Quantum simulators

A new tool to tackle computational quantum many-body problems

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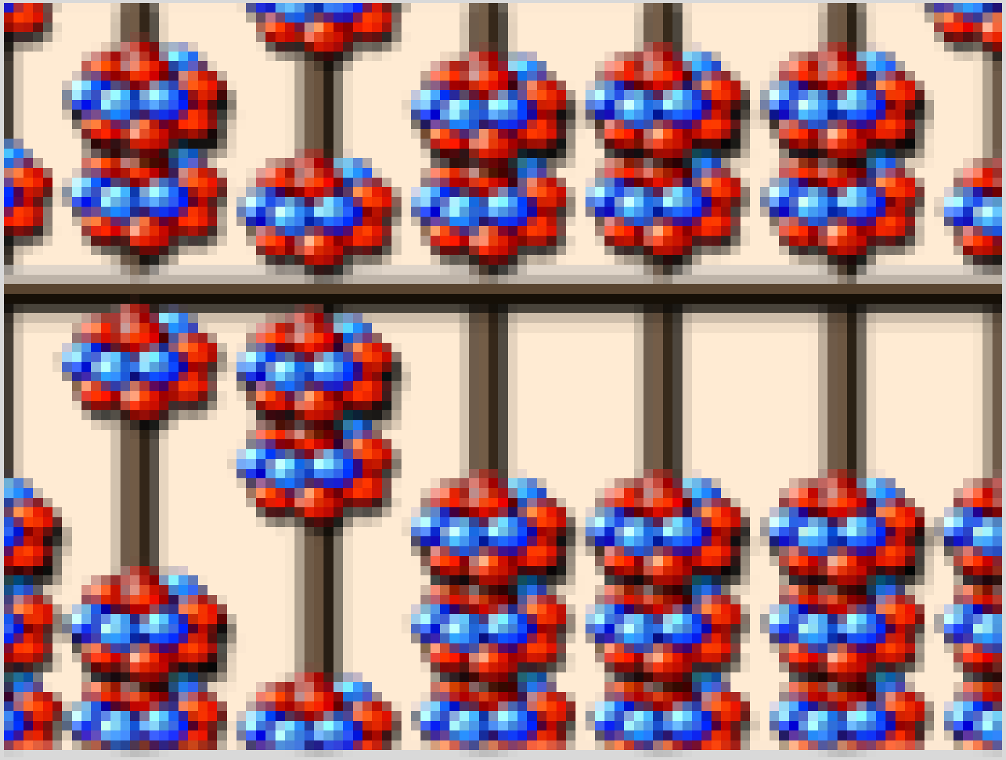
Stockholm “Computational Challenges in Nuclear and Many-Body Physics” ?/10-2014



Computational Challenges in Nuclear and Many-Body Physics

Motivation

"Take-home-message"

- **Scenario 1**: A quantum wire described by a Heisenberg XYZ chain in an external field

$$\hat{H}_{XYZ} = \sum_i \left(J_x \hat{\sigma}^x{}_i \hat{\sigma}^x{}_{i+1} + J_y \hat{\sigma}^y{}_i \hat{\sigma}^y{}_{i+1} + J_z \hat{\sigma}^z{}_i \hat{\sigma}^z{}_{i+1} + h \hat{\sigma}^z{}_i \right).$$

Local perturbation/quench. How is entanglement building up?

- *DMRG* and *MPS* is doing the job for us… Up till some point! After that….

Motivation

“Take-home-message”

- **Scenario 2**: Map out the phase diagram of the 3D Heisenberg XYZ model in an external field. Determine the critical exponents!

- N sites → Hilbert space dimension $D = 2^N$. $N = 1000$ gives $D = 300000000000000000000000000000000$!

Exponential growth of memory resources!

(Record 2007: $N = 36$)

FORGET IT!!!

Motivation

"Take-home-message"

- **Scenario 3**: Ground state of the Fermi-Hubbard model in 2D and 3D. "*Sign problem*" causes a mess for Monte-Carlo.

Motivation

“Take-home-message”

Think twice which quantum problem you tell your student to solve/simulate!

Motivation

“Take-home-message”

”*Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws.*”

Richard Feynman

[1] R. P. Feynman, *Int. J. Theor. Phys.* **21**, 467 (1982).

