Nucleon Pair Approximation to the nuclear Shell Model

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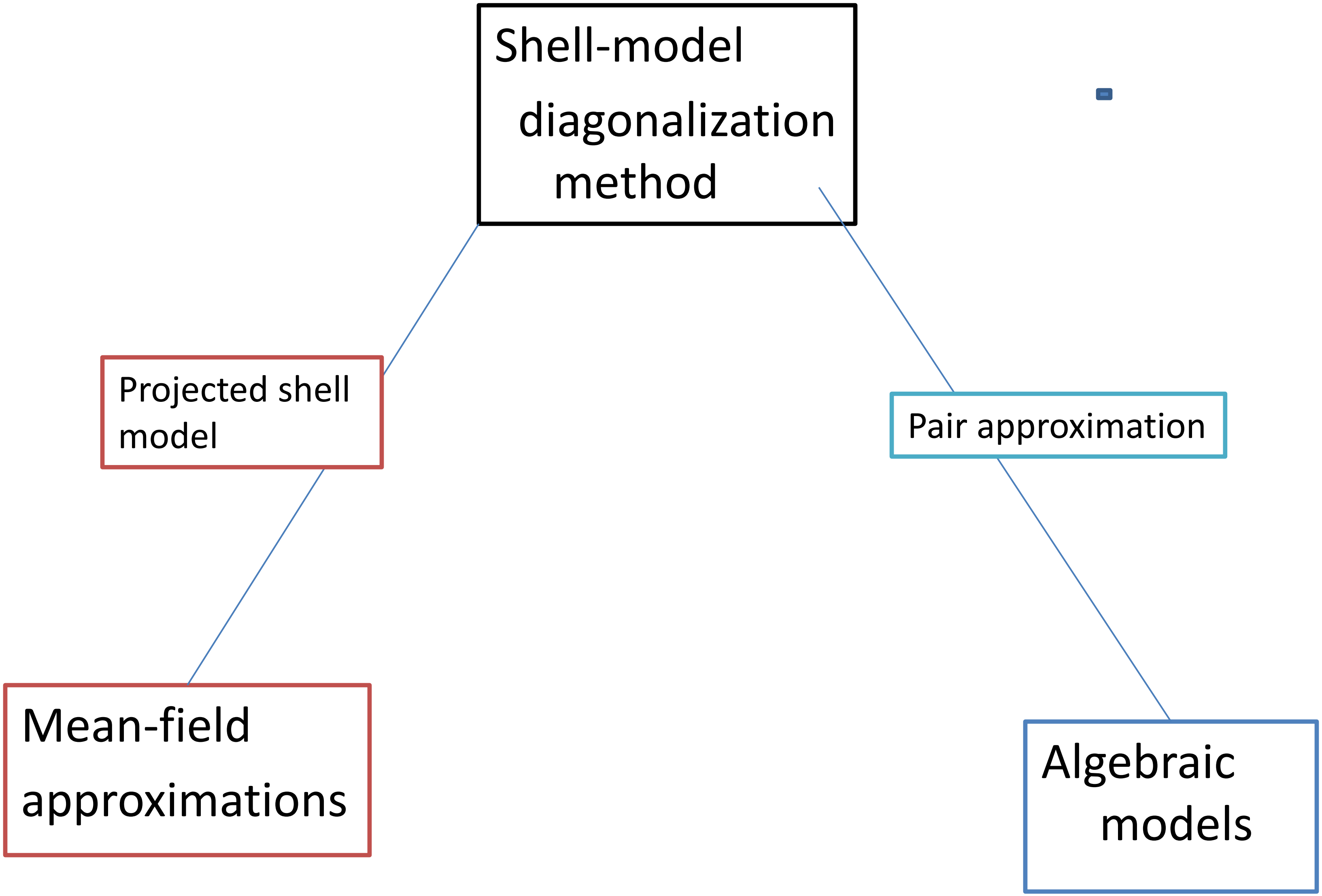
in collaboration with my teachers (J. Q. Chen from Nanjing, and A. Arima from Tokyo), my collaborators (N. Yoshinaga from Saitama, S. Yamaji from RIKEN), and my students (Y. Lei, Z. Y. Xu, H. Jiang, G. J. Fu) over the last 20 years.

For nuclear structure, exact solutions are impossible and approximations based on truncation schemes are indispensable.

The other day Yang SUN’s talk presented one possible approach, i.e., truncation based on combination of mean field approach and the shell model.

Nuclear structure models

- Shell-model diagonalization method
  - Based on quantum mechanical principles
  - Growing computer power helps extending applications
  - A single configuration contains no physics
  - Huge basis dimension required, severe limit in applications
- Mean-field approximations
  - Applicable to any size of systems
  - Fruitful physics around minima of energy surfaces
  - No configuration mixing, results depend on quality of mean-field
  - States with broken symmetry, cannot study transitions
- Algebraic models
  - Based on symmetries, simple and elegant
  - Serve as important guidance for complicated calculations



Shell-model diagonalization method
Projected shell model
Pair approximation
Mean-field approximations
Algebraic models

Here the motivation is the same: a wise selection of relevant configurations from the enormous shell model space. This method is called pair approximation which is based on the pairing phenomenon in nuclear structure, originated from the short-range and attractive feature of interactions between nucleons.

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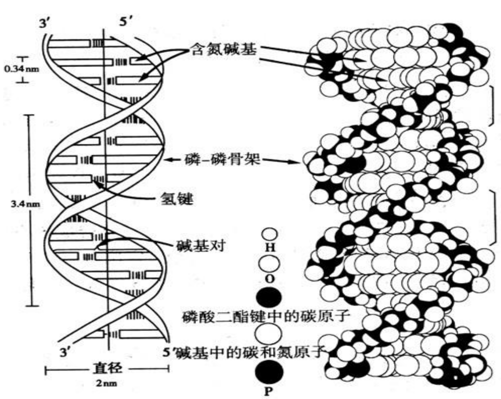
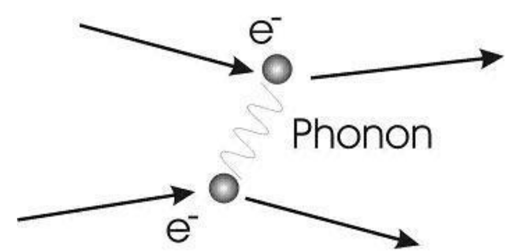

3′
5′
0.34nm
含氮碱基
小沟
磷-磷骨架
氢键
3.4nm
大沟
碱基对
H
O
磷酸二酯键中的碳原子
3′
5′
碱基中的碳和氮原子
P
直径
2nm


e⁻
Phonon
e⁻

Cooper pair

General picture

The common feature seems to be the existence of an attractive (effective) interaction between the individual “building blocks” of the system. Then the system achieves the smallest energy in the pairing configuration and becomes more stable.

This is particularly true for atomic nuclei which we shall discuss today.

Outline

- introduction: why ?
- formulation: how to do ?
- Validity: how good ?
- Applications and current situation
- Future and perspective
- (regularity of low-lying states with random interactions) if I have time

I. introduction:
why nucleon-pair approximation for low-lying states of atomic nuclei ?

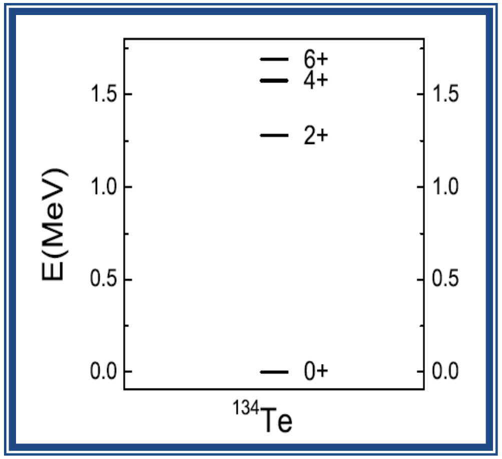
The strong interaction between nucleons are short-range and attractive.

Why short range ? The total binding energy of atomic nuclei is, at a very rough approximation, proportional to the mass number A (i.e., the number of total nucleons, protons and neutrons). If the interaction were long-range, then the total binding energy would have been approximately proportional to A^2-A, or roughly A^2.
One simple model for short-range interaction is delta force.

Why attractive ? There exists a strong repulsion due to Coulomb force between protons . In order that atomic nuclei are bond states, the strong interaction between nucleons have to be attractive to have the balance against the Coulomb force.

The interaction between nucleons can therefore take the following form:

$$V= - G \delta (\vec{r}_1 - \vec{r}_2)$$



two valence protons outside Sn-132

Pair approximation

J=0,2 have the lowest energy

Pairs with lowest energy are key building blocks

This explains why pair approximation are useful.

If all possible pairs are considered, the pair basis present the exact shell model results.

Nucleon pair approximation

A long history since the nuclear shell model:

1952：single-j S

1971：many-j S

1981：one or two broken pairs

1992：SD pairs for even systems

2000 ： odd-A and odd-odd nuclei

2013 ： pair approximation with isospin-symmetry

II.Formulation

Many people including a number of people in this workshop have tried pair approximations for different purposes. Today I have to skip their efforts and concentrate on what we have done these years … …

The details of the NPA formulation can be found in the papers as below. Here I would like to point out something which might be useful to others in general and the key part.

[1] J. Q. Chen, Nucl. Phys. A 626, 686 (1997).

[2] Y. M. Zhao, N. Yoshinaga, S. Yamaji, J. Q. Chen, and A. Arima, Phys. Rev. C 62, 14304 (2000).

[3] G. J. Fu, Y. M. Zhao, and A. Arima, Phys. Rev. C 87, 044310 (2013).

[1] Factorization of commutators: The Wick theorem for coupled operators
J. Q. Chen, B. Q. Chen, and A. Klein, Nucl. Phys. A 554, 61 (1993);

$$[(F^a\times F^b)^e, F^c]^d = \sum_f U(abdc;ef)(F^a\times[F^b,F^c]^f)^d$$
$$+\sum_f U(abcd;ef)$$
$$\times(-1)^{a+d-e-f}([F^a,F^c]^f\times F^b)^d,$$

$$A^{r\dagger}_\nu = \sum_{ab} y(abr)A^{r\dagger}_\nu(ab), \quad A^{r\dagger}_\nu(ab) = (C_a^\dagger\times C_b^\dagger)^r_\nu,$$

$$A^{J_N\dagger}_{M_N}(\tau)|0\rangle \equiv A^{J_N\dagger}_{M_N}(r_0r_1r_2\cdots r_N, J_1J_2\cdots J_N)|0\rangle$$
$$= \left(\cdots\left(\left((A^{r_0\dagger}\times A^{r_1\dagger})^{(J_1)}\times A^{r_2\dagger}\right)^{(J_2)}\times A^{r_3\dagger}\right)^{(J_3)}\times\cdots\times A^{r_N\dagger}\right)^{(J_N)}_{M_N}|0\rangle,$$

$$[\tilde{A}^{J_N}(r_i,J_i),Q^t]^{L_N}_{M_N} = \sum_{M'_N\sigma} C^{L_N\,M_N}_{J_N M'_N,\,t\sigma}\left[\tilde{A}^{J_N}_{M'_N}(r_i,J_i),Q^t_\sigma\right]$$
$$=\sum_{k=N}^{1}\sum_{r'_k L_k\ldots L_{N-1}} Q_N(t)\ldots Q_{k+1}(t)$$
$$\times\tilde{A}(r_1\ldots r'_k\ldots r_N; J_1\ldots J_{k-1}L_k\ldots L_N M_N),$$

[2] The Wick theorem for coupled fermion clusters
J. Q. Chen, Nucl. Phys. A 562, 218 (1993).

