Odd frequency pairing in hybrids and muliband superconductors

A. V. Balatsky

Los Alamos NATIONAL LABORATORY

http://theory.lanl.gov

NORDITA

Outline

- Introduction to odd-frequency pairing
- Odd-frequency pairing in Dirac Materials: e.g. topological insulators (TIs)-SC hybrid structures
- Odd-frequency pairing in multiband superconductors
- Odd frequency BEC

Symmetry of the Order Parameter

The superconducting anomalous pairing function is $\Delta_{\alpha\beta}(k,\tau)=\langle T c_{\alpha}(\tau) c_{\beta}(0)\rangle$. Due to Fermi statistics

$$\Delta_{\alpha\beta}(\mathbf{k}) = -\Delta_{\beta\alpha}(-\mathbf{k})$$

$$\Delta_{\alpha\beta}(\mathbf{k}) = \Delta_0 e^{i\varphi}\eta(\mathbf{k})\chi_{\alpha\beta}$$

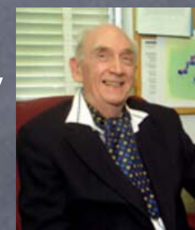
orbital

spin

spin-singlet *s*-wave $\Delta_{\alpha\beta}(\mathbf{k},\omega) = -\Delta_{\beta\alpha}(-\mathbf{k},-\omega)$

The pair function can also be odd in time/frequency: [1]

[1]: Berezinskii, JETP Lett. 20, 287 (1974)

- P. Coleman  V. Emery 
- A. Tsvelik  S. Kivelson 
- Y. Tanaka  T. Kirkpatrick  D.Belitz 
- M. Eschrig  K. Efetov

Bifurcating symmetries of SC order: odd frequency class

V. L. Berezinskii, JETP Lett. **20**, 287(1974)

$$\Delta_{\alpha\beta}(\tau,k) = \langle T_\tau c_{\alpha,k}(\tau) c_{\beta,-k}(0)\rangle,$$

$$\Delta(\tau,k) = \epsilon_{\alpha\beta}\Delta_{\alpha\beta}(\tau,k),$$

$$\vec{\Delta}(\tau,k) = (i\hat{\sigma}\vec{\sigma})_{\alpha\beta}\Delta_{\alpha\beta}(\tau,k).$$

$$\Delta(k,\tau) = \Delta(-k,-\tau), S=0$$

$$PT\Delta(k,\tau) = \Delta(k,\tau)$$

$$PT = 1$$

$$\vec{\Delta}(k,\tau) = -\vec{\Delta}(-k,-\tau), S=1$$

BCS class:
P =+1
T =+1
(even parity singlet)

Odd-frequency SC
P = -1
T = -1
(odd parity singlet)

EA, AVB
PRB45, p13125 (1992)

Classification

$$\mathrm{PT}[\Delta(\mathbf{r},\tau)]=\Delta(\mathrm{r},\tau)$$

$$PT=+1$$

$$S=0$$

$P=+1, T=+1$

$s-wave$

$time$ even

BCS pairing

$P=-1, T=-1$

$p-wave$

$time$ odd

odd-frequency pairing

$$\mathrm{PT}[\vec{\Delta}(\mathbf{r},\tau)]=\vec{\Delta}(\mathrm{r},\tau)$$

$$PT=-1$$

$$S=1$$

$P=-1, T=+1$

$p-wave$

time even

BCS triplet

$P=+1, T=-1$

$s-wave$

time odd triplet

odd-frequency pairing

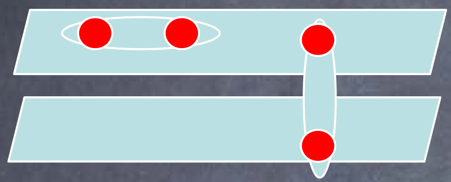
Possible symmetries

	ODD-frequency	BCS
S=0 PT=1	$(-1)^k(-1)^\omega$	$(+1)^k(+1)^\omega$
S=1 PT=-1	$(+1)^k(-1)^\omega$	$(-1)^k(+1)^\omega$

In simple terms

- Exists: spin singlet p-wave, odd-frequency
- Exists: spin triplet s-wave, odd frequency

Two layer(band) SC: parity vs spin



Layer index a,b = 1,2

$$< C_{a\sigma}(r)C_{a\sigma'}(r) >\sim \varepsilon_{\sigma\sigma'}F_e(r) - even$$

$$< C_{a\sigma}(r)C_{b\sigma'}(r') >\sim \varepsilon_{ab}\varepsilon_{\sigma\sigma'}F_o(r) - odd$$

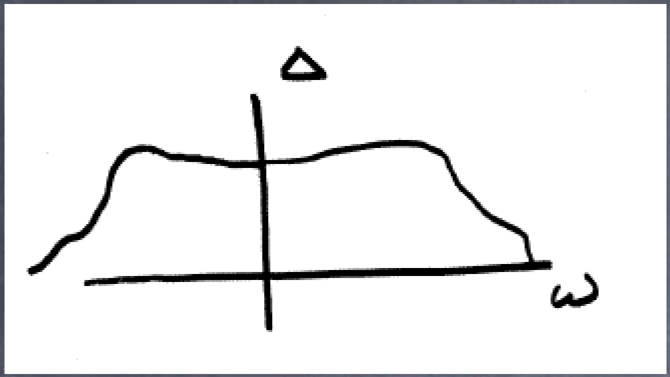
singlet that is odd in layer (orbital index) and is Podd(p wave)

A.Leggett, ~1995

One always needs another index:
1.extra neutral boson
2.orbital or band
3. or time

Mechanism - e-boson interaction

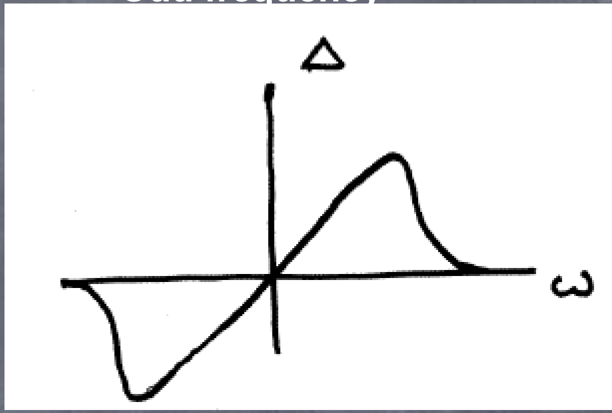
Even frequency



$$g \log\left(\frac{\omega_c}{T}\right) = 1$$

$$T_c \sim \omega_c \exp\left(\frac{-1}{g}\right)$$

Odd frequency



$$\frac{g}{g_c}\left(1 - \frac{T^2}{\omega_c^2}\right) = 1$$

$$T_c \sim \omega_c (g - g_c)^{1/2}$$

$$D\left(\vec{k}, \omega\right) = \int \frac{A_{\vec{k}}\left(\tilde{\omega}\right) d\tilde{\omega}}{\tilde{\omega}^2 + \omega^2}$$

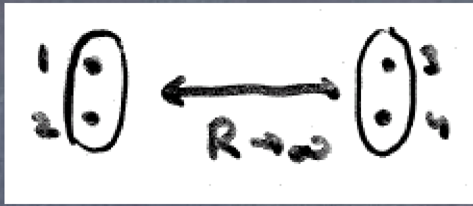
$$A_{\vec{k}-\vec{k}'}(\omega) = J_0 \omega_D^2 \delta(\omega - \omega_D) + J_1 \hat{k} \cdot \hat{k}' \omega_D^2 \delta(\omega - \omega_D)$$

IR theory

No IR div

Absence of conventional ODLRO

$$\left\langle \psi_{\uparrow}^{\dagger}\left(r_1 t_1\right)\psi_{\downarrow}^{\dagger}\left(r_2 t_2\right)\psi_{\uparrow}\left(r_3 t_3\right)\psi_{\downarrow}\left(r_4 t_4\right)\right\rangle \sim \left\langle \psi_{\uparrow}^{\dagger}\left(r_1 t_1\right)\psi_{\downarrow}^{\dagger}\left(r_2 t_2\right)\right\rangle - \left\langle \psi_{\uparrow}\left(r_3 t_3\right)\psi_{\downarrow}\left(r_4 t_4\right)\right\rangle$$



$\Delta\left(r,t\right)=0$ at $t=0$

No Cooper wave function

Why odd frequency state is interesting:
Satisfies no double occupancy constraints

$$\left\langle \psi^{+}_{\uparrow}\left(r_1 t\right)\psi^{+}_{\downarrow}\left(r_1 t\right)\right\rangle = \sum_{k,\omega} \Delta(k,\omega) = 0$$

Insensitive to pseudo Coulomb μ^*
In gap equation in contrast to standard BCS S-wave

Numerical Evidence for Odd Frequency SC a ground state

AF order is suppressed by geometrical frustration.

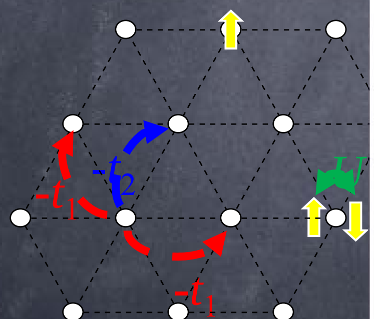
Nesting condition is not good any more.

