Low Scale Minimal Seesaw Models

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Outline

- Motivations
- Minimal seesaw models vs Neff
 - General Bounds on 3+2 models
 General Bounds on 3+3 models (work in progress)
 P. Hernandez, M. Kekic, JLP
 arXiv:11311.2614 (PRD ... (2014)) + Work in progress

• Minimal seesaw models vs oscillation anomalies A. Donini, P. Hernandez, JLP, M. Maltoni, T. Schwetz arXiv:1205.5230 (JHEP 1207 (2012) 161)

Conclusions

Motivation

• The recent LHC results seem to indicate that the Higgs mechanism, with $m_H\sim 125\,$ GeV, is the responsible of the mass generation of the SM particles. ATLAS, CMS 2012

• However, the origin of light neutrino masses, which existence is supported by neutrino oscillation experiments, still remains unknown.

• Although the light neutrino masses could also be generated through the Higgs mechanism, their smallness in comparison with the SM particles might be calling for a different explanation

•••which is the simplest extension of the SM that can account for neutrino masses?

As simple as just adding singlet fermions (sterile neutrinos) to the SM field content.

If lepton number conservation is not impossed, the most general Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm kin} - \frac{1}{2} \overline{\nu_{si}} M_{ij} \nu_{sj}^c - (Y)_{i\alpha} \overline{\nu_{si}} \widetilde{\phi}^{\dagger} L_{\alpha} + \text{h.c.}$$

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•The Majorana mass scale constitutes a New Physics scale introduced to account for the light neutrino masses.

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Dirac neutrino limit

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Dirac neutrino limit Minkowski 77; Yanagida 79; Gell-Mann, Ramond, Slansky 79 Mohapatra, Senjanovic 80.

Standard Type-I

sesaw limit

- It is often assumed that the scale M is much higher than the electroweak scale. But it is also worth to explore other possibilities:
 - $M \sim eV \qquad \mbox{Could provide an explanation to neutrino anomalies pointed} \\ \mbox{out mainly by LSND and reactor experiments.} \label{eq:M}$

 $M\sim KeV$ Could still be a valid candidate for warm DM. Moreover, after the recent X-ray signal/hint. Bulbul et al (arXiv:1402.2301) Dodelson and Widrow 1994 Shi and Fuller 1999

 $M\sim GeV~$ Could account for baryon asymmetry in the Universe. Akhmedov, Rubakov, Smirnov 1998 Asaka, Blanchet, Shaposhnikov 2005

- Small M technically natural since in the limit $\,M \to 0\,$ a global lepton number symmetry is recovered.

• A low Majorana scale does not worsen the Higgs mass hierarchy problem.

 $[\delta m_H^2]_{\nu_B} \propto M^2$

Vissani 1998 hep-ph/9709409

• We start from the lowest level of complexity. Minimum number of extra fermionic degrees of freedom (fermion singlets) n_R

 $n_R = 1$ Excluded by neutrino oscillation data. Donini, Hernandez, JLP, Maltoni 2011

 $n_R = 2$ In agreement with neutrino oscillation data.

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 $n_R = 2$ In agreement with neutrino oscillation data.

We do not assume any hierarchy for the new parameters of the model.

What is the New Physics scale?

• Can we obtain general bounds on the Majorana scale without assuming a priori anything about the parameters of the model?

• We performed a global analysis of neutrino oscillation experiments, studying the whole parameter space for $n_R = 2$ with degenerate Majorana masses.

$$M \lesssim 10^{-9} (10^{-10}) eV$$

bound mainly from solar data Dirac limit Gouvea, Huang, Jenkins 2009 Donini, Hernandez, JLP, Maltoni 2011

$$M \gtrsim 0.6(1.6) eV$$

constraint mainly from LBL and reactor data

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3+2 Minimal Seesaw Model vs Cosmology

P. Hernandez, M. Kekic, JLP 2013 ArXiv:1311.2614

Extending Casas-Ibarra parameterization

Donini, Hernandez, JLP, Maltoni, Schwetz 2012; arXiv:1205.5230

$$U = \begin{pmatrix} U_{aa} & U_{as} \\ U_{sa} & U_{ss} \end{pmatrix}$$

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 $U_{aa} = U_{PMNS} \begin{pmatrix} 1 & 0 \\ 0 & H \end{pmatrix},$

