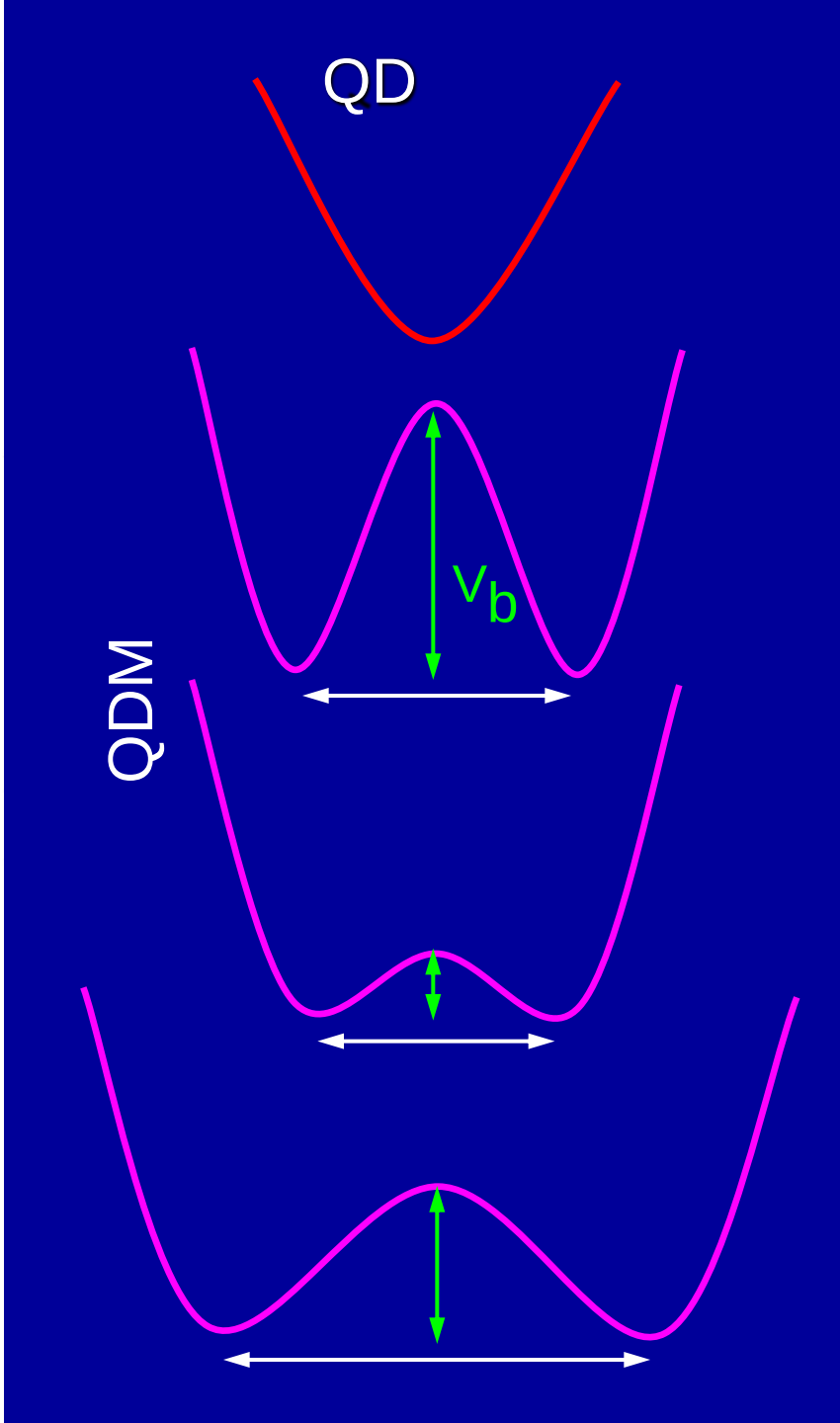
Two-center oscillator with V_b control } QDM

$$[(\vec{p}_i - e\vec{A}_i/c)^2 - \vec{p}_i^{\,2}]/2m^* + g^* \mu_B \vec{B}\cdot\vec{S}_i/\hbar$$

$$\vec{A}_i = B\,(-y_i, x_i, 0)/2$$

Zeeman

$\mathcal{H}$ can be generalized to:
Multi-component systems



HAMILTONIAN FOR CLEAN 2D QD'S AND QDM'S

$$\mathcal{H} = \sum_{i=1}^{N_e} H(i) + \sum_{i=1}^{N_e} \sum_{j>i} g\,\delta(\mathbf{r}_i - \mathbf{r}_j)$$

$$H(i) = H_0(i) + H_B(i)$$

$$\frac{\vec{p}_i^{\,2}}{2m^*} + V(x_i, y_i)$$

External confinement

Parabolic, single QD

Two-center oscillator with V_b control } QDM

Ω

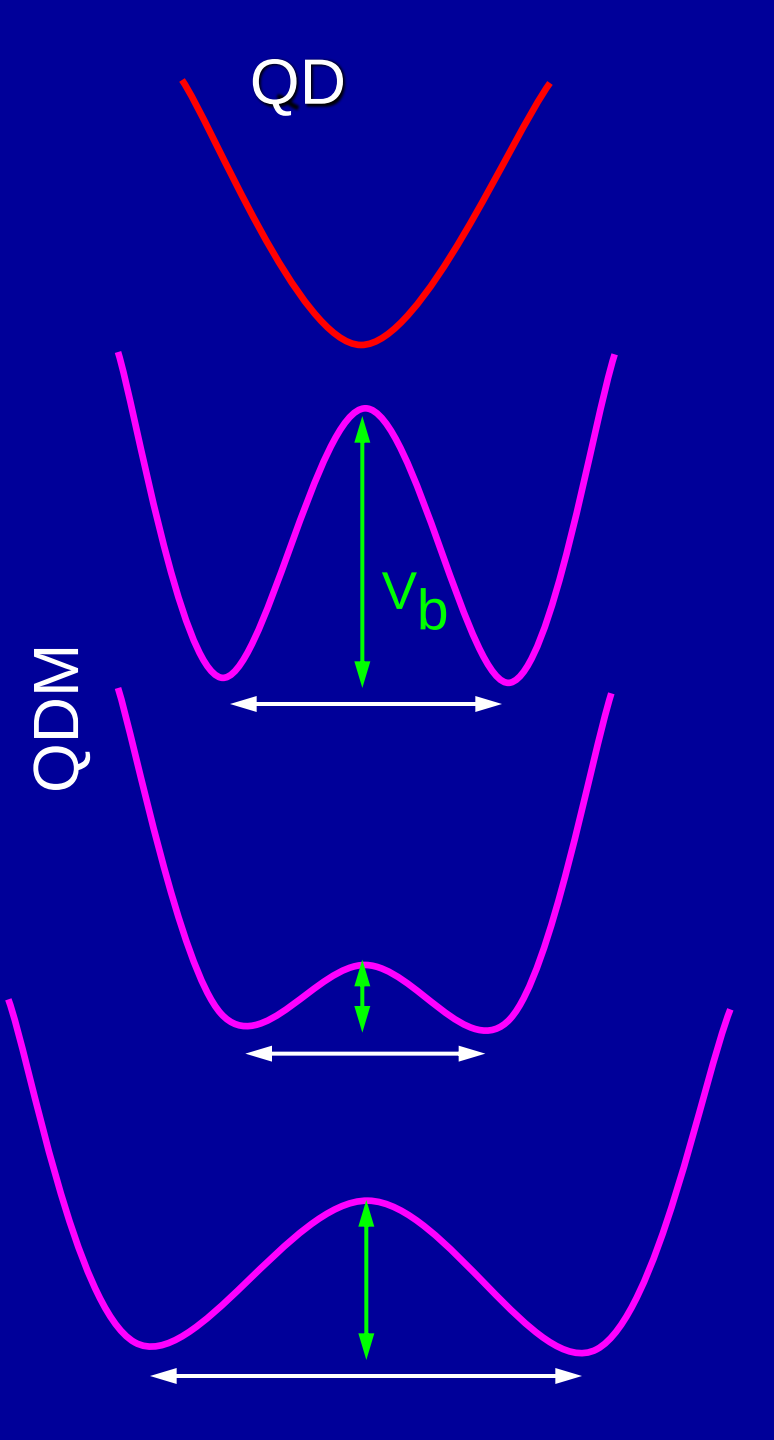
$$[(\vec{p}_i - e\vec{A}_i/c)^2 - \vec{p}_i^{\,2}]/2m^* + g^* \mu_B \vec{B}\cdot\vec{S}_i/\hbar$$

$$\vec{A}_i = B\,(-y_i, x_i, 0)/2$$

Zeeman

$\mathcal{H}$ can be generalized to Multi-component syste

Neutral Bosonic systems



CONTROL PARAMETERS FOR SYMMETRY BREAKING

IN SINGLE QD'S: WIGNER CRYSTALLIZATION

- Essential Parameter at B=0: (parabolic confinement)

$$R_W = (e^2/\kappa l_0)/\hbar\omega_0 \sim 1/(\hbar^3\omega_0)^{1/2}$$

e-e Coulomb repulsion — kinetic energy

$$l_0 = (\hbar/m^*\omega_0)^{1/2}$$ } Spatial Extent of 1s s.p. state

κ : dielectric const. (12.9)

m^* : e effective mass (0.067 m_e) GaAS

$\hbar\omega_0$ (5 - 1 meV) => R_W (1.48 - 3.31)

