Superconductivity as a Universal Emergent Phenomenon in Diverse Physical Systems

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This simple, reasonably-unified picture was shattered by a series of discoveries beginning in in the 1970s:

- Heavy-fermion superconductivity
- Superfluidity in Helium-3
- Organic superconductors
- Cuprate high-temperature superconductors
- Iron-based high-temperature superconductors

For these systems there was strong evidence that

- The SC or SF was not phonon-mediated.
- Pairing was *unconventional and not s-wave* like in conventional BCS.
- There were *doubts that BCS was adequate* to describe the systems, even using unconventional pairing formfactors.
- In at least some cases the parent state was *not a Fermi liquid*.

Increasingly, attempts to understand the new forms of superconductivity became localized around specific microscopic details in various subfields. For example, many papers were devoted to ideas like

- The two-dimensional nature of the SC, or
- The *d*-wave nature of the pairing, or
- Mott-insulator parent states, or
- Quantum critical points having mystical powers being the magical ingredient crucial to explaining the

surprising new cuprate high- T_c results.

But it is clear that a unified understanding of superconductivity and superfluidity *cannot* be built on traditional microscopic approaches for two basic reasons:

- The SC and SF states are *emergent* and not connected by infinitesimal steps to the microscopic normal system. It is then conjecture to assume that any tendency of the microscopic normal system is reflected in the SC state.
- The standard microscopic approaches differ radically in length scales, energy scales, and methodology between fields, and thus cannot unify across fields.

Yet SC/SF similarity across fields can hardly be accidental.

I would first like to survey the broad range of superconducting behavior in a variety of physical systems, and then propose an approach capable of unifying this varied superconducting behavior.

Our initial survey will not be exhaustive, but rather is intended to give a flavor for the broad range of superconductivity that is now known to exist.

Since conventional BCS superconductivity is well documented and well understood, our emphasis will be on unconventional SC/SF.

Cuprate High- T_c Superconductivity

Superconductivity in the cuprates typically results from doping of electrons, or electron holes in the most common cases, into a parent compound that is a Mott insulator with a half- filled electron band.

They are unconventional, beyond doubt.

Iron-Based Superconductors

Superconductors based on Fe compounds were discovered in 2008. Their phase diagrams have many similarities with that of the cuprates.

They are almost certainly unconventional.

Organic Superconductors

Various organics and salts exhibit superconductivity, with phase diagrams in which pressure often replaces doping that resemble cuprates. Probably unconventional.

Buckyball (Alkali-Doped Fullerene) Superconductors

It was thought that fullerenes are strong-coupled conventional superconductors, but some exhibit a phase diagram similar to cuprates, suggesting unconventional superconductivity.

Heavy-Fermion Superconductors

Heavy-fermion compounds exhibit superconductivity, again with phase diagrams similar to the cuprates (their SC and its unusual properties were discovered before that of the cuprates, but T_c is not very high).

They are widely believed to be unconventional.

Superconductivity in Atomic Nuclei

2-neutron transfer cross sections in collision of two heavy ions. Enhancement factor *F* defined by

 $F = \frac{P_{2n}}{P_{1n}P_{1n}}$

 $P_{n} = 1n$ transfer probability

 P_{2n} = 2n transfer probability

Thus *F* measures the degree to which the transfer cross section is enhanced by pairing correlations.

X. T. Liu, D. Cline, T. Czosnpka, M. W. Guidry, X. L. Han, A. E. Kavka, W. J. Kernan, R. W. Kincaid, S. P. Sorensen, E. G. Vogt, and C. Y. Wu, Phys. Rev. **C43**, R1 (1991)

A Nuclear Structure "Gap" Diagram

Phases are not sharp because of the finite valence particle number which causes statistical fluctuations.

SC and SF in nuclei clearly involve more than s-wave BCS pairing and thus are not "conventional".

Superfluid Phases of Helium-3

Superconductivity in Neutron Stars

- At birth neutron stars may have $T \sim 10^{11}$ K and may have $T > 10^8$ K for $\sim 10^5$ years.
- However, nucleons are *very degenerate* in this period.
- Because of *high densities,* the critical temperatures are large.
- Typically $T_c \sim 10^8 10^{10}$ K.
- Thus, $kT_c \sim 0.001 = 1.0$ MeV.
- These are the real high-*T*_c superconductors!
- SC/SF likely unconventional.

Thus, onset of SC/SF is expected for a neutron star's interior within *minutes to thousands of years after its birth.*

Cas A Supernova Remnant in X-rays

Chandra X-ray Observatory

Cassiopeia A (Cas A) is the expanding remnant of a corecollapse supernova that exploded about 330 years ago in the constellation Cassiopeia.

