UNIVERSITY OF JYVÄSKYLÄ
JYVÄSKYLÄN YLIOPISTO

NORDITA
September 2014

Nuclear Field Theory as a complete description of Nuclear Structure

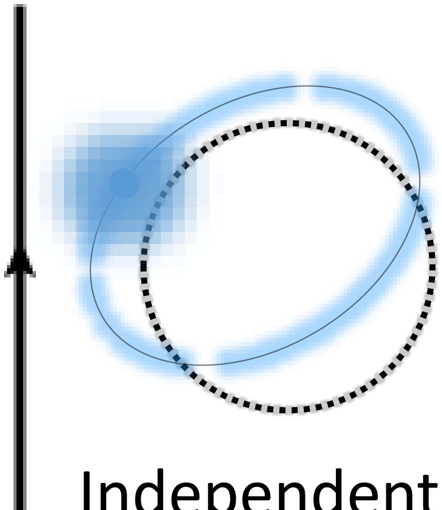
Andrea Idini

F. Barranco, R. A. Broglia, G. Potel, E. Vigezzi
B. A. Brown, K. Langanke, G. Martinez-Pinedo

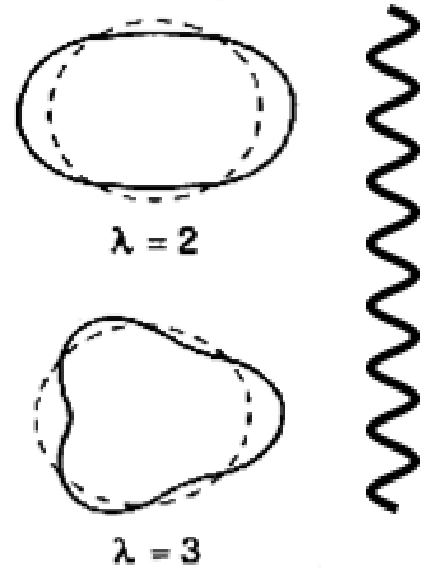
A.I. et al. PRC 85:014331 (2012)
A.I. et al. **arXiv:1404.7365 [nucl-th]**
A.I. PhD Thesis «Renormalization Effects in Nuclei»

Computational Challenges in Nuclear and Manybody Physics

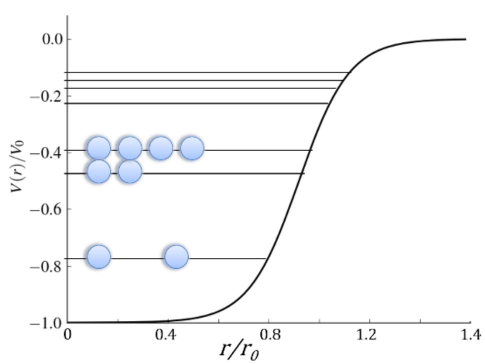
We can see Nuclei organize as a **many-body** system using **Two Degrees of Freedom**



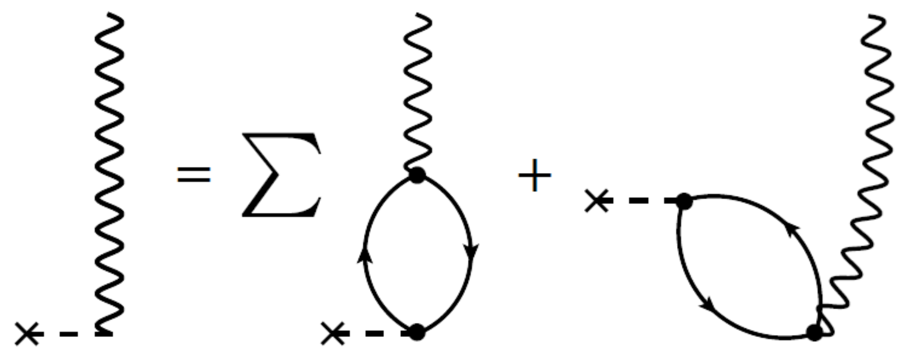
Independent Particle



Collective Phonon

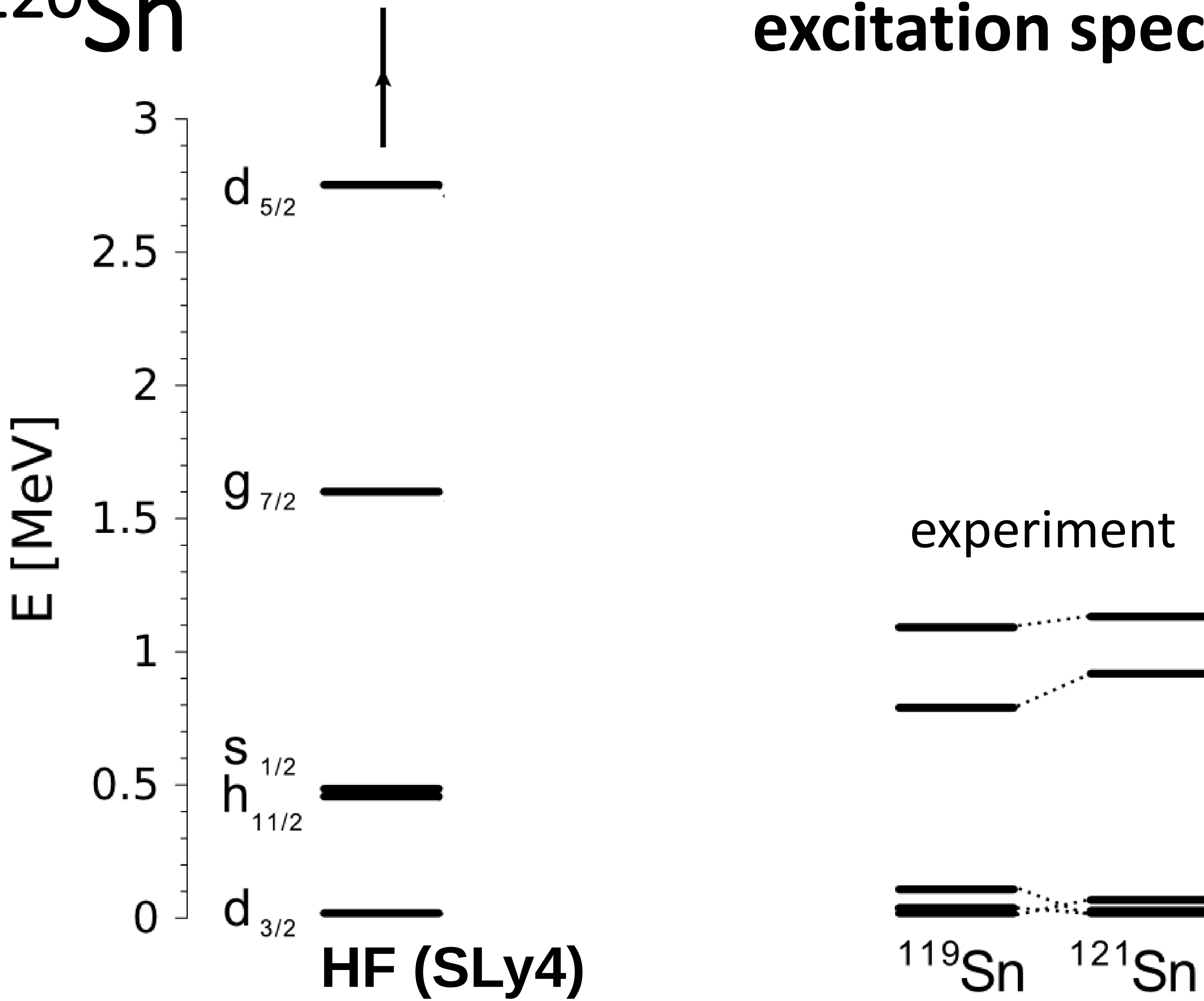


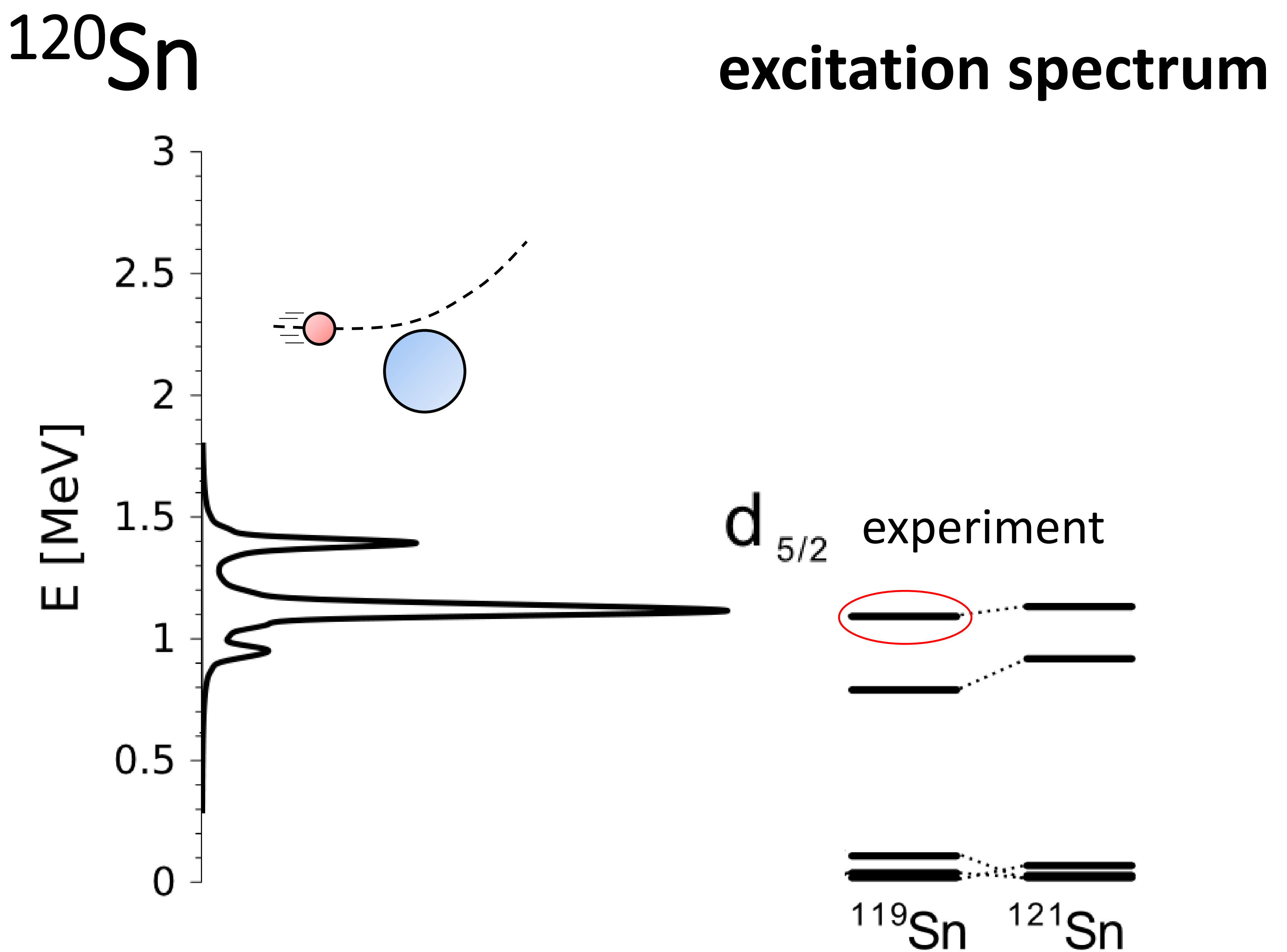
Hartree-Fock mean Field



Random Phase Approximation

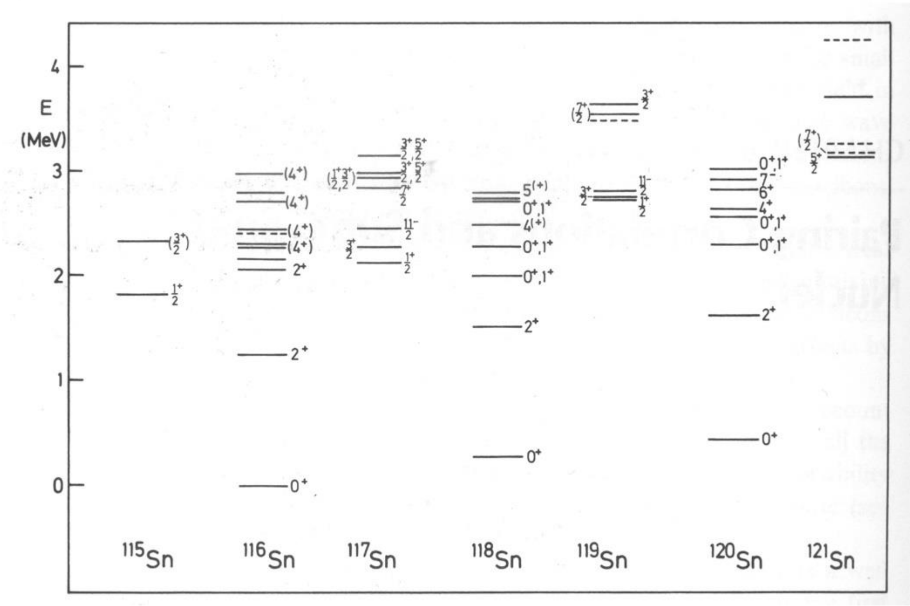
^{120}Sn excitation spectrum





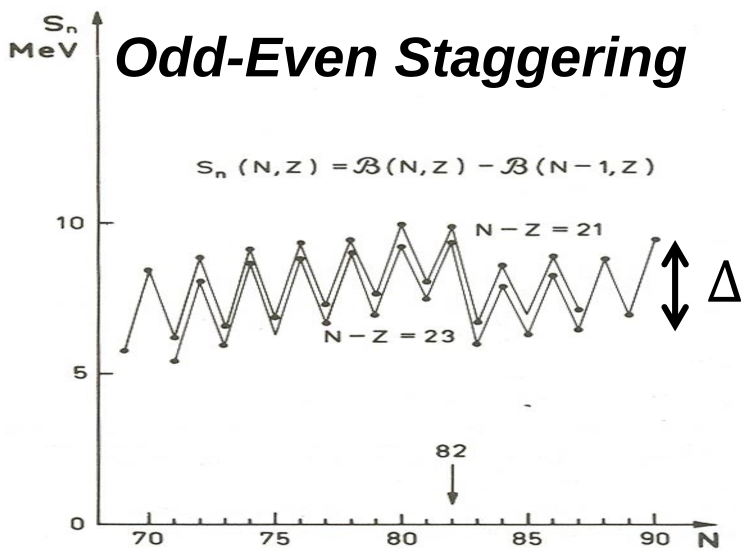
120Sn
excitation spectrum
E [MeV]
3
2.5
2
1.5
1
0.5
0
d 5/2
experiment
119Sn
121Sn

Superfluidity



Different behaviour from **even** to **odd** nuclei!

<u>Fermions pair together</u> in bound states (Cooper pairs): (quasi)bosons

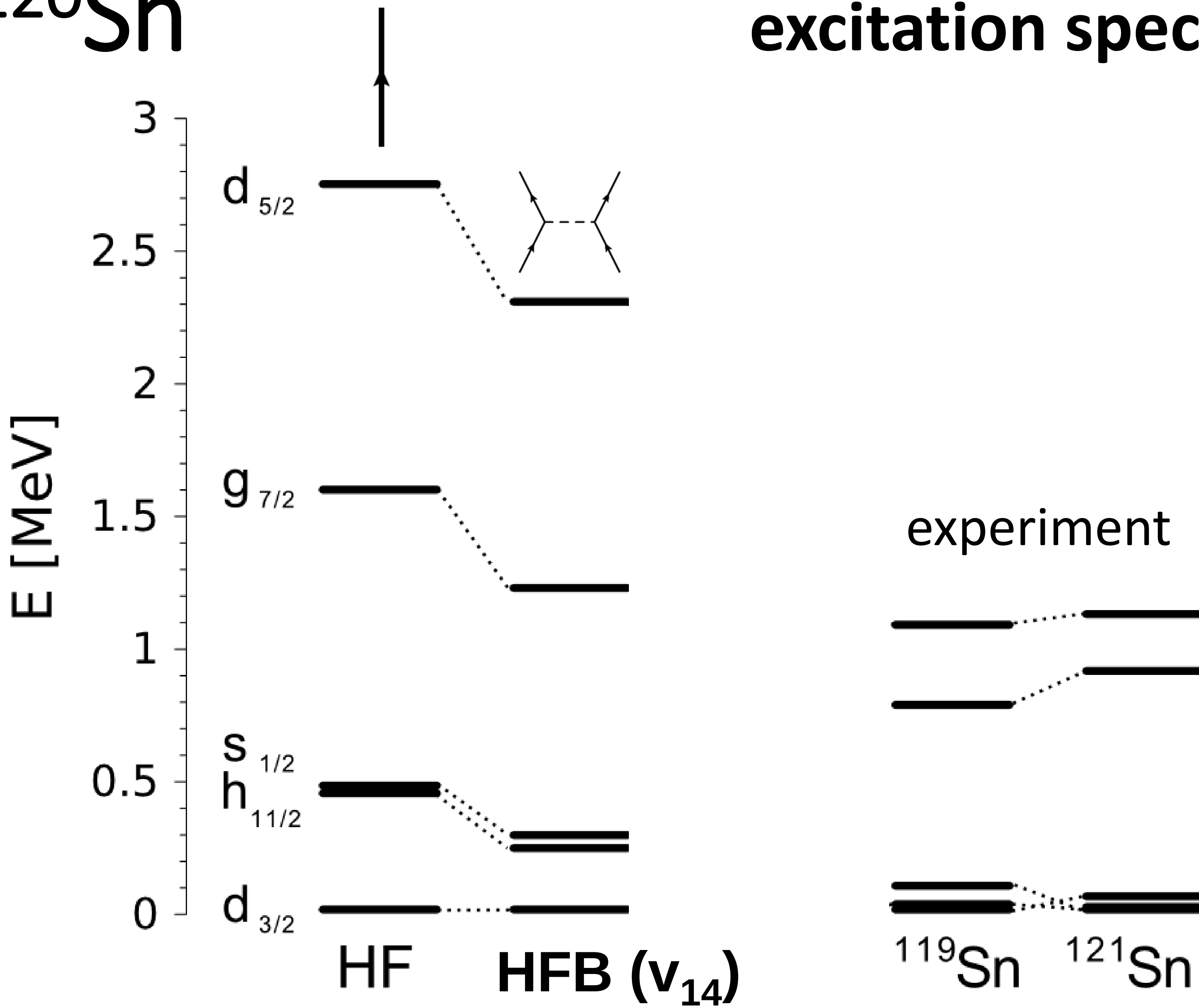


3-point formula

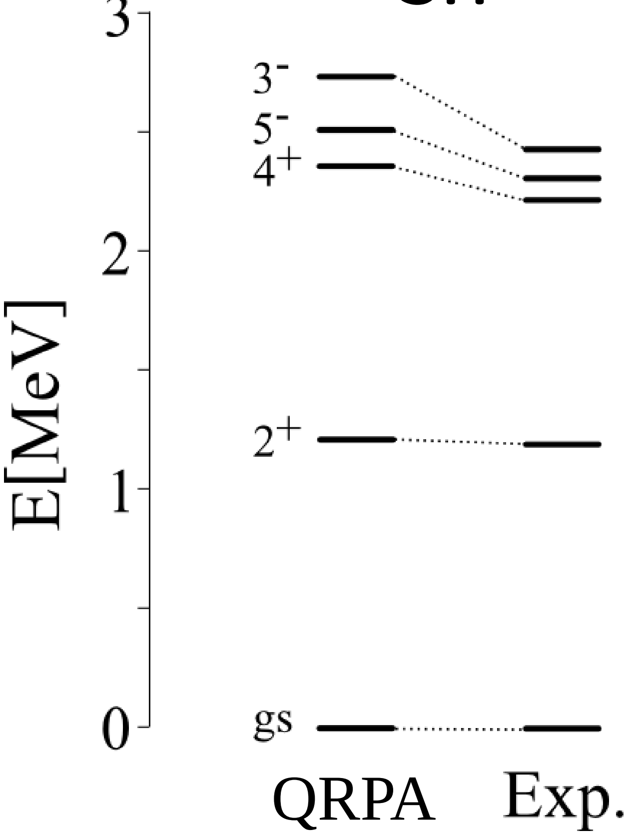
$$\Delta(A) = \frac{BE(A-1) + BE(A+1) - 2BE(A)}{2}$$

^{120}Sn

excitation spectrum

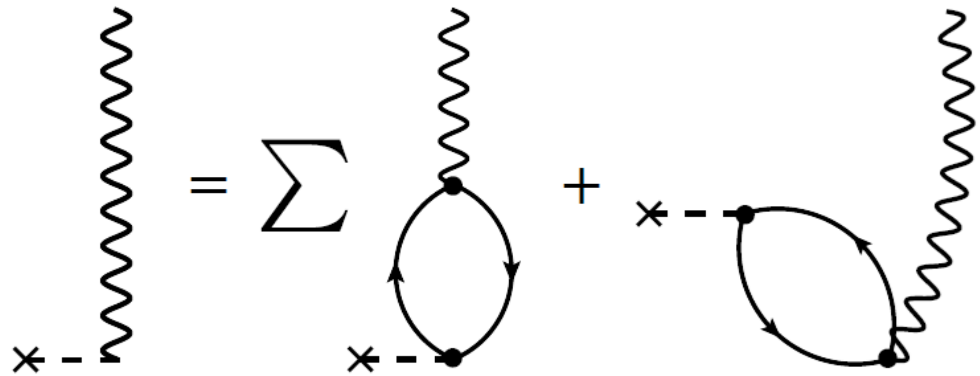


^{120}Sn



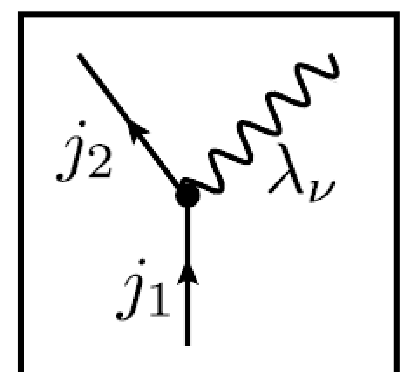
Correlated Particle-Hole excitations give

Collective Excitations.



(Quasi) Random Phase Approximation

Scattering Vertex

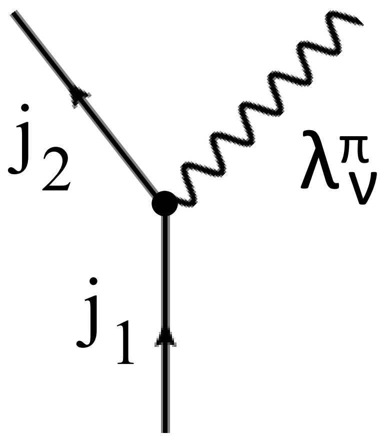


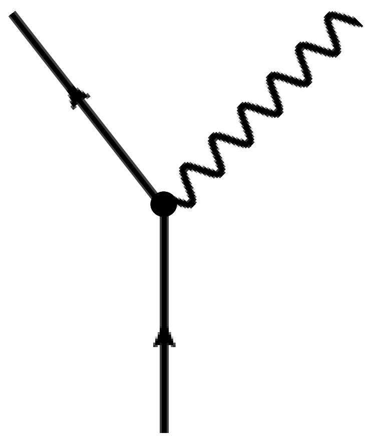
$$f(j_1, j_2, \lambda_\nu^\pi) = \langle j_2 | k_{\lambda^\pi} | j_1 \rangle \langle \lambda | \alpha_\lambda | 0 \rangle$$

$$g(j_1, j_2, \lambda_\nu^\pi L) = \langle j_1 m_1 | (G_0(r) + G_0'(r')\tau \cdot \tau') \times \sigma \cdot \sigma' \delta(r - r') | [j_2 \otimes \lambda_\nu^\pi L]_{j_1 m_1} \rangle$$

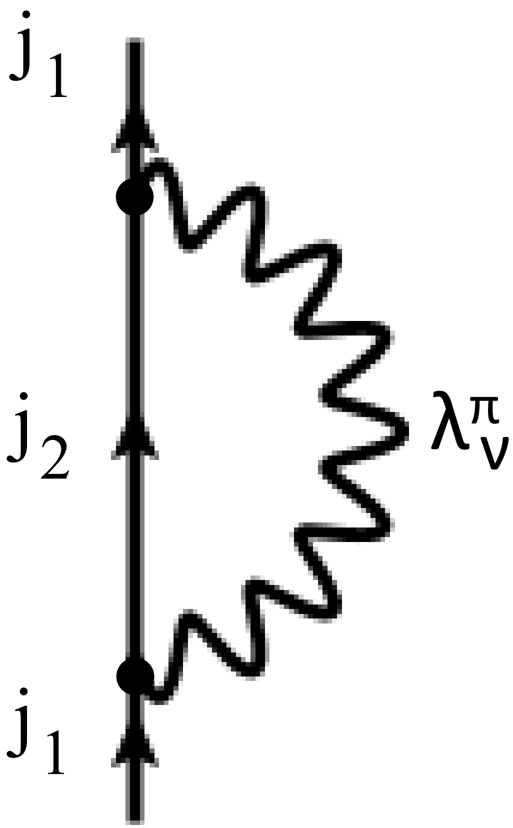
The Two Degrees of Freedom Talk between each other…

Scattering Vertex
(QRPA)





…and the correlations renormalize the properties of the system.