- In a magnetic field, essential parameter is B itself

IN QDM'S: DISSOCIATION (Electron puddles, Mott transition)

Essential parameters: Separation (d)
Potential barrier (V_b)
Magnetic field (B)

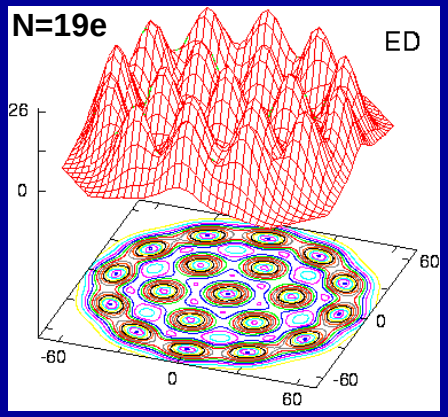
$$R_\delta = gm/(2\pi\hbar^2)$$

Neutral bosons

Circular external confinement

$B = 0$

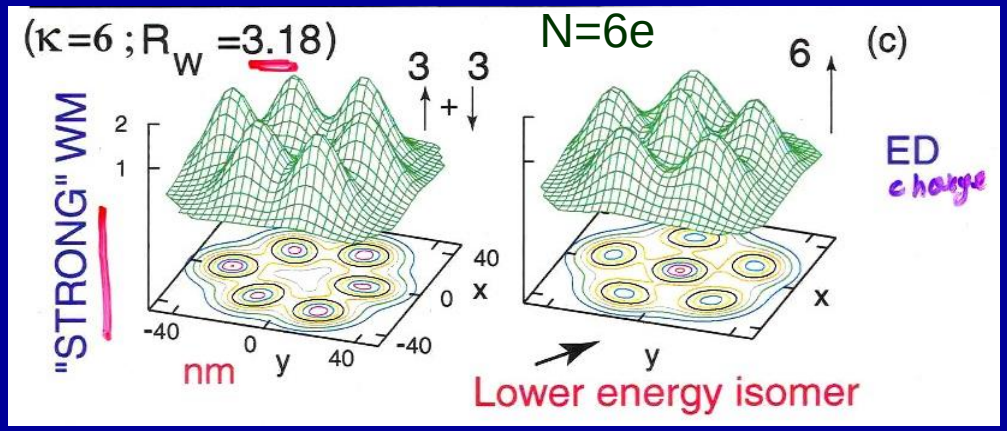
$R_W = 5$



Wigner molecule in a 2D circular QD.
Electron density (ED) from
Unrestricted Hartree-Fock (UHF).
Symmetry breaking (localized orbitals).
Concentric polygonal rings

Concentric rings: (1,6,12)

Y&L,
PRB 68, 035325 (2003)



Concentric rings: (0,6) left, (1,5) right

Y&L, PRL 82, 5325 (1999)

Exact electron densities are circular! No symmetries are broken! (N, small, large?)

Restoration of symmetry ➡ Quantum crystal

Rotating Boson Molecules (Circular trap)

Ground states: Energy, angular momentum and probability densities.

$R_\delta = 50$

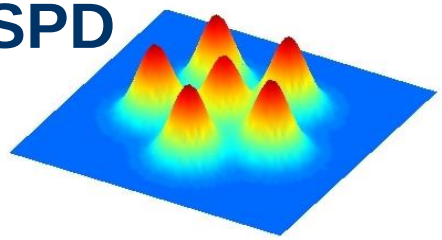
$R_W = 10$

Probability densities

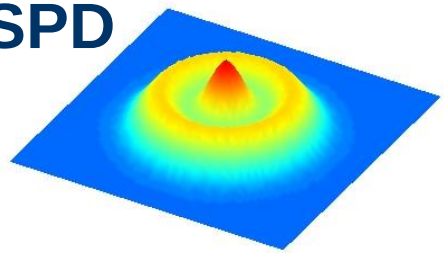
Rotating Frame

Magnetic Field

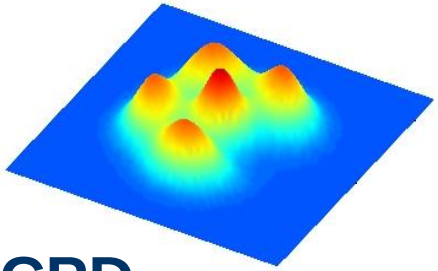
SPD

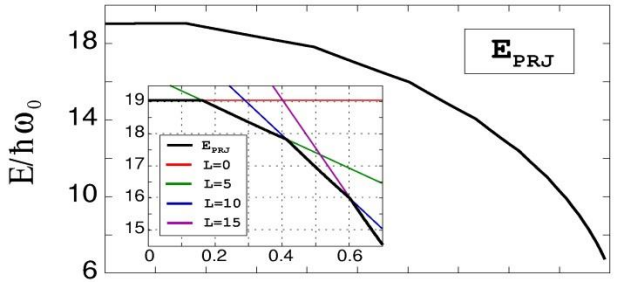


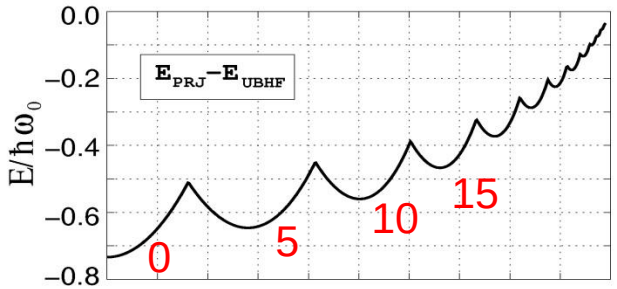
SPD

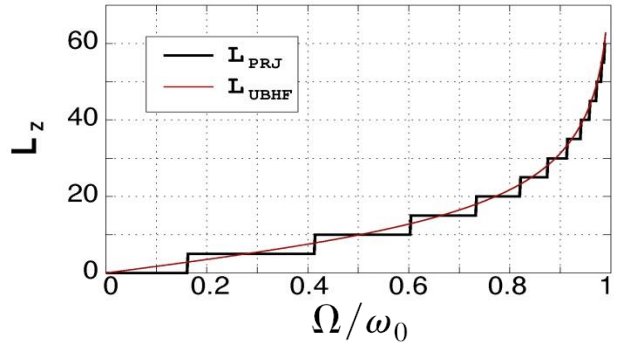


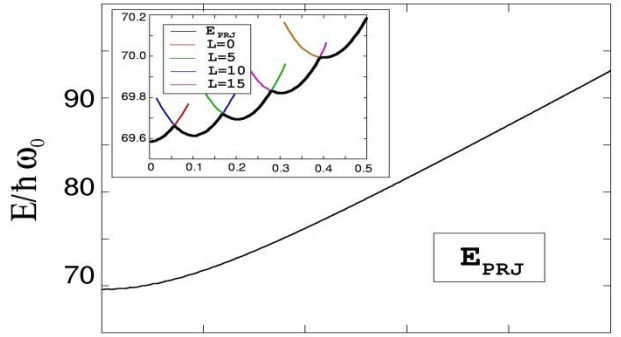
CPD

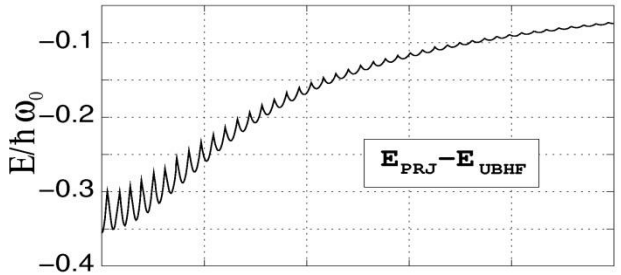


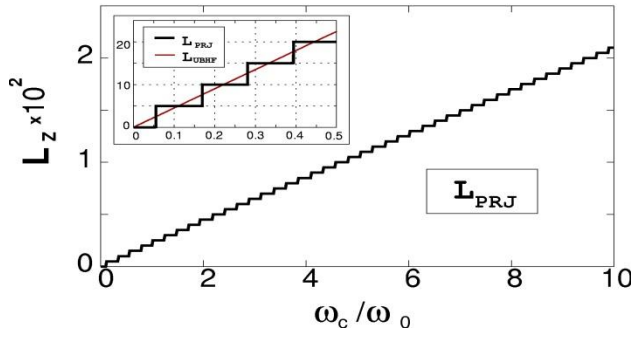












Rotating Boson Molecules (Circular trap)

Ground states: Energy, angular momentum and probability densities.

