Defects in Nuclear Pasta

C. J. Horowitz, Indiana University
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Nuclear Pasta

- Nuclear matter, at somewhat below $\rho_0$, forms complex shapes because of competition between short range nuclear attraction and long range Coulomb repulsion $\rightarrow$ “Coulomb frustration”.

- Gravitational collapse during SN increases density: nuclei -> nuclear pasta -> uniform nuclear matter -> neutron star.

- Nuclear pasta expected in neutron stars at base of crust about 1 km below surface at $\sim 1/3 \rho_0$.

- During NS merger tidal excitation decreases density: uniform nuclear matter $\rightarrow$ nuclear pasta $\rightarrow$ nuclei + n $\rightarrow$ r-process.

- **Quantum density functional** calculations. [Sergey Postnikov, Irina Sagert]

- **Semiclassical molecular dynamics model:**
  \[ v(r) = a e^{-r^2/\Lambda} + b_{ij} e^{-r^2/2\Lambda} + e_i e_j e^{-r/\Lambda}/r \]
  Parameters of short range interaction fit to binding E and density of nuclear matter. [Andre Schneider]
Core of massive star collapses to form proto-neutron star. Neutrinos form neutron star energizes shock that ejects outer 90% of star.

Proto-neutron star: hot, e rich

Shock

Neutrino Sphere

Envelope

Crab nebula

July 5, 1054

Crab Pulsar

Audio: Jordal Bank
Neutron stars

- Mass \( \sim 1.4 \, M_{\text{sun}} \), Radius \( \sim 10 \, \text{km} \)
- Solid crust \( \sim 1\,\text{km} \) thick over liquid (outer) core of neutron rich matter.
- Possible exotic phase in center: de-confined quark matter, strange matter, meson condensates, color superconductor...
- Structure determined by Equation of State (pressure vs density) of n rich matter.
- Hyperons could reduce p at high densities but not too much or collapse to black hole. Repulsive three body forces could limit hyperon effects.
- Figure: Dany Page, UNAM
\[ v(r) = a e^{-r^2/\Lambda} + b_{ij} e^{-r^2/2\Lambda} + e_i e_j e^{-r/\lambda}/r \]

51200 nucleons,
\( Y_p = 0.4 \),
\( T = 1 \) MeV,
\( \lambda = 10 \) fm,
\( \xi = 2 \times 10^{-8} \) c/fm,
\( L_0 = 80 \) fm

\[ n = 0.1200 \text{fm}^{-3} \]

Andre Schneider
Average curvature of pasta shapes

Decreasing expansion rate, black to red to blue curves, leads to more abrupt changes in curvature.

- Average curvature vs density. Abrupt transitions for slow expansion suggest series of first order phase transitions.
Minkowski functionals to characterize shapes

\[ V = \int_{\partial K} dA \]
\[ A = \int_{\partial K} \frac{\kappa_1 + \kappa_2}{4\pi} dA \]
\[ B = \int_{\partial K} \frac{\kappa_1 \cdot \kappa_2}{4\pi} dA \]

Volume
Surface Area
Mean Breadth
Euler Characteristic

Graph showing the relationship between \( n \) (in \( \text{fm}^{-3} \)) and \( 10^3 \chi/A \) (in \( \text{fm}^{-2} \)) for different values of \( \xi \) and \( \xi' \).
MD code IUMD

• Use GPUs for calculation of long range screened coulomb interactions. [Fast multipole expansions could improve code for very large systems (but complicated to scale to many GPUs?)]

• Short range nuclear interaction calculated on CPUs with nearest neighbor lists (or localized cubes).

• MPI+OpenMP+CUDA for GPU accelerated multiple nodes. GPUs do coulomb and CPUs do nuclear force concurrently.

Don Berry
Scaling on Big Red II a Cray XE6/XK7

- Speed up compared to performance on a single 2 CPU node (with 32 cores) for pasta simulation with 409,600 particles.

- Each GPU node (with 16 core CPU+GPU) has about 5 times performance of a 2 CPU node.
Simulation Size

• Use computational advances to simulate larger systems, for longer times, and or calculate more complicated observables.