This **<u>Self-Energy</u>** process **change** the particle **wavefunction**, thus:

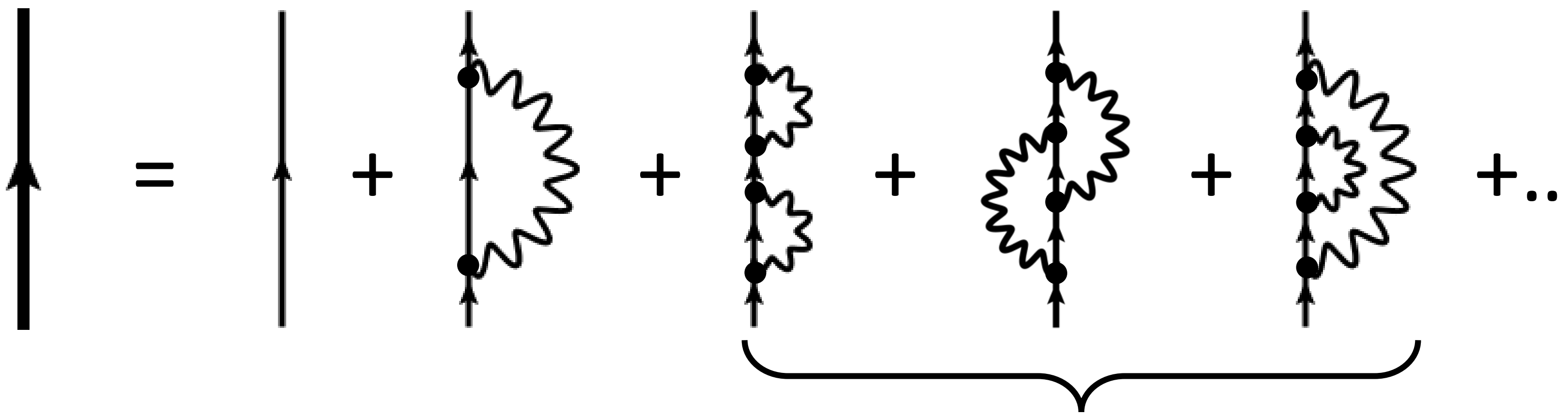
Change Energy

Fragment Single Particle Strength

Open other reaction channels

<u>Challenge the shortcomings of Mean Field!</u>

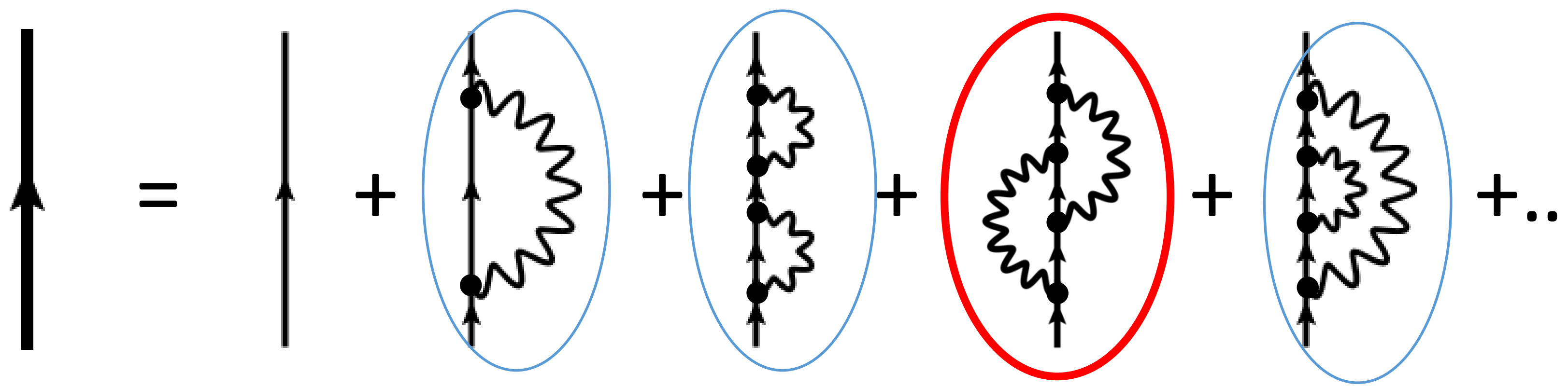
Nuclear Field Theory Approach

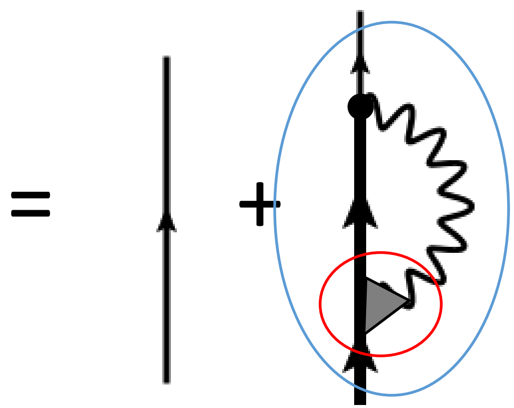


Ist order IInd order

Green’s function can consistently propagate perturbation processes to the **infinite order**:

Nuclear Field Theory Approach

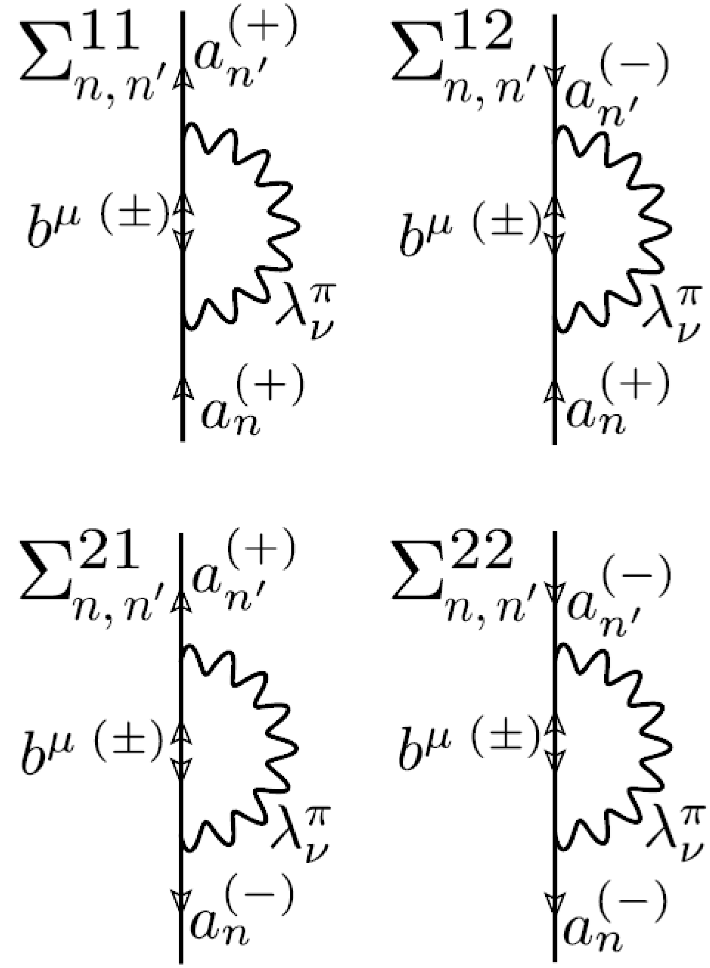




Dyson Equation

$$G_i = G_i^{HF} + G_i \Sigma G_i^{HF} \Rightarrow G_i^{-1} = G_i^{HF^{-1}} - \Sigma$$

$$
\left(\begin{array}{cc||cc}
\Sigma^{11}_{hh} & \Sigma^{11}_{hp} & \Sigma^{12}_{hh} & \Sigma^{12}_{hp} \\ \hline
\Sigma^{11}_{ph} & \Sigma^{11}_{pp} & \Sigma^{12}_{ph} & \Sigma^{12}_{pp} \\ \hline\hline
\Sigma^{21}_{hh} & \Sigma^{21}_{hp} & \Sigma^{22}_{hh} & \Sigma^{22}_{hp} \\ \hline
\Sigma^{21}_{ph} & \Sigma^{21}_{pp} & \Sigma^{22}_{ph} & \Sigma^{22}_{pp}
\end{array}\right)
$$



$$G_i^{\;-1} = G_i^{HF\,-1} - \Sigma$$

Lehmann Representation of Green's Functions

$$\Sigma^{11}(a,\omega) = \sum_{b,\lambda_\nu^\pi} \int_0^{+\infty} \mathrm{d}\omega' \frac{V^2(a,b,\lambda_\nu^\pi,\omega')}{\omega - \omega' - \hbar\omega_{\lambda_\nu^\pi}} + \frac{W^2(a,b,\lambda_\nu^\pi,\omega')}{\omega + \omega' + \hbar\omega_{\lambda_\nu^\pi}},$$

$$\Sigma^{22}(a,\omega) = \sum_{b,\lambda_\nu^\pi} \int_0^{+\infty} \mathrm{d}\omega' \frac{V^2(a,b,\lambda_\nu^\pi,\omega')}{\omega + \omega' + \hbar\omega_{\lambda_\nu^\pi}} + \frac{W^2(a,b,\lambda_\nu^\pi,\omega')}{\omega - \omega' - \hbar\omega_{\lambda_\nu^\pi}},$$

$$\Sigma^{12}(a,\omega) = \sum_{b,\lambda_\nu^\pi} \int_0^{+\infty} \mathrm{d}\omega' VW(a,b,\lambda_\nu^\pi,\omega') \left[\frac{1}{\omega + \omega' + \hbar\omega_{\lambda_\nu^\pi}} - \frac{1}{\omega - \omega' - \hbar\omega_{\lambda_\nu^\pi}}\right].$$

$$G_i^{-1} = G_i^{HF\,-1} - \Sigma$$

Energy dependent Vertices

$$
\begin{aligned}
V^2(a,b,\lambda_\nu^\pi,\omega) = \; & (f(a,b,\lambda_\nu^\pi) + g(a,b,\lambda_\nu^\pi))^2 u^2 a S^+(b,\omega) \\
+ \; & (f(a,b,\lambda_\nu^\pi) - g(a,b,\lambda_\nu^\pi))^2 v_a^2 S^-(b,\omega) \\
- \; & 2(f(a,b,\lambda_\nu^\pi) + g(a,b,\lambda_\nu^\pi))(f(a,b,\lambda_\nu^\pi) - g(a,b,\lambda_\nu^\pi)) u_a v_a \widetilde{S}(b,\omega)
\end{aligned}
$$

$$
\begin{aligned}
W^2(a,b,\lambda_\nu^\pi,\omega) = \; & (f(a,b,\lambda_\nu^\pi) - g(a,b,\lambda_\nu^\pi))^2 u_a^2 S^-(b,\omega) \\
+ \; & (f(a,b,\lambda_\nu^\pi) + g(a,b,\lambda_\nu^\pi))^2 v_a^2 S^+(b,\omega) \\
- \; & 2(f(a,b,\lambda_\nu^\pi) - g(a,b,\lambda_\nu^\pi))(f(a,b,\lambda_\nu^\pi) + g(a,b,\lambda_\nu^\pi)) u_a v_a \widetilde{S}(b,\omega)
\end{aligned}
$$

$$
\begin{aligned}
VW(a,b,\lambda_\nu^\pi,\omega) = \; & (f(a,b,\lambda_\nu^\pi) + g(a,b,\lambda_\nu^\pi))^2 u_a v_a S^+(b,\omega) \\
- \; & (f(a,b,\lambda_\nu^\pi) - g(a,b,\lambda_\nu^\pi))^2 u_a v_a S^-(b,\omega) \\
+ & (f(a,b,\lambda_\nu^\pi) + g(a,b,\lambda_\nu^\pi))(f(a,b,\lambda_\nu^\pi) - g(a,b,\lambda_\nu^\pi))(u_a^2 - u_a^2) \widetilde{S}(b,\omega)
\end{aligned}
$$

Strength Function

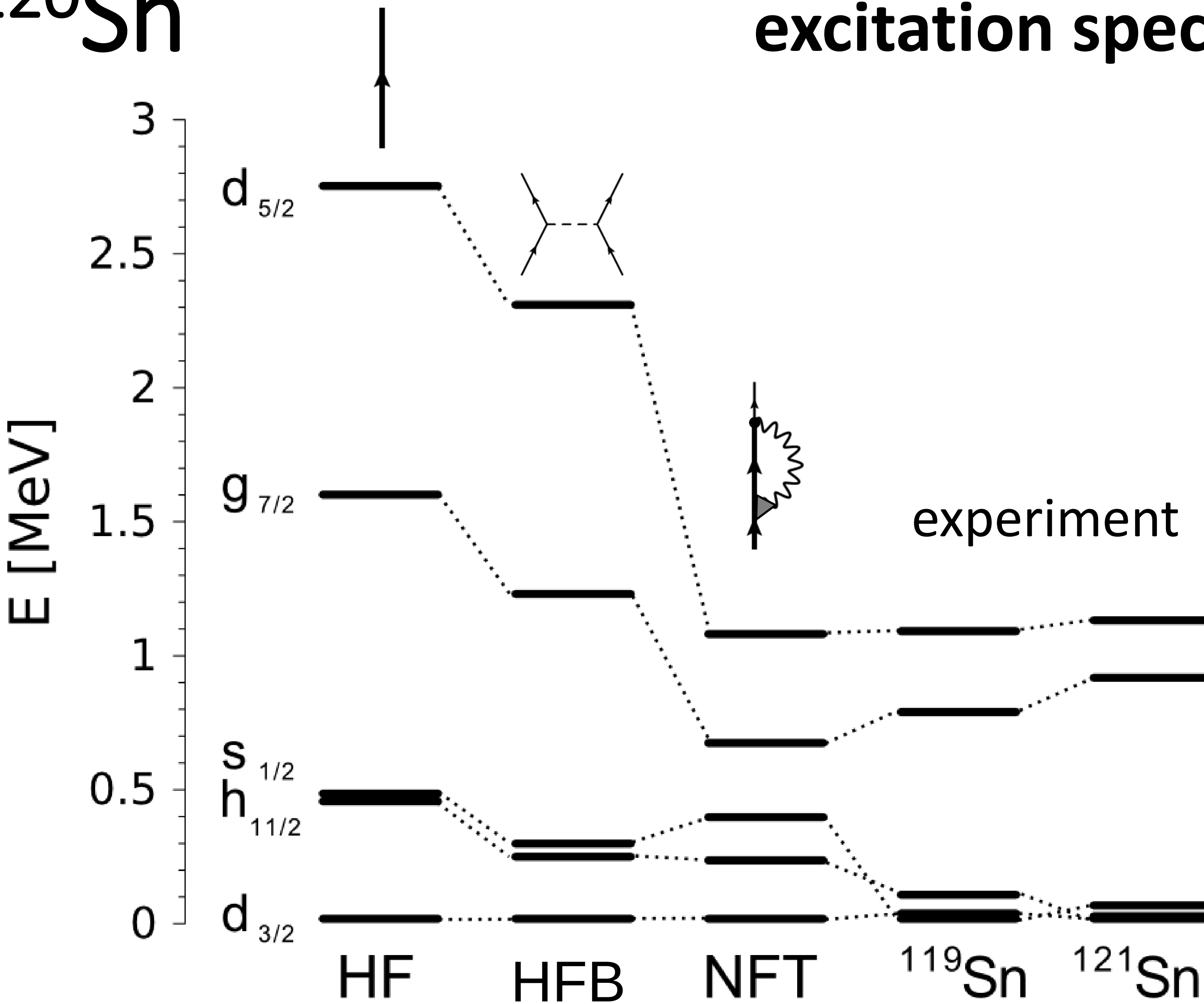
$$S^+(a,\omega) = S(a,\omega) \qquad \text{and} \qquad S^-(a,\omega) = S(a,-\omega).$$

$$S(a,\omega) = -\Im m\{u_a^2 \widetilde{G}_{11}(a,\omega+i\eta) + v_a^2 \widetilde{G}_{22}(a,\omega+i\eta) - 2u_a v_a \widetilde{G}_{12}(a,\omega+i\eta)\}/\pi,$$

$$\widetilde{S}(a,\omega) = -\Im m\{u_a v_a(\widetilde{G}_{11}(a,\omega+i\eta) - \widetilde{G}_{22}(a,\omega+i\eta)) + (u_a^2 - v_a^2)\widetilde{G}_{12}(a,\omega+i\eta)\}/\pi.$$

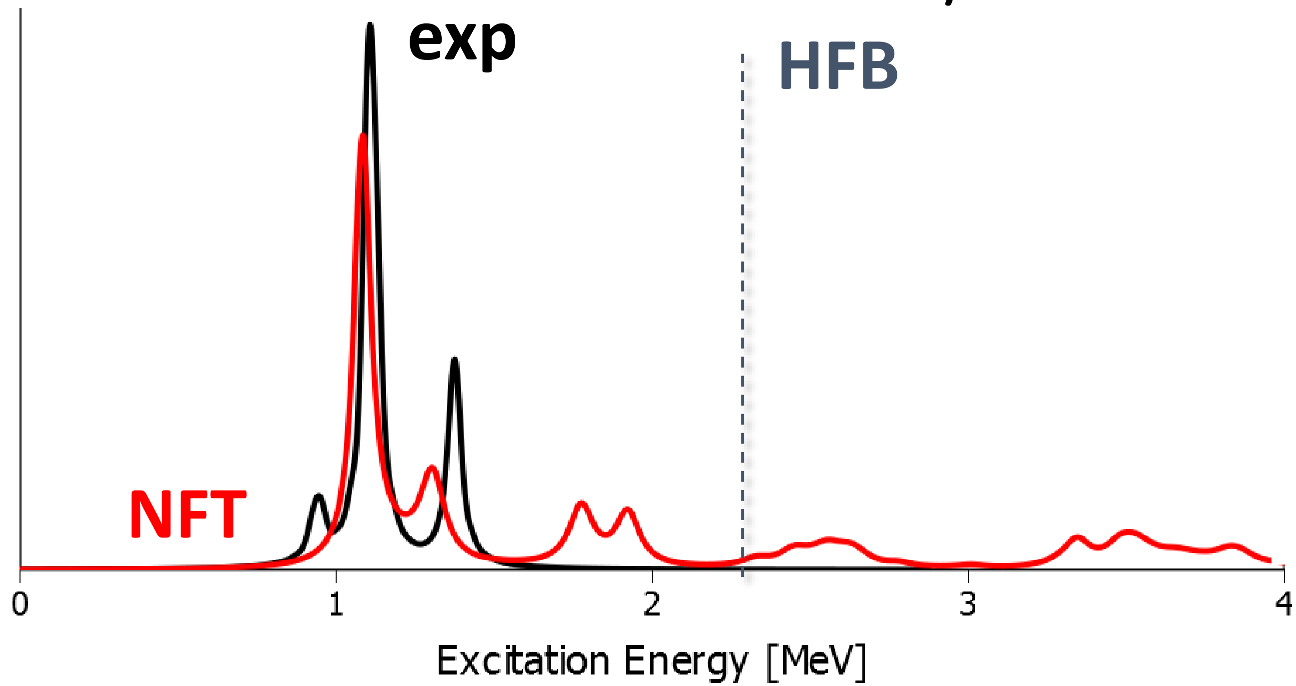
^{120}Sn

excitation spectrum

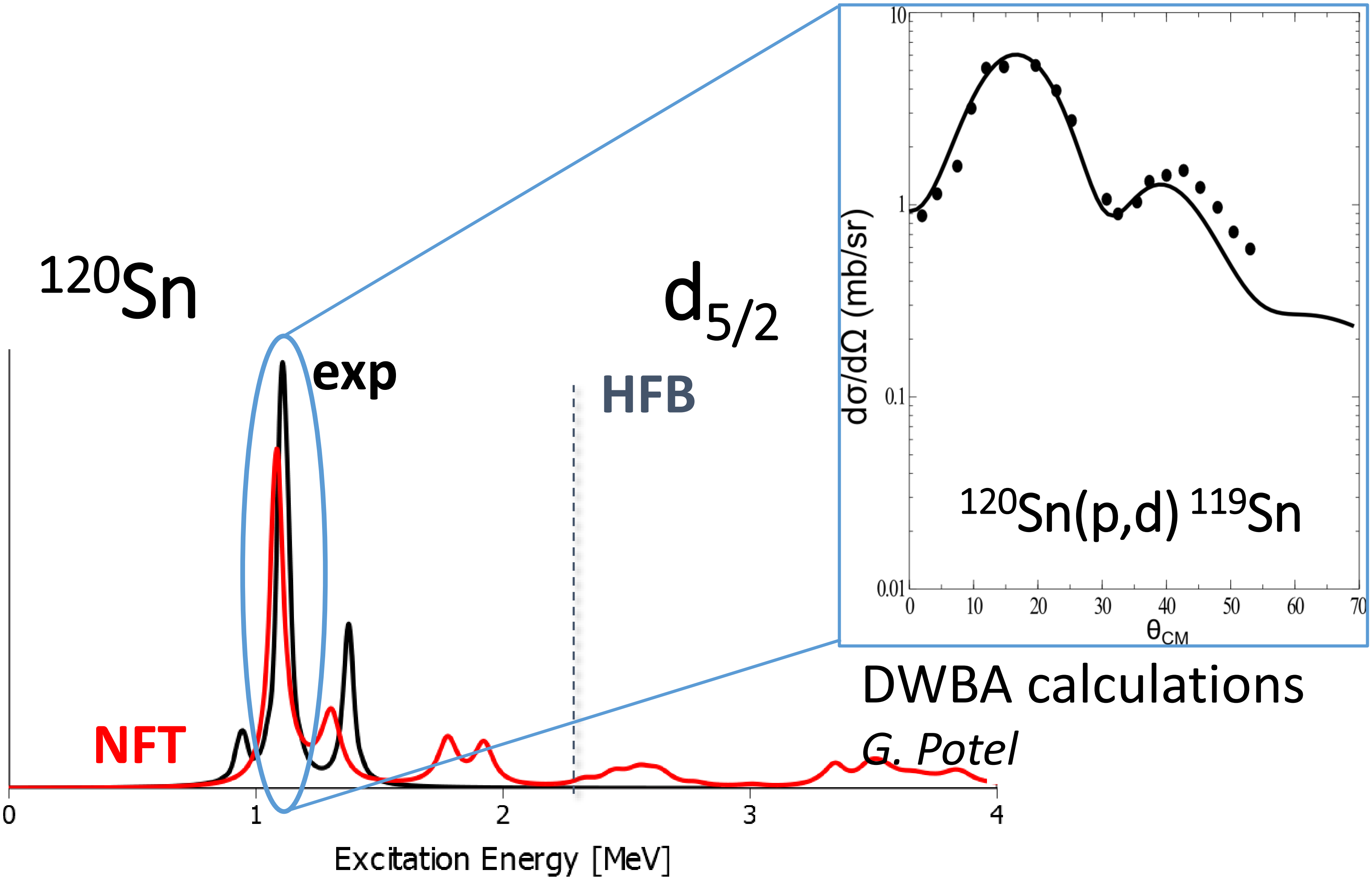


Quasiparticle strength can now be compared with experiment

^{120}Sn $d_{5/2}$



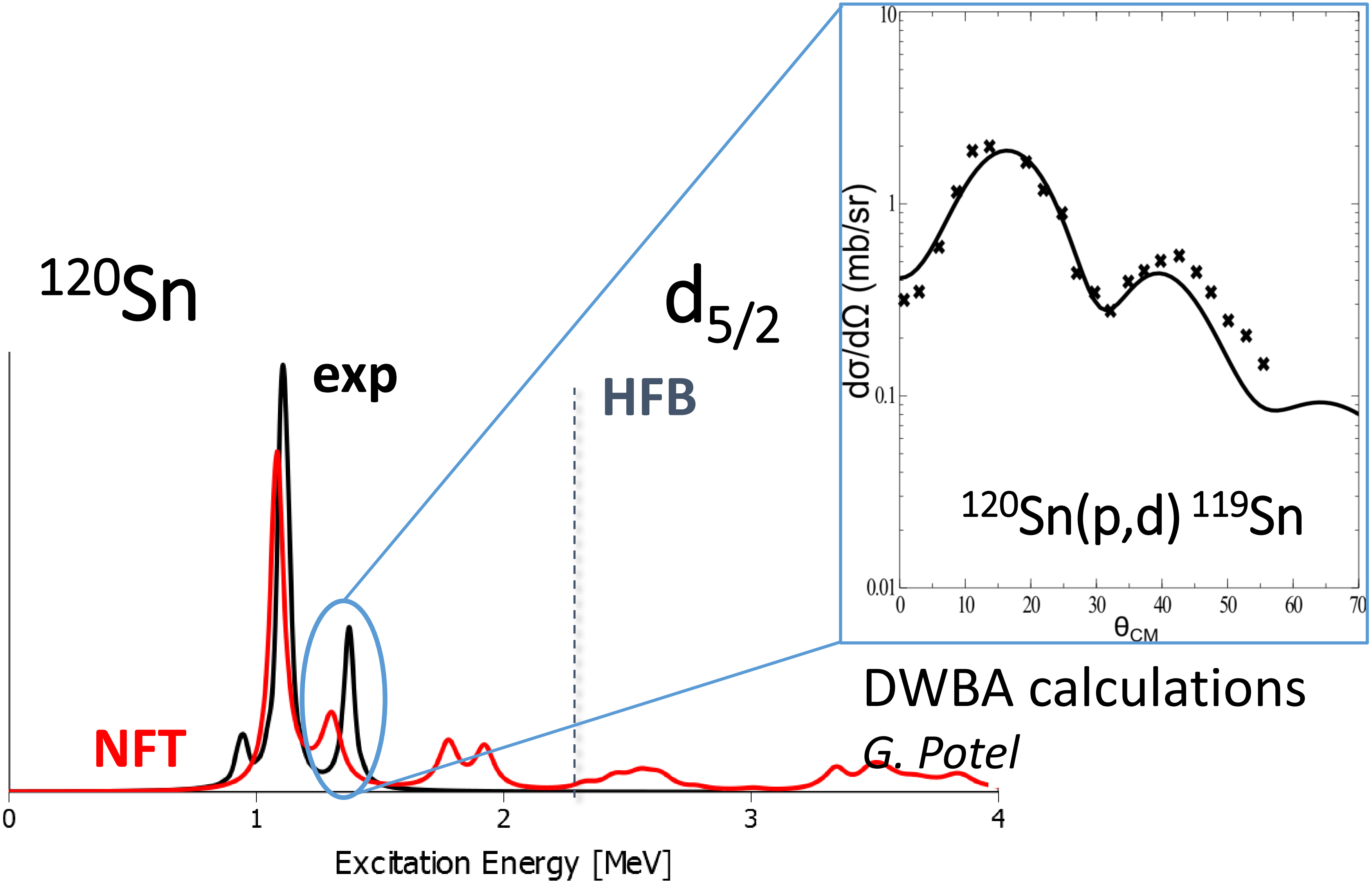
Quasiparticle strength can now be compared with experiment



DWBA calculations

G. Potel

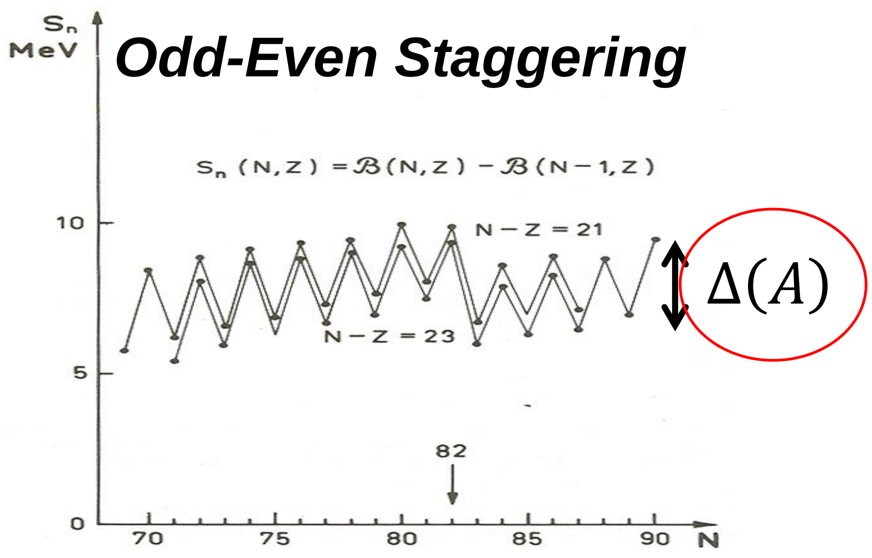
Quasiparticle strength can now be compared with experiment