$R_\delta = 50$

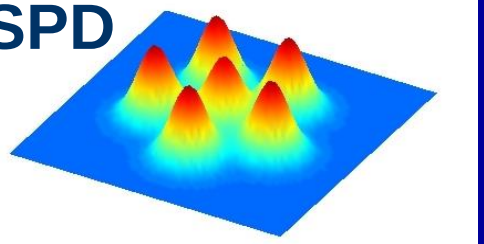
$R_W = 10$

Probability densities

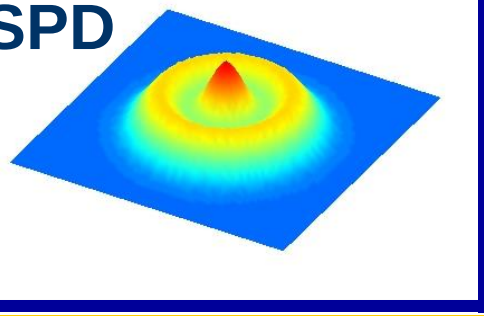
Rotating Frame

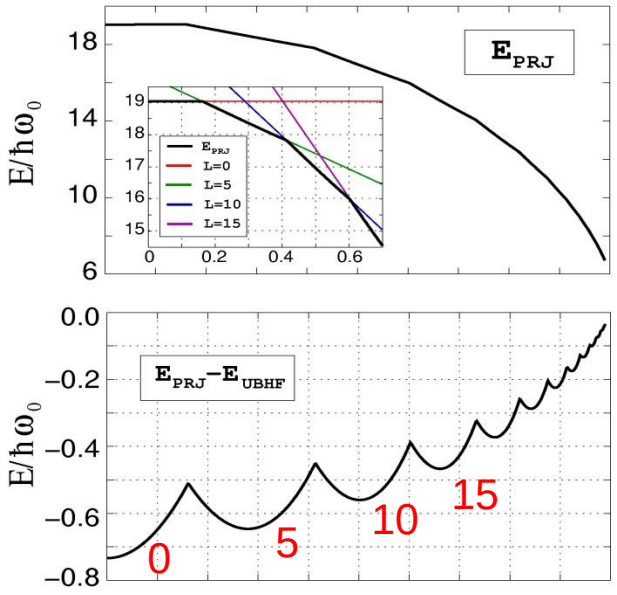
Magnetic Field

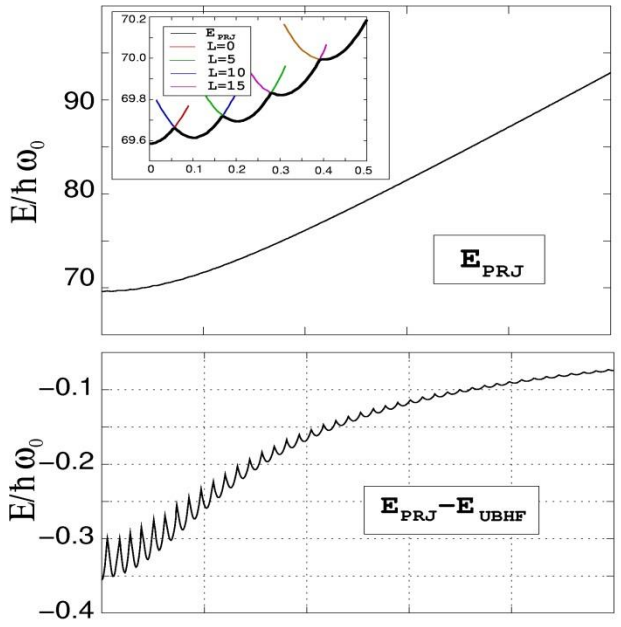
SPD

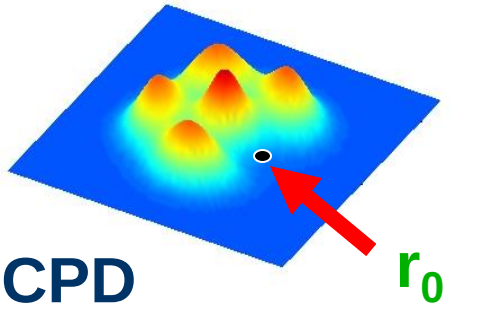


SPD









The hidden crystalline structure in the projected function can be revealed through the use of conditional probability density (CPD).

$$\rho(\mathbf{r}|\mathbf{r_0}) = \langle\Phi| \sum_{i\neq j} \delta(\mathbf{r}_i - \mathbf{r})\delta(\mathbf{r}_j - \mathbf{r}_0)|\Phi\rangle / \langle\Phi|\Phi\rangle$$

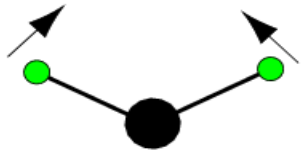
Exact

Y&L, PRL 85, 1726 (2000)

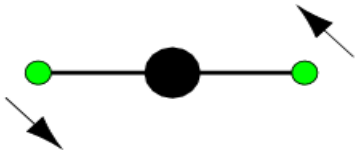
Quantum Dot Helium

2e QD, $R_w = 200$

COLLECTIVE MOTION OF RIGID "TRIATOMIC" MOLECULE



Bending Vibration



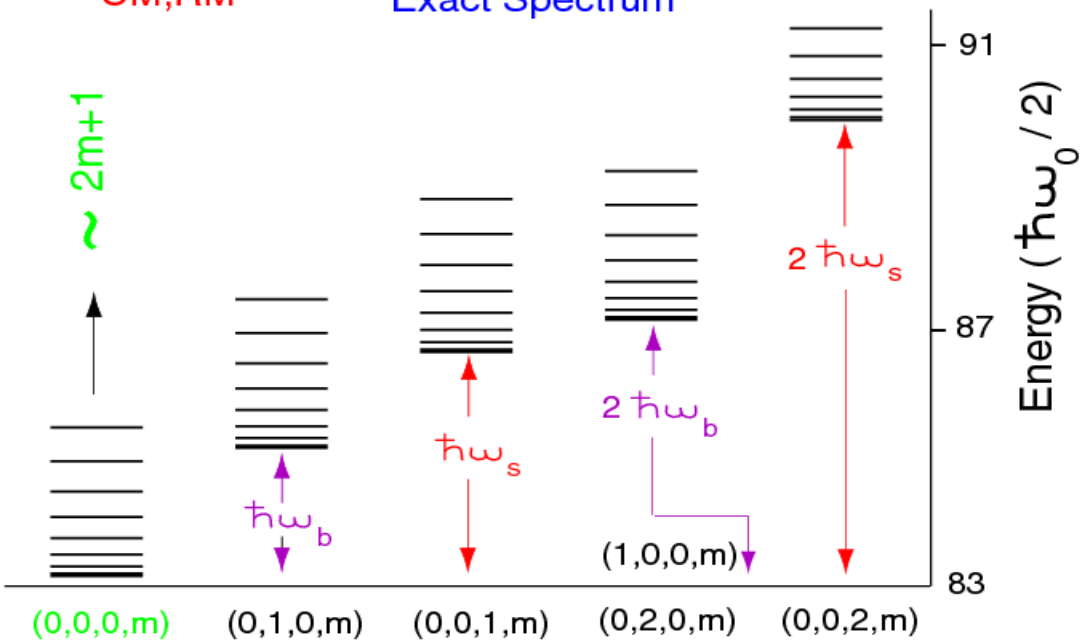
Rotation

Stretching Vibration

$$E_{NM,nm} = Cm^2 + (n+1/2)\,\hbar\omega_s + (2N+M+1)\,\hbar\omega_b$$

CM,RM

Exact Spectrum



RIGID ROTOR

B=0

Natural Helium Doubly excited States/ Kellman/Herrick Phys. Rev. A **22**, 1536 (1980).

