Hall Viscosity of Hierarchical Quantum Hall States

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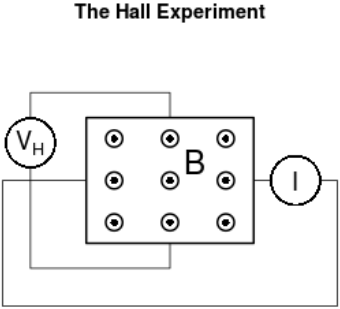
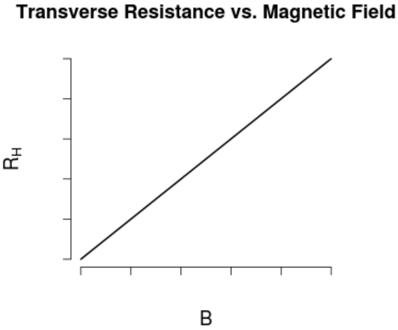
Outline

The Hall Effect

- Edwin Hall: 1879
- Transverse resistance: $R_H = \frac{V_H}{I} \propto B$.

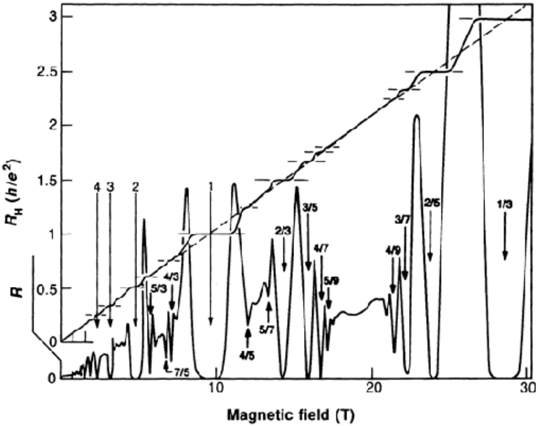




The Quantum Hall Effect

- 2D interface, low temperatures, clean samples, high magnetic fields

  Klitzing, Dorda & Pepper 80'



Stormer 92'

- Plateaus at $R_H = \frac{1}{\nu}\frac{h}{e^2}$, with a rational number $\nu = \frac{p}{q}$

Electrons in a Magnetic Field: Landau Levels

- Landau Hamiltonian

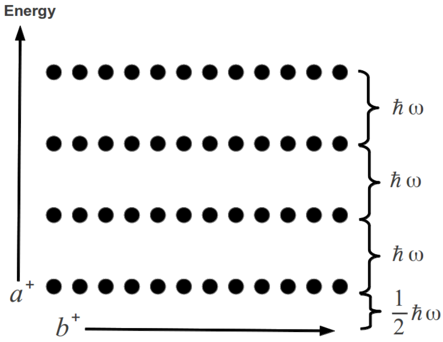
$$H = \frac{1}{2m} \sum_{j=x,y} (\hat{p}_j - eA_j)^2$$

- Harmonic oscillator solution

$$H = \hbar\omega(a^\dagger a + \frac{1}{2})$$

$$\omega = \frac{eB}{mc}, \; \ell_B = \sqrt{\frac{\hbar}{eB}}$$

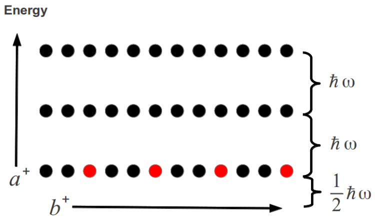
- Each energy level is called a Landau Level (LL)



Many-body Hamiltonian for the FQHE

- Fractional Quantum Hall Effect:
  Rational filling, $\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9} \dots$
  Flat bands $\Rightarrow$ Kinetic energy zero.

$$H_{\mathrm{Int}} = \sum_{i<j} V\left(\mathbf{r}_i - \mathbf{r}_j\right)$$



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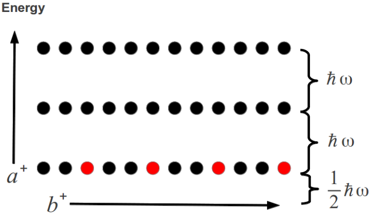
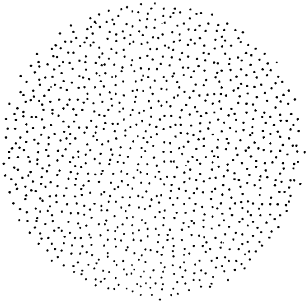
- No small parameter!

