Sensitivity to the ordering of neutrino masses at oscillation experiments

Pilar Coloma

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Based on the collaboration:
M. Blennow, P. Coloma, P. Huber and T. Schwetz,
JHEP 1403 (2014) 028, arXiv: 1311.1822 [hep-ph]

News in Neutrino Physics
NORDITA, Stockholm,
April 7, 2014

Outline

- Introduction and motivation
- Statistical issues
- Ways to determine the mass ordering for a large θ_{13}
    - Interference effects in vacuum
    - Matter effects
    - Precise determination of mass splittings
- Conclusions

The two-family approximation

In the two-family approximation:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta_{\alpha\alpha} \sin^2 \left(\frac{\Delta m^2_{\alpha\alpha} L}{4E} \right)$$

The two-family approximation

In the two-family approximation:

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right) \qquad \text{(KamLAND)}$$

$\theta_{12} \sim 33^\circ$

$\Delta m^2_{21} \sim 7.5 \times 10^{-5}\mathrm{eV}^2$

The two-family approximation

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$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$ (Daya Bay, RENO, D-CHOOZ)

$\sim 9^\circ$

$\sim 2.5 \times 10^{-3}\text{eV}^2$

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$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

(KamLAND)

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

(Daya Bay, RENO, D-CHOOZ)

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

(K2K, MINOS)

$$\sim 40^\circ - 50^\circ$$

$$\sim 2.5 \times 10^{-3} \mathrm{eV}^2$$

The two-family approximation

In the two-family approximation:

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m^2_{32} L}{4E} \right)$$

Currently holds the largest uncertainty.

$$\theta_{23} \gtreqless 45^\circ ?$$

See for instance:
Fogli et al, 1205.5254, 1312.1878,
Forero et al, 1205.4018,
Gonzalez-Garcia et al, 1209.3023,
www.nu-fit.org

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Important for the flavor puzzle:

- bimaximal, tri-bimaximal, etc
- golden ratio
- quark-lepton complementarity

...

Some nice reviews:

King et al, 1402.4271 [hep-ph]

Altarelli et al, 1205.5133 [hep-ph], 1002.0211 [hep-ph]

The two-family approximation

In the two-family approximation:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{\alpha\alpha} L}{4E}\right)$$

No sensitivity to CP violation (δ)!!

- Is CP violated only in the quark sector?
- Is leptogenesis viable?
- Model building

...

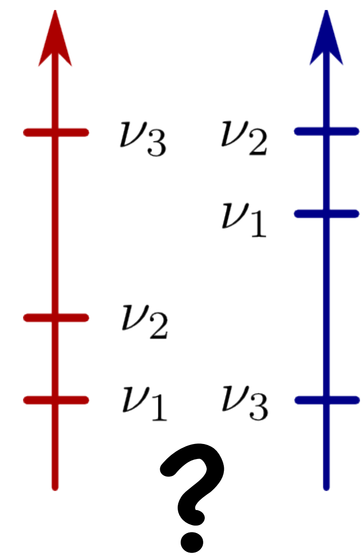
Note that an appearance experiment is needed to observe CP violation

The two-family approximation

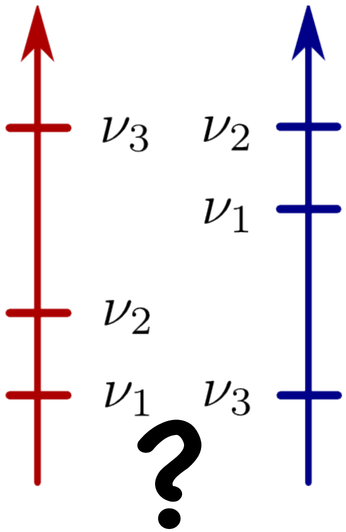
In the two-family approximation:

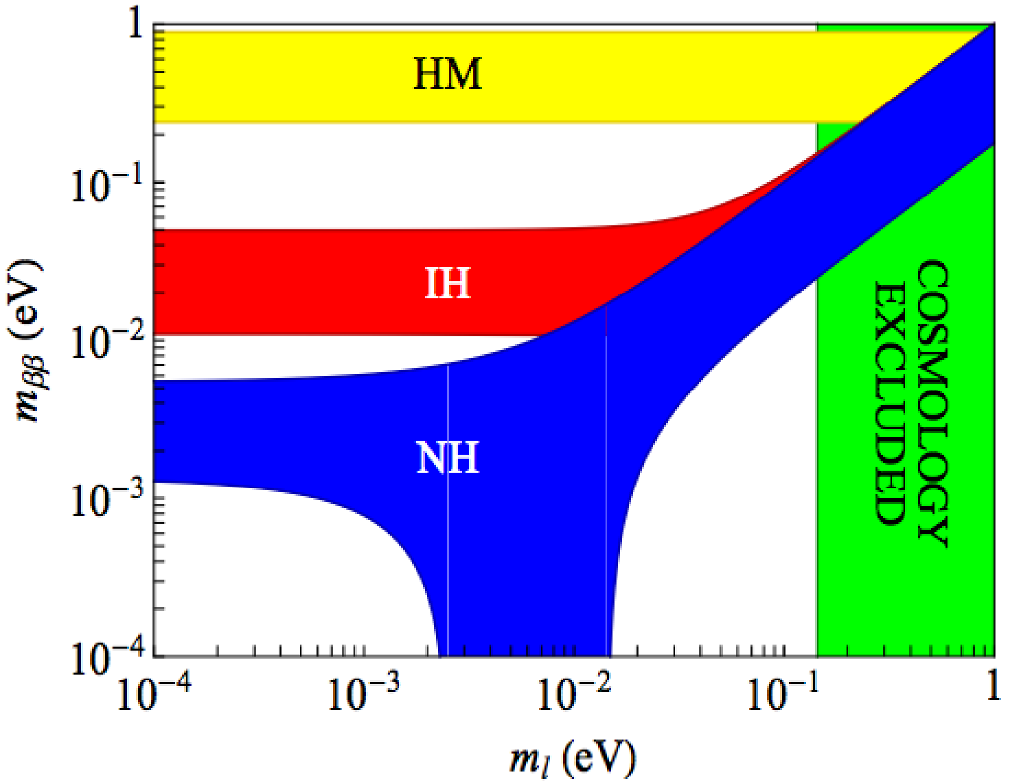
$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{\alpha\alpha} L}{4E}\right)$$

$\Delta m^2 \gtrless 0?$



Mass ordering and $0\nu\beta\beta$





An independent measurement of the hierarchy is extremely useful as a double-check of $0\nu\beta\beta$ and new physics

(see, for instance, Blennow et al, 1005.3240 [hep-ph])

Mass ordering and CP violation

Three family golden oscillation probability:

$$
\begin{aligned}
P_{e\mu}^{\pm}(\theta_{13}, \delta) = X_{\pm} \sin^2 2\theta_{13} \\
+ Y_{\pm} \cos\theta_{13} \sin 2\theta_{13} \cos\left(\pm\delta - \frac{\Delta m_{31}^2 L}{4E}\right) \\
+ Z
\end{aligned}
$$

Cervera et al, hep-ph/0002108

An unknown hierarchy usually leads to a reduced ability to observe CP violation

Minakata, Nunokawa, hep-ph/0108085

Barger, Marfatia, Whisnant, hep-ph/0112119

Statistical issues

Parameter estimation and sensitivities

i. Define a test statistic. For instance:

$$\Delta\chi^2(\theta) = \chi^2(\theta) - \chi^2_{min}$$

ii. Wilks' theorem tells us that this test statistic will be χ^2 distributed with p dof , where p is the number of parameters estimated from the data

S. S. Wilks, Annals Math. Statist. 9, no. 1, 60 (1938)

iii. Use the Asimov data set to get the median value of $\Delta\chi^2$ This gives the median sensitivity of a given experiment to θ

Statistical issues with mass ordering

One of the requirements of Wilks' theorem is that the parameter begin tested needs to be continuous, but the mass ordering is not!

$\rightarrow$ What happens then?

Qian et al, 1210.3651 [hep-ph]
Ciuffoli, Evslin and Zhang, 1305.5150 [hep-ph]
Capozzi, Lisi and Marrone, 1309.1638 [hep-ph]
Vittels and Read, 1311.4076 [hep-ex]
Blennow et al, 1311.1822 [hep-ph]
Blennow, 1311.3183 [hep-ph]
LBNO collaboration, 1312.6520 [hep-ph]

Hypothesis testing

Pick up a test statistic. Several possibilities:

$$\Delta\chi^2 = \chi^2_{\mathrm{NO}} - \chi^2_{min}$$

$$T = \chi^2_{\mathrm{IO}} - \chi^2_{\mathrm{NO}}$$

$$T' = \chi^2(\theta) - \min\left\{\chi^2\right\}$$

...(or any other possibility you can think of)