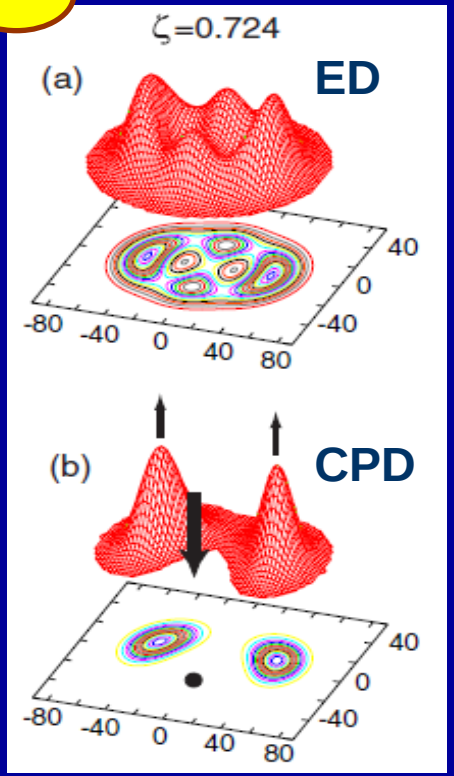
Three electron anisotropic QD: pinning

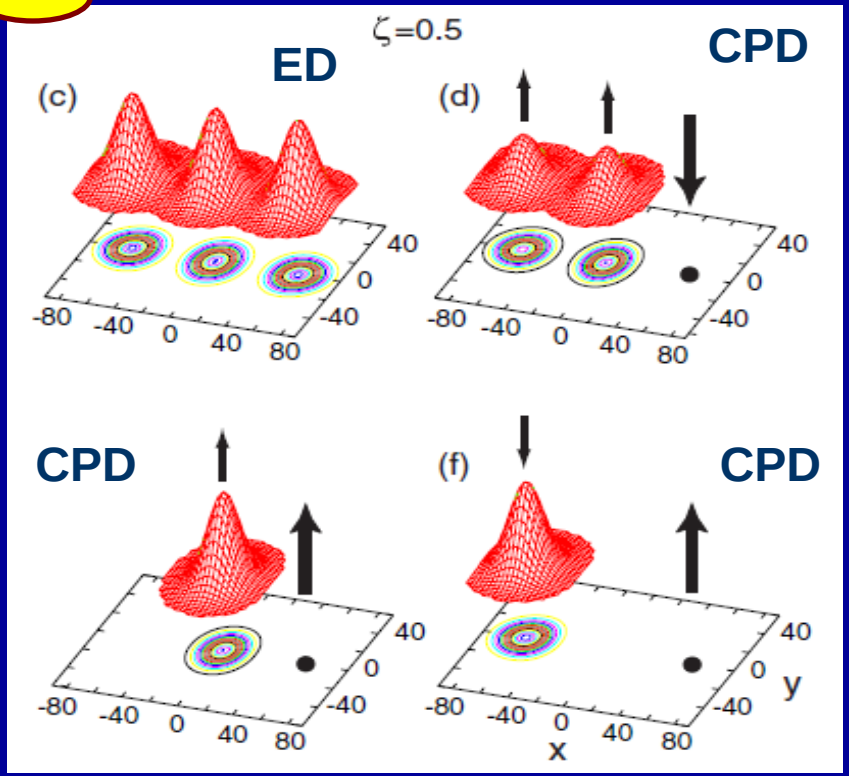
Method: ***Exact Diagonalization (EXD)***

Anisotropic confinement

Electron Density (ED)

(spin resolved) Conditional Probability Distribution (CPD)





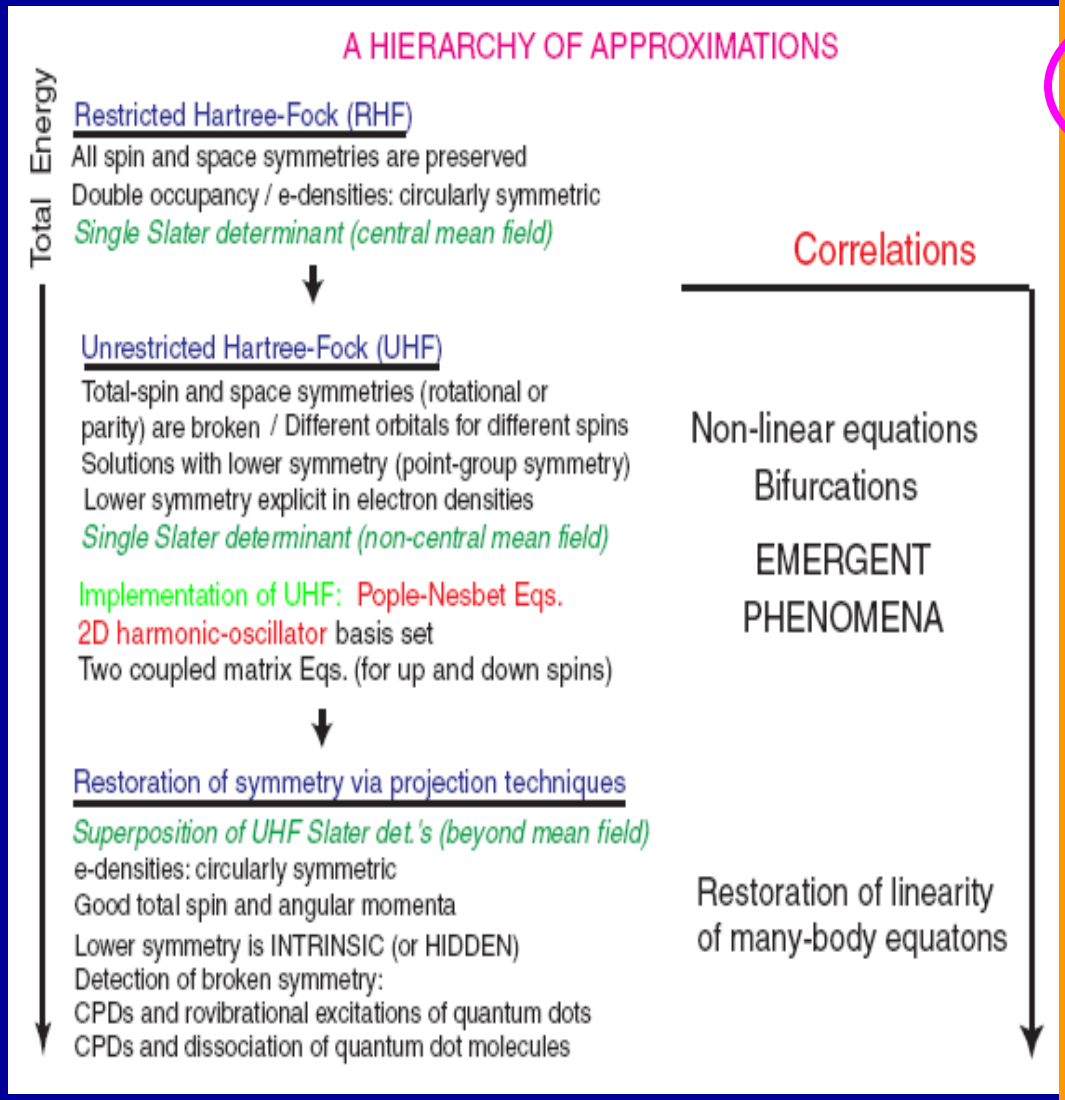
Yuesong Li, Y&L, Phys. Rev. B **76,** 245310 (2007)

EXD wf ~ $|\downarrow\uparrow\uparrow\rangle - |\uparrow\uparrow\downarrow\rangle$

Entangled three-qubit W-states

WAVE-FUNCTION BASED APPROACHES

TWO-STEP METHOD



EXACT DIAGONALIZATION
(Full Configuration Interaction)

When possible (small N): High numerical accuracy

Physics less transparent compared to "THE TWO-STEP"

Pair correlation functions, CPDs

Yannouleas and Landman, Rep. Prog. Phys. ***70****, 2067 (2007)*

Restoration of Broken Rotational Symmetry

- To restore the good angular momentum of the wave function one can use the projection operator

$$\hat{P}_L = \frac{1}{2\pi}\int_0^{2\pi} d\theta e^{i\theta\left(L-\hat{L}\right)} = \delta(L-\hat{L})$$

- Projected wave functions can be written as a Fourier transform of unprojected wave function

$$\left|\Phi^{\mathbf{PRJ}}_{N,L}\right\rangle = \hat{P}_L\left|\Phi_N\right\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\theta \left|\Phi_N(\theta)\right\rangle e^{i\theta L}$$

Here $|\Phi_N(\theta)\rangle$ is the original UBHF permanent, rotated by an azimutal angle. The wave function $|\Phi_{PRJ}\rangle$ has not only good angular momentum, but also its energy is lower than the energy of $|\Phi_N\rangle$

Romanovsky, Yannouleas, and Landman
Phys. Rev. Lett. 93, 230405 (2004) (RBMs)

Romanovsky, Yannouleas, Baksmaty, Landman
Phys. Rev. Lett. 97, 090401 (2006) (RBMs)

TWO-STEP METHOD

SECOND STEP:
RESTORATION OF SYMMETRIES VIA PROJECTION

TOTAL SPIN:

$$P_s \equiv \prod_{s' \neq s} \frac{S^2 - s'(s'+1)\hbar^2}{[s(s+1) - s'(s'+1)]\hbar^2}$$

$$S^2 \Phi_{\mathrm{UHF}} = \hbar^2 \left[(N_\alpha - N_\beta)^2/4 + N/2 + \sum_{i<j} \varpi_{ij} \right] \Phi_{\mathrm{UHF}}$$

ϖ_{ij} interchanges spins

Two electrons in a DQD:

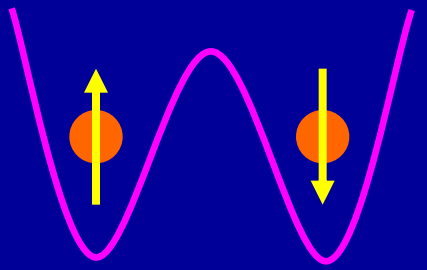
$$\Psi^s_{\mathrm{GVB}}(1,2) = n_s \sqrt{2} P_0 \Psi_{\mathrm{UHF}}(1,2)$$ ← singlet

$$2\sqrt{2} P_0 \Psi_{\mathrm{UHF}}(1,2) = (1 - \varpi_{12}) \sqrt{2} \Psi_{\mathrm{UHF}}(1,2)$$
$$= |u(1)\bar{v}(2)\rangle - |\bar{u}(1)v(2)|.$$ two det.'s

GVB, Generalized Valence Bond
GHL, Generalized Heitler London

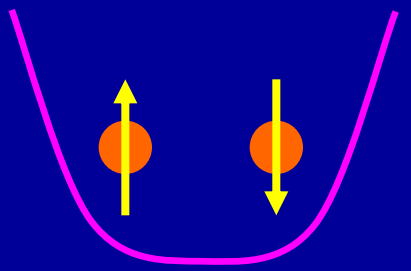
Y&L, Eur. Phys. J. D 16, 373 (2001)
Int. J. Quantum Chem. 90, 699 (2002)

DQD



localized orbitals

Elongated QD



No circular symmetry

RESOLUTION OF SYMMETRY DILEMMA:

RESTORATION OF BROKEN SYMMETRY BEYOND MEAN FIELD (Projection)!

