Quantum simulators
A new tool to tackle computational quantum many-body problems

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Computational Challenges in Nuclear and Many-Body Physics
Motivation
“Take-home-message”

- Scenario 1: A quantum wire described by a Heisenberg $XYZ$ chain in an external field

$$\hat{H}_{XYZ} = \sum_i (J_x \hat{\sigma}^x_i \hat{\sigma}^x_{i+1} + J_y \hat{\sigma}^y_i \hat{\sigma}^y_{i+1} + J_z \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} + h \hat{\sigma}^z_i).$$

Local perturbation/quench. How is entanglement building up?

- DMRG and MPS is doing the job for us… Up till some point! After that….
Motivation
“Take-home-message”

- **Scenario 2**: Map out the phase diagram of the 3D Heisenberg $XYZ$ model in an external field. Determine the critical exponents!

- $N$ sites $\rightarrow$ Hilbert space dimension $D = 2^N$. $N = 1000$ gives $D = 3000000000000000000000000!$

  **Exponential growth of memory resources**!
  (Record 2007: $N = 36$)

  **FORGET IT!!!**
Motivation

“Take-home-message”

- **Scenario 3**: Ground state of the Fermi-Hubbard model in 2D and 3D. "Sign problem" causes a mess for Monte-Carlo.
Motivation

“Take-home-message”

Think twice which quantum problem you tell your student to solve/simulate!
Motivation

“Take-home-message”

“Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws.”

Richard Feynman

Motivation
“Take-home-message”

- ”Quantum simulators” outrun classical computers (today!).

- We will learn ”new physics” thanks to quantum simulators (soon).

- ”There’s more to the picture than meets the eye”. There are not only quantum simulators that will result from this story…
1. Quantum computers.
2. Quantum simulators.
Quantum computers
The idea

Digital quantum computer

- Bits ”0” and ”1” → qubits $|0\rangle$ and $|1\rangle$.

”01001100101110 ...” → $|\psi\rangle = \sum_{\{i\}=0,1} c_i |i_1 i_2 ... i_N\rangle$

- Logic gates → quantum logic gate operations

$$|\psi_\text{out}\rangle = \prod_i \hat{U}^{(i)} |\psi_\text{in}\rangle.$$  

Analog (continuous) quantum computer

$$\psi_\text{out}(\{x\}, t_f) = \hat{U}(\{x\}, \{p\}, t_f) \psi_\text{in}(\{x\}, 0) = e^{-i \hat{H} t_f} \psi_\text{in}(\{x\}, 0).$$

Adiabatic quantum computer

$$\psi_0(t);$$

$$\hat{H}(t) \psi_0(t) = E_0(t) \psi_0(t), \quad \hat{H}(t) = t\hat{H}_1 + (1-t)\hat{H}_2.$$
What we need

**Loss-DiVencezo criteria**

i. Well-defined qubits,

ii. State preparation,

iii. Low decoherence/scaleability,

iv. Gate operations,

v. Measurement protocols.

When does it become practical?

<table>
<thead>
<tr>
<th>Factorizing (Shor)</th>
<th>Classical computer (laptop)</th>
<th>Quantum computer</th>
</tr>
</thead>
<tbody>
<tr>
<td>193 digits</td>
<td>few months.</td>
<td>0.1 second</td>
</tr>
<tr>
<td>500 digits</td>
<td>$10^{12}$ years</td>
<td>2 minutes</td>
</tr>
<tr>
<td>2048 digits</td>
<td>Supercomputer; size of Sweden, $10^6$ trillion $, consumes world’s supply of fossil fuels in on day. 10 years.</td>
<td>16 hours (10^6 qubits, 10^8 $)</td>
</tr>
</tbody>
</table>
What we need

**Loss-DiVencezo criteria**

i. **Well-defined qubits** – Quantum dots, ions,…

ii. **State preparation** – questionable.

iii. **Low decoherence/scaleability** – No! Ions: 8-14 qubits (Blatt), Qdots: 5 qubits (Martinis).

iv. **Gate operations** (to some degree).

v. **Measurement protocols** – questionable.

- **Quantum error correction.** Encode the qubit in collective states of many ”physical” qubits. → Increasing number of qubits.

- **Fault tolerance.** How much errors do we afford and still achieve the goal? (> 99% gate fidelities).
Never say never

- Topological quantum computing.
- Circuit QED. Fault tolerance single gates.
- Topological quantum computing.
Quantum simulators
Digital quantum computers $\rightarrow$ quantum simulators

- Seth Lloyd:

   Any (local) Hamiltonian many-body evolution can be effectively simulated on a digital quantum computer via Trotter-decomposition.