In these two papers, Chen studied the algorithm of evaluating the commutators of **coupled** fermions (and bosons). This is a very useful technique in practice.

The formulation of the nucleon pair approximation is an application of this technique. Here the basis and the Hamiltonian are written in terms of coupled pairs. The key calculation is how to calculate the matrix elements in the basis which is built up by using nucleon pairs.

Phenomenological (pairing + Quadruple):

$$H = H_1 + \sum_{\sigma} \left(\sum_{s=0,2} G_{s\sigma} \mathcal{A}^{s\sigma\,\dagger} \cdot \mathcal{A}^{s\sigma} + \kappa_\sigma Q_\sigma \cdot Q_\sigma \right) + \kappa_{\pi\nu} Q_\nu \cdot Q_\pi$$

$$A^{J_N\,\dagger}_{M_{J_N}}(r_1 r_2 \cdots r_N, J_1 J_2 \cdots J_N)|0\rangle \equiv A^{J_N\,\dagger}_{M_{J_N}}|0\rangle = \left(\cdots \left(\left(A^{r_1\dagger} \times A^{r_2\dagger} \right)^{(J_2)} \times A^{r_3\dagger} \right)^{(J_3)} \times \cdots \times A^{r_N\dagger} \right)^{(J_N)}_{M_{J_N}} |0\rangle.$$

effective interaction :

$$H_2 = \frac{1}{4} \sum_{abcd,JT} \sqrt{(1+\delta_{ab})(1+\delta_{cd})}\, G_{JT}(ab,cd) \sum_{M_J M_T} A^{JT\dagger}_{M_J M_T}(ab)\, A^{JT}_{M_J M_T}(cd)$$

$$\begin{aligned} A^{J_N T_N\dagger}_{M_N M_T}(\tau)|0\rangle &\equiv A^{J_N T_N\dagger}_{M_N M_T}(r_0 t_0, r_1 t_1, r_2 t_2, \ldots, r_N t_N; J_1 T_1, J_2 T_2, \ldots, J_N T_N)|0\rangle \\ &= \left(\cdots \left(\left(\left(A^{(r_0 t_0)\dagger} \times A^{(r_1 t_1)\dagger} \right)^{(J_1 T_1)} \times A^{(r_2 t_2)\dagger} \right)^{(J_2 T_2)} \times A^{(r_3 t_3)\dagger} \right)^{(J_3 T_3)} \times \cdots \times A^{(r_N t_N)\dagger} \right)^{(J_N T_N)}_{M_N M_T} |0\rangle. \end{aligned}$$

$$
\begin{aligned}
&\langle jr_1\cdots r_N, J_1\cdots J_N|V_{\mathrm{P}}^{(s)}|j_0s_1\cdots s_N, J_1'\cdots J_N'\rangle\\
&\quad = G_s\delta_{J_NJ_N'}\sum_{k=N}^{1}\Bigg[\hat{s}\varphi_0\delta_{r_k,s}\langle jr_1\cdots r_{k-1}sr_{k+1}\cdots r_N, J_1\cdots J_N|j_0s_1\cdots s_N, J_1'\cdots J_N'\rangle\\
&\qquad - G_s\sum_{i=k-1}^{0}\sum_{tr_i'L_i\cdots L_{k-1}}(-1)^{t-s-r_k}\frac{\hat{t}}{\hat{r}_k}U(J_{k-1}tJ_ks;\, L_{k-1}r_k)Q_{k-1}(t)\cdots Q_{i+1}(t)\bar{M}_i(tr_i')\\
&\qquad \times\langle jr_1\cdots(r_i')_{\mathcal{B}}\cdots r_{k-1}sr_{k+1}\cdots r_N, J_1\cdots J_{i-1}L_i\cdots L_{k-1}J_k\cdots J_N|j_0s_1\cdots s_N, J_1'\cdots J_N'\rangle\Bigg].
\end{aligned}
$$

$(r_i')_{\mathcal{B}}$ represents the pair $\tilde{\mathcal{B}}^{r_i'} = -\big[\tilde{A}^{r_i}, [\tilde{A}^{r_k}, \mathcal{A}^{s_N\dagger}]^t\big]$

$$
\begin{aligned}
&\langle jr_1\cdots r_N, J_1\cdots J_N|\kappa_tQ^t\cdot Q^t|j_0s_1\cdots s_N, J_1'\cdots J_N'\rangle\\
&\quad = \kappa_t\sum_{k=N}^{0}\sum_{r_k'}(-1)^{r_k'-r_k}\frac{\hat{r}_k'}{\hat{r}_k}\langle jr_1\cdots(r_k)_{\mathbf{B}}\, r_{k+1}\cdots r_N, J_1\cdots J_N|j_0s_1\cdots s_N, J_1'\cdots J_N'\rangle\\
&\qquad + \kappa_t(-)^t\sum_{k=N}^{1}\sum_{i=k-1}^{0}\sum_{r_i'r_k'L_i\cdots L_{k-1}}2\frac{\hat{r}_k'}{\hat{r}_k}U(J_{k-1}tJ_kr_k';\, L_{k-1}r_k)Q_{k-1}(t)\cdots Q_{i+1}(t)\bar{M}_i(tr_i')\\
&\qquad \times\langle jr_1\cdots(\mathbf{r}_i')r_{i+1}\cdots(\mathbf{r}_k')r_{k+1}\cdots r_N, J_1\cdots J_{i-1}L_i\cdots L_{k-1}J_k\cdots J_N|j_0s_1\cdots s_N, J_1'\cdots J_N'\rangle.
\end{aligned}
$$

Wick theorem for operators with double couplings

$$\left((A\times B)^{(e)},C\right)^{(d)} = \left\|\sum_f (-)^{a+b+c+d}\,\hat{e}\hat{f}\begin{Bmatrix} a & b & e \\ c & d & f \end{Bmatrix}\right\| (A\times(B,C)^{(f)})^{(d)}$$
$$+\theta_{bc}\left\|\sum_f (-)^{b+c+e+f}\,\hat{e}\hat{f}\begin{Bmatrix} a & b & e \\ d & c & f \end{Bmatrix}\right\| ((A,C)^{(f)}\times B)^{(d)}$$

$$\left\|\sum_f (-)^{a+b+c+d}\,\hat{e}\hat{f}\begin{Bmatrix} a & b & e \\ c & d & f \end{Bmatrix}\right\| \equiv \sum_{J_f T_f} (-)^{J_a+J_b+J_c+J_d}\,(-)^{T_a+T_b+T_c+T_d}\,\hat{J}_e\hat{J}_f\hat{T}_e\hat{T}_f\begin{Bmatrix} J_a & J_b & J_e \\ J_c & J_d & J_f \end{Bmatrix}\begin{Bmatrix} T_a & T_b & T_e \\ T_c & T_d & T_f \end{Bmatrix}$$

$$\mathrm{V} = \sum_{JT}\sum_{j_1\leqslant j_2}\sum_{j_3\leqslant j_4}\frac{V_{JT}(j_1j_2j_3j_4)}{\sqrt{(1+\delta_{j_1j_2})(1+\delta_{j_3j_4})}}\sum_{m\tau}A^{(JT)}_{m\tau}(j_1j_2)^{\dagger}A^{(JT)}_{m\tau}(j_3j_4)$$
$$= \sum_{JT}\hat{J}\hat{T}\sum_{j_1\leqslant j_2}\sum_{j_3\leqslant j_4}\frac{V_{JT}(j_1j_2j_3j_4)}{\sqrt{(1+\delta_{j_1j_2})(1+\delta_{j_3j_4})}}\left(A^{(JT)}(j_1j_2)^{\dagger}\times\tilde{A}^{(JT)}(j_3j_4)\right)^{(0)}$$

Examples of calculating the two-body matrix elements

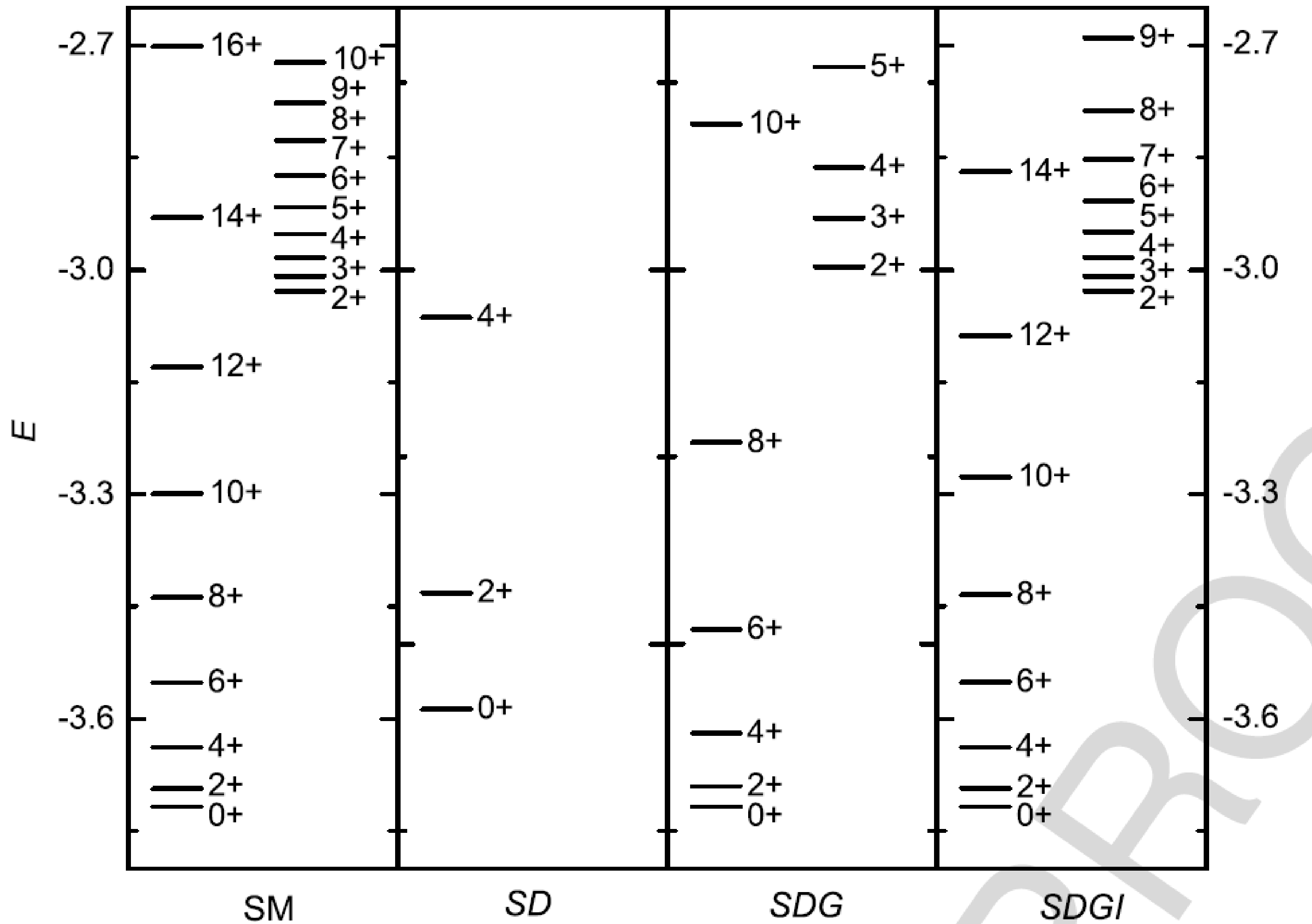
$$\begin{aligned}&\langle r_1\cdots r_N,\mathbb{J}_1\cdots\mathbb{J}_N|(A_1^{(s)\dagger}\times\tilde{A}_2^{(s)})^{(0)}|s_1\cdots s_N,\mathbb{J}'_1\cdots\mathbb{J}'_N\rangle\\ =\;&\left\|\delta_{\mathbb{J}_N\mathbb{J}'_N}\right\|\langle 0\;|\mathcal{A}^{(\mathbb{J}_N)}A^{(\mathbb{J}'_N)\dagger}(s_1\cdots s_N,\mathbb{J}'_1\cdots\mathbb{J}'_N)|\;0\rangle\;,\end{aligned}$$

