Three-Nucleon Forces and Exotic Oxygen and Calcium Isotopes









Drip Lines and Magic Numbers: The Nuclear Landscape Toward the Extremes

Exploring the frontiers of nuclear science:

Worldwide joint experimental/theoretical effort

What are the properties of proton/neutron-rich matter?

What are the limits of nuclear existence? 82

How do magic numbers form and evolve?

neutrons

Advances in many-body methods



Coupled Cluster (Hagen, Papenbrock, Dean, Roth) In-Medium SRG (Bogner, Hergert, JDH, Schwenk) Many-Body Perturbation Theory (JDH, Hjorth-Jensen, Schwenk) Self-Consistent Green's Function (Barbieri, Soma, Duguet)

3N forces essential for exotic nuclei







Oxygen Isotopes

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References

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- Otsuka, Suzuki, JDH, Schwenk, Akaishi PRL 105, 032501 (2010)
- JDH, Menendez, Schwenk, EPJA **49**, 39 (2013)
- Caesar et al. (R3B), Simonis, JDH, Menendez, Schwenk PRC 88, 034313 (2013)
- Bogner, Hergert, JDH, Schwenk et al., PRL, 113, 142501 (2014)



Key physics problems:

- Location of dripline
- Properties of new closed-shell nuclei ^{22,24}O
- Physics beyond the neutron dripline

The Nuclear Many-Body Problem

Nuclei understood as many-body system starting from closed shell, add nucleons Calculate **valence-space** Hamiltonian inputs from nuclear forces **Interaction matrix elements Single-particle energies (SPEs)**



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- \pm 1) Effective interaction: sum excitations outside valence space to 3rd order
- \star 2) Single-particle energies calculated self consistently
 - 3) Harmonic-oscillator basis of 13-15 major shells: converged
 - 4) NN and 3N forces from chiral $EFT to 3^{rd}$ -order MBPT
 - 5) Explore extended valence spaces



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Clear convergence with HO basis size Promising order-by-order behavior

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G-matrix – no signs of convergence (similar in pf-shell)

Chiral Effective Field Theory: Nuclear Forces



Nucleons interact via pion exchanges and contact interactions

Consistent treatment of NN, 3N,...

3N couplings fit to properties of light nuclei at low momentum

Improve convergence of many-body methods:

$$V_{{
m low}\,k}$$
 or $V_{{
m SRG}}$

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meissner,...

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NN matrix elements

- Chiral N³LO (Machleidt, $\Lambda_{NN} = 500$ MeV); smooth-regulator $V_{low k}(\Lambda)$

3N force contributions

- Chiral N²LO

c_D, c_E fit to properties of light nuclei with $V_{\text{low }k}$ ($\Lambda = \Lambda_{3N} = 2.0 \text{ fm}^{-1}$)

- Included to 5 major HO shells

- 1) Effective interaction: sum excitations outside valence space to 3rd order
- 2) Single-particle energies calculated self consistently
- 3) Harmonic-oscillator basis of 13-15 major shells: converged
- 4) NN and 3N forces from chiral $EFT to 3^{rd}$ -order MBPT
- **★**5) Explore **extended valence spaces**

Philosophy: diagonalize in largest possible valence space (where orbits relevant)



Treats higher orbits nonperturbatively When important for exotic nuclei?

Limits of Nuclear Existence: Oxygen Anomaly



Limits of Nuclear Existence: Oxygen Anomaly



Mass Number A

3N Forces for Valence-Shell Theories

Normal-ordered 3N: contribution to valence-space Hamiltonian

Effective one-body

Effective two-body



Combine with NN (Third Order): no empirical adjustments

Oxygen Anomaly



Otsuka, Suzuki, JDH, Schwenk, Akaishi, PRL (2010)

Ground-State Energies of Oxygen Isotopes

Valence-space interaction and SPEs from NN+3N



JDH, Menendez, Schwenk, EPJA (2013)

Repulsive character improves agreement with experiment *sd*-shell results underbound; improved in **extended space**

Impact on Spectra: ²³O

Neutron-rich oxygen spectra with NN+3N

 $5/2^+$, $3/2^+$ energies reflect ^{22,24}O shell closures



Experimental Connection: Beyond the Dripline

Hoffman, Kanungo, Lunderberg... PRLs (2008+)

Valence-space Hamiltonian from NN + 3N + residual 3N



Repulsion more pronounced for neutron-rich systems: 400 keV at ²⁶O Improved agreement with new data beyond ²⁴O dripline Future: include coupling to continuum

In-Medium SRG: Basics

In-Medium SRG applies continuous unitary transformation to drive offdiagonal physics to zero Tsukiyama, Bogner, Schwenk, PRL (2011)

$$H(s) = U(s)HU^{\dagger}(s) \equiv H^{d}(s) + H^{od}(s) \rightarrow H^{d}(\infty)$$