Motivation

“Take-home-message”

- ”*Quantum simulators*” outrun classical computers (today!).
- We will learn ”new physics” thanks to quantum simulators (soon).
- ”*There’s more to the picture than meets the eye*”. There are not only quantum simulators that will result from this story…

Outline

1. Quantum computers.
2. Quantum simulators.
3. Realizations – State-of-the-art.
4. Proposal for simulating spin models.

Quantum computers

The idea

Digital quantum computer

- Bits ”0” and ”1” → qubits $|0\rangle$ and $|1\rangle$.

$$\text{"01001100101110 ..."} \rightarrow |\psi\rangle = \sum_{\{i\}=0,1} c_i |i_1 i_2 \dots i_N\rangle$$

- Logic gates → quantum logic gate operations

$$|\psi_{out}\rangle = \prod_i \widehat{U}^{(i)} |\psi_{in}\rangle.$$

Analog (continuous) quantum computer

$$\psi_{out}\left(\{\boldsymbol{x}\}, t_f\right) = \widehat{U}\left(\{\boldsymbol{x}\}, \{\boldsymbol{p}\}, t_f\right)\psi_{in}(\{\boldsymbol{x}\}, 0) = e^{-i\widehat{H}t_f}\psi_{in}(\{\boldsymbol{x}\}, 0).$$

Adiabatic quantum computer

$$\psi_0(t);$$

$$\widehat{H}(t)\psi_0(t) = E_0(t)\psi_0(t), \qquad \widehat{H}(t) = t\widehat{H}_1 + (1-t)\widehat{H}_2.$$

What we need

Loss-DiVencezo criteria

i. Well-defined qubits,
ii. State preparation,
iii. Low decoherence/scaleability,
iv. Gate operations,
v. Measurement protocols.

When does it become practical?

Factorizing (Shor)	**Classical computer (laptop)**	**Quantum computer**
193 digits	few months.	0.1 second
500 digits	10^{12} years	2 minutes
2048 digits	Supercomputer; size of Sweden, 10^6 trillion $, consumes world's supply of fossil fuels in on day. 10 years.	16 hours (10^6 qubits, 10^8 $)

What we need

Loss-DiVencezo criteria

i. *Well-defined qubits*  – Quantum dots, ions,…

ii. *State preparation* – questionable.

iii. *Low decoherence/scaleability* – No! Ions: 8-14 qubits (Blatt), Qdots: 5 qubits (Martinis).

iv. *Gate operations*  (to some degree).

v. *Measurement protocols* – questionable.

- ***Quantum error correction***. Encode the qubit in collective states of many ”phyical” qubits. $\rightarrow$ Increasing number of qubits.

- ***Fault tolerance***. How much errors do we afford and still achieve the goal? ($> 99\%$ gate fidelities).

Never say never

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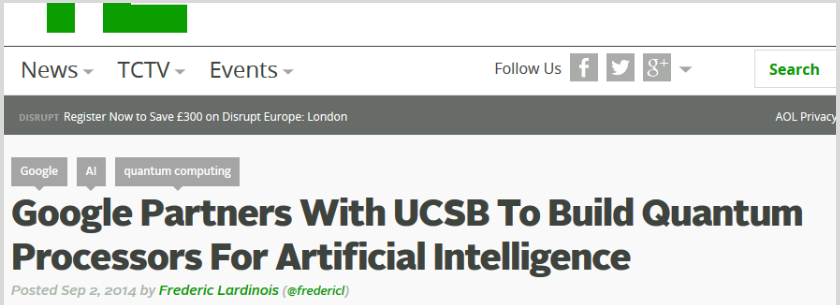
Science

Microsoft delves deep into quantum computing research

- Topological quantum computing.



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Google AI quantum computing

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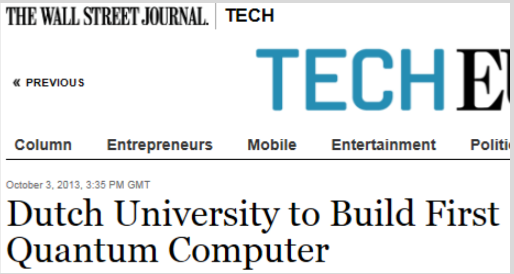
Posted Sep 2, 2014 by Frederic Lardinois (@fredericl)

- Circuit QED. Fault tolerance single gates.



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October 3, 2013, 3:35 PM GMT

Dutch University to Build First Quantum Computer

- Topological quantum computing.