Hubbard model

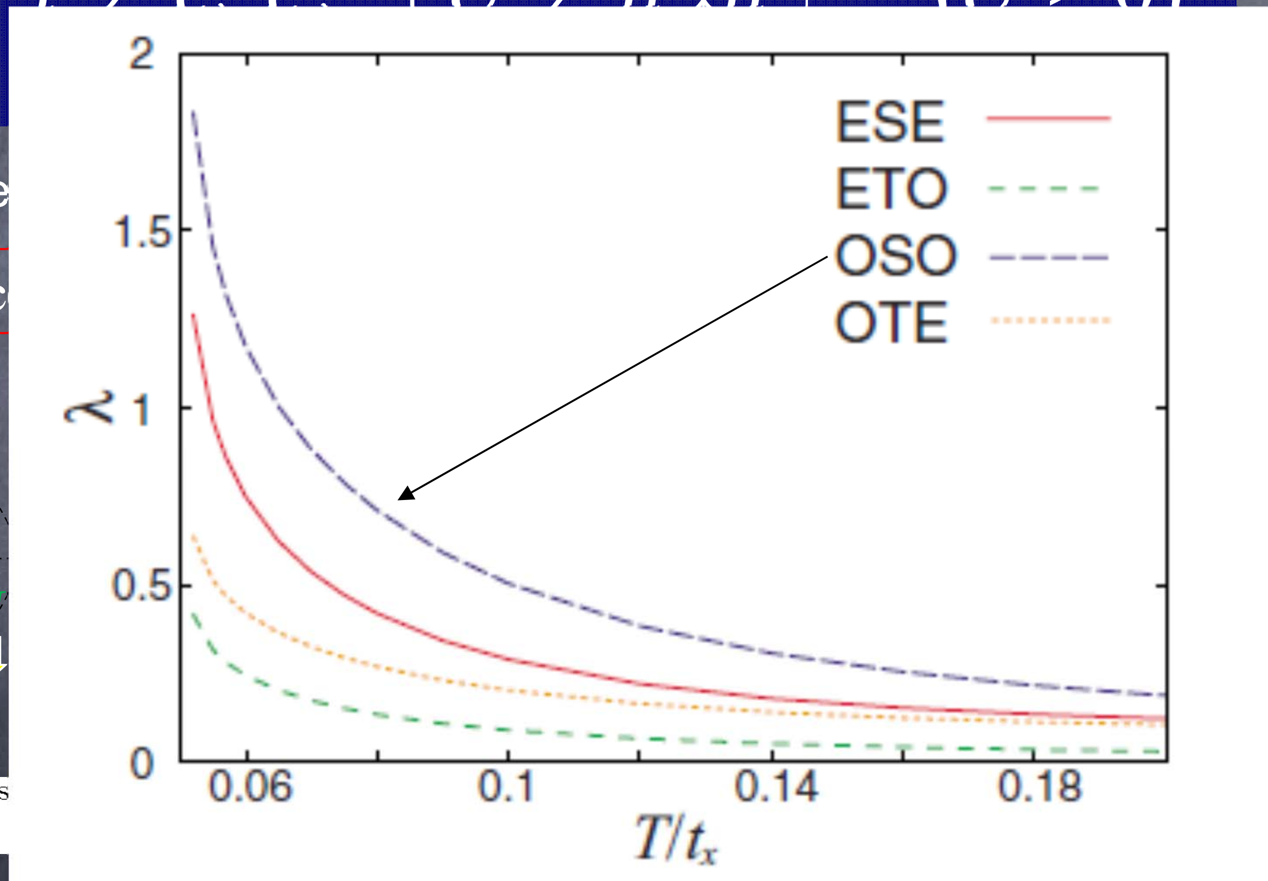
$$H = -t\sum c_i^+ c_j \; +U\sum n_\uparrow n_\downarrow \qquad (U>0)$$

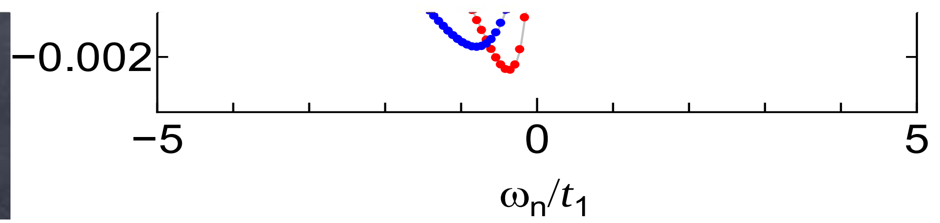
Kine

Triangular lattic



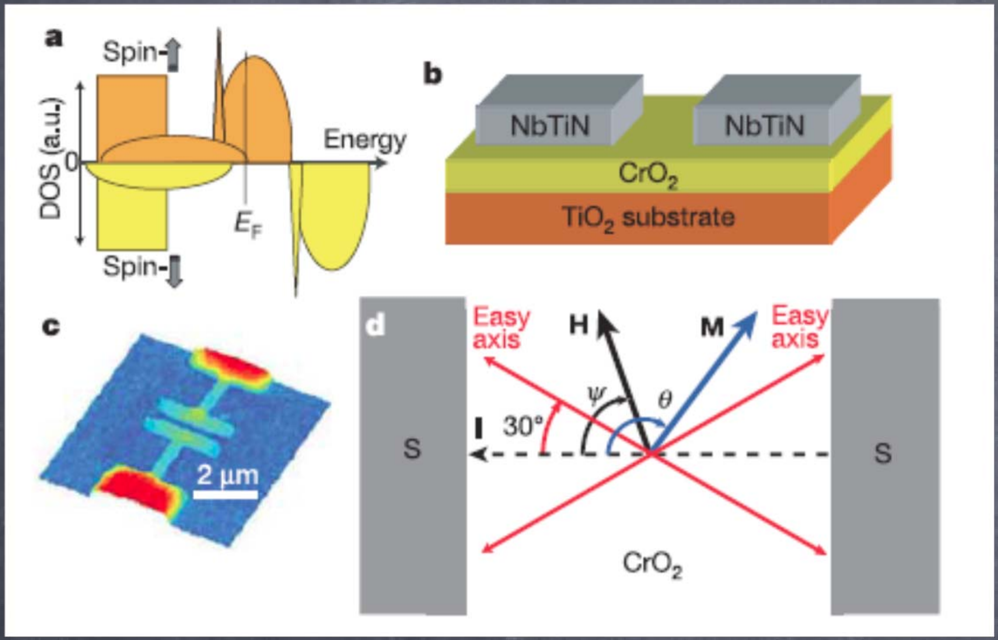
$\varepsilon_{\mathbf{k}} \;=\; -2t_1\left(\cos(k_x)+\cos\right.$
$\qquad - 2t_2\cos(k_x+k_y)$

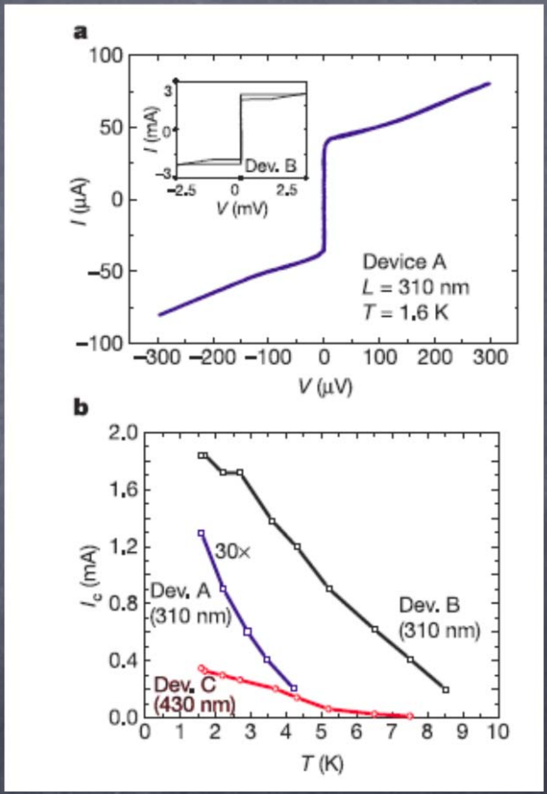




K. Shigeta, S. Onari, K. Yada and Y. Tanaka, Phys. Rev. B 79 174507 (2009).

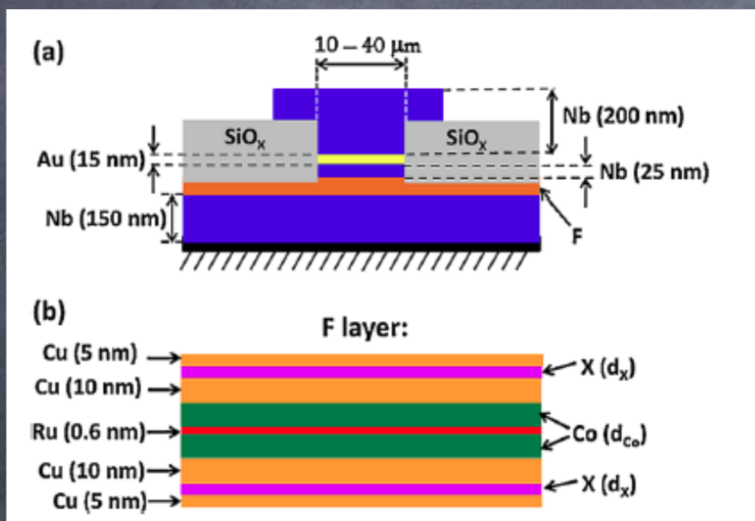
Experimental Evidence: examples

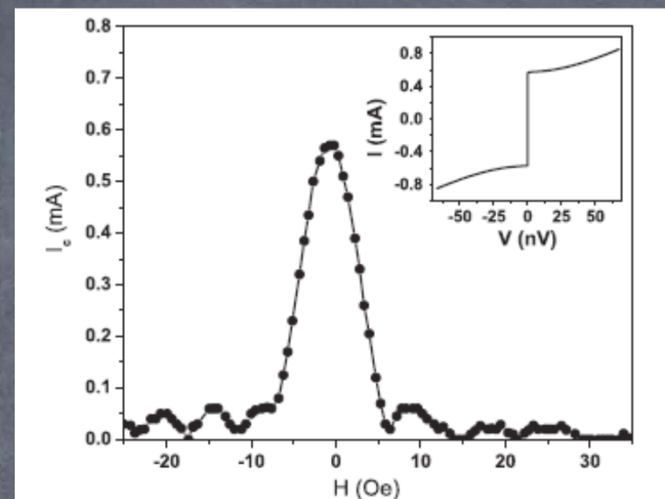


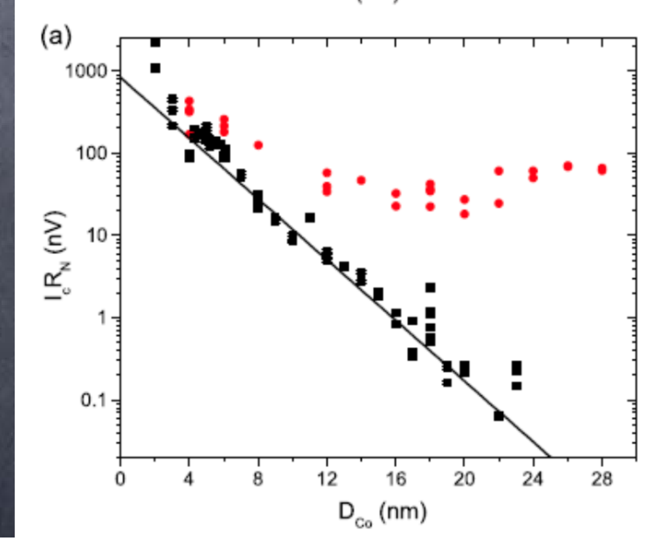


R. S. Keizer, S. T. B. Goennenwein, T. M. Klapwijk, G. Miao, G. Xiao, and A. Gupta, Nature (London) 439, 825 (2006).

Experimental Evidence: examples 2







N. Birge etal, PRL 104, 137002 (2010)

[Low-Field Superconducting Phase of (TMTSF)2ClO4]

