The Many-body localization transition

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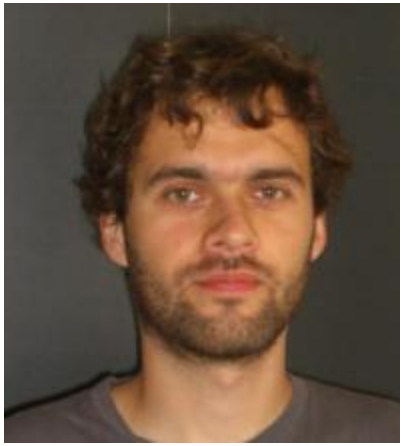
[PRL **113** 107204]

mpipks



Jens H. Bardarson

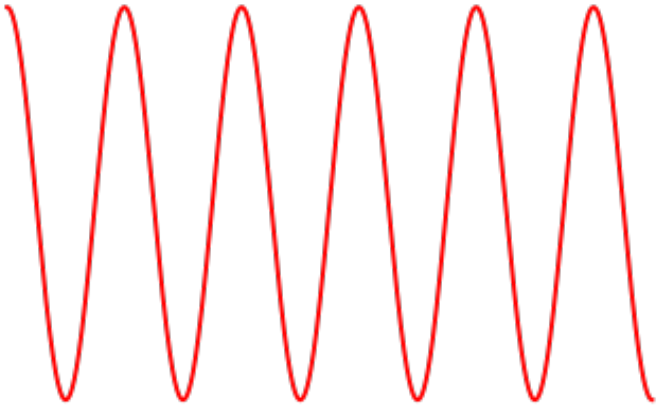
Frank Pollmann

MAX-PLANCK-GESELLSCHAFT

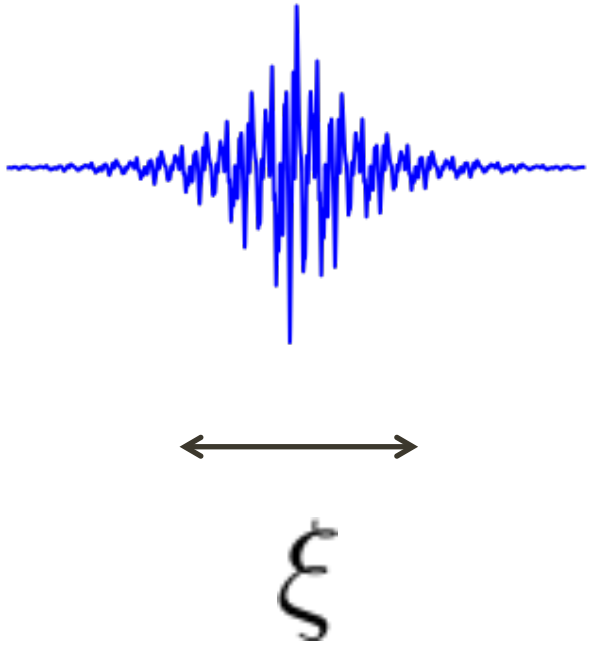
Anderson localization

Single particle

free

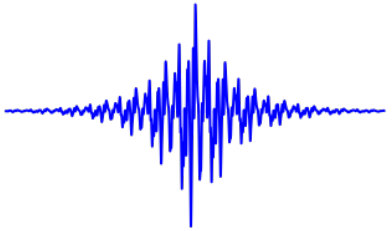


localized



Anderson PR 1958

Anderson localization



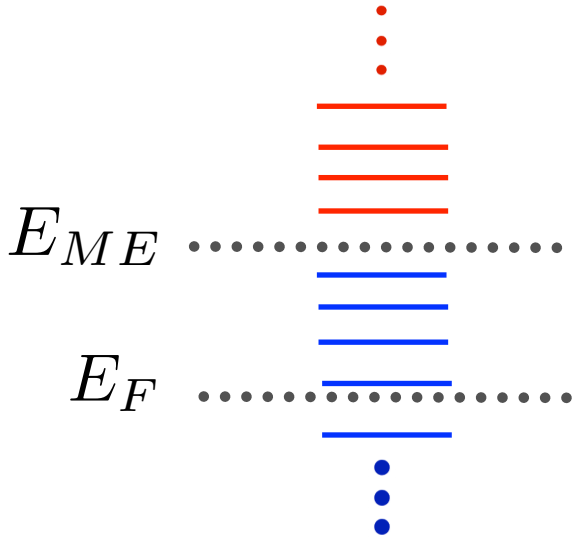
- 1D and 2D: All single particle states localized for any amount of quenched disorder.

$$\sigma(T) = 0$$

- 3D: A *Mobility edge* separates localized and extended states.

$$\sigma(T) \propto (-(E_{ME} - E_F)/T)$$

Arrhenius law



Thouless, 1977; Abrahams et al. , 1979

No localization when coupled to a bath

what about electron-phonon interactions?

phonons: $$\sigma(T) \propto \exp\left[-(\Delta_\xi/T)^{1/(d+1)}\right]$$

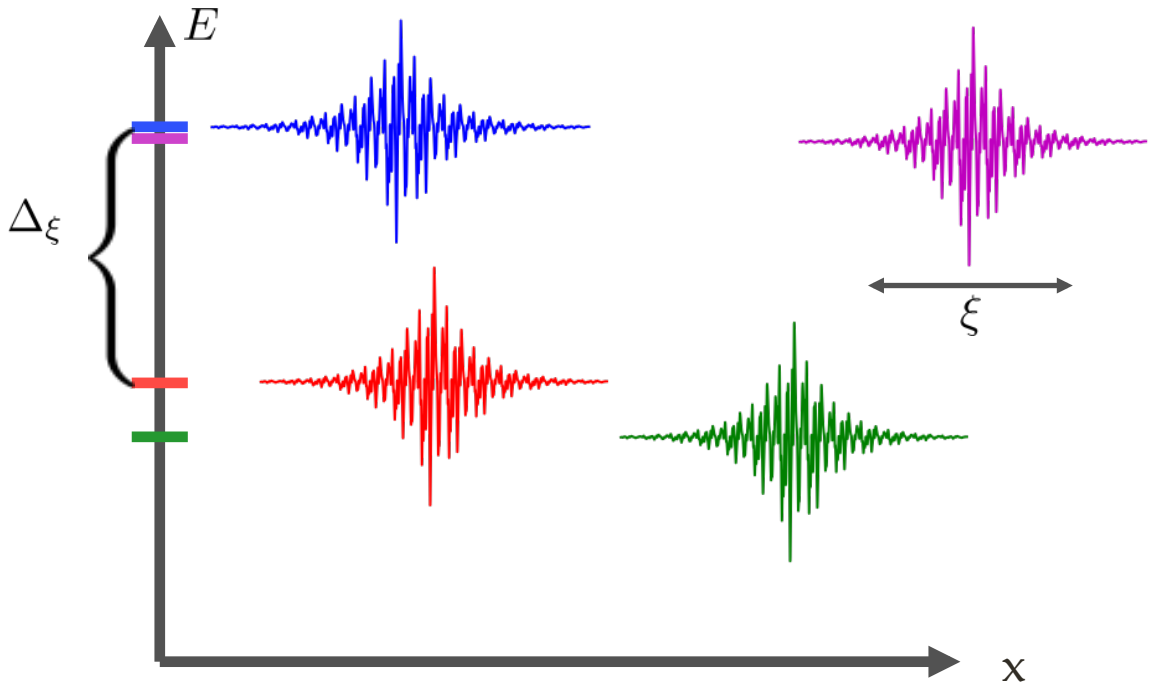
Mott 1968

what about electron-electron interactions?

Many-body localization!

Many-body localization

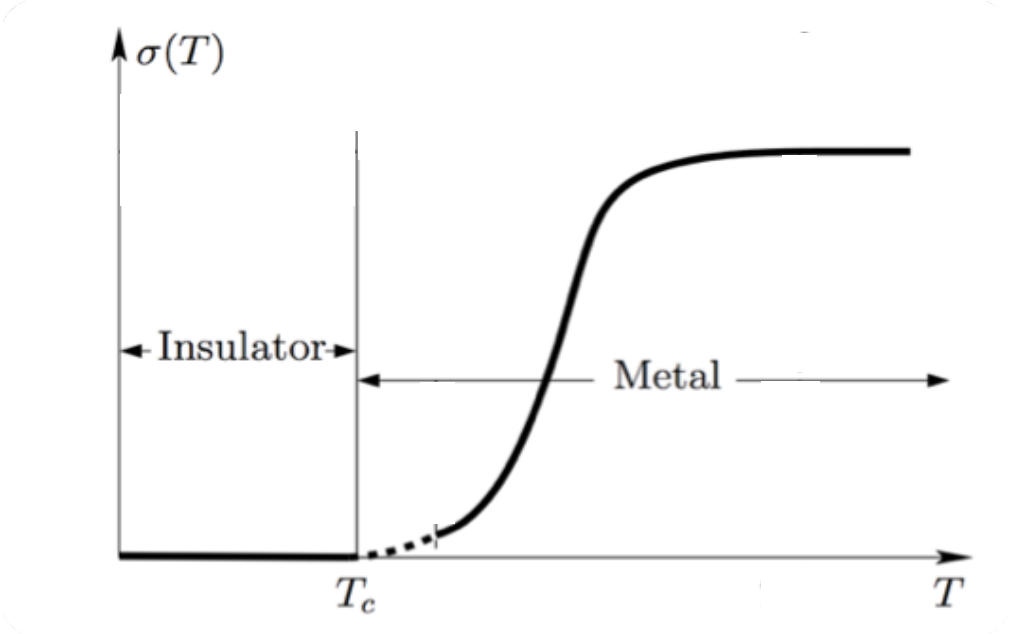
what about electron-electron interactions?



Basko, Aleiner, and Altshuler, Ann. Phys. 2006

See also: Altshuler, Gefen, Kamenev, Levitov PRL 1997;
Gornyi, Mirlin, Polyakov PRL 2005