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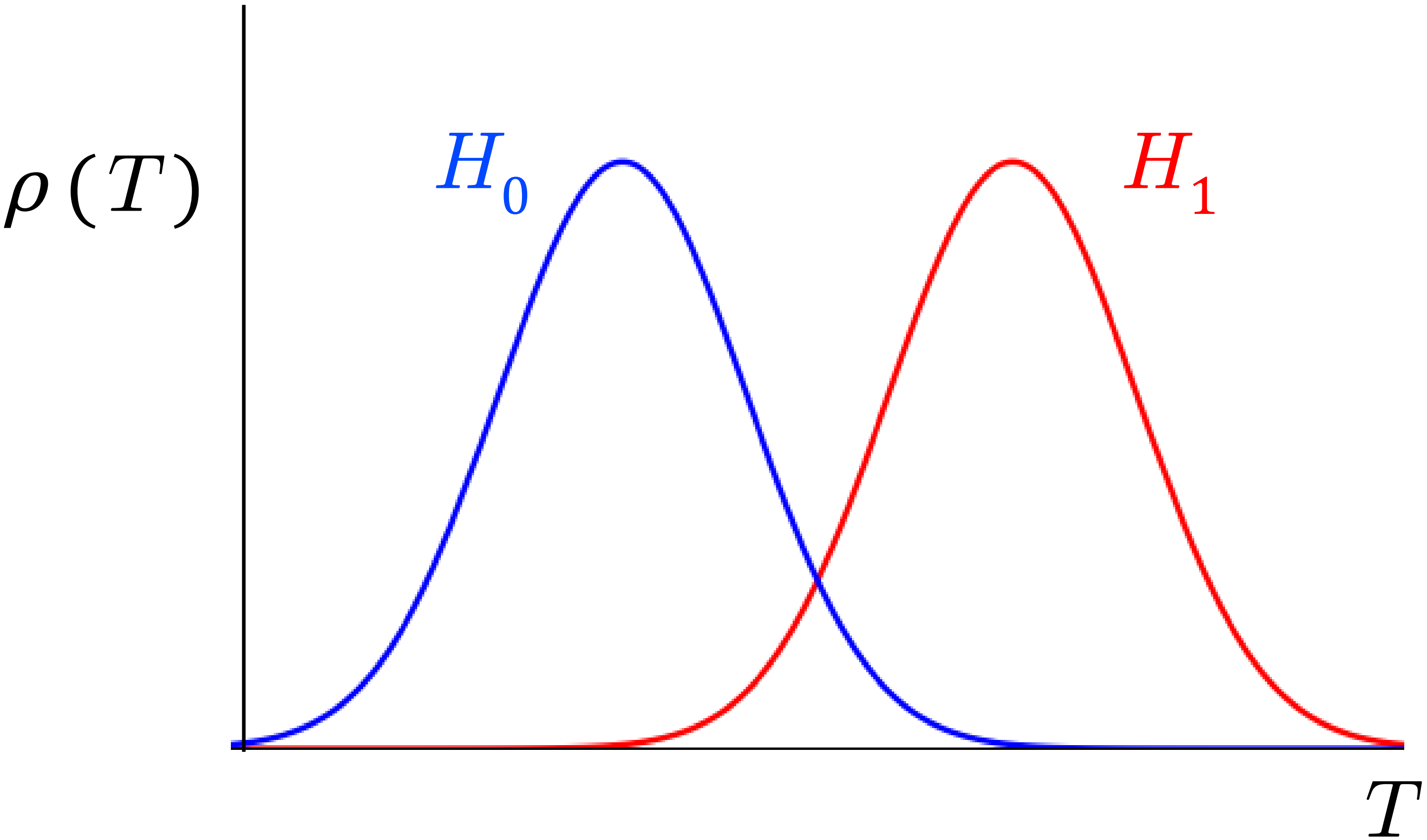
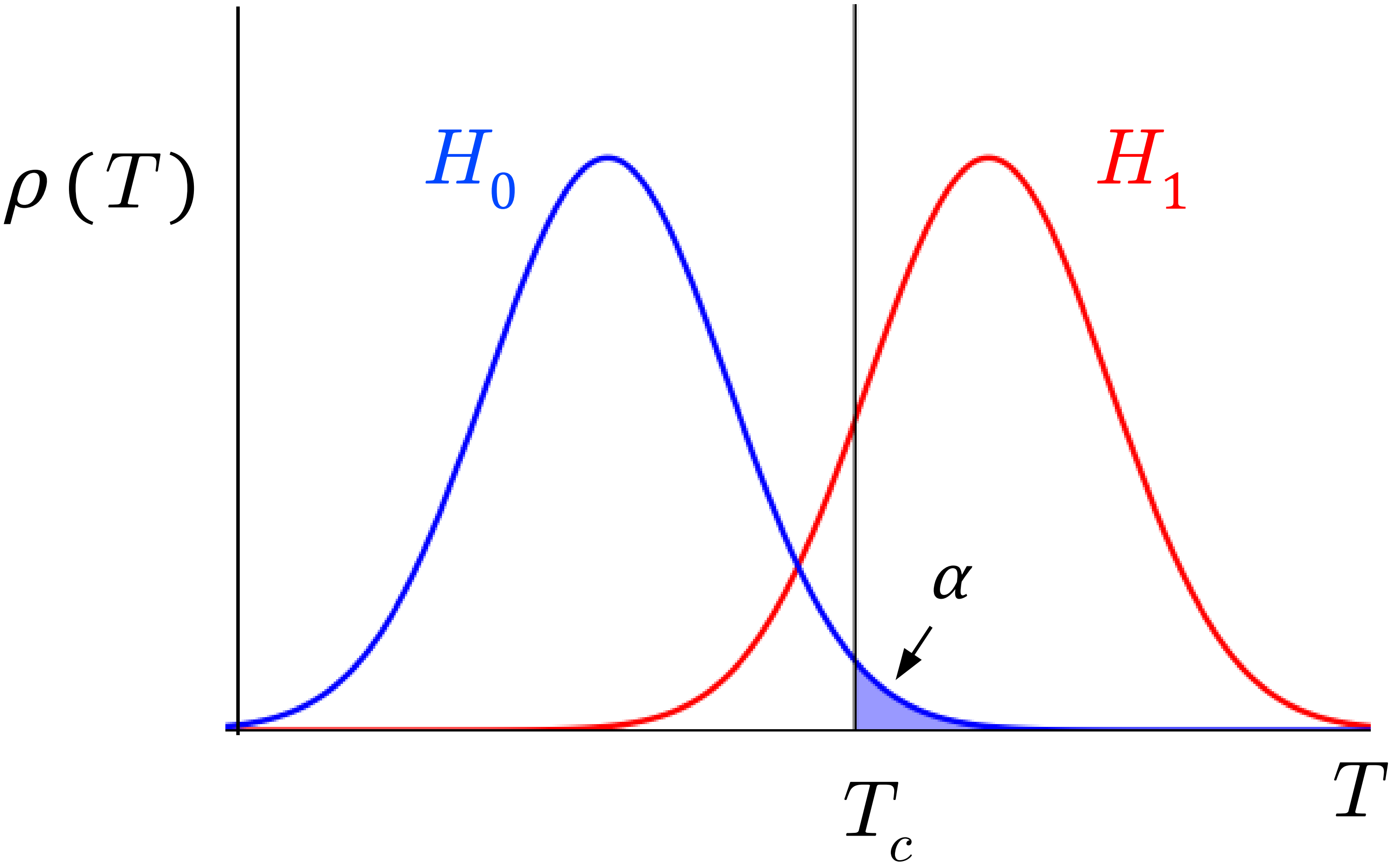
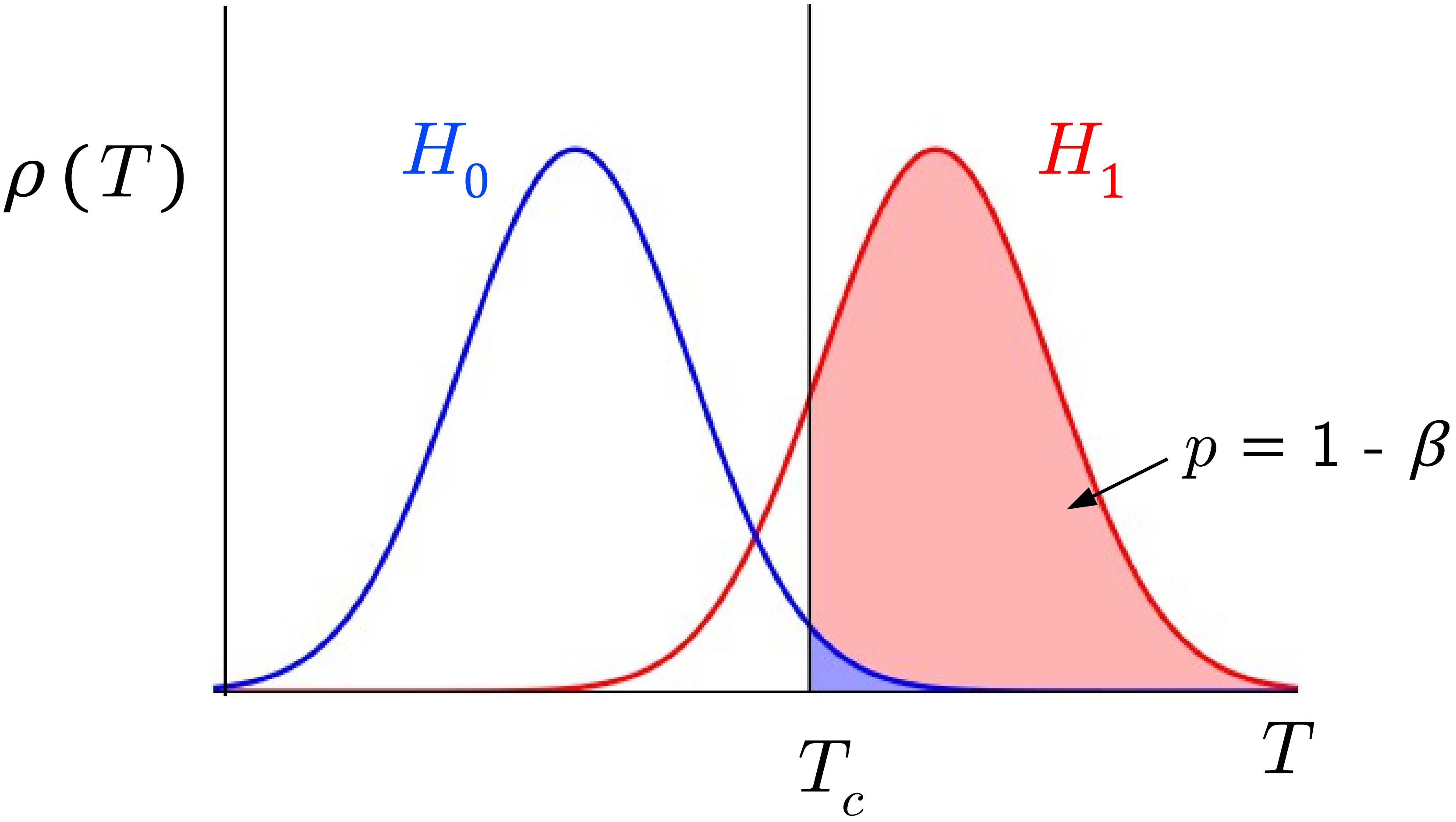
Hypothesis testing



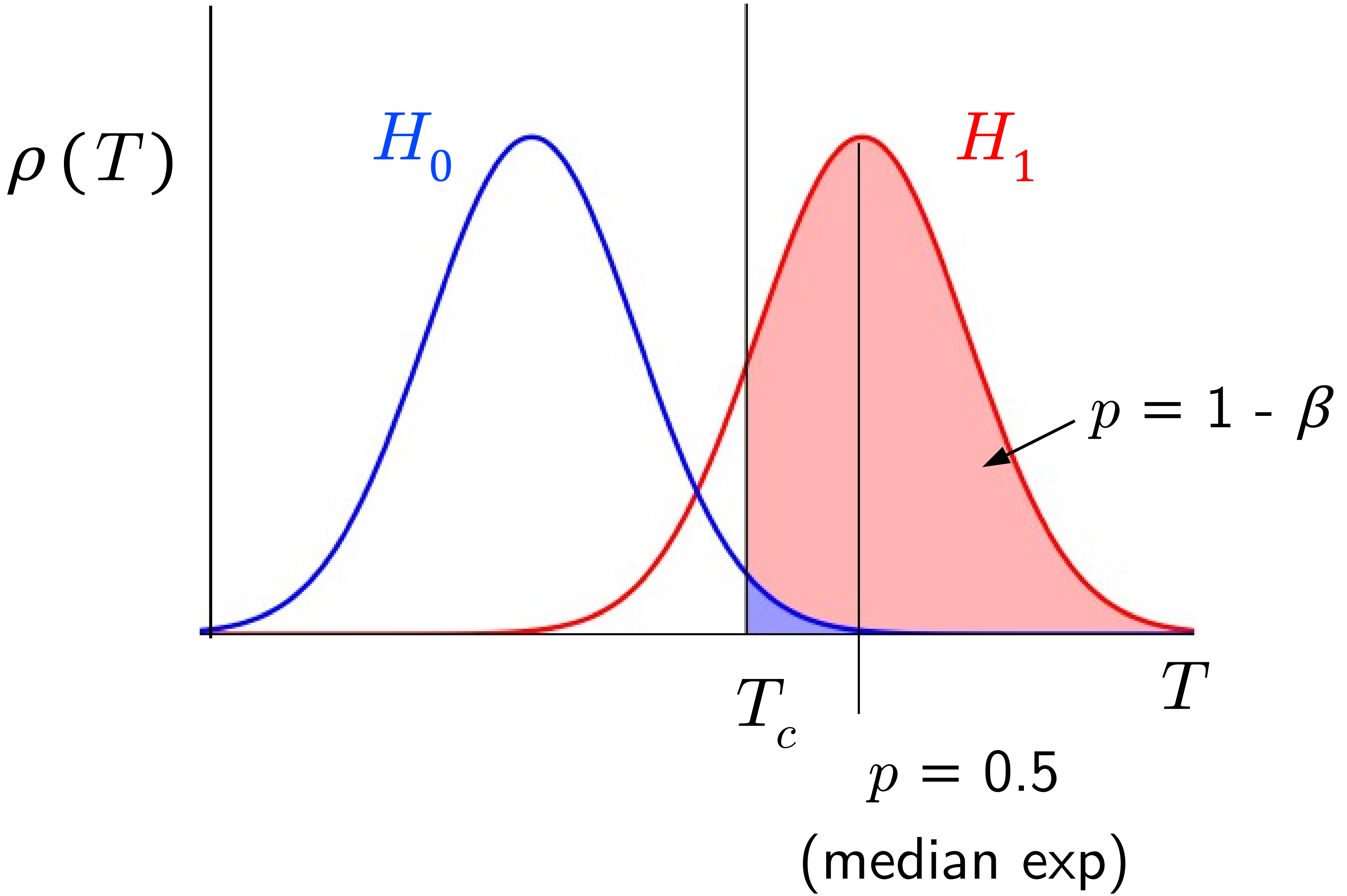
Hypothesis testing



Hypothesis testing



Hypothesis testing



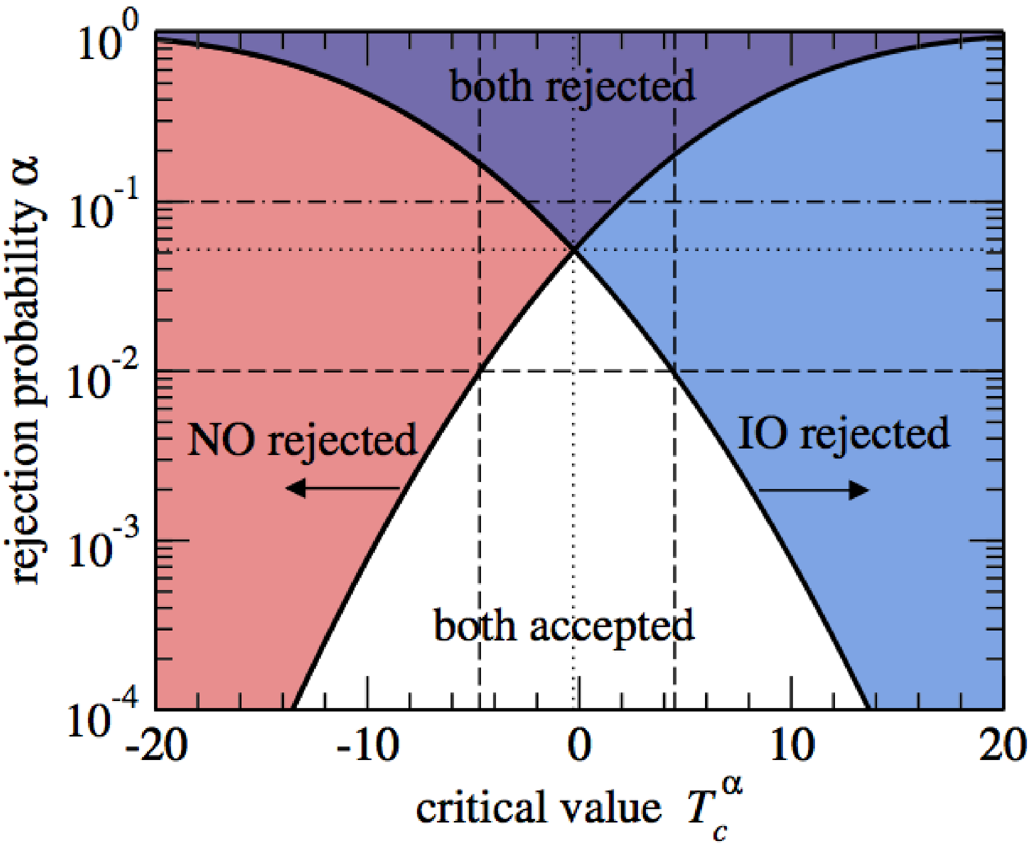
Hypothesis testing

Three possible outcomes are in principle possible:

1) Reject exactly one hypothesis

2) Reject both hypotheses

3) Accept both hypotheses



Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

Gaussian approximation

- Under the gaussian approximation:

$$T = \mathcal{N}\left(T_0, 2\sqrt{T_0}\right)$$

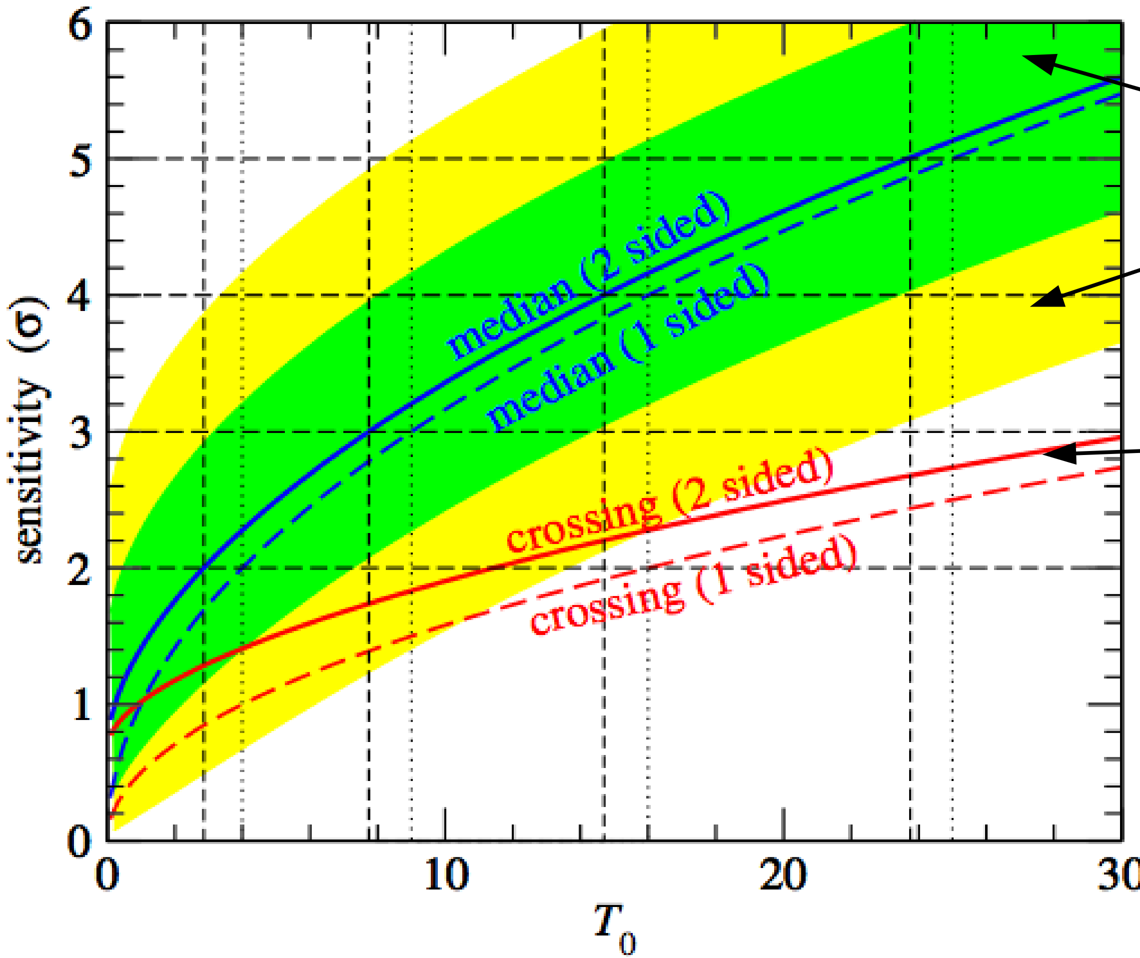
One can obtain expressions for type I and type II error rates as a function of T_0, which turns into a relation between α and β.

- Then, setting β=0.5 one can then get the expression for the number of sigmas for the median experiment in the gaussian case:

$$n = \sqrt{2}\mathrm{erfc}^{-1}\left(\frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{T_0}{2}}\right)\right)$$

Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

Gaussian approximation



$\beta = \frac{1}{2} \pm \frac{0.6827}{2}$

$\beta = \frac{1}{2} \pm \frac{0.9545}{2}$

$\beta = \alpha$

Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

Does the gaussian approximation hold?

Ways to measure the mass ordering

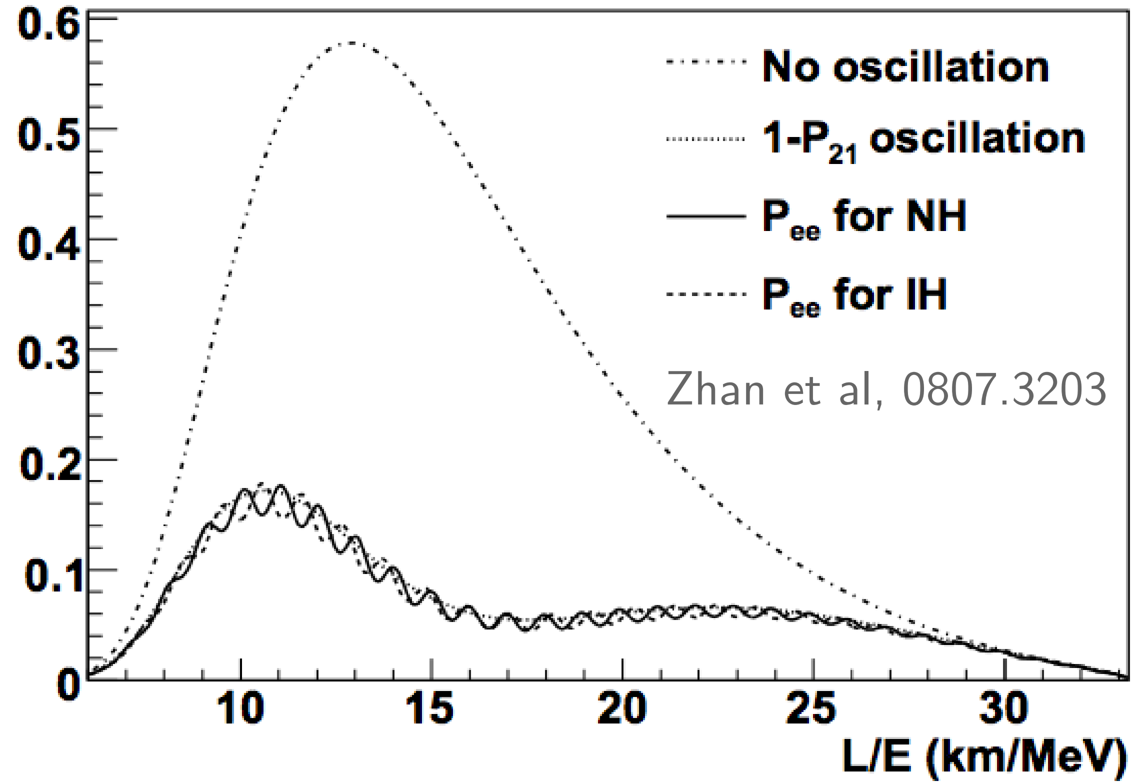
A large θ_{13} opens multiple ways:

i. Interference effects between solar and atmospheric oscillations

$\rightarrow$ reactors at medium baselines

Reactor experiments at medium baselines

Petcov, Piai, hep-ph/01102074
Choubey, Petcov, Piai,
hep-ph/0306017



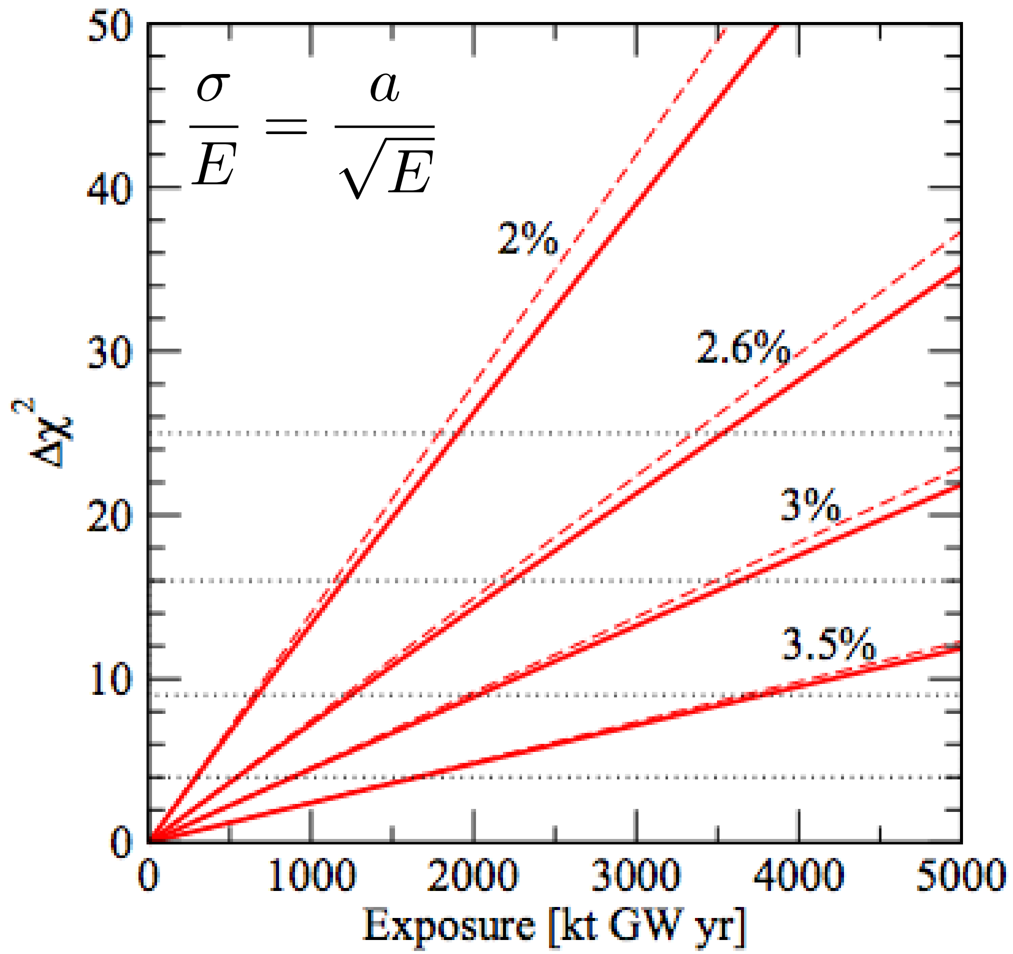
$$
\begin{aligned}
P_{ee} &= 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) \\
&- \sin^2 2\theta_{13}\left[c_{12}^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) + s_{12}^2 \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right)\right]
\end{aligned}
$$

Reactor experiments at medium baselines

Two major proposals: RENO-50 and JUNO

Technical challenges:

- energy resolution
- energy non-linearity
- reactor distribution



Blennow and Schwetz 1306.3988 [hep-ph]

See also:

Zhan et al, 0807.3203, 0901.2976

Qian et al, 1208.1551

Kettell et al, 1307.7419

Learned et al, hep-ex/0612022

Ciuffoli et al, 1209.2227,1308.0591

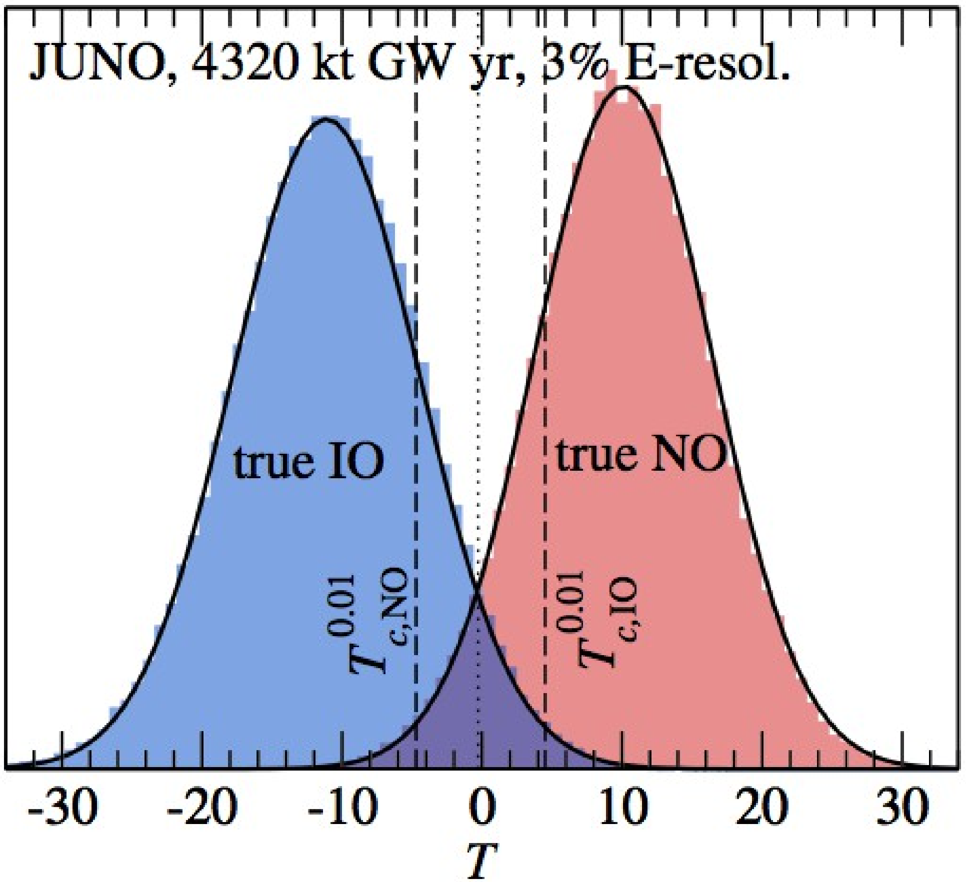
Ge et al, 1210.8141

...