$\mathcal{A}^{(\mathbb{J}_N)}$ is the annihilation operator corresponding to $(\tilde{A}^{(\mathbb{J}_N)}(r_1\cdots r_N,\,\mathbb{J}_1\cdots\mathbb{J}_N),\,(A_1^{(s)\dagger}\times\tilde{A}_2^{(s)})^{(0)})^{(\mathbb{J}_N)}$

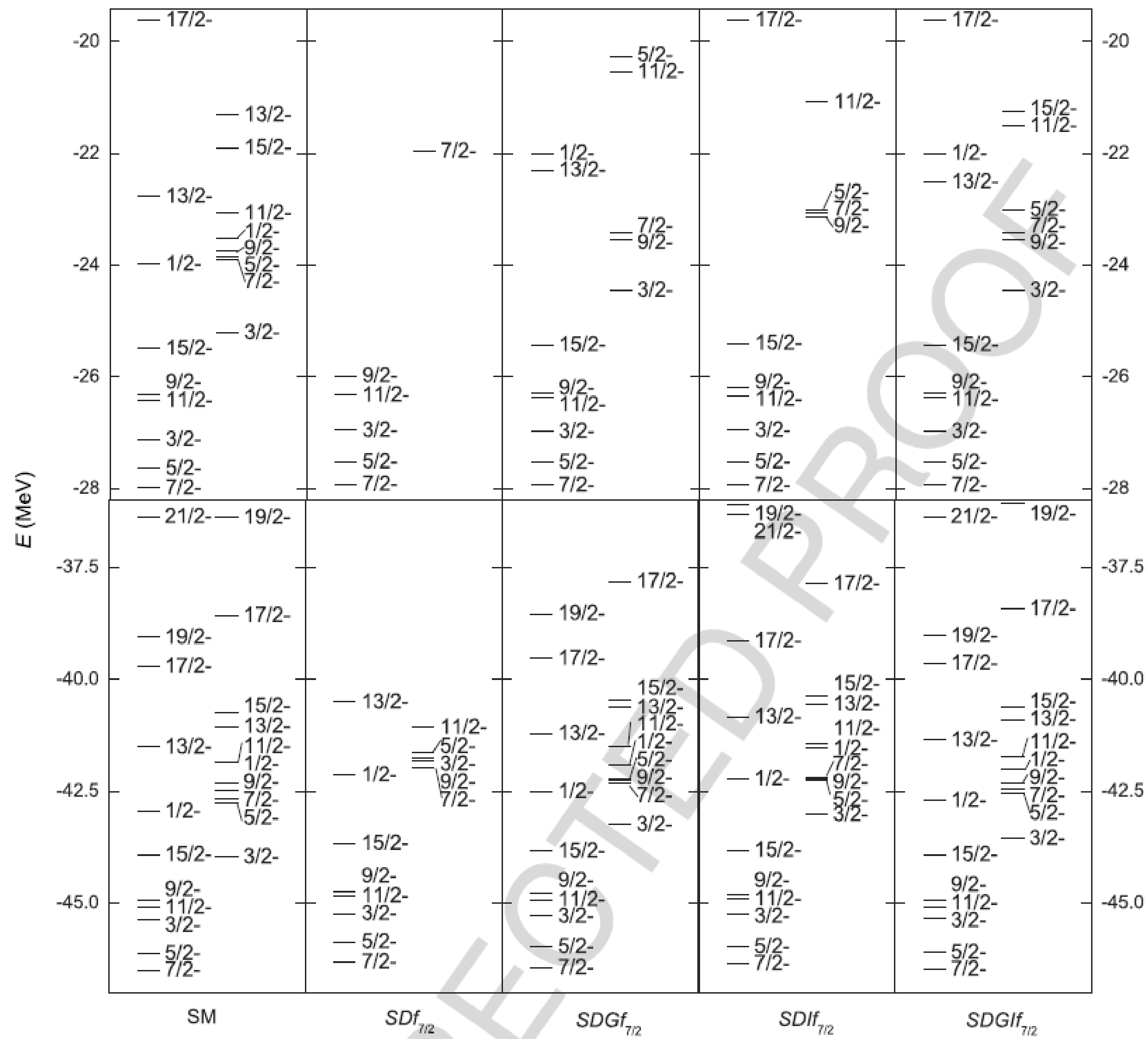
$$\begin{aligned}&\left(\tilde{A}^{(\mathbb{J}_N)}(r_1\cdots r_N,\mathbb{J}_1\cdots\mathbb{J}_N),(A_1^{(s)\dagger}\times\tilde{A}_2^{(s)})^{(0)}\right)^{(\mathbb{J}_N)}\\
=&\sum_{i=N}^{1}\left(\cdots\left(\left(\tilde{A}^{(\mathbb{J}_{i-1})}(r_1\cdots r_{i-1},\mathbb{J}_1\cdots\mathbb{J}_{i-1})\times\left(\tilde{A}^{(r_i)},(A_1^{(s)\dagger}\times\tilde{A}_2^{(s)})^{(0)}\right)^{(r_i)}\right)^{(\mathbb{J}_i)}\times\tilde{A}^{(r_{i+1})}\right)^{(\mathbb{J}_{i+1})}\times\cdots\times\tilde{A}^{(r_N)}\right)^{(\mathbb{J}_N)}\\
=&\;2\times\sum_{i=N}^{1}\left\|\frac{\delta_{sr_i}}{\hat{s}}\right\|\sum_{ab}y(abr_i)y_1(abs)\times\tilde{A}^{(\mathbb{J}_N)}(r_1\cdots r_{i-1}(\mathrm{s})r_{i+1}\cdots r_N,\mathbb{J}_1\cdots\mathbb{J}_N)\\
&-4\times\sum_{i=N}^{2}\left\|\sum_f(-)^{r_i+f+s}\frac{\hat{f}}{\hat{r}_i\hat{s}}\sum_{t_{i-1}}U(\mathbb{J}_{i-1}f\mathbb{J}_ist_{i-1}r_i)\right\|\sum_{k=i-1}^{1}\left\|\sum_{r'_kt_k\cdots t_{i-2}}(\overline{Q}(f,t_{i-1})\cdots\overline{Q}(f,t_{k+1})\overline{M}(f,r'_k))\right\|\\
&\times\left(\cdots\left(\left(\cdots\left((\tilde{A}^{(\mathbb{J}_{k-1})}(r_1\cdots r_{k-1},\mathbb{J}_1\cdots\mathbb{J}_{k-1})\times\tilde{\mathrm{A}}^{(r'_k)})^{(t_k)}\times\tilde{A}^{(r_{k+1})}\right)^{(t_{k+1})}\times\cdots\right.\right.\right.\\
&\left.\left.\left.\times\tilde{A}^{(r_{i-1})}\right)^{(t_{i-1})}\times\tilde{A}_2^{(s)}\right)^{(\mathbb{J}_i)}\times\cdots\times\tilde{A}^{(r_N)}\right)^{(\mathbb{J}_N)}\;,\qquad(36)\end{aligned}$$

III. Validity of the NPA

In this part, I would like to show a few typical cases on how the NPA works, i.e., how well the NPA results reproduce the exact shell model results.



Calculated energy levels of six nucleons in a single-j ($j = 19/2$) shell, with an attractive quadrupole–quadrupole interaction

