Cabibbo's dream

Belén Gavela

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

(Alonso, Gavela, Isidori, Maiani)

invisibles
neutrinos, dark matter & dark energy physics

Cabibbo's dream:

dynamical origin of mass

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BSM because

1) Experimental evidence for new particle physics:

*** **Neutrino masses**
*** **Dark matter**
*** **CMB polarization ?**

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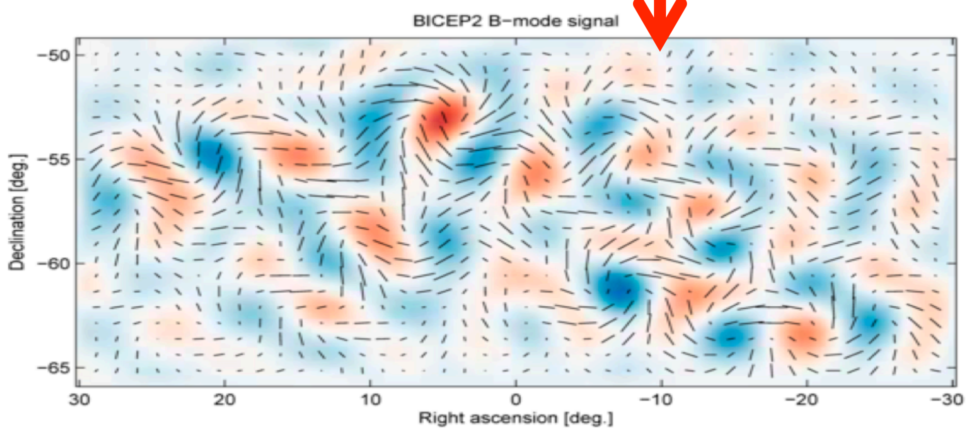
Bicep2



BICEP2 Collaboration

B-mode polarization image

The first detection of gravitational waves from the very early universe. This image shows the orientation of "B-mode" polarization (lines) of the cosmic microwave background light, superimposed on the strength of the polarization (colors). The pinwheel pattern characteristic of a "B-mode" signal comes from a combination of gravitational lensing in the local universe and primordial gravitational waves.

BSM because

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Bicep2

New energy-density scale **?** $V \sim 5\ 10^{16}$ GeV $\sim m_\phi^{\,2}\, \phi^{\,2}$

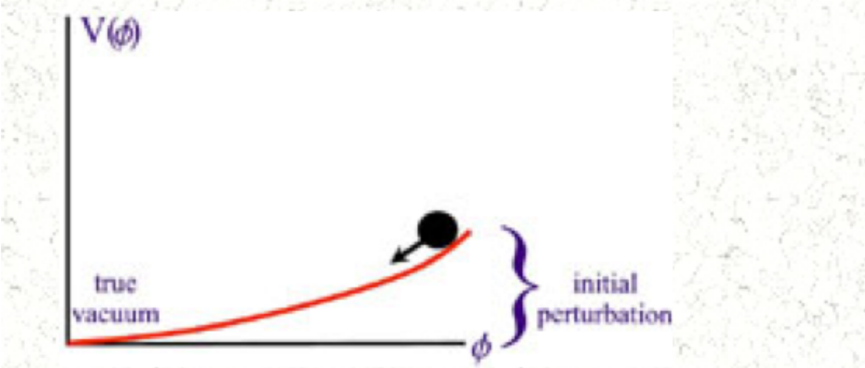

Figure 9. Toy Model For Chaotic Inflation

→ $m_\phi \sim 10^{13}$ GeV

or higher mass scale

BSM because

1) **Experimental evidence** for new particle physics:

*** **Neutrino masses**
*** **Dark matter**
*** **CMB polarization ?**

2) **SM fine-tunings/uneasiness**

$\boxed{\text{SU(3) x SU(2) x U(1) x classical gravity}}$

We ~understand ordinary particles= excitations over the vacuum

We DO NOT understand the vacuum = state of lowest energy:

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The Higgs excitation has the quantum numbers of the EW vacuum

BSM because

1) Experimental evidence for new particle physics:

***** Neutrino masses**
***** Dark matter**
***** CMB polarization ?**

2) SM fine-tunings/uneasiness, i.e. in electroweak:

***** Hierarchy problem**
***** Flavour puzzle**

BSM electroweak

* **HIERARCHY PROBLEM**

fine-tuning issue: if there is BSM physics,
why is the Higgs so light?