Many-body localization



MBL Metallic

$$\sigma = 0 \quad (T < T_c)$$

Basko, Aleiner, and Altshuler, Ann. Phys. 2006

Many-body localization

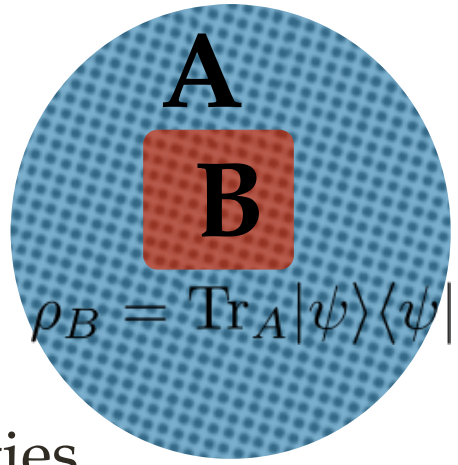
Isolated systems with quenched disorder

- No thermalization

eigenstate thermalization hypothesis does not apply

$$\rho_B \sim e^{-\beta H_B}$$

Deutsch PRA 1991,
Srednicki PRE 1994, …



- System is "quantum" at high energy densities

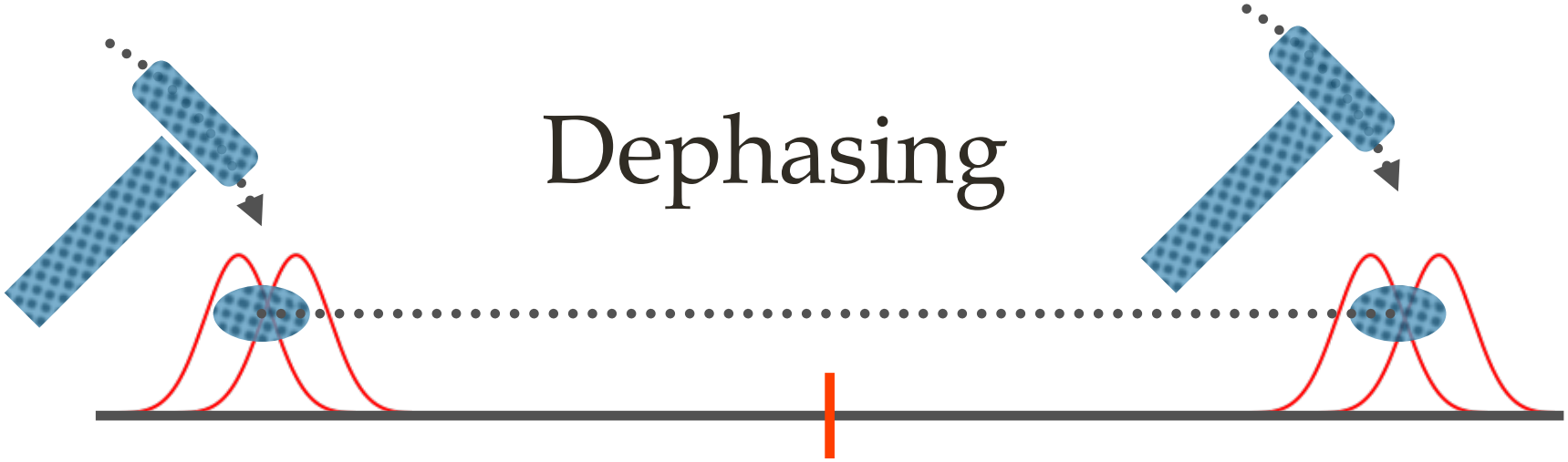
The disorder can protect quantum order at high temepratures

Huse et. al. PRB 2013

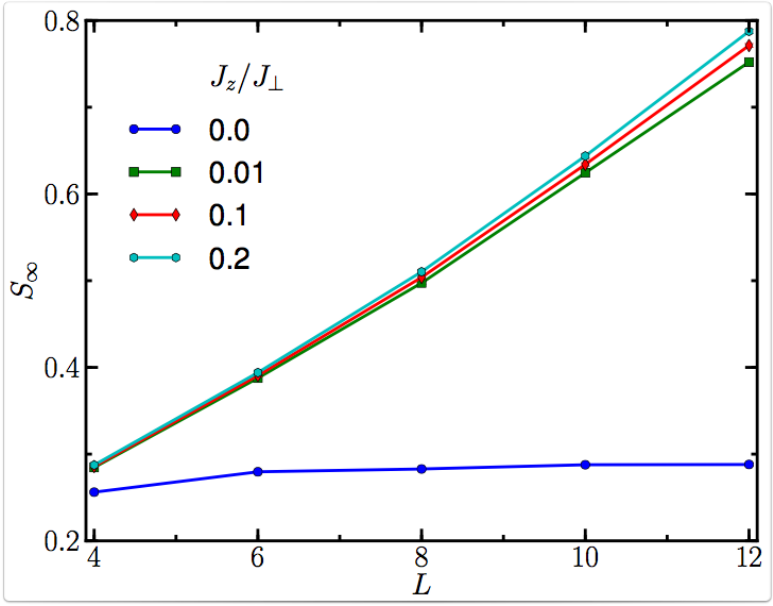
- No conductivity

There can be entanglement growth from dephasing.

Dephasing



Time evolution in an excited disordered XXZ chain.



Logarithmically slow entanglement build up. ("weak" volume law")

Bardarson, Pollmann and Moore PRL 2012

Serbyn, Papic, Abanin PRL 2013

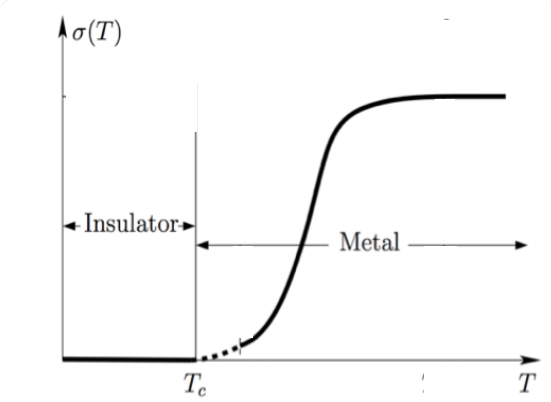
Many-body mobility edge

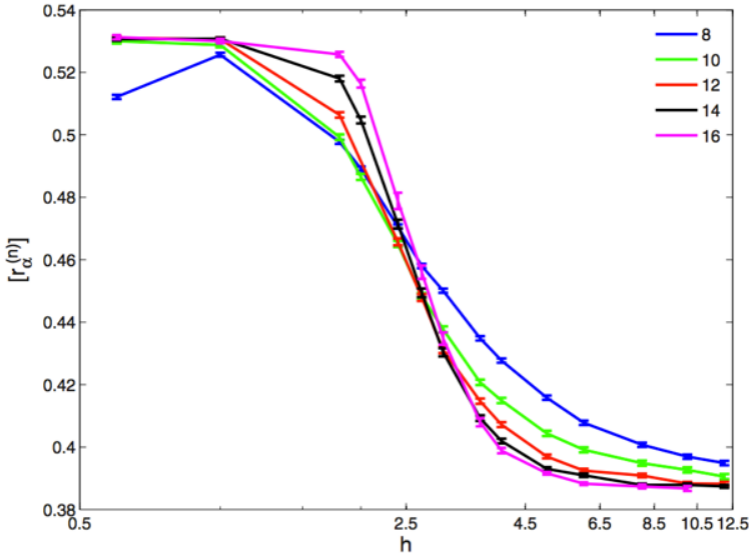
- Eigenstate phase transition
  -Dynamic

- Fully MBL
  -All eigenstates are localized

$$\sigma(T) = 0$$

Oganesyan et. al. 2007, Pal et. al. 2010, Huse et. al. PRB 2013





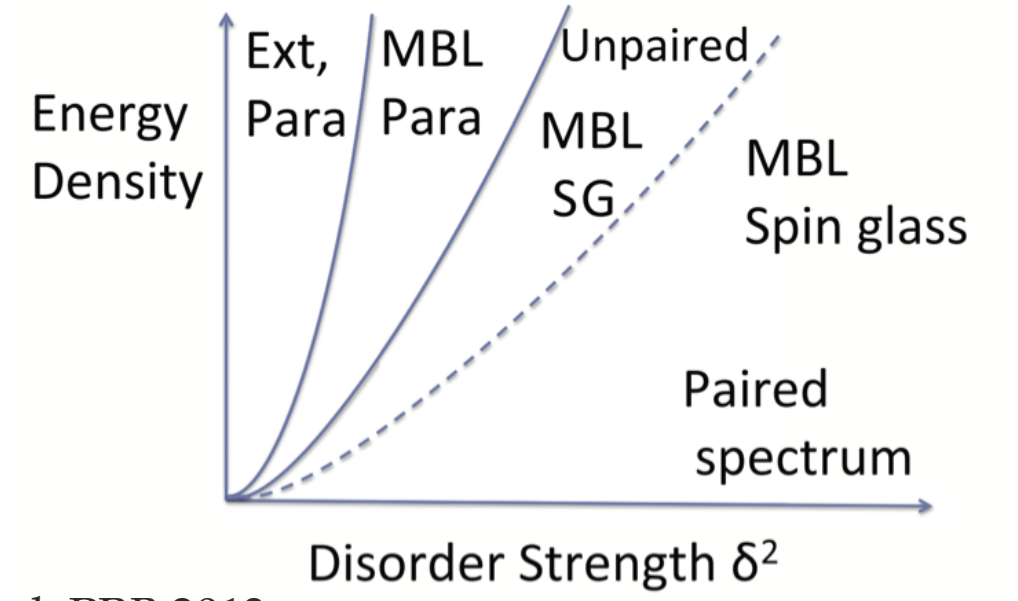
XXZ chain in a random transverse field.

Disorder-protected quantum order

PHYSICAL REVIEW B **88**, 014206 (2013)

Localization-protected quantum order

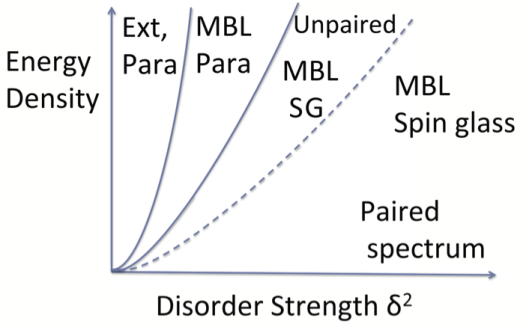
David A. Huse,[1,2] Rahul Nandkishore,[1] Vadim Oganesyan,[3,4] Arijeet Pal,[5] and S. L. Sondhi[2]



Huse et. al. PRB 2013

See also: Pekker, Refael, Altman, Demler and Oganesyan (2013)
Bahri, Vosk, Altman, and Vishwanath (2013)
Chandran, Khemani, Laumann, and Sondhi (2013)
Bauer and Nayak (2013)

Motivation



* Detection of the MBL phase transition

  Challenging to detect in the accessible system sizes.

  What quantity to study?

* Localization protected order

  When the excitations in the high energy states are localized, the spontaneously broken symmetry of the ground state is protected.

  Can we observe separate MBL phase transitions?

* Mobility edge

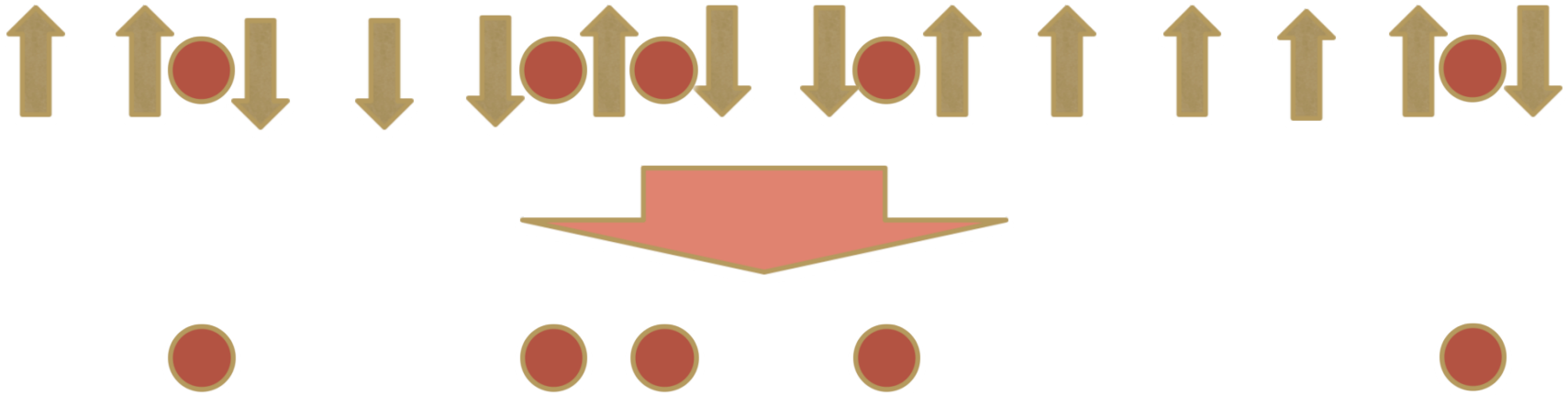
  The energy spectra has a sharp edge, with states extended above and localized below.

  Can we observe signatures of it?