MC results for JUNO

T is gaussian distributed up to very good accuracy:

$$T = \mathcal{N}\left(T_0, 2\sqrt{T_0}\right)$$



Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

(Similar distributions found for instance in 1210.3651)

Ways to measure the mass ordering

A large θ_{13} opens multiple ways:

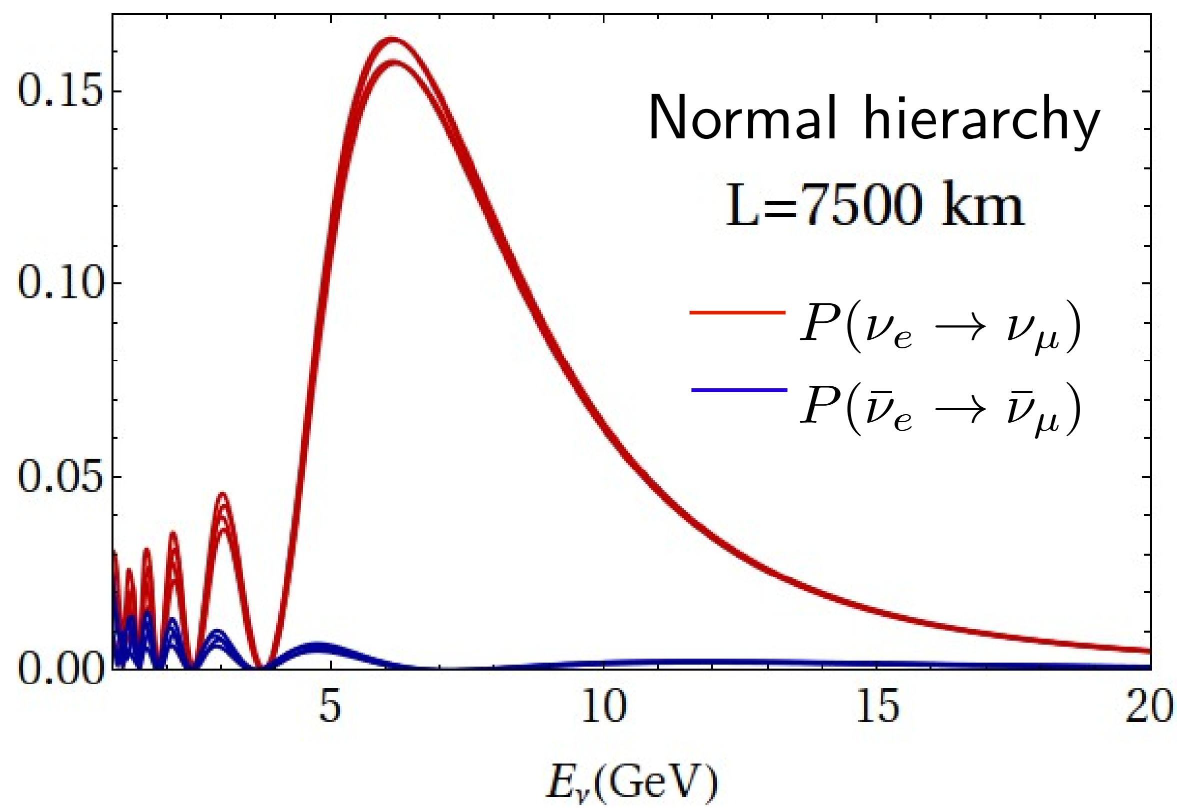
i. Interference effects between solar and atmospheric oscillations
$\rightarrow$ reactors at medium baselines

ii. Matter effects

- In appearance $\rightarrow$ beams
- In disappearance $\rightarrow$ atmospheric neutrinos

Matter effects in appearance (beams)

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos\theta - A)^2}\,; \quad A = \frac{2EV}{\Delta m^2}$$



Wolfenstein ('78), Barger et al ('80), Mikheev and Smirnov ('85)

Matter effects in appearance (beams)

Types of neutrino beams:

- Based on pion-decay (NOνA, T2K, LBNE, LBNO, ESSnuSB)

$$\pi^+ \to \mu^+ \nu_\mu \qquad\qquad \nu_\mu \longrightarrow \nu_e$$

Technology well-known; but intrinsic backgrounds and typically large systematics

- Based on muon decay (IDS-NF, NuMAX)

$$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e \qquad\qquad \begin{array}{c} (\,\nu_\mu \longrightarrow \nu_e\,) \\ \nu_e \longrightarrow \nu_\mu (\nu_\tau) \end{array}$$

Very clean, low systematics, flavor rich; but technically challenging and requires charge discrimination at detector

Matter effects in appearance (beams)