Cas A Supernova Remnant in X-rays

Chandra X-ray Observatory

Quite remarkably, X-ray satellites have identified the neutron star left behind at the center of the Cas A supernova explosion.

Cooling for the Cas A Neutron Star

The most direct way to measure superconducting properties in neutron stars is to *measure their cooling rates*.

This is possible using X-ray observatories such as Chandra.

The adjacent plot shows cooling curves measured by Chandra for the Cas A neutron star, which was born about 330 years ago.

Generic Origin of Superconductivity

The Cooper instability is easily recognized across all these different subfields of physics.

- It is difficult to believe that this is accidental.
- Microscopically these are fundamentally different systems, but we note three common features:

(1) The SC/SF results from a collective condensate of Cooper pairs.

(2) The SC/SF is *emergent*, and thus depends only abstractly on the underlying microscopic physics.

(3) The normal state involves many degrees of freedom but the SC/SF state has only a few (*enormous Hilbert space truncation*).

Generic Origin of Superconductivity

A theory accounting for this diversity of superconducting and superfluid (SC/SF) behavior must exhibit

1. A robust Cooper instability arising in both Fermi-liquid and other contexts,

2. Accommodation of SC/SF and other emergent modes, implying quantum phase transitions among these modes.

3. Limits corresponding to pure SC and to pure collective modes competing with SC.

Such a theory can depend only *parametrically and not fundamentally* on the underlying microscopic physics.

Analogy with General Relativity

General Relativity:

1. Gravity is a property of space, not of microscopic physics within that space.

2. Thus gravity is universal, independent of the microscopic structure of masses.

Superconductivity / Superfluidity:

1. SC/SF is a property of the (collective Hilbert) space, not of microscopic physics.

2. Thus SC/SF is universal, with similar properties for all microscopic structure.

In some sense, the collective subspaces of these very different systems must be the "same subspace".

This can't be true literally, but can be *true effectively* if the collective subspaces correspond to *truncations dictated by the same dynamical symmetries*. Then

- *SC/SF matrix elements* <f |Q| f> *can be equivalent, even though*
- the operators Q are very different and
- the wavefunctions f are very different.

Generic Origin of Superconductivity

We propose that all fermionic SC and SF results from a spontaneous reorganization of the Hilbert space that transcends microscopic details of the normal system.

- The generic structure of this reorganized space accounts for SC and SF in its myriad forms.
- Normal-state structure influences the reorganized space, but only parametrically.
- The pair condensate competes for Hilbert space with other emergent modes.
- Thus conventional BCS states are *highly atypical*, representing limiting cases where the space factorizes and SC/SF decouples from other emergent modes.

We propose that the reorganized low-energy Hilbert space is characterized by a symmetry, and that this symmetry accounts for the highly universal properties observed for fermionic superconductivity and superfluidity.

Generic Origin of Superconductivity

This implies a relationship that can be expressed in terms of *symmetries of the effective Hamiltonian* for the emergent states.

Extensive experience in many-body physics then suggests that

- The symmetries are *dynamical*.
- Hence they are described by *Lie algebras and Lie groups*.

The group generators are in *one-to-one correspondence with physical operators* for emergent states.

Since we typically deal with bound states, possibilities are further restricted to *compact Lie algebras*.

The Compact Groups with Fewer than 50 Generators

Group	Instances	Generators	Dimensionality*
SO(2 <i>n</i> +1)	SO(3)	3	$Dim = 2n^2 + n$
	SO(5)	10	
	SO(7)	21	
	SO(9)	36	
SO(2 <i>n</i>)	SO(2)	1	$\text{Dim} = 2n^2 - n$
	SO(4)	6	
	SO(6)	15	
	SO(8)	28	
	SO(10)	45	
SU(<i>n</i>)	SU(2)	3	$Dim = n^2 - 1$
	SU(3)	8	
	SU(4)	15	
	SU(5)	25	
	SU(6)	35	
	SU(7)	48	
Sp(2 <i>n</i>)	Sp(2)	3	$Dim = 2n^2 + n$
	Sp(4)	10	
	Sp(6)	21	
	Sp(8)	36	

*General expression for dimensionality of the generator space.

Restricting the Highest Symmetries

Dynamical Symmetries and Puzzles

The right figure is analogous to the highly constrained way in which (say) antiferromagnetism and superconductivity can fit together within an SU(4) algebra, when they compete for the same Hilbert space in a high-temperature superconductor.