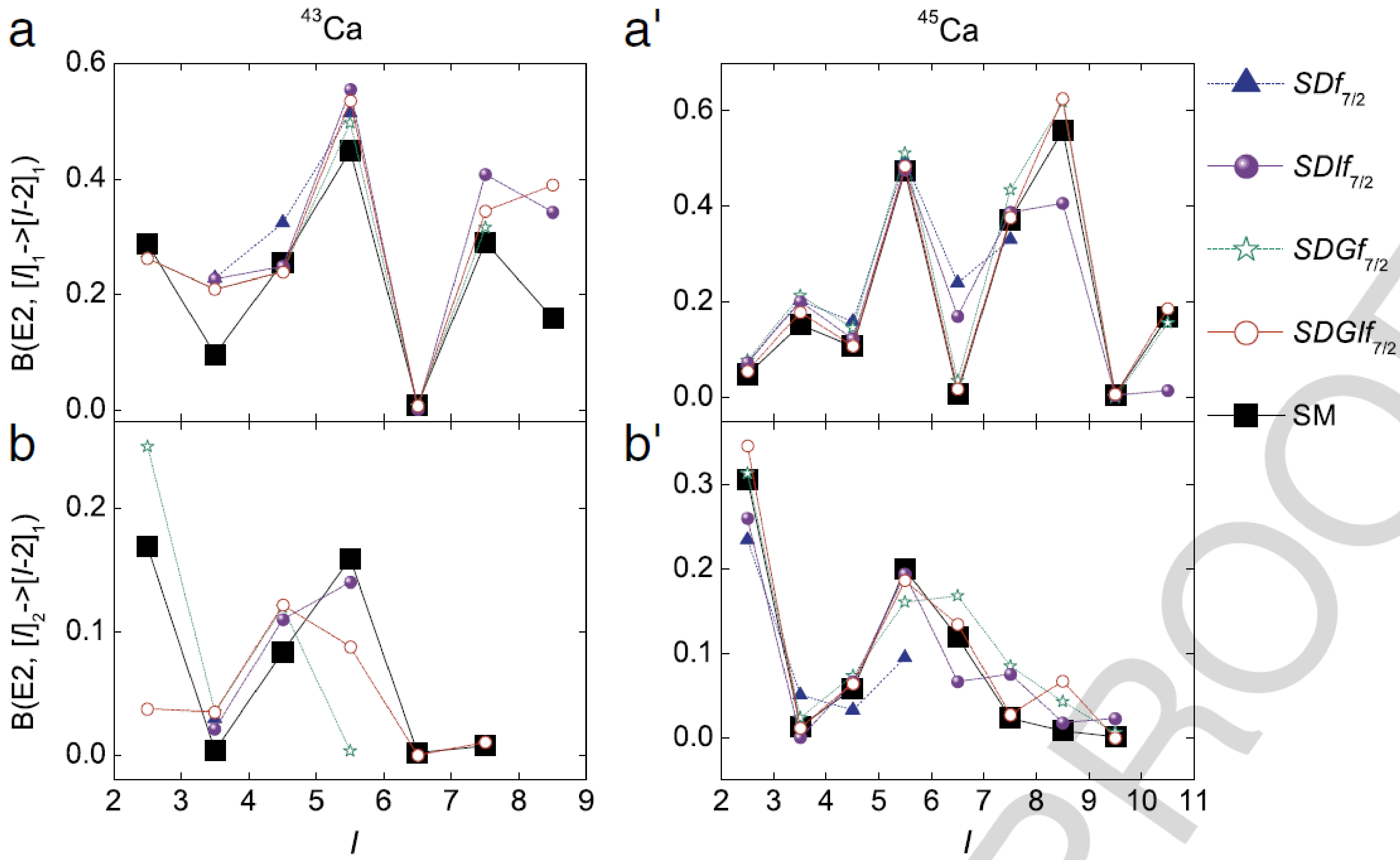


low-lying spectra of ^{43}Ca and ^{45}Ca

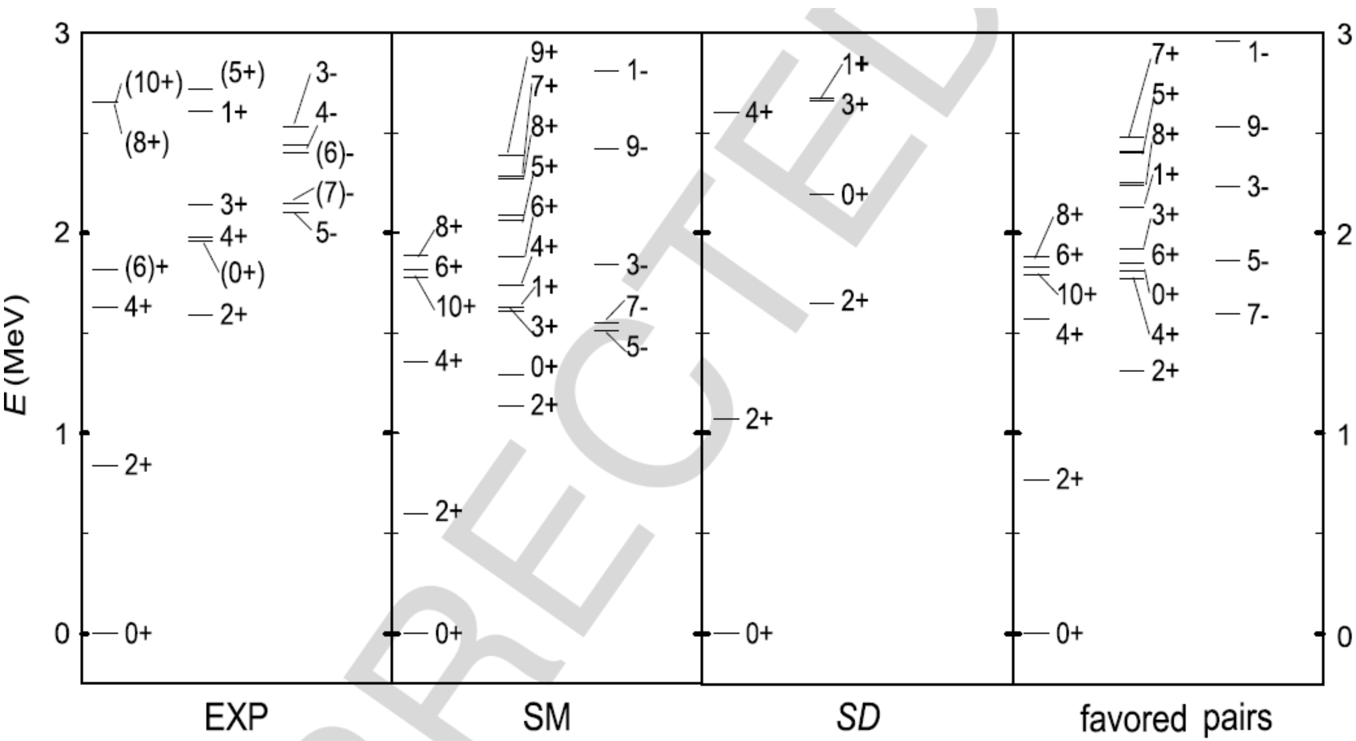
the GXPF1A interaction

The dimension of the configuration space

	SM	$SDf_{7/2}$	$SDGf_{7/2}$	$SDIf_{7/2}$	$SDGIf_{7/2}$
	^{43}Ca				
$1/2^-$	12		1		1
$3/2^-$	25	1	2	1	2
$5/2^-$	28	1	2	2	3
$7/2^-$	27	2	3	3	4
$9/2^-$	23	1	2	2	3
$11/2^-$	16	1	2	2	3
$13/2^-$	8		1	1	2
$15/2^-$	5		1	1	2
$17/2^-$	1			1	1
	^{45}Ca				
$1/2^-$	107	1	5	3	9
$3/2^-$	198	3	10	7	18
$5/2^-$	253	3	12	10	24
$7/2^-$	271	5	15	13	29
$9/2^-$	252	3	13	12	29
$11/2^-$	211	3	13	12	29
$13/2^-$	153	1	9	10	25
$15/2^-$	105	1	8	9	20
$17/2^-$	58		4	6	15
$19/2^-$	29		3	4	11
$21/2^-$	11		1	2	6



a
^{43}Ca
a'
^{45}Ca
b
b'
B(E2, [I]$_1$->[I-2]$_1$)
B(E2, [I]$_2$->[I-2]$_1$)
I
SDf$_{7/2}$
SDIf$_{7/2}$
SDGf$_{7/2}$
SDGIf$_{7/2}$
SM



E (MeV)
3
2
1
0
0+
2+
4+
(6)+
(8+)
(10+)
2+
(0+)
4+
3+
1+
(5+)
5-
(7)-
(6)-
4-
3-
EXP
0+
2+
4+
10+
6+
8+
2+
0+
3+
1+
4+
6+
5+
8+
7+
9+
5-
7-
3-
9-
1-
SM
0+
2+
4+
2+
0+
3+
1+
SD
0+
2+
4+
10+
6+
8+
2+
4+
0+
6+
3+
1+
8+
5+
7+
7-
5-
3-
9-
1-
favored pairs

Te-130

$\mathrm{spin}^{\mathrm{parity}}$	SM	*SD*	FP	$\mathrm{spin}^{\mathrm{parity}}$	SM	*SD*	FP
0^+	8 316	4	64	1^+	23 471	2	99
2^+	37 219	7	213	3^+	47 159	3	225
4^+	54 190	4	307	5^+	57 032	1	289
6^+	57 309		336	7^+	54 503		295
8^+	50 319		313	9^+	44 439		260
10^+	38 242		257				

$\mathrm{spin}^{\mathrm{parity}}$	*SD*	Favored pairs	$\mathrm{spin}^{\mathrm{parity}}$	*SD*	Favored pairs
0_1^+	0.961	0.980	2_2^+	0.209	0.585
2_1^+	0.813	0.945	3_1^+	0.021	0.842
4_1^+	0.533	0.903	1_1^+	0.038	0.116
6_1^+	0.046	0.956	4_2^+	0.212	0.845
8_1^+		0.961	6_2^+		0.956
10_1^+		0.965	5_1^+	0.080	0.925
			8_2^+		0.927
7_1^-		0.936	7_1^+		0.944
5_1^-		0.828	9_1^+		0.934
3_1^-		0.730	10_2^+		0.942
9_1^-		0.904			
1_1^-		0.555			

IV.Applications

There have been many numerical calculations, and the calculated results have been well received by experimentalists. This is just expected because the NPA basis is very simple. Only essential configurations are used.

Here I give a few examples.