→SUSY ?, strong-int. Higgs ?, extra-dim ?….

In practice, none without further fine-tunings

• **FLAVOUR PUZZLE**

The Flavour Puzzle

u	ν_e	c	ν_μ	t	ν_τ
d	e	s	μ	b	τ

Why so diferent masses and mixing angles?

The Flavour Puzzle

u	ν_e	c	ν_μ	t	ν_τ
d	e	s	μ	b	τ

Why has nature chosen the number and properties of **(quark)** **families so as to allow for CP violation... and explain nothing?** (i.e. not enough for matter-antimatter asymmetry)

BSM electroweak

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- **FLAVOUR PUZZLE**: no progress

Understanding stalled since 30 years.
Only new B physics data AND neutrino masses and mixings

BSMs tend to make it worse

BSM electroweak

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$\Lambda_{electroweak} \sim 1$ **TeV ?**

• **FLAVOUR PUZZLE**: no progress

Understanding stalled since 30 years.
Only new B physics data AND neutrino masses and mixings

BSMs tend to make it worse

$\Lambda_{flavour} \sim 100$ **TeVs ???**

The FLAVOUR WALL for BSM

BSM theories usually die or are very unsatisfactory when confronted with:

i) **Electric dipole moments** **(quarks and leptons)**

ii) **FCNC processes** **(quarks and leptons)**

iii) **Strong CP problem**

iii) **Matter-antimatter asymmetry**

competing with SM at one-loop

BSM because

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***** Dark matter**
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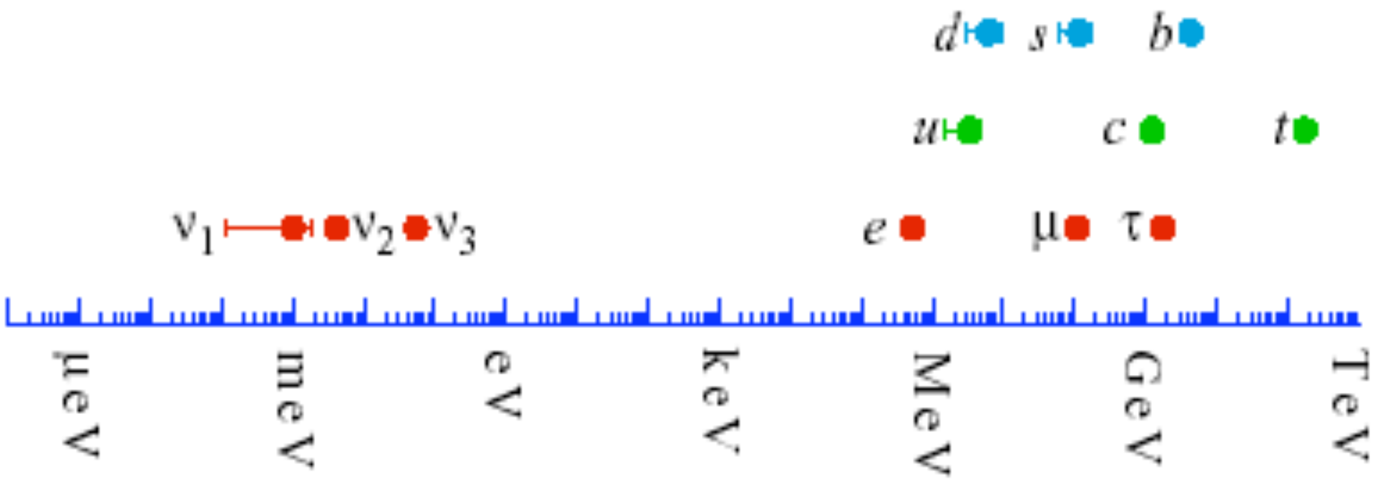
2) SM fine-tunings/uneasiness, i.e. in electroweak:

***** Hierarchy problem**
***** Flavour puzzle**

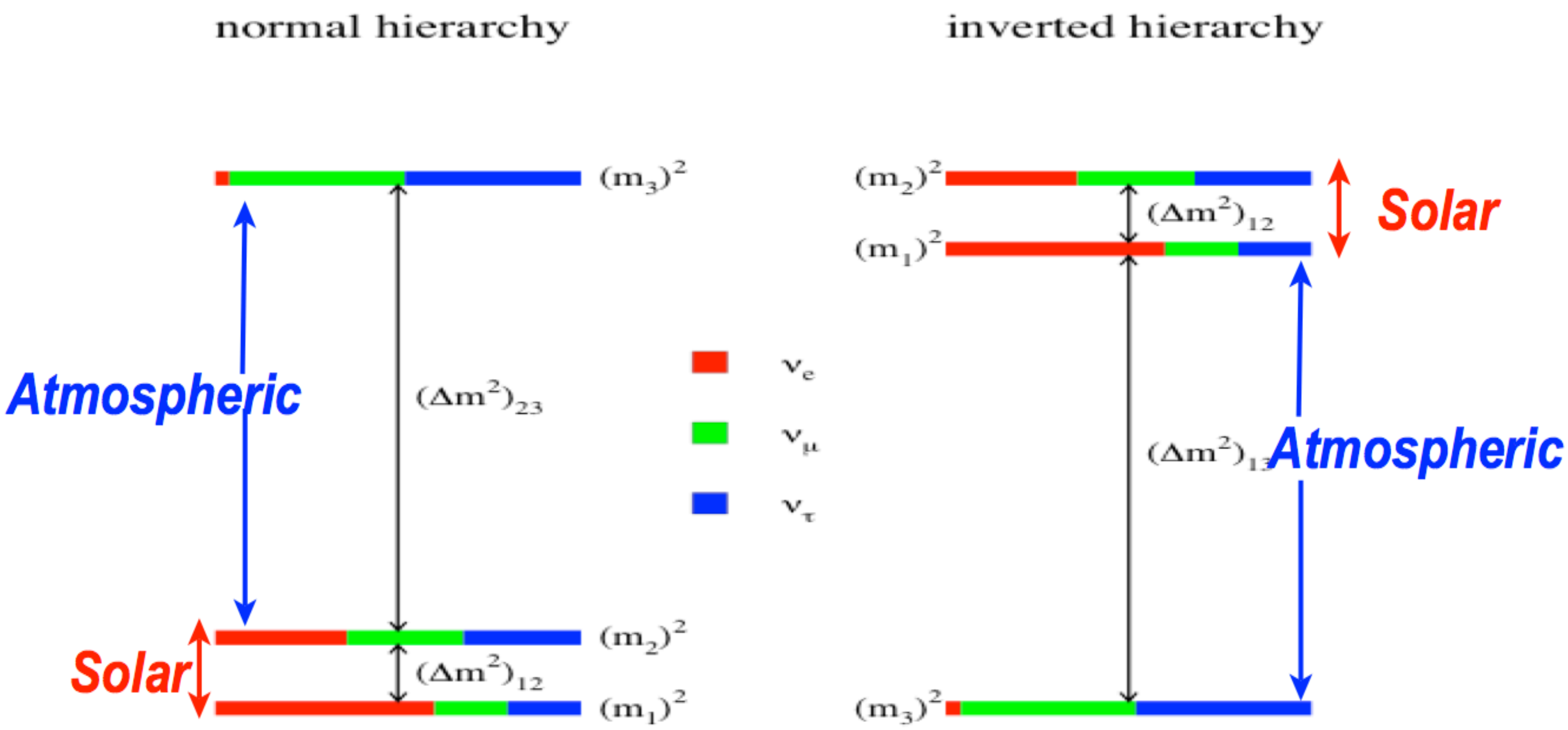
$\downarrow$

Pattern of masses and mixings in visible universe

Masses of the matter particles of the visible universe

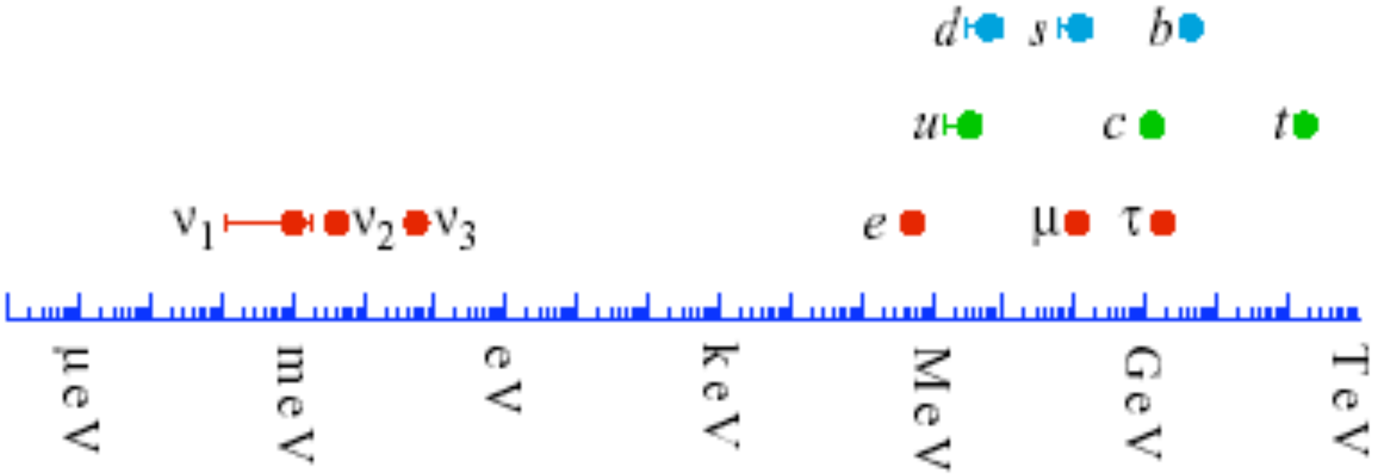