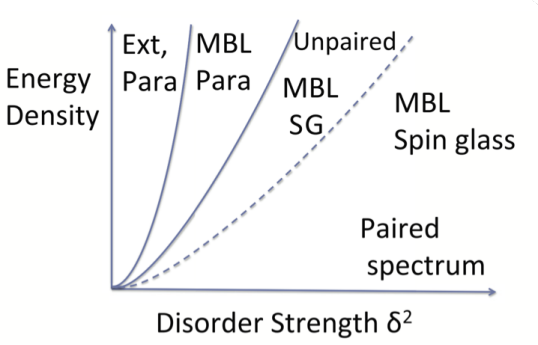
Model

$$H = -\sum_{i=1}^{L-1}(J + \delta J_i)\sigma_i^z\sigma_{i+1}^z + J_2\sum_{i=1}^{L-2}\sigma_i^z\sigma_{i+2}^z + h\sum_{i=1}^{L}\sigma_i^x$$

$$J = 1 \qquad h/2 = J_2 = 0.3 \quad \delta J_i \in [-\delta J, \delta J]$$

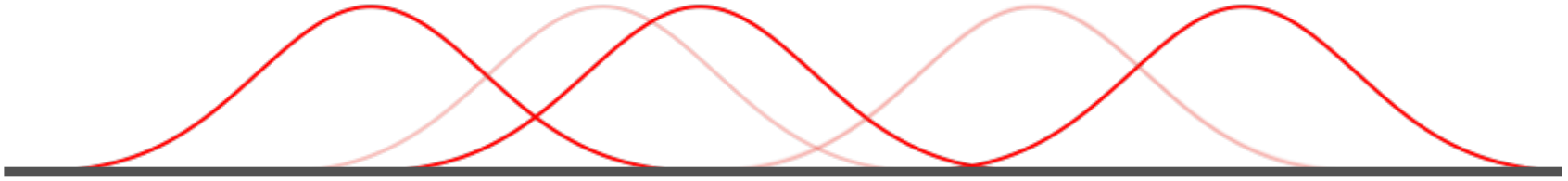


Phases

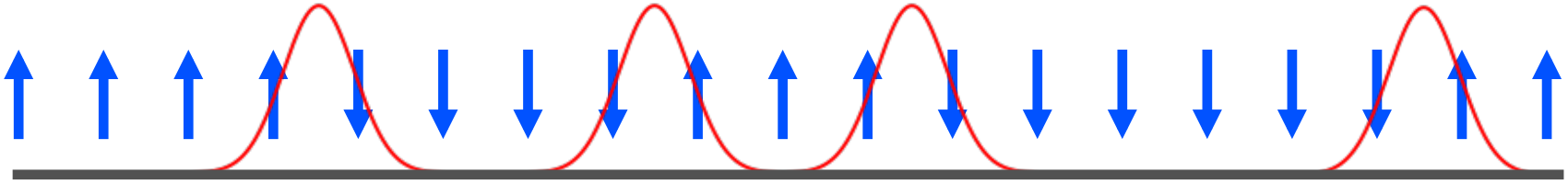


extended

localized, paramagnetic



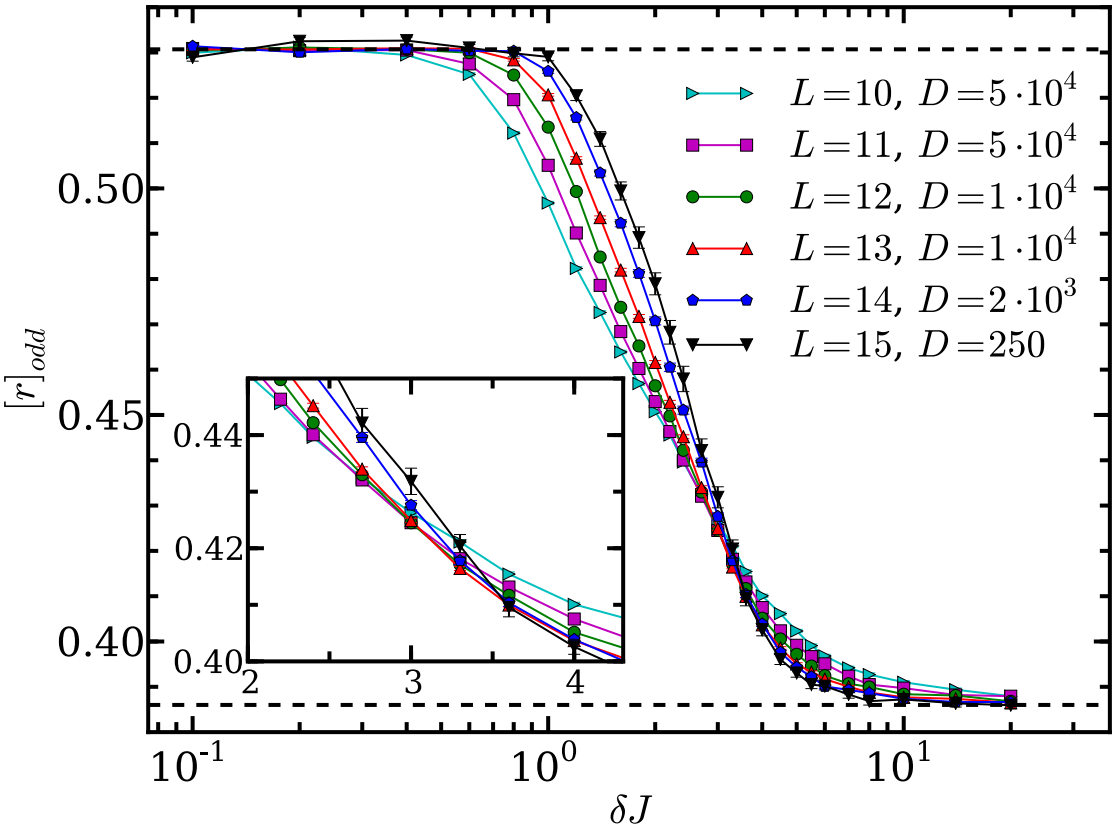
localized, spin-glass



Level spacing statistics

(All data from the middle of the energy spectra unless otherwise noted)

$$r = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \qquad \delta_n = E_{n+1} - E_n$$