- A digital quantum computer with a universal set of gates $\rightarrow$ Universal digital quantum simulator (unitary Hamiltonian evolution).

- Quantum error correction possible but costly (number of gate operations increases and simulations become slow, state-of-the-art systems can imply time-scales of years!).

- Non-local interactions problematic.

- Generalizations to non-universal digital and open quantum simulators. Error corrections?

### Definition - *quantum simulators*

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td><strong>Relevance</strong></td>
<td>Simulated systems/models should have physical applications. Address open questions.</td>
</tr>
<tr>
<td><strong>Controllability</strong></td>
<td>System parameters tunable, control of preparation/initialization, evolution/manipulation and detection.</td>
</tr>
<tr>
<td><strong>Reliability</strong></td>
<td>Measured results should be trustworthy.</td>
</tr>
<tr>
<td><strong>Efficiency</strong></td>
<td>The solved problem should be difficult to solve on a classical computer.</td>
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Analog quantum simulators

- Simulate time-evolution: $\hat{\rho}(0) \rightarrow \hat{\rho}(t)$.

- Closed quantum system, engineer $\hat{H}$ such that $|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle$.

- Continuous time-evolution, no Trotter-decomposition but also no error correction.

- Note, we imagine also ground-state simulations $t \rightarrow -it$. 
Realizations – State-of-the-art
Trapped ions

- Singled trapped ion, dressed with a laser
  \[ \hat{H}_{1on} = \omega \hat{a}^+ \hat{a} + \frac{A}{2} \hat{\sigma}_z + g(\hat{\sigma}^+ e^{-i\eta (\hat{a}^+ \hat{a})} + \hat{\sigma}^- e^{i\eta (\hat{a}^+ \hat{a})}) \]

- Single out certain transitions (Lamb-Dicke regime, \( \eta \ll 1 \))

  1. \( \hat{H}_{JC} = \omega \hat{a}^+ \hat{a} + \frac{A}{2} \hat{\sigma}_z + g(\hat{\sigma}^+ \hat{a} + \hat{a}^+ \hat{\sigma}^-) \), \textit{Red sideband}
  2. \( \hat{H}_{aJC} = \omega \hat{a}^+ \hat{a} + \frac{A}{2} \hat{\sigma}_z + g(\hat{\sigma}^+ \hat{a}^+ + \hat{\sigma}^- \hat{a}) \), \textit{Blue sideband}
  3. \( \hat{H}_{car} = \omega \hat{a}^+ \hat{a} + \frac{A}{2} \hat{\sigma}_z + g(\hat{\sigma}^+ + \hat{\sigma}^-) \), \textit{Carrier.}

- Enormous control! Gate fidelities of 99.9\%. 
Trapped ions

- Quantum simulators $\rightarrow$ many ions.
- *Paul trapps* $\rightarrow$ linear ion chains.
- Blatt’s *Insbruck-group*. Controlled entanglement generation of up to 14 qubits! Full state tomography of 8 qubits (600 000 experimental repetitions!).

Trapped ions

- Coloumb interaction → collective vibrational modes.
- Eliminate vibrational modes:

\[ \hat{H}_{\text{eff}} = \sum_{\alpha,i,j} J^{\alpha}_{ij} \hat{\sigma}^{\alpha}_i \hat{\sigma}^{\alpha}_j, \quad J^{\alpha}_{ij} \propto \frac{1}{|q_i - q_j|^\gamma} \]

- The power \(0 \leq \gamma \leq 3\) is in general controlable.
- Monroe group: Frustration and signatures of phase transitions in 3-16 ion chains (\(\gamma = 1\)).

**Relevance** – Probably.

**Controllability** – Not fully.

**Reliability** – Yes.

**Efficiency** – No.

Trapped ions

- NIST group: ~300 ions in a Penning trap, $0 \leq \gamma \leq 1.4$.

- Coherent evolution by measuring $M = \sum_i \langle \hat{z}_i \rangle$.

- Relevance – Probably.
- Controllability – No.
- Reliability – Yes.
- Efficiency – No.