NuMAX 1290 km

~300/60 events/ch

LBNE 1290 km

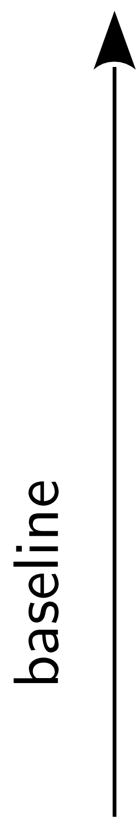
~200/60 events

NOvA 810 km

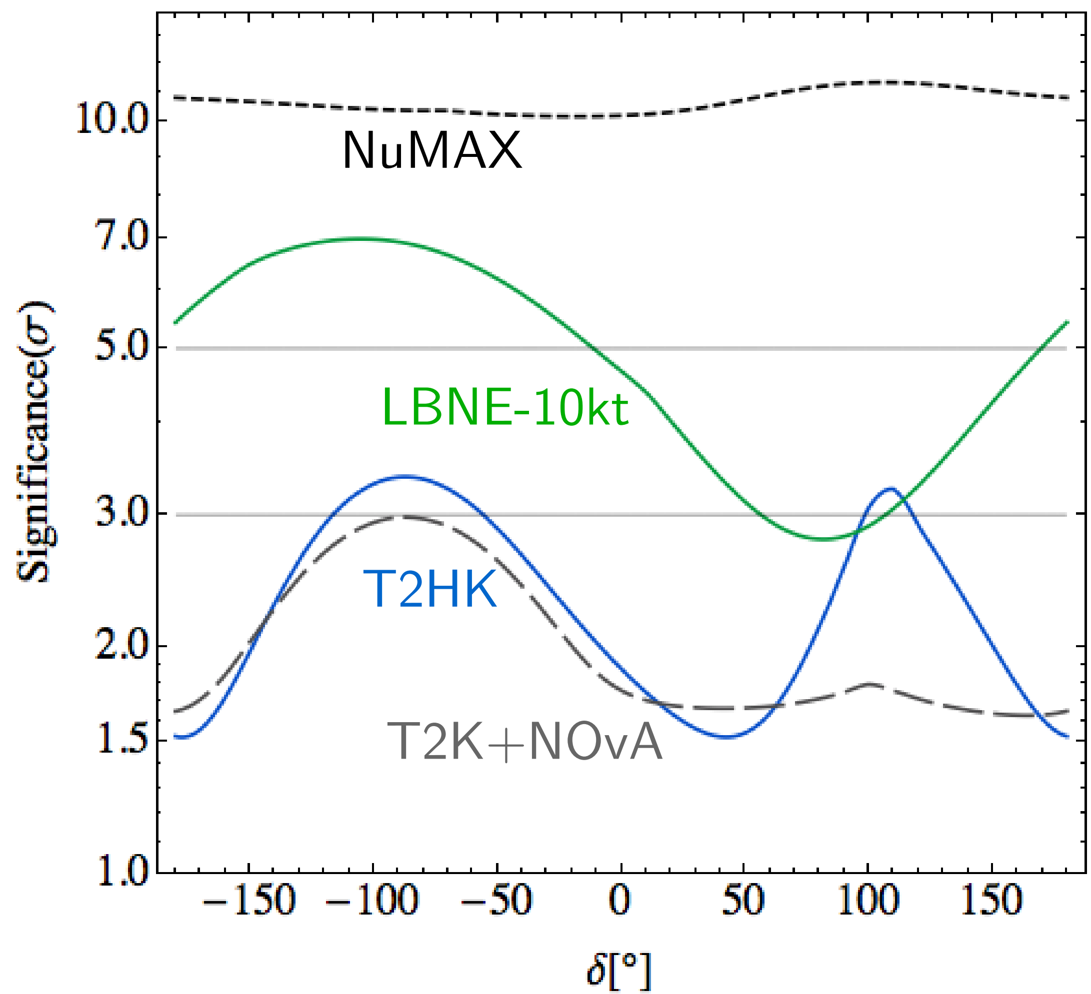
~80/23 events

T2(H)K 295 km

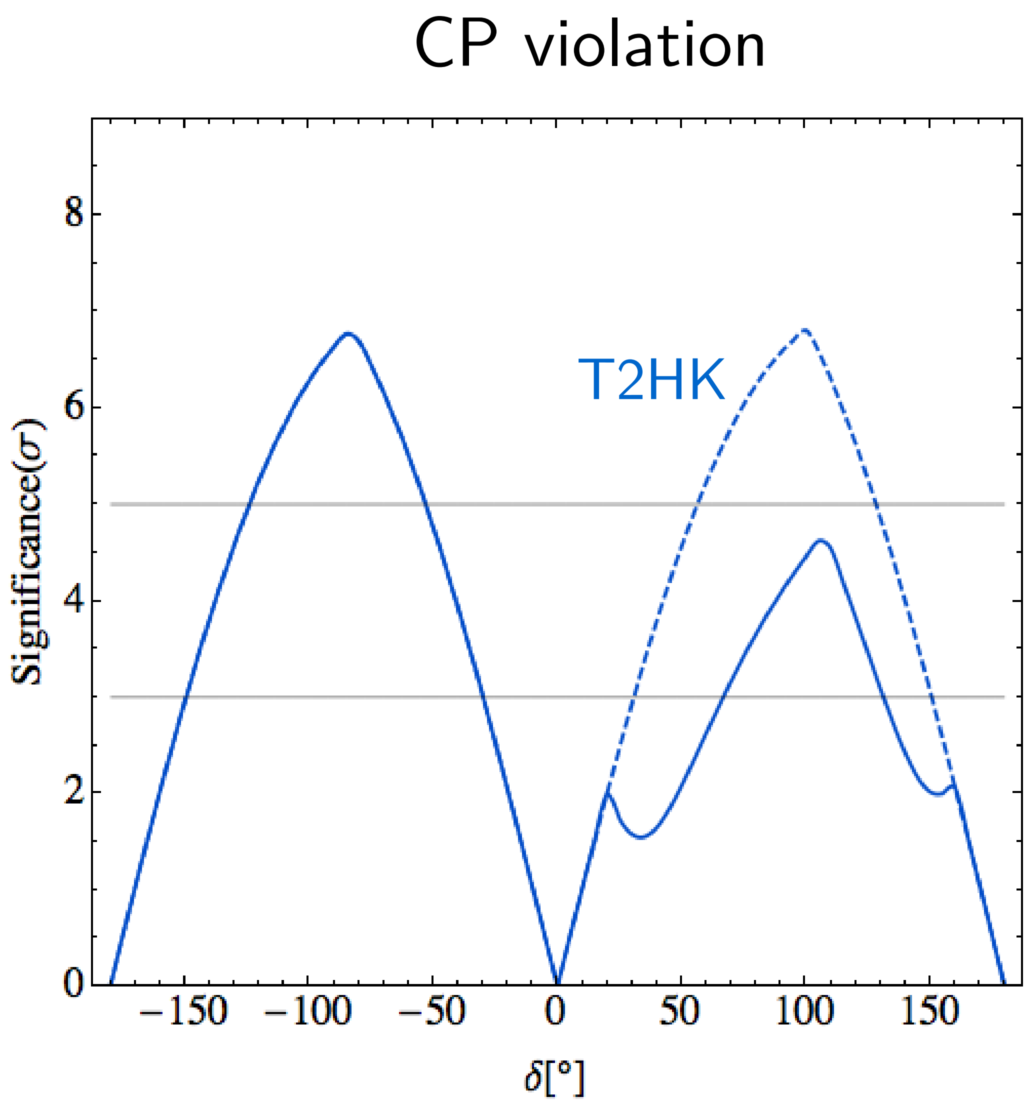
~4000/2200 events

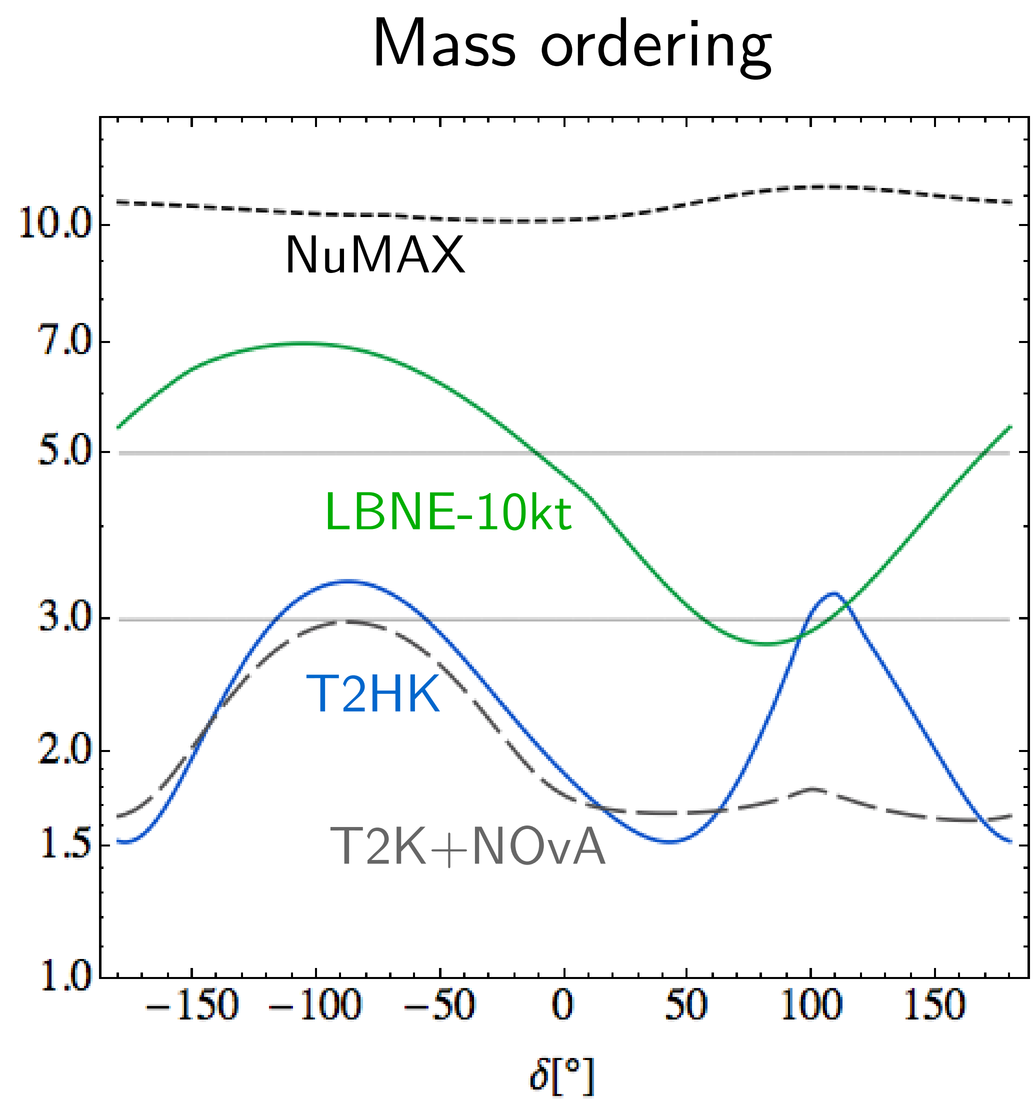


Mass ordering



Matter effects in appearance (beams)



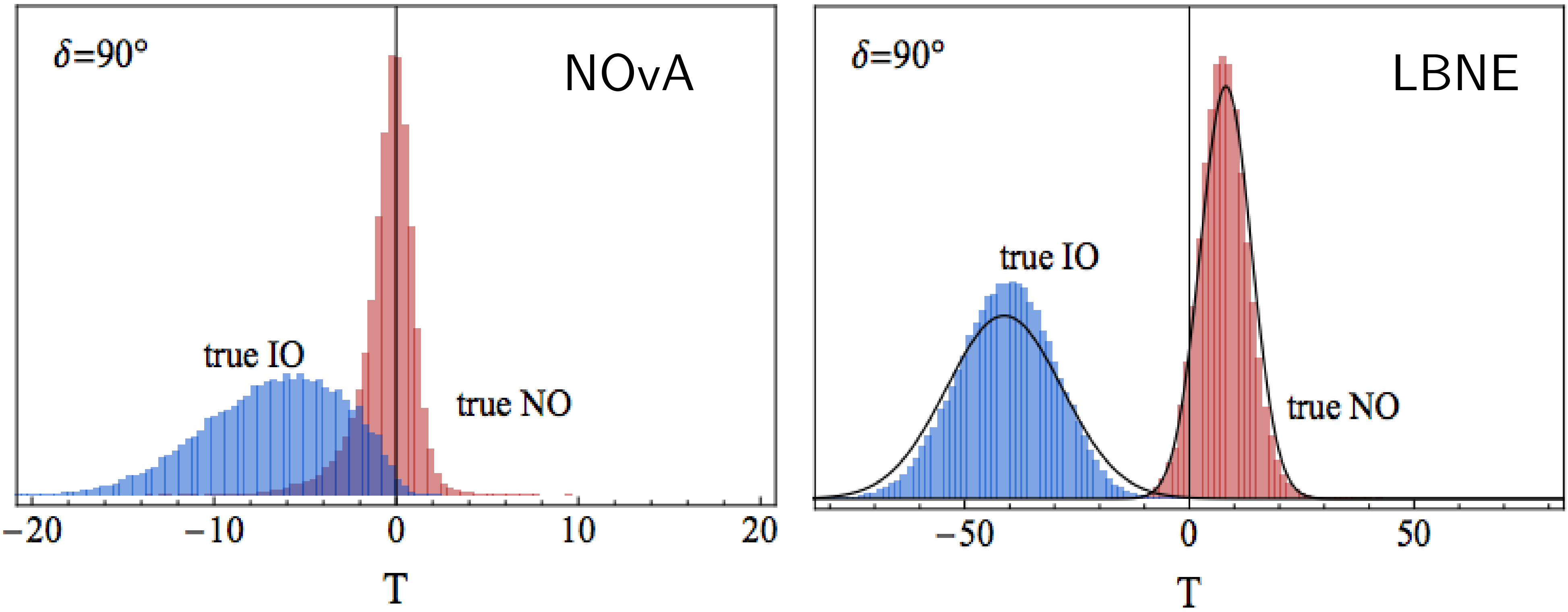


Simple vs composite hypotheses

- For composite hypotheses, the distribution of T depends on some parameter:
  - θ_{23} and δ in the case of long baselines
  - θ_{23} in the case of atmospheric neutrinos
- The null hypothesis has to be rejected for all values of the parameter:

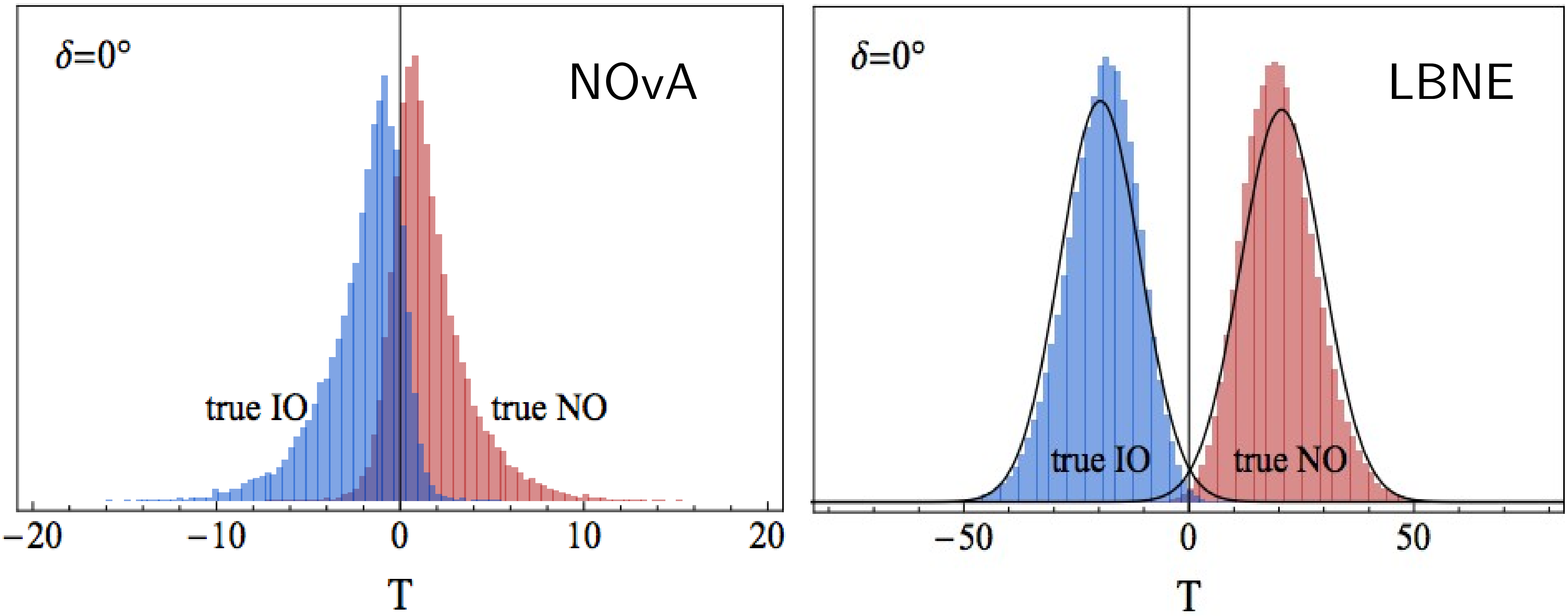
$$T_c = \min_\theta T_c(\theta)$$

Simple vs composite hypotheses



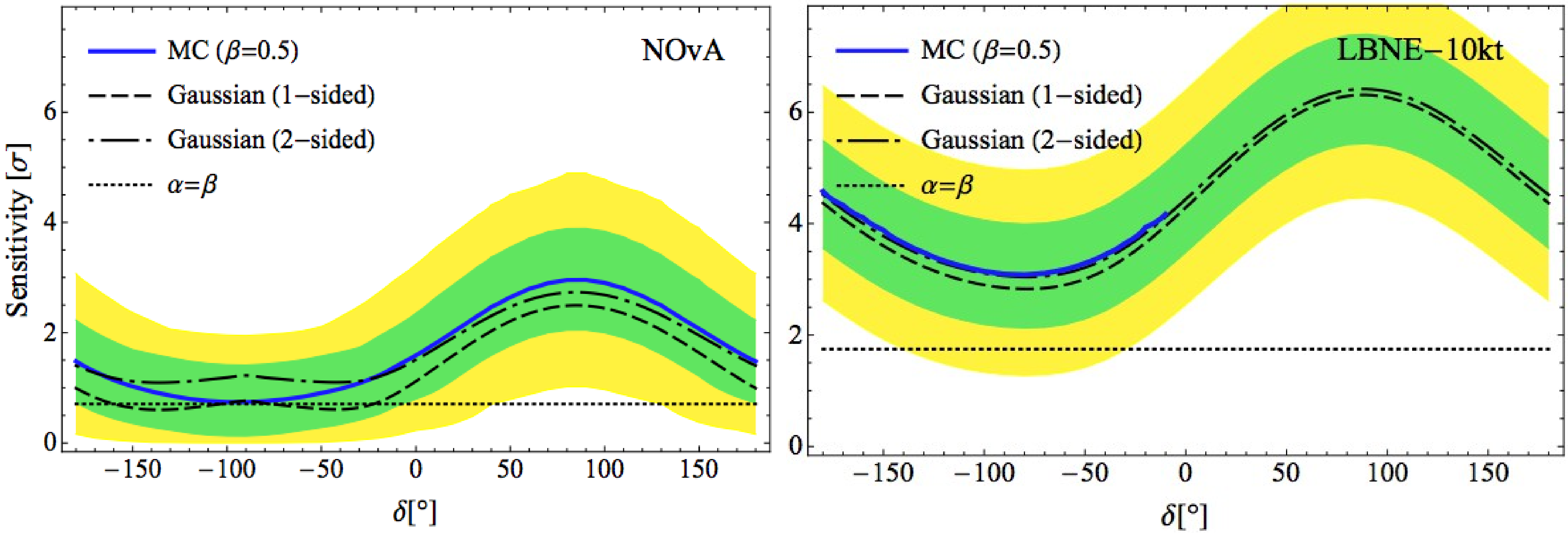
Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