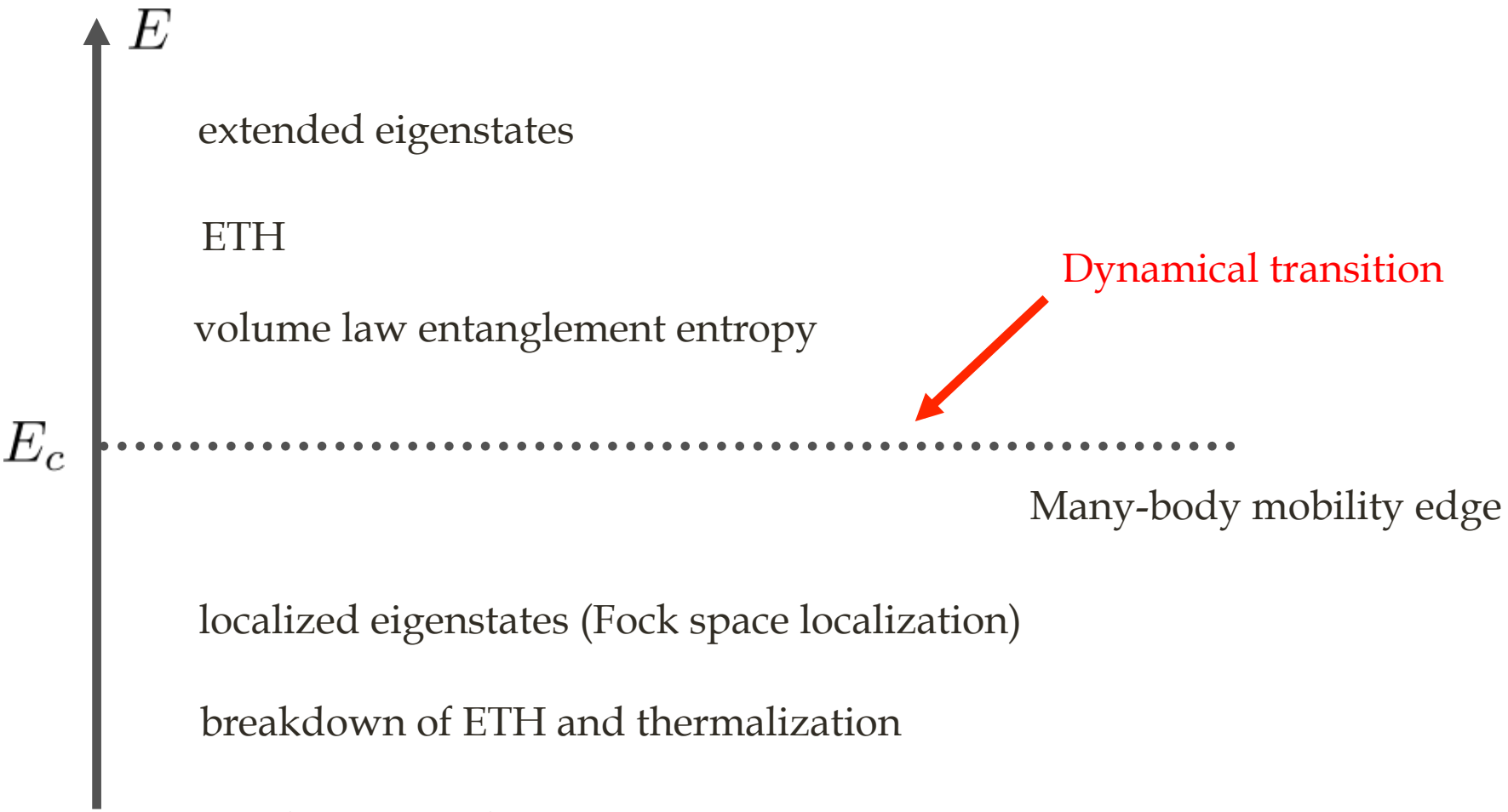


0.531- Wigner-Dyson

0.386 - Poisson

Oganesyan and Huse, PRB 2007; Atas, Bogomolny, Giraud, Roux PRL 2013

Localized vs. Extended



Deutsch PRA 1991, Srednicki PRE 1994, …
Huse et. al. PRB 2013, …

Entanglement

$$S = -\mathrm{Tr}_{\mathrm{L}}\rho\ln\rho$$

$$\rho = \mathrm{Tr}_{\mathrm{R}}|\psi\rangle\langle\psi|$$

Entanglement

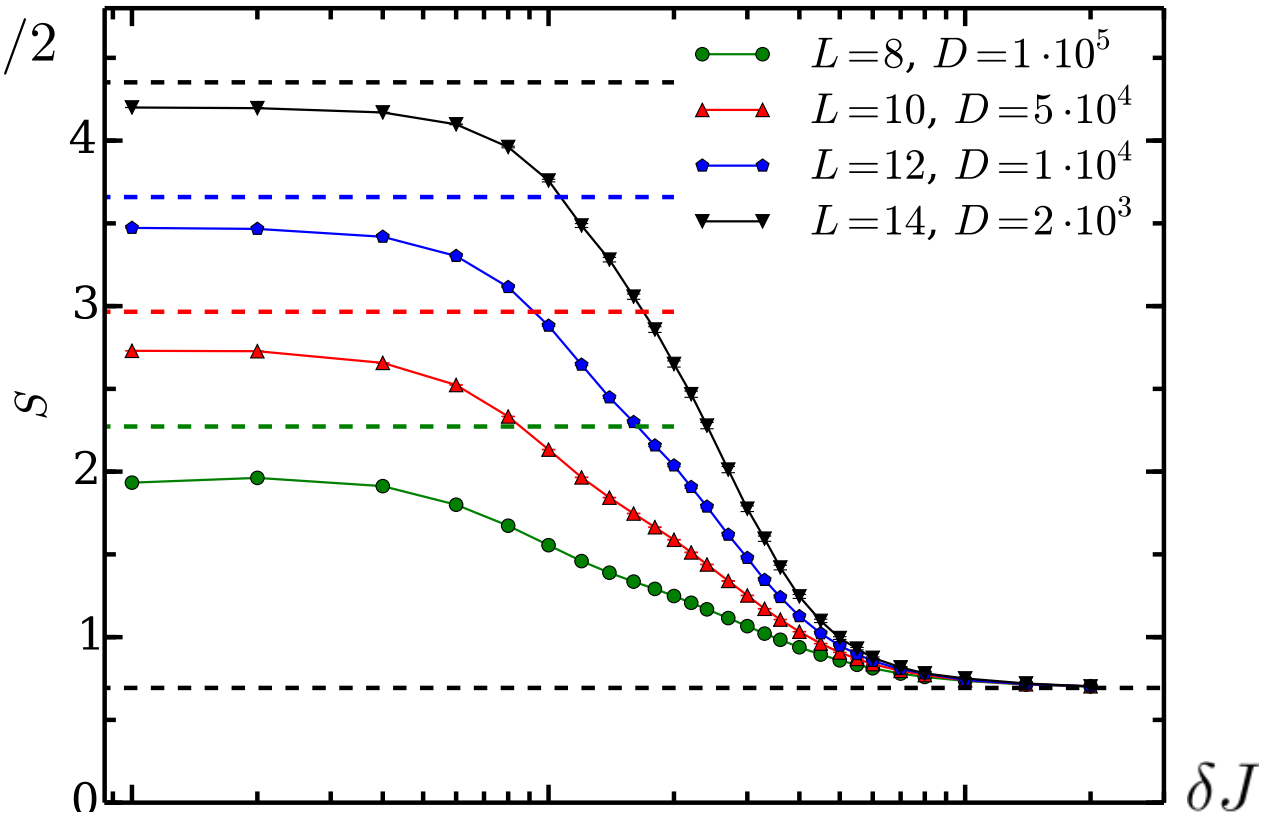
$$|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle \pm |\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$$

$$|\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\rangle \pm |\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\rangle$$

$S = (L\ln 2 - 1)/2$

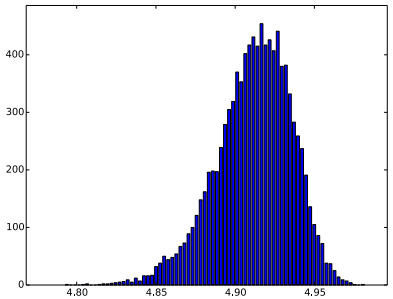
Page PRL 1993

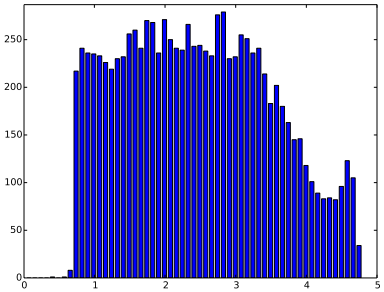
(Volume law)

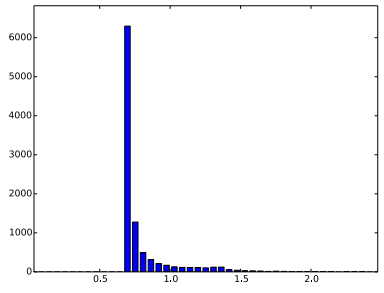


$S = \ln 2$

(Area law)

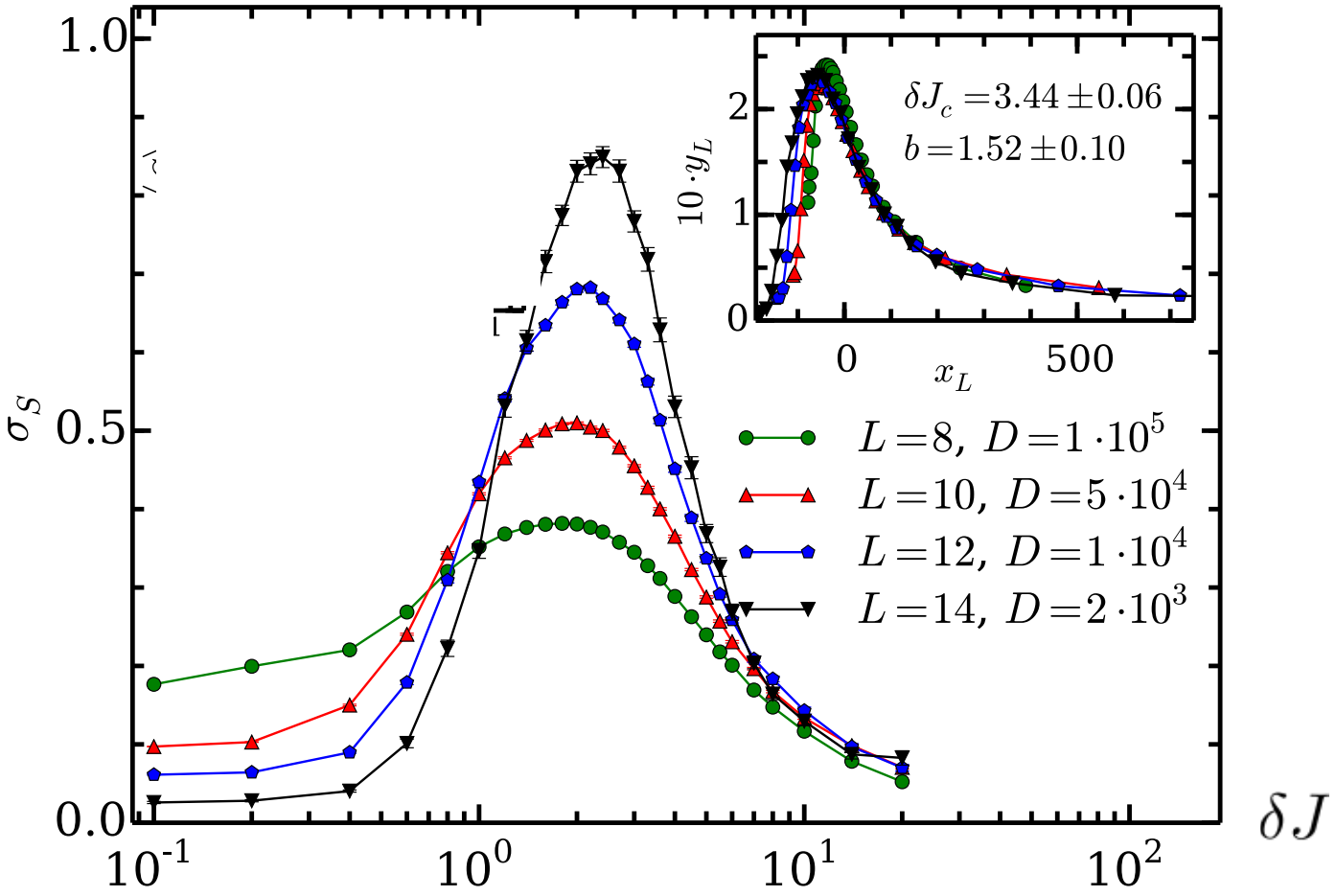






Entanglement variance

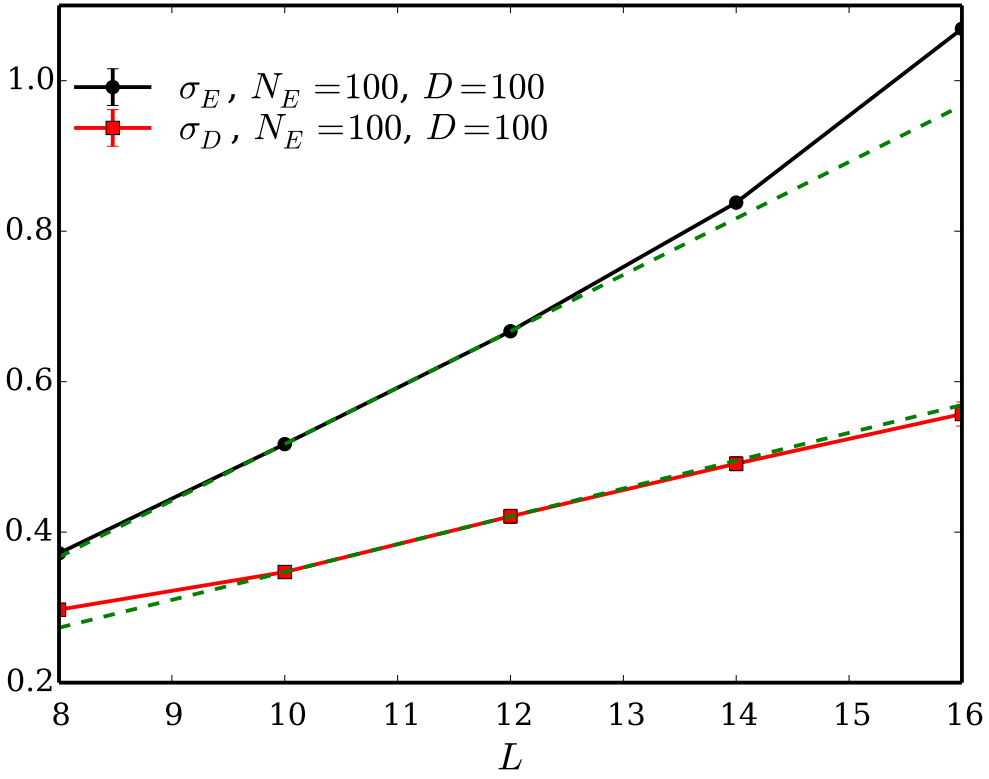
over different disorder realizations

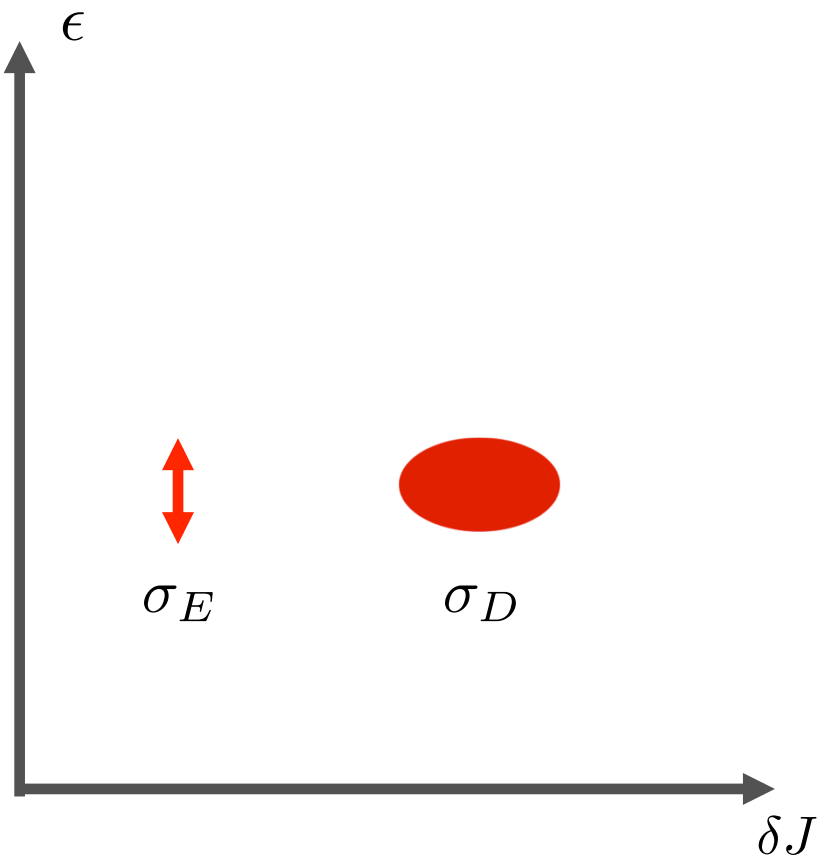


$$Q(L, \delta J) = g(L) f[(\delta J - \delta J_c) L^b] \qquad g(L) = [(L-2)\log 2 - 1]/2$$