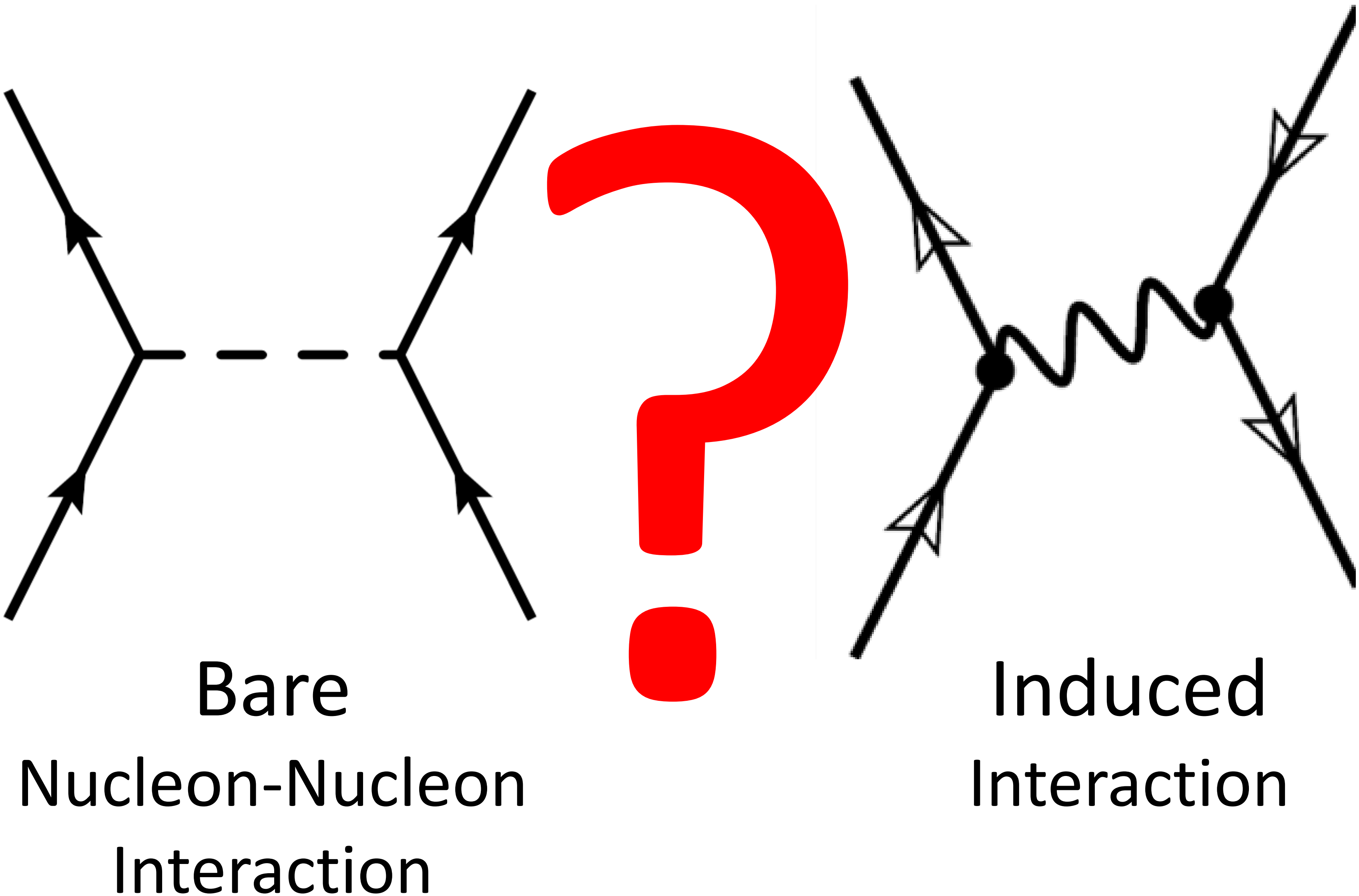
DWBA calculations

G. Potel

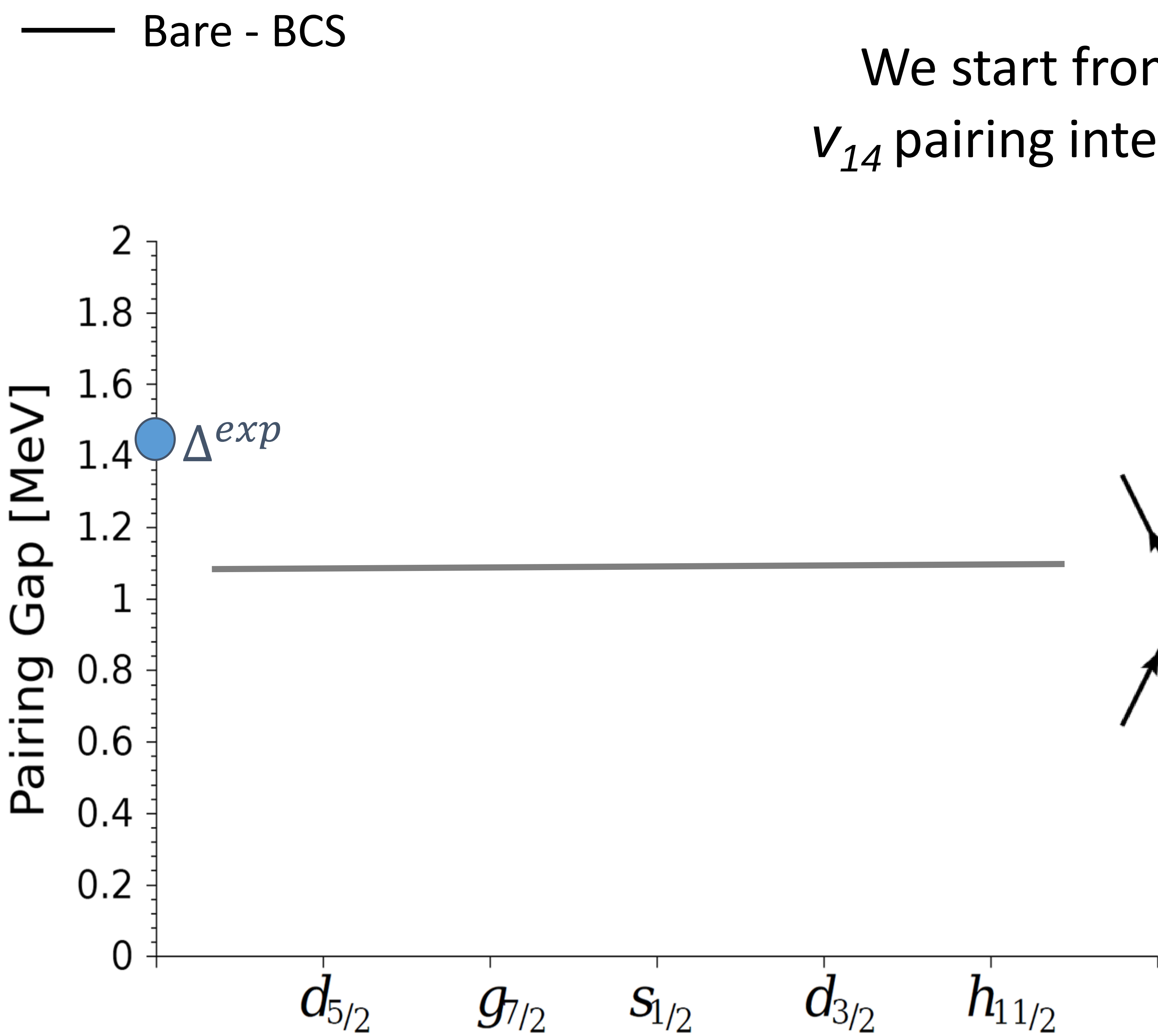
Odd-Even Staggering

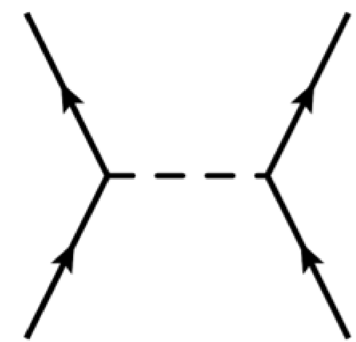


?

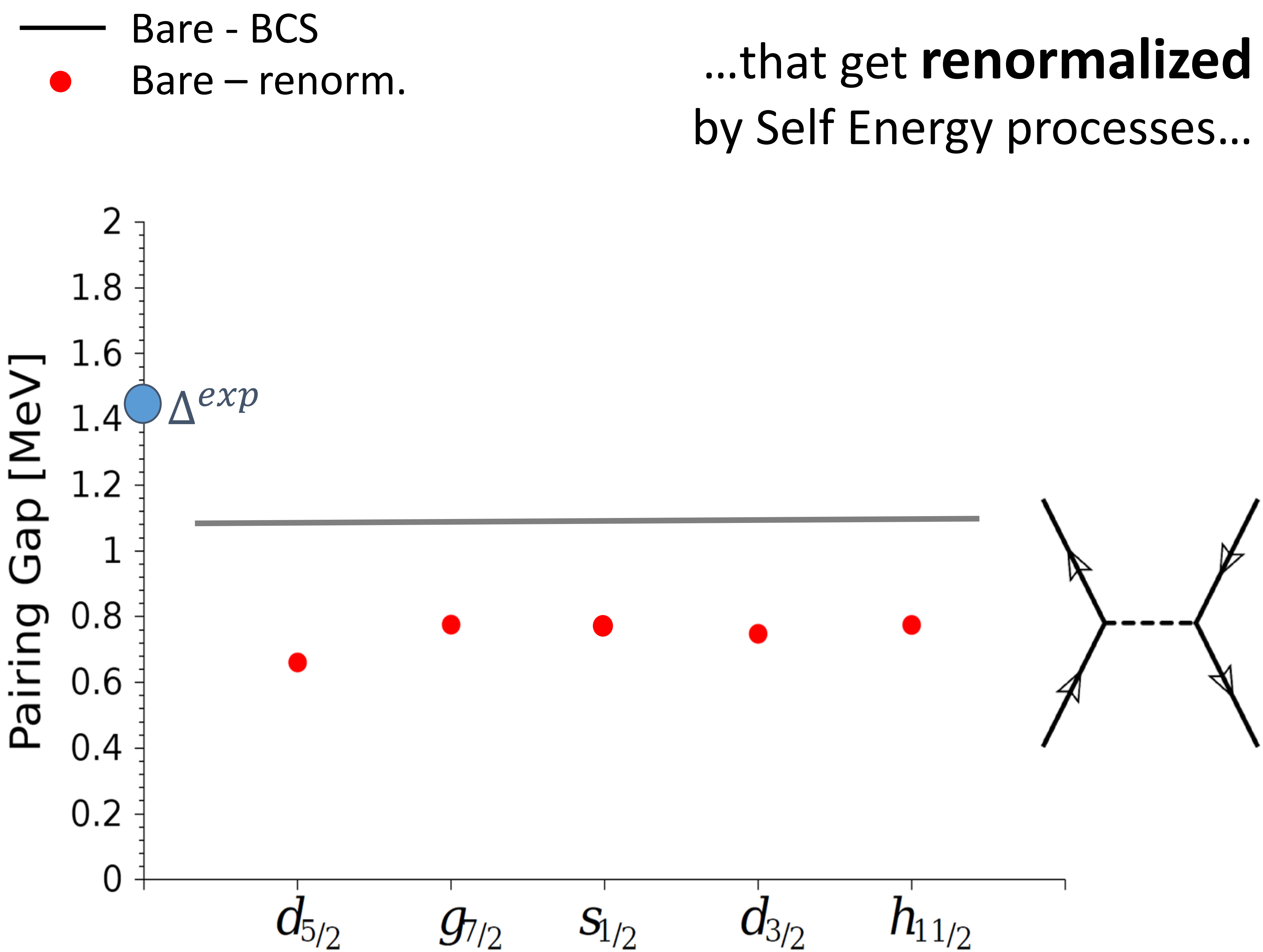

Bare
Nucleon-Nucleon
Interaction
Induced
Interaction

We start from a **bare**
v_{14} pairing interaction…

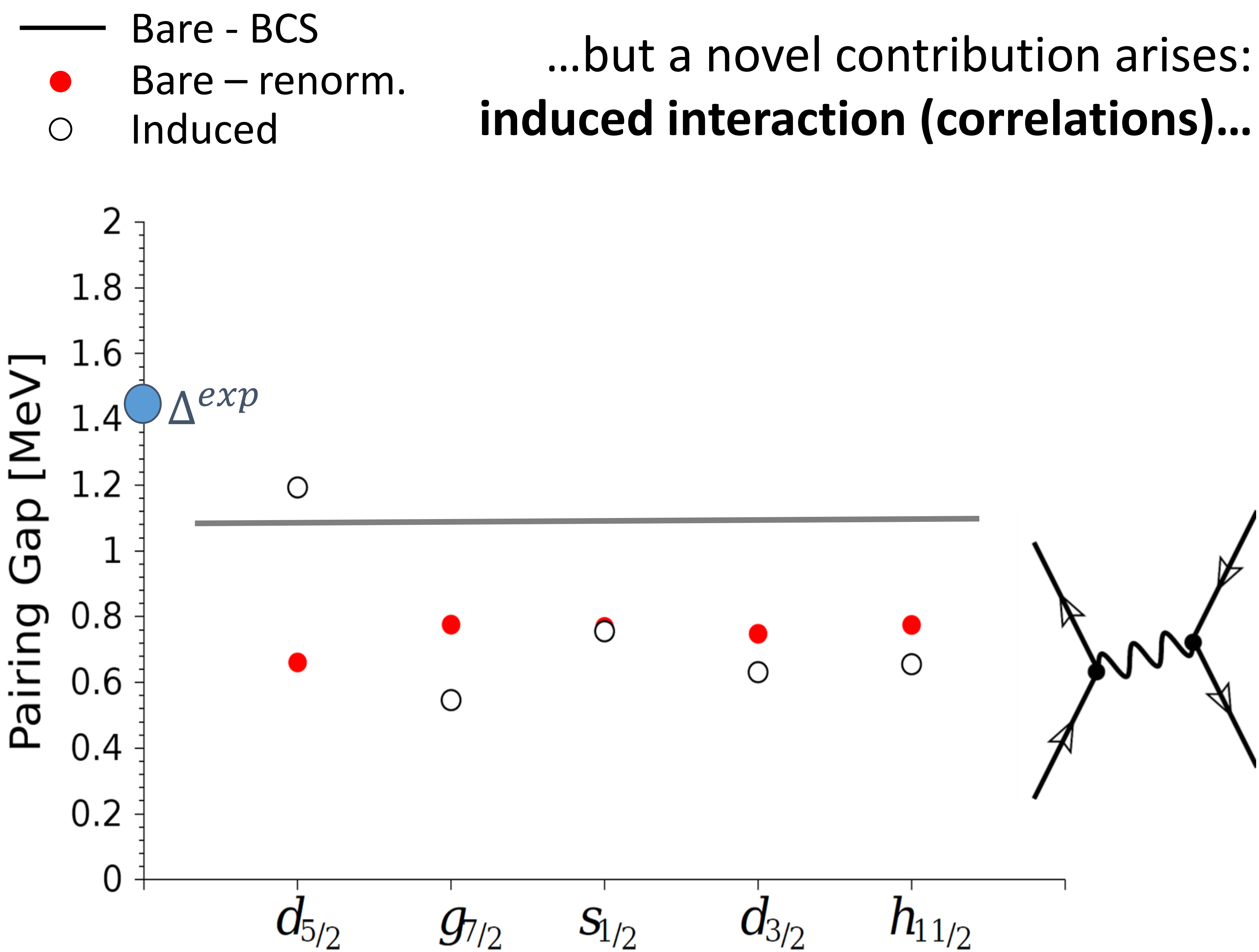




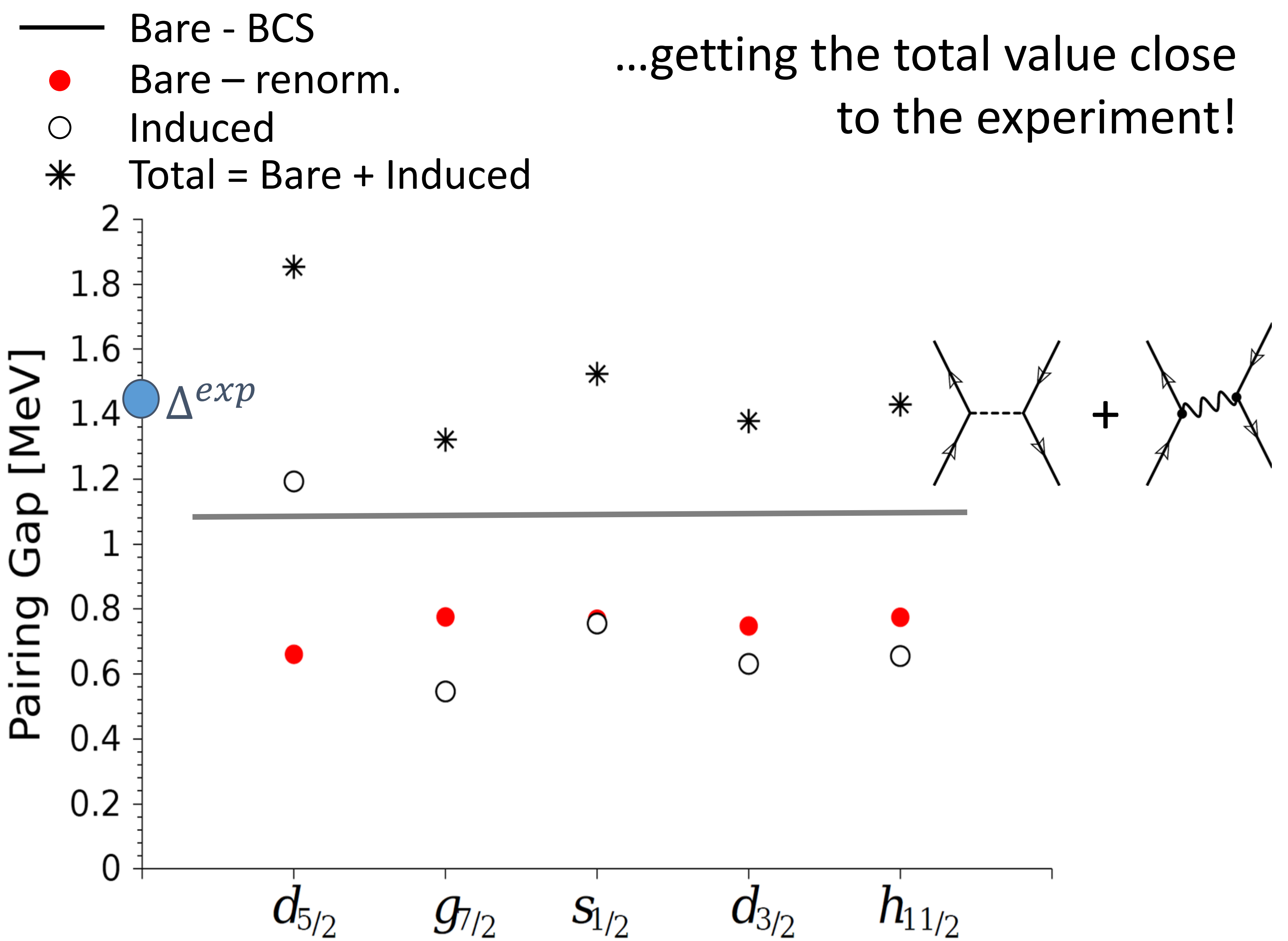
...that get **renormalized** by Self Energy processes...



...but a novel contribution arises:
induced interaction (correlations)...



...getting the total value close to the experiment!



We have a then consistent picture that explains

Independent particle **excitation energies**

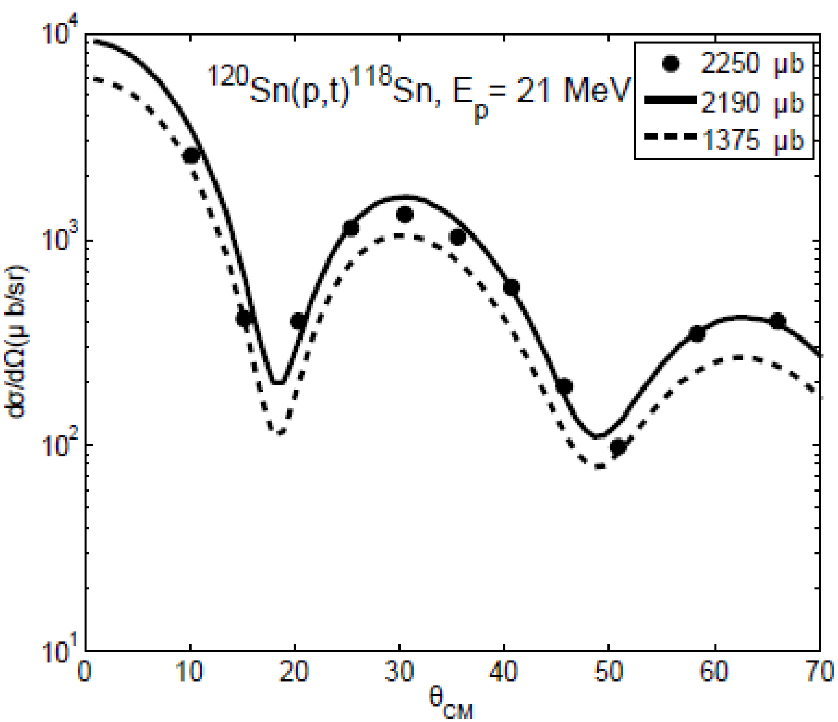
$$\tilde{E}_j$$

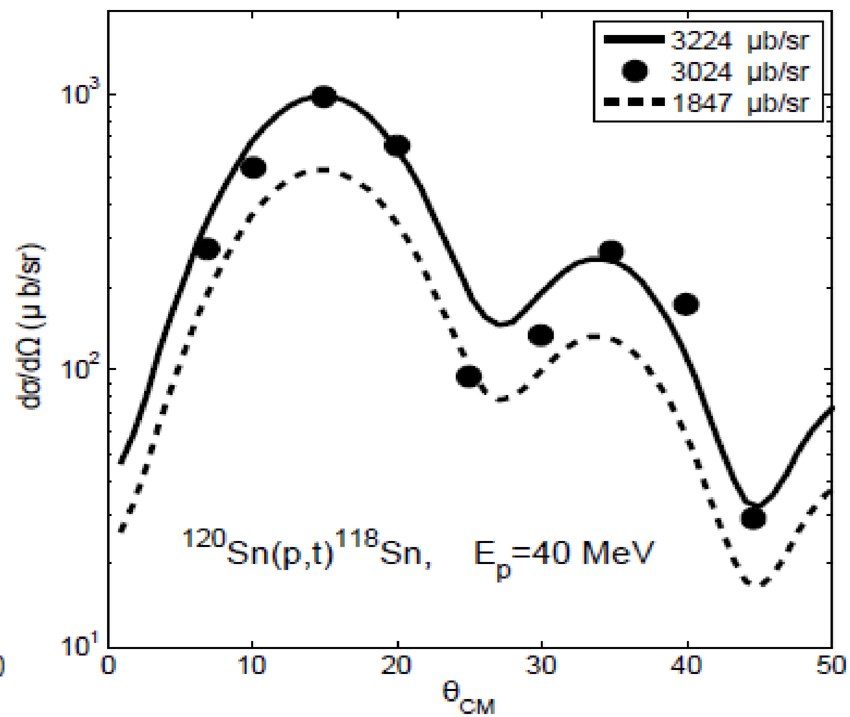
$$\left.\begin{array}{l}\text{Spectroscopic Factors}\\ \text{Pairing Gap}\end{array}\right\} \qquad \tilde{u}_j \tilde{v}_j$$

These are inputs for…

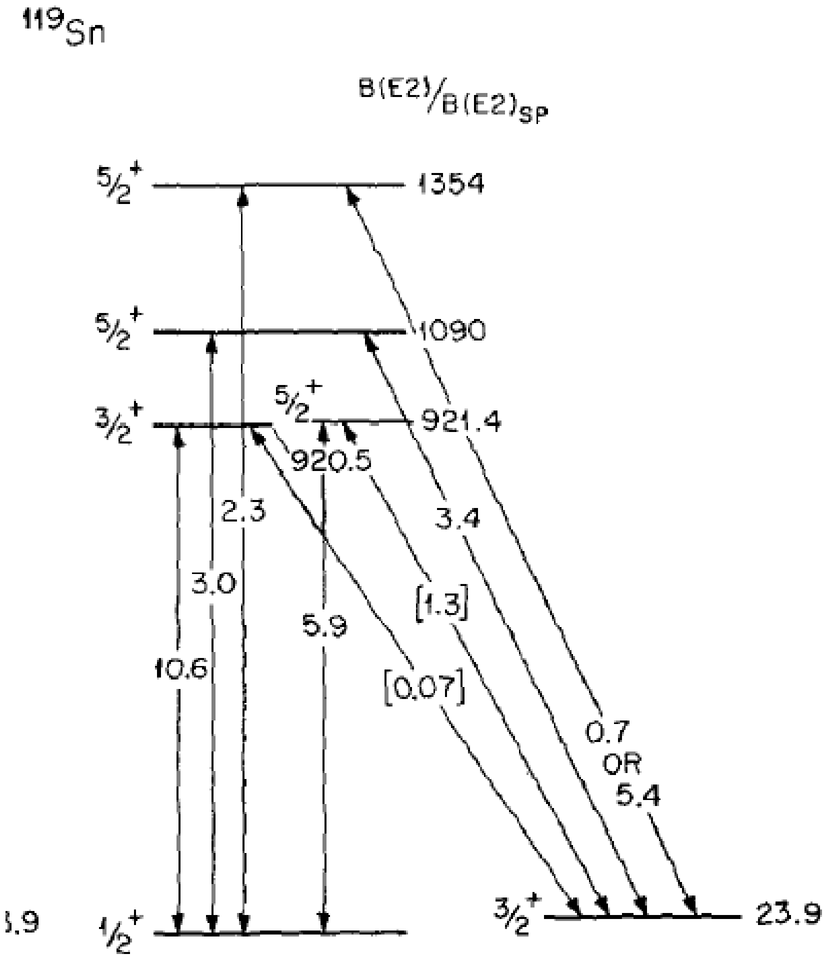
Two particle transfer cross section
IInd order DWBA

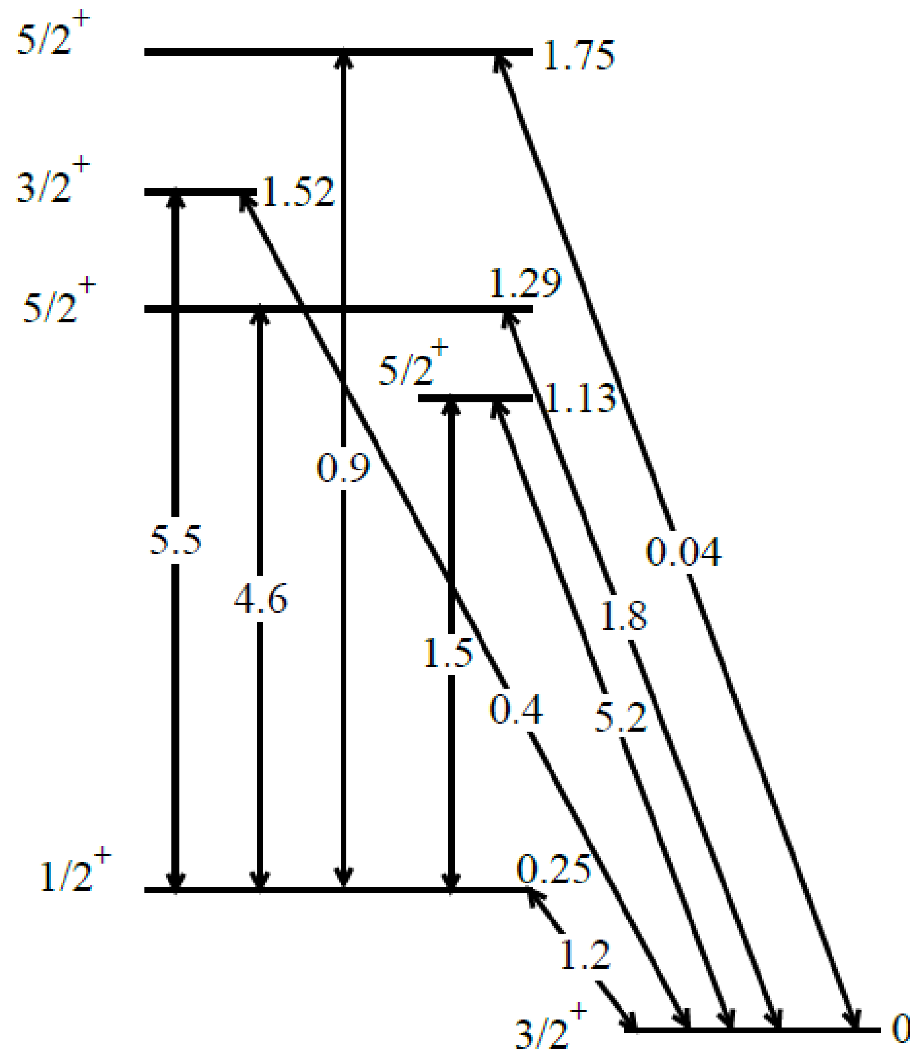
	$\underline{\mathbf{E_p=21\ MeV}}$		$\underline{\mathbf{E_p=40\ MeV}}$
····	$1375\ \mu b$	Only Bare	$1847\ \mu b$
▬	$2190\ \mu b$	Total Pairing	$3224\ \mu b$
●	$2250 \pm 352\ \mu b$	Experimental	$3024 \pm 907\ \mu b$