- Per-Olov Löwdin
  (Chemistry - Spin)
- R.E. Peierls and J. Yoccoz
  (Nuclear Physics – *L, rotations*)

Ch. 11 in the book by P. Ring and P. Schuck
Note: Example in 2D

Yannouleas, Landman, Rep. Prog. Phys. **70,** 2067 (2007)

ETH single QD

PRL 96, 126806 (2006) PHYSICAL REVIEW LETTERS week ending 31 MARCH 2006

Excitation Spectrum of Two Correlated Electrons in a Lateral Quantum Dot with Negligible Zeeman Splitting

C. Ellenberger,[1] T. Ihn,[1] C. Yannouleas,[2] U. Landman,[2] K. Ensslin,[1] D. Driscoll,[3] and A. C. Gossard[3]
[1]*Solid State Physics, ETH Zurich, 8093 Zurich, Switzerland*
[2]*School of Physics, Georgia Instititute of Technology, Atlanta, Georgia 30332-0430, USA*
[3]*Materials Department, University of California, Santa Barbara, California 93106, USA*
(Received 16 December 2005; published 30 March 2006)

basis of an avoided crossing with the first excited singlet state at finite fields. The measured spectra are in remarkable agreement with exact-diagonalization calculations. The results prove the significance of electron correlations and suggest the formation of a state with Wigner-molecular properties at low magnetic fields.

Carbon nanotube

ARTICLES
PUBLISHED ONLINE: 28 JULY 2013 | DOI: 10.1038/NPHYS2692
nature physics

Observation and spectroscopy of a two-electron Wigner molecule in an ultraclean carbon nanotube

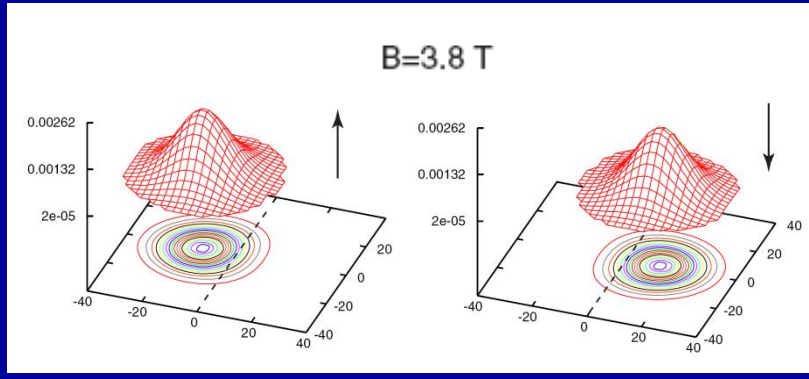
S. Pecker[1†], F. Kuemmeth[2†], A. Secchi[3,4‡], M. Rontani[3], D. C. Ralph[5,6], P. L. McEuen[5,6] and S. Ilani[1*]

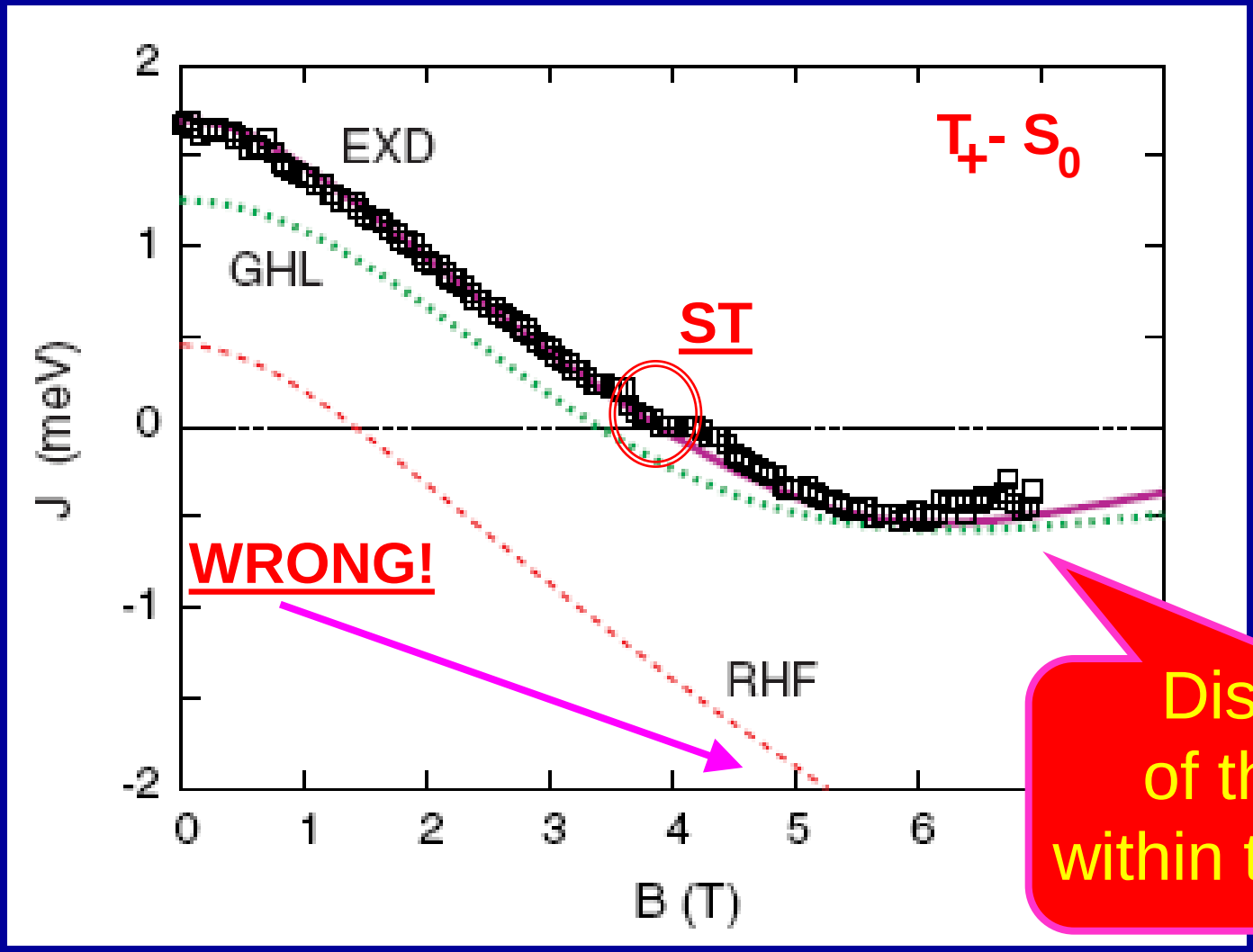
1 Weizmann Institute of Science, Israel 2 Niels Bohr Institute, Denmark
5 Physics Department, Cornell University, Ithaca, New York

ETH single QD

hwx=4.23 meV; hwy=5.84 meV;

m*=0.070; κ=12.5; γ=0.86





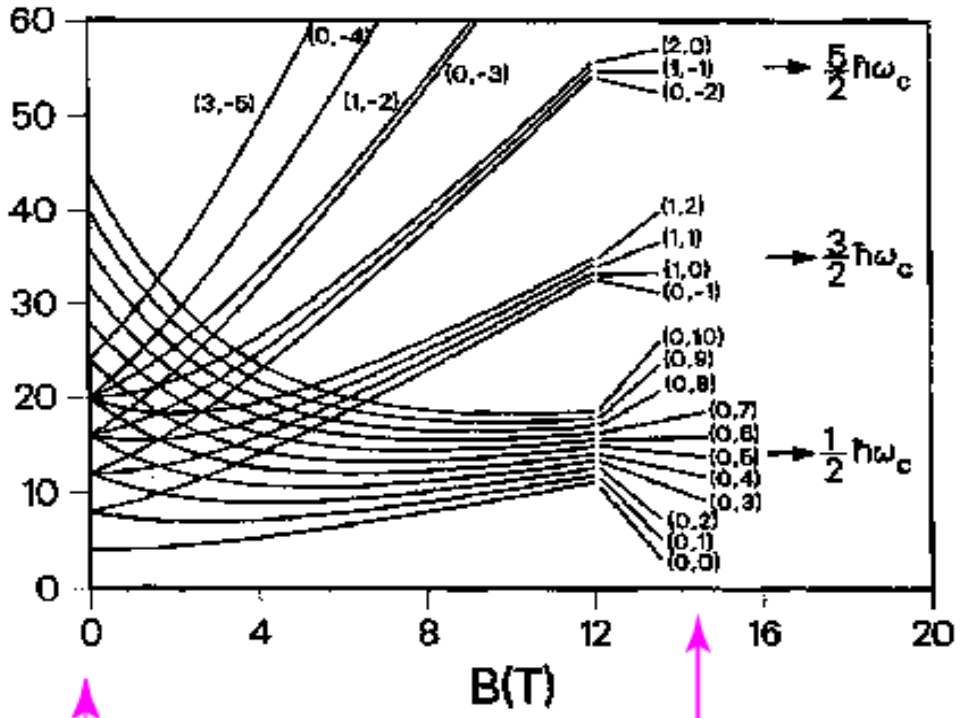
UHF broken symmetry orbitals used to construct the GHL wave function