Entanglement variance

disorder distributions vs. energy

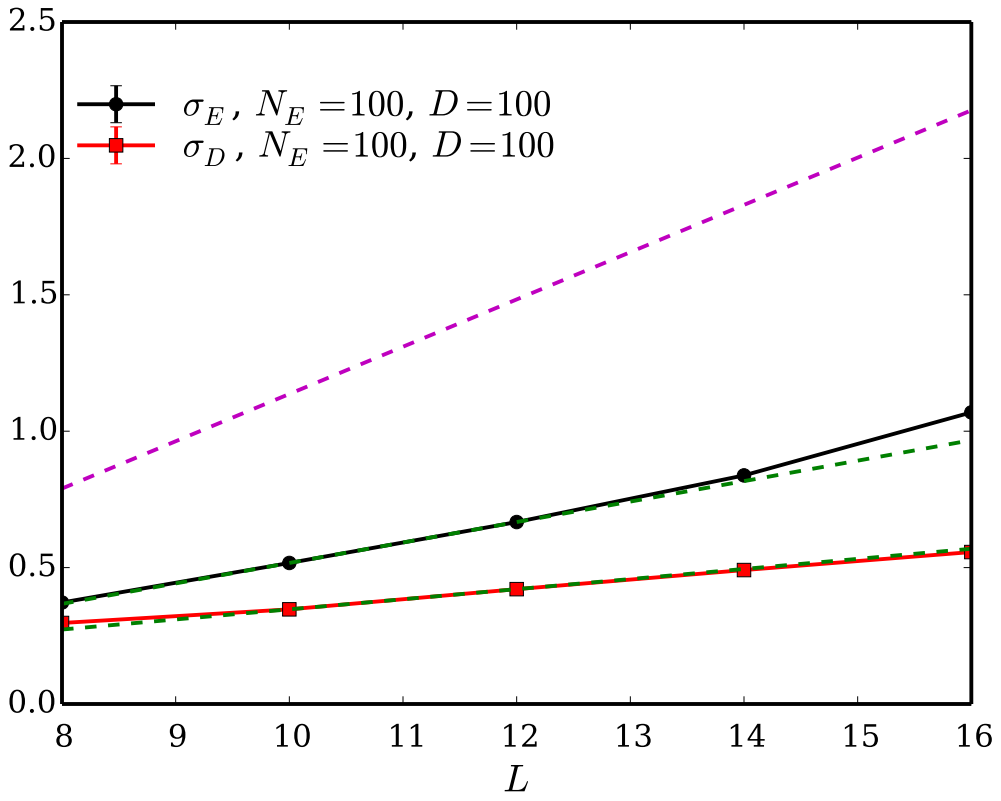


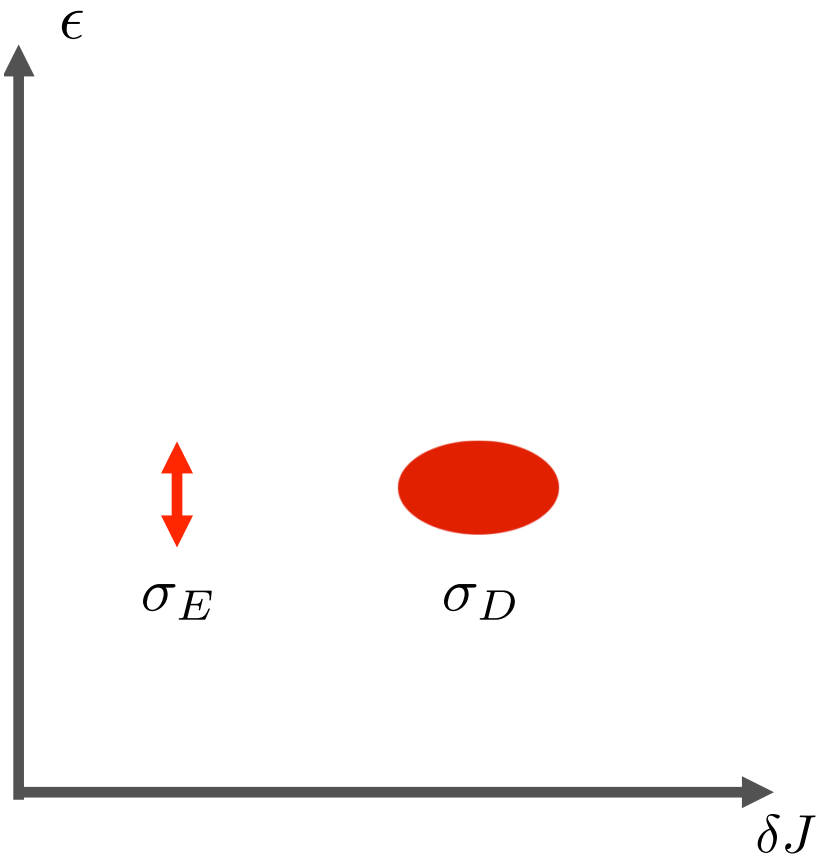


Sparse matrix -> Lanczos: $\sqrt{\langle H^2\rangle - \langle H\rangle^2} \sim 10^{-7}$ $\Delta E \sim 10^{-4}$

Entanglement variance

disorder distributions vs. energy



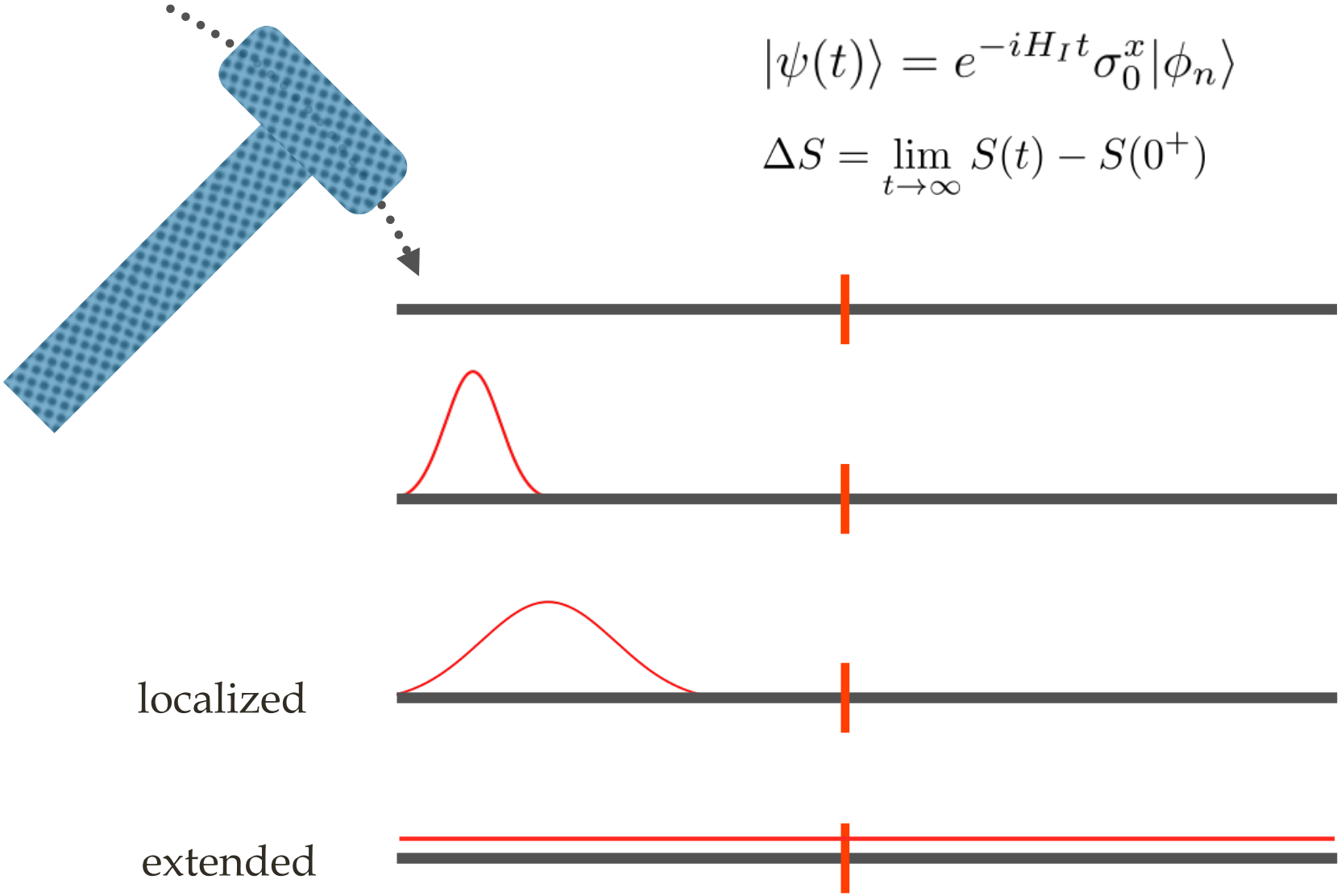


$$\delta J_i \in [-10, 10] \rightarrow \{-4, 2, 3, -1, -5, 1, 0, 5, -2, 4 - 3\}$$

Local quench

$$|\psi(t)\rangle = e^{-iH_I t}\sigma_0^x|\phi_n\rangle$$

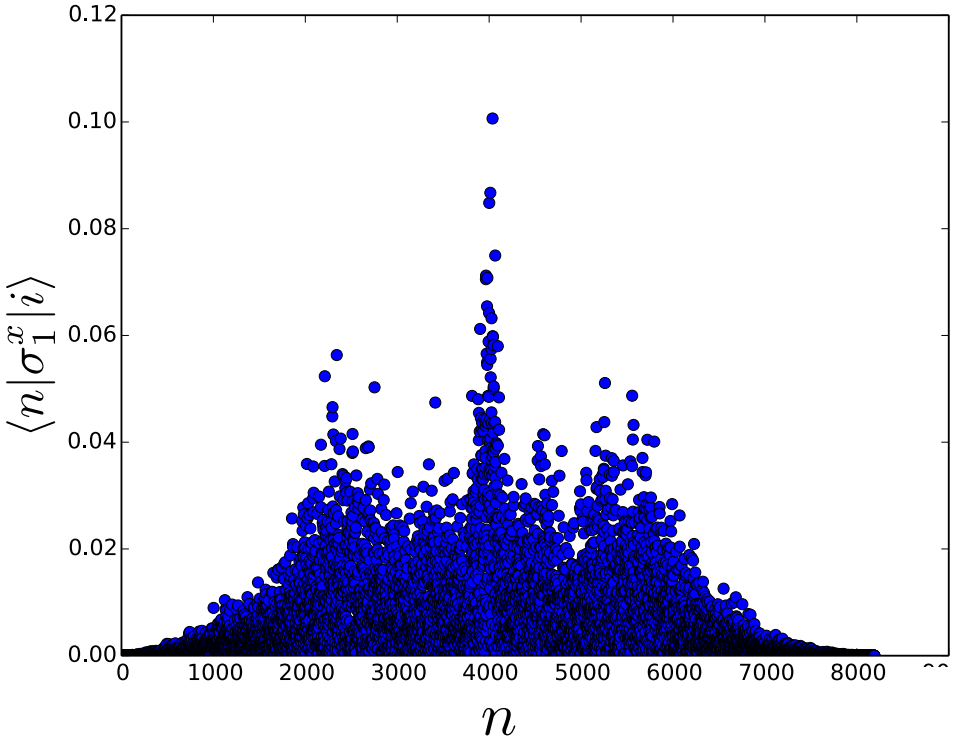
$$\Delta S = \lim_{t\to\infty} S(t) - S(0^+)$$