Electromagnetic Spectrum

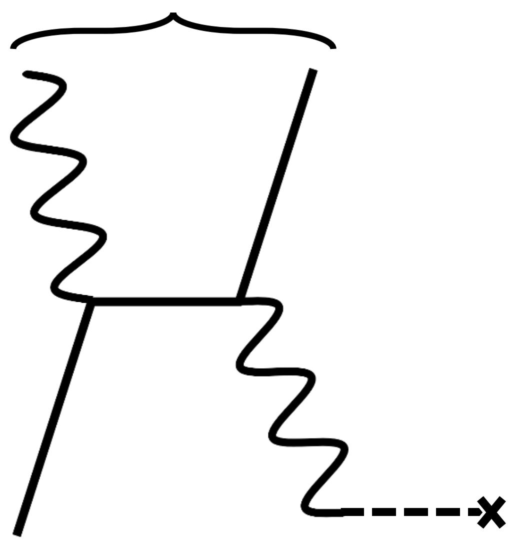




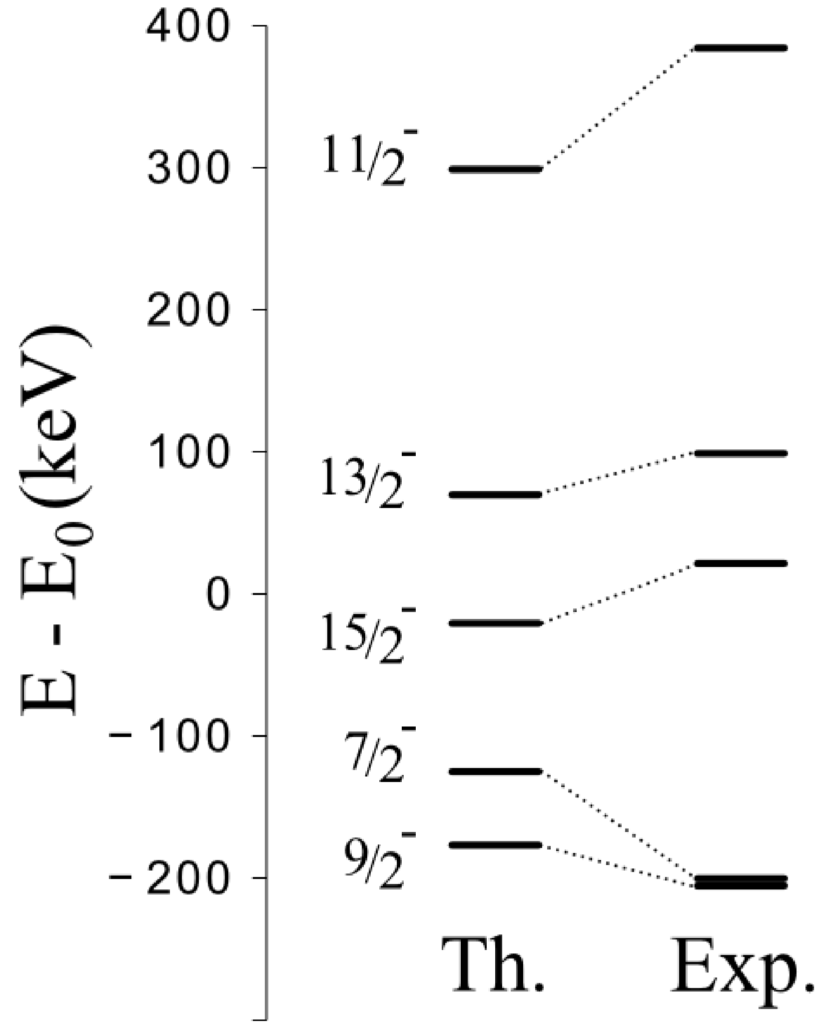
Reaction Channel: Multiplet

quasiparticle state
coupled
to the core vibrations

$$\left(E_{qp}(j) \otimes \lambda\right)_{J'}$$

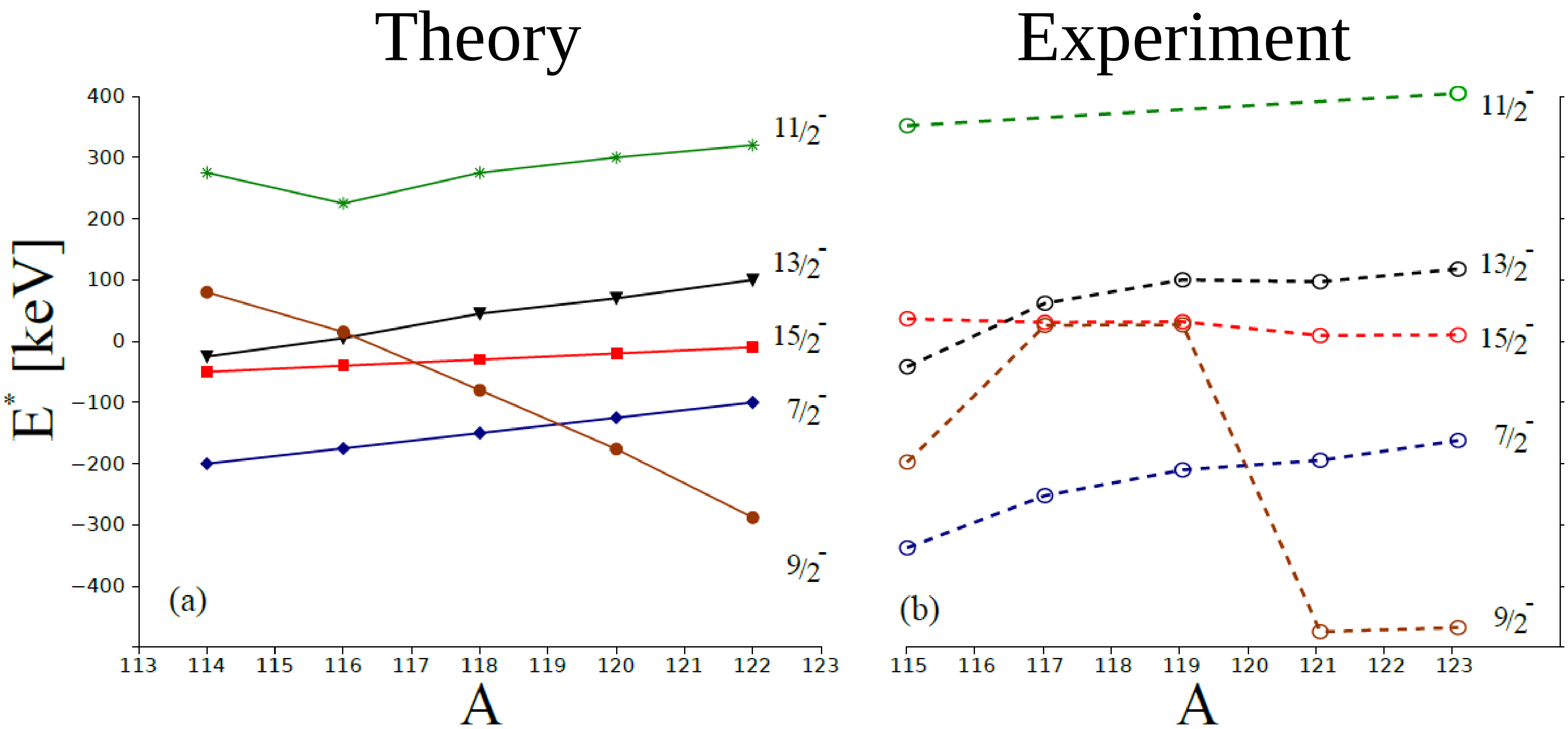


$$\left(h_{11/2} \otimes 2\right)_{7/2-15/2}$$



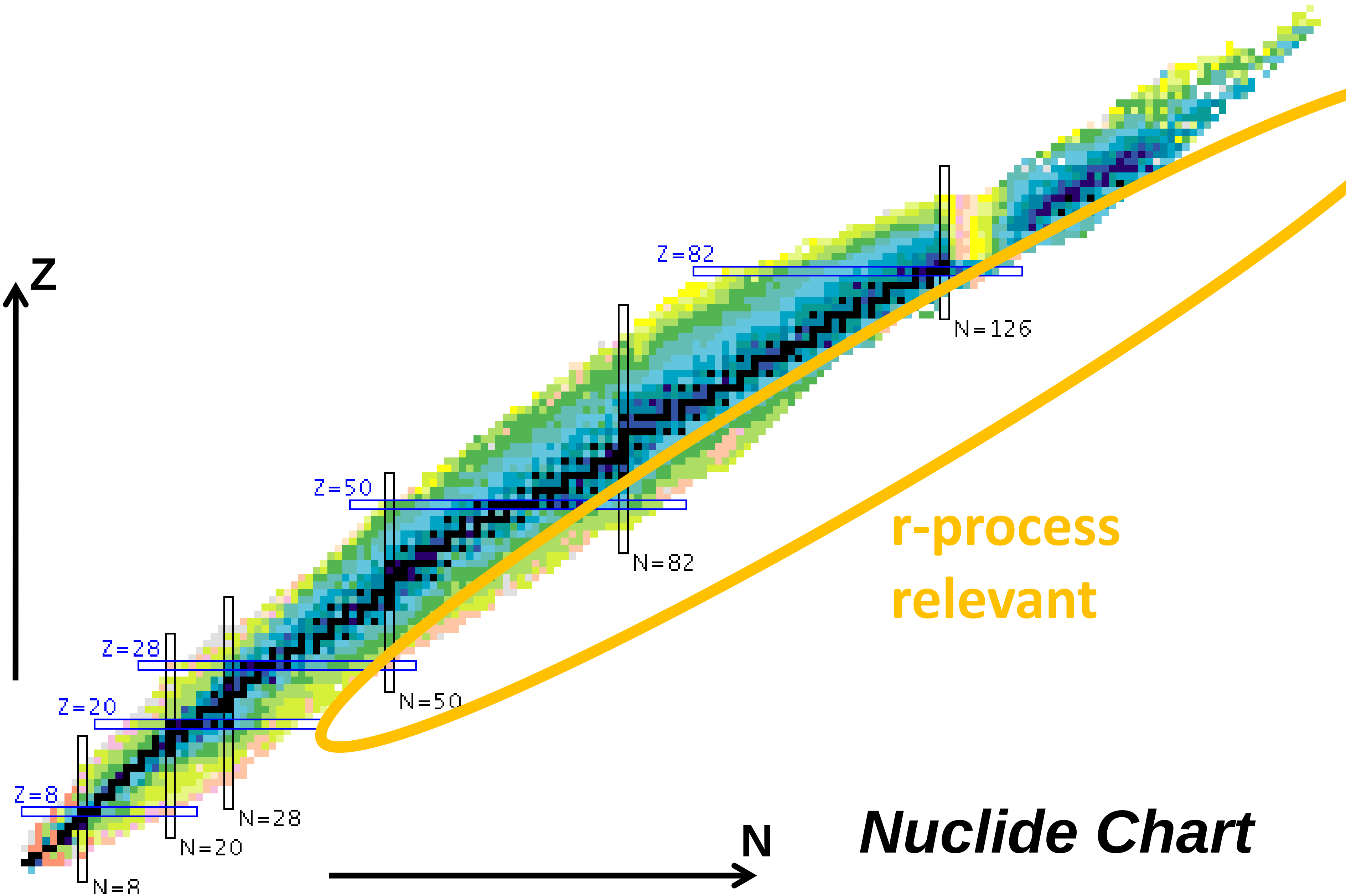
Reaction Channel: Multiplet

$$\left(h_{11/2} \otimes 2\right)_{7/2-15/2}$$

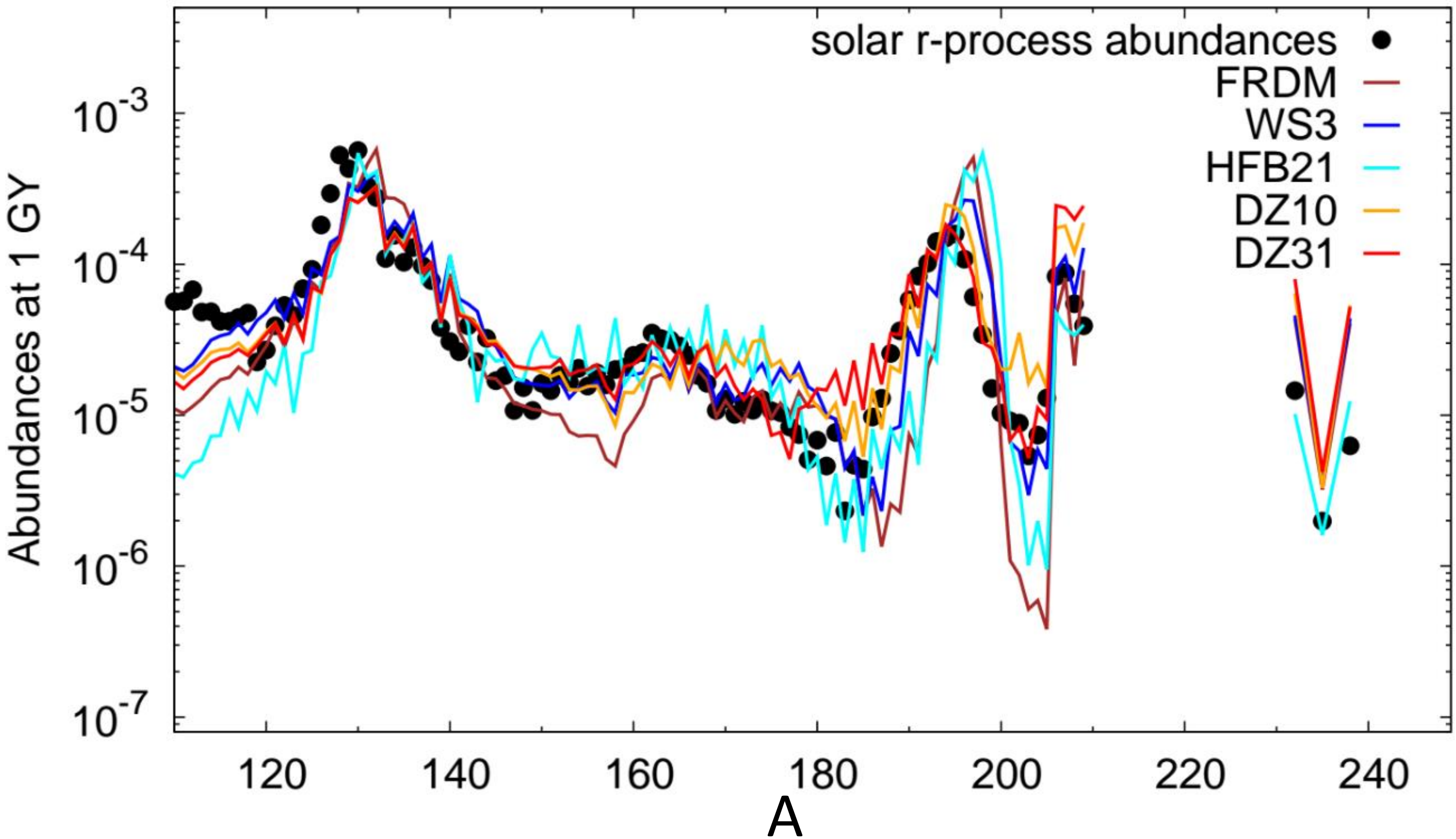


Considering correlations between valence and the core by the means of NFT we have:

- ☑ Increased Hartree-Fock excitation spectrum density.
- ☑ Introduced fragmentation of quasiparticle strength and compared with experimental 1 particle transfer cross sections.
- ☑ Increased the pairing correlations and pairing gap energy of realistic bare interaction closer to the experimental value, and reproduced the 2-particle transfer cross sections.
- ☑ Opened other reaction channels, like coupling of core excitations and quasiparticles.

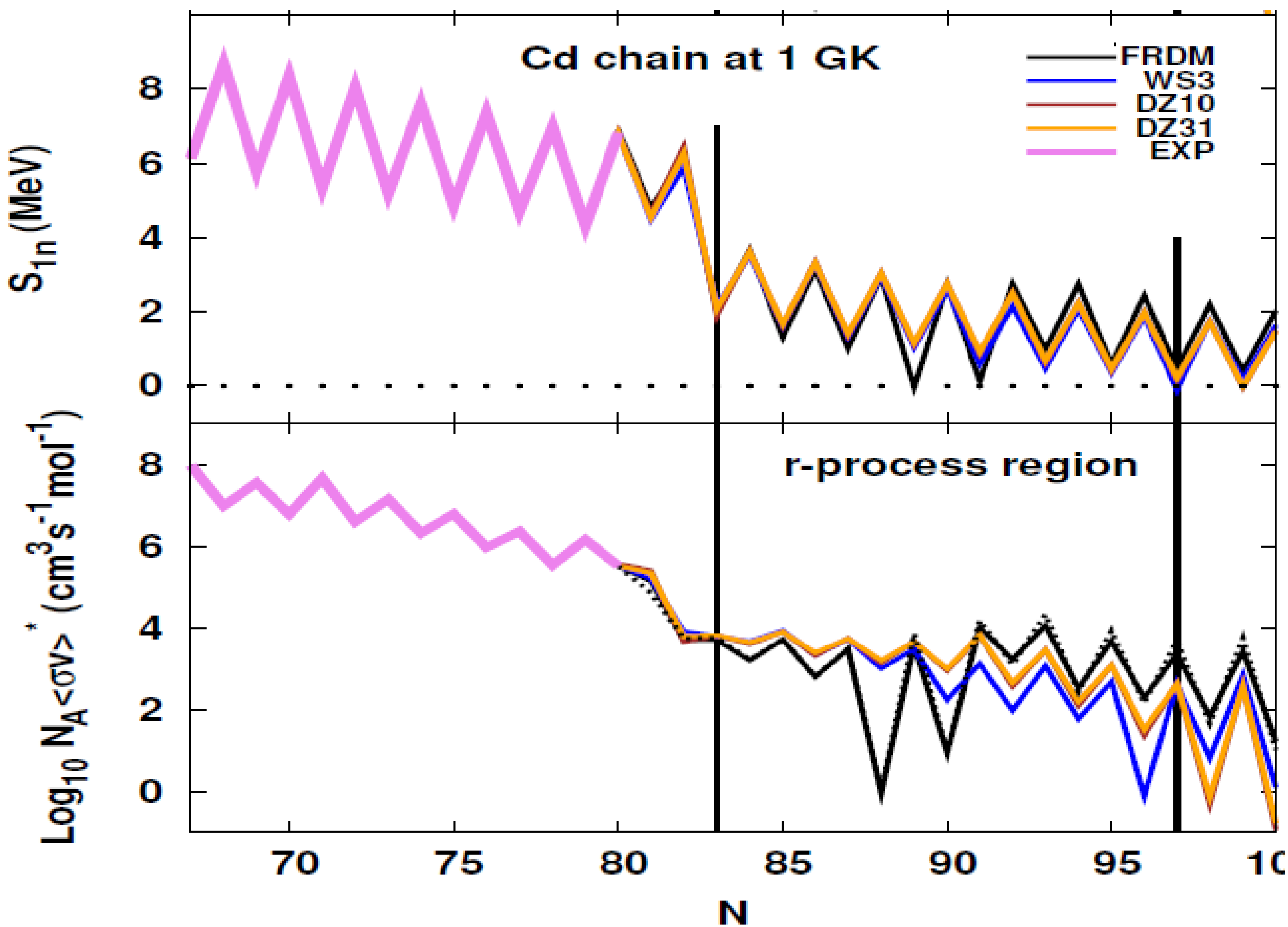

Z
N
Z=8
Z=20
Z=28
Z=50
Z=82
N=8
N=20
N=28
N=50
N=82
N=126
r-process
relevant
Nuclide Chart

r-process is Mass Model sensitive



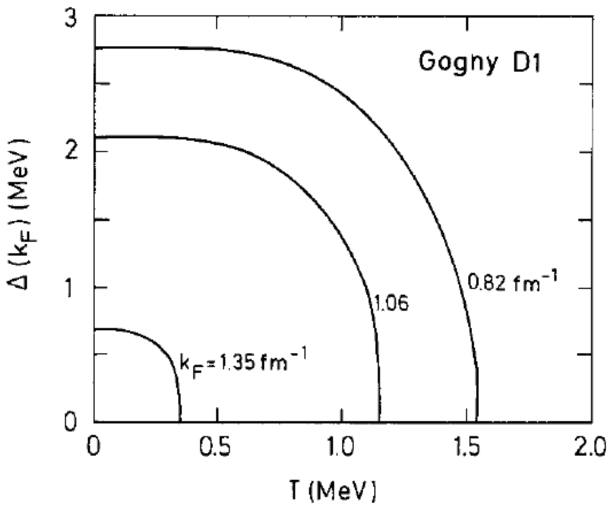
J. Mendoza-Temis, PhD Thesis, 2014

r-process is **Pairing** sensitive



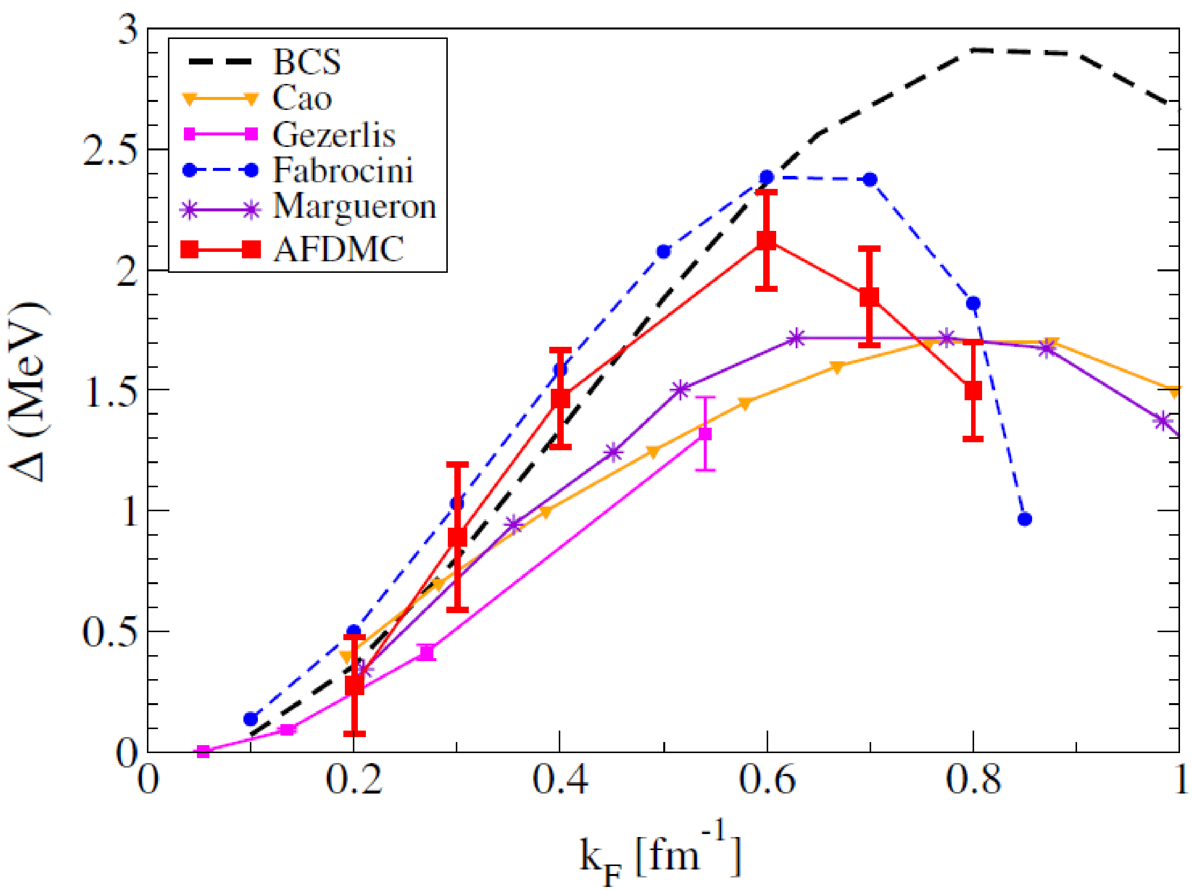
Like a feather on
an unstable Rock

K. Kucharek et al. ZPA334 (1989)

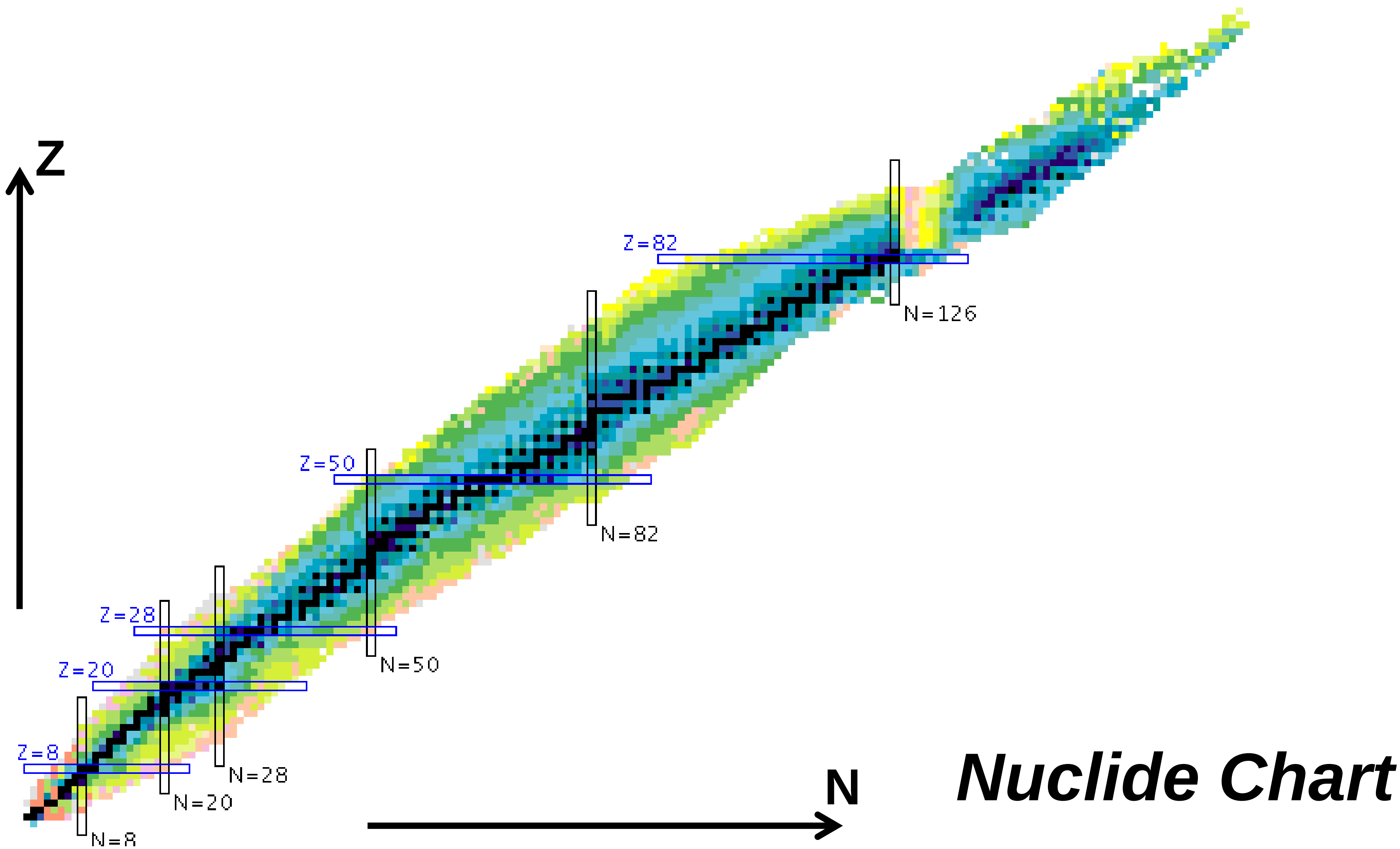


PRL **101,** 132501 (2008)

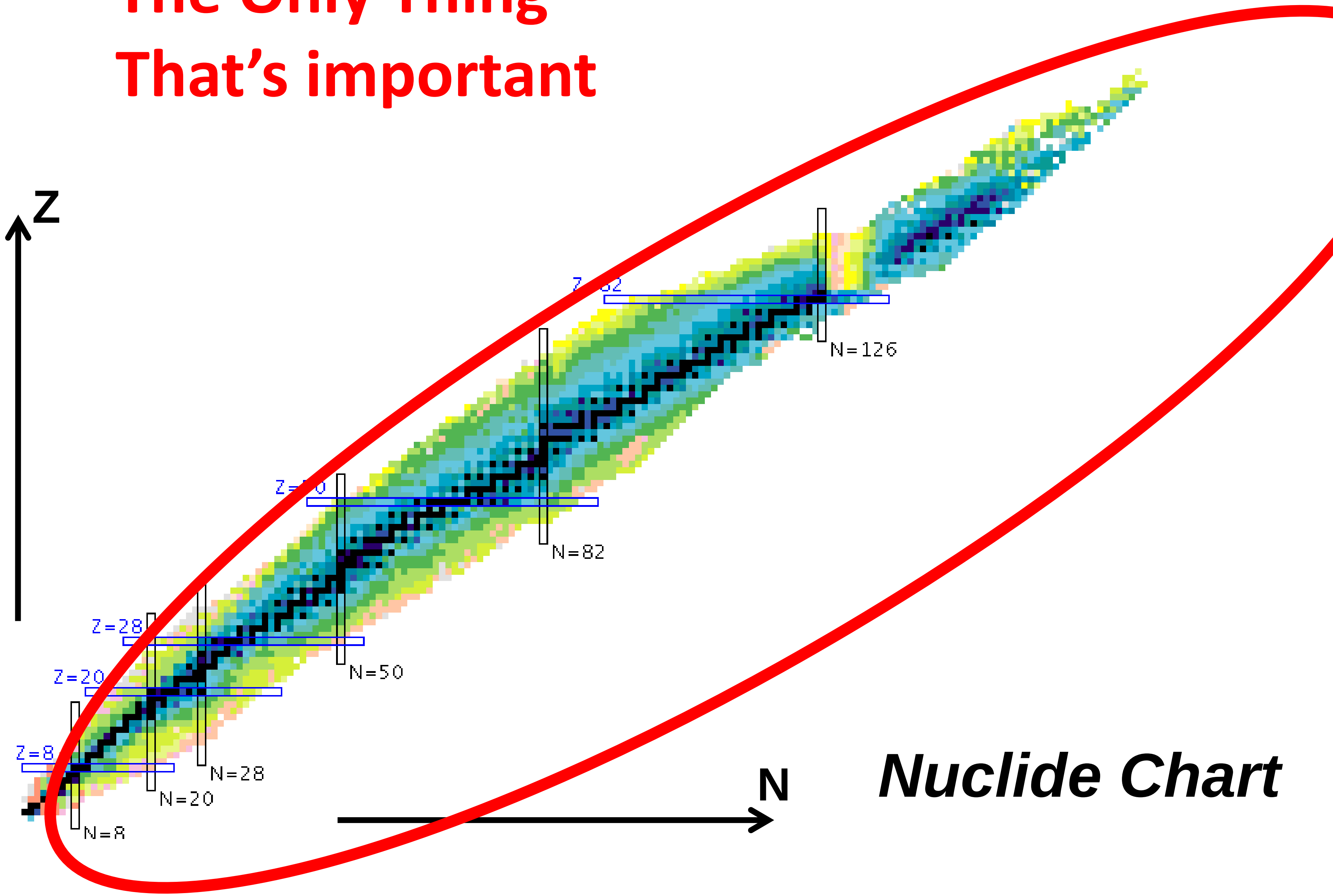
S. Gandolfi et al.