We should at least measure the 3 active ν mass matrix



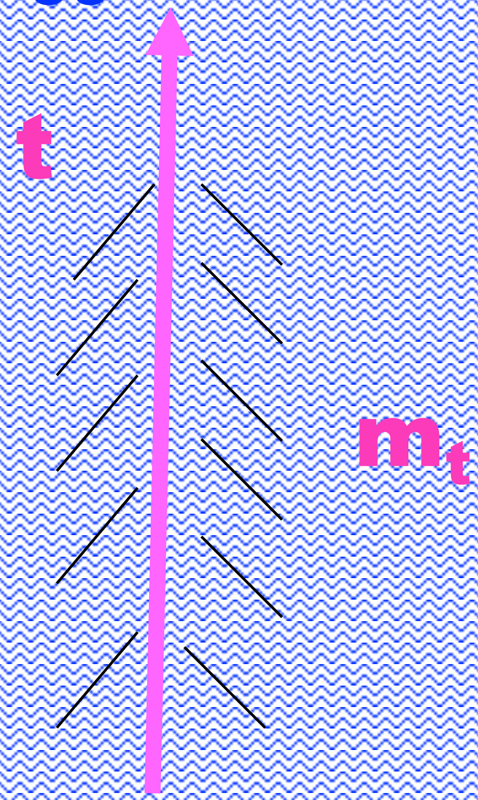
Masses
$m_1^2 < m_2^2$, m_3^2

Masses of the matter particles of the visible universe



Masses in SM

In SM, mass is a sort of friction with a field that permeates vaccuum: the Higgs field