F. Pratt, T. Lancaster, S. Blundell, and C. Baines

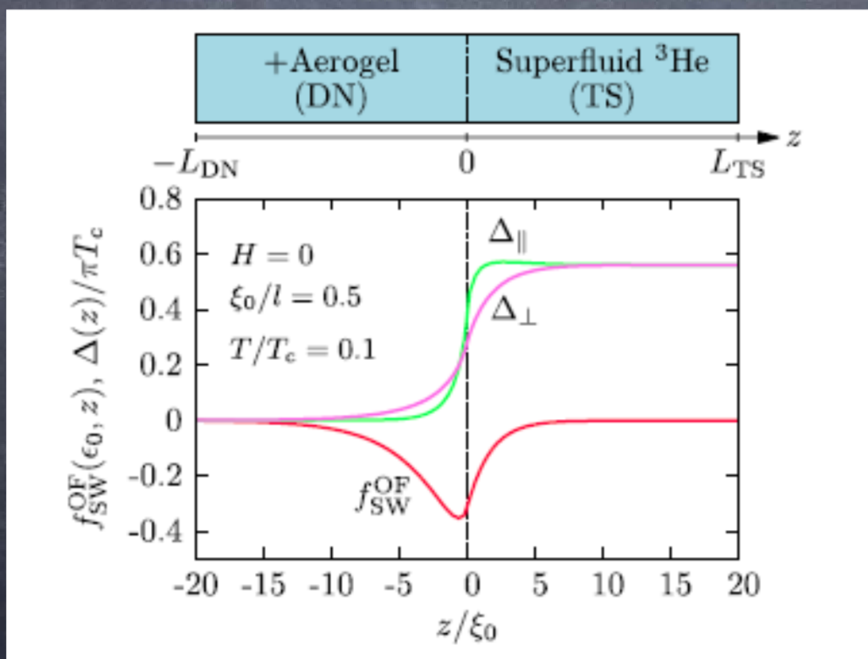
Phys. Rev. Lett. **110** 107005 (2013)

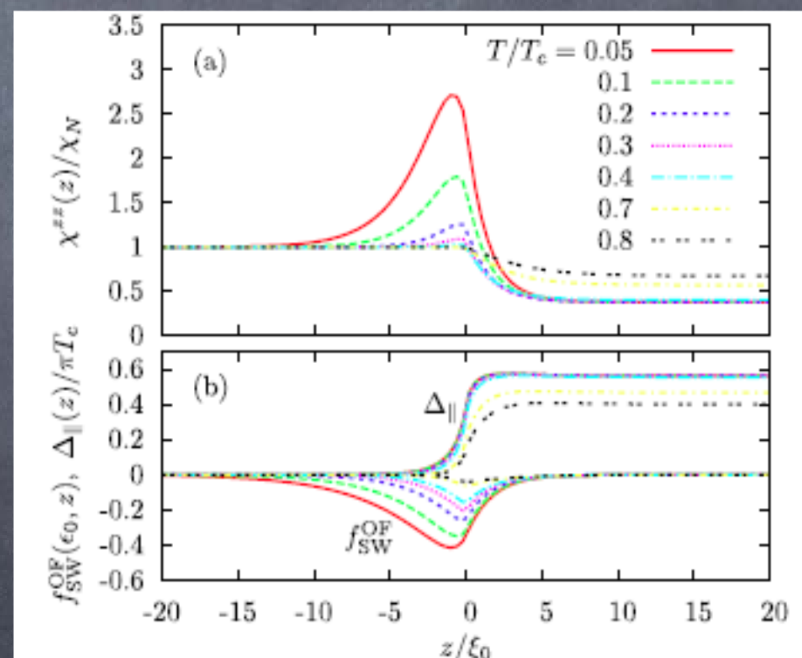
3He Superfluid example

PRL **110,** 175301 (2013) PHYSICAL REVIEW LETTERS week ending 26 APRIL 2013

Magnetic Response of Odd-Frequency *s*-Wave Cooper Pairs in a Superfluid Proximity System

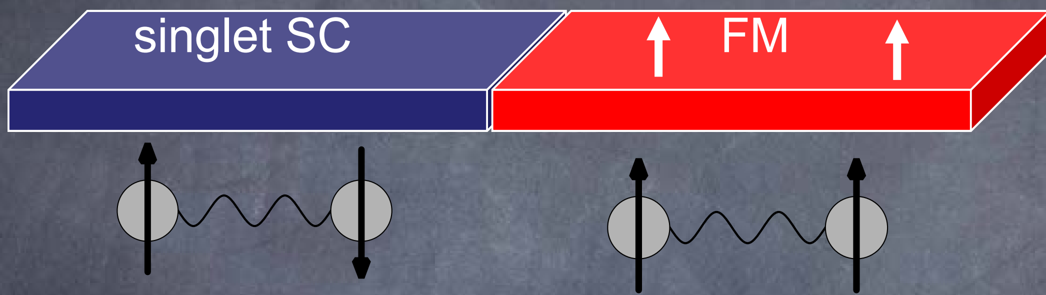
S. Higashitani, H. Takeuchi, S. Matsuo, Y. Nagato, and K. Nagai

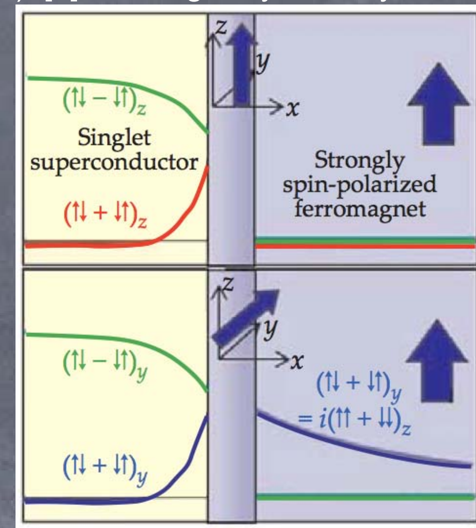




S|F Interface

Bergeret et al, RMP, 77, 1321 (2005), [1]: Eschrig, Phys. Today 64, 43 (2011)





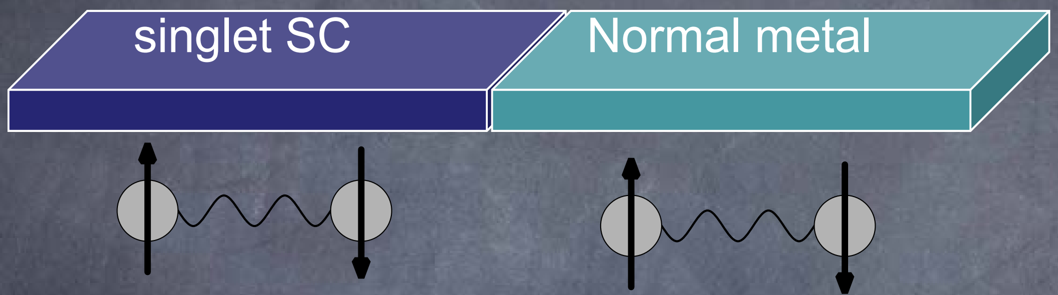
[1]

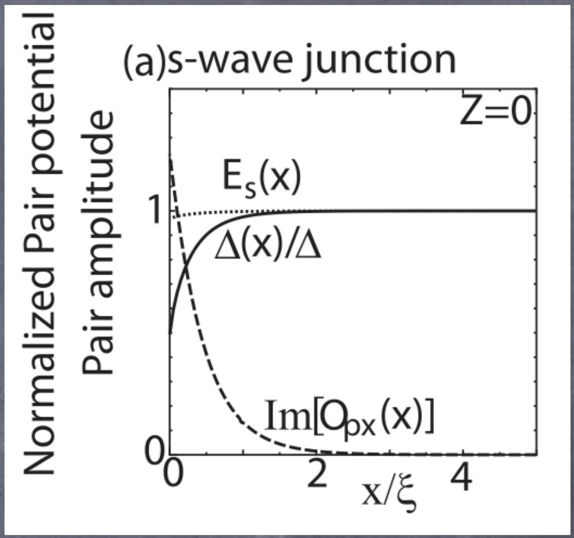
Spin-singlet *s*-wave pairing in the SC converted into

odd-frequency spin-triplet s-wave pairing in the FM

- Requires magnetic inhomogeneity or interface magnetization
- Long-range superconducting proximity effect in the FM
- *s*-wave = robust against impurities

S|N Interface





[1]

Spin-singlet *s*-wave pairing in the SC

converted into

odd-frequency spin-singlet p-wave pairing

- Interface generates *p*-wave pairing
- Only high-transparency junctions
- *p*-wave = only in ballistic systems

[1]: Tanaka et al, PRL 99, 037005 (2007)

Remaining and unexpected puzzles: known and unknown unknowns

Remaining issues, absent in the discussion

- Any observed phase transition typically triggers the questions:
- Order parameter of odd-frequency SC, equal time expectation value
- Wave function and GL functional