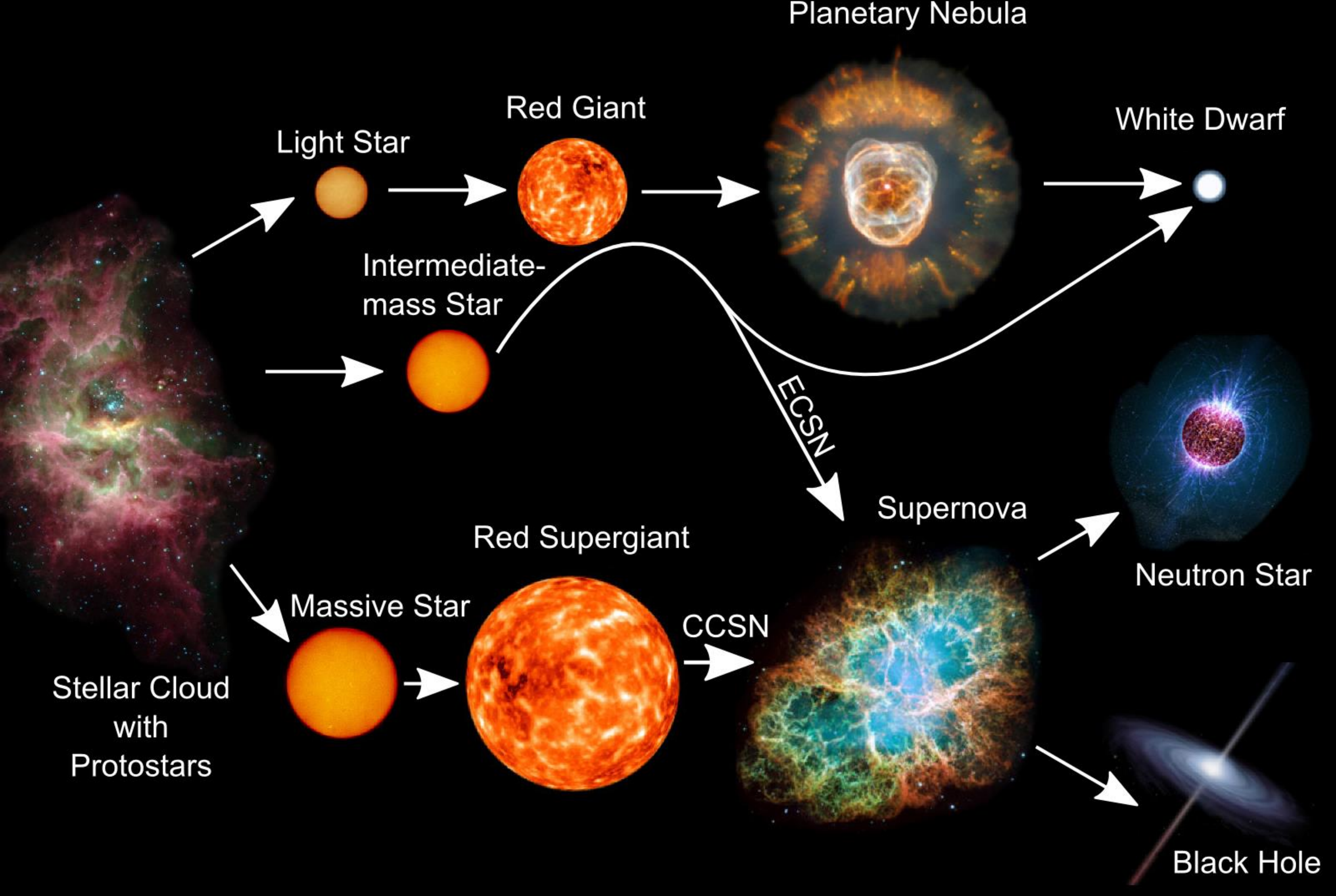
Not everything is r-process



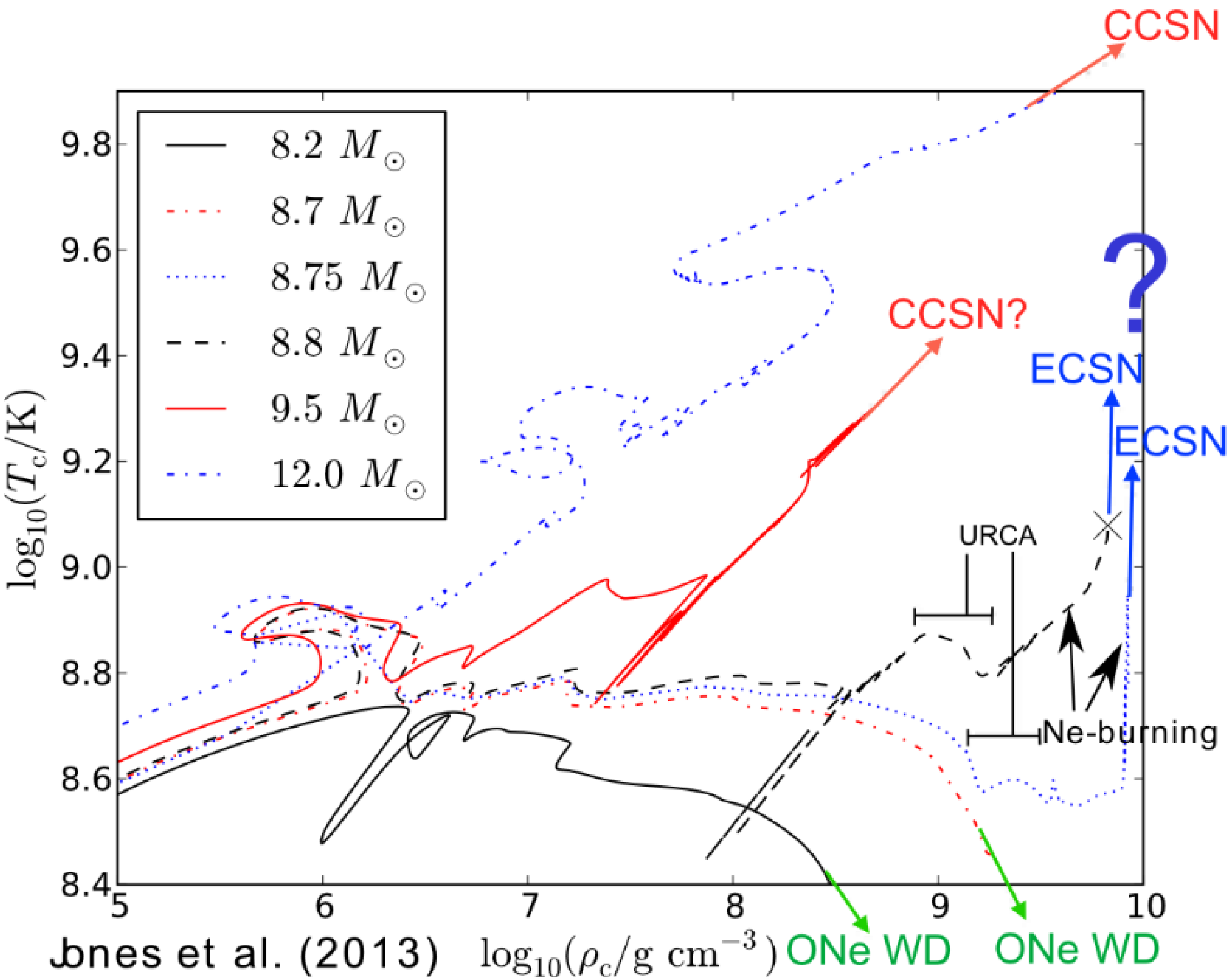
The Only Thing That's important



Nuclide Chart



Planetary Nebula
Red Giant
Light Star
White Dwarf
Intermediate-
mass Star
ECSN
Supernova
Red Supergiant
Neutron Star
Massive Star
CCSN
Stellar Cloud
with
Protostars
Black Hole



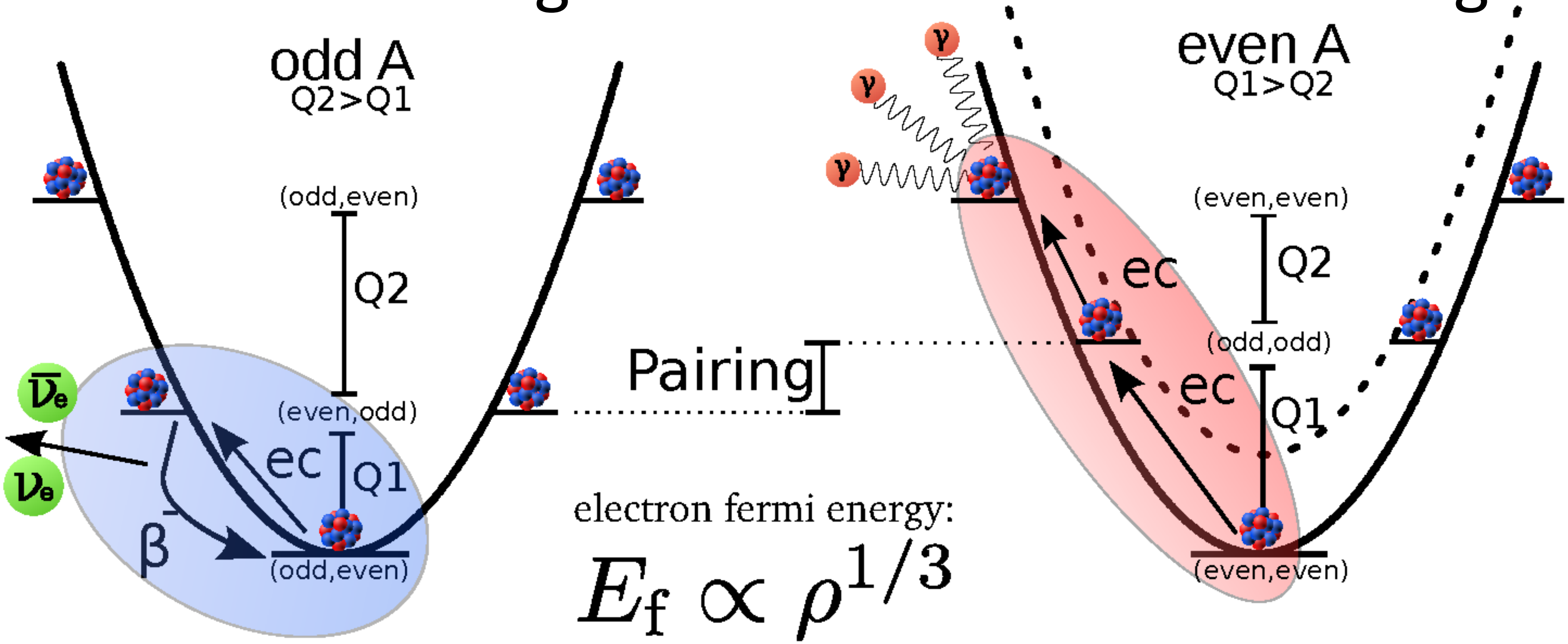
$$M_C = 1.4 \times 2Y_e \ M_S$$

$$^{20}Ne\,(Q_{ec} = -7.5\ \mathrm{MeV})$$

$$^{24}Mg(Q_{ec} = -3.4\ \mathrm{MeV})$$

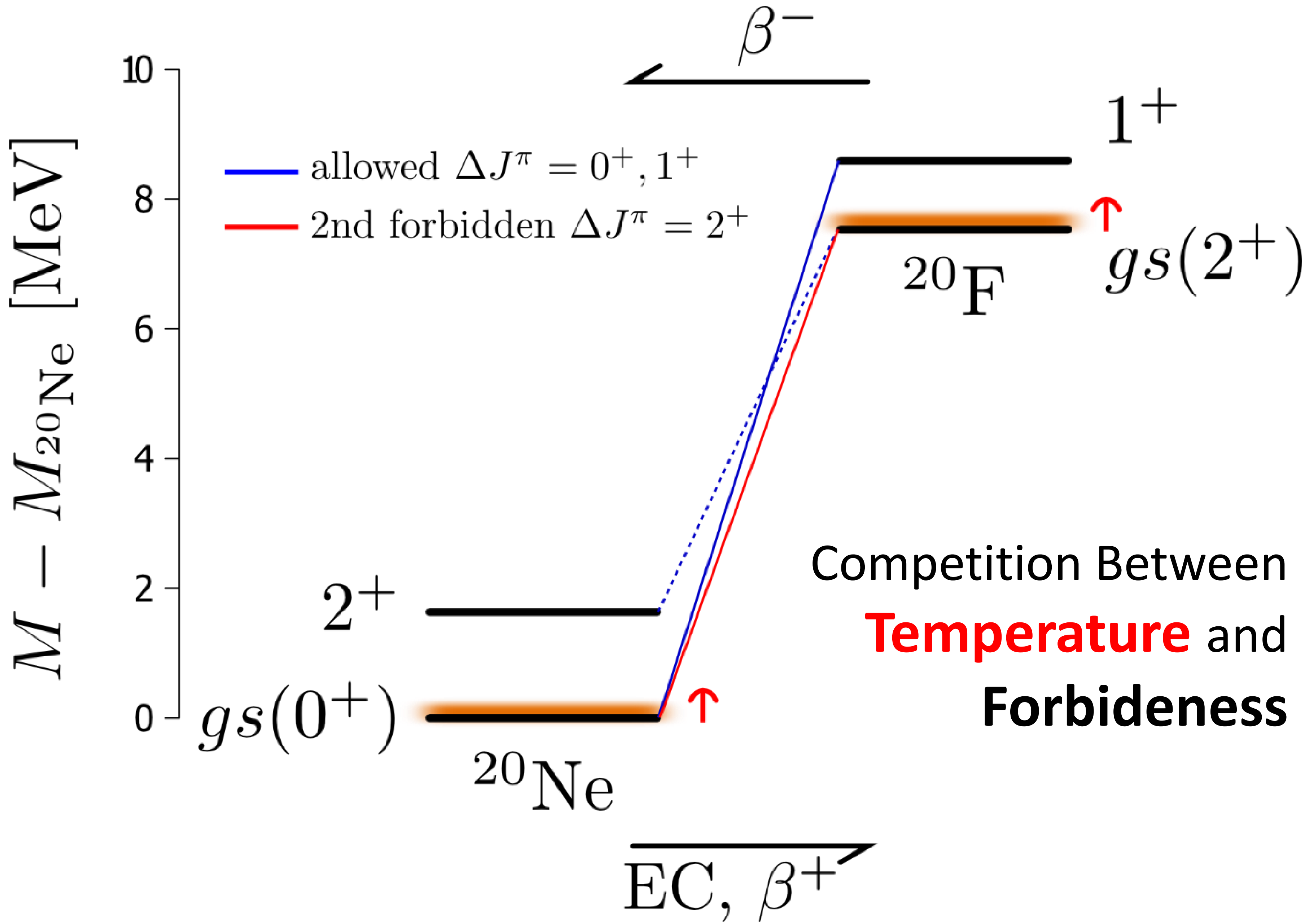
Urca **Cooling** — e^- conserving

Heating — e^- decreasing



$$(\mathrm{A,Z}) + e^- = (\mathrm{A,Z\text{-}1}) + \nu_e \qquad (\mathrm{A,Z}) + e^- = (\mathrm{A,Z\text{-}1}) + \nu_e$$

$$(\mathrm{A,Z\text{-}1}) = (\mathrm{A,Z}) + e^- + \bar{\nu}_e \qquad (\mathrm{A,Z\text{-}1}) + e^- = (\mathrm{A,Z\text{-}2})^* + \nu_e + \gamma$$



β^-
1^+
allowed $\Delta J^\pi = 0^+, 1^+$
2nd forbidden $\Delta J^\pi = 2^+$
$gs(2^+)$
^{20}F
$M - M_{^{20}\mathrm{Ne}}$ [MeV]
10
8
6
4
2
0
2^+
$gs(0^+)$
^{20}Ne
EC, β^+
Competition Between
Temperature and
Forbideness

Shape Factor
(Nuclear Physics)

Fermi Distribution
(Statistical Description)

$$\lambda^{ec} = \frac{ln2}{K} \int_{|Q|}^{+\infty} E_\nu^2\, C(E)\, Ep\, f(E, kT, \mu)\, F(Z, E)\, dE$$

Kinematic

Fermi Function

$$C(E) \propto \left| \langle f | H_\beta | i \rangle \right|^2$$

$C = const$ (allowed)

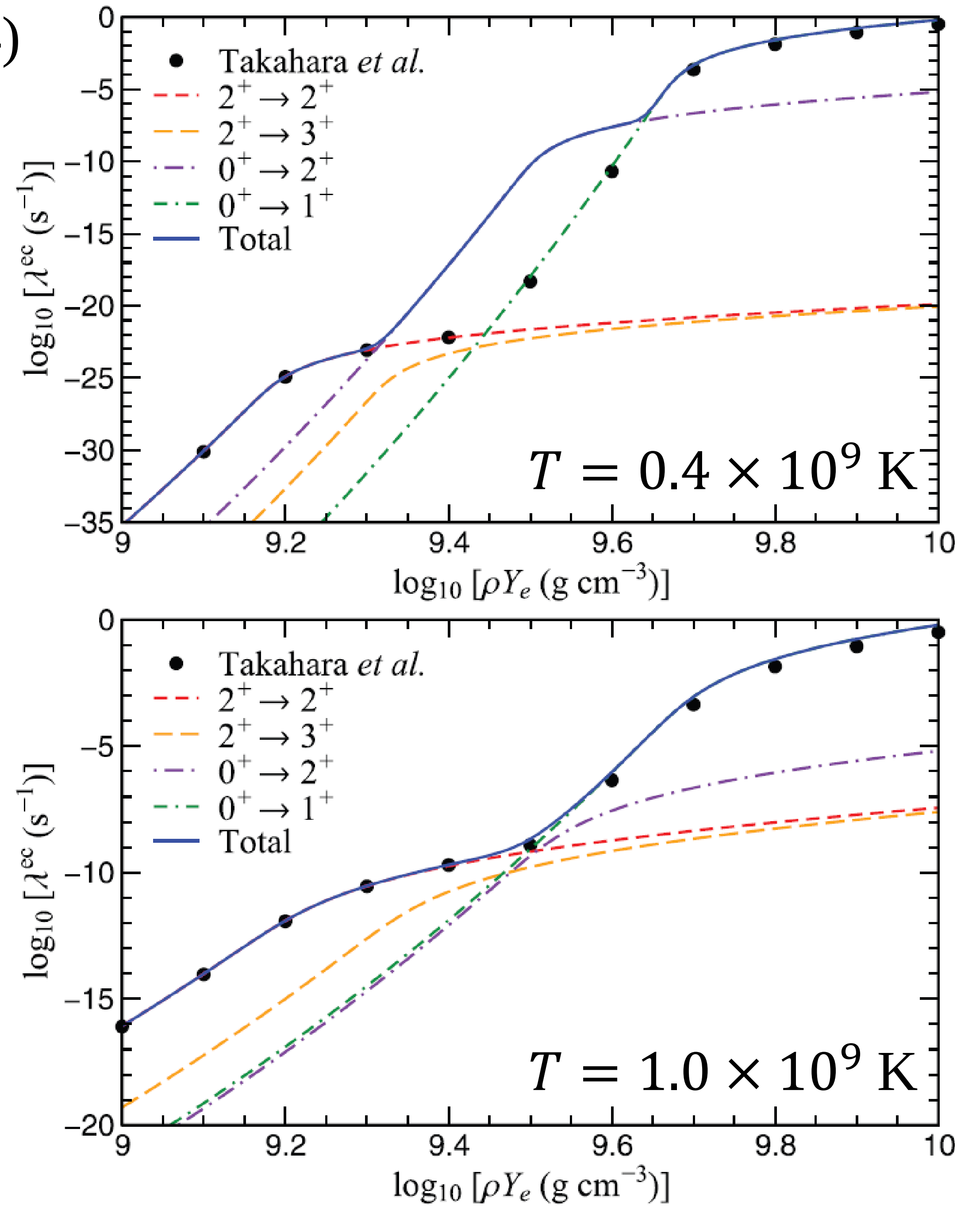
$C(E)$ (forbidden)

$C \sim \ E^4$ (2° forbidden)

Martinez-Pinedo et al. (2014)
PRC 89:045806

$C = 1.23 \times 10^{-6}$
(exp. upper limit)

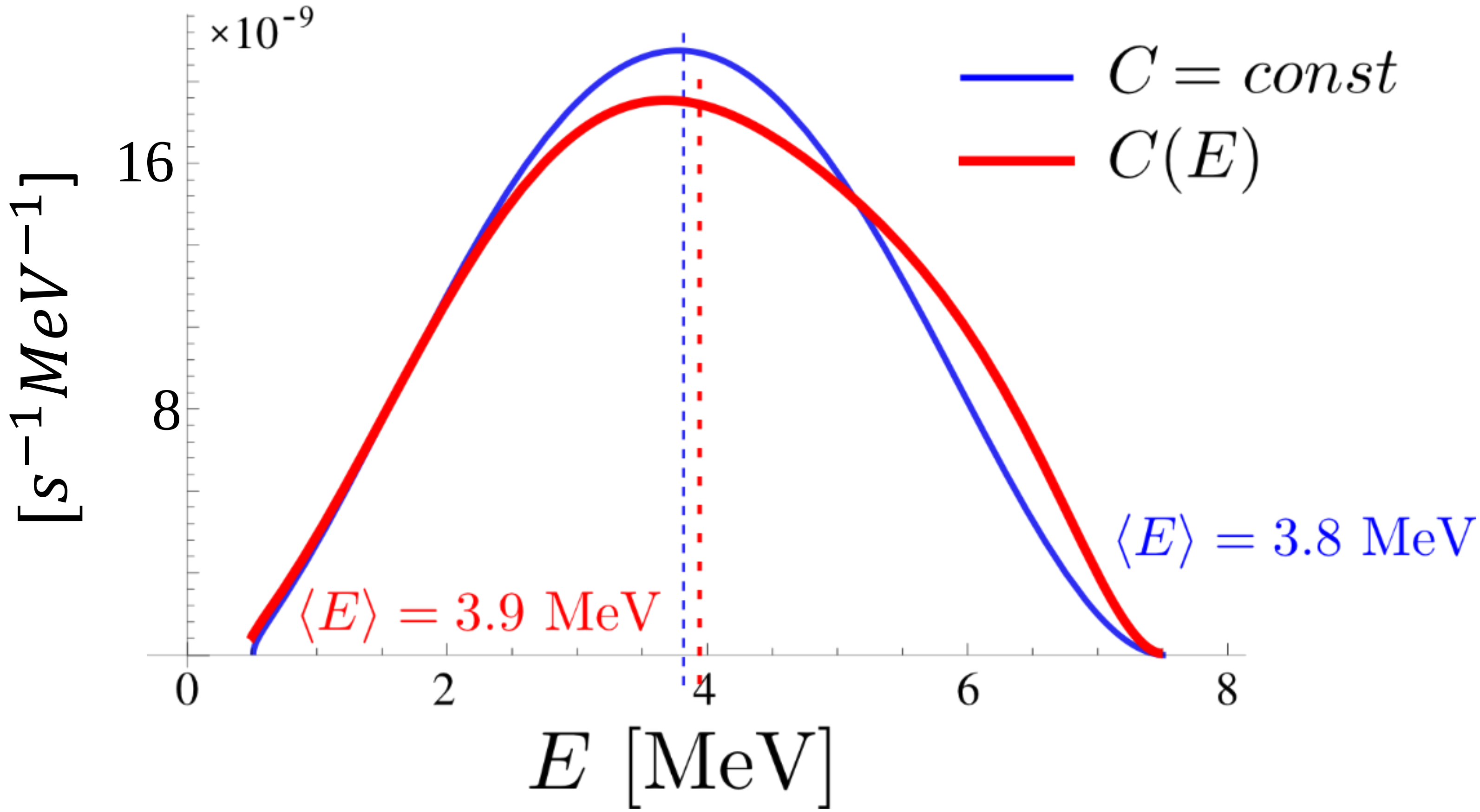
Estimate needed!
(possibly experiment)