Simple vs composite hypotheses



Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

MC results for beam experiments



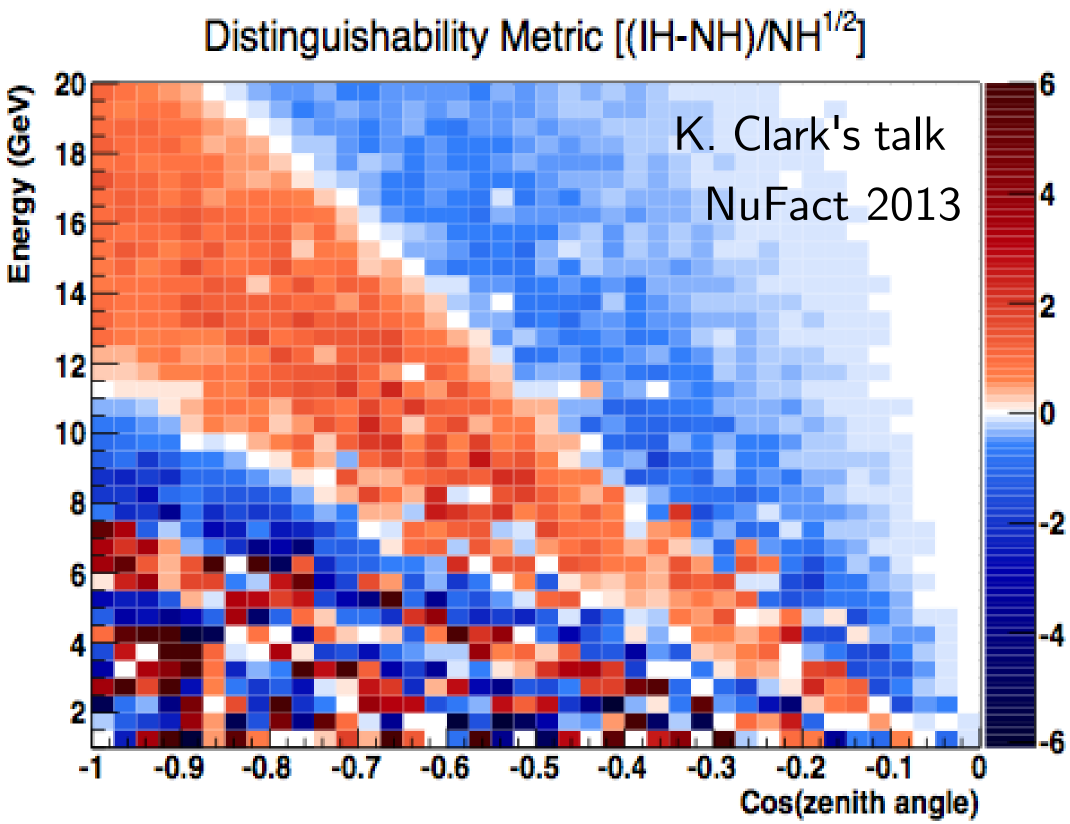
Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

Matter effects in disappearance

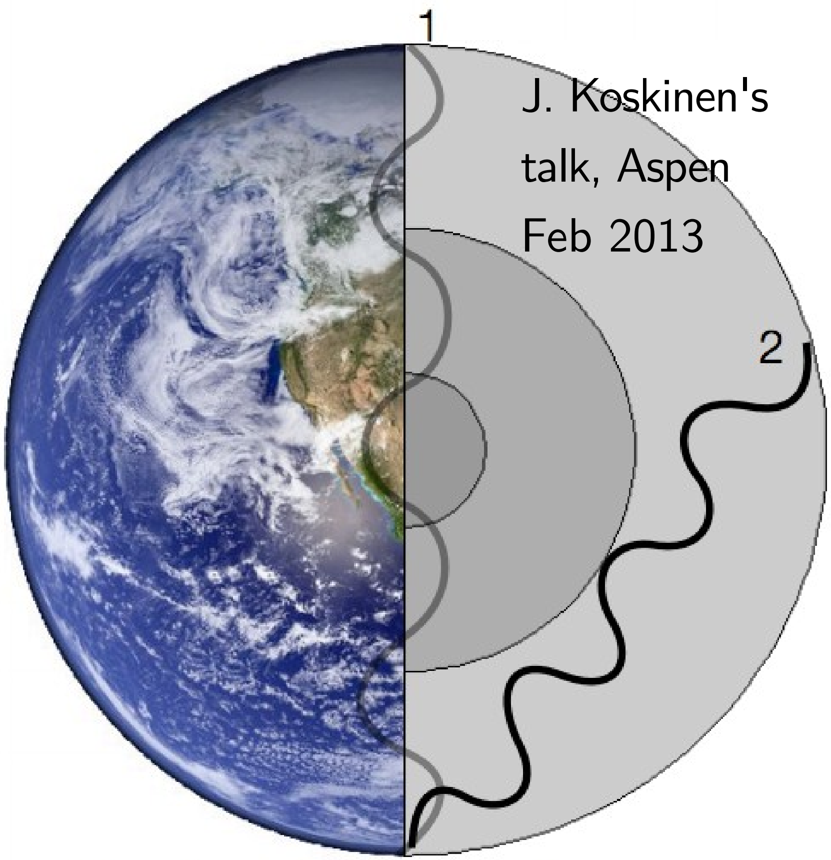
Petov,hep-ph/9805262

Akhmedov,hep-ph/9805272

$$P^{\pm}_{\mu\mu}(\theta_{13},\delta) = 1 - \chi_{\pm}\sin^2 2\theta_{13} - \psi_{\pm}\sin 2\theta_{13}\cos\delta - \omega$$



Perfect detector resolution



Matter effects in disappearance

Many possibilities:

- ORCA @ KM3NET

(see e.g. 1402.1022 [astro-ph.IM]);

- Hyper-Kamiokande

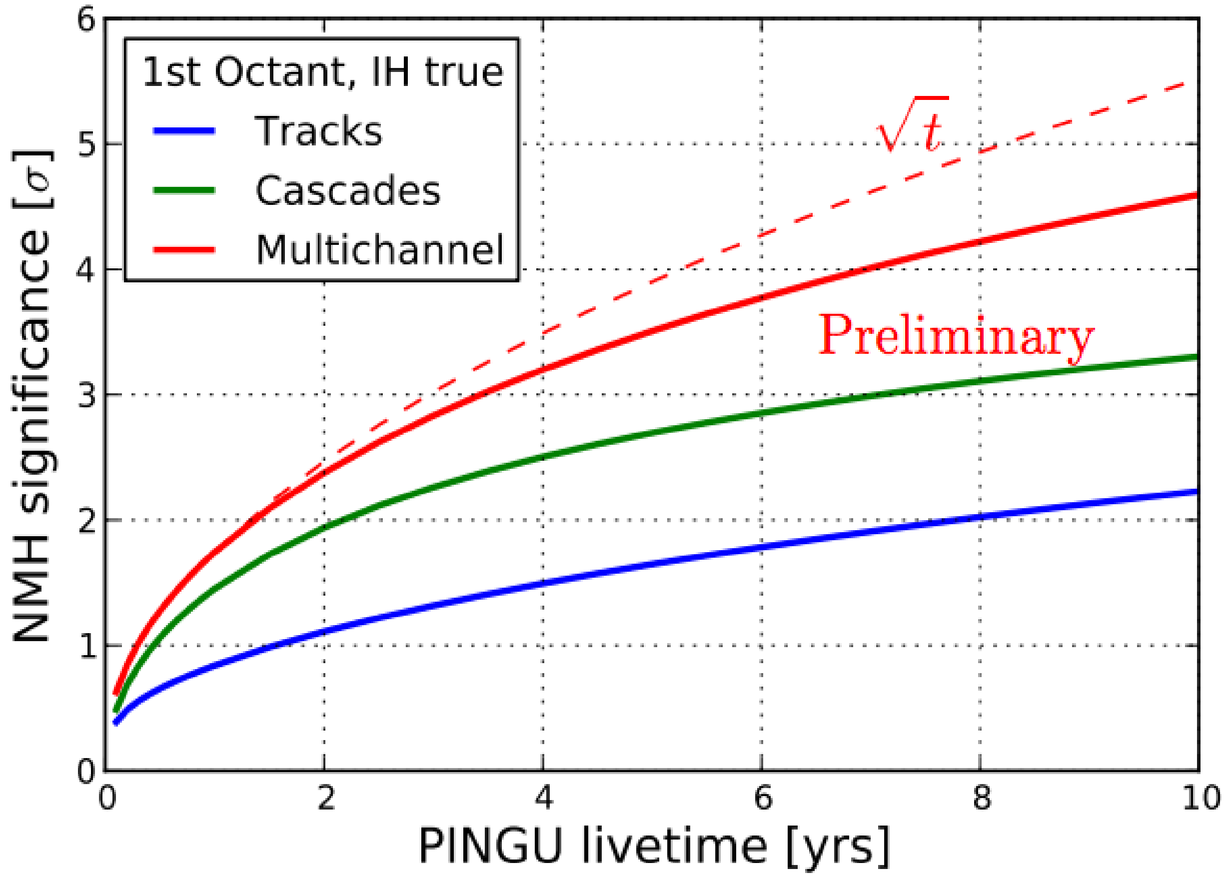
(1109.3262 [hep-ex], 1309.0184 [hep-ex]);

- INO @ ICAL

(see e.g. Ghosh and Choubey, 1306.1423 [hep-ph])

- 50 kt LAr detector

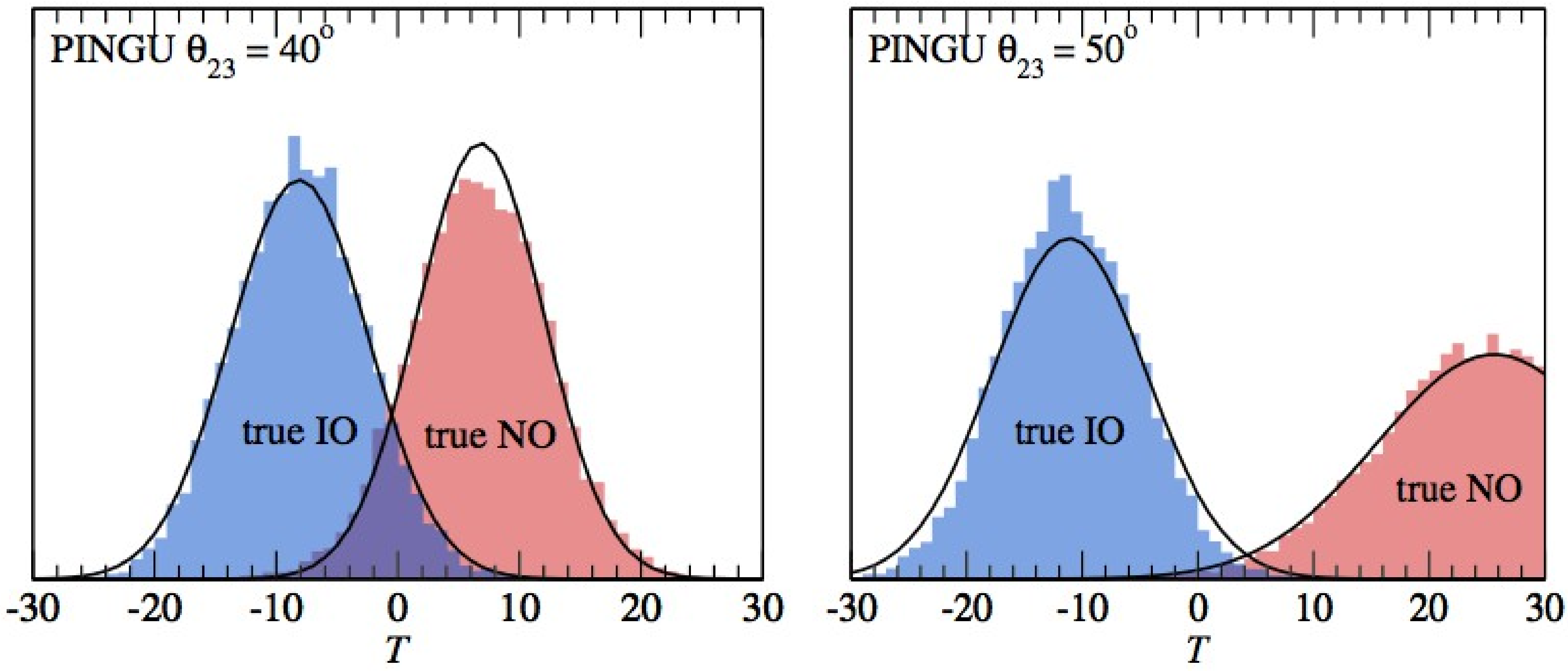
(Barger et al, 1203.6012 [hep-ph])



PINGU coll., 1401.2046 [hep-ex]