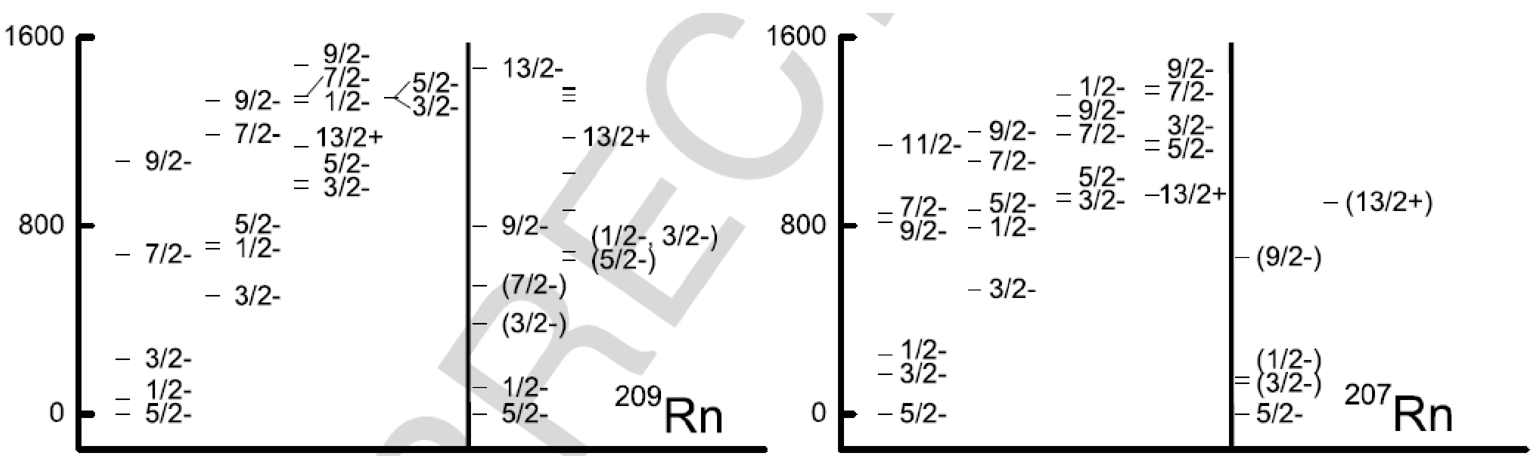
References	Collective pairs	Nuclei
Caprio, Luo et al. [54]	$v \le 3$	even and odd $^{21-39}$Ca.
Caprio, Luo et al. [55]	$v_\pi \le 2,\ v_\nu \le 2$	even–even $^{42-58}$Ca, even–even $^{44-60}$Ti, even–even $^{46-62}$Cr.
Rowe, Rosensteel [56]	$v = 0, 2, 4$	^{92}Mo, ^{94}Ru, ^{96}Pd.
Monnoye, Pittel et al. [65]	$v_\pi \le 2,\ v_\nu \le 3$	odd-mass $^{57-69}$Ni.
Sandulescu, Blomqvist et al. [69]	$S^N + S^{N-1}D$	even–even $^{104-112}$Sn.
Morales, Isacker et al. [70]	$S^N + S^{N-1}D$	even–even $^{102-130}$Sn.
Qi, Blomqvist et al. [116]	$(g_{9/2}g_{9/2})^{(9)}, T = 0$	^{92}Pd.
Xu, Qi et al. [117]	$(g_{9/2}g_{9/2})^{(9)}, T = 0$	^{92}Pd, ^{94}Ag, ^{96}Cd.
Fu, Shen et al. [115]	$(g_{9/2}g_{9/2})^{(9)}, T = 0$ and $SD, T = 1$	^{92}Pd, ^{94}Ag, ^{96}Cd.
Kwasniewicz, Brzostowski et al. [121]	$SS'DD'D''G$	$A = 59 - 62$nuclei, with $T = 0$ and 1.
Kartamyshev, Engeland et al. [149]	$v \le 2$	even and odd mass $^{134-142}$Sn.
Xu, Lei et al. [150]	$SD + G_\pi I_\pi K_\pi G_\nu$	even–even $^{202-206}$Pb, $^{204-210}$Po, $^{212-206}$Rn, $^{214-208}$Ra, odd-A $^{203-207}$Bi, $^{205-211}$At, $^{209-213}$Fr, odd-A ^{203,205}Pb, $^{205-209}$Po, $^{207-211}$Rn, ^{211,213}Ra.
Jia, Zhang et al. [143,151]	SD	both even and odd-A $^{124-139}$Sn, $^{126-142}$Te $^{128-144}$Xe, both even and odd-A $^{130-146}$Ba, $^{132-148}$Ce.
Jia, Zhang et al. [151]	SD	odd-mass $^{125-141}$Sb, $^{127-143}$I, $^{129-145}$Cs, $^{129-145}$La.
Pan, Ping et al. [152]	SO(6)-SD	$^{126-132}$Xe, $^{131-137}$Ba.
Chen, Luo et al. [153,154]	SD	even–even $^{128-134}$Xe, $^{130-136}$Ba, $^{126-132}$Te, $^{132-138}$Ce.
Luo, Zhang et al. [155]	SD	^{134}Ba.
Meng, Wang et al. [156]	SD	^{126}Xe, ^{128}Ba.
Zhao, Yamaji et al. [157]	SD	even–even $^{124-140}$Sn, $^{126-132}$Te, $^{128-144}$Xe, $^{130-136}$Ba, $^{132-138}$Ce.
Yoshinaga, Higashiyama [158]	SD	even–even $^{126-134}$Xe, $^{128-136}$Ba, $^{130-138}$Ce, $^{132-140}$Nd, odd-mass $^{129-133}$Xe, $^{131-135}$Ba, $^{133-137}$Ce.
Higashiyama, Yoshinaga et al. [159]	$SD + (h_{\frac{11}{2}}h_{\frac{11}{2}})^{(J)}_\nu$	^{132}Ba.
Takahashi, Yoshinaga et al. [160]	$SD + (h_{\frac{11}{2}}h_{\frac{11}{2}})^{(10)}_\nu$	^{132,134,136}Ce.
Higashiyama, Yoshinaga [161]	SD	even–even $^{126-134}$Xe, $^{128-136}$Ba, $^{130-138}$Ce, $^{132-140}$Nd, odd-mass $^{129-135}$Xe, $^{131-137}$Ba, $^{133-139}$Ce, $^{129-135}$Cs, odd-mass $^{131-137}$La, odd–odd ^{130,132}Cs, ^{132,134}La.
Lei, Xu et al. [162]	$SD + A^{(J-)}_\nu, J = 5, 6$	^{132}Ba.
Lei, Fu et al. [163]	$SD + (h_{\frac{11}{2}}h_{\frac{11}{2}})^{(J)}_\nu$ $+H^-I^-$	$^{128-136}$Ba, $^{130-138}$Ce.
Higashiyama, Yoshinaga et al. [164]	$SD + (h_{\frac{11}{2}}h_{\frac{11}{2}})^{(J)}_\nu$	^{130,132}Cs, ^{132,134}La.
Jiang, Lei et al. [165]	$SD + G_\nu$	even–even $^{102-130}$Sn.
Jiang, Fu et al. [166]	SDG	even–even $^{72-80}$Zn, odd-A $^{73-81}$Ga.
Zhang, Luo et al. [167]	SD	even–even $^{94-100}$Mo.
Yoshinaga, Higashiyama et al. [168]	$SDG + (g_{9/2}g_{9/2})^{(J)}$	even–even $^{76-80}$Ge, ^{78}Se.
Jiang, Shen et al. [169]	$SD + G_\nu$	even–even $^{198-202}$Pt, $^{200-204}$Hg, odd-A $^{199-203}$Pt, $^{201-205}$Hg, odd-A $^{197-201}$Ir, $^{199-203}$Au, $^{201-205}$Tl, odd–odd $^{198-202}$Ir, $^{200-204}$Tl, $^{202-206}$Tl.
Zerguine, Isacker [170]	$(g_{9/2}g_{9/2})^{(9)}, T = 0$	^{96}Cd.
Higashiyama, Yoshinaga [171]	SD	^{130,132}Cs, ^{132,134}La.
Jiang, Qi et al. [172]	SD	^{108,109}Te, ^{109}I.
Jiang, Lei et al. [173]	SD	even–even $^{102-130}$Sn, odd-A $^{101-109}$Sn and $^{123-131}$Sn.

$$G_{0\sigma} = -\alpha_\sigma \frac{27}{A}, \qquad G_{2\sigma} = \beta_\sigma \frac{G^0_\sigma}{r_0^4}, \qquad \kappa_\sigma = -\frac{1}{2}\chi \eta_\sigma^2, \qquad \kappa_{\nu\pi} = -\chi \eta_\nu \eta_\pi,$$

$$\eta_\nu = -\gamma_\nu \left[\frac{2(A-Z)}{A}\right]^{\frac{2}{3}}, \qquad \eta_\pi = \gamma_\pi \left(\frac{2Z}{A}\right)^{\frac{2}{3}},$$

$$\chi A^{\frac{5}{3}} = -\left\{242\left[1+\left(\frac{2}{3A}\right)^{\frac{1}{3}}\right] - \frac{10.9}{A^{\frac{1}{3}}}\left[\left[1+2\left(\frac{2}{3A}\right)^{\frac{1}{3}}\right]\left(19-0.36\frac{Z^2}{A}\right)\right]\right\}$$

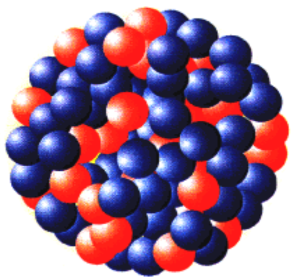
	α_π	β_π	γ_π		α_ν	β_ν	γ_ν
^{210}Po	1.1	3	0.6	^{206}Pb	0.9	2.7	0.6
^{212}Rn	1.1	2.5	0.8	^{204}Pb	0.85	2	0.6
^{214}Ra	1.1	2.5	0.8	^{202}Pb	0.9	2.7	0.7



$$|(\nu f_{5/2})^{-1} \times (2_1^+, \text{even–even core})\rangle$$

J^π	exp	cal	overlap	exp	cal	overlap
	^{205}Pb			^{207}Po		
$1/2^-$	2.33	47.4	0.728	68.57	54.9	0.718
$3/2^-$	576.19	512.2	0.970	392.99	505.2	0.966
$5/2^-$	751.43	728.2	0.935	685.79	723.8	0.927
$7/2^-$	703.43	693.1	0.991	588.31	682.5	0.988
$9/2^-$	987.63	1100.5	0.921	814.42	1060.7	0.873
	^{209}Rn			^{211}Ra		
$1/2^-$	110.3	63.09	0.730	134	68.58	0.736
$3/2^-$	382.3	499.31	0.962	295	503.03	0.962
$5/2^-$	652.6	724.47	0.926	–	726.99	0.929
$7/2^-$	547.1	676.1	0.985	–	676.22	0.985
$9/2^-$	797.8	1070.89	0.934	–	1071.76	0.930

	μ moment		$\mathcal{Q}$ moment			μ moment		$\mathcal{Q}$ moment	
	cal	exp	cal	exp		cal	exp	cal	exp
^{205}Pb					^{203}Pb				
$3/2_1^-$	−1.039		+0.147		$3/2_1^-$	−1.208		+0.143	
$5/2_1^-$	+0.771	+0.7117(4)[a]	+0.229	+0.226(37)[a]	$5/2_1^-$	+0.803	+0.6864(5)[d]	+0.064	+0.095(52)[d]
$7/2_1^-$	+0.656		+0.148		$7/2_1^-$	+0.015		−0.064	
$13/2_1^-$	+0.546		+0.505		$13/2_1^-$	+0.668		+0.229	
$17/2_1^-$	+2.314		+0.431		$17/2_1^-$	−0.131		+0.322	
$13/2_1^+$	−1.518	−0.975(40)[b]	+0.384	0.30(5)[c]	$13/2_1^+$	−1.517		+0.398	
^{209}Po					^{207}Po				
$1/2_1^-$	+0.426	0.68(8)[s]	–		$1/2_1^-$	+0.427		–	
$3/2_1^-$	−1.286		+0.165		$3/2_1^-$	−1.035		+0.169	
$5/2_1^-$	+0.882		+0.231		$5/2_1^-$	+0.776	+0.79(6)[d]	+0.268	+0.28[e]
$13/2_1^-$	+5.803	6.13(9)[d]	−0.148		$13/2_1^-$	+4.629		+0.320	
$17/2_1^-$	+7.655	7.75(5)[d]	−0.706	0.39(8)[d]	$17/2_1^-$	+6.258		+0.002	
$13/2_1^+$	−1.518		+0.369		$13/2_1^+$	−1.516	−0.910(14)[d]	+0.453	

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NATURE | LETTER
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Evidence for a spin-aligned neutron–proton paired phase from the level structure of ^{92}Pd

B. Cederwall, F. Ghazi Moradi, T. Bäck, A. Johnson, J. Blomqvist, E. Clément, G. de France, R. Wadsworth, K. Andgren, K. Lagergren, A. Dijon, G. Jaworski, R. Liotta, C. Qi, B. M. Nyakó, J. Nyberg, M. Palacz, H. Al-Azri, A. Algora, G. de Angelis, A. Ataç, S. Bhattacharyya, T. Brock, J. R. Brown, P. Davies, A. Di Nitto, Zs. Dombrádi, A. Gadea, J. Gál, B. Hadinia, F. Johnston-Theasby, P. Joshi, K. Juhász, R. Julin, A. Jungclaus, G. Kalinka, S. O. Kara, A. Khaplanov, J. Kownacki, G. La Rana, S. M. Lenzi, J. Molnár, R. Moro, D. R. Napoli, B. S. Nara Singh, A. Persson, F. Recchia, M. Sandzelius, J.-N. Scheurer, G. Sletten, D. Sohler, P.-A. Söderström, M. J. Taylor, J. Timár, J. J. Valiente-Dobón, E. Vardaci & S. Williams − Show fewer authors