[1]: Abrahams et al, PRB 52, 1271 (1995) [2]: Dahal et al, NJP 11, 065005 (2009),

Odd-frequency Pairing Order Parameter

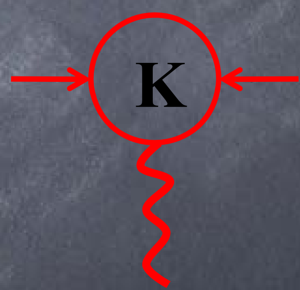
BCS order parameter:

$$F(\mathbf{r},t;\mathbf{r}',t'\to t)=\langle\psi(\mathbf{r},t)\psi(\mathbf{r}',t'\to t)\rangle$$

Equal-time odd-frequency order composite operator : pair+Boson[1,2] is an order parameter:

$$\left.\frac{\mathrm{d}F(\mathbf{r},t;\mathbf{r}',t')}{\mathrm{d}t}\right|_{t\to t'}$$

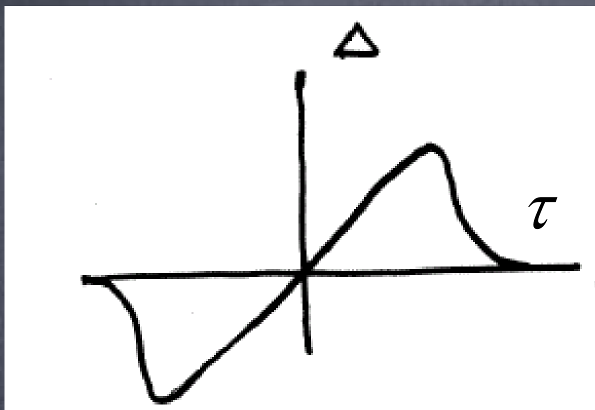
~



Theory proposals for odd-frequency bulk superconductors exists (e.g. composite boson condensate: Cooper pair + magnon [1,2]) but most interest have been focused on interfaces …

[1]: Abrahams et al, PRB 52, 1271 (1995) [2]: Dahal et al, NJP 11, 065005 (2009) [3] Coleman Tsvelik 1994, [4] Tanaka Golubov 2007

Order parameter in odd-frequency SC- composite boson



Thus we can take time derivative to obtain equal time expectation value

$$K = \partial_\tau \Delta(\tau)|_{\tau\to 0} = \left\langle T_\tau \partial_\tau c_\uparrow(\tau) c_\downarrow(0) \right\rangle = \left\langle T_\tau [H, c_\uparrow], c_\downarrow \right\rangle |_{\tau=0}$$

$$[H, c_{\alpha i}] = -t_{ij} c_{\alpha j} + J\vec{S}\vec{\sigma}_{\alpha\beta} c_{\beta i}$$

Janes Bonca+ AVB, PRB (95)

Composite boson condensate

$$i\left.\frac{\partial F_{\mathbf{k}}}{\partial \tau}\right|_{\tau=0} = \langle i\dot{c}_{\mathbf{k}\uparrow}c_{-\mathbf{k}\downarrow}\rangle = \sum_{n=-\infty}^{\infty} \omega_n F_{\mathbf{k}}(\omega_n)$$

For the spin-fermion Hamiltonian

$$H = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} \eta_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} J_{\mathbf{q}} \mathbf{S}_{\mathbf{q}} \cdot \sigma_{\alpha\beta} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}-\mathbf{q},\beta}$$

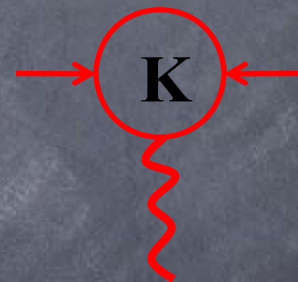
one has

$$i\dot{c}_{\mathbf{k}\uparrow} = \varepsilon_{\mathbf{k}} c_{\mathbf{k}\uparrow} + \sum_{\mathbf{q}} J_{\mathbf{q}} \mathbf{S}_{\mathbf{q}} \cdot \sigma_{\uparrow\beta} c_{\mathbf{k}-\mathbf{q},\beta}$$

Therefore,

$$\left(i\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}\right) F_{\mathbf{k}}$$

$$= \bar{\lambda}_{\mathbf{k}}(\tau) \equiv \sum_{\mathbf{q}} J_{\mathbf{q}} \langle \mathbf{S}_{\mathbf{q}} \cdot \sigma_{\uparrow\beta} c_{\mathbf{k}-\mathbf{q},\beta} c_{-\mathbf{k}\downarrow} \rangle$$



2e pair + boson

2e pair + 2 boson

Bonca PRB'92
J R Schrieffer et al,
J Superconductivity '95

In simple terms

- Exist: spin singlet p-wave, odd-frequency

- Exist: spin triplet s-wave, odd frequency

Equal-time operator (odd-frequency)

(t-J model case)

OSO (odd-frequency spin-singlet odd-parity)

$$\Delta^{odd}_{singlet} \propto (\vec{S}_{i-1} + \vec{S}_{i+2})(\sigma^y \vec{\sigma})_{\alpha\beta} c_{i,\alpha} c_{i+1,\beta}$$

Berezinskii (odd-frequency spin-triplet even-parity)

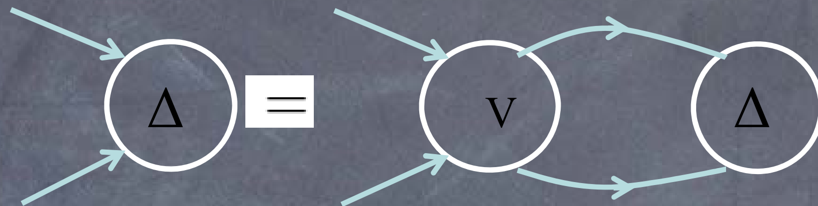
$$\Delta^{odd}_{triplet,Sz=0} \propto (\vec{S}_{i-1}(\sigma^x \vec{\sigma})_{\alpha\beta} - \vec{S}_{i+2}(\sigma^x \vec{\sigma})_{\beta\alpha}) c_{i,\alpha} c_{i+1,\beta}$$

$$\Delta^{odd}_{triplet,Sz=\pm 1} \propto (\vec{S}_{i-1}(\sigma \pm \sigma^z \vec{\sigma})_{\alpha\beta} - \vec{S}_{i+2}(\sigma \pm \sigma^z \vec{\sigma})_{\beta\alpha}) c_{i,\alpha} c_{i+1,\beta}$$

Balatsky Bonca PRB 48 7445 (1992)

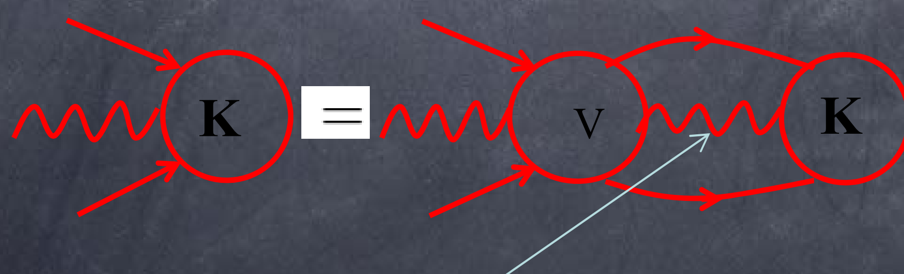
BCS case

$$\mathbf{1} = g\,\log\frac{\hbar\omega_c+\sqrt{\Delta^2+\hbar\omega_c^2}}{\Delta}$$



$$1 = V\int p^2\,dp\,\langle S^- S^+\rangle_p\,\log\frac{\varepsilon_C(p)+\sqrt{K^2+\varepsilon_C(p)^2}}{f(p)+\sqrt{K^2+f(p)^2}}$$

$$\varepsilon_C(p)=\frac{\varepsilon_{k+\frac{q}{2}}+\varepsilon_{k-\frac{q}{2}}}{2}+\frac{\omega_q\langle S^-S^+\rangle_q}{2}$$



Integral over momentum corresponding to this extra diagram softens the BCS log-divergence.

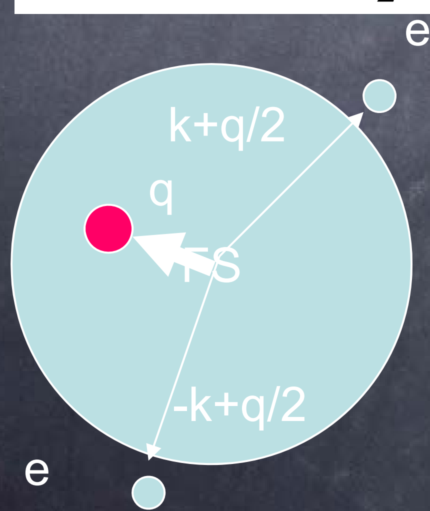
Coherent state

$$\left|\psi_{ODD,S=0}\right\rangle = \prod_{kq}\left(u_{kq} + v_{kq} c^{+}_{k+\frac{q}{2}\downarrow} c^{+}_{-k+\frac{q}{2}\downarrow} S^{+}_{q}\right)\left|0\right\rangle$$

$$\left|0\right\rangle = \left|vac_{fermion}\right\rangle\left|PM_{spin}\right\rangle$$

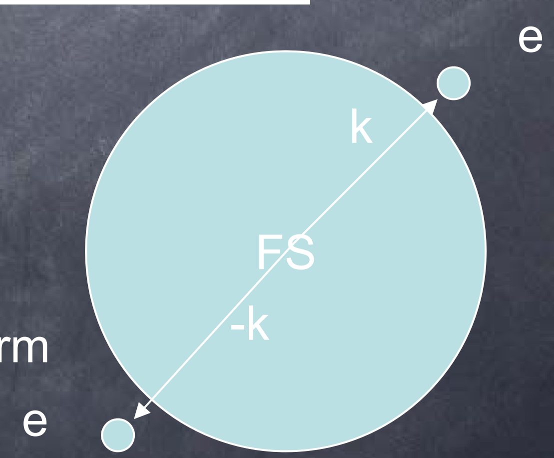
Coherent state that describes condensate 2e and spin triplet

$$\left\langle \Psi_{odd,S=0}\right| c^{+}_{k+\frac{q}{2}\downarrow} c^{+}_{-k+\frac{q}{2}\downarrow} S^{+}_{q} \left|\Psi_{odd,S=0}\right\rangle = K \quad \text{ODLRO}$$