Extending Casas-Ibarra parameterization

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$$U = \begin{pmatrix} U_{aa} & U_{as} \\ U_{sa} & U_{ss} \end{pmatrix} \xrightarrow{} active-sterile mixing$$

$$U_{aa} = U_{PMNS} \begin{pmatrix} 1 & 0\\ 0 & H \end{pmatrix},$$

$$U_{as} = iU_{PMNS} \begin{pmatrix} 0\\ Hm^{1/2}R^{\dagger}M^{-1/2} \end{pmatrix},$$

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$$R = \begin{pmatrix} \cos(\theta_{45} + i\gamma_{45}) & \sin(\theta_{45} + i\gamma_{45}) \\ -\sin(\theta_{45} + i\gamma_{45}) & \cos(\theta_{45} + i\gamma_{45}) \end{pmatrix}$$
 Casas-Ibarra complex orthogonal matrix

$$H^{-2} = I + m^{1/2} R^{\dagger} M^{-1/2} R m^{1/2}$$

Gives deviations from 3x3 unitarity

Parameters of the model $\theta_{23}, \theta_{12}, \theta_{13}, m_2, m_3, M_1, M_2, \delta, \alpha, \theta_{45}, \gamma_{45}$

Advantages of our parameterization

- EXACT. No expansion made.
- It does not explode for large values of the imaginary part of the complex angle. No necessary to introduce any extra theoretical constraint.
- Transparent. Easy to recover the Casas-Ibarra parameterization (at leading order in the seesaw expansion, $\mathcal{O}(m/M)$, H = H = 1)
- Corrections are important for very low Majorana mass scales , $\mathcal{O}\left(eV
 ight)$
- An alternative general parameterization was found by Blennow and Fernández-Martínez.
 Blennow, Fernández-Martínez 2011 arXiv:1107.3992

The energy density of the extra sterile neutrino species is usually quantified in terms of

$$N_{eff} = \frac{\rho_s + \rho_\nu}{\rho_{1\nu}^0}$$

$$N_{eff}^{BBN} = 3.68(3.80)^{+0.80}_{-0.70} (2\sigma)$$
 Izotov, Thuan 2010 (arXiv:1303.076)

$$N_{eff}^{BBN}=3.5\pm0.2[1\sigma]~~\left(N_{eff}^{BBN}<4~[~2.2\sigma]\right)$$
 Cooke et al; arXiv:1308.3240

Extra radiation, Neff

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Planck Collboration 2013 (arXiv:1303.076)

CMB



Planck Collboration 2013 (arXiv:1303.076)

- The 3 active neutrinos contribute with $N_{eff}^{SM}pprox 3$

- One fully thermal extra sterile state that decouples being relativistic contributes $\Delta N_{eff}\approx 1$ when decouples.

Can sterile neutrinos escape from thermalization in the 3+2 Minimal Seesaw Models?

• Sterile neutrino thermalization is controlled by:

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Sterile neutrino collision rate

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Sterile neutrino collision rate

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Hubble expansion rate

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Sterile neutrino collision rate

$$H\left(T\right) = \sqrt{\frac{4\pi^{3}g_{*}(T)}{45}} \frac{T^{2}}{M_{Planck}}$$

Hubble expansion rate

• The sterile neutrinos thermalize if $f_s(T) \ge 1$

• $f_s(T)$ reaches a maximum at some temperature T_{max} and if the maximum is larger than one, thermalization will be achieved. At decoupling we can estimate:

$$N_{eff} \approx N_{eff}^{SM} + \sum_{j} \left(1 - exp \left(-\alpha f_{s_j}(Tmax) \right) \right)$$
$$\bigcup_{\Delta N_{eff}}$$



- Same result for both sterile neutrinos, N_1 and N_2

- Thermalization rate basically indepent of the seesaw scale. The small dependece is modulated by $g_\ast(M)$

$$f_{s_j}(T_{max}) \sim \frac{\sum_{\alpha} |U_{\alpha s_j}|^2 M_j}{\sqrt{g_*(T_{max})}} \qquad |U_{\alpha s_j}|^2 \sim m/M_j$$

• In the 3+2 MM, for the whole parmeter space, the sterile neutrinos always thermalize at some point of the thermal history.

• This is because all favours participate in oscillations. The mixing with the three different flavous can not be small enough at the same time due to the correlation.


Sterile Neutrino Thermalization

• In the 3+2 Minimal Seesaw Model sterile neutrinos always thermalize.

- Each sterile neutrino contributes with $\Delta N_{eff}\approx 1$ when they decouple from the thermal bath. Therefore,

 $\Delta N_{eff} \approx 2$ @ decopling (sterile neutrino freeze out)

Can we thus rule out the 3+2 minimal seesaw model?