Quantum simulators

Digital quantum computers → quantum simulators

- Seth Lloyd:

Any (local) Hamiltonian many-body evolution can be effectively simulated on a digital quantum computer via Trotter-decomposition.

- A digital quantum computer with a universal set of gates → Universal *digital quantum simulator* (unitary Hamiltonian evolution).
- Quantum error correction possible but costly (number of gate operations increases and simulations become slow, state-of-the art systems can imply time-scales of years!).
- Non-local interactions problematic.
- Generalizations to *non-universal digital* and *open quantum simulators*. Error corrections?

S. Lloyd, *Science* 1996.

Definition - *quantum simulators*

Relevance – Simulated systems/models should have physical applications. Address open questions.

Controllability – System parameters tunable, contol of preparation/initialization, evolution/manipulation and detection.

Reliability – Measured results should be trustworthy.

Efficiency – The solved problem should be difficult to solve on a classical computer.

P. Hauke *et al.*, Rep. Prog. Phys. 2012.

Analog quantum simulators

- Simulate time-evolution: $\hat{\rho}(0) \to \hat{\rho}(t)$.
- Closed quantum system, engineer $\hat{H}$ such that $|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle$.
- Continuous time-evolution, no Trotter-decomposition but also no error correction.
- Note, we imagine also ground-state simulations $t \to -it$.

Realizations – State-of-the-art

Trapped ions

- Singled trapped ion, dressed with a laser

$$\hat{H}_{Ion} = \omega \hat{a}^{+}\hat{a} + \frac{\Delta}{2}\hat{\sigma}_z + g\left(\hat{\sigma}^{+} e^{-i\eta\left(\hat{a}^{+}+\hat{a}\right)} + \hat{\sigma}^{-} e^{i\eta\left(\hat{a}^{+}+\hat{a}\right)}\right)$$

- Single out certain transions (*Lamb-Dicke regime*, $\eta \ll 1$)

i. $\hat{H}_{JC} = \omega \hat{a}^{+}\hat{a} + \frac{\Delta}{2}\hat{\sigma}_z + g(\hat{\sigma}^{+}\hat{a} + \hat{a}^{+}\hat{\sigma}^{-})$, *Red sideband*

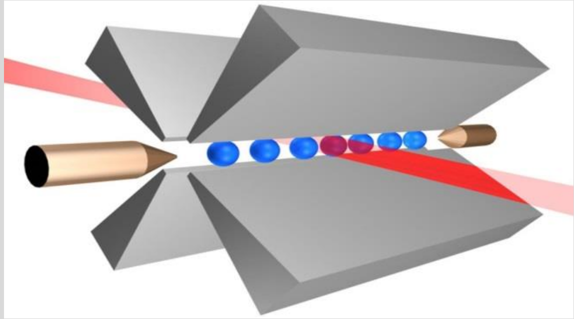
ii. $\hat{H}_{aJC} = \omega \hat{a}^{+}\hat{a} + \frac{\Delta}{2}\hat{\sigma}_z + g(\hat{\sigma}^{+}\hat{a}^{+} + \hat{\sigma}^{-}\hat{a})$, *Blue sideband*

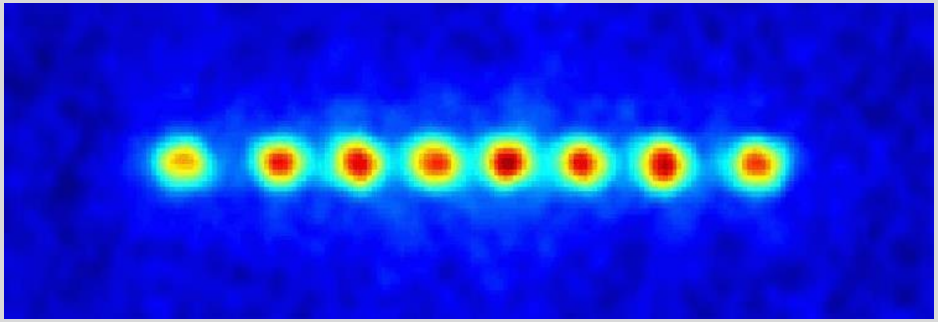
iii. $\hat{H}_{car} = \omega \hat{a}^{+}\hat{a} + \frac{\Delta}{2}\hat{\sigma}_z + g(\hat{\sigma}^{+} + \hat{\sigma}^{-})$, *Carrier*.