Composite K order
(No Cooper)
Instability

Takes three particles to form
A condensate



BCS (Cooper)
instability

Explicit proof of existence of a new class of SC:

$$\left|\Psi_{ODD,S=0}\right\rangle = \prod_{kq}(u_{kq} + v_{kq} c^{+}_{k+\frac{q}{2}\downarrow} c^{+}_{-k+\frac{q}{2}\downarrow} S^{+}_{q})\left|0\right\rangle$$

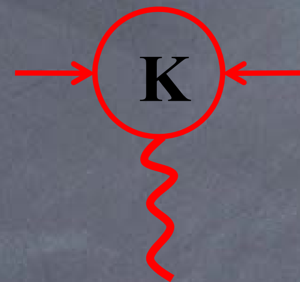
$$\left|\Psi_{BCS,S=0}\right\rangle = \prod_{k}(u_{k} + v_{k} c^{+}_{k\uparrow} c^{+}_{-k\downarrow})\left|0\right\rangle$$

$$\left\langle\Psi_{ODD,S=0}\right|\left|\Psi_{BCS,S=0}\right\rangle = \left\langle 0 \mid 0\right\rangle \prod_{kk'q}(u_{k} u_{k'q}) \to 0_{N\to\infty}\,!$$

Both states are spin singlet S =0 yet they have zero overlap with BCS wf for macrosopic state.
This proves that there is a spin singlet state that is qualitatively different then
The conventional BSC state! It is an odd frequency S = 0 superconductor with the
wave function

$$\left|\Psi_{ODD,S=0}\right\rangle = \prod_{kq}(u_{kq} + v_{kq} c^{+}_{k+\frac{q}{2}\downarrow} c^{+}_{-k+\frac{q}{2}\downarrow} S^{+}_{q})\left|0\right\rangle$$

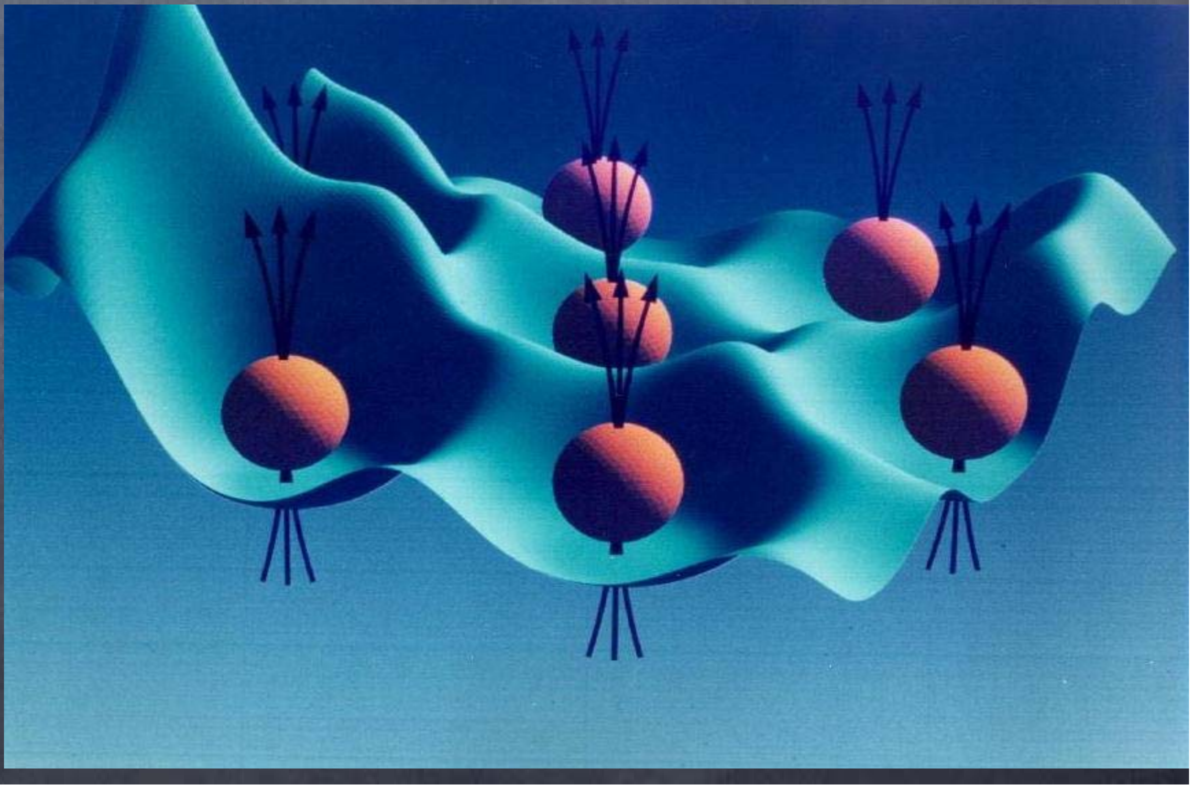
How to get BCS results from the current formulation



BCS limit can be recovered at any stage of this analysis if we assume that spin correlators are factorized and have a peak at $\mathbf{q} = 0$. This limit corresponds to the condensation of spin field $\langle S^- S^+ \rangle_{\mathbf{q}} = \langle S^- \rangle_{\mathbf{q}} \langle S^+ \rangle_{\mathbf{q}} \delta_{\mathbf{q},0}$. In this limit additional summation over $\mathbf{q}$ drops out and we recover standard BCS logarithm in selfconsistency equation along with other features of BCS solution. This limit corresponds to the factorizitation of composite boson into product $\langle \psi | c^\dagger_{\mathbf{k}+\frac{\mathbf{q}}{2}\uparrow} c^\dagger_{-\mathbf{k}+\frac{\mathbf{q}}{2}\downarrow} S^+_{\mathbf{q}} | \psi \rangle \rightarrow \langle \psi | c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} | \psi \rangle \langle \psi | S^+_{\mathbf{q}} | \psi \rangle \delta_{\mathbf{q},\mathbf{0}}$.

Composite Fermions

- QHE: Composite state of Fermion + Boson (Flux tubes)



More then one way to extend Cooper pairing as a paradigm for SC

Conventional discussion →

Boson attached to pair ↓

$$2e \qquad 4e \qquad 6e$$

$$2e + neutral\ boson \qquad 4e + boson \qquad 6e + boson$$

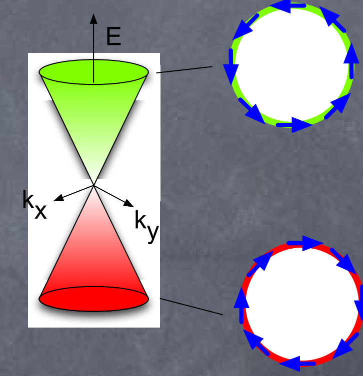
$$2e + 2boson \qquad 4e + 2boson \qquad 6e + 2boson$$

Topological Insulator (TI)

Surface state of a topological insulator

– Dirac Material = nodal spectrum

– Momentum locked to spin



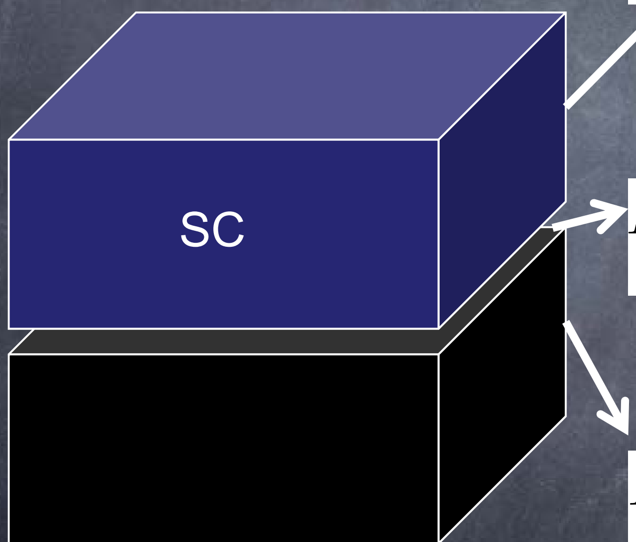
Spin-singlet *s*-wave superconductor + TI:

→ odd-frequency spin-triplet *s*-wave pairing

Symmetry breaking through:

- Dirac surface state
- In-surface gradient in the superconducting gap

TI – SC Hybrid Structure



$$H_{\mathrm{SC}} = \sum_{\mathbf{k},\alpha,\beta} \varepsilon(\mathbf{k}) d^{\dagger}_{\alpha,\mathbf{k}} d_{\alpha,\mathbf{k}} + \sum_{i,\alpha,\beta} \Delta(i)_{\alpha\beta} d^{\dagger}_{\alpha,i} d^{\dagger}_{\beta,i} + \mathrm{H.c.}$$

Conventional *s*-wave SC with position (*i*) dependent order parameter

$$H_T = \sum_{\alpha} T_i c^{\dagger}_{\alpha,i} d_{\alpha,i} + \mathrm{H.c.}$$

Local tunneling

$$H_{\mathrm{TI}} = \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\alpha,\mathbf{k}} \mathbf{k} \cdot \sigma_{\alpha\beta} c_{\beta,\mathbf{k}}$$

TI surface state

Black-Schaffer and Balatsky, PRB 86, 144506 (2012)

Analytic Derivation

Anomalous Green's function in the TI:

$$F_{\mathrm{TI},\alpha\beta}(\tau|\mathbf{k},\mathbf{k}') = -i\langle T_\tau c_\alpha(\tau,\mathbf{k})c_\beta(0,\mathbf{k}')\rangle$$

$$\hat{F}_{\mathrm{TI}}(\omega_n|i,i) = -|T|^2\sum_{j,l}\hat{G}^0(\omega_n|i,j)\hat{F}(j,l)\hat{G}^0(\omega_n|l,i)$$

$$\hat{G}^0(\omega_n,\mathbf{k}) = \frac{\mathbf{k}\cdot\sigma - i\omega_n}{\mathbf{k}^2+\omega_n^2}$$

$$\hat{F}(\omega_n,\mathbf{k}) = \frac{\hat{\Delta}(i)}{\omega_n^2+\varepsilon^2(\mathbf{k})+\hat{\Delta}(i)^2}$$

$$\Delta(\mathbf{k}) = \Delta_0\delta_{\mathbf{k},0} + i\frac{\partial\Delta}{\partial x}|_0\partial_{\mathbf{k}_x}$$