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NO! But a huge portion of the seesaw scale (8 orders of magnitude) is excluded!!!







- Above $\sim 1 GeV$, there is Boltzman suppression. The bounds do not apply for

 $M\gtrsim 1GeV$

- Moreover, after sterile neutrino decoupling two effects could modify ΔN_{eff} , before BBN:

(i) Dilution

(ii) Decay

Entropy dilution



Entropy dilution



Entropy dilution

- Dilution effects allow to relax the bounds for the range of masses $10 KeV \lesssim M \lesssim 1 GeV$

• However, those sterile neutrinos would give a huge contribution to the energy density when they become non-relativistic later, modyfing in a drastic way CMB and structure formation.

• The only way CMB and BBN bounds can be evaded for this range of masses is if the sterile neutinos decay before BBN.

sterile neutrino decay

• For sufficiently large M the sterile neutrino could decay before BBN and our analysis does not apply to this case.

$$\tau \sim 6 \times 10^{11} s \left(\frac{MeV}{M}\right)^4 \frac{0.05 eV}{|U_{\alpha s}|^2 M}$$

• For natural choices of the mixing decay takes place after BBN. However, for extreme mixings of $\mathcal{O}(1)$, sterile neutrinos as light as 10 MeV could decay before BBN.

sterile neutrino decay

- Bounds on short-lived sterile neutrinos with masses on the range $\left[10MeV,140MeV\right]$ have been studied by

Dolgov, Hansen, Raffelt, Semikoz 2000 Fuller, Kishimoto, Kusenko, 2011 Ruchayskiy, Ivashko, 2012

• Very strong bounds found combining BBN and direct acelerator searches, excluding the sterile neutrino decay before BBN in the minimal model for $M \lesssim \mathcal{O}\left(100 MeV\right)$ Ruchayskiy, Ivashko, 2012

Summary 3+2 vs cosmology

• In summary, cosmology allow us to exlude a huge part of the parameter space and the seesaw scale (8 orders of magnitude!) of the 3+2 MM.



3+3 Minimal Seesaw Model Vs Cosmology (work in progress)

In collaboration with P. Hernandez and M. Kekic

3+3 Minimal Seesaw Model

 Lager parameter space: 3 light masses + 3 heavy masses +6 angles + 6 CP-phases.

• We have explored the whole parameter space allowed by neutrino oscillation data.

• In spite of the larger parameter space, only one sterile neutrino can escape from thermalization. The thermalization being basically controlled by the lightest neutrino mass.

3+3 Minimal Seesaw Model



• Analytical lower bound on thermalization rate

$$f_{s_j}(T_{max}) \ge g(M_j)h_j \ge g(M_j)m_1$$
$$h_j \equiv \sum_{\alpha} |U_{\alpha s_j}|^2 M_j = \sum_i |R_{ij}|^2 m_i \ge m_1$$

• Analytical lower bound on thermalization rate

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$$h_j \equiv \sum_{\alpha} |U_{\alpha s_j}|^2 M_j = \sum_i |R_{ij}|^2 m_i \ge m_1$$
$$m_1 \ge g(M_j)^{-1} \approx 1(4) \times 10^{-3} eV \text{ for } M_1 = eV(1GeV)$$
$$\bigcup_{f_{s_j}(T_{max}) \ge 1}$$

Lower bound almost independent of M Mainly controled by lightest neutrino mass

How many sterile neutrinos can simultaneusly satisfy this thermalization bound?



- Only one sterile neutrino can escape from thermalization.
- The thermalization is controlled by the lightest neutrino mass.
- If $m_1 \ge \mathcal{O}\left(10^{-3}eV\right)$ the three sterile neutrinos thermalize!

Possible scenarios

• $m_1 \geq \mathcal{O}\left(10^{-3} eV\right)$: the three sterile neutrinos thermalize.



Possible scenarios

• $m_1 \leq \mathcal{O}\left(10^{-3} eV\right)$: one sterile neutrino does not thermalize. The other two contribute as in the 3+2 model.



Allowed sterile neutrino spectra

Impact on neutrinoless double beta decay



Only a sub-eV sterile neutrino can give a significant contribution!

3+2 Minimal Seesaw Model vs Neutrino Anomaly

A. Donini, P. Hernandez, JLP, M. Maltoni, T. Schwetz 2012 arXiv:1205.5230

Neutrino anomaly & Sterile neutrinos

- LSND/reactors anomaly requires at least an extra $\,\Delta m^2_{LSND} \sim 1 eV^2$

• Tension between appearance and disappearance experiments. Difficult to accommodate all data.