Cold atoms in optical lattices

- Optical lattices:
  a. Ultracold atoms, bosons, fermions or mixtures.
  b. Standing wave laser fields $\rightarrow$ dipole coupling $\rightarrow$ periodic Stark shift potentials.
  d. Tight-binding approximation: tunneling to nearest neighbour.
  e. Onsite atom-atom interaction.

\[ \hat{H}_{BH} = -t \sum_{\langle ij \rangle} (\hat{a}_i^+ \hat{a}_j + h.c.) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \hat{N} \]

$\mu$ = chemical potential

Cold atoms in optical lattices

- "Mott-superfluid phase transition". Ground state:

\[ U \gg t \rightarrow |\psi_0(\mu)\rangle \approx |n, n, \ldots, n\rangle \quad \text{"Mott-insulator state"} \]
\[ t \gg U \rightarrow |\psi_0(\mu)\rangle \propto (\hat{a}^+_{k=0})^N |0\rangle \quad \text{"Superfluid state"} \]
- "Time-of-flight" measurements.

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Cold atoms in optical lattices

- Initialize

\[ |\psi(0)\rangle = |1,0,1,0,1, ... \rangle \]

- Single-site-addressing – population of every even site.
- DMRG calculations, no fitting parameters!

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Proposal for simulating spin models
Cold atoms in excited bands

- Spin models → we need quasi degenerate (atomic) levels.
  
  1) Internal Zeeman levels (L.-M. Duan et al., PRL 2003). Typically $XXZ$-models.
  
  

- Use the quasi degenerate states of excited bands, $p$-bands.
Cold atoms in excited bands

- Two dimensional square isotropic lattice, bosons.

- $p$-band: Two degenerate atomic orbitals, $p_x$-orbital and $p_y$-orbital.

- Tunneling anisotropic due to orbital shape.

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F. Pinheiro et al., PRL 2013.
Cold atoms in excited bands

- Kinetic part

\[ \hat{H}_{\text{kin}} = - \sum_{\alpha, \beta} \sum_{\langle ij \rangle} t_{\alpha \beta} \hat{a}^+_{\alpha i} \hat{a}_{\alpha j}. \]

- Interaction parts

\[ \hat{H}_{\text{dens}} = \sum_\alpha \sum_i \frac{v_{\alpha \alpha}}{2} \hat{n}_{\alpha i} (\hat{n}_{\alpha i} - 1) + \sum_{\alpha \neq \beta} \sum_i U_{\alpha \beta} \hat{n}_{\alpha i} \hat{n}_{\beta i}, \]

\[ \hat{H}_{\text{oc}} = \sum_{\alpha \neq \beta} \sum_i \frac{v_{\alpha \beta}}{4} (\hat{a}^+_{\alpha i} \hat{a}^+_{\alpha i} \hat{a}_{\beta i} \hat{a}_{\beta i} + \text{h.c.}). \]

- \[ \hat{H}_{\text{oc}} \] - “orbital changing term” (Two \( \alpha \)-orbital atoms scatter into two \( \beta \)-orbital atoms).

F. Pinheiro et al., PRL 2013.
Cold atoms in excited bands

- Recepie:
  1) Mott-insulator \((n_i = 1)\).
  2) Perturbation theory in \(t/U\).
  3) Schwinger spin-boson mapping.

- Result: Heisenberg \(XYZ\)-model

\[
\hat{H}_{XYZ} = J \sum_{\langle ij \rangle} [(1 + \gamma) \hat{\sigma}^x_i \hat{\sigma}^x_j + (1 - \gamma) \hat{\sigma}^y_i \hat{\sigma}^y_j] + \Delta \sum_{\langle ij \rangle} \hat{\sigma}^z_i \hat{\sigma}^z_j + h \sum_i \hat{\sigma}^z_i.
\]

- Non-integrable in the general case \(\rightarrow\) promising quantum simulator.
Cold atoms in excited bands

- Comments:
  1. Phase diagram in 1D fairly known.
  2. Beyond tight-binding → \textit{Dzyaloshinskii-Morya} terms.
  3. Different lattice configurations → \textit{Dzyaloshinskii-Morya} terms.
  4. Three dimensions → $SU(3)$ models.
  5. Spinor atoms → $SU(n) \times SU(m)$ models.
  6. $d$-band → spin-1 models (also for $n = 2$ Mott on the $p$-band).
  7. Including s-band atoms → disordered models (\textit{many-body localization}).
Classical vs. Quantum chaos.

Possibilities to study closed quantum dynamics.

Equilibration and thermalization.

Not well understood:

i. Criteria for equilibration/thermalization.

ii. Mechanism behind thermalization.

iii. Definition for "Quantum integrability".

iv. Open systems...

Thanks!