Order parameter for odd-frequency pairing:

$$\hat{F}_{\mathrm{TI}}(\omega_n|i=0) = \sum_{\mathbf{k}}\frac{|T|^2\omega_n\hat{\sigma}\partial_x\hat{\Delta}|_0}{2[\omega_n^2+\varepsilon(\mathbf{k})^2+\Delta^2(0)](\omega_n^2+\mathbf{k}^2)^2}$$
$$\sim |T|^2\omega_n\sigma^z\partial_x\Delta|_0/(E_F^2|\omega_n|^2)$$

➡

$$\partial_\tau\hat{F}_{\mathrm{TI}}(\tau|i)|_0 \sim \frac{\partial\Delta}{\partial x}$$

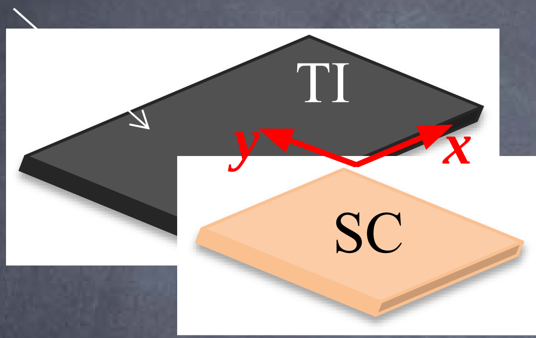
→ Odd-frequency spin-triplet *s*-wave pairing:

- For spatially inhomogenous SCs
- $1/\omega$ dependence

Black-Schaffer and Balatsky, PRB 86, 144506 (2012)

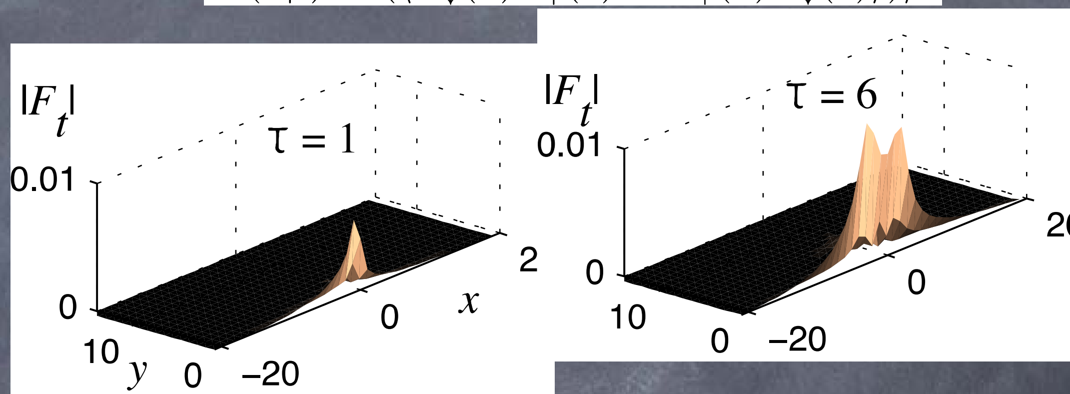
S|N Junction in a 2D TI

Kane-Mele 2D TI



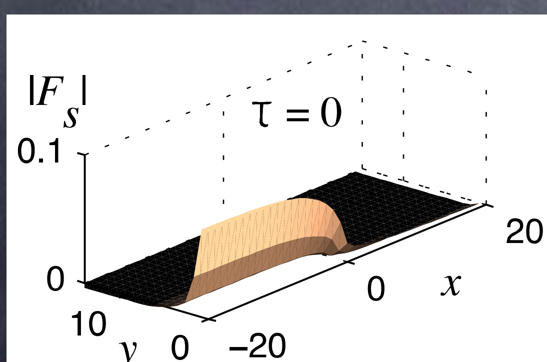
Spin-triplet *s*-wave pairing:

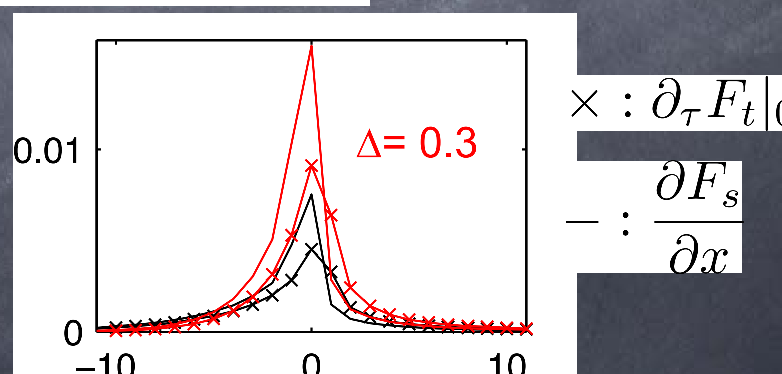
$F_t(\tau|i) = (\langle c_{i\downarrow}(\tau)c_{i\uparrow}(0) + c_{i\uparrow}(\tau)c_{i\downarrow}(0)\rangle)/2$



Spin-singlet *s*-wave pairing:

$F_s(\tau|i) = (\langle c_{i\downarrow}(\tau)c_{i\uparrow}(0) - c_{i\uparrow}(\tau)c_{i\downarrow}(0)\rangle)/2$





$$\partial_\tau F_t|_0 \sim \frac{\partial F_s}{\partial x}$$

Black-Schaffer and Balatsky, PRB 86, 144506 (2012)

Odd frequency Multiband Superconductors

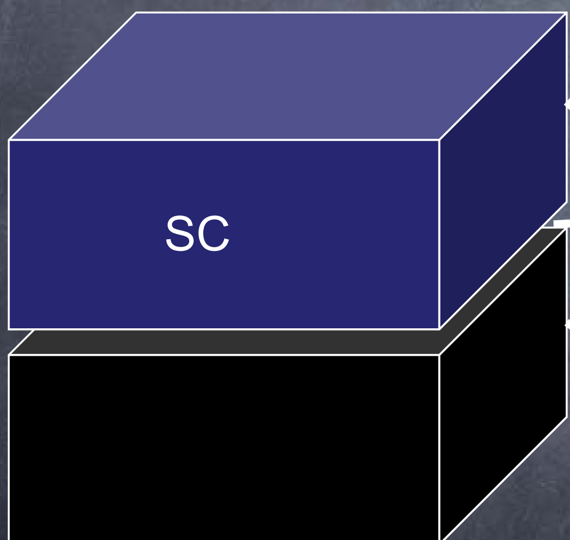
- Spin-singlet (S = 0) or spin-triplet (S = 1)
- Space parity P (*s*-,*d*- or *p*-wave)
- Even or odd-frequency dependence T
- Orbital parity O (band index permutation)

$S = 0$	P	T	O	$S = 1$	P	T	O
even-ω	$+$	$+$	$+$	even-ω	$-$	$+$	$+$
even-ω	$-$	$+$	$-$	even-ω	$+$	$+$	$-$
odd-ω	$+$	$-$	$-$	odd-ω	$+$	$-$	$+$
odd-ω	$-$	$-$	$+$	odd-ω	$-$	$-$	$-$

Spin-singlet *s*-wave: TO = +1

Black-Schaffer and Balatsky, arXiv:1305.4593, PRB 2013

Bi_2Se_3 – SC Hybrid Structure



2D superconductor (D_{4h} symmetry):

$$H_{\rm SC} = \sum_{\mathbf{k},\sigma} (-2\cos(k_x a) - 2\cos(k_y a) + \mu_{\rm SC}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{2}\sum_{\mathbf{k},\sigma,\sigma'} \Delta_{\sigma\sigma'}(\mathbf{k}) c^\dagger_{\mathbf{k}\sigma} c^\dagger_{-\mathbf{k}\sigma'} - \Delta^*_{\sigma\sigma'}(-\mathbf{k}) c_{-\mathbf{k}\sigma} c_{\mathbf{k}\sigma'}$$

Local tunneling:

$$H_T = -\sum_{\mathbf{k},\sigma} T_1 c^\dagger_{\mathbf{k}\sigma} b_{1\mathbf{k}\sigma} + T_2 c^\dagger_{\mathbf{k}\sigma} b_{2\mathbf{k}\sigma} + \text{H.c.}$$

Bi_2Se_3 on a cubic lattice (2 orbitals per site (τ)): [1]

$$H_{\rm TI} = \gamma_0 - 2\sum_{\mathbf{k},i} \gamma_i \cos(k_i a) + \sum_{\mathbf{k},\mu} d_\mu \Gamma_\mu$$

$$d_0 = \epsilon - 2\sum_i t_i \cos(k_i a),\ d_i = -2\lambda_i \sin(k_i a)$$

$$\Gamma_0 = \tau_x \otimes \sigma_0,\ \Gamma_x = -\tau_z \otimes \sigma_y,\ \Gamma_y = \tau_z \otimes \sigma_x,\ \Gamma_z = \tau_y \otimes \sigma_0$$

Black-Schaffer and Balatsky, PRB 87, 220506(R) (2013), [1]: Rosenberg and Franz, PRB 85, 195119 (2012)

Odd frequency SC in any Generic Two-Band SC? Yes.