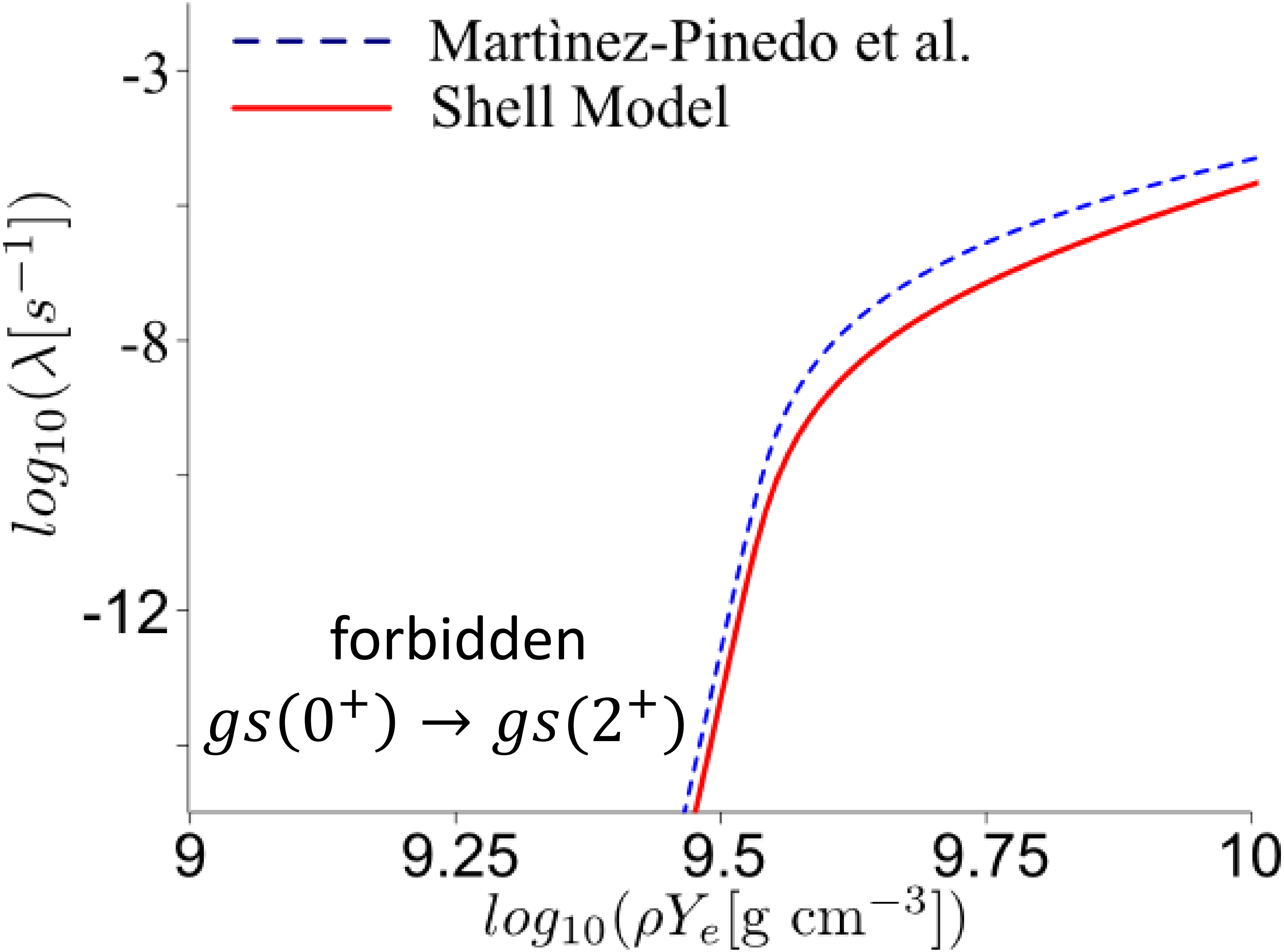
$$\lambda^{\beta^-} = \frac{ln2}{K} \int_1^Q \underbrace{E_\nu^2\, Ep\, C(E) F(Z,E) dE}_{\beta\text{-spectrum}}$$

$$^{20}F(gs(2^+)) \rightarrow {}^{20}Ne(gs(0^+))$$

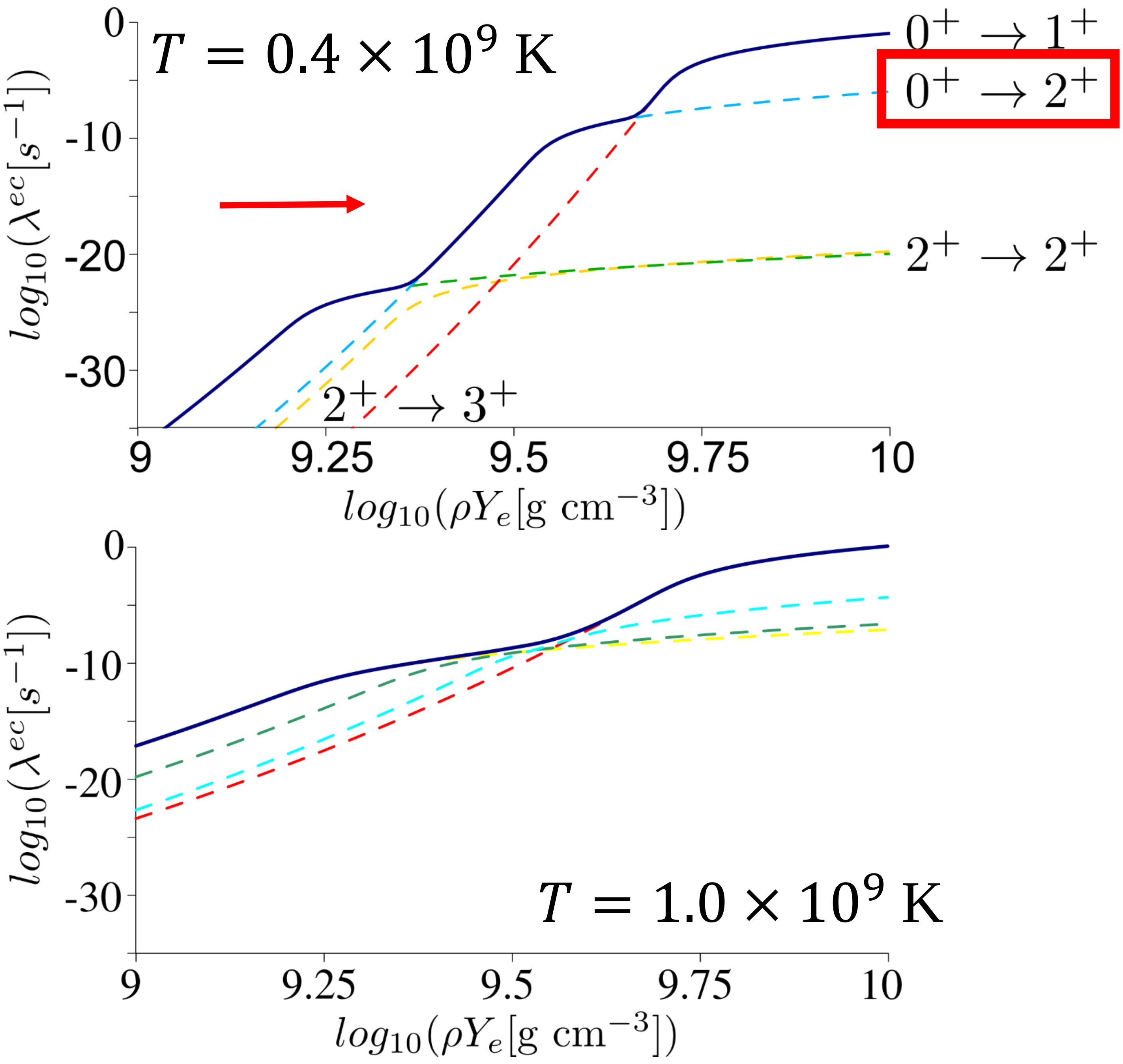
$$BR = 1.3 \times 10^{-6}$$



Electron Capture Decay Rate



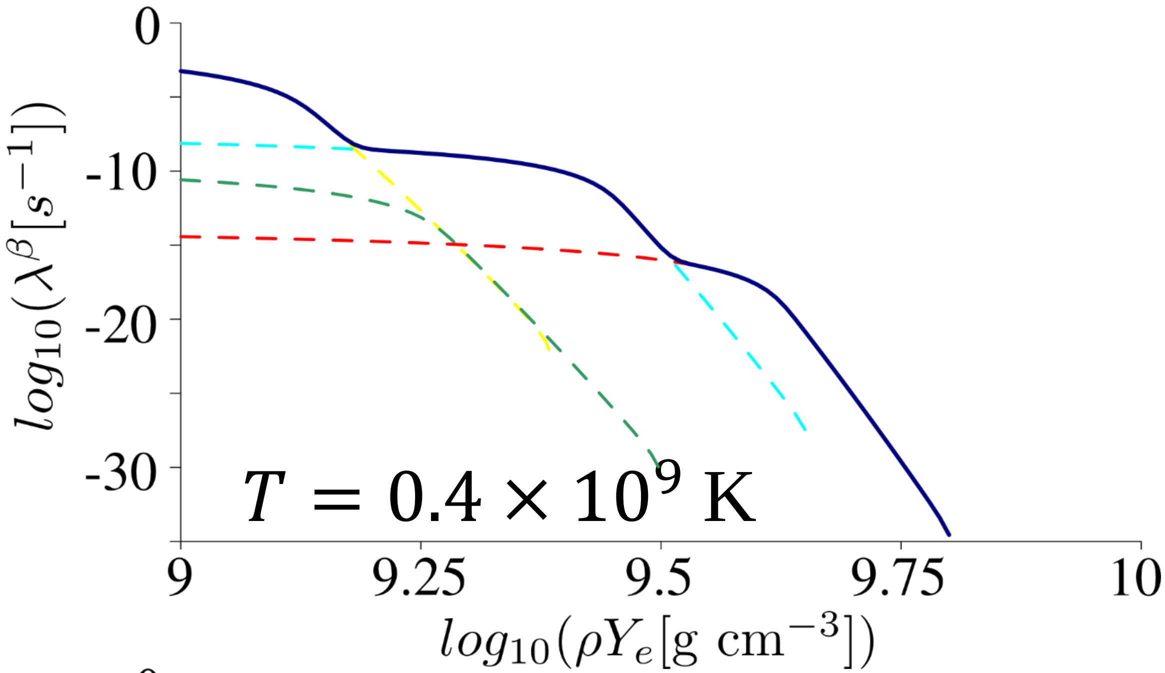
ec

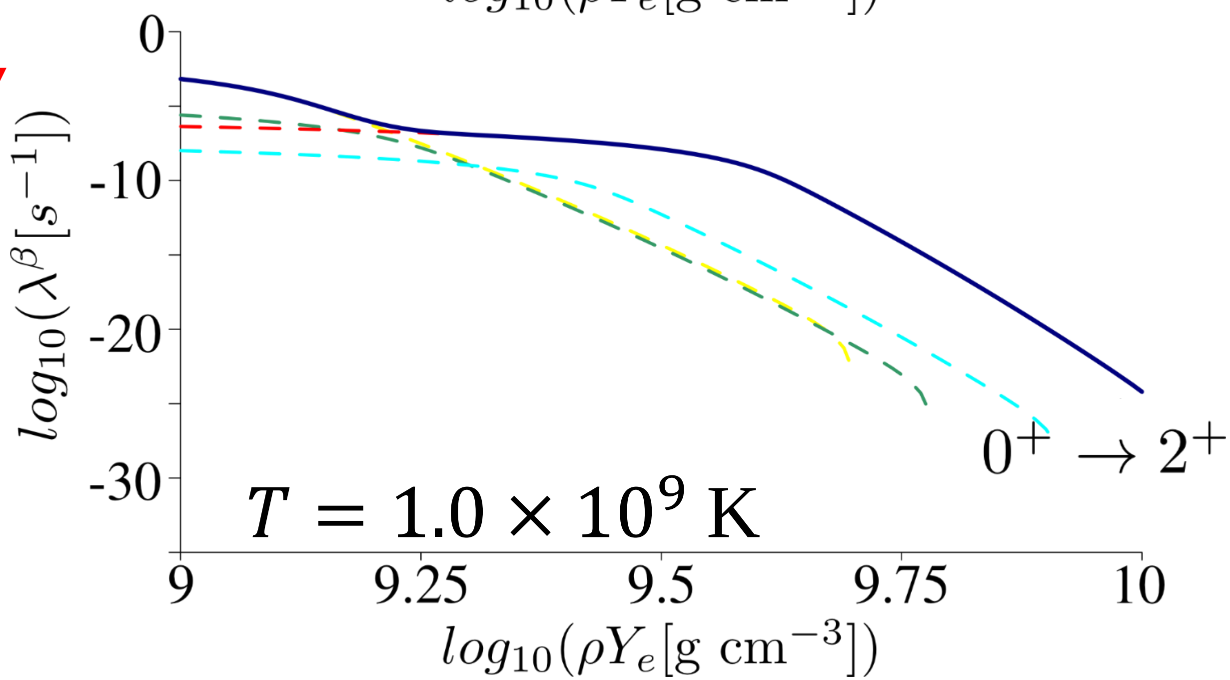


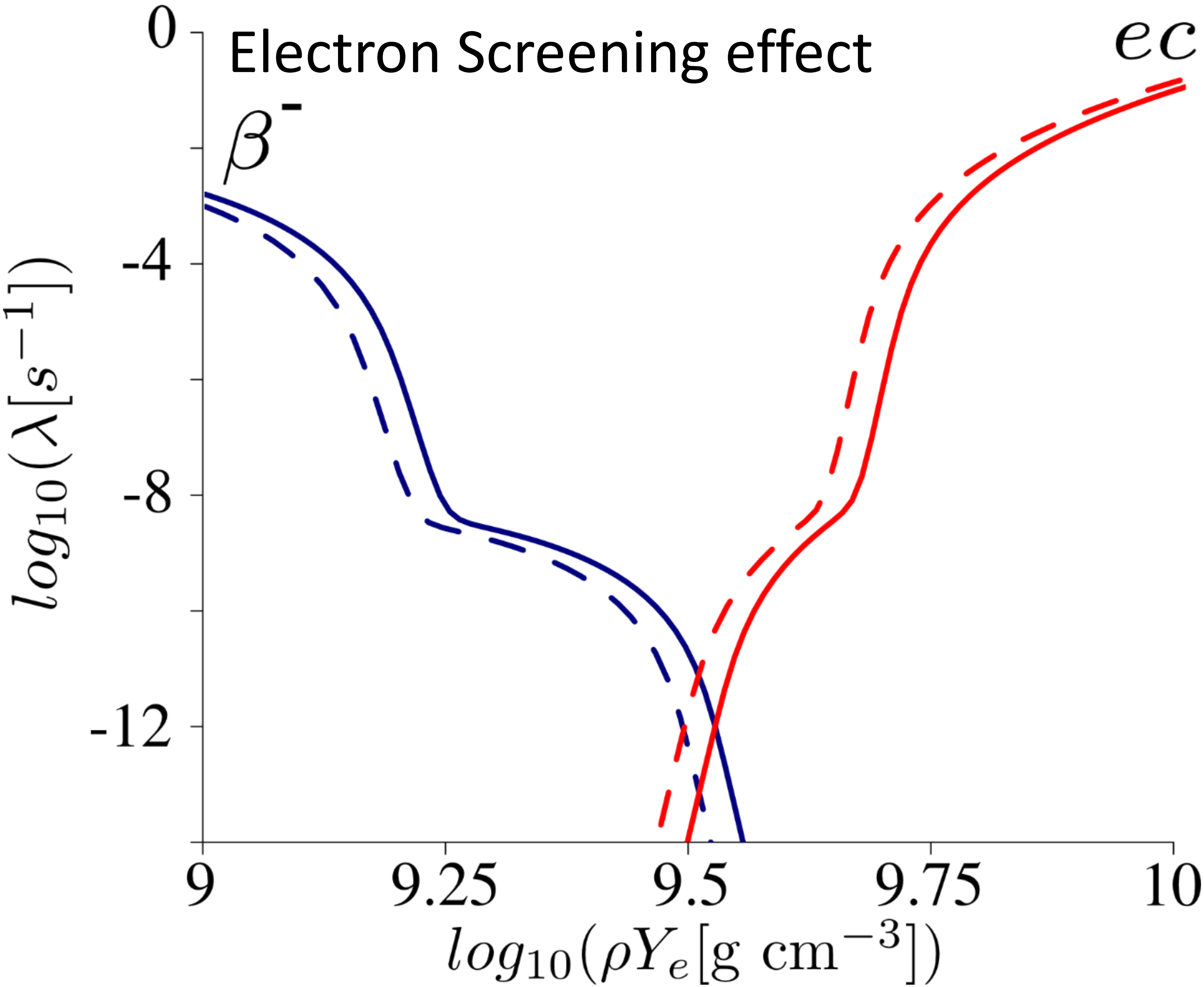
$T = 0.4 \times 10^9$ K
$0^+ \rightarrow 1^+$
$0^+ \rightarrow 2^+$
$2^+ \rightarrow 2^+$
$2^+ \rightarrow 3^+$
0
-10
-20
-30
9
9.25
9.5
9.75
10
$log_{10}(\lambda^{ec}[s^{-1}])$
$log_{10}(\rho Y_e[\mathrm{g\ cm}^{-3}])$
$T = 1.0 \times 10^9$ K
0
-10
-20
-30
9
9.25
9.5
9.75
10
$log_{10}(\lambda^{ec}[s^{-1}])$
$log_{10}(\rho Y_e[\mathrm{g\ cm}^{-3}])$

β^-

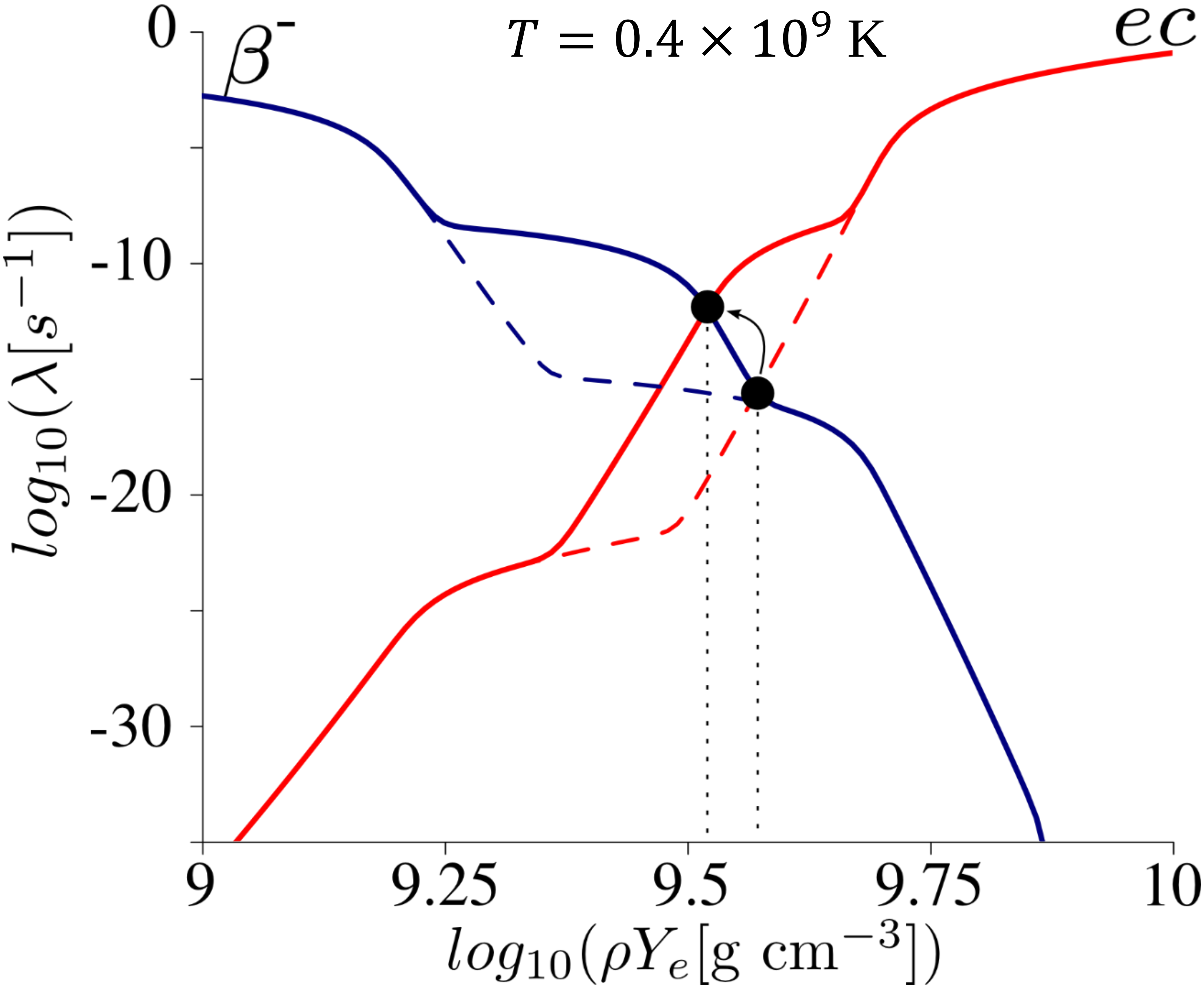
$0^+ \rightarrow 2^+$
dominant until
$T \sim 0.7 \times 10^9$ K



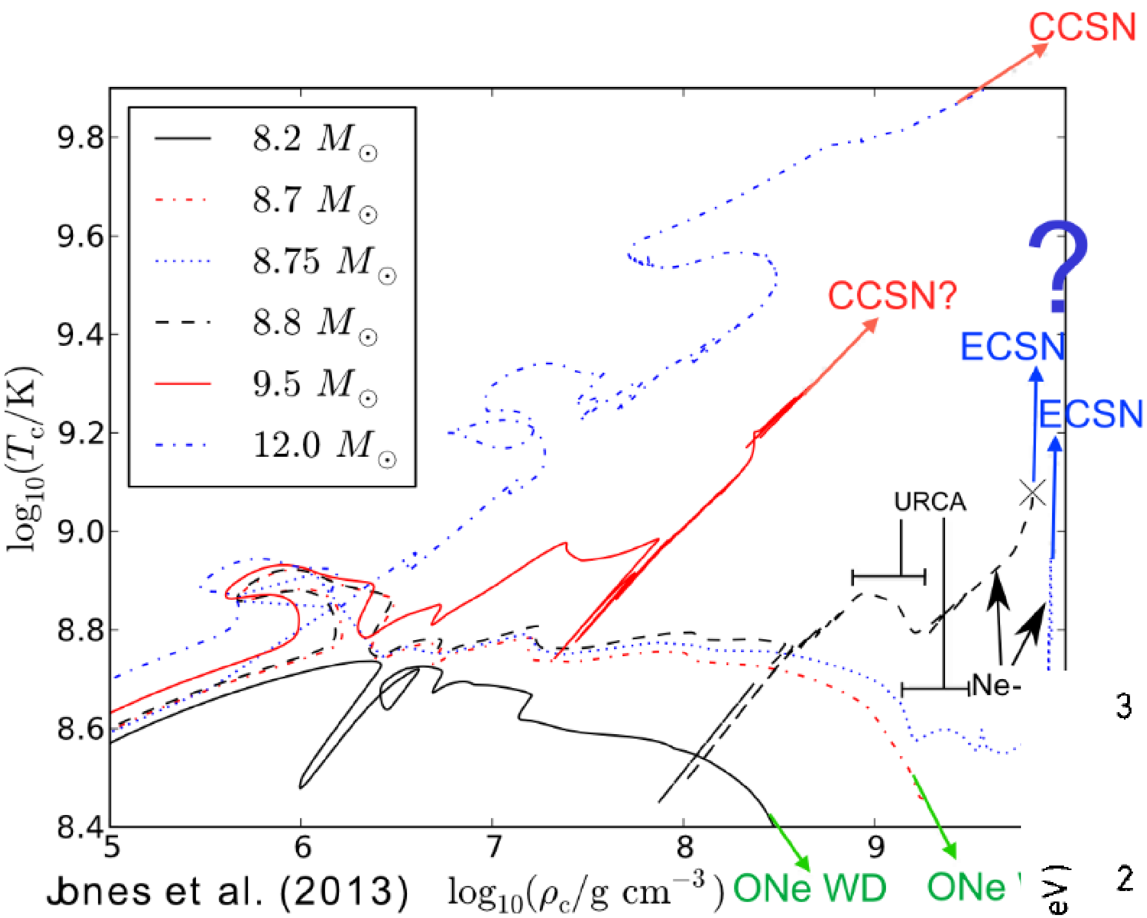




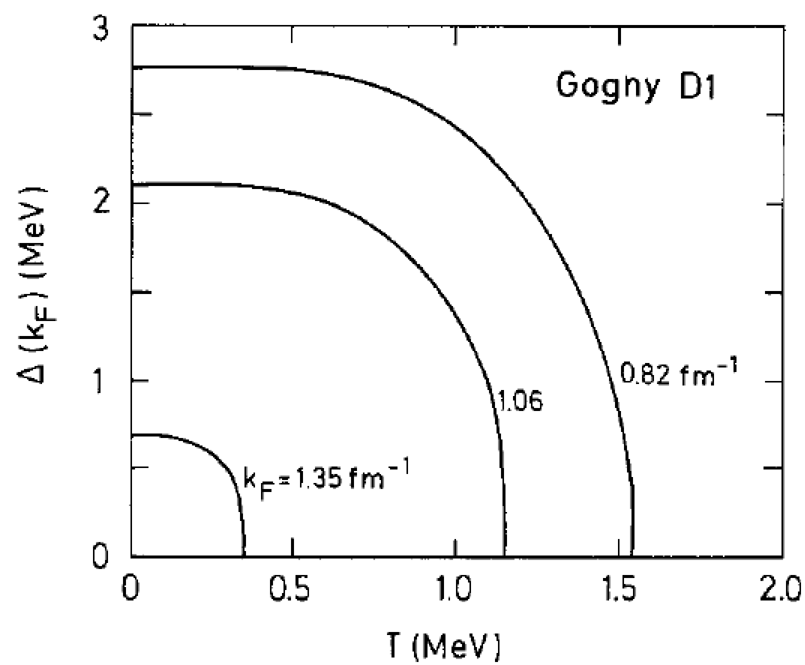
Electron Screening effect
β^-
ec
0
-4
-8
-12
9
9.25
9.5
9.75
10
$log_{10}(\lambda[s^{-1}])$
$log_{10}(\rho Y_e[\mathrm{g\ cm}^{-3}])$



$T = 0.4 \times 10^9$ K
β^-
ec
0
-10
-20
-30
9
9.25
9.5
9.75
10
$log_{10}(\lambda[s^{-1}])$
$log_{10}(\rho Y_e[\mathrm{g\ cm^{-3}}])$

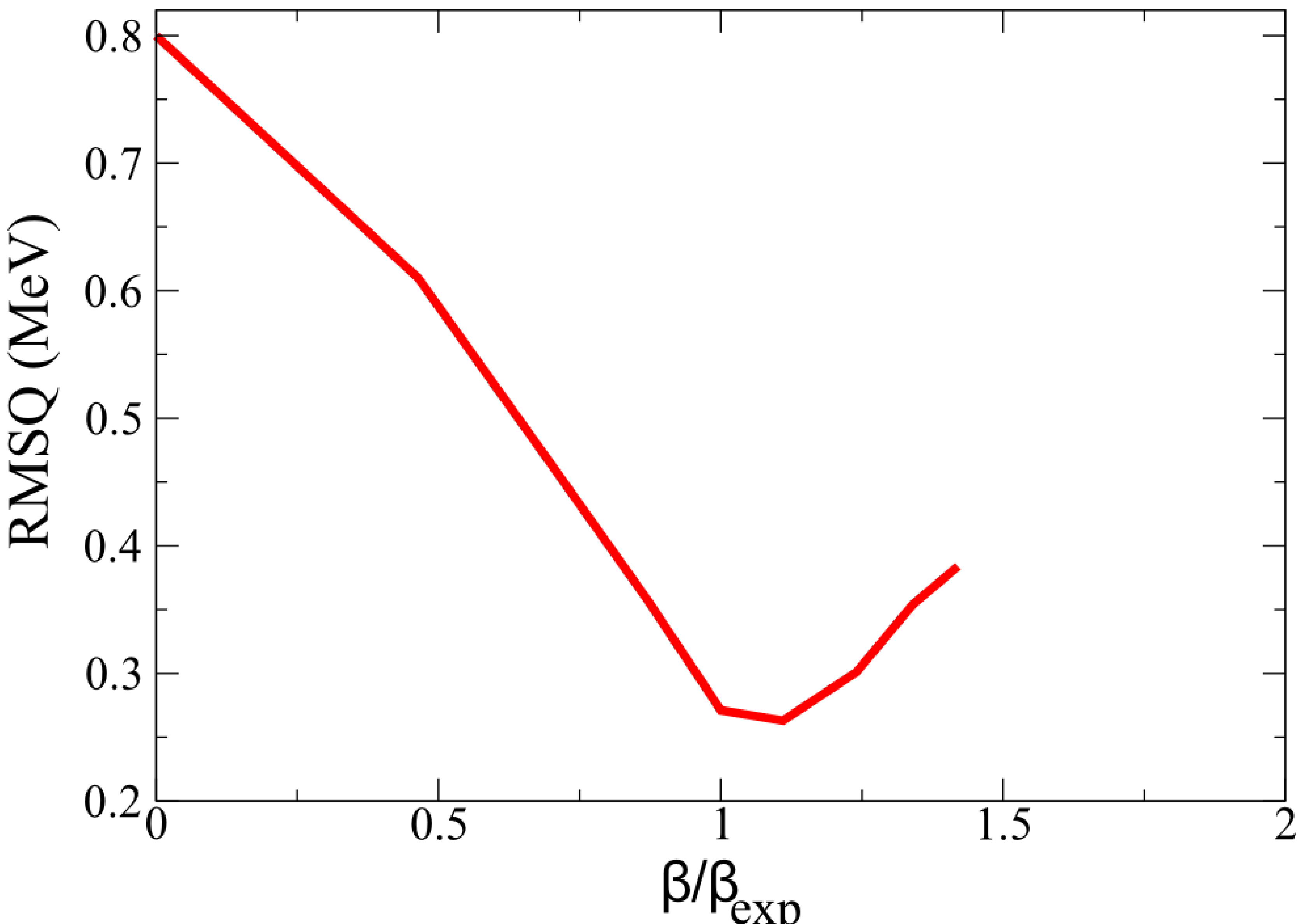


CCSN
9.8
9.6
9.4
9.2
9.0
8.8
8.6
8.4
$\log_{10}(T_c/\mathrm{K})$
$8.2\ M_\odot$
$8.7\ M_\odot$
$8.75\ M_\odot$
$8.8\ M_\odot$
$9.5\ M_\odot$
$12.0\ M_\odot$
?
CCSN?
ECSN
ECSN
URCA
Ne-
5
6
7
8
9
Jones et al. (2013)
$\log_{10}(\rho_c/\mathrm{g\ cm^{-3}})$
ONe WD
ONe