Dissociation of the 2e WM within the single QD

Circular 2D QD NO e-e INTERACTION

DARWIN-FOCK S.P. LEVELS AT ANY B

NO MAGNETIC FIELD



STRONG MAGNETIC FIELD

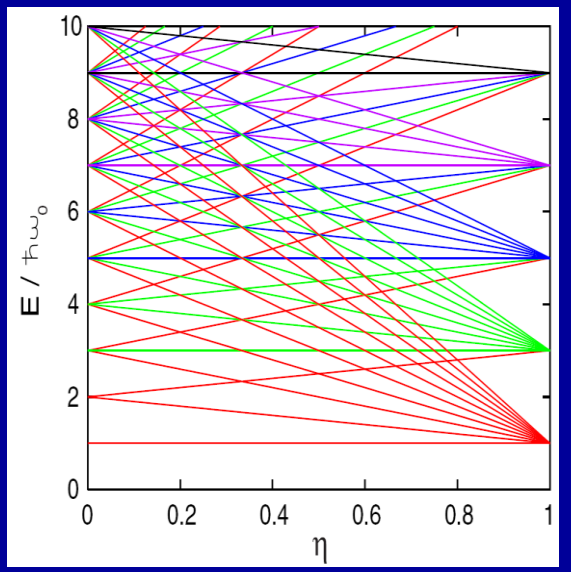
Closed Shells:
2, 6, 12, 20, ...