- Enormous control! Gate fidelities of 99.9%.

Trapped ions

- Quantum simulators → many ions.
- *Paul trapps* → linear ion chains.





- Blatt's *Insbruck-group*. Controlled entanglement generation of up to 14 qubits! Full state tomography of 8 qubits (600 000 experimental repetitions!).

H. Häffner *et al.*, Nature 2005.

Trapped ions

Example 1

- Coloumb interaction → collective vibrational modes.
- Eliminate vibrational modes:

$$\hat{H}_{eff} = \sum_{\alpha,i,j} J^{\alpha}{}_{ij}\hat{\sigma}^{\alpha}{}_{i}\hat{\sigma}^{\alpha}{}_{j}\,, \qquad J^{\alpha}{}_{ij} \propto \frac{1}{|q_i - q_j|^{\gamma}}$$

- The power $0 \le \gamma \le 3$ is in general controlable.
- Monroe group: Frustration and signatures of phase transitions in 3-16 ion chains ($\gamma = 1$).

Relevance – Probably.

Controllability – Not fully.

Reliability – Yes.

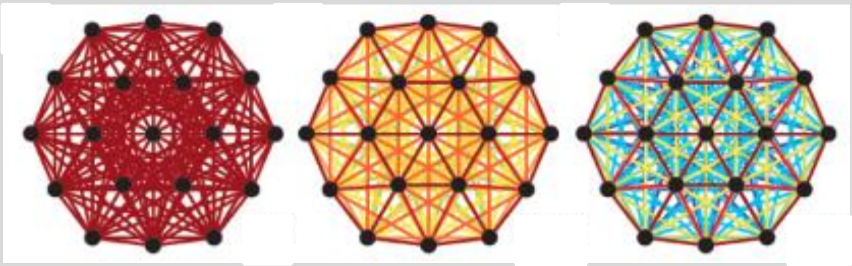
Efficiency – No.

D. Porras and I. J. Cirac., PRL 2004.

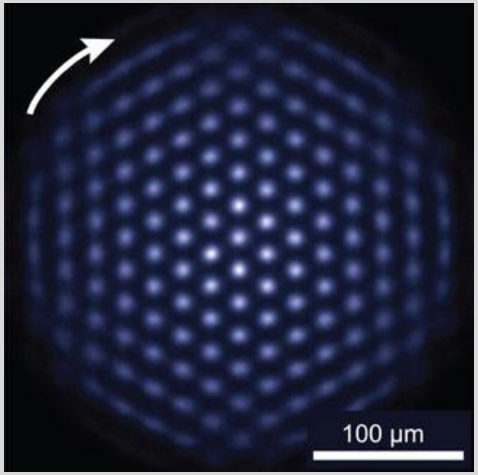
Example 2

Trapped ions

- NIST group: ~300 ions in a Penning trap, $0 \leq \gamma \leq 1.4$.



$\gamma = 0$ $\gamma = 3/2$ $\gamma = 3$



- Coherent evolution by measuring $M = \sum_i \langle \hat{\sigma}^z{}_i \rangle$.

Relevance – Probably.

Controllability – No.

Reliability – Yes.

Efficiency – No.

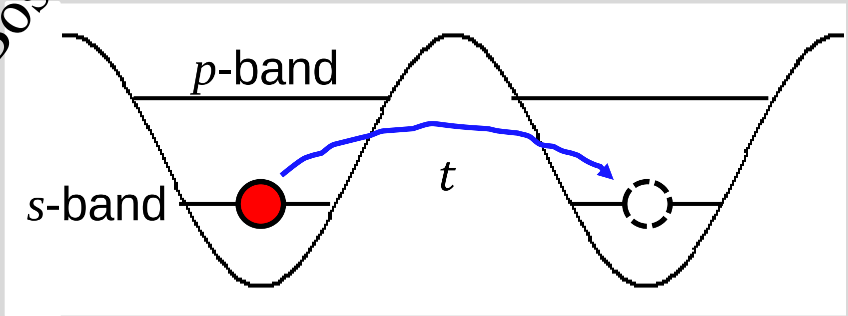
J. W. Britton *et al*, Nature 2013.