  Numerically hard, as the electrons are strongly interacting





Trial Wave Function for the FQHE

- Use representative wave functions for particular ν.
- Laughlin: $\nu = \frac{1}{q}$

Laughlin 83'

$$\psi_{\frac{1}{q}} = e^{-\frac{1}{4}\sum_i |z_i|^2} \prod_{i<j} (z_i - z_j)^q$$

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- Moore-Read: $\nu = 5/2$

Moore & Read 91'

$$\psi_{\mathrm{MR}} = e^{-\frac{1}{4}\sum_i |z_i|^2} \prod_{i<j} (z_i - z_j)^2 \times \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)$$

- Typically fractionally charged excitations and anyonic statistics!

Trial Wave Function for the FQHE II

- A whole zoo of different wave functions.
- Two layer version of Moore-Read: $\nu = 5/2$

Cappelli, Georgiev & Todorov 01'

$$\psi_{\frac{5}{2}} = \mathcal{A}\, e^{-\frac{1}{4}\sum_i \left(|z_i|^2 + |w_i|^2\right)} \prod_{i,j} (z_i - w_j) \times \prod_{i<j} (z_i - z_j)^3 \prod_{i<j} (w_i - w_j)^3$$

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- Composite fermions: $\nu = \frac{2}{5}$

Jain 89'

$$\psi_{\frac{2}{5}} = \mathcal{A}\, e^{-\frac{1}{4}\sum_i\left(|z_i|^2+|w_i|^2\right)} \prod_i \partial_{z_i} \prod_{i,j}(z_i - w_j)^2 \times \prod_{i<j}(z_i - z_j)^3 \prod_{i<j}(w_i - w_j)^3$$

Testing Wave Functions

- What kind of test can I perform on my trial wave functions?
- Compute energy expectation value

$$\langle E \rangle = \int_V d^N\mathbf{r}\,\psi^\star(\mathbf{r})V(\mathbf{r})\psi(\mathbf{r})$$

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- But, two trial wave functions Ψ_A and Ψ_B can not necessarily be compared directly.

The Shift

- Filling fraction is a thermodynamic quantity

$$\nu = \frac{N_e}{N_\Phi} \qquad N_e, N_\Phi \to \infty$$

- For finite systems

$$\nu \left(N_\phi + \mathcal{S}\right) = N_e \qquad \text{Shift } \mathcal{S}!$$

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- The shift is a topological property of a FQH state.

Filled lowest LL	$\nu = 1$	$\mathcal{S} = 1$
Laughlin	$\nu = \frac{1}{q}$	$\mathcal{S} = q$
Composite Fermions	$\nu = \frac{m}{2m+1}$	$\mathcal{S} = 2 + m$

The Sphere and Shift II

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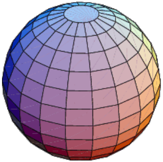
- ψ_A and ψ_B at different shifts can *not* be compared directly, since N_ϕ and N_e do not match!
- Solution: Choose geometry where $\mathcal{S} = 0$.
  Shift is manifest though the quantum Hall viscosity η

A Flat Geometry

- Sphere:



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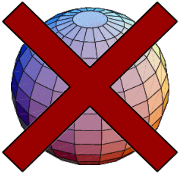

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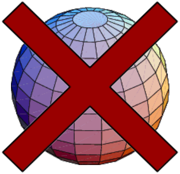

- Plane/Disc:

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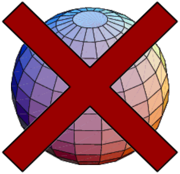

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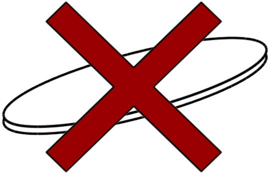

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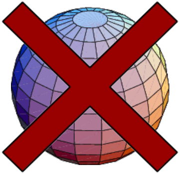

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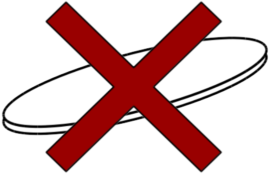

- Torus: Flat geometry $\Rightarrow \mathcal{S} = 0$
- Easy access to geometry parameter τ

Ordinary Viscosity

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  Liquid nitrogen: (at 77 K)
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  Pitch tar: $\eta = 2.3 \cdot 10^8$ Pa·s.