See for instance: Kopp, Machado, Maltoni and Schwetz 2013, arXiv:1303.3011

See light Sterile Neutrinos White Paper, Abazajian et al arXiv: 1204.5379 and refs. therein

Neutrino anomaly & Sterile neutrinos

• Tension between appearance and disappearance experiments

$$\begin{split} P_{\mu e} &\sim |U_{se}|^2 |U_{s\mu}|^2 & \textcircled{ISND/MB signal} \\ P_{ee} &\sim |U_{se}|^2 & \textcircled{ISND/MB signal} \\ P_{\mu \mu} &\sim |U_{s\mu}|^2 & \swarrow & \texttt{no signal...} \end{split}$$

• Difficult to accommodate all data. Convincing signal should appear in all these channels.

3+2 Phenomenological Models (PM)

• Best fits in the 3+2 phenomenological model (PM). Number of free parameters: 9 angles + 5 phases + 4 neutrino mass differences.

3+2 PM	$ \Delta m^2_{41} (\mathrm{eV}^2)$	$ \Delta m_{51}^2 (\mathrm{eV}^2)$	$ U_{e4} $	$ U_{e5} $	$ U_{\mu4} $	$ U_{\mu 5} $	ϕ_{45}
$\mathrm{KMS}[13]$	0.47	0.87	0.128	0.138	0.165	0.148	1.64 π
$\operatorname{GL}[14]$	0.9	1.61	0.13	0.13	0.14	0.078	$1.51 \ \pi$

Kopp, Maltoni, Schwetz (KMS) arXiv:1103.4570 Giunti, Laveder, (GL) arXiv:1107.1452

See also more recent analysis by Kopp, Machado, Maltoni and Schwetz 2013, arXiv:1303.3011

• Phenomenological models are model independent. Realistic models are much more constrained.

3+2 Mini Seesaw Model

• Mini-seesaw model (MM). Minimal model accounting for neutrino masses.

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm kin} - \frac{1}{2} \overline{\nu_{si}} M_{ij} \nu_{sj}^c - (Y)_{i\alpha} \overline{\nu_{si}} \widetilde{\phi}^{\dagger} L_{\alpha} + \text{h.c.}$$

- More predictive than the phenomenological models (PM).
- Mini-Seesaw have 7 parameters less than the PM !

Model	$\# \Delta m^2$	# Angles	# Phases	Total
3ν	2	3	1	6
3+2 MM	4	4	3	11
3+2 PM	4	9	5	18

...and, in the Mini-Seesaw, active-sterile mixing and mass parameters are strongly correlated!!

3+2 Mini Seesaw Model

- We performed a global fit in the context of the 3+2 mini-seesaw model.
- LBL and SBL can not be decoupled. Correlation between active and sterile mixing is not negligible. Active-sterile mixing and heavy squared mass diferences strongly correlated.
- In the analysis we fix the values of $M_1 {\rm and}\, M_2 {\rm to}$ the results of the above PM best fits.
- We use a general parameterization which generalizes Casas-Ibarra to the case in which corrections are important (as in this case)

Global fit



Best fit: active-sterile mixing

Prediction: large tau-mixing with extra states for NH!



Minimal Sterile Neutrino Model

• But large tension appearance/disappearance in μ sector remains.



Conclusions

- We have studied in detail the simplest low scale models that can accomodate light neutrino masses: just adding singlet fermions (sterile neutrinos) to the SM.
- The minimal model requires two sterile neutrinos and is able to explain the neutrino anomaly at the same level as the phenomenological models.
- However, low scale 3+2 minimal seesaw models are strongly constrained by cosmology since the sterile neutrinos can not scape from thermalization.
- A huge part of the parameter space and the seesaw scale (8 orders of magnitude) of the 3+2 MM can be exluded thanks to cosmology.
- Low scale 3+3 minimal seesaw models are also very constrained by cosmology. **Only one sterile neutrino might escape from thermalization.** Thermalization is controlled by the lightest neutrino mass, being the threshold: $m_1 = O(10^{-3} eV)$
- Strong impact of the cosmological bounds on neutrinoless double beta decay.

Tack!