• At typical pasta density $\rho=0.05 \text{ fm}^{-3} \left(1/3 \rho_0\right)$

**Box size**

- 25 fm
- 50
- 100
- 200

**Number of nucleons**

- 781
- 6,250
- 50,000
- 400,000

Typical HF calc.  First Sky3D calc by S. Postnikov

Our typical MD simulation  Larger MD simulation to check finite size effects

We have tested IUMD for over 6,400,000 nucleons in 500+ fm box

(10 sec/ MD step on 64 nodes with $Y_p=0.2$)
Pasta configuration with 409,600 nucleons

Waffle configuration each 2D plane has a lattice of holes

Run for 15 million MD time steps

\( t = 29.80 \times 10^6 \text{ fm/c} \)

\( Y_p = 0.3, \ T = 1 \text{ MeV} \)

Pasta configuration with 409,600 nucleons
Nuclear Waffles

Some different pasta configurations that depend on density (how much below saturation density of $n_0=0.16$ fm$^{-3}$) and proton fraction (ratio of coulomb/ surface E).
FIG. 1: (Color online) Charge density isosurfaces of runs with proton fractions $Y_p = 0.10, 0.20, 0.30$ and $0.40$ after $10^7$ fm/$c$ evolution time. The golden surfaces represent isosurfaces of charge density with $n_{ch} = 0.03$ fm$^{-3}$ while the cream color shows regions such that $n_{ch} > 0.03$ fm$^{-3}$. 
Plot of one waffle and its nearest neighbor (system cooled to 0.5 MeV)
Plot of a Waffle and its next nearest neighbor
How to smell the pasta?
Observables sensitive to pasta shapes

- Coherent $\nu$-pasta scattering gives $\nu$ opacity for supernova simulations. Depends on static structure factor $S_n(q)=<\rho_n(q)\rho_n(q)>$ or dynamical response function $S_n(q,w)$ [Classical MD $\rightarrow$ dynamical response]

- Coherent electron-pasta scattering gives shear viscosity, thermal conductivity, and electrical conductivity of pasta in NS crusts.

- Hysteresis in pasta shapes with density changes gives bulk viscosity. Could be important for damping of neutron star r-mode oscillations.

- Response to small deformations of simulation volume gives shear modulus -- determines neutron star oscillation frequencies.

- Response to large deformations gives breaking strain. Pasta strength important for star quakes (crust breaking), magnetar giant flares, and mountain heights. Deform simulation volume and look at stress vs strain.
Static structure factor

\[ S_a(q) = \langle \rho_a^*(q) \rho_a(q) \rangle - \langle \rho_a^*(q) \rangle \langle \rho_a(q) \rangle, \]

\[ \rho_a(q) = \frac{1}{\sqrt{N_a}} \sum_{i=1}^{N_a} e^{i q \cdot r_i} \]

n = 0.05 fm\(^{-3}\)
T = 1 MeV
• Jose Pons et al speculate [Nature Physics, 9, 431 (2013)] that an “impure” pasta layer with a low electrical conductivity leads to magnetic field decay (in of order a million years) in neutron stars. This could explain why no X-ray pulsars are observed with rotation periods longer than 20 sec.

• They assumed a crystal lattice of some average charge with random impurities of different charges and required a significant spread in charges to produce enough election-pasta scattering for a low conductivity.

• Note this likely also decreases the thermal conductivity which should be observable in X-ray light curves of crust cooling.

• How to describe the amount of disorder in pasta, and could it be large enough to give low electrical and thermal conductivities?
Pasta Defects

• How to describe pasta imperfections?
  – Impurities—unlikely because nucleons may be free to flow and equal out composition.
  – Topological defects—could be very long lived excitations that increase electron-pasta scattering.
  – Grain boundaries between different pasta phases and orientations.

• Need pasta molding over larger length scales to find elementary excitations and their impact on transport.
Screw defect in Lasagne

75,000 nucleon configuration ($Y_p=0.4$, $T=1$ MeV, $\rho=0.05$ fm$^{-3}$) that was started from random positions and quickly formed screw defect. Defect then stable for over 10 million MD time steps.
Plane of screw defects

50,000 nucleon configuration of lasagne planes (yellow) with a 2D array of screw defects (gray)
Try and characterize possible nuclear pasta defects. Then calculate effects of defects on electron transport.
Defects in Nuclear Pasta

- Nuclear pasta expected near base of neutron star crust.
- X-ray observations of NS radii probe liquid n rich matter.
- Energies of supernova neutrinos and antineutrinos, important for nucleosynthesis, depend on properties of gaseous n rich matter.
- Continuous gravitational waves from mountains on NS can probe solid n rich matter.

- Collaborators: D. Berry, A. Schneider, M. Caplan, C. Briggs
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