Ordinary Viscosity

- Viscosity η and elastic modulus λ, relate stress tensor σ to the strain u, and the strain rate $\dot{u}$.

$$\sigma_{\alpha,\beta} = \sum_{\gamma,\delta} \lambda_{\alpha,\beta,\gamma,\delta} u_{\gamma,\delta} + \sum_{\gamma,\delta} \eta_{\alpha,\beta,\gamma,\delta} \dot{u}_{\gamma,\delta}$$

$\alpha, \beta, \gamma, \delta = 1, \ldots, d$ in d dimensions.

- The viscosity tensor may be divided as

$$\eta = \eta^S + \eta^A$$

where

$$\begin{aligned} \eta^S_{\alpha,\beta,\gamma,\delta} &= \eta^S_{\gamma,\delta,\alpha,\beta} \\ \eta^A_{\alpha,\beta,\gamma,\delta} &= -\eta^A_{\gamma,\delta,\alpha,\beta} \end{aligned}$$

Normal Viscosity and Dissipation

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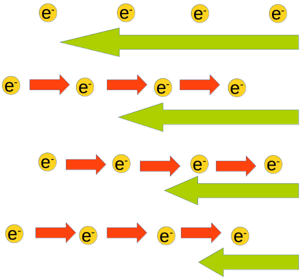
  ... and in ordinary three dimensions (or higher) that is natural.
- The familiar viscous dissipation comes from the symmetric tensor η^S.
  Force and displacement are parallel $\Rightarrow$ dissipation



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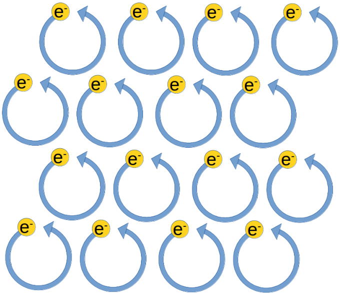
  ... like in the quantum Halls system.



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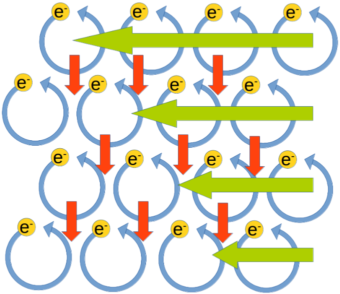

- Now, force and displacement are orthogonal, and no dissipation may occur.
- η^A carries information about the shift $\mathcal{S}$.

Analytic Quantum Hall Viscosity

Viscosity has been computed for:

- The filled lowest Landau level: $\eta = \frac{1}{4}\hbar\bar{n}$

Avron, Seiler & Zograf 95'

$\mathcal{S} = 1$

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  Read 09'

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  Read 09'

  $\mathcal{S} = q.$

- The (conjectured) general relation is

  Read 09'

$$\eta = \frac{1}{4}\hbar\bar{n}\mathcal{S}$$

- Viscosity can distinguish between different states.

Recapitulation

- Sphere:

  Curved geometry $\Rightarrow \mathcal{S} \neq 0$

- Plane/Disc:

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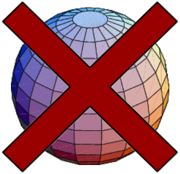

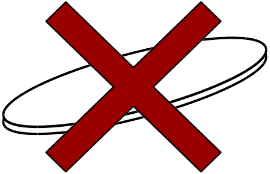

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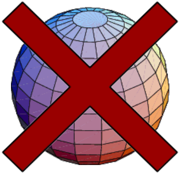

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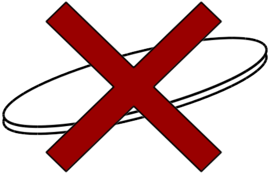


- Torus: Flat geometry $\Rightarrow \mathcal{S} = 0$
  Easy access to geometry parameter τ