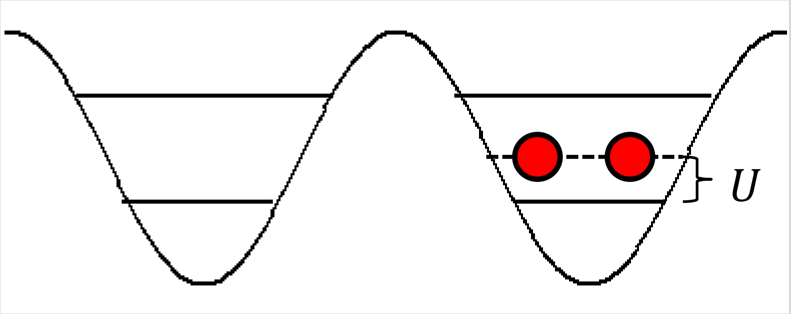
Cold atoms in optical lattices

- Optical lattices:

  a. Ultracold atoms, bosons, fermions or mixtures.
  b. Standing wave laser fields → dipole coupling → periodic Stark shift potentials.
  c. *Single-band approximation*: atoms populate one energy band.
  d. *Tight-binding approximation*: tunneling to nearest neighbour.
  e. Onsite atom-atom interaction.

Bosons!





$$\hat{H}_{BH} = -t \sum_{\langle ij \rangle} \left(\hat{a}^{+}{}_{i} \hat{a}_{j} + h.c. \right) + \frac{U}{2} \sum_{i} \hat{n}_{i} (\hat{n}_{i} - 1) - \mu \hat{N}$$

Bose-Hubbard model

μ = chemical potential

I. Bloch *et al.*, Rev. Mod. Phys. 2008.

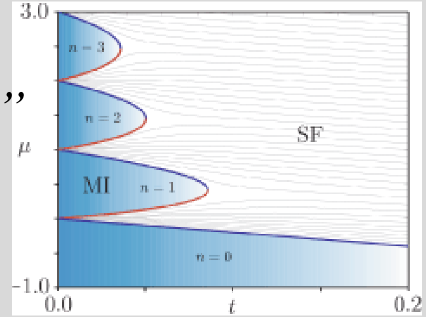
Example 1

Cold atoms in optical lattices

- *"Mott-superfluid phase transition".* Ground state:

$U \gg t \quad \rightarrow \quad |\psi_0(\mu)\rangle \approx |n, n, \dots, n\rangle$ *"Mott-insulator state"*

$t \gg U \quad \rightarrow \quad |\psi_0(\mu)\rangle \propto (\hat{a}^+{}_{k=0})^N |0\rangle$ *"Superfluid state"*



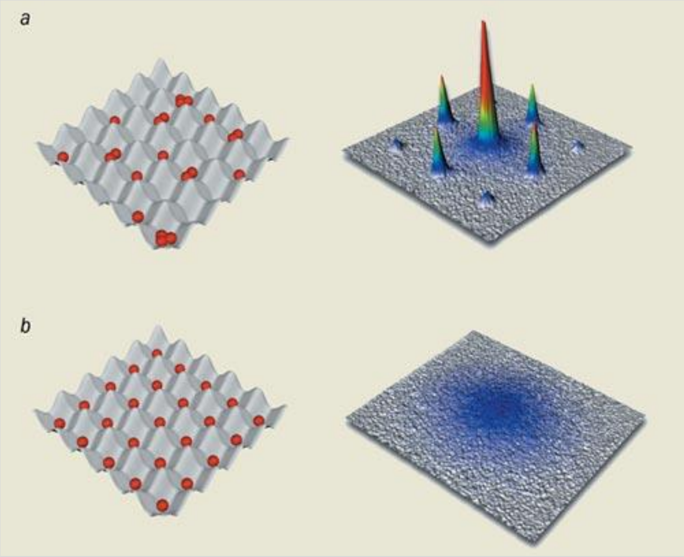
- *"Time-of-flight"* measurements.

Relevance – Maybe.

Controllability – Yes.

Reliability – Yes.

Efficiency – No.