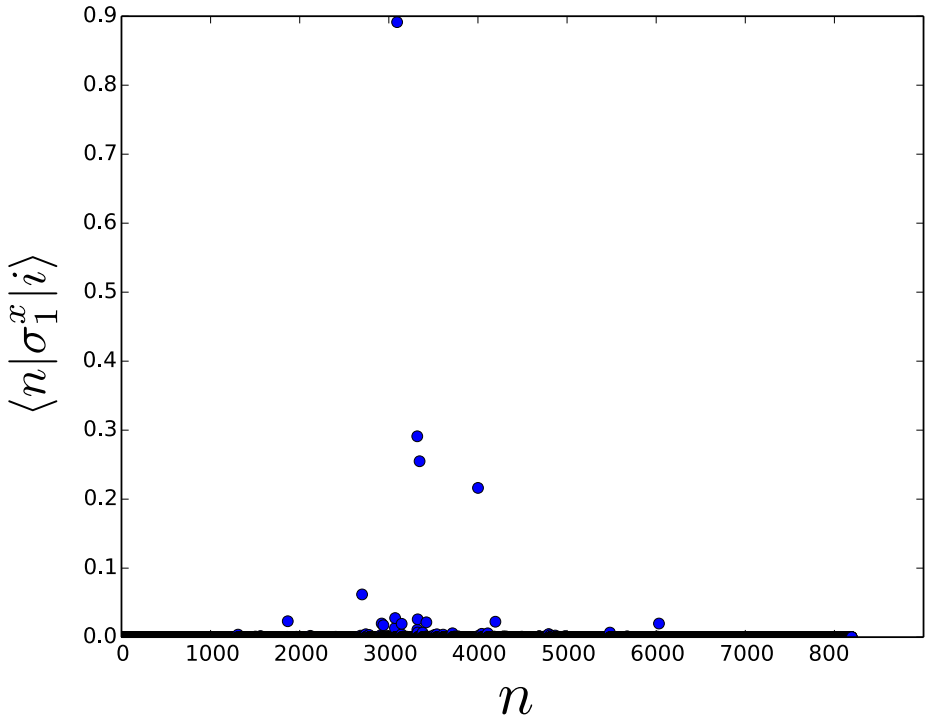
Local quench thermal vs. localized

Flipping the edge spin of an eigenstate in the middle of the spectra

$\delta J = 0.4$

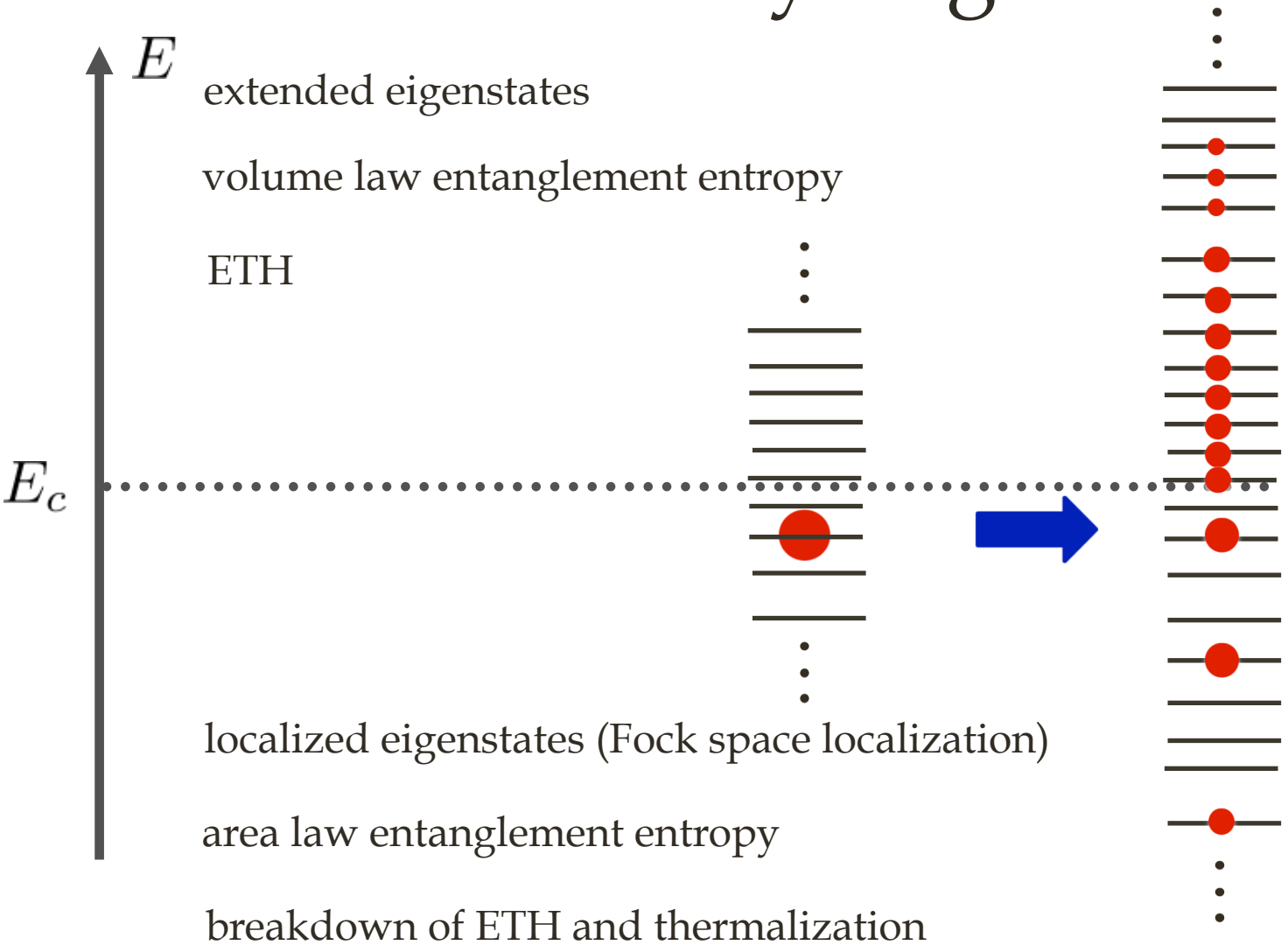


$\delta J = 8.0$



L=14

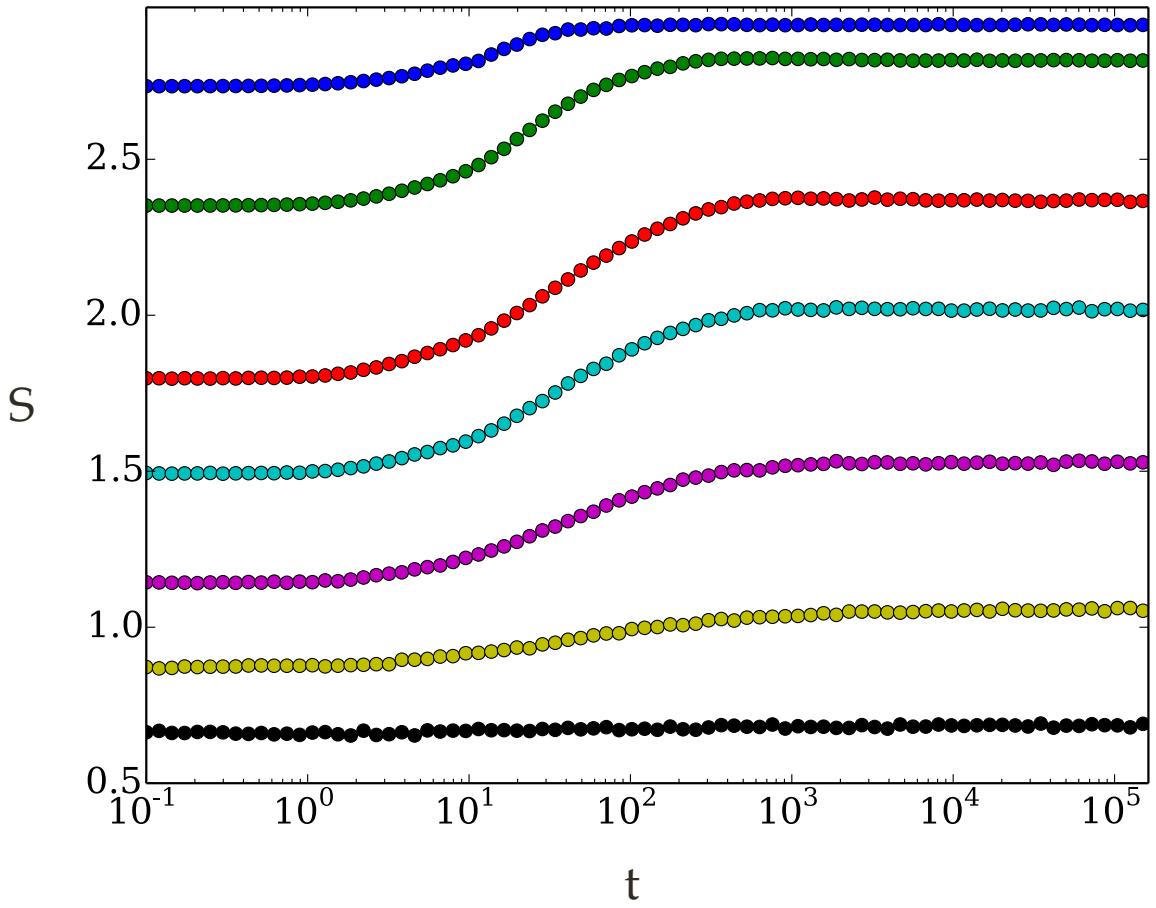
Mobility edge



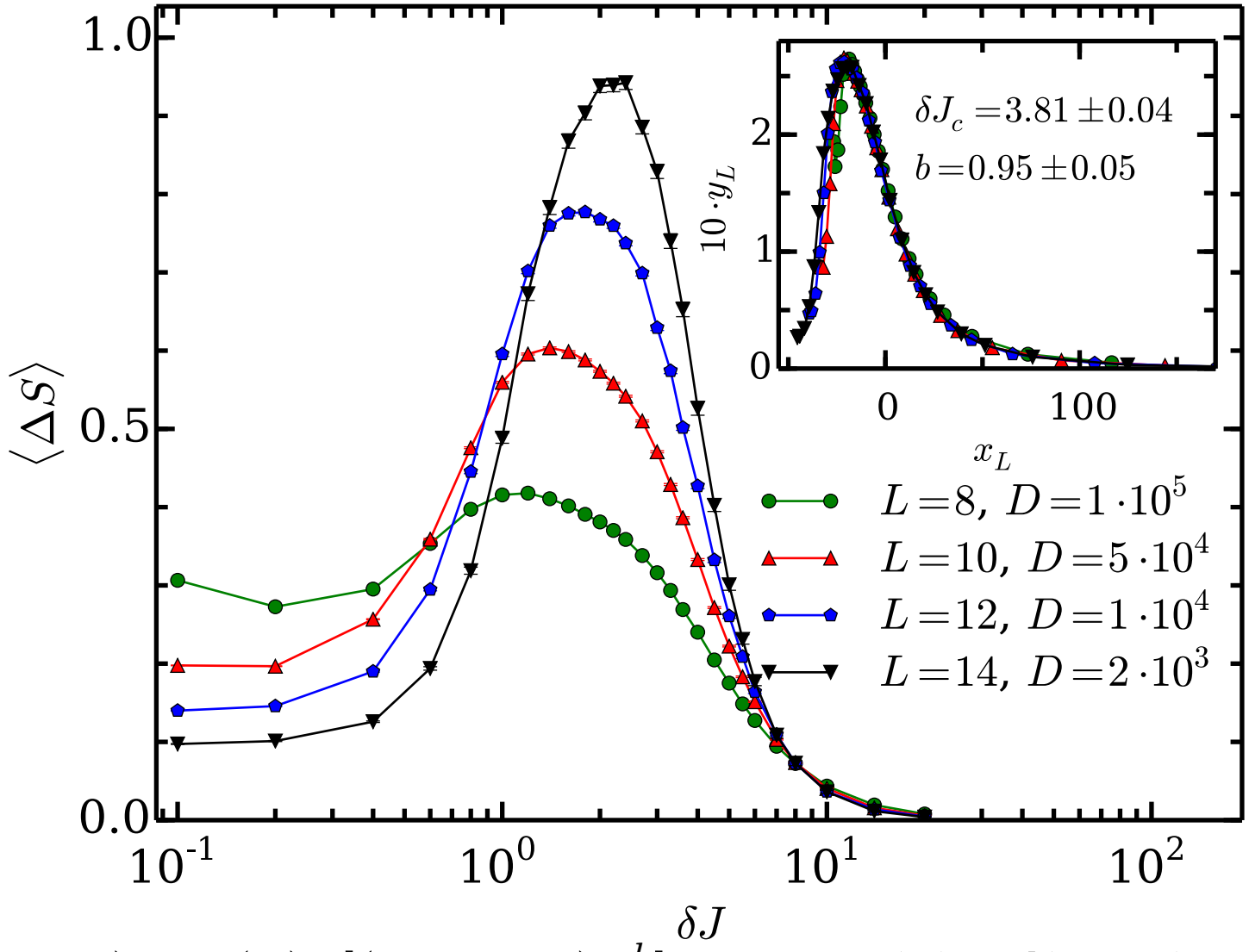
Entanglement evolution after the local quench