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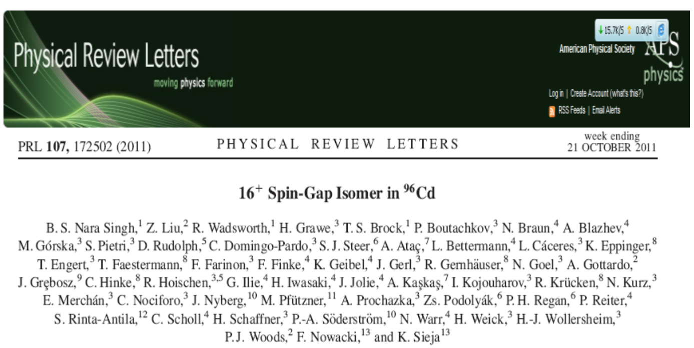
Nature 469, 68–71 (06 January 2011) | doi:10.1038/nature09644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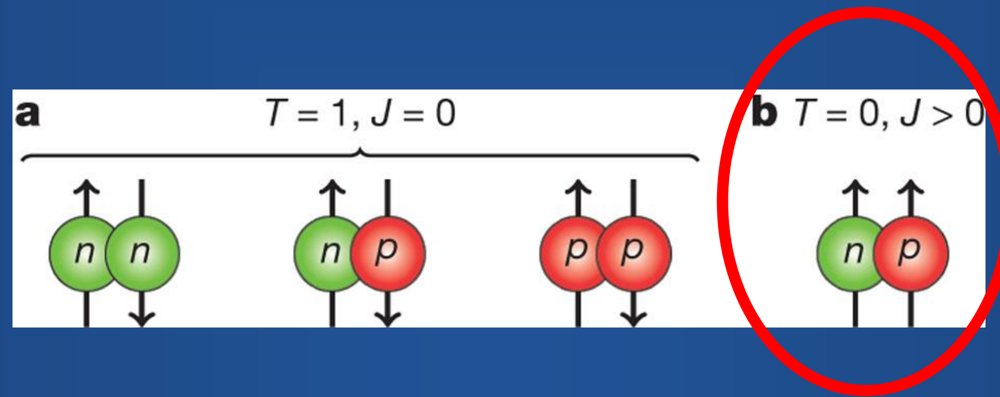
PRL 107, 172502 (2011) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2011

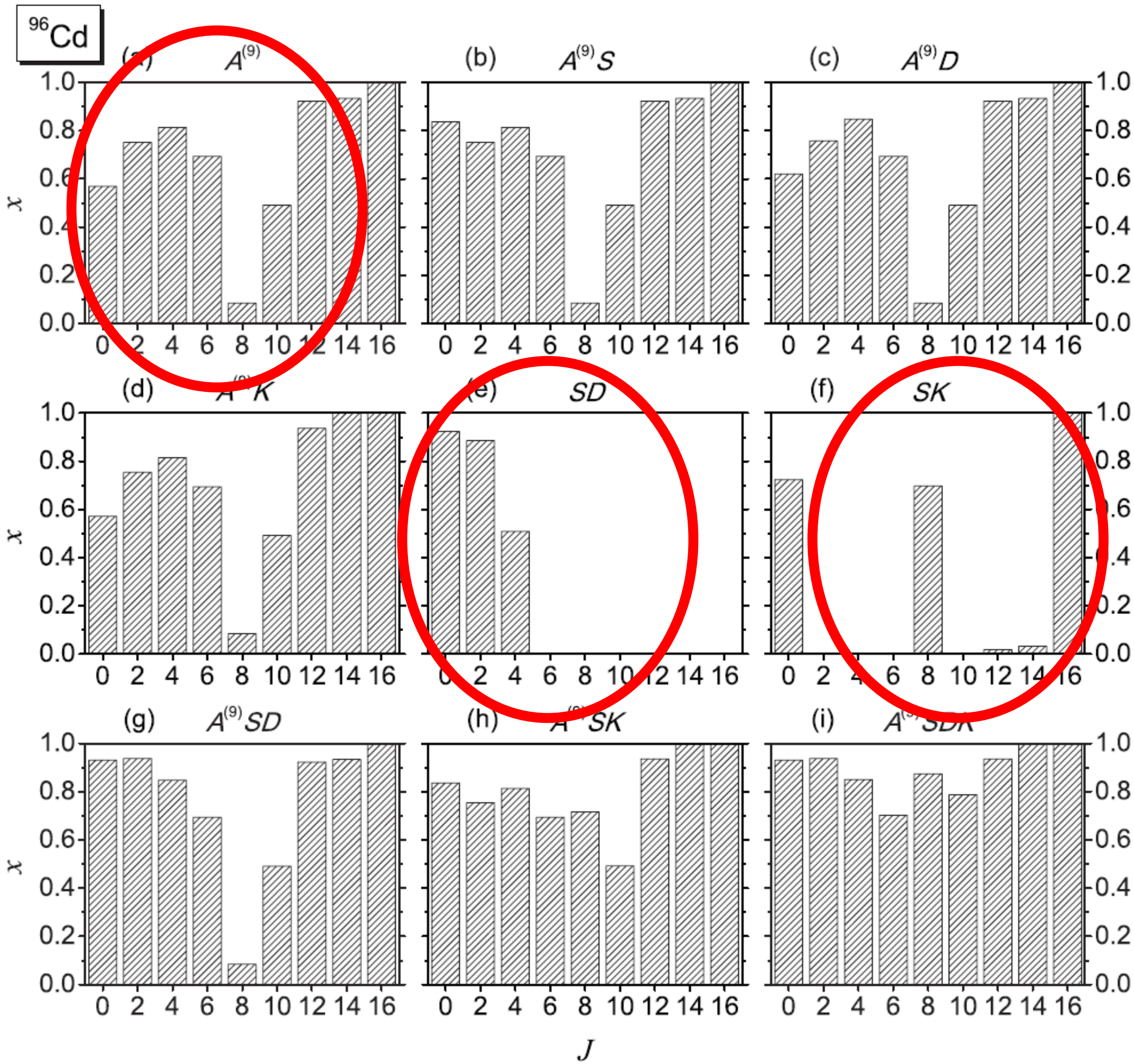
16^+ Spin-Gap Isomer in ^{96}Cd

B. S. Nara Singh,[1] Z. Liu,[2] R. Wadsworth,[1] H. Grawe,[3] T. S. Brock,[1] P. Boutachkov,[3] N. Braun,[4] A. Blazhev,[4] M. Górska,[3] S. Pietri,[3] D. Rudolph,[5] C. Domingo-Pardo,[3] S. J. Steer,[6] A. Ataç,[7] L. Bettermann,[4] L. Cáceres,[3] K. Eppinger,[8] T. Engert,[3] T. Faestermann,[8] F. Farinon,[3] F. Finke,[4] K. Geibel,[4] J. Gerl,[3] R. Gernhäuser,[8] N. Goel,[3] A. Gottardo,[2] J. Grębosz,[9] C. Hinke,[8] R. Hoischen,[3,5] G. Ilie,[4] H. Iwasaki,[4] J. Jolie,[4] A. Kaşkaş,[7] I. Kojouharov,[3] R. Krücken,[8] N. Kurz,[3] E. Merchán,[3] C. Nociforo,[3] J. Nyberg,[10] M. Pfützner,[11] A. Prochazka,[3] Zs. Podolyák,[6] P. H. Regan,[6] P. Reiter,[4] S. Rinta-Antila,[12] C. Scholl,[4] H. Schaffner,[3] P.-A. Söderström,[10] N. Warr,[4] H. Weick,[3] H.-J. Wollersheim,[3] P. J. Woods,[2] F. Nowacki,[13] and K. Sieja[13]

Spin-aligned neutron-proton pair

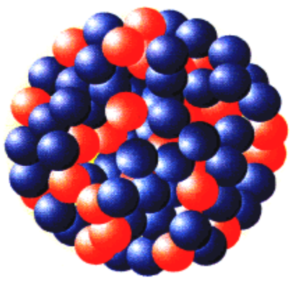


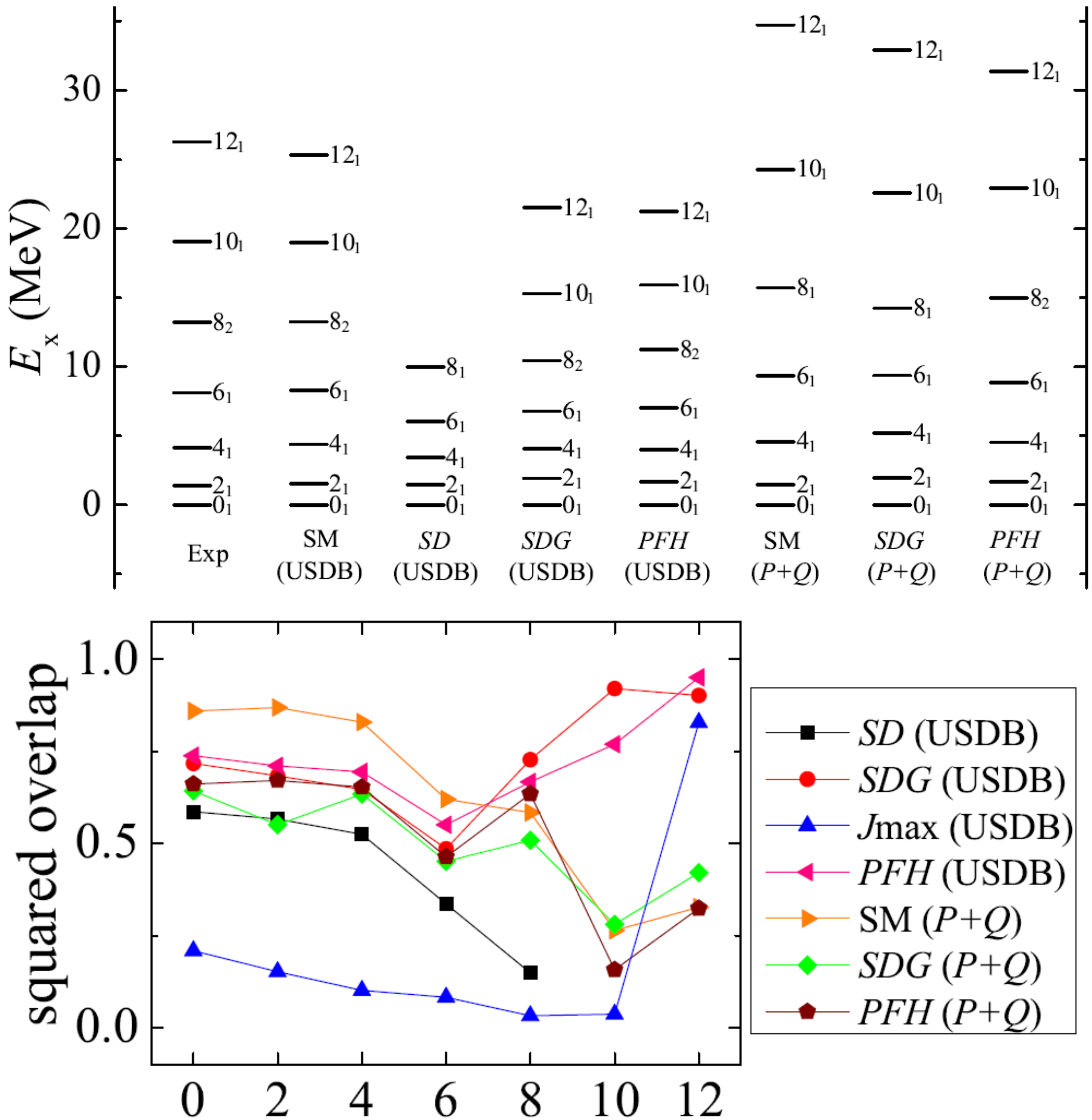
The NPA with isospin is the direct approach to study this phenomenon.