3
2
1
0
Gogny D1
$\Delta(k_F)$ (MeV)
$0.82\ \mathrm{fm}^{-1}$
1.06
$k_F = 1.35\ \mathrm{fm}^{-1}$
0
0.5
1.0
1.5
2.0
T (MeV)

Thank you



RMSQ (MeV)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.5
1
1.5
2
β/β_{exp}

For higher forbidden transitions, we have

$$M_L(k_x, k_\nu^{(1)}) = K_L(p_x R)^{k_x-1}(q_x R)^{k_\nu^{(1)}-1}\{-[(2L+1)/L]^{1/2}\,{}^VF^0_{LL-11} + (2k_x+1)^{-1/2}\alpha Z\,{}^VF^0_{LL0}(k_x, 1, 1, 1)$$
$$+[(2k_x+1)^{-1}(2k_\nu^{(1)}+1)^{-1}q_x R]\,{}^VF^0_{LL0} - (2k_x+1)^{-1}\alpha Z[(L+1)/L]^{1/2}\,{}^AF^0_{LL1}(k_x, 1, 1, 1)$$
$$-[(2k_x+1)^{-1}W_x R + (2k_\nu^{(1)}+1)^{-1}q_x R][(L+1)/L]^{1/2}\,{}^AF^0_{LL1}\}, \quad (2.106a)$$

$$m_L(k_x, k_\nu^{(1)}) = K_L(p_x R)^{k_x-1}(q_x R)^{k_\nu^{(1)}-1}(2k_x+1)^{-1}R\{{}^VF^0_{LL0} - [(L+1)/L]^{1/2}\,{}^AF^0_{LL1}\}, \quad (2.106b)$$

$$M_L(k_x, k_\nu^{(2)}) = -\tilde{K}_L(p_x R)^{k_x-1}(q_x R)^{k_\nu^{(2)}-1}(L+1)^{1/2}[(2k_x-1)(2k_\nu^{(2)}-1)]^{-1/2}\{{}^VF^0_{LL0} + (k_x - k_\nu^{(2)})(L+1)^{-1}[(L+1)/L]^{1/2}\,{}^AF^0_{LL1}\}, \quad (2.106c)$$

$$M_{L+1}(k_x, k_\nu^{(2)}) = -\tilde{K}_L(p_x R)^{k_x-1}(q_x R)^{k_\nu^{(2)}-1}\,{}^AF^0_{(L+1)L1}. \quad (2.106d)$$

$$
{}^{V}\mathfrak{M}^{N}_{KK0}(k_x, m, n, \rho) = \sqrt{2}(2J_i+1)^{-1/2}\Bigg\{G_{KK0}(\kappa_f, \kappa_i) \int_0^\infty g_f(r, \kappa_f)\left(\frac{r}{R}\right)^{K+2N} I(k_x, m, n, \rho; r) g_i(r, \kappa_i) r^2\, dr
$$

$$
+ \operatorname{sign}(\kappa_f)\operatorname{sign}(\kappa_i) G_{KK0}(-\kappa_f, -\kappa_i) \int_0^\infty f_f(r, \kappa_f)\left(\frac{r}{R}\right)^{K+2N} I(k_x, m, n, \rho; r) f_i(r, \kappa_i) r^2\, dr\Bigg\}, \tag{2.84a}
$$

$$
{}^{A}\mathfrak{M}^{N}_{KL1}(k_x, m, n, \rho) = \sqrt{2}(2J_i+1)^{-1/2}\Bigg\{G_{KL1}(\kappa_f, \kappa_i) \int_0^\infty g_f(r, \kappa_f)\left(\frac{r}{R}\right)^{L+2N} I(k_x, m, n, \rho; r) g_i(r, \kappa_i) r^2\, dr
$$

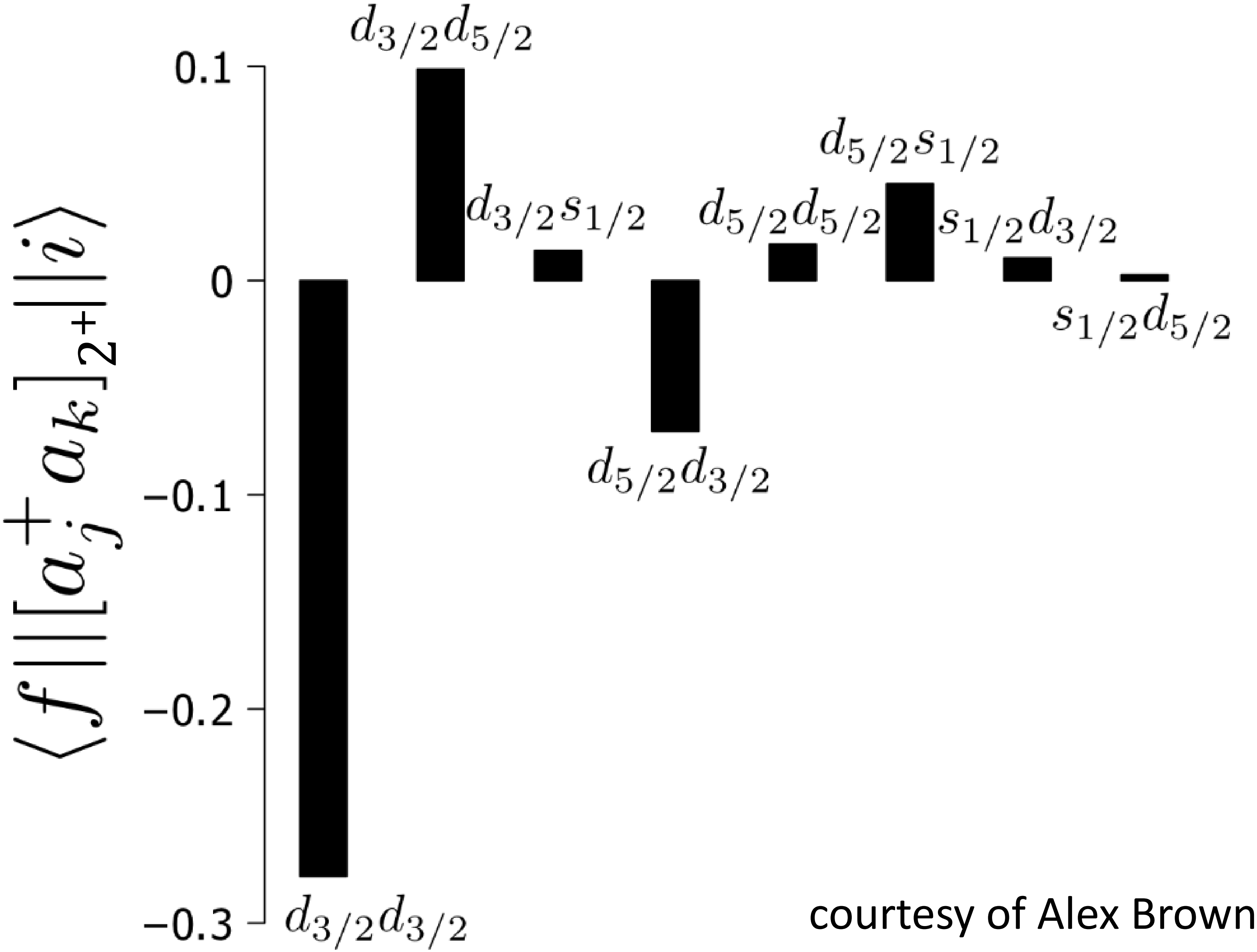
$$
+ \operatorname{sign}(\kappa_f)\operatorname{sign}(\kappa_i) G_{KL1}(-\kappa_f, -\kappa_i) \int_0^\infty f_f(r, \kappa_f)\left(\frac{r}{R}\right)^{L+2N} I(k_x, m, n, \rho; r) f_i(r, \kappa_i) r^2\, dr\Bigg\}, \tag{2.84b}
$$

$$
{}^{A}\mathfrak{M}^{N}_{KK0}(k_x, m, n, \rho) = \sqrt{2}(2J_i+1)^{-1/2}\Bigg\{\operatorname{sign}(\kappa_i) G_{KK0}(\kappa_f, -\kappa_i) \int_0^\infty g_f(r, \kappa_f)\left(\frac{r}{R}\right)^{K+2N} I(k_x, m, n, \rho; r) f_i(r, \kappa_i) r^2\, dr
$$

$$
+ \operatorname{sign}(\kappa_f) G_{KK0}(-\kappa_f, \kappa_i) \int_0^\infty f_f(r, \kappa_f)\left(\frac{r}{R}\right)^{K+2N} I(k_x, m, n, \rho; r) g_i(r, \kappa_i) r^2\, dr\Bigg\}, \tag{2.84c}
$$

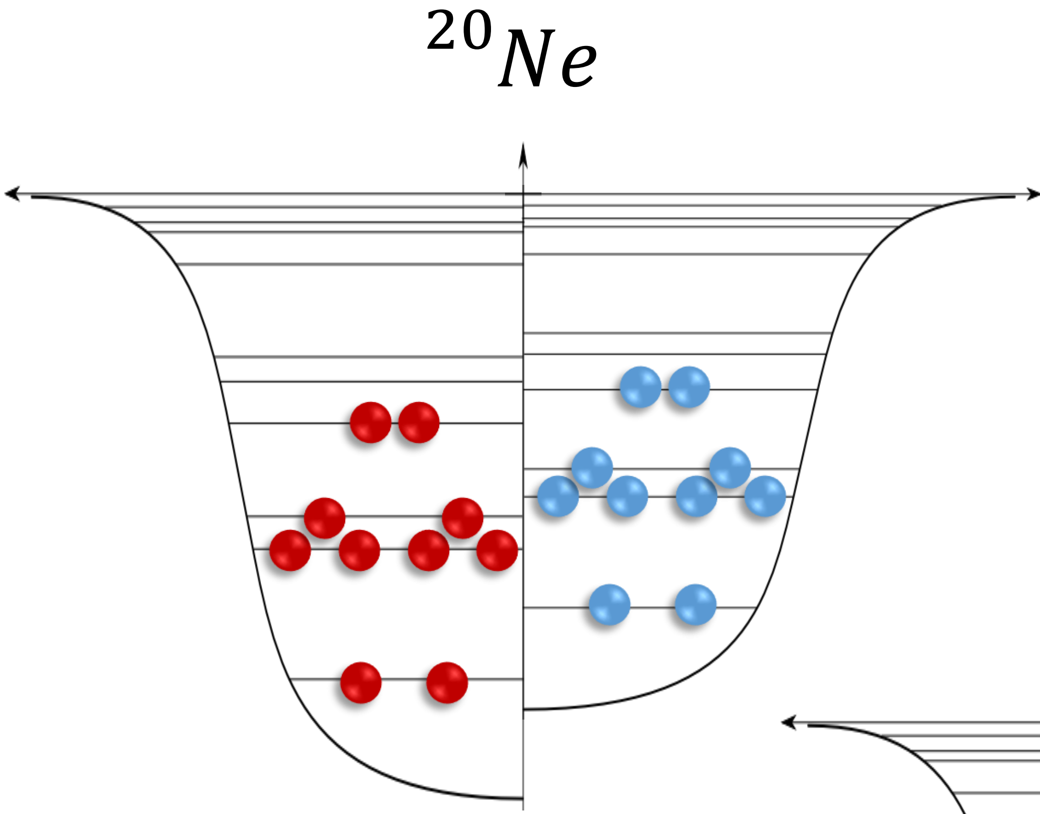
$$
{}^{V}\mathfrak{M}^{N}_{KL1}(k_x, m, n, \rho) = \sqrt{2}(2J_i+1)^{-1/2}\Bigg\{\operatorname{sign}(\kappa_i) G_{KL1}(\kappa_f, -\kappa_i) \int_0^\infty g_f(r, \kappa_f)\left(\frac{r}{R}\right)^{L+2N} I(k_x, m, n, \rho; r) f_i(r, \kappa_i) r^2\, dr
$$

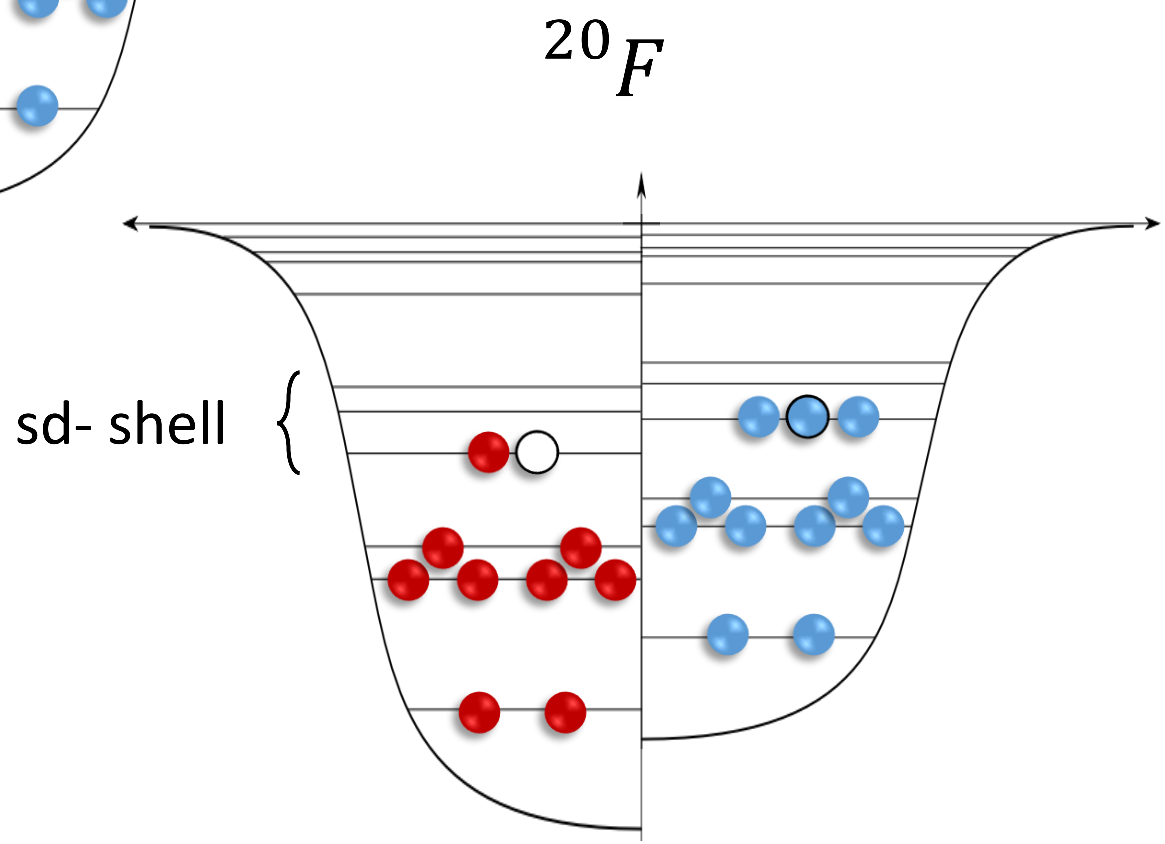
$$
+ \operatorname{sign}(\kappa_f) G_{KL1}(-\kappa_f, \kappa_i) \int_0^\infty f_f(r, \kappa_f)\left(\frac{r}{R}\right)^{L+2N} I(k_x, m, n, \rho; r) g_i(r, \kappa_i) r^2\, dr\Bigg\}. \tag{2.84d}
$$



$\langle f || [a_j^+ a_k]_{2^+} || i \rangle$
0.1
0
−0.1
−0.2
−0.3
$d_{3/2}d_{5/2}$
$d_{3/2}s_{1/2}$
$d_{5/2}d_{5/2}$
$d_{5/2}s_{1/2}$
$s_{1/2}d_{3/2}$
$s_{1/2}d_{5/2}$
$d_{5/2}d_{3/2}$
$d_{3/2}d_{3/2}$
courtesy of Alex Brown

Shell Model

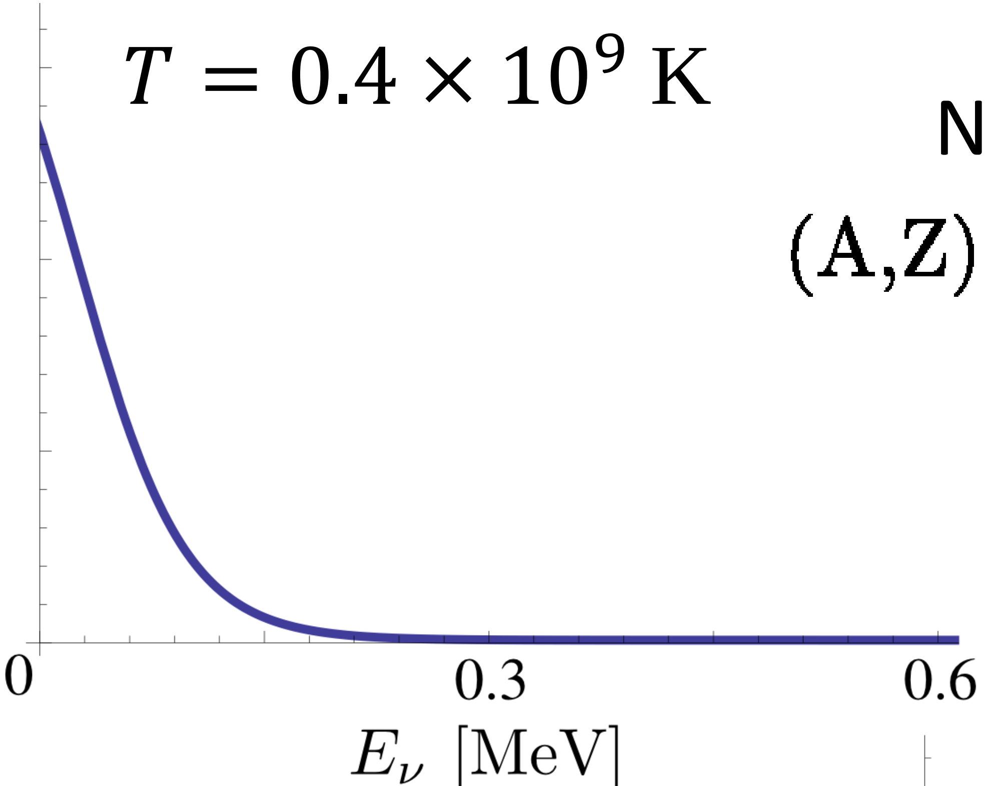




Neutrino EC-Spectrum

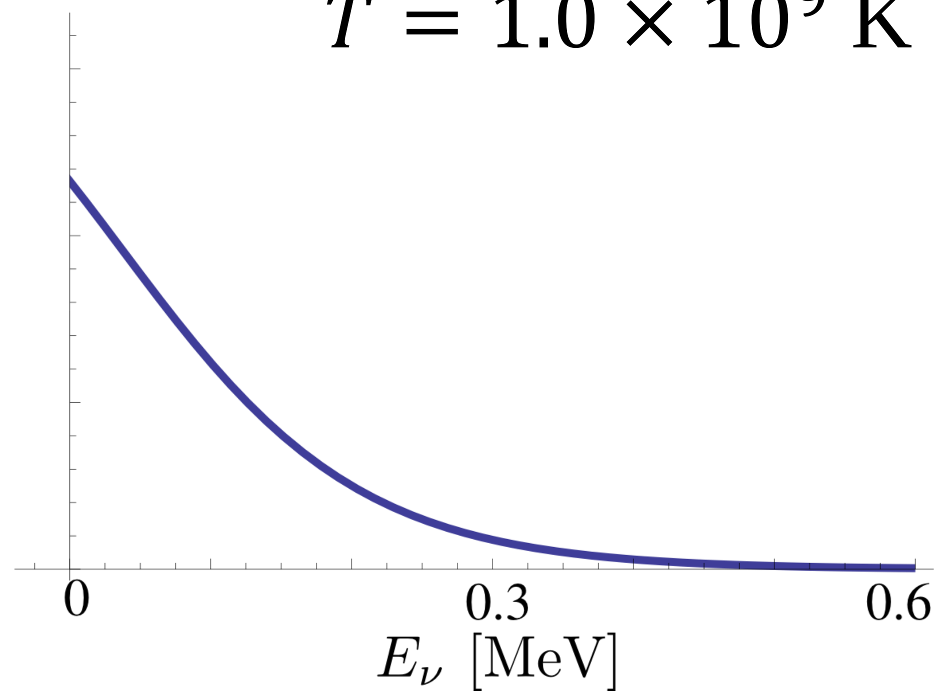
$$(\mathbf{A},\mathbf{Z}) + e^- = (\mathbf{A},\mathbf{Z}\text{-}\mathbf{1}) + \nu_e$$

$T = 0.4 \times 10^9$ K

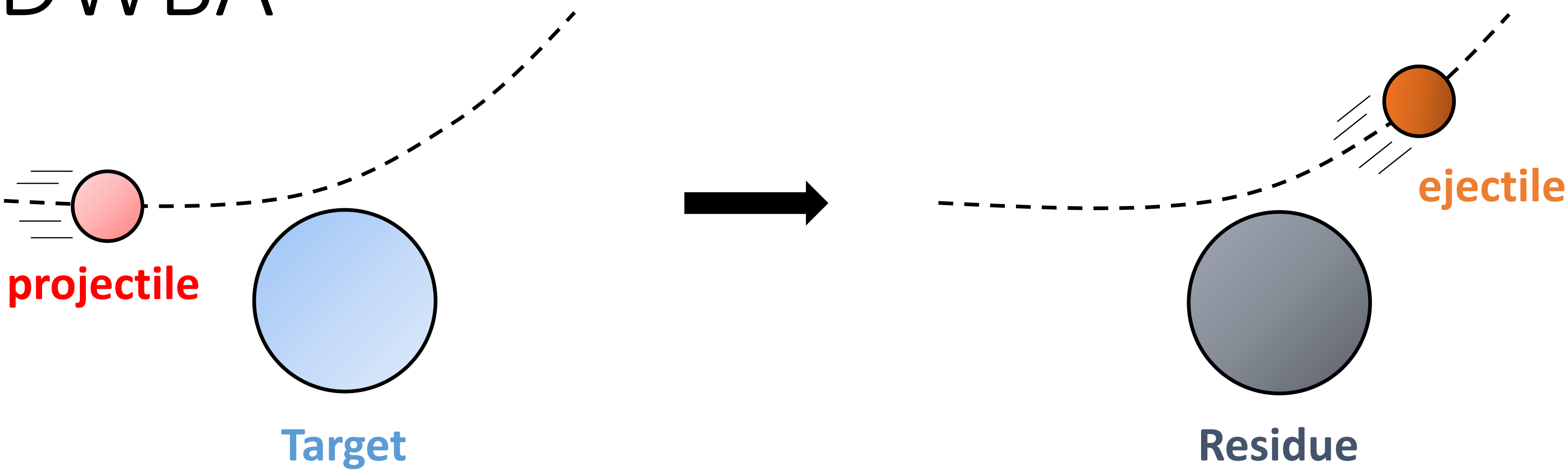


$\rho Y_e = 10^{9.5} g/cm^3$

$T = 1.0 \times 10^9$ K



DWBA



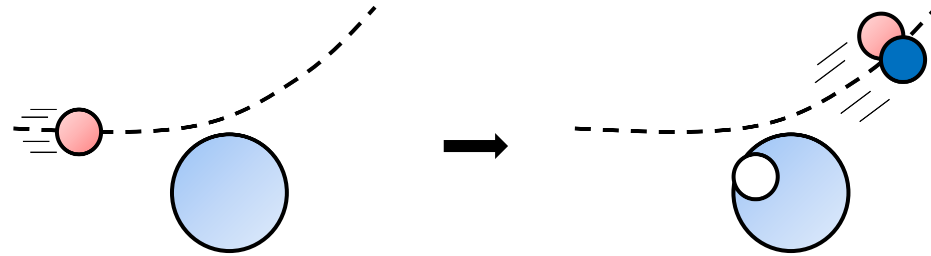
Entrance Channel **Exit Channel**

$$\frac{d\sigma}{d\Omega} \sim \left| \left\langle \phi_\alpha(\xi_\alpha)\, \chi_\alpha(r_\alpha) \middle| V_{int} \middle| \phi_\beta(\xi_\beta) \chi_\beta(r_\beta) \right\rangle \right|^2$$

Internal Structure Wavefunction **Relative Motion: Distorted Wave**

DWBA approximate the entrance channel as a factorization of internal and relative coordinates, consider relative motion as Distorted Plain Wave, and calculate matrix element between this approximated ***<intial| and |final>*** state.