$$|\psi(t)\rangle = e^{-iH_I t}\sigma_0^x|\phi_n\rangle$$

$$\delta J = 0.1, 0.8, 1.6, 2.4, 3.6, 5.5, 20.0$$

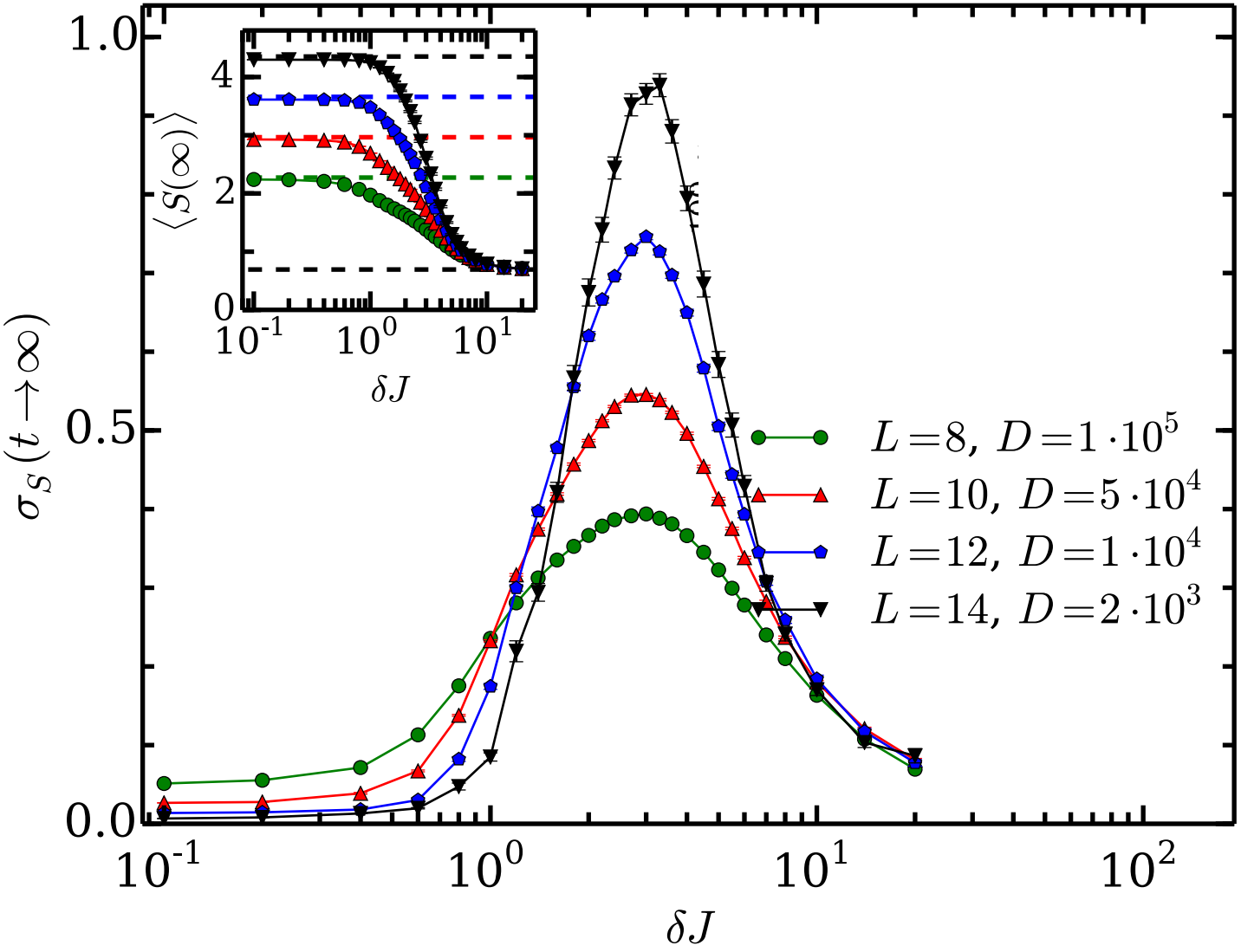


Entanglement difference after a quench



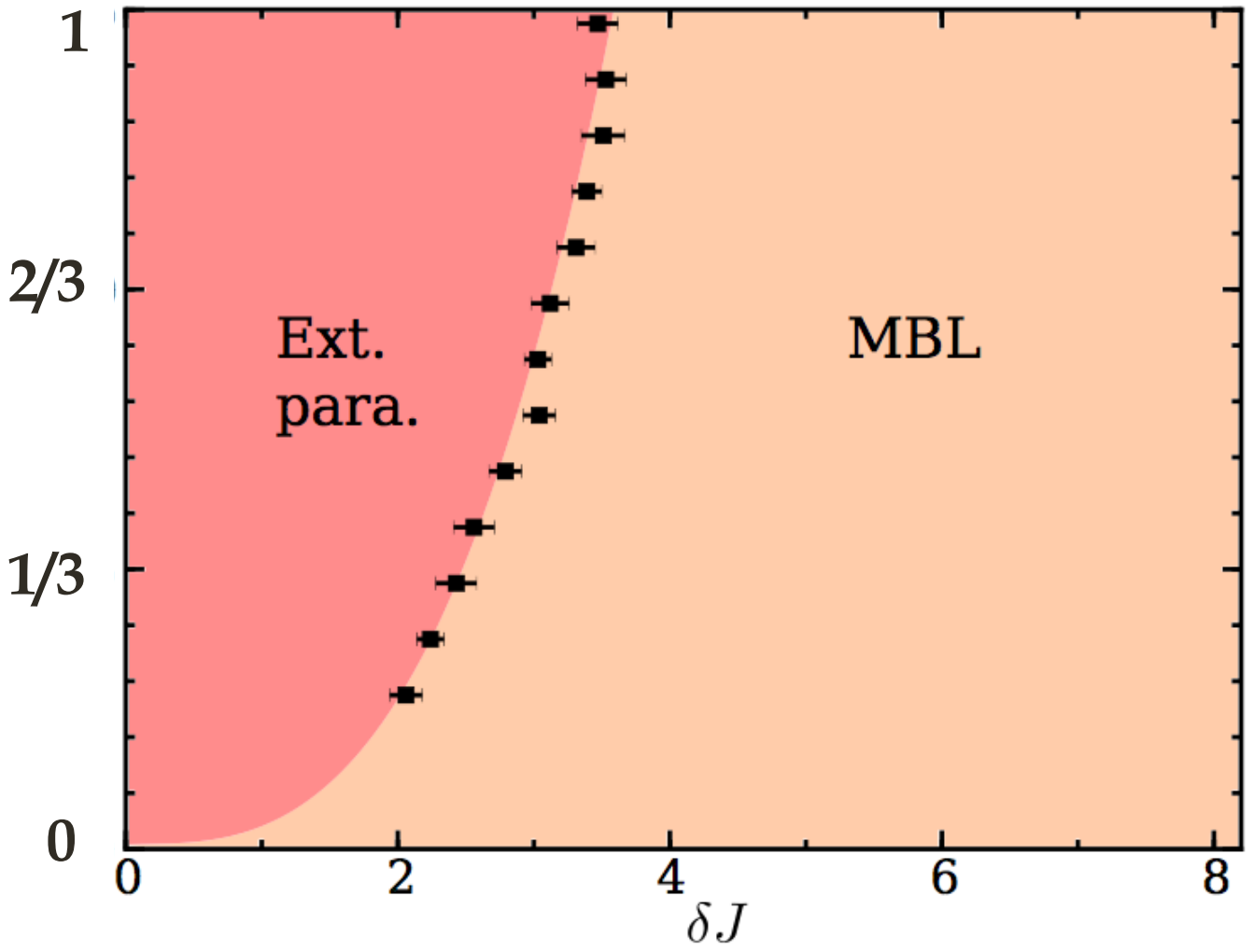
$$Q(L, \delta J) = g(L) f[(\delta J - \delta J_c) L^b] \qquad g(L) = [(L-2)\log 2 - 1]/2$$

Entanglement variance (t→∞)