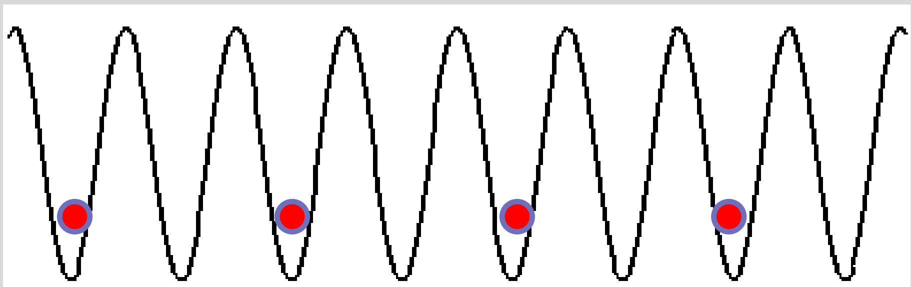


M. Greiner *et al.*, Nature 2002.

Example 2

Cold atoms in optical lattices

- Initialize



$$|\psi(0)\rangle = |1,0,1,0,1,\ldots\rangle$$

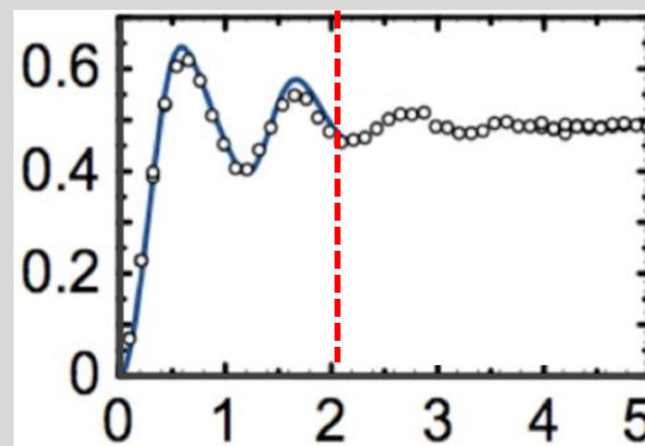
- *Single-site-addressing* – population of every even site.
- *DMRG* calculations, no fitting parameters!

Relevance – Maybe.

Controllability – Yes.

Reliability – Yes.

Efficiency – Yes.



A. Flesch *et al.*, Nature 2012.

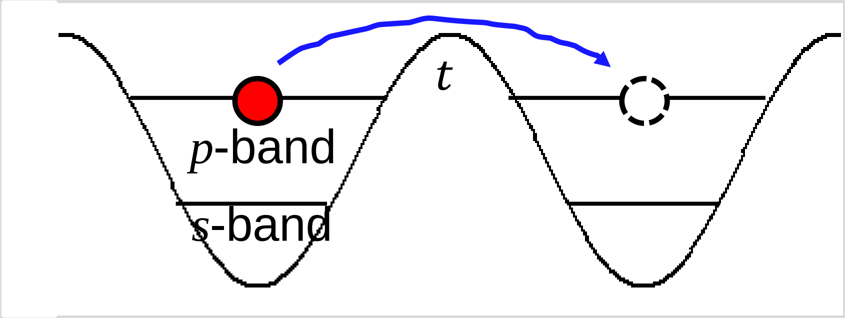
Proposal for simulating spin models

Cold atoms in excited bands

- Spin models → we need quasi degenerate (atomic) levels.

    1) Internal Zeeman levels (L.-M. Duan *et al.*, PRL 2003). Typically XXZ–models.
    2) Tilted lattices. Transverse *Ising*-model (J. Simon *et al.*, Nature 2011). One dimension.
    3) Polar molecules in optical lattices (A. Micheli *et al.*, Nature 2006). Inherently “long-range”.

- Use the quasi degenerate states of excited bands, *p*-bands.

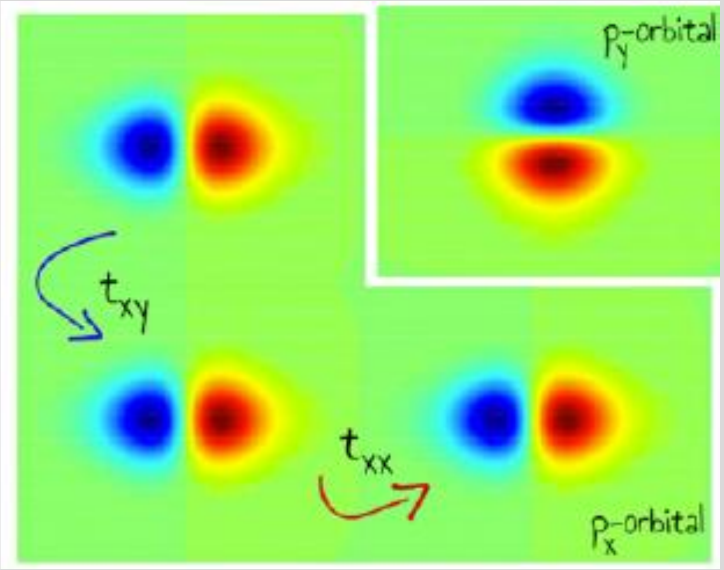


F. Pinheiro *et al.*, PRL 2013.

Cold atoms in excited bands

- Two dimensional square <u>isotropic</u> lattice, <u>bosons</u>.