The Mathematical Torus

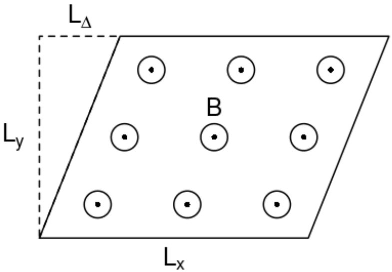
- The torus is periodic in two directions: L_x and $\tau L_x = L_\Delta + \imath L_y$.
- Torus area: $2\pi N_s \ell^2 = L_x L_y$.
- Torus geometry:

$$\tau = \tau_1 + \imath\tau_2 = \frac{L_\Delta + \imath L_y}{L_x}$$

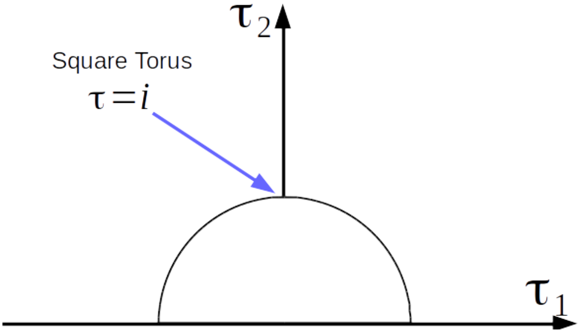
τ_2 : Aspect ratio

τ_1 : Skewness ratio

N_s: Number of flux quanta



The Complex τ-plane



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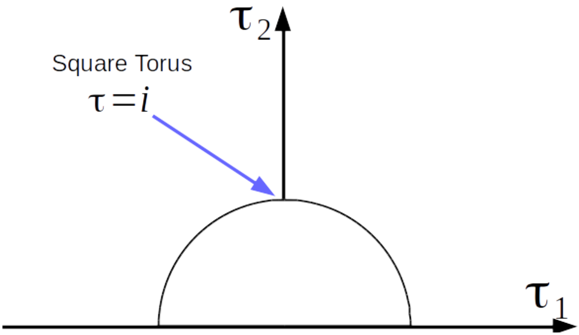
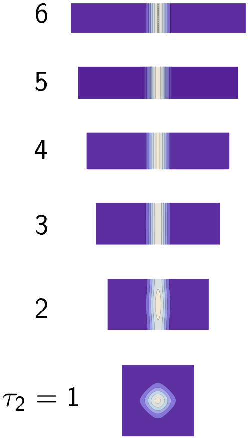


Fremling 13'



Low $\leftarrow |\psi|^2 \rightarrow$ High

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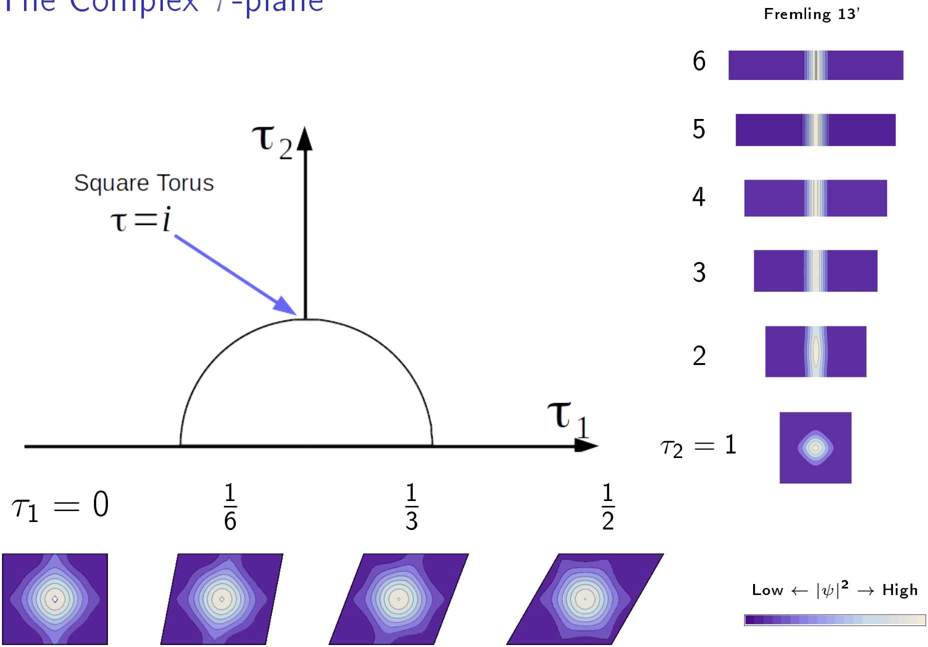