Possible scenarios

• $m_1 \leq \mathcal{O}\left(10^{-3} eV\right)$: one sterile neutrino does not thermalize. The other two contribute as in the 3+2 model.


$$\dot{\rho} = -i[H,\rho] - \frac{1}{2} \{\Gamma, \rho - \rho_{eq} I_A\};$$

$$\dot{\rho}_{A} = -i(H_{A}\rho_{A} - \rho_{A}H_{A} + H_{AS}\rho_{AS}^{\dagger} - \rho_{AS}H_{AS}^{\dagger}) - \frac{1}{2}\{\Gamma_{A}, \rho_{A} - \rho_{eq}I_{A}\}$$
$$\dot{\rho}_{AS} = -i(H_{A}\rho_{AS} + H_{AS}\rho_{S} - \rho_{AS}H_{S}) - \frac{1}{2}\Gamma_{A}\rho_{AS},$$
$$\dot{\rho}_{S} = -i(H_{AS}^{\dagger}\rho_{AS} - \rho_{AS}^{\dagger}H_{AS} + H_{S}\rho_{S} - \rho_{S}H_{S}).$$
$$\Gamma_{\nu_{\alpha}} \gg H \longrightarrow \dot{\rho}_{A} = \dot{\rho}_{AS} = 0$$

$$\dot{\rho}_{ss} = -\left(H_{AS}^{\dagger}\left\{\frac{\Gamma_{AA}}{(H_{AA} - H_{ss})^2 + \Gamma_{AA}^2/4}\right\}H_{AS}\right)_{ss}\tilde{\rho}_{ss}$$
$$\simeq -\frac{1}{2}\sum_{a}\langle P(\nu_s \to \nu_a)\rangle\Gamma_a\tilde{\rho}_{ss},$$
$$\tilde{\rho}_S \equiv \rho_S - \rho_{eq}I_S$$

Sterile Neutrino Thermalization

• Necesary to go beyond the two family approximation. Expanding over m/M we have computed the time-averaged probability:

$$f_{s_j}(T) = \frac{\Gamma_{\nu_e}(T)}{H(T)} \sum_{\alpha = e, \mu, \tau} n_\alpha \left(\frac{M_j^2}{2pV_\alpha - M_j^2}\right)^2 |(U_{as})_{\alpha j}|^2$$

 $\begin{cases} (\tau) \ T \gtrsim 180 \text{ MeV: } n_{\alpha} = 1 \text{ and } V_{\alpha} = A T^{4} p, \\ (\mu) \ 20 \ \text{MeV} \lesssim T \lesssim 180 \ \text{MeV: } n_{e} = n_{\mu} = 1, \ n_{\tau} = 9/13, \ V_{e} = V_{\mu} = A T^{4} p \text{ and} \\ V_{\tau} = B T^{4} p, \end{cases}$ $(e) \ T \lesssim 20 \ \text{MeV: } n_{e} = 1, \ n_{\mu} = n_{\tau} = 9/13, \ V_{e} = A T^{4} p \text{ and } V_{\mu} = V_{\tau} = B T^{4} p. \end{cases}$





Global fit

• DATA

LBL: KamLAND, MINOS, T2K

SBL: LSND, MiniBooNE, KARMEN, NOMAD, CDHS, Bugey, ROVNO, Krasnoyarsk, Gösgen, CHOOZ, Palo Verde, Double CHOOZ, Daya Bay, RENO.



Analytical lower bound

• Analytical lower bound on thermalization rate!

$$f_{s_j}(T) \ge \frac{9}{16} \frac{\Gamma_{\nu_e}(T_{max}^{\tau})}{H(T_{max}^{\tau})} \sum_i |U_{\alpha s_j}|^2 \propto \frac{\sum_{\alpha} |U_{\alpha s_j}|^2 M_j}{\sqrt{g_*(T_{max}^{\tau})}}$$

$$h_j \equiv \sum_{\alpha} |U_{\alpha s_j}|^2 M_j = \sum_i |R_{ij}|^2 m_i \ge m_1$$

Independent of PMNS parameters

Extending Casas-Ibarra parameterization

$$U = \begin{pmatrix} U_{aa} & U_{as} \\ U_{sa} & U_{ss} \end{pmatrix}$$

 $U_{aa} = U_{PMNS} \begin{pmatrix} 1 & 0 \\ 0 & H \end{pmatrix}, \qquad U_{as} = i \left(0 \quad \overline{H} M^{-1/2} R m^{-1/2} \right),$

$$U_{as} = i U_{PMNS} \begin{pmatrix} 0\\ Hm^{1/2}R^{\dagger}M^{-1/2} \end{pmatrix}, \qquad \qquad U_{ss} = \overline{H}$$