Bands (orbitals) *a* & *b* with finite interband hybridization/scattering Γ:

$$H_{ab} = \sum_{\mathbf{k}\sigma} \varepsilon_a(\mathbf{k}) a^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \varepsilon_b(\mathbf{k}) b^\dagger_{\mathbf{k}\sigma} b_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\sigma} \Gamma(\mathbf{k}) a^\dagger_{\mathbf{k}\sigma} b_{\mathbf{k}\sigma} + \text{H.c.}$$
$$+ \sum_{\mathbf{k}} \Delta_a(\mathbf{k}) a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} + \Delta_b(\mathbf{k}) b^\dagger_{\mathbf{k}\uparrow} b^\dagger_{-\mathbf{k}\downarrow} + \text{H.c.}$$

$$H_{cd} = \sum_{\mathbf{k}\sigma} \varepsilon_c(\mathbf{k}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \varepsilon_d(\mathbf{k}) d^\dagger_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma}$$
$$+ \sum_{\mathbf{k}} \Delta_c(\mathbf{k}) c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \Delta_d(\mathbf{k}) d^\dagger_{\mathbf{k}\uparrow} d^\dagger_{-\mathbf{k}\downarrow} + \text{H.c.}$$
$$+ \sum_{\mathbf{k}} \Delta_{cd}(\mathbf{k}) (c^\dagger_{\mathbf{k}\uparrow} d^\dagger_{-\mathbf{k}\downarrow} + d^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}) + \text{H.c.}$$

Diagonal bands

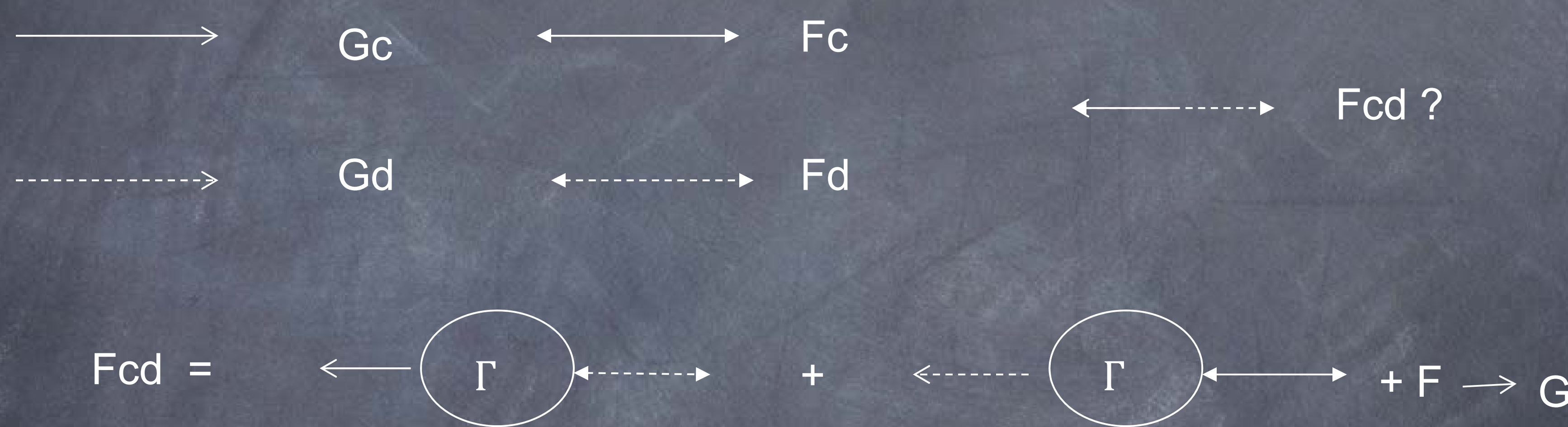
Intraband pairing

Interband pairing =

$$\Delta_{cd} = \frac{(\Delta_b - \Delta_a)|\Gamma|}{\sqrt{(\varepsilon_a - \varepsilon_b)^2 + 4|\Gamma|^2}}$$

Black-Schaffer and Balatsky, arXiv:1305.4593, PRB 2014

Feynman Diagrams



$$F_{cd}(\omega,r) \sim G_c(-\omega,-r)\Gamma F_d(\omega,r) + G_c(\omega,r)\Gamma F_d(-\omega,-r)$$

$$\sim (-i\omega-\varepsilon_c)\Gamma\Delta_d + (i\omega-\varepsilon_d)\Gamma\Delta_c + \ldots \sim i\omega\Gamma(\Delta_d-\Delta_c)$$

Odd frequency means hybridization induced component here

Time-dependent Pairing

Time-ordered s-wave interband pairing:

$$F^{\pm}(\tau) = \frac{1}{2N_{\mathbf{k}}} \sum_{\mathbf{k}} \mathcal{T}_\tau \langle c_{-\mathbf{k}\downarrow}(\tau) d_{\mathbf{k}\uparrow}(0) \pm d_{-\mathbf{k}\downarrow}(\tau) c_{\mathbf{k}\uparrow}(0) \rangle$$

$$F^e = F^+(\tau \to 0^+)$$

Even-frequency, even-interband pairing

$$F^o_\omega = \left. \frac{\partial F^-}{\partial \tau} \right|_{\tau \to 0^+}$$

Odd-frequency, odd-interband pairing

PTO = +1 or -1

Black-Schaffer and Balatsky, arXiv:1305.4593

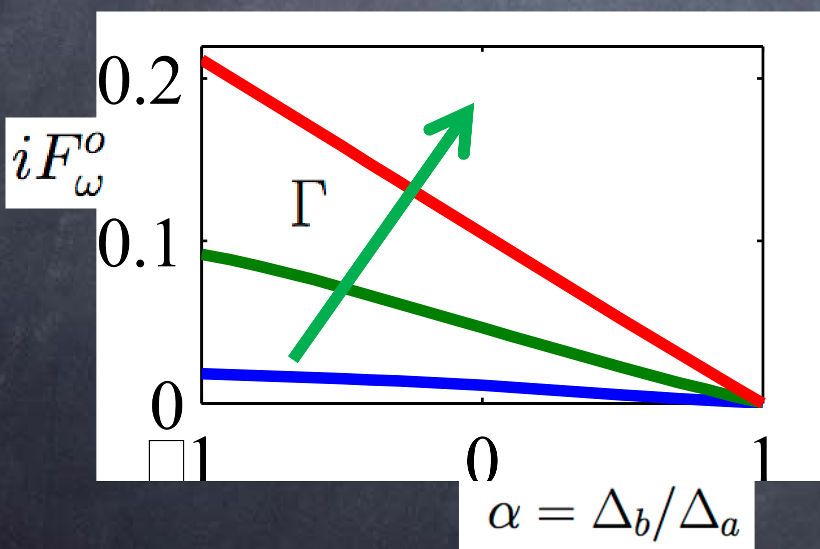
Odd-Frequency, Odd-Interband Pairing

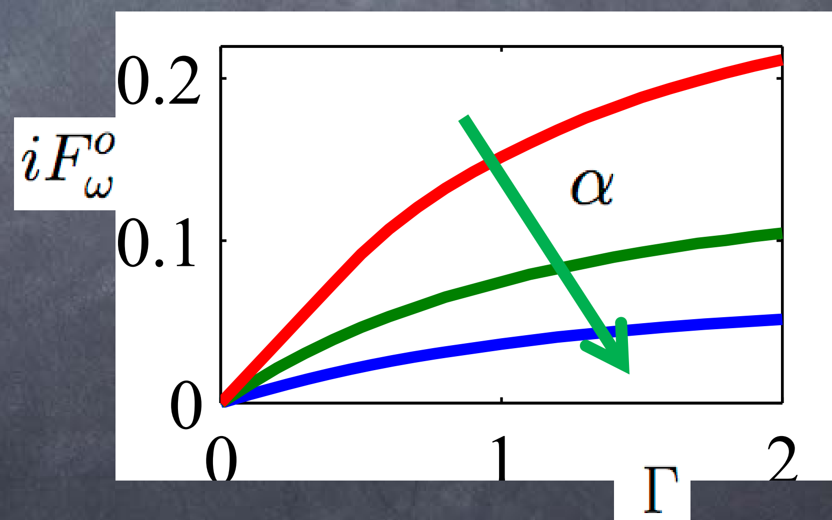
For $\begin{cases} \varepsilon_a = \varepsilon_b \\ \Delta_a = -\Delta_b \end{cases}$ → $\begin{array}{l} \varepsilon_{c,d} = \varepsilon_a \mp \Gamma \\ \Delta_c = \Delta_d = 0 \\ \Delta_{cd} = \Delta_a \end{array}$ $\xrightarrow{\Gamma < \Delta_a}$ $F^e = -\frac{1}{2N_{\mathbf{k}}} \sum_{\mathbf{k}} \frac{\Delta_a}{\sqrt{\varepsilon_a^2 + |\Delta_a|^2}}$

Only interband pairing

BCS equation

$F^o_\omega = i\Gamma F^e$ Odd-frequency component



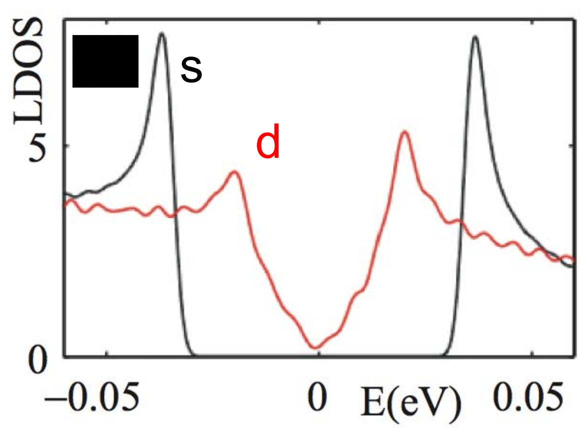


Black-Schaffer and Balatsky, arXiv:1305.4593

LDOS of *p*-wave SCs - Bi_2Se_3

p-wave SC in TI surface is gapless: $\varepsilon(\mathbf{k}) = \pm\left[v_F|\mathbf{k}| \pm \sqrt{\mu^2 + |\Delta(\mathbf{k})|^2}\right]$ [1]