Computing Viscosity

- Viscosity is a response to strain rate, such as a velocity gradient.
- Is related to a Berry phase under changes in geometry.

$$\mathcal{F} = \imath\partial_{\bar{\tau}} A_{\tau} - \imath\partial_{\tau} A_{\bar{\tau}}$$

$$\eta^{H} = -\frac{2\tau_2^2}{A_{\mathrm{Torus}}}\mathcal{F}$$

$A_{\mathrm{Torus}} = L_x L_y = 2\pi N_s \ell_B^2$. Berry connection $A_\mu = \imath \langle \varphi | \partial_\mu \varphi \rangle$

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- Numerically: Discretizing path in τ-space

$$W = e^{\imath A_\Omega \bar{\mathcal{F}}} = e^{\imath \oint A_\mu(\lambda)\, d\lambda_\mu} \approx \prod_j \langle \varphi_j | \varphi_{j+1} \rangle$$

giving

$$\eta^H = -\frac{2\tau_2^2}{A_{\mathrm{Torus}}} \frac{\Im(W)}{A_\Omega}$$

A_Ω : Area of circle.

Computing Viscosity from Diagonalization

$$W \approx \prod_j \langle \varphi_j | \varphi_{j+1} \rangle$$

$$|\varphi_n\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}}^{(n)} |\mathbf{k}\rangle$$

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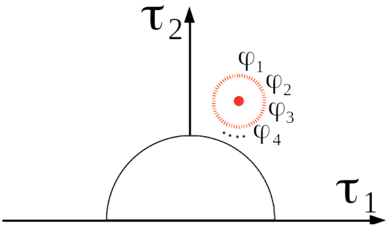

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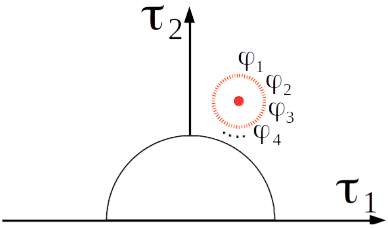
Extra piece with Fock basis

$$\eta^H = -\frac{2\tau_2^2}{A_{\mathrm{Torus}}} \frac{\Im(W)}{A_\Omega} + \frac{1}{4}\hbar\bar{n}$$



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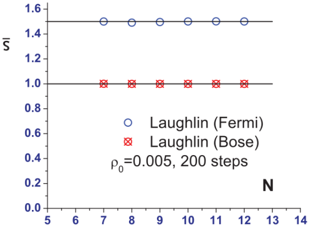
Read & Rezayii 11'



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- Moore - Read $\nu = 1$, $\nu = 5/2$

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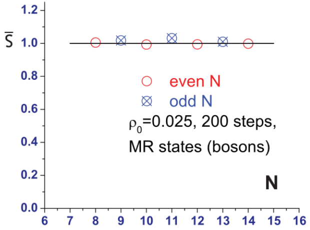

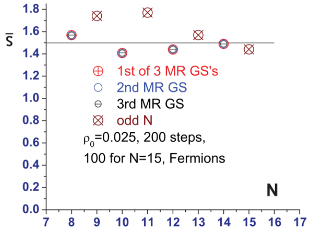

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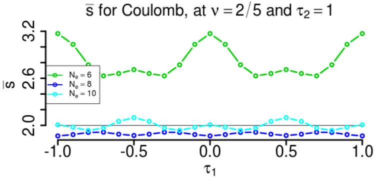

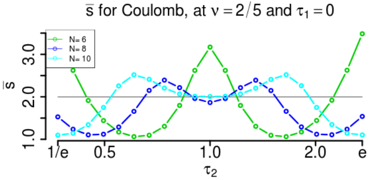

Viscosity from Exact Diagonalization

- Numerically computed viscosity $\bar{s} = \mathcal{S}/2$ for the exact ground state
- Laughlin $\nu = 1/2$, $\nu = 1/3$

  Read & Rezayii 11'
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- But what about the trial wave functions?