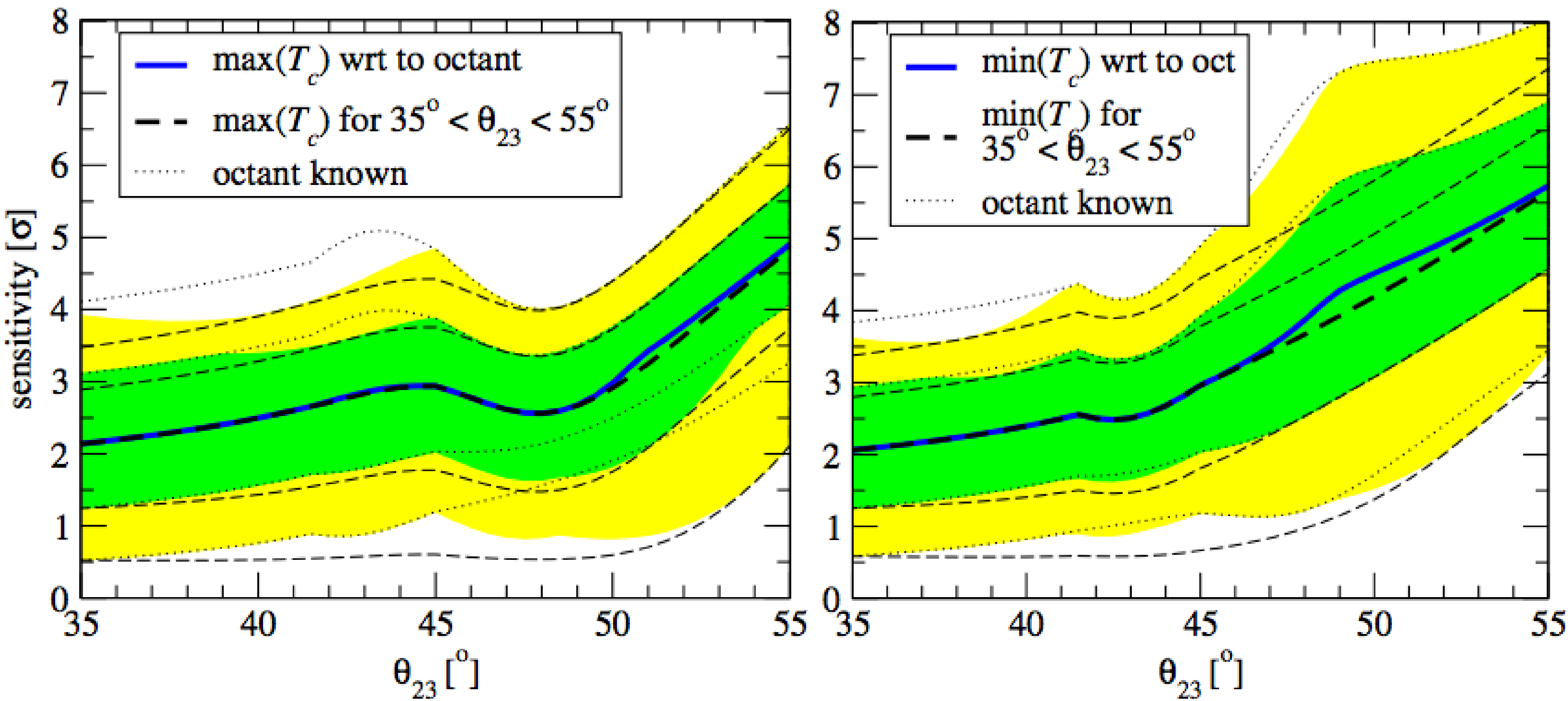
(see also Mena, Mocioiu, Razzaque, 0803.3044[hep-ph] and Akhmedov, Razzaque, Smirnov, 1205.7071 [hep-ph])

Matter effects in disappearance



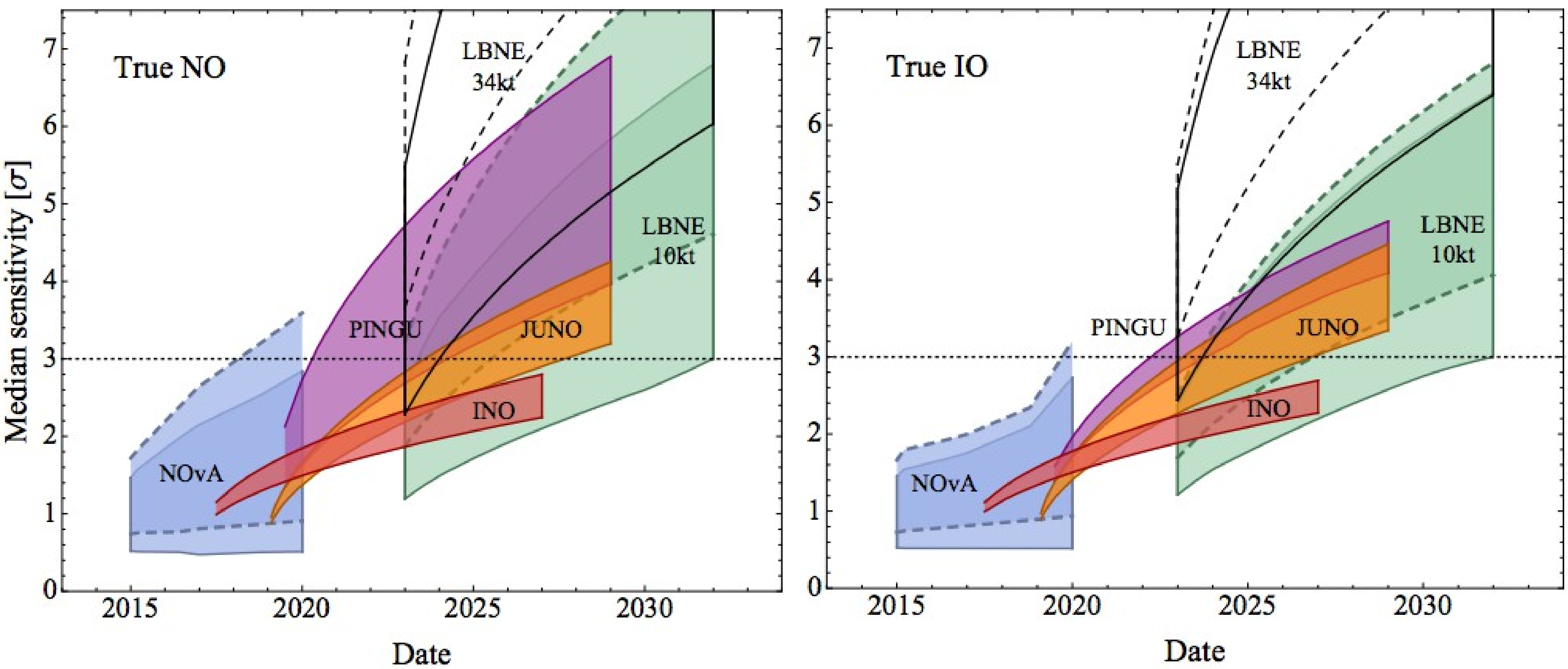
Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

MC results for PINGU



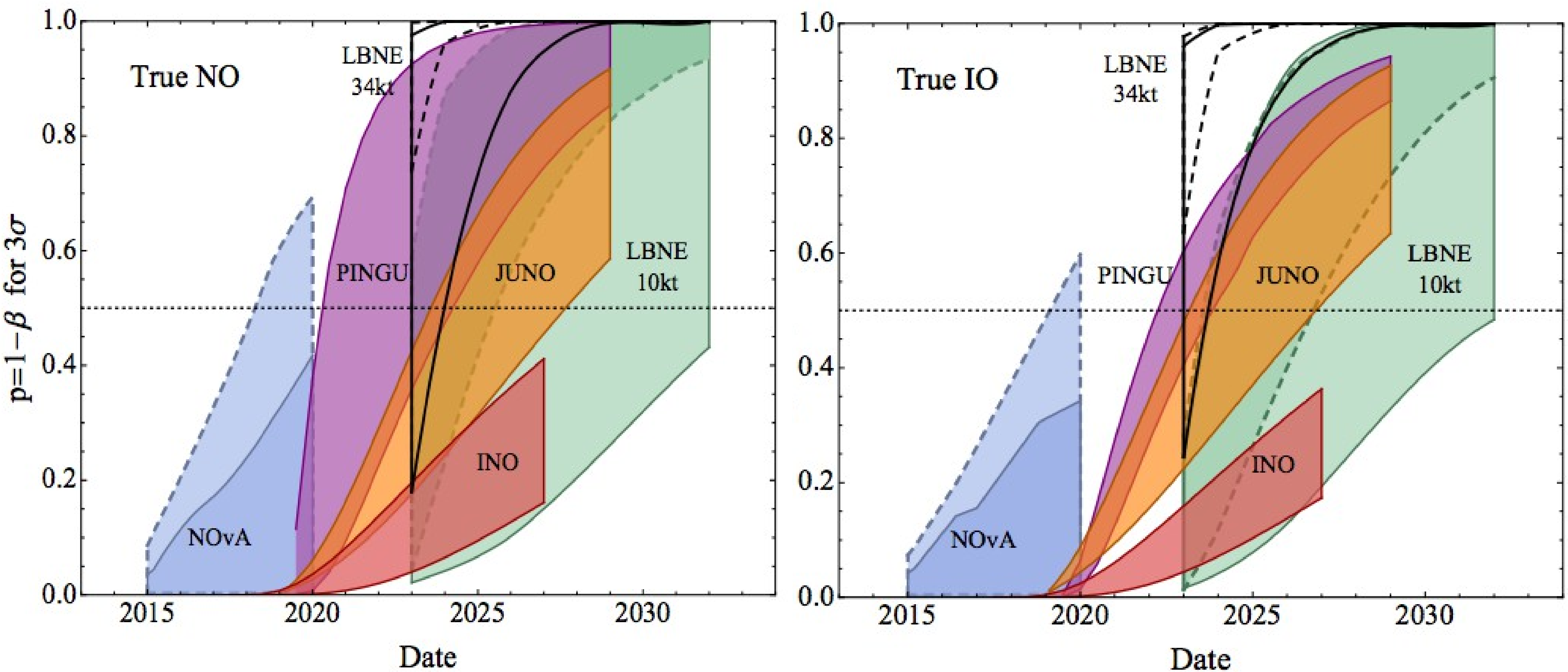
Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

Present and future prospects



Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

Present and future prospects



Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

Precise measurements of mass splittings

Disappearance experiments measure an effective mass splitting which depends on the neutrino flavor, even in vacuum:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta_{\alpha\alpha} \sin^2 \left(\frac{\Delta m^2_{\alpha\alpha} L}{4E} \right)$$

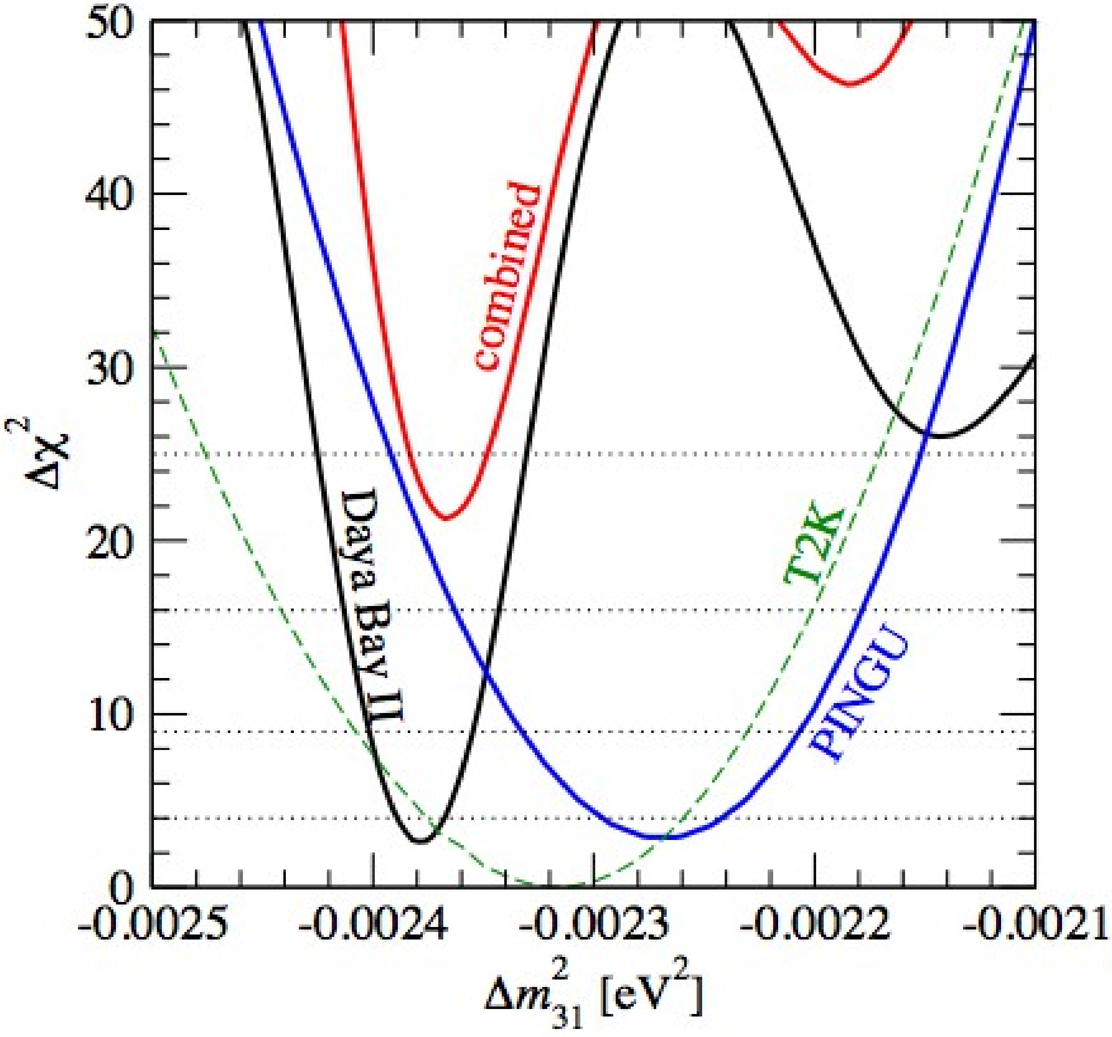
$$\Delta_{e\mu} \equiv \Delta m^2_{ee} - \Delta m^2_{\mu\mu} = \pm \Delta m^2_{21} (r_e - r_\mu)$$

$$r_e - r_\mu = \cos 2\theta_{12} - \cos\delta \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23} + \mathcal{O}\left(\sin^2\theta_{13}\right)$$

Nunokawa, Parke, Zukanovich Funchal, hep-ph/0503283
Minakata, Nunokawa, Parke, Zukanovich Funchal, hep-ph/0607284
De Gouvea, Jenkins, Kayser, hep-ph/0503079

Precise measurements of mass splittings

Physics in this case is more involved, but the observable effect is similar.