Fu, Zhao, Arima, PRC88, 054303 (2013).

Mg-24





Fu, Zhao, Arima, submitted to PRC (2014).

the Shell model versus *SD* pair approximation

0^+ states of the *pf*-shell nuclei

for a single CPU with main frequency 2.66 giga-Hertz and memory four giga-bytes.

(N_p, N_n)	(2, 2)	(2, 4)	(2, 6)	(4, 4)	(4, 6)	(6, 6)
SM: dimension/time	158/0.5	2343/3	14177/33	41355/256	267054/–	1777116/–
NPA: dimension/time	2/2	4/2	6/6	9/2	14/6	24/11

perspective

1、low-lying states of heavy nuclei

2、quartet structure

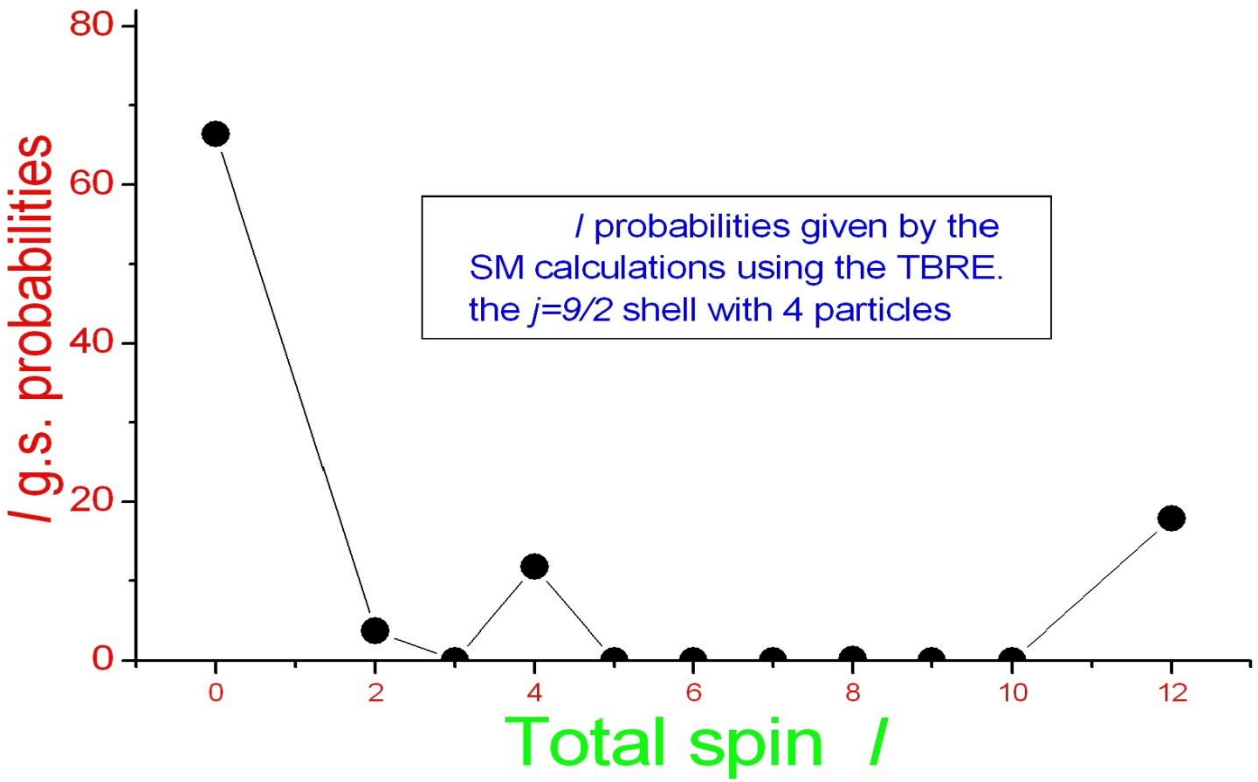
3、particle-hole excitations

4、unbound states

V. Collective motion under random interactions

Recent results by us

$$j = \frac{9}{2}, \quad n = 4$$



spin 自旋	dimension 维数
0	2
2	2
3	1
4	3
5	1
6	3
7	1
8	2
9	1
10	1
12	1

Zhao, Arima, Yoshinaga, Phys. Rep. 400, 1 (2004); Zelevinsky and Volya, Phys. Rep. 391, 311 (2004); Weidenmueller and Mitchell, Rev. Mod. Phys. 81, 539 (2009).

Parity distribution in the ground states

- (A) Both protons and neutrons are in the $f_{5/2}p_{1/2}g_{9/2}$ shell which corresponds to nuclei with both proton number Z and neutron number N ~40;
- (B) Protons in the $f_{5/2}p_{1/2}g_{9/2}$ shell and neutrons in the $g_{7/2}d_{5/2}$ shell which correspond to nuclei with Z~40 and N~50;
- (C) Both protons and neutrons are in the $h_{11/2}s_{1/2}d_{3/2}$ shell which correspond to nuclei with Z and N~82;
- (D) Protons in the $g_{7/2}d_{5/2}$ shell and neutrons in the $h_{11/2}s_{1/2}d_{3/2}$ shell which correspond to nuclei with Z~50 and N~82.

(单位: %)

Basis (A) $\Leftarrow$

(0,4)	(0,6)	(2,2)	(2,4)	(2,6)		
86.6	86.2	93.1	81.8	88.8		
(2,3)	(1,4)	(0,5)	(6,1)	(2,1)	(1,3)	(1,5)
42.8	38.6	45.0	38.4	31.2	77.1	69.8

Basis (B) $\Leftarrow$

(2,2)	(2,4)	(4,2)					
72.7	80.5	81.0					
(3,4)	(2,3)	(3,2)	(4,1)	(1,4)	(5,0)	(3,3)	(5,1)
42.5	72.4	39.1	75.1	26.4	44.1	79.4	42.9

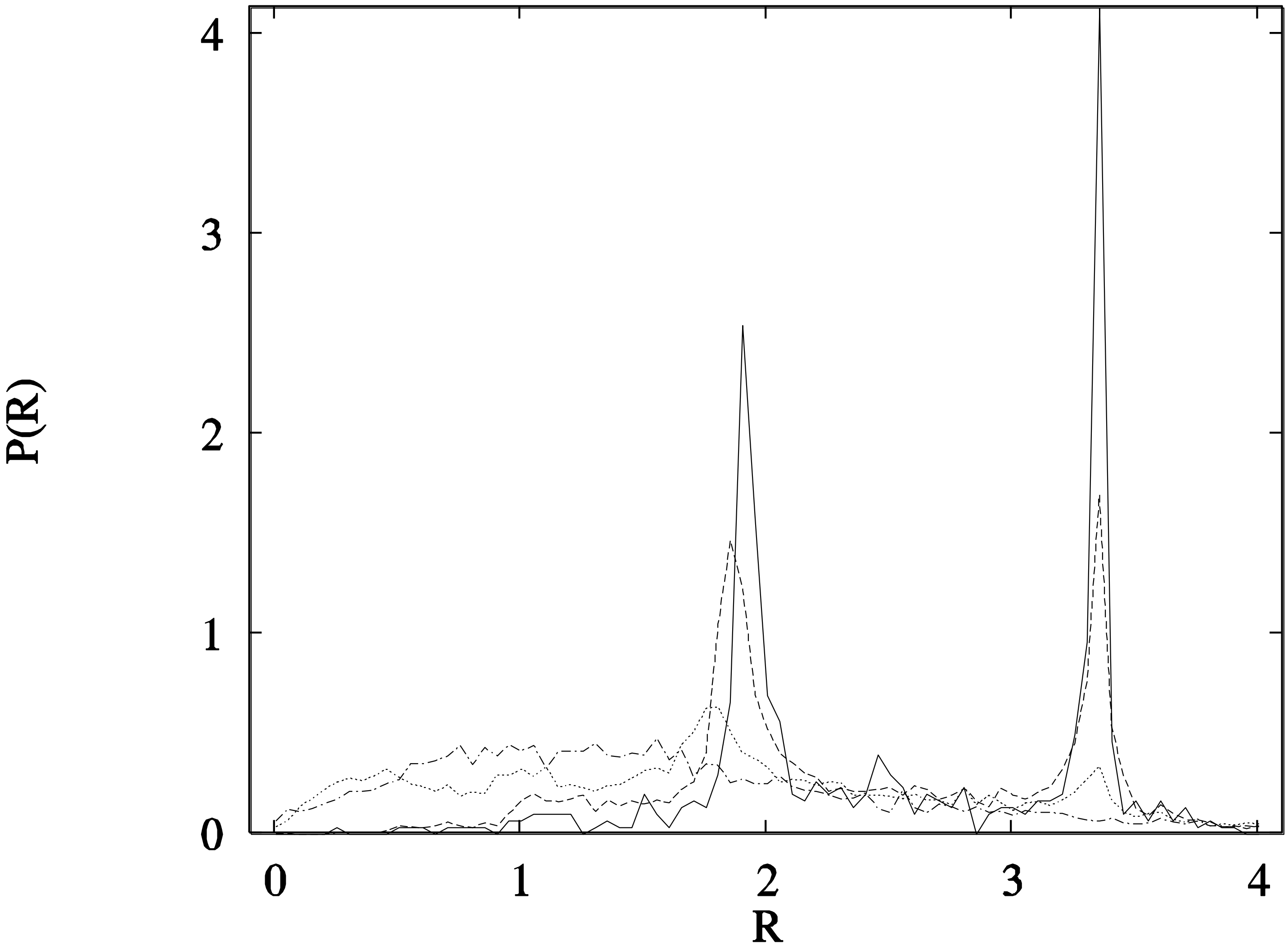
Basis (C) $\Leftarrow$

(2,2)	(2,4)	(4,0)	(6,0)	
92.2	81.1	80.9	82.4	
(2,3)	(5,0)	(4,1)	(1,5)	(1,3)
52.0	42.6	56.5	64.4	73.0