Landau levels
Full polarization
Open shells

Fractional Quantum Hall Effect

B -> Omega

Rotating atomic traps



$$\eta = \Omega/\omega_0$$

Composite-Fermion (CF) vs. RWM

EXD, LLL, Coulomb, N=6e

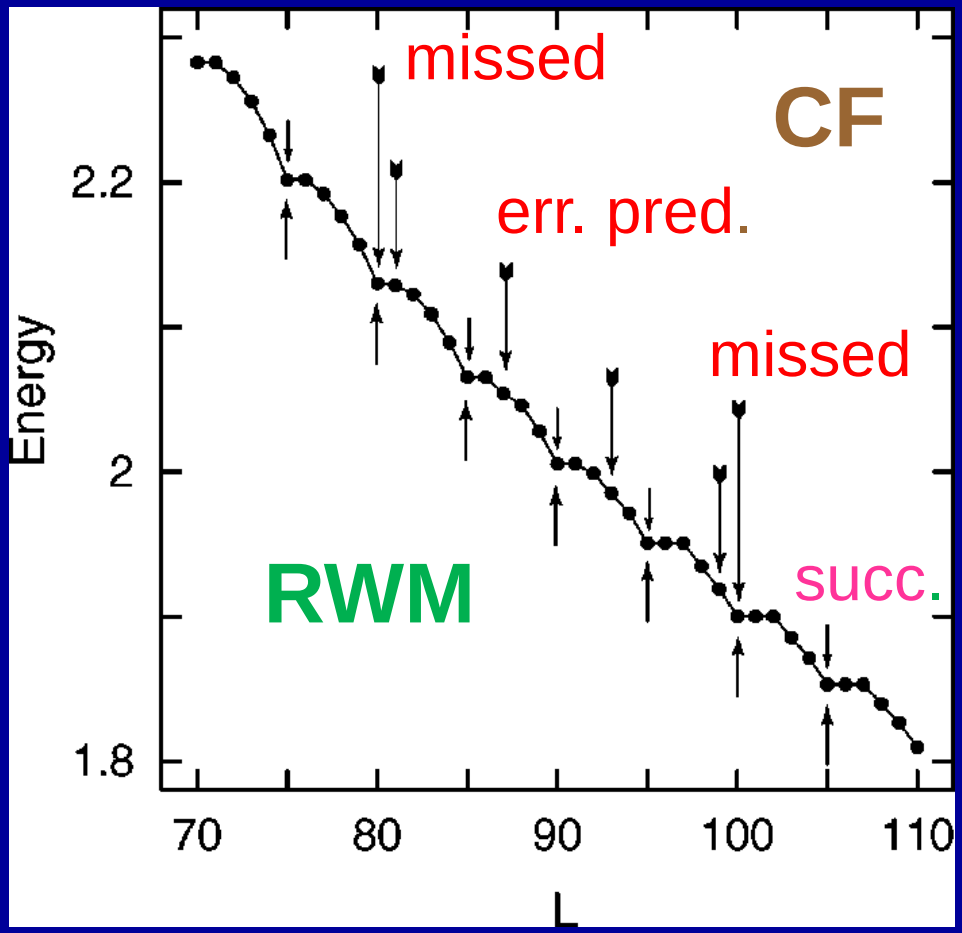
fractional filling

$$\nu = \frac{N(N-1)}{2L_m}$$

m -> magic

Cusp states -> precursor states of FQHE

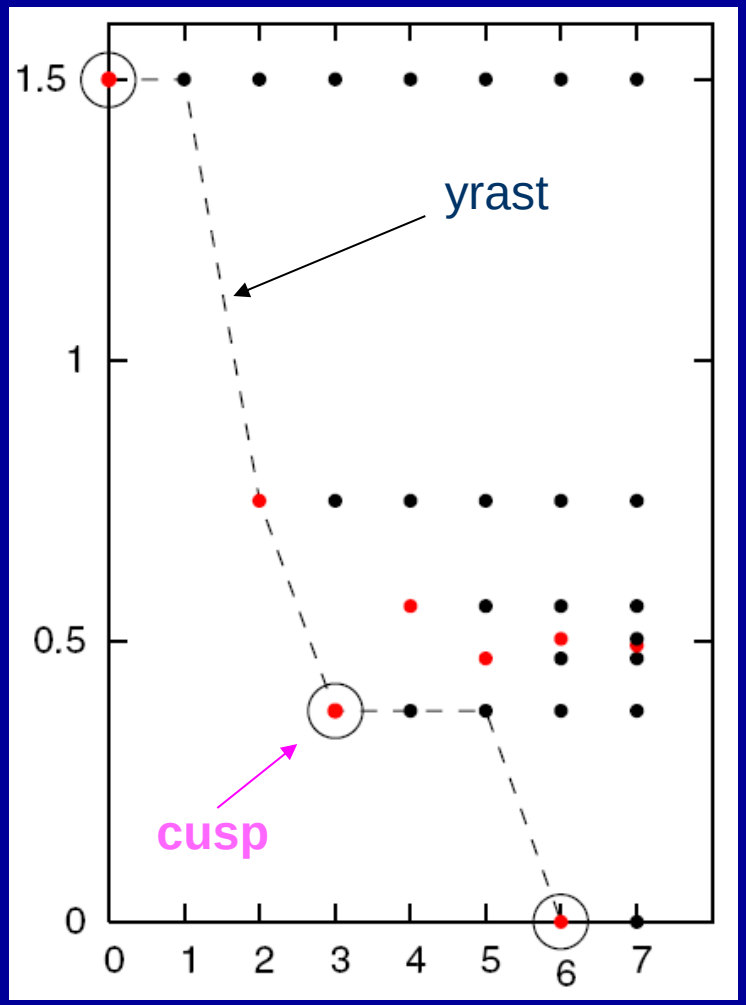
(1,5) polygonal ring



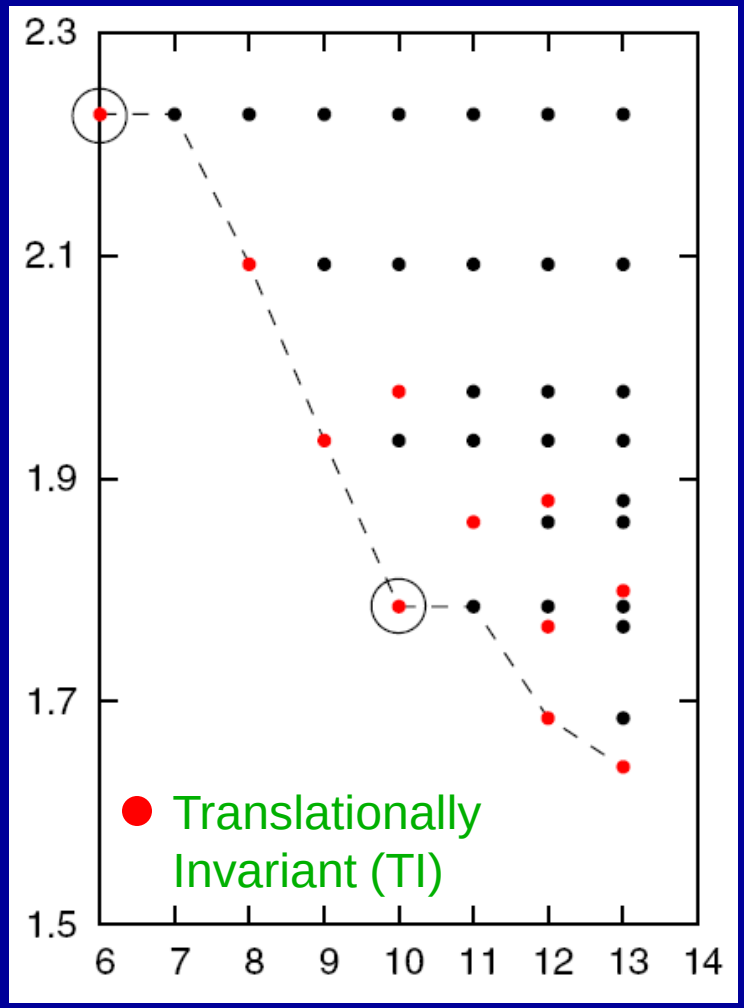
C.Y. & U.L., PRB **68** (2003) 035326

Full LLL spectra (interaction only)

N=3 bosons (delta)



N=4 fermions (Coulomb)



TRIAL WFs ARE HIGHLY CORRELATED; THEY CAPTURE THE PHYSICS OF CUSP STATES AND CERTAIN SELECTIVE EXCITATIONS (Jastrow-Laughlin, compact CF, Moore-Read, REM)

TO DESCRIBE LARGER PARTS OF LLL SPECTRA: USE TRIAL WFs TO FORM A COMPLETE CORRELATED BASIS

Practical: Controlled improvements of variational WF

Conceptual: Completeness/ Properties of basis are reflected in exact WFs

EXAMPLE: quantum liquids

[J.W. Clark and E. Feenberg, Phys. Rev. 113, 388 (1959)]

EXAMPLE: CF BASIS (mainly the full yrast band)

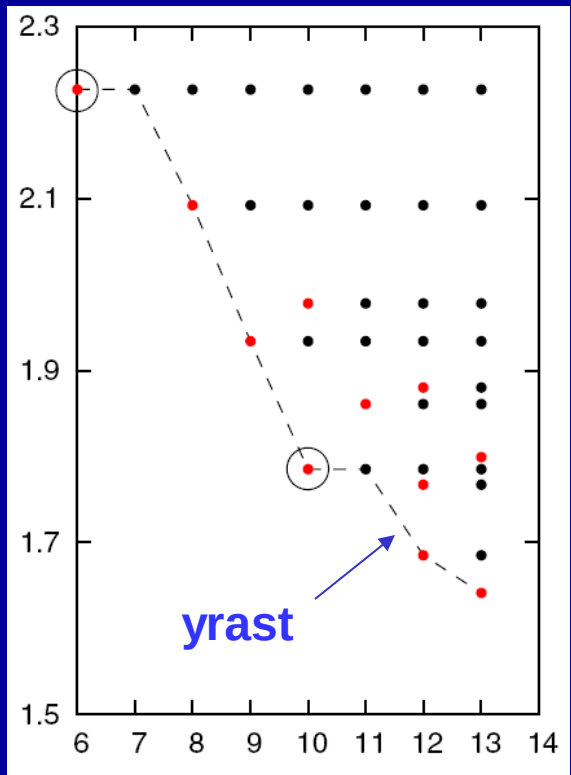
[G.S. Jeon et al, PRB 69, 241304(R) (2004); Eur. Phys. J. B 55, 271 (2007)]

EXAMPLE (this talk):

RO-VIBRATIONAL MOLECULAR (RVM) BASIS

[*Y & L.*, Phys. Rev. A **81,** 023609 (2010)]

THE FULL LLL SPECTRUM



RVM trial functions:

RXM -> RBM or REM

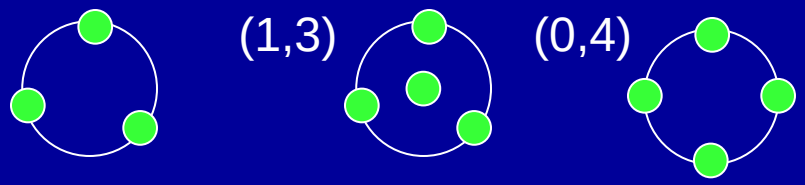
$$\Phi_{\mathcal{L}}^{\mathrm{RXM}}(n_1,n_2)Q_\lambda^m|0>$$

$$L=\mathcal{L}+\lambda m$$

Pure rotations (cusp, vibrationless)

(molecular point-group symmetries)

(0,3) (1,3) (0,4)



RBM

$$\Phi_{\mathcal{L}}^{\mathrm{RBM}}(0,3)=\sum_{0\le l_1\le l_2\le l_3}^{l_1+l_2+l_3=\mathcal{L}} C(l_1,l_2,l_3)\ \mathrm{Perm}[z_1^{l_1},z_2^{l_2},z_3^{l_3}]$$

$$C(l_1,l_2,l_3) = \left(\prod_{i=1}^{3} l_i!\right)^{-1}\left(\prod_{k=1}^{M} p_k!\right)^{-1} \times\left(\sum_{1\le i<j\le 3}\cos\left[\frac{2\pi(l_i-l_j)}{3}\right]\right)$$

$$\mathcal{L}=\mathcal{L}_0+n_1k_1+n_2k_2$$

MAGIC ANGULAR MOMENTA

Vibrations

$$Q_\lambda=\sum_{i=1}^{N}(z_i-z_c)^\lambda$$

$$z_c=(1/N)\sum_{i=1}^{N} z_i$$

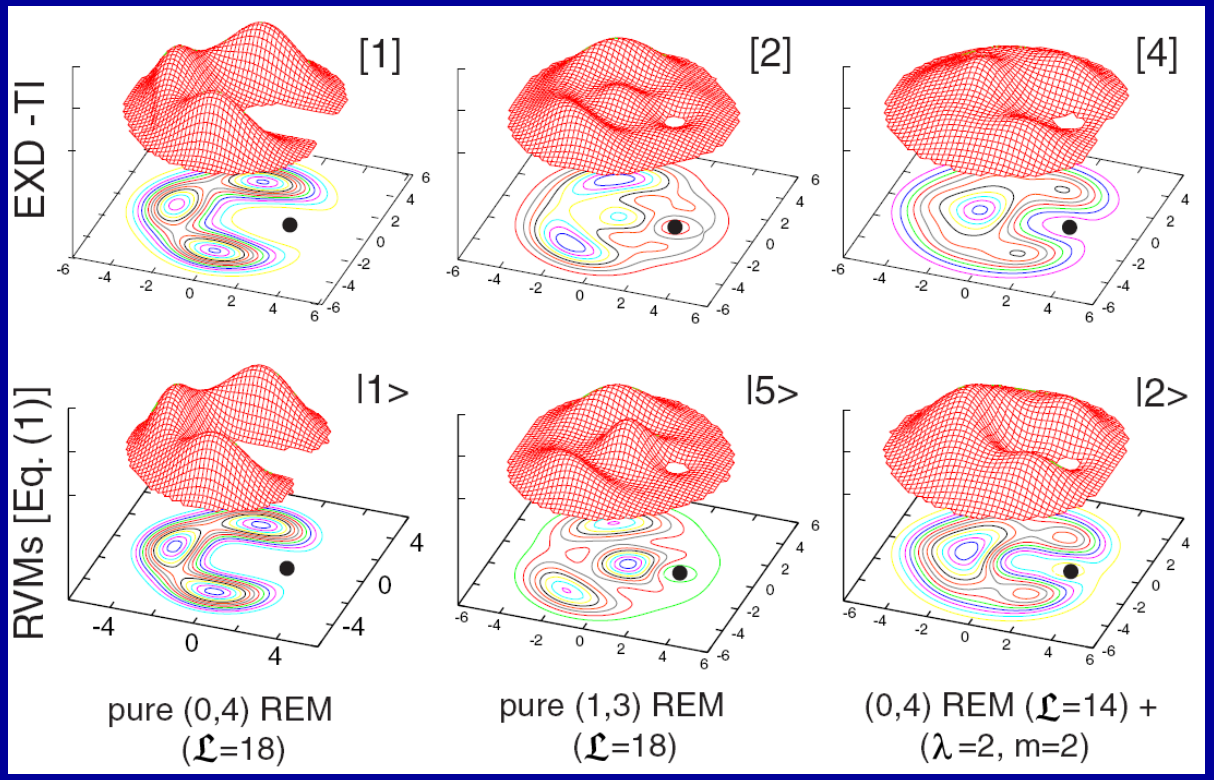
REMs (analytic):
Yannouleas and Landman,
PRB **66**, 115315 (2002);
Rep. Prog. Phys. **70**, 2067 (2007)