Blennow, Schwetz, 1306.3988 [hep-ph]

(see also Li *et al*, 1303.6733 [hep-ph], for instance)

Conclusions

- The large value of θ_{13} recently measured has opened a door to measure the neutrino mass spectrum in many different ways
  - Huge number of possibilities (short-, mid- and long-term): PINGU, ORCA, HyperK, JUNO, RENO50, ICAL, NOvA, LBNE,…
- The usual sensitivity estimates for the median experiment are valid
- Synergies between different proposals exist

Thank you for your attention!

Backup

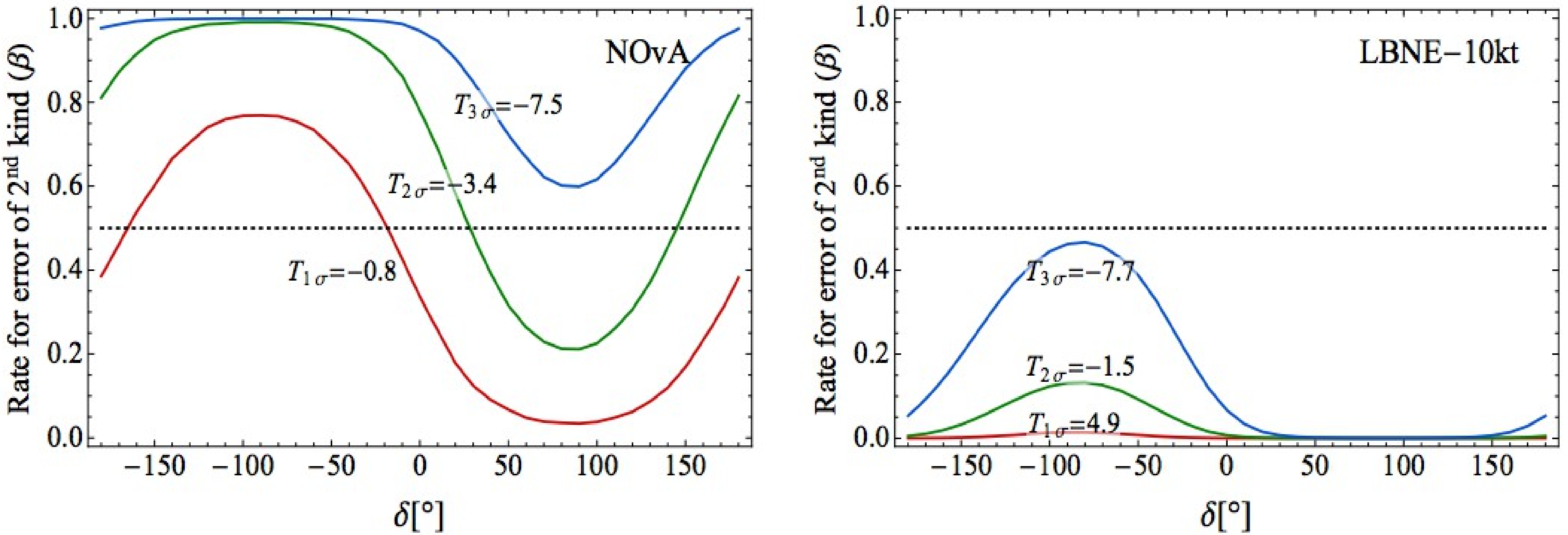
JUNO:

energy resolution	$3\%\sqrt{1\,\mathrm{MeV}/E}$		$3.5\%\sqrt{1\,\mathrm{MeV}/E}$	
	normal	inverted	normal	inverted
$T_0\,(\sqrt{T_0}\sigma)$	10.1 (3.2σ)	11.1 (3.3σ)	5.4 (2.3σ)	5.9 (2.4σ)
median sens.	7.3×10^{-4} (3.4σ)	4.3×10^{-4} (3.5σ)	1.0×10^{-2} (2.5σ)	7.5×10^{-3} (2.7σ)
crossing sens.	5.2% (1.9σ)		12% (1.6σ)	

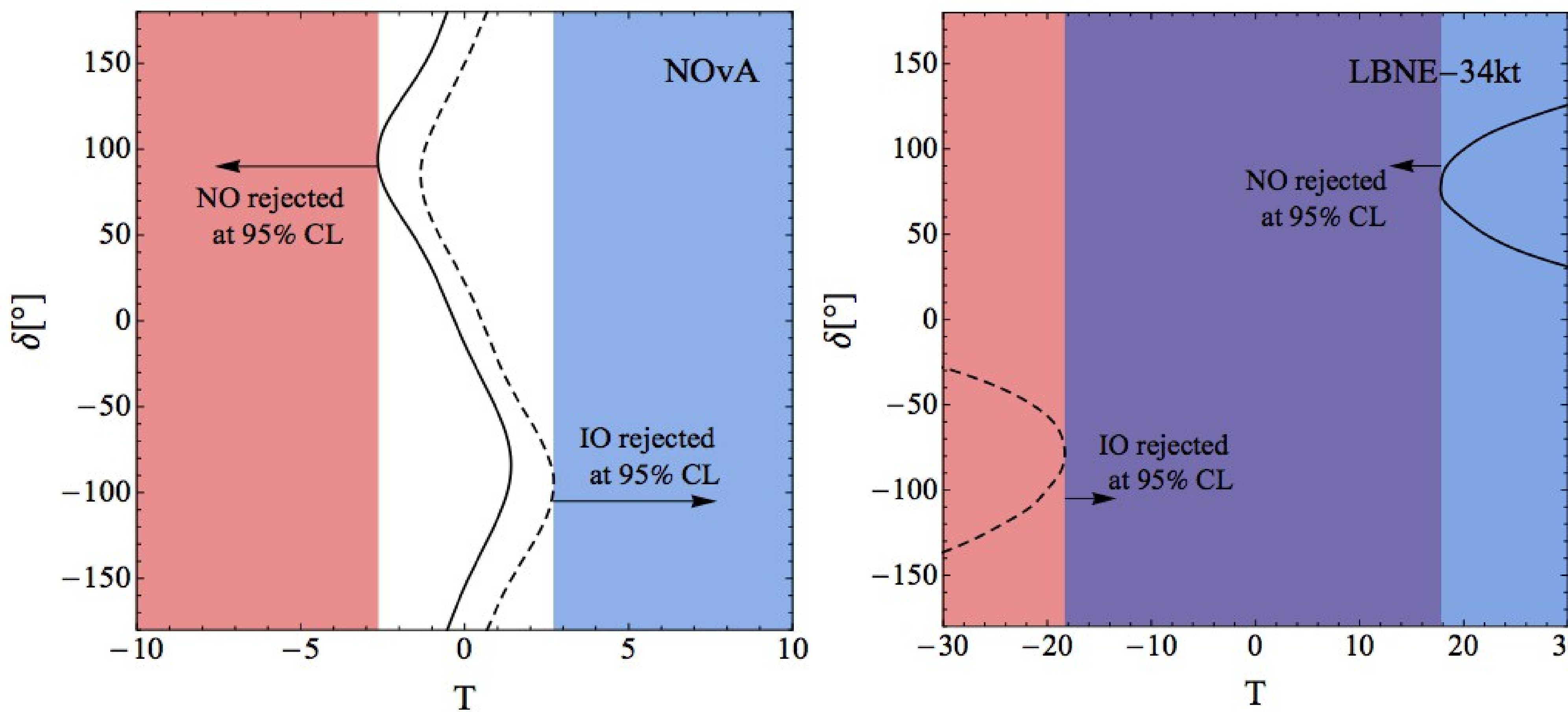
	σ_{E_ν}	σ_{θ_ν}	exposure	T_0^{NO} (med. sens.)	T_0^{IO} (med. sens.)
INO	$0.1E_\nu$	10°	10 yr × 50 kt	5.5 (2.6σ)	5.4 (2.6σ)
PINGU	$0.2E_\nu$	$29^\circ/\sqrt{E_\nu/\mathrm{GeV}}$	5 yr	12.5 (3.7σ)	12.0 (3.6σ)

	L (km)	Off-axis angle	ν flux peak	Detector	M(kt)	Years $(\nu,\bar{\nu})$
NOνA	810	14 mrad	2 GeV	TASD	13 kt	(3,3)
LBNE-10(34) kt	1290	–	2.5 GeV	LAr	10(34) kt	(5,5)

Long-baseline experiments

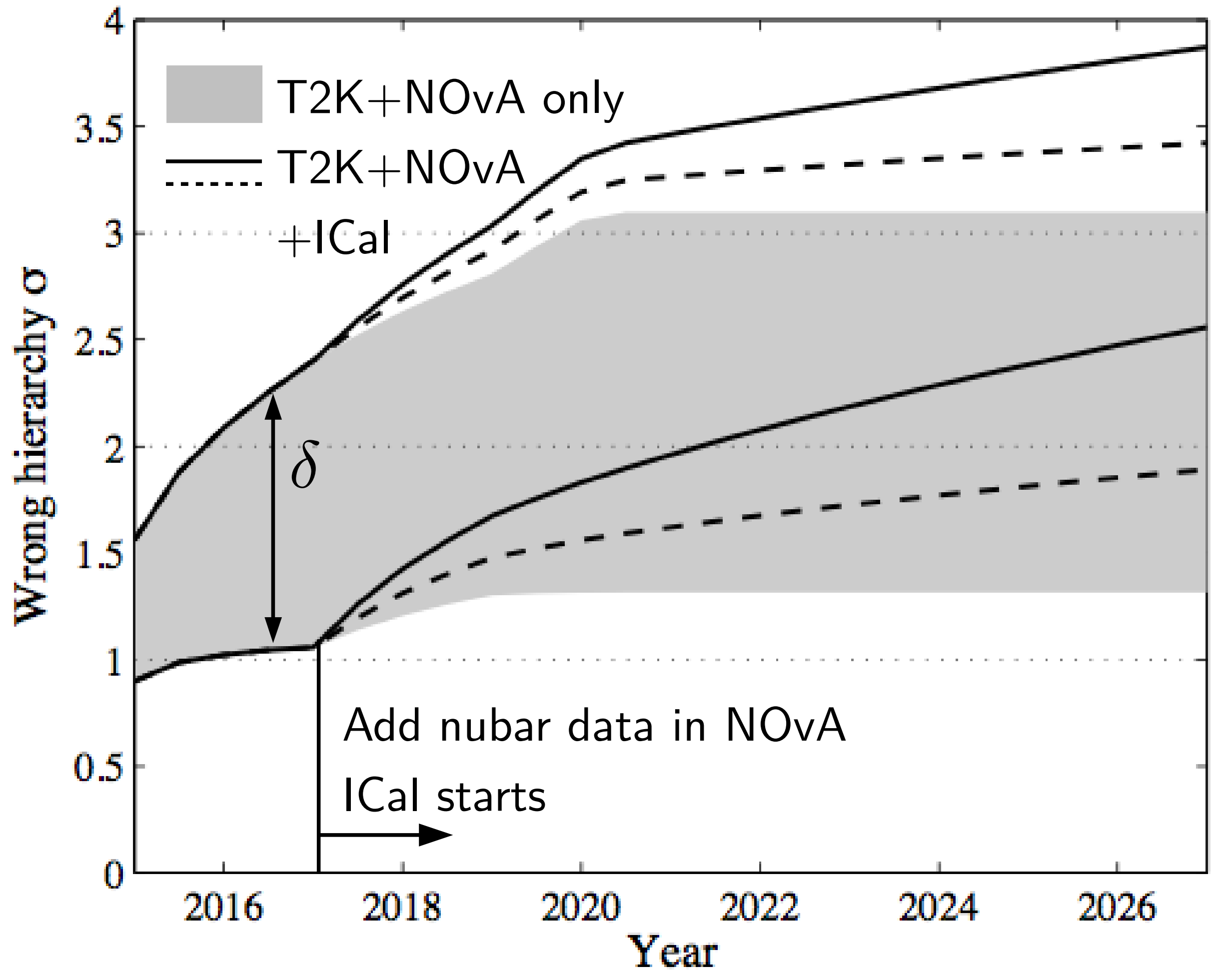


Long-baseline experiments



Blennow, Coloma, Huber and Schwetz, 1311.1822 [hep-ph]

Synergies between different experiments



Blennow, Schwetz, 1203.3388 [hep-ph]

(see also Ghosh, Thakore, Choubey, 1212.1305 [hep-ph])