Decouples reference state from excitations $\langle npnh | H(\infty) | \Phi_c \rangle = 0$



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$$H(s) = U(s)HU^{\dagger}(s) \equiv H^{d}(s) + H^{od}(s) \rightarrow H^{d}(\infty)$$

Where U is defined by the generator:

 $\eta(s) \equiv (dU(s)/ds)U^{\dagger}(s)$ chosen for desired decoupling behavior

Taking

$$\eta(s) = \left[H^{d}(s), H(s)\right] = \left[H^{d}(s), H^{od}(s)\right]$$

Drives H^{od} to 0 (Wegner, 1994)

Closed-shell reference state: drives all n-particle n-hole couplings to 0 $\frac{dH(s)}{ds} = \left[\eta(s), H(s)\right] \qquad \langle npnh | H(\infty) | \Phi_c \rangle = 0$

IM-SRG for Valence-Space Hamiltonians

In-Medium SRG applies continuous unitary transformation to drive offdiagonal physics to zero Tsukiyama, Bogner, Schwenk, PRC (2012)

Open shell systems:

split particle states into valence states, v, and those above valence space, qRedefine "off-diagonal" to exclude valence particles



 $H(s=0) \rightarrow H(\infty)$

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 $H(s = 0) \rightarrow H(\infty)$

Defines new effective valence-space Hamiltonian $H_{\rm eff}$ States outside valence space are decoupled

- 1) Effective interaction: nonperturbative from IM-SRG
- 2) Single-particle energies: nonperturbative from IM-SRG
- 3) Hartree-Fock basis of $e_{\text{max}} = 2n + l = 14$ **converged**
- \star 4) NN and 3N forces from chiral EFT
 - 5) Explore extended valence spaces in progress

NN matrix elements

- Chiral N³LO (Machleidt, Λ_{NN} = 500MeV); free-space SRG evolution
- Cutoff variation $\lambda_{\text{SRG}} = 1.88 2.24 \text{ fm}^{-1}$
- -Vary $\hbar \omega = 20 24 \text{MeV}$
- Consistently include 3N forces induced by SRG evolution (NN+3N-ind)

Initial 3N force contributions

- Chiral N²LO Λ_{3N} = 400MeV (NN+3N-full)
- Included with cut: $e_1 + e_2 + e_3 \leq E_{3\max} = 14$

IM-SRG Oxygen Ground-State Energies

Valence-space interaction and SPEs from IM-SRG in *sd* shell



NN+3N-induced reproduce exp well, not dripline NN+3N-full modestly overbound – good behavior past dripline Good dripline properties Very weak $\hbar\omega$ dependence

IM-SRG Oxygen Ground-State Energies

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Comparison with Large-Space Methods

Large-space methods with same SRG-evolved NN+3N forces



Clear improvement with full NN+3N Confirms valence-space results Remarkable agreement with same forces



Dripline Mechanism

Compare to large-space methods with same SRG-evolved NN+3N forces



Robust mechanism driving dripline behavior

3N repulsion raises $d_{3/2}$, lessens decrease across shell Similar to initial MBPT NN+3N calculations in oxygen

IM-SRG Oxygen Spectra

Oxygen spectra: extended-space MBPT and IM-SRG



Clear improvement with NN+3N-full IM-SRG: comparable with phenomenology

IM-SRG Oxygen Spectra

Oxygen spectra: extended-space MBPT and IM-SRG



Clear improvement with NN+3N-full

Continuum neglected: expect to lower $d_{3/2}$

IM-SRG Oxygen Spectra

Oxygen spectra: IM-SRG predictions beyond the dripline



²⁴O closed shell (too high 2^+)

Continuum neglected: expect to lower spectrum Only one excited state in ²⁶O below 6.5MeV

Experimental Connection: ²⁶O Spectrum

Oxygen spectra: IM-SRG predictions beyond the dripline



New measurement at RIKEN on excited states in ²⁶O

Existence of excited state "just over 1.0 MeV" (uncertainty not finalized)

Towards Full sd-Shell with MBPT: Fluorine

Next challenge: valence protons + neutrons

Neutron-rich fluorine and neon



sd shell filled at 29 F/ 30 Ne

Need extended-space orbits

Towards Full sd-Shell with MBPT: Fluorine

Next challenge: valence protons + neutrons

Neutron-rich fluorine and neon



NN only: severe overbinding

NN+3N: good experimental agreement through 29 F Sharp increase in ground-state energies beyond 29 F: incorrect dripline