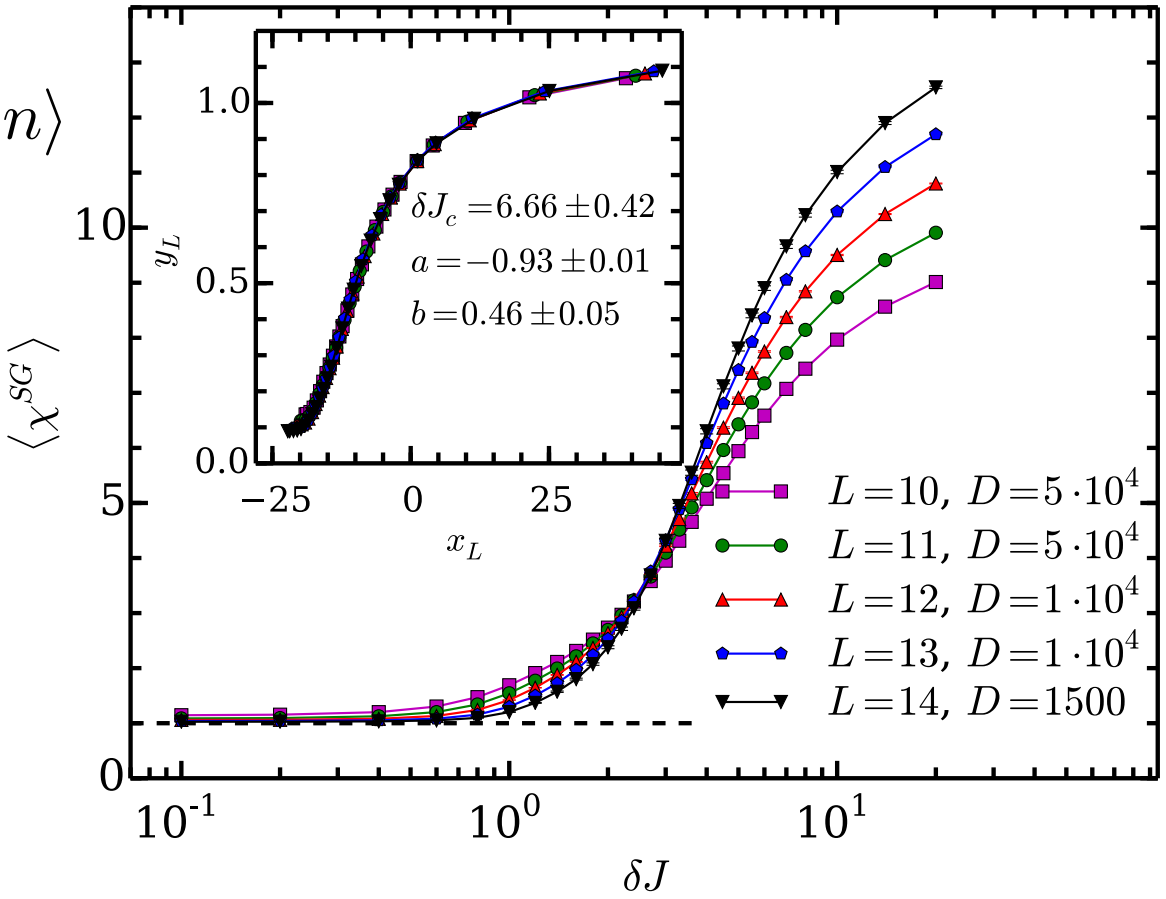
Localized-delocalized phase transition

$$\epsilon = 2\frac{E - E_{\min}}{E_{\max} - E_{\min}}$$



Spin-glass phase

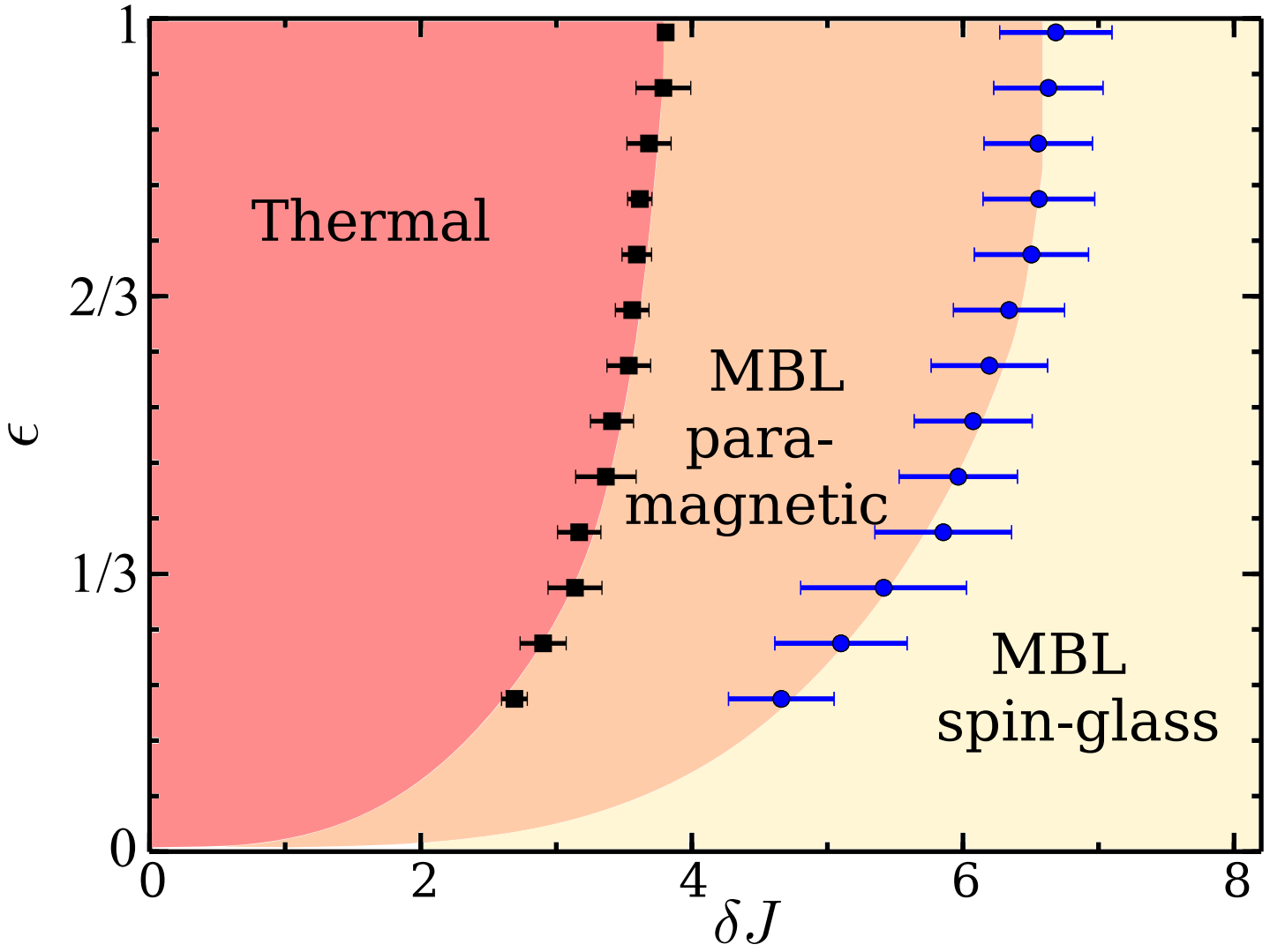
$$\chi_n^{\mathrm{SG}} = \frac{1}{L}\sum_{i,j=1}^{L} \langle n|\sigma_i^z\sigma_j^z|n\rangle$$



$$\langle \chi_{\mathrm{SG}} \rangle = L^a f[(\delta J - \delta J_c)L^b]$$

See also: Pekker, Refael, Altman, Demler and Oganesyan (2013)

Energy-disorder phase diagram



Results

- Entanglement properties good at detecting the MBL phase transition:

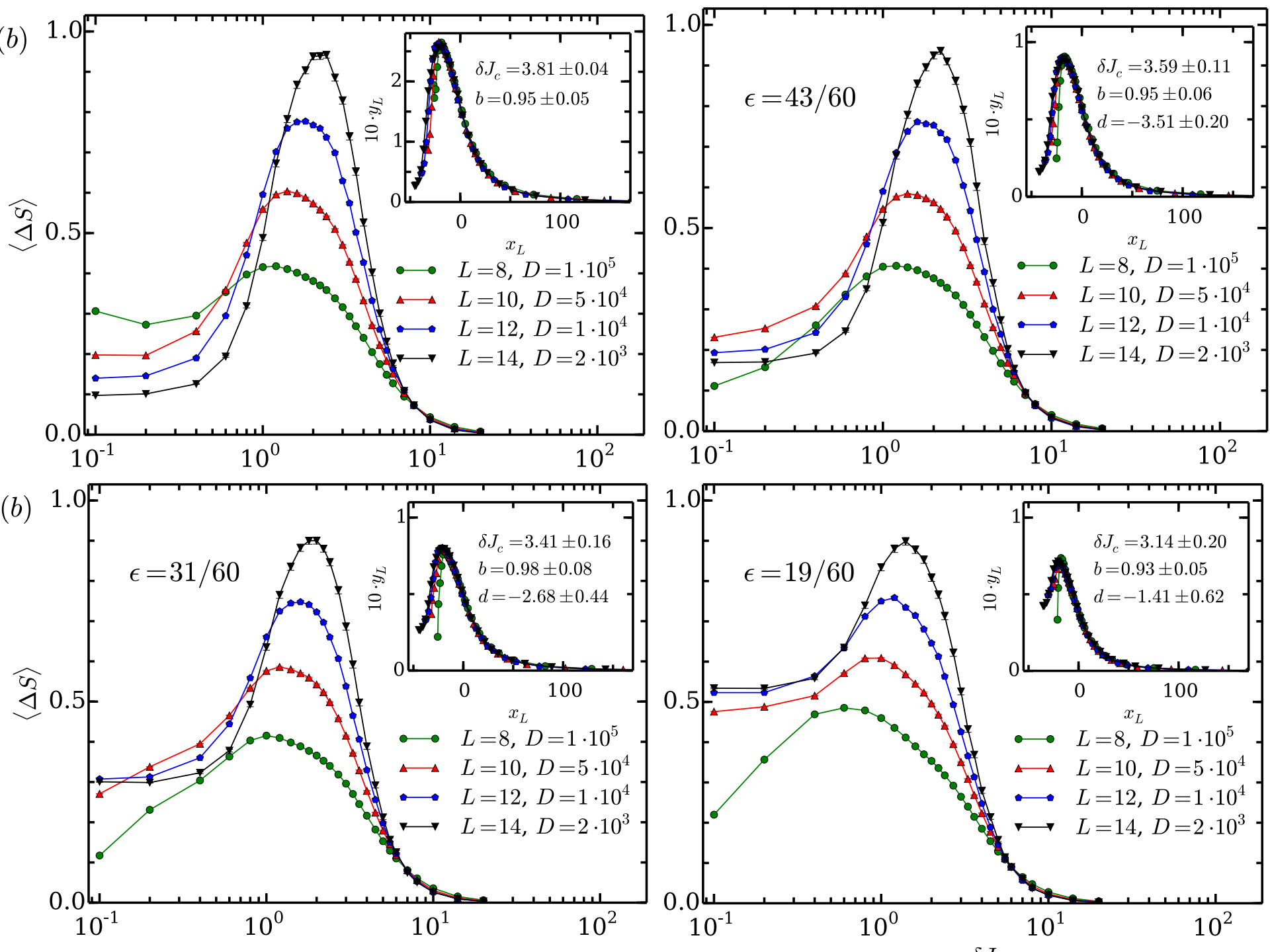
  -Variance.

  -Entanglement difference after a local quench.

- Mapped out the phase diagram of a disordered ferromagnetic quantum Ising chain as a function of disorder strength and energy.

- The spin-glass phase transition is separated from the MBL phase transition.

[JAK, J.H. Bardarson and F. Pollmann, PRL **113** 107204]


(b)
$\langle \Delta S \rangle$
$\epsilon = 43/60$
$\delta J_c = 3.59 \pm 0.11$
$b = 0.95 \pm 0.06$
$d = -3.51 \pm 0.20$
$10 \cdot y_L$
x_L
$L = 8,\ D = 1 \cdot 10^5$
$L = 10,\ D = 5 \cdot 10^4$
$L = 12,\ D = 1 \cdot 10^4$
$L = 14,\ D = 2 \cdot 10^3$
(b)
$\epsilon = 31/60$
$\delta J_c = 3.41 \pm 0.16$
$b = 0.98 \pm 0.08$
$d = -2.68 \pm 0.44$
$\epsilon = 19/60$
$\delta J_c = 3.14 \pm 0.20$
$b = 0.93 \pm 0.05$
$d = -1.41 \pm 0.62$