Wave Function on the Torus

- Next, compute viscosity of trial wave function.
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$$\psi_{\frac{1}{q}}(\{z\}) = e^{-\frac{1}{4}\sum_j |z_j|^2} \prod_{i<j} \vartheta_1(z_i - z_j|\tau)^q \mathcal{F}_s(Z)$$

$$Z = \sum_i z_i \qquad \mathcal{F}_s(Z) = \vartheta\left[\begin{array}{c} \frac{s}{q} \\ 0 \end{array}\right](qz|q\tau)$$

The Conformal Field Theory (CFT) Construction I

- Simplest approach works for Laughlin, Moore-Read, but then difficult. The center of mass piece $\mathcal{F}$ is hard to guess.

- Idea: Use CFT to construct Laughlin wave function instead
- On the plane

$$\psi_{\frac{1}{q}}(\{z\}) = \left\langle \mathcal{O} \prod_{j=1}^{N_e} V(z_j) \right\rangle$$

- Electron operator

$$V(z) =: e^{\imath\sqrt{q}\phi(z)} :$$

$\mathcal{O}$: Background operator

$\phi(z)$: Scalar boson

The Conformal Field Theory (CFT) Construction II

- Works also on the torus, but with some more work.
- Both chiralities in ϕ

$$V(z,\bar{z}) =: e^{\imath\sqrt{q}\phi(z,\bar{z})} :$$

$$\left\langle \mathcal{O} \prod_{j=1}^{N_e} V(z_j,\bar{z}_j) \right\rangle = \sum_{\mathbf{F}} \Psi_{\mathbf{F}}(\{z\})\bar{\Psi}_{\mathbf{F}}(\{\bar{z}\})$$

- The Laughlin wave function is a linear combination

$$\psi_{\frac{1}{q}}(\{z\}) = \sum_{\mathbf{F}} e^{\imath 2\pi\lambda_{\mathbf{F}}} \Psi_{\mathbf{F}}(\{z\})$$

- Yes, I'm hiding a lot of ugly details!

The Normalization

- Good news, can be done for more complicated states than Laughlin and Moore-Read.
- Eg. positive Jain series $\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \ldots$
- Essential Bonus:
  Gives control over the τ-dependence of the normalization constant.
- For Laughlin

$$\mathcal{N} = \mathcal{N}_0 \left(\sqrt{\tau_2} \eta(\tau)^2 \right)^{qN_e/2}$$

  $\mathcal{N}_0$: τ-independent (assumption).

  $\eta(\tau)$: Dedekind's η-function.

Wave Function for $\nu = \frac{2}{5}$ on the Torus

- We have applied the CFT construction to the $\nu = \frac{2}{5}$ wave function

$$\psi_{\frac{2}{5}} = \mathcal{A}\, e^{-\frac{1}{4}\sum_i\left(|z_i|^2+|w_i|^2\right)}\prod_i \partial_{z_i}\prod_{i,j}(z_i - w_j)^2 \times\prod_{i<j}(z_i - z_j)^3\prod_{i<j}(w_i - w_j)^3$$

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- On the torus becomes

$$\psi_{\frac{2}{5}} = \mathcal{A} \sum_{m,n} D_{m,n}^{(z,\tau)} e^{-\frac{1}{4}\sum_i\left(|z_i|^2+|w_i|^2\right)} \prod_{i,j} \vartheta_1(z_i - w_j|\,\tau)^2 \times \prod_{i<j} \vartheta_1(z_i - z_j|\,\tau)^3 \prod_{i<j} \vartheta_1(w_i - w_j|\,\tau)^3 \times \mathcal{F}(Z, W)$$