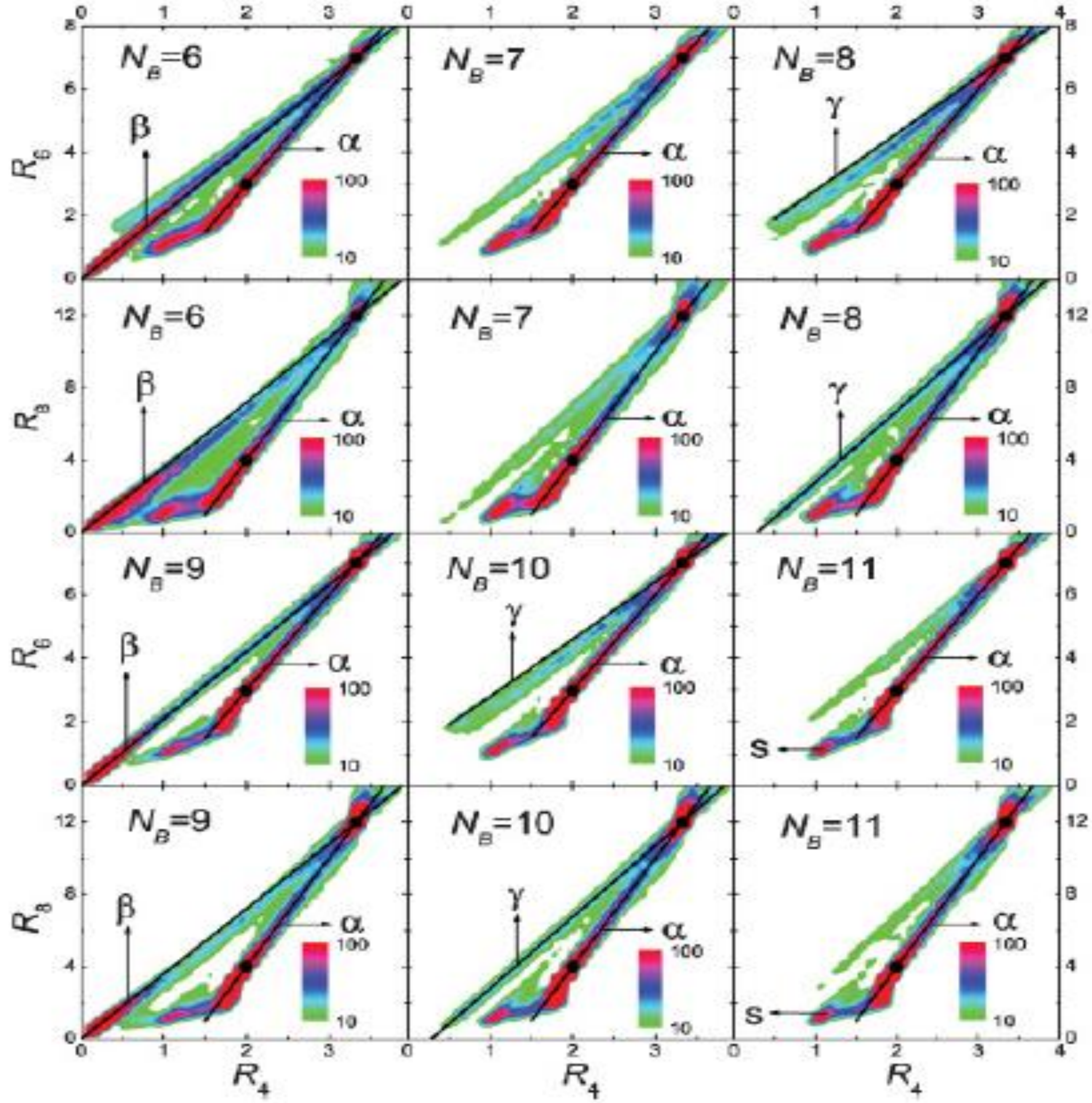
Basis (D) $\Leftarrow$

(2,2)	(4,2)	(2,4)	(0,6)
67.2	76.1	74.6	83.0
(3,2)	(2,3)	(0,5)	(3,3)
54.2	54.0	45.9	54.5

My favorite figures

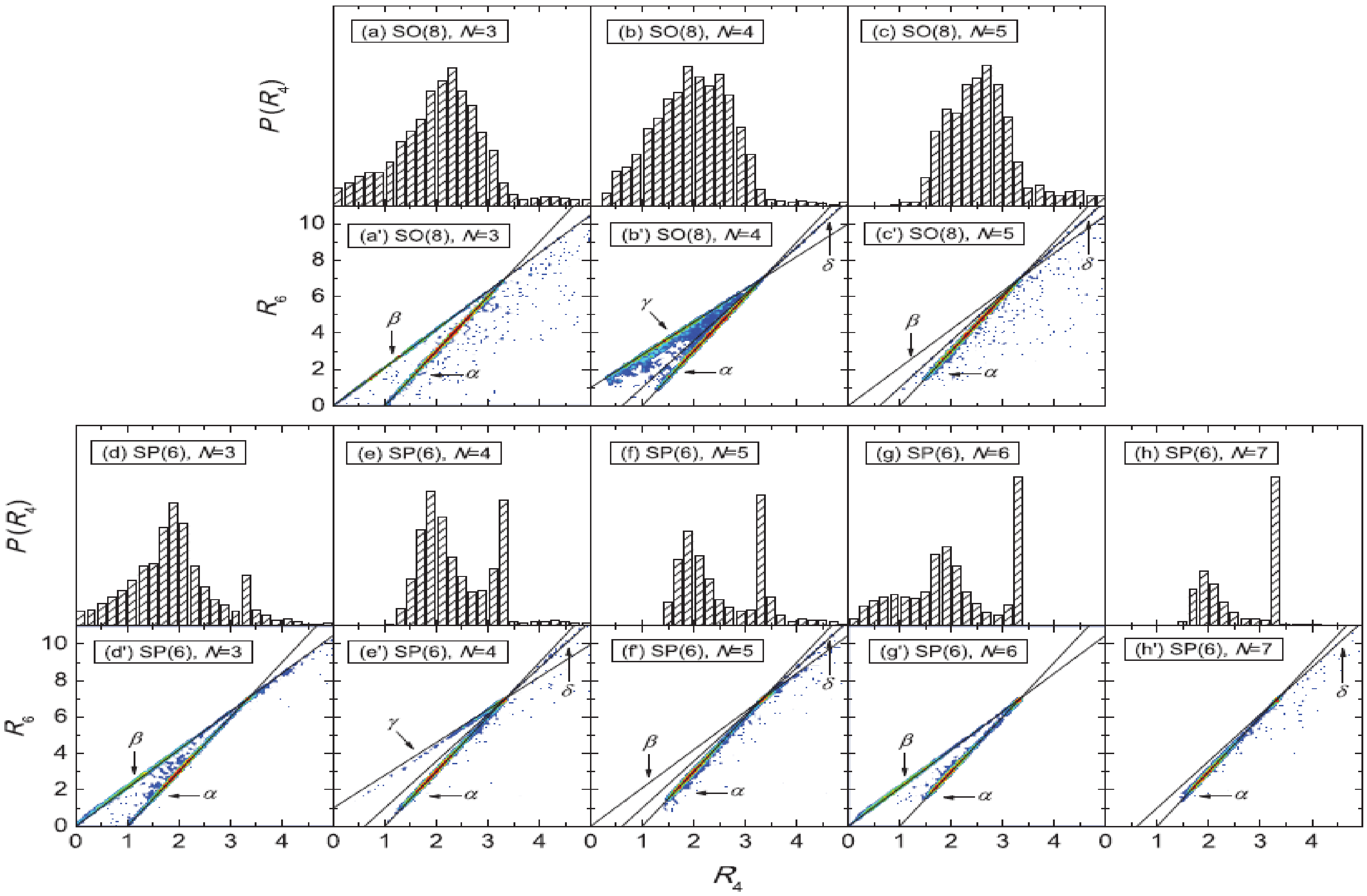


从PRC62,014303(2000)摘录. 原作者:R. Bijker and A. Frank

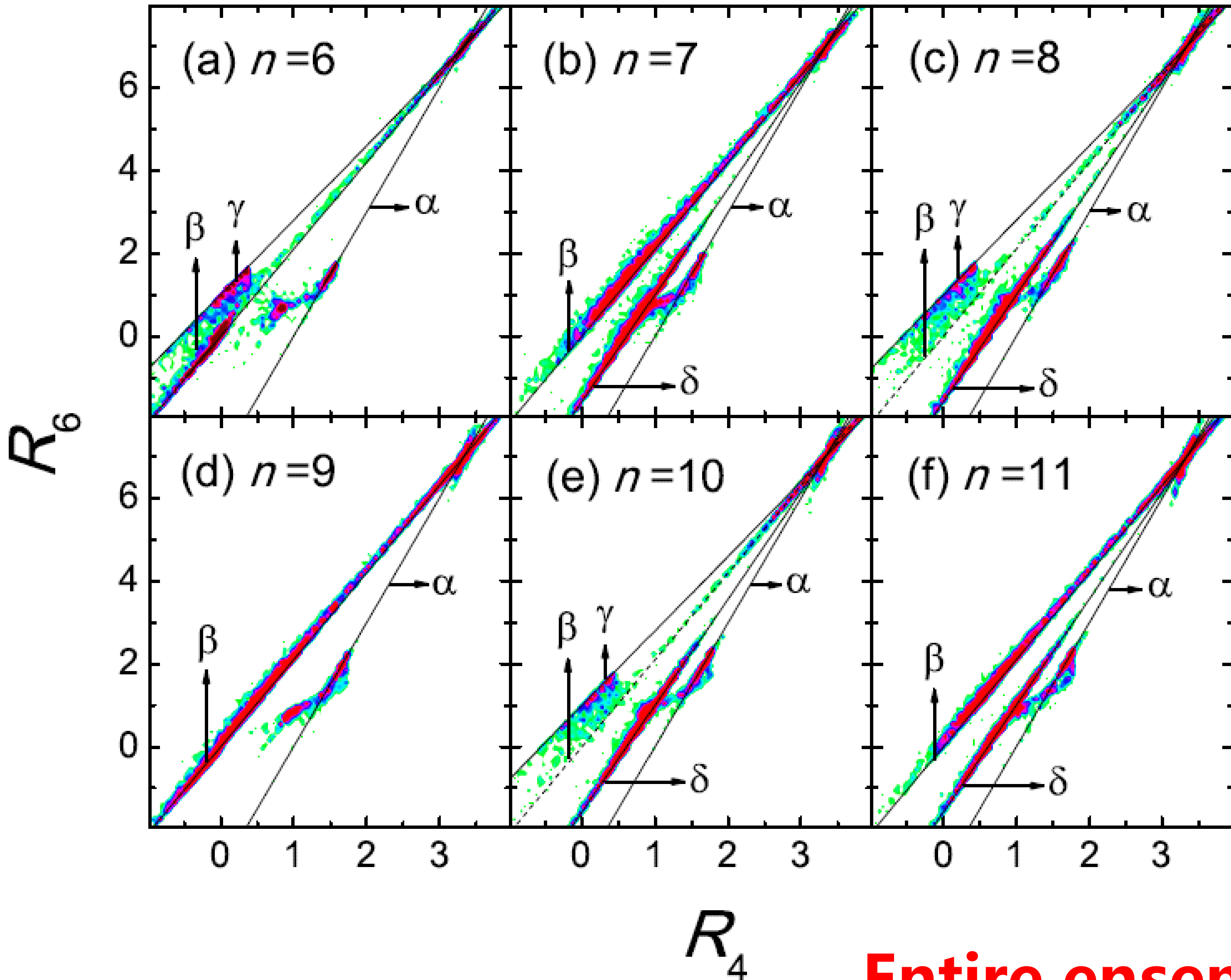


Malmann plot, R6-R4 for sd bosons,
Lei, Zhao, Yoshida, Arima, PRC (2011).

FDSM-SD pairs



Malmann plot, R6-R4 for FDSM-SD pairs, Fu, Zhao, Arima, PRC (2013).



Lu, Zhao, Arima, to be published.

Entire ensemble

Summary

I introduced very briefly the background, history, formulation, validity and applications of the nucleon pair approximation to the shell model.

In addition, I explained our recent results of collective motion under random interactions.

THANK YOU FOR YOUR ATTENTION !