Towards Full sd-Shell with MBPT: Neon

Next challenge: valence protons + neutrons

Neutron-rich fluorine and neon



Similar behavior in Neon isotopes

Revisit cross-shell valence space theory – **non-degenerate valence spaces** Tsunoda, Hjorth-Jensen, Otsuka IM-SRG energies overbound in F/Ne

Experimental Connection: ²⁴F Spectrum

Fluorine spectra: extended-space MBPT and (sd-shell) IM-SRG



New measurements from GANIL

IM-SRG: comparable with phenomenology in good agreement with new data

²⁵Ne Spectrum

Neon spectra: extended-space MBPT and (sd-shell) IM-SRG



Limited experimental data

IM-SRG: comparable with phenomenology, good agreement with data

Calcium Isotopes



Calcium Isotopes: Magic Numbers



GXPF1: Honma, Otsuka, Brown, Mizusaki (2004) KB3G: Poves, Sanchez-Solano, Caurier, Nowacki (2001)



Phenomenological Forces

Large gap at ⁴⁸Ca
Discrepancy at N=34

Microscopic NN Theory

Small gap at ⁴⁸Ca

N=28: first standard magic

number not reproduced
in microscopic NN theories

Calcium Ground State Energies and Dripline

Signatures of shell evolution from ground-state energies?



No clear dripline; flat behavior past ⁵⁴Ca – Halos beyond ⁶⁰Ca?

 $S_{2n} = -[BE(N,Z) - BE(N-2,Z)] \text{ sharp decrease indicates shell closure}$ $\Delta_n^{(3)} = \frac{(-1)^N}{2} [BE(N+1,Z) + BE(N-1,Z) - 2BE(N,Z)] \text{ peak indicates shell closure}$

Two-Neutron Separation Energies: Mass of 52Ca

Compare with AME2003 data



NN+3N Predictions

Reproduce ⁴⁸Ca shell closure

Predictions too bound past ⁵⁰Ca

Experimental Connection: Mass of 52Ca

New mass measurements of ^{51,52}Ca at **TITAN**: Penning trap experiment





TITAN Measurement

⁵²Ca mass 1.75MeV *more* bound than AME2003 value

NN+3N Predictions

Confirmed with new measurements

Good reproduction of pairing gaps

Pairing for Shell Evolution N=28



Peak in pairing gaps: complementary signature for shell closure Compare with 2^+ energies for Ca Agreement with CC throughout chain Hagen et al. PRL (2012)

N=28 strong peak

Pairing for Shell Evolution N=32



Peak in pairing gaps: complementary signature for shell closure Compare with 2^+ energies for Ca Agreement with CC throughout chain Hagen et al. PRL (2012) N=28 strong peak N=32 moderate peak Close to data with new TITAN value Experimental measurement of ⁵³Ca mass needed to reduce uncertainty

Experimental Connection: Mass of 54Ca

New precision mass measurement of ^{53,54}Ca at **ISOLTRAP**: multi-reflection ToF





ISOLTRAP *Measurement* Sharp decrease past ⁵²Ca Unambiguous closed-shell ⁵²Ca Test predictions of various models

MBPT NN+3N

Excellent agreement with new data Reproduces closed-shell ^{48,52}Ca Weak closed sell signature past ⁵⁴Ca

Experimental Connection: Mass of 54Ca

New precision mass measurement of ^{53,54}Ca at **ISOLTRAP**: multi-reflection ToF



Pairing for Shell Evolution N=34



Peak in pairing gaps: complementary signature for shell closure Compare with 2⁺ energies for Ca Agreement with CC throughout chain Hagen et al. PRL (2012)

N=28 strong peak

N=32 moderate peak

N=34 weak signature

3N forces suppress closed-shell feature

JDH, Menendez, Schwenk, JPG (2013)

Neutron-Rich Ca Spectra Near N=34

Neutron-rich calcium spectra with NN+3N



JDH, Menendez, Schwenk, JPG (2013) JDH, Menendez, Simonis, Schwenk, PRC (2014)

Phenomenology: inconsistent predictions

NN+3N: signature of new *N*=34 magic number (also predicted in CC theory) **Agrees with new measurements from RIKEN**

Steppenbeck et al., Nature (2013)

Neutron-Rich Ca Spectra Near N=34

Neutron-rich calcium spectra with NN+3N



Steppenbeck et al., Nature (2013)

New Directions and Outlook

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Heavier semi-magic chains: Nickel and Tin

Ab initio valence-shell Hamiltonians

Towards full sd- and pf-shells Revisit cross-shell theory

Moving beyond stability

Continuum effects near driplines

Islands of inversion in sd/pf regions Exotic cores

 $\mathbf{28}$

neutrons

20

50

Map driplines in sd region?

protons

8

2

28

20

8

2

Fundamental symmetries

Non-empirical calculation of 0vββ decay Effective electroweak operators WIMP-nucleus scattering

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