- *p*-band: Two degenerate atomic orbitals, p_x-orbital and p_y-orbital.



- Tunneling anisotropic due to orbital shape.

F. Pinheiro *et al.*, PRL 2013.

Cold atoms in excited bands

- Kinetic part

$$\hat{H}_{kin} = -\sum_{\alpha,\beta} \sum_{\langle ij \rangle} t_{\alpha\beta} \hat{a}^{+}{}_{\alpha i} \hat{a}_{\alpha j}.$$

- Interaction parts

$$\hat{H}_{dens} = \sum_{\alpha} \sum_{i} \frac{U_{\alpha\alpha}}{2} \hat{n}_{\alpha i}(\hat{n}_{\alpha i} - 1) + \sum_{\alpha \neq \beta} \sum_{i} U_{\alpha\beta} \hat{n}_{\alpha i} \hat{n}_{\beta i},$$

$$\hat{H}_{oc} = \sum_{\alpha \neq \beta} \sum_{i} \frac{U_{\alpha\beta}}{4} \left(\hat{a}^{+}{}_{\alpha i} \hat{a}^{+}{}_{\alpha i} \hat{a}_{\beta i} \hat{a}_{\beta i} + h.c. \right).$$

- $\hat{H}_{oc}$ - “orbital changing term” (Two α-orbital atoms scatter into two β-orbital atoms).

F. Pinheiro *et al.*, PRL 2013.

Cold atoms in excited bands

- Recepie:

  1) Mott-insulator ($n_i = 1$).
  2) Perturbation theory in $^t/_U$.
  3) *Schwinger spin-boson mapping.*

- Result: Heisenberg XYZ-model

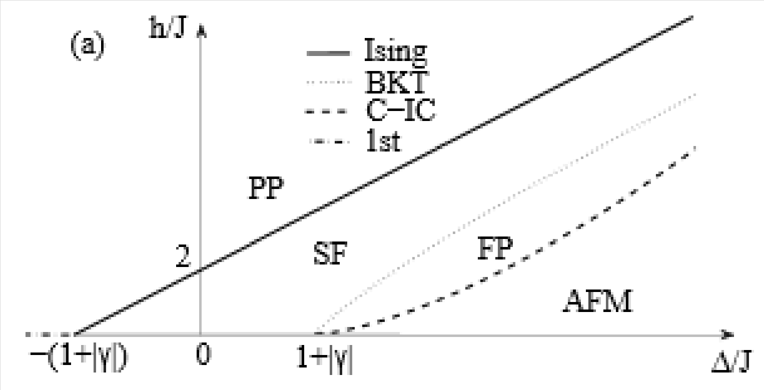
$$\hat{H}_{XYZ} = J\sum_{\langle ij\rangle}\left[(1+\gamma)\hat{\sigma}^x{}_i\hat{\sigma}^x{}_j + (1-\gamma)\hat{\sigma}^y{}_i\hat{\sigma}^y{}_j\right] + \Delta\sum_{\langle ij\rangle}\hat{\sigma}^z{}_i\hat{\sigma}^z{}_j + h\sum_i\hat{\sigma}^z{}_i.$$

- Non-integrable in the general case → promising quantum simulator.

F. Pinheiro *et al.*, PRL 2013.

Cold atoms in excited bands

- Comments:



1. Phase diagram in 1D fairly known.
2. Beyond tight-binding → *Dzyaloshinskii-Morya* terms.
3. Different lattice configurations → *Dzyaloshinskii-Morya* terms.
4. Three dimensions → $SU(3)$ models.
5. Spinor atoms → $SU(n) \times SU(m)$ models.
6. *d*-band → spin-1 models (also for $n = 2$ Mott on the *p*-band).
7. Including *s*-band atoms → disordered models (*many-body localization*).

F. Pinheiro *et al.*, PRL 2013.


Thanks!