CPDs

N=4 e L=18 ν=1/3

EXD-TI ➔

RVMs that DOMINATE EXPANSION ➔



RVM	EXD-TI [1]	EXD-TI [2]	EXD-TI [4]
$\|1>$	0.9294	-0.3430	0.0903
$\|2>$	-0.1188	-0.0693	0.8930
$\|3>$	0.0067	0.0382	-0.2596
$\|4>$	0.0137	0.0191	-0.0968
$\|5>$	0.2540	0.8486	0.1519
$\|6>$	0.0211	0.0283	0.3097
$\|7>$	-0.2387	-0.3935	0.0877

LLL, pinned Wigner crystal (broken symmetry) ν= 1/3

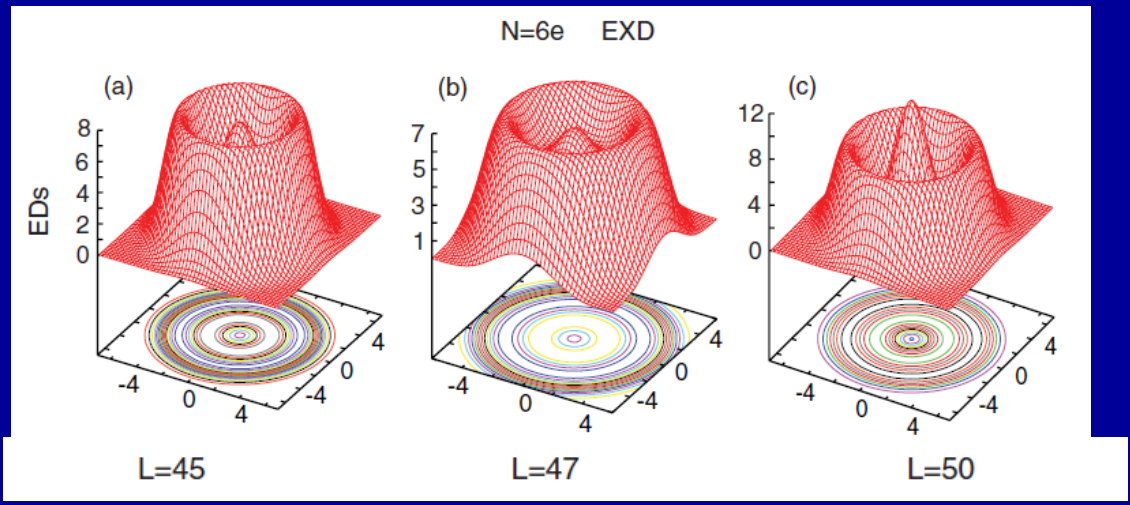
PRL 105, 126803 (2010) PHYSICAL REVIEW LETTERS week ending 17 SEPTEMBER 2010

Observation of a Pinning Mode in a Wigner Solid with $\nu = 1/3$ Fractional Quantum Hall Excitations

Han Zhu,[1,2] Yong P. Chen,[3] P. Jiang,[2,1] L. W. Engel,[2] D. C. Tsui,[1] L. N. Pfeiffer,[1] and K. W. West[1]
[1]*Princeton University, Princeton, New Jersey 08544, USA*
[2]*National High Magnetic Field Laboratory, Tallahassee, Florida 32310, USA*
[3]*Purdue University, West Lafayette, Indiana 47907, USA*
(Received 11 June 2010; published 17 September 2010)

Laughlin liquid versus Wigner crystal : different wave functions
Crossover between 1/5 to 1/7 -- At 1/3 liquid lower than WC

RVMs, EXD
Good symmetry, crystalline correlations are hidden



LLL, quantum Wigner crystal (broken symmetry) $\nu = 1/3$

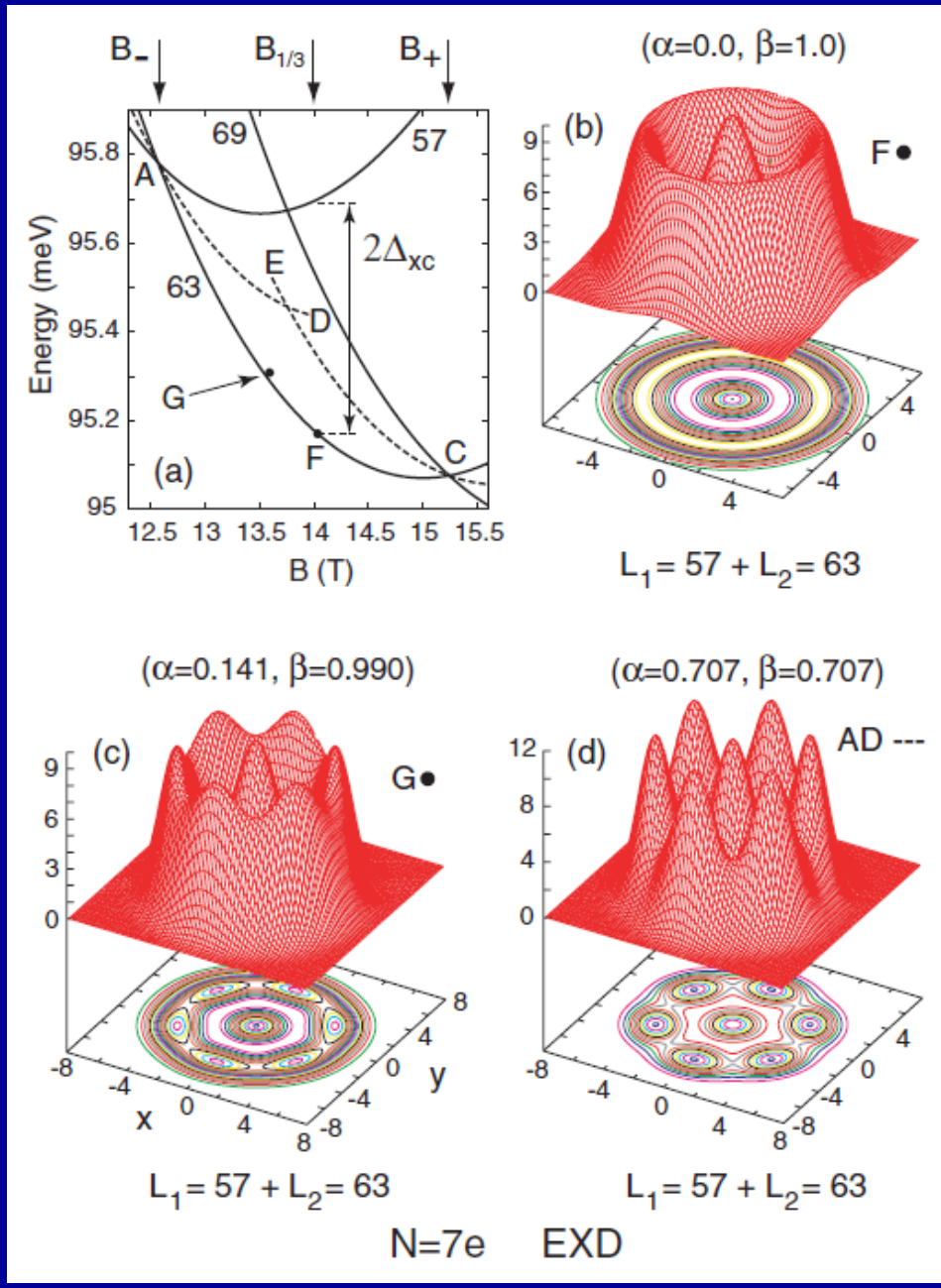
External confinement

Pinning -> Impurity -> Linear superposition:

$$\Phi^{\mathrm{PIN}}(L_1, L_2; \alpha, \beta) = \alpha \Phi_{L_1} + \beta e^{i\theta} \Phi_{L_2}$$

Energy lower than HF
(1 det) crystal,
Maki & Zotos, etc.

C.Y. & U.L.,
PRB **84**, 165327 (2011)



SUMMARY (Symmetry Restoration)

Under appropriate conditions, 2D electrons (and ultracold repelling bosons) exhibit localization (hidden or explicit) and organize themselves in geometric shells, forming Rotating (or pinned) Wigner Molecules (Quantum Crystallites) (semiconductor Quantum Dots, Ultracold rotating bosonic traps, Dissociation of natural molecules)

Instead of:

For electrons: organizing in electronic shells associated with a confining central potential (Cluster physics/ jellium model)

For bosons: forming a Bose-Einstein condensate

In the LLL: Rovibrational molecular theory offers alternative description to Laughlin and composite-fermion approaches for the fractional quantum Hall effect