Superconductor			Even-frequency	
Γ	Basis function	J_z	Even-orbital	Odd-orbital
A_{1u}	$\mathbf{d} = (k_x, k_y, 0)$	0	A_{1u} triplet $(m_s = \pm 1)$	A_{1g} triplet $(m_s = 0)$ **ungapped** [2]
A_{2u}	$\mathbf{d} = (k_y, -k_x, 0)$	0	A_{2u} triplet $(m_s = \pm 1)$, A_{1g} singlet **gapped**	-
B_{1u}	$\mathbf{d} = (k_x, -k_y, 0)$	± 2	B_{1u} triplet $(m_s = \pm 1)$, B_{2g} singlet	B_{1g} triplet $(m_s = 0)$
B_{2u}	$\mathbf{d} = (k_y, k_x, 0)$	± 2	B_{2u} triplet $(m_s = \pm 1)$, B_{1g} singlet **nodal**	B_{2g} triplet $(m_s = 0)$
E^+_{2u}	$\mathbf{d} = (0, 0, k_x + ik_y)$	1	E^+_{2u} triplet $(m_s = 0)$	A_{1g} triplet $(m_s = 1)$, $B_{1g}+iB_{2g}$ triplet $(m_s = -1)$ **ungapped**
E^-_{2u}	$\mathbf{d} = (0, 0, k_x - ik_y)$	-1	E^-_{2u} triplet $(m_s = 0)$	A_{1g} triplet $(m_s = -1)$, $B_{1g}-iB_{2g}$ triplet $(m_s = 1)$ **ungapped**



Black-Schaffer and Balatsky, PRB 87, 220506(R) (2013)
[1]: Linder et al PRL 104, 067001 (2010), [2]: Hao and Lee, PRB 83, 134516 (2011)

Reciprocity
Odd-f and odd-band

- PTO = -1(F) +1(B)
- Choose particular P = +1
- TO = -1(F) +1(B)
- T ->-T implies O->-O

Odd frequency BEC

Start with the multicomponent(multilayers, multi band)
boson gas, b_a(r,t) operators
a- flavor, layer, spin , hyperfine state index. Analogous to spin-nematic (Andereev Grischuk 1984)

$$1. < b_a >= 0$$

$$2. D_{ab}(r,t) = < T_t b_a(r,t) b_b(0,0) > \neq 0$$

can be nontrivial function, but as long as it is finite there is a condensate.

$$\mathrm{b_a}(r,t) \to e^{i\theta} \mathrm{b_a}(r,t)$$

$$D_{ab}(r,t) \to e^{i2\theta} D_{ab}(r,t) - \text{boson - nematic order}$$

similar to fermions

$$D_{ab}(r,t) = D_{ba}(-r,-t)$$

$$\mathrm{PTO} = +1$$

T	P	O
$+1$	$+1$	$+1$
$+1$	-1	-1
-1	$+1$	-1
-1	-1	$+1$

Balatsky, arXiv:1409.4875 Odd-frequency Two Particle Bose-Einstein Condensate

Definition of the order parameter for odd frequency pair BEC

$H = \varepsilon_a b^*{}_a b_a + V_{ab} n_a n_b + t_{ab}(b^*{}_a b_b + hc)$

$D_{ab}(r, t = 0) = 0$

$\partial_t D_{ab}(r - r', t = 0) \neq 0 \rightarrow$

$d_{ab} = \langle \partial_t b_a(r,0) b_b(r',0) \rangle \neq 0$ is an order parameter

$\partial_t b_a = b_a V_{ab} n_b + t_{ab} b_b$

$d_{ab} = \langle b_a(r,0) b_b(r',0) \rangle \rightarrow$ off diagonal interflavor (interlayer) condensate!

this state can exist.

Hence odd frequency boson nematic state can exist.

Reciprocity for bosons

Even interband BEC even-f BEC

↓

Odd interband BEC odd-f BEC

Multiband interacting boson models
would be a good candidate.

Odd fellows

References of more works in odd frequency superconductivity

- "Realization of Odd-Frequency *p*-Wave Spin–Singlet Superconductivity Coexisting with Antiferromagnetic Order near Quantum Critical Point" Yuki Fuseya, Hiroshi Kohno, and Kazumasa Miyake,J. Phys. Soc. Jpn. **72**, 2914(2003)
- "Possible realization of odd-frequency pairing in heavy fermion compounds", P. Coleman, E. Miranda, and A. Tsevelik, Phys. Rev. Lett. **70**, 2960 - 2963 (1993).
- "Odd-frequency pairing in the Kondo lattice", P. Coleman, and E. Miranda, Phys. Rev. B **49**, 8955 - 8982 (1994)
- "Identifying the odd-frequency pairing state of superconductors by a field-induced Josephson effect" Jacob Linder, Takehito Yokoyama, and Asle Sudbo, Phys. Rev. B **77**, 174507 (2008).
- "Nesting, spin fluctuations, and odd-gap superconductivity in NaxCoO2.yH2O.", Johannes MD, Mazin II, Singh DJ, Papaconstantopoulos DA, Phys. Rev. Lett. **93**, 097005(2004).
- "Quantum transport in a normal metal/odd-frequency superconductor junction", Jacob Linder, Takehito Yokoyama, Yukio Tanaka, Yasuhiro Asano, and Asle Subdo, Cond-mat/0712.0443.
- 4) "Manifestation of the odd-frequency spin-triplet pairing state in diffusive ferromagnet/superconductor junctions." T. Yokoyama, Y. Tanaka, and A. A. Golubov, Phys. Rev. B **75**, 134510(2007).
- "Anomalous Josephson Effect between Even- and Odd-Frequency Superconductors", Yuko Tanaka, Alexander A. Golubov, Satoshi Kashiwaya, and Masahito Ueda, Phys. Rev. Lett. **99,** 037005(2007).
- "Odd-frequency pairing in normal-metal/superconductor junctions." Y. Tanaka, Y. Tanuma, and A. A. Golubov, Phys. Rev. B 76, 054522(2007).

"Odd-frequency pairing in binary boson-fermion atom mixture", Ryan M. Kalas, Alexander Balatsky, and Dimitry Mozyrsky, cond-mat/0806.0419., PRB 2009.

Summary of previous works establishes the idea of the composite pairing

1) New class of singlet superconductors which break the time reversal and parity,
Alexander Balatsky, and Elihu Abrahams, PRB **45**, 13125(1992).
Proposal of the odd-frequency spin-singlet p-wave superconductivity. e-ph interaction is tried and realized that it can not mediate the odd frequency superconductivity

2) Interactions for odd-ω-gap singlet superconductors
Elihu Abrahams, Alexander Balatsky, J. R. Schrieffer, and Philip B. Allen, PRB 47, 513(1993)
Proposal of a spin dependent electron-electron interaction. Does not face the problem that e-ph interaction faces.

3) Odd Frequency pairing in Superconductors,
J. R. Schrieffer, Alexander Balatsky, Elihu Abrahas, and Douglas J. Scalapino, Journal of Superconductiviy **7**, 501(1994).
Using equation of motion it is shown that the condensate of the odd-frequency pairing consists of two fermions and a boson (composite).

4) Even- and odd-frequency pairing correlations in the one-dimensional *t-J-h* model: A comparative study, A. V. Balatsky, and J. Bonca, Phys. Rev. B **48**, 7445 - 7449 (1993).
Binding of Cooper pairs with magnetization fluctuations naturally appears in this model.

5) Properties of odd-gap superconductors,
Elihu Abrahams, Alexander Balatsky, D. J. Scalapino and J. R. Schrieffer, Phys. Rev. B **52**, 1271 (1995).

6) H. Daha et al, New J. Phys. 11 (2009) 065005. – wave function and QP energy and DOS calculation

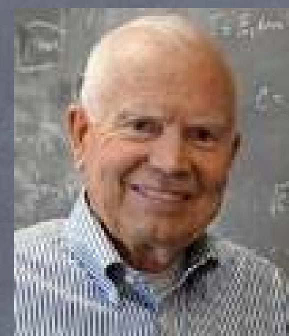
7) A. Black-Schaffer and A.V. Balatsky, PRB 86, 144506 (2012)

8) A. Black-Schaffer and A.V. Balatsky, PRB 87, 220506(R) (2013)

=> Odd Fellows

V. Berezinskii

E. Abrahams



D. Scalapino

J.R. Schrieffer

A. Black-Schaffer, UU
H. Dahal APS
J. Bonca JSI
D. Mozyrsky LANL
E. Abrahams, Rutgers/UCLA
JR Schrieffer U Florida
D. Scalapino UCSB

Summary

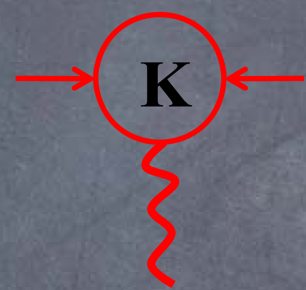
- Odd-frequency pairing in multiband superconductors
  - Odd-frequency, odd-interband pairing always exists if there is finite interband hybridization
    - Finite interband hybridization (scattering) + non-identical intraband pairing
    - TI + SC hybrid structures
    - Graphene with sublattice symmetry breaking
    - Iron-pnictides, heavy fermion superconductors, MgB_2?

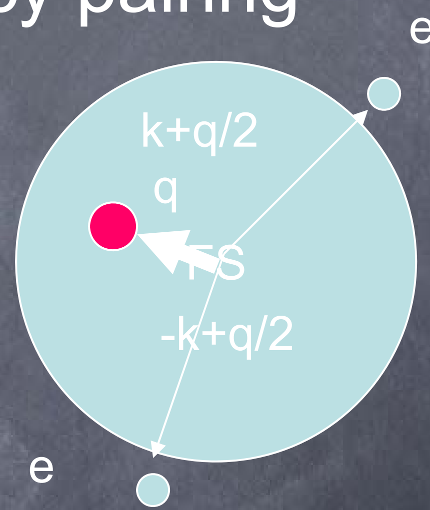
Black-Schaffer and Balatsky, arXiv:1305.4593

Summary and future

- Ubiquity of odd-frequency states. Need spectroscopy: Fish are the last to notice the water
- Order parameter and wf for odd-frequency pairing

$$\left.\frac{\mathrm{d}F(\mathbf{r},t;\mathbf{r}',t')}{\mathrm{d}t}\right|_{t\to t'}$$





- Odd frequency is ubuquitous in multiband superconductors: TO= +1, FeSe, MgB2, Gr/substrate.
- Investigate the nature of SC and competing states at interfaces (LaAlO/STO – multiband SC).