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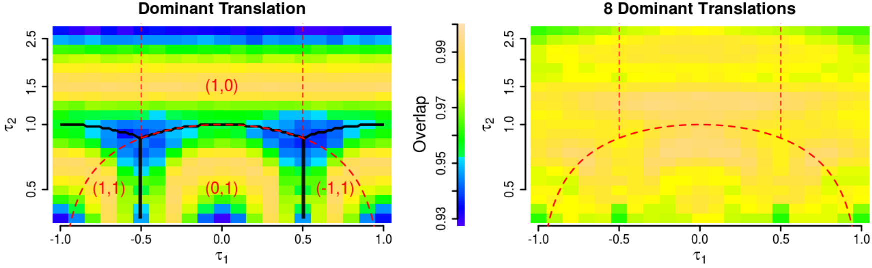
- $D_{m,n}$: Translation operator on z
- $\mathcal{F}(Z, W)$: Center of mass function
- $\mathcal{S} = 4$, $\eta = \frac{1}{4}\,\hbar\,\bar{n}\,4$

Overlap with Coulomb: The Whole τ-plane

- Different terms $D_{m,n}$ are dominant in different regions

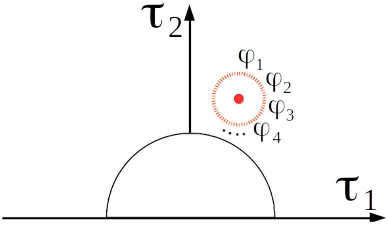


- Red lines: Fundamental $\mathrm{SL}(2,\mathbb{Z})$ domain
  Black lines: Boundaries for the dominating $D_{m,n}$
- The anzats for very well in the entire τ-plane.
  Above 99% overlap by taking the 8 most dominant terms.
- Fantastic!

Computing Viscosity from Real-space Wave Functions

$$W \approx \prod_j \langle \varphi_j | \varphi_{j+1} \rangle \qquad \langle \varphi_{n+1} | \varphi_n \rangle = \int_\Omega \varphi^\star_{n+1}(\mathbf{z})\varphi_n(\mathbf{z})$$

$$\eta^H = -\frac{2\tau_2^2}{A_{\text{Torus}}} \frac{\Im(W)}{A_\Omega}$$



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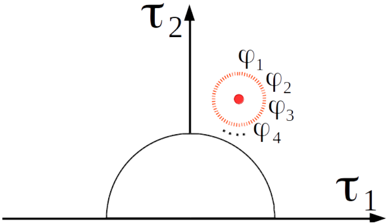

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- All MC integrals use the same set of $\mathbf{z}_m$ for numerical stability

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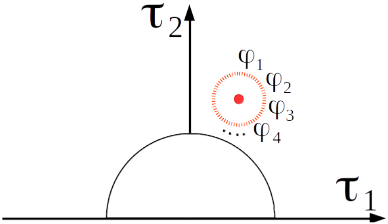
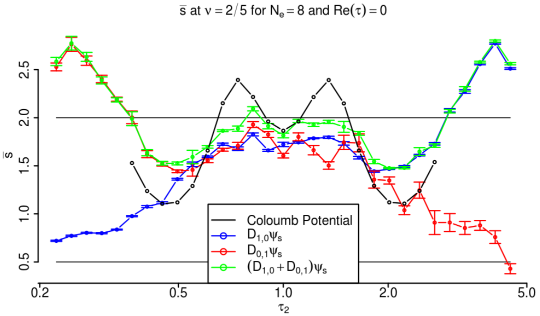


Viscosity at $\nu = 2/5$

- We then numerically calculate $\bar{s}$ for a trial wave function we believe describes the ground state



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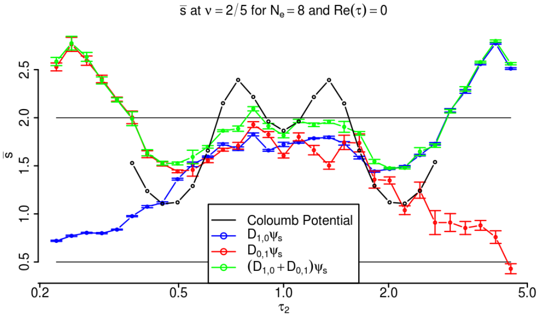

- Again, $\bar{s}$ is not constant in the τ-plane, but converges on $\bar{s} = 2$ as $N_\phi \to \infty$.

Summary

- We have constructed torus wave functions for $\nu = 2/5$ (among many other)
- The construction is valid for all τ
- We have qualitative understanding of with terms $D_{m,mn}$ are dominant
- We have computed the viscosity for these state and it coincided with the Coulomb ground state, and theoretical predictions.

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Thank you!