DWBA



cross section is

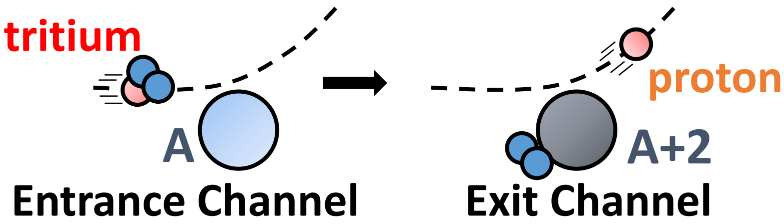
$$\frac{d\sigma}{d\Omega} = \frac{k_f}{k_i}\frac{\mu_i\mu_f}{4\pi^2\hbar^4}\frac{1}{(2J_A+1)(2J_a+1)}$$

$$\times \sum_{\substack{M_A,M_a\\M_B,M_b}} \left| \sum_{m_i,m_f} \langle J_b\ j_i\ M_b\ m_i | J_a\ M_a\rangle \langle J_A\ j_f\ M_A\ m_f | J_B\ M_B\rangle T_{m_i,m_f} \right|^2 .$$

The transition amplitude T_{m_i,m_f} is

$$T_{m_i,m_f} = \sum_\sigma \int d\mathbf{r}_f d\mathbf{r}_{An} \chi^{(-)*}(\mathbf{r}_f) \psi_{m_f}^{j_f*}(\mathbf{r}_{An},\sigma) V(r_{bn}) \psi_{m_i}^{j_i}(\mathbf{r}_{bn},\sigma) \chi^{(+)}(\mathbf{r}_i).$$

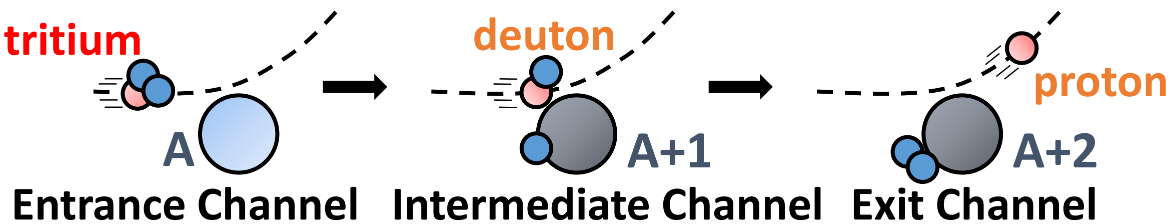
2-particle transfer DWBA



Ist order DWBA:

Simultaneus transfer

$$\langle (A,t)|V_{int}|(A+2,p)\rangle$$



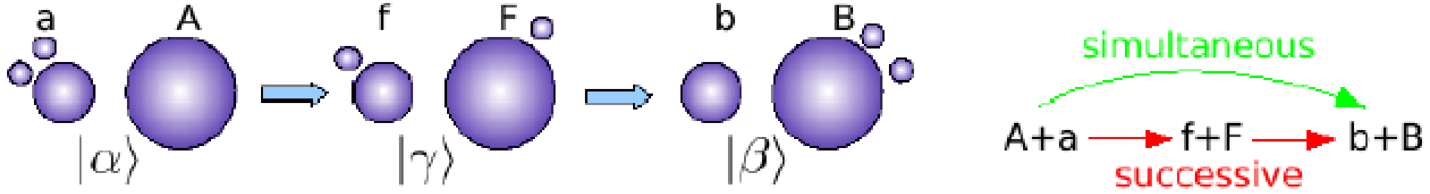
IInd order DWBA:

Successive transfer

We need the structure information to calculate the correlation between the two transferred neucleons, so the probability of 1 neutron in the target and 1 in the ejectile, in the intermediate state

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for the two–neutron transfer (^{1}H(^{11}Li,^{9}Li)^{3}H) reaction



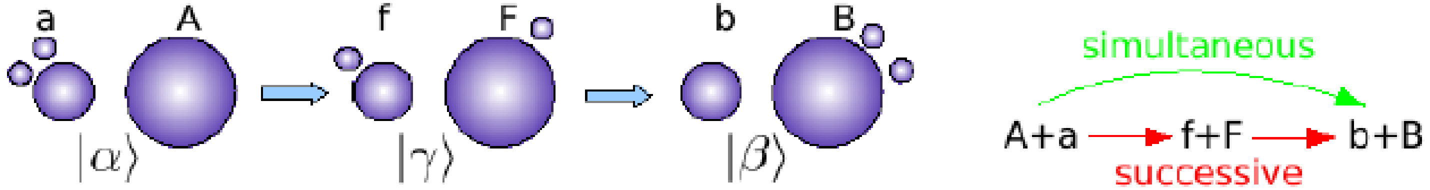
Simultaneous transfer

$$T^{(1)}(j_i,j_f) = 2\sum_{\sigma_1\sigma_2}\int d\mathbf{r}_{fF}\,d\mathbf{r}_{b1}\,d\mathbf{r}_{A2}[\Psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\Psi^{j_f}(\mathbf{r}_{A2},\sigma_2)]_0^{0*}\chi_{bB}^{(-)*}(\mathbf{r}_{bB})$$

$$\times\, v(\mathbf{r}_{b1})[\Psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\Psi^{j_i}(\mathbf{r}_{b2},\sigma_2)]_0^{0}\chi_{aA}^{(+)}(\mathbf{r}_{aA})$$

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for the two–neutron transfer ($^1H(^{11}Li,^9Li)^3H$) reaction

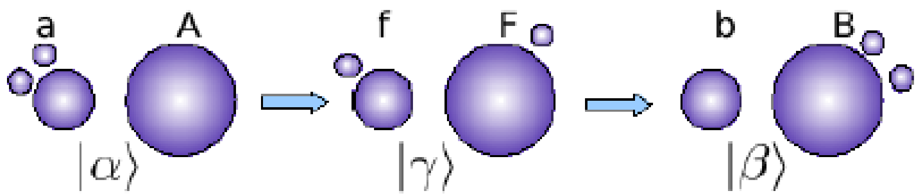


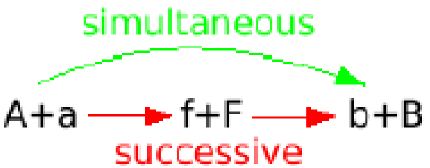
Successive transfer

$$T^{(2)}_{succ}(j_i,j_f) = 2\sum_{K,M}\sum_{\substack{\sigma_1\sigma_2\\ \sigma_1'\sigma_2'}}\int d\mathbf{r}_{fF}\,d\mathbf{r}_{b1}\,d\mathbf{r}_{A2}[\Psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\Psi^{j_f}(\mathbf{r}_{A2},\sigma_2)]^{0*}_{0}$$
$$\times\,\chi^{(-)*}_{bB}(\mathbf{r}_{bB})v(\mathbf{r}_{b1})[\Psi^{j_f}(\mathbf{r}_{A2},\sigma_2)\Psi^{j_i}(\mathbf{r}_{b1},\sigma_1)]^K_M$$
$$\times\int d\mathbf{r}'_{fF}\,d\mathbf{r}'_{b1}\,d\mathbf{r}'_{A2}\,G(\mathbf{r}_{fF},\mathbf{r}'_{fF})[\Psi^{j_f}(\mathbf{r}'_{A2},\sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)]^K_M$$
$$\times\,\frac{2\mu_{fF}}{\hbar^2}v(\mathbf{r}'_{f2})[\Psi^{j_i}(\mathbf{r}'_{A2},\sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1},\sigma'_1)]^0_0\chi^{(+)}_{aA}(\mathbf{r}'_{aA})$$

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for the two–neutron transfer ($^{1}H(^{11}Li,^{9}Li)^{3}H$) reaction

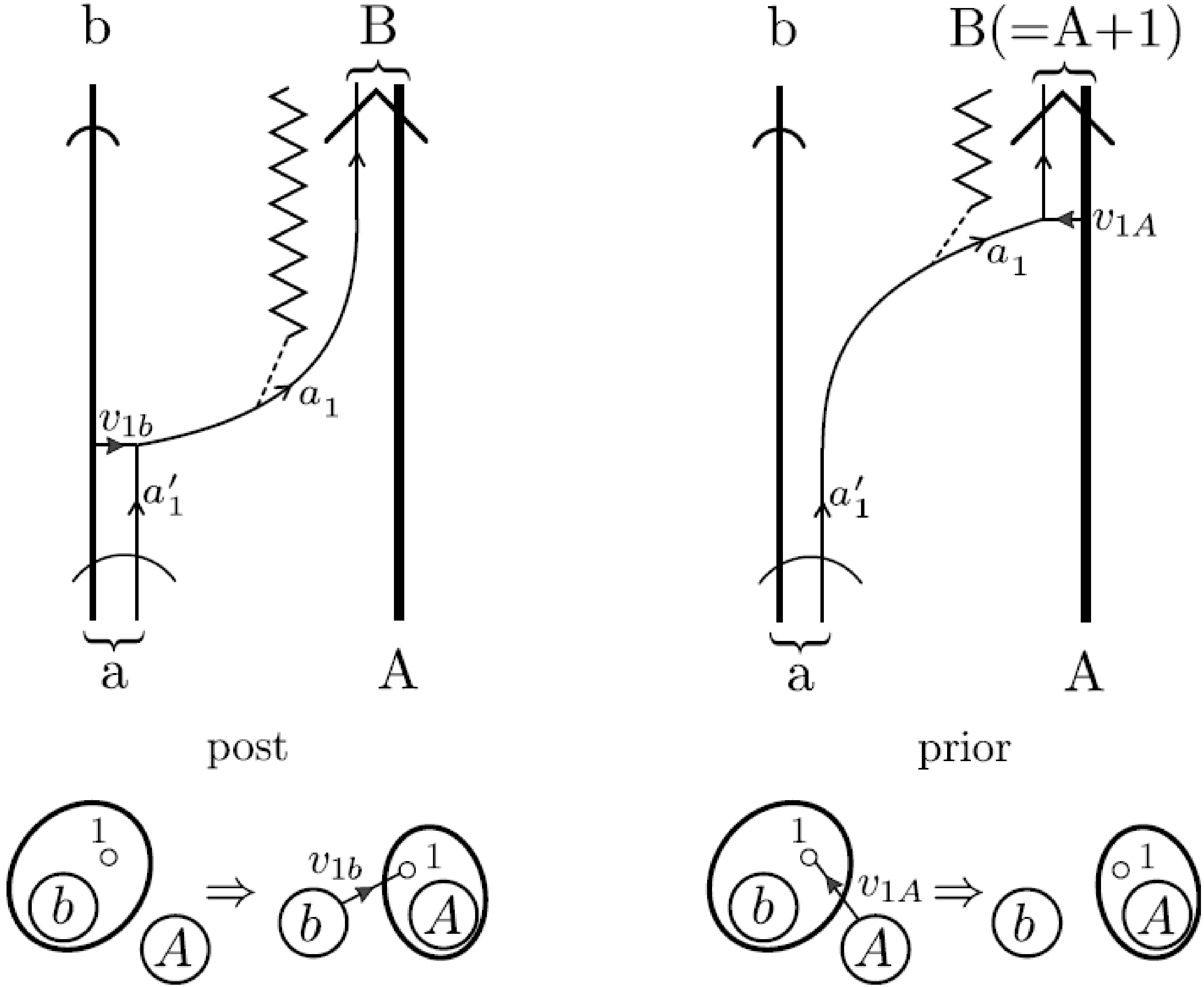


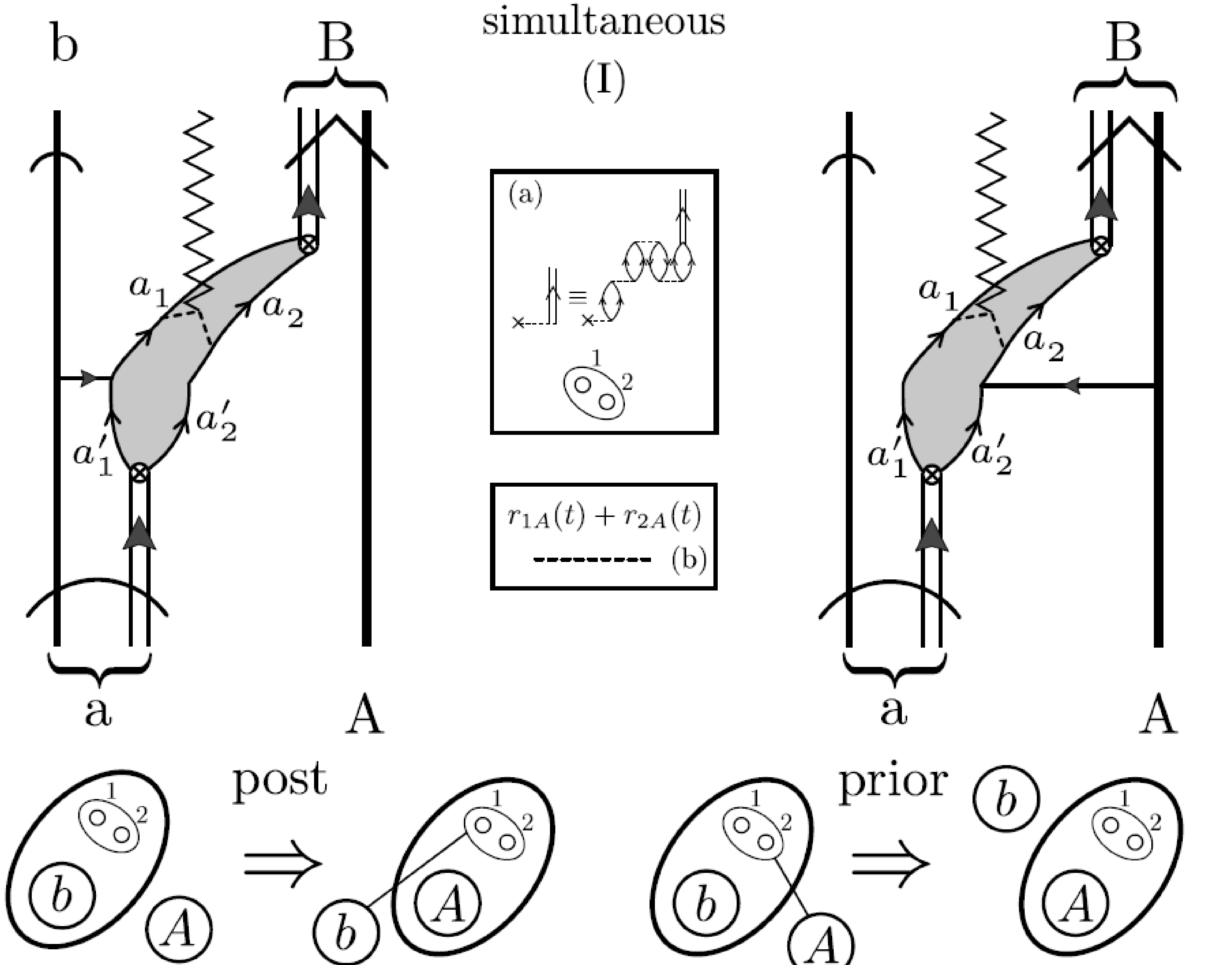


Non–orthogonality term

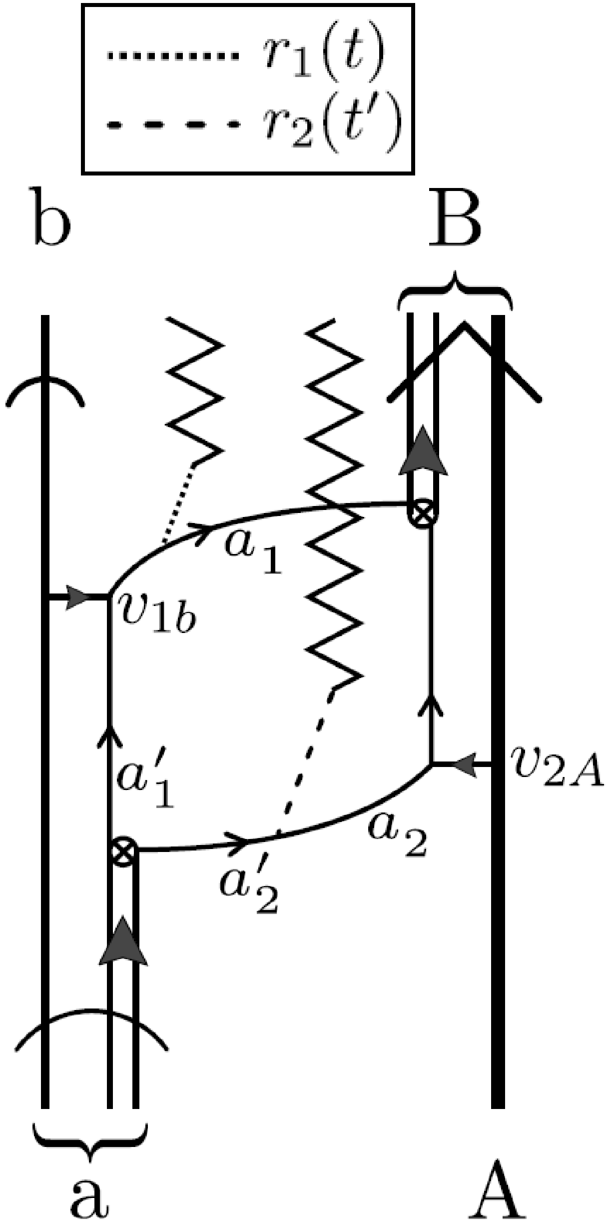
$$
\begin{aligned}
T^{(2)}_{NO}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1\sigma_2 \\ \sigma'_1\sigma'_2}} \int & d\mathbf{r}_{fF}\, d\mathbf{r}_{b1}\, d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_{0} \\
& \times \chi^{(-)*}_{bB}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]^{K}_{M} \\
& \times \int d\mathbf{r}'_{b1}\, d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^{K}_{M} \\
& \times [\Psi^{j_i}(\mathbf{r}'_{A2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^{0}_{0} \chi^{(+)}_{aA}(\mathbf{r}'_{aA})
\end{aligned}
$$

one-particle transfer

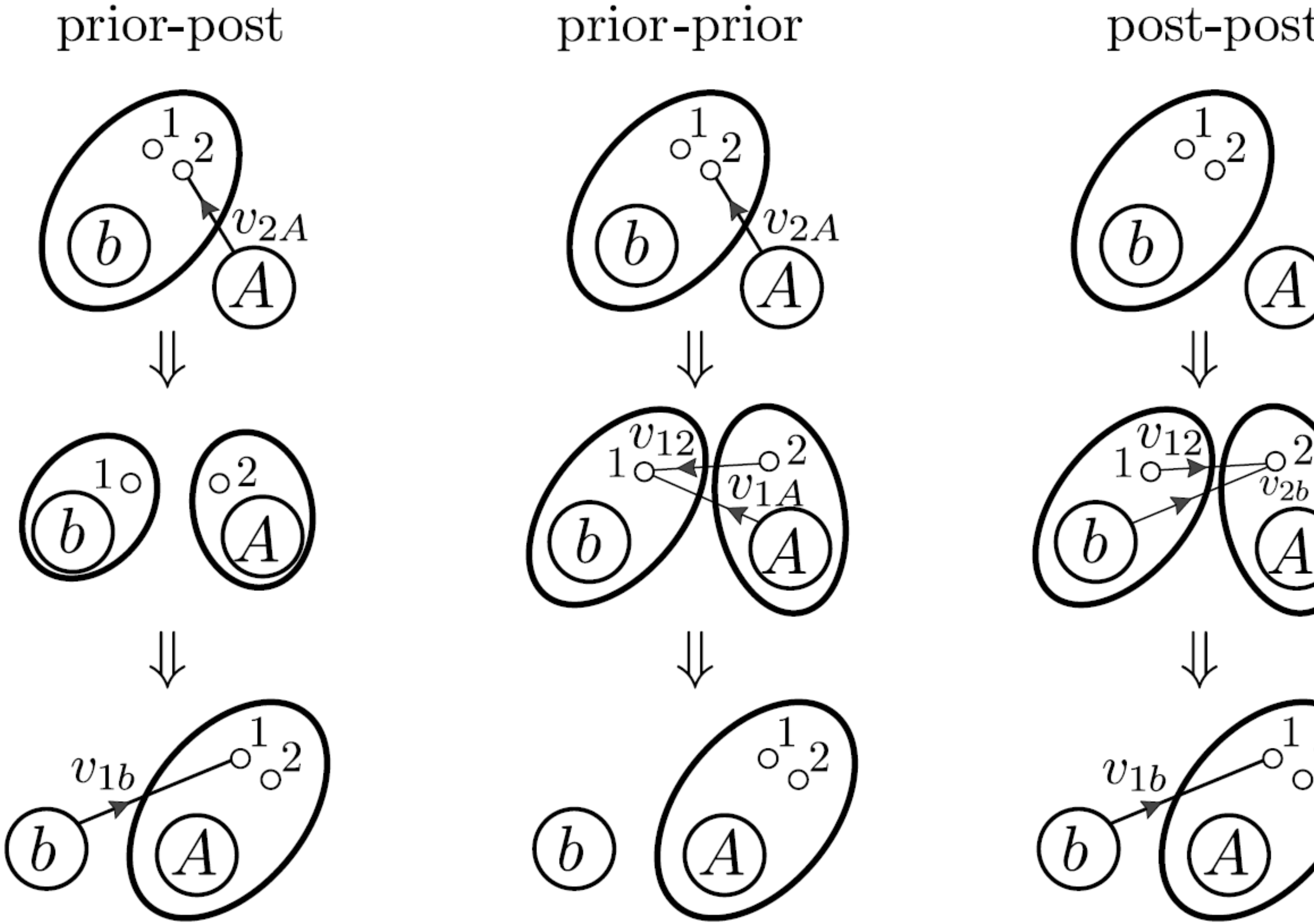


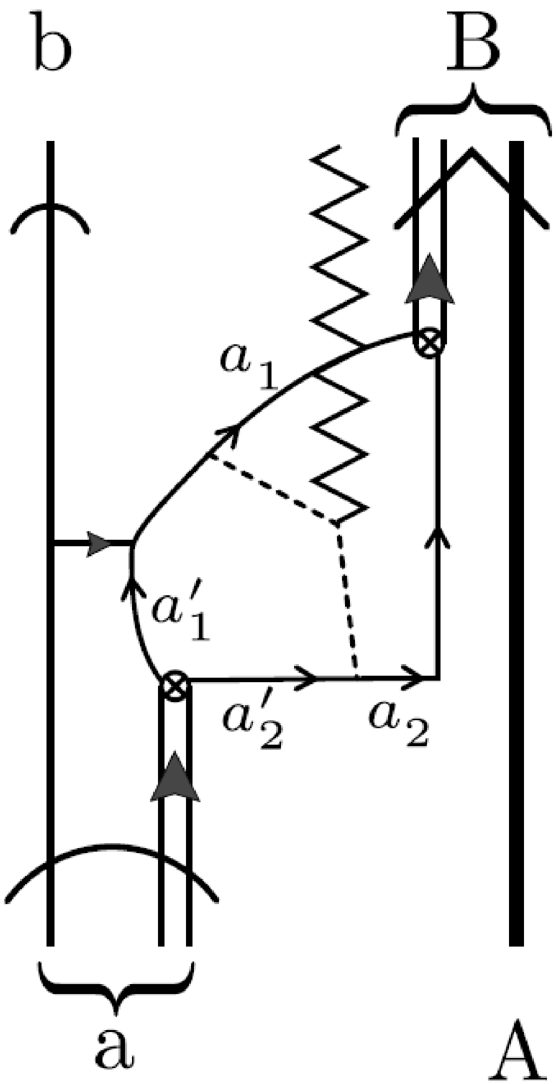


simultaneous
(I)
b
B
a_1
a_2
a'_1
a'_2
a
A
(a)
1
2
$r_{1A}(t) + r_{2A}(t)$
(b)
post
prior
b
A



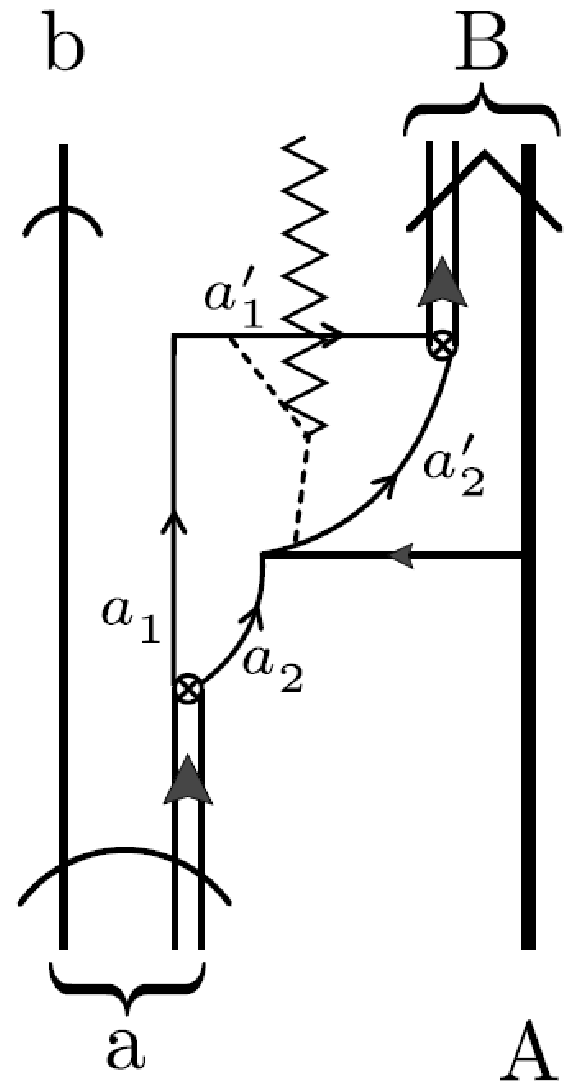
$r_1(t)$
$r_2(t')$
successive
(III)
b
B
a
A
a_1
v_{1b}
a'_1
a'_2
a_2
v_{2A}
v_{12}
v_{1A}
v_{2b}


prior-post
prior-prior
post-post
1
2
b
A
v_{2A}
v_{12}
v_{1A}
v_{2b}
v_{1b}



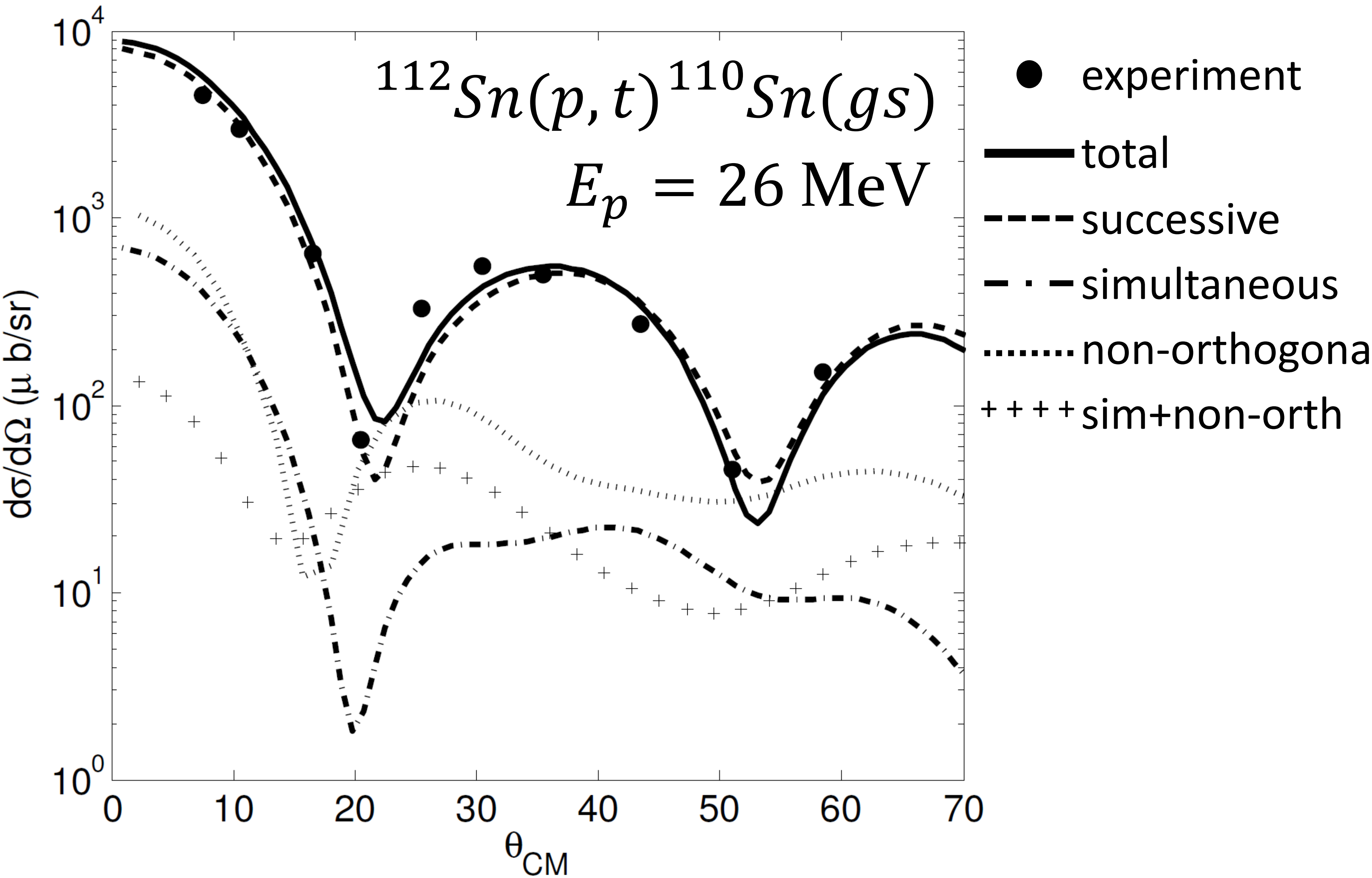
$$\langle \varphi^{(b)}_{a'_1} | \varphi^{(A)}_{a_1} \rangle \neq 0$$

$$\langle \varphi^{(b)}_{a'_2} | \varphi^{(A)}_{a_2} \rangle \neq 0$$



(II)

non-orthogonality



$^{112}Sn(p,t)^{110}Sn(gs)$
$E_p = 26$ MeV
experiment
total
successive
simultaneous
non-orthogonal
sim+non-orth
10^4
10^3
10^2
10^1
10^0
dσ/dΩ (μ b/sr)
0
10
20
30
40
50
60
70
θ_{CM}