More Than Taxonomy Is Required

A unified picture of superconductivity and superfluidity must do more than classify states. For example, it must

- 1. Permit matrix elements of physical observables to be calculated systematically across fields.
- 2. Account dynamically for how the Cooper instabilities and other instabilities arise in a natural way, in systems that are not necessarily Fermi liquids (for example, in Mott insulators).
- 3. Explain the corresponding quantum phase transitions.
- 4. Explain the relationship between conventional BCS SC and the variety of unconventional SC/SF behaviors.

We now give examples illustrating all of these properties for dynamical symmetries of the SC/SF state.

The FDSM in Nuclear Structure

All heavy nuclei seem to be described by one of two highest symmetries, *Sp(6) or SO(8)*, and associated dynamical-symmetry subgroup chains.

Two *dynamical symmetry subgroup chains* are indicated by shading.

Each chain corresponds physically to a *collective (emergent) mode* of the system.

Rare-Earth B(E2) Values

B(E2) electromagnetic transition rates between ground-band 0⁺ and 2⁺ states in the heavy rare-earth nuclei.

Solid lines are FDSM calculations with the only adjustable parameter an overall scale (effective charge)

Dashed lines indicate an approximation treating the fermion pairs as bosons: *Cooper pairs are not bosons!*

SU(4) Cuprate Phase Diagram

Quantum Phase Transitions and Fundamental Instabilities

Staggered Magnetization Q

AF

x=x_q Doping x → Antiferromagnetic instability

$$\frac{\partial Q}{\partial x}\bigg|_{x=x_q} = -\frac{1}{4} \frac{x_q + x_q^{-1} - 2x}{[(x_q - x)(x_q^{-1} - x)]^{1/2}}\bigg|_{x=x_q} = -\infty$$

Quantum phase transition between ground states having finite AF correlation and those having vanishing AF correlations. Dynamical symmetries imply *quantum phase transitions.*

In some cases these lead to *quantum critical points* and to *quantum critical phases.*

Example: A great deal of cuprate behavior is explained by the two SU(4) instabilities illustrated adjacent left.

Quantum critical behavior does not "cause" SC, but it is a consequence of the emergent dynamical symmetry that does.

SO(5) Quantum Critical Phase and the Pseudogap State

The SO(5) critical dynamical symmetry implies a *quantum critical phase:*

(1) Generalizes the idea of a quantum critical point to a quantum critical phase.

(2) Leads to large fluctuations in both SC and AF, and to extreme susceptibility to perturbations in the underdoped region.

(3) Leads to a pseudogap and dominates its behavior.

An Aside: Generalization to other Strongly-Correlated Fermionic Systems

An SU(4) \supset SO(5) critical dynamical symmetry implying a *quantum critical phase*.

Graphene SU(4) \supset SO(5) critical phase interpolates between *antiferromagnetic* and *Kekulé distortion* modes for graphene in a strong perpendicular magnetic field.

Recovering Conventional and Unconventional BCS

Symmetry-Dictated Truncation

Dynamical symmetries reflect a truncation of the Hilbert space to a tiny subspace of very specific structure (defined by a symmetry).

It is our contention that the relatively universal properties of all forms of SC and SF are a consequence of the symmetry of the emergent collective subspace.

Summary

- The varieties of observed SC/SF *cannot* be unified through the underlying microscopic physics, which can differ radically between systems.
- It *can* be understood in terms of dynamical symmetries implemented through Lie algebras having a relatively small number of generators.
- The Lie algebras represent *dynamical symmetries of the emergent states*, with the microscopic structure entering only *parametrically*.
- Through such methods we have thus far demonstrated a comprehensive and unified description of superconductivity in
 - Normal BCS superconductors in condensed matter physics
 - Cuprate and iron-based superconductors in condensed matter physics
 - A broad range of nuclear structure physics

We propose that these methods can provide *a unified description of all superconductivity and superfluidity,* from condensed matter, to nuclei, to neutron stars.

Analogy with Renormalization Group

Renormalization Group:

- 1. Operators can be classified as *relevant* and as *irrelevant*..
- 2. Under RG flow to a critical point, the irrelevant operators lose their influence, leaving the properties determined only by the relevant operators.
- 3. Physically, the irrelevant operators can play at most a parametric role because large fluctuations near the critical point permit only their averages to contribute.

Superconductity / Superfluidity:

- 1. Operators can be classified as *relevant* and as *irrelevant*..
- 2. Under "flow" in the group space (taking successive subgroups), irrelevant operators drop out, leaving only the relevant operators (generators of the final SC/SF symmetry).
- 3. Physically, the irrelevant operators can play only a parametric role because they can at most renormalize the parameters of the collective subspace.