Higgs

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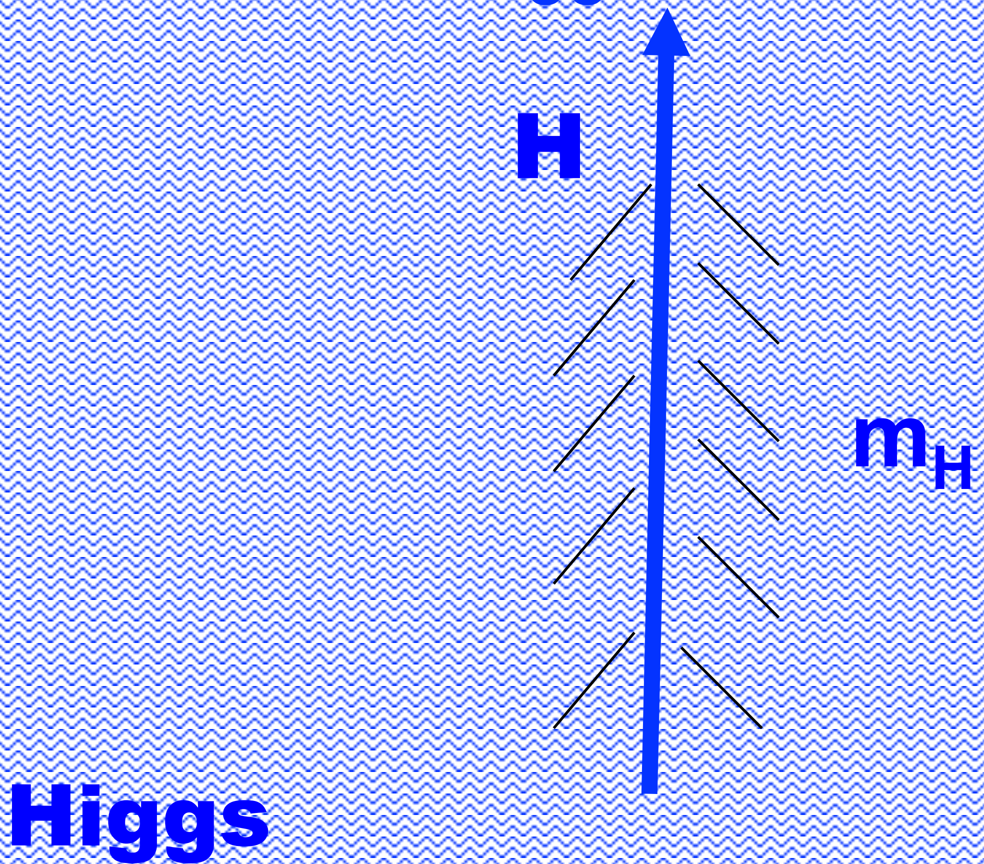
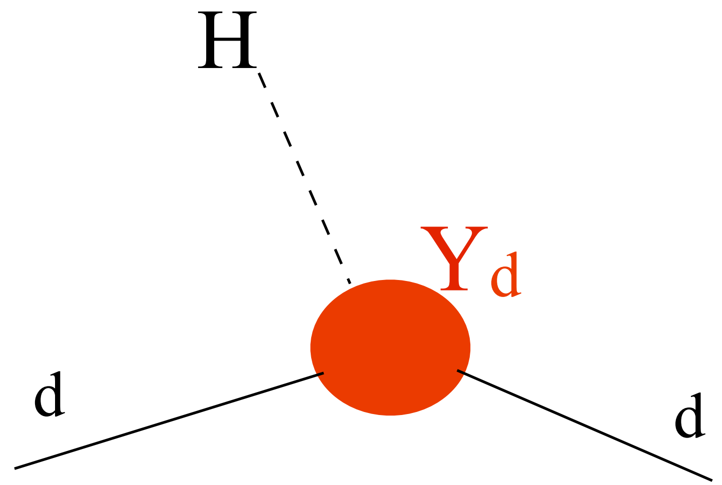
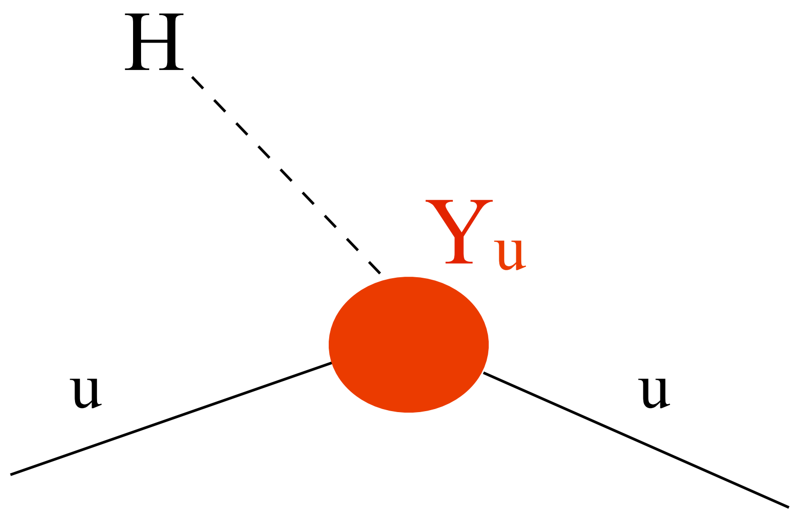


The Higgs field is the source of masses in the SM

$$\delta\mathcal{L}_m = Q\, Y_d\, H\, d + Q\, Y_u\, \tilde{H}\, u + \text{h.c.}$$



$m_d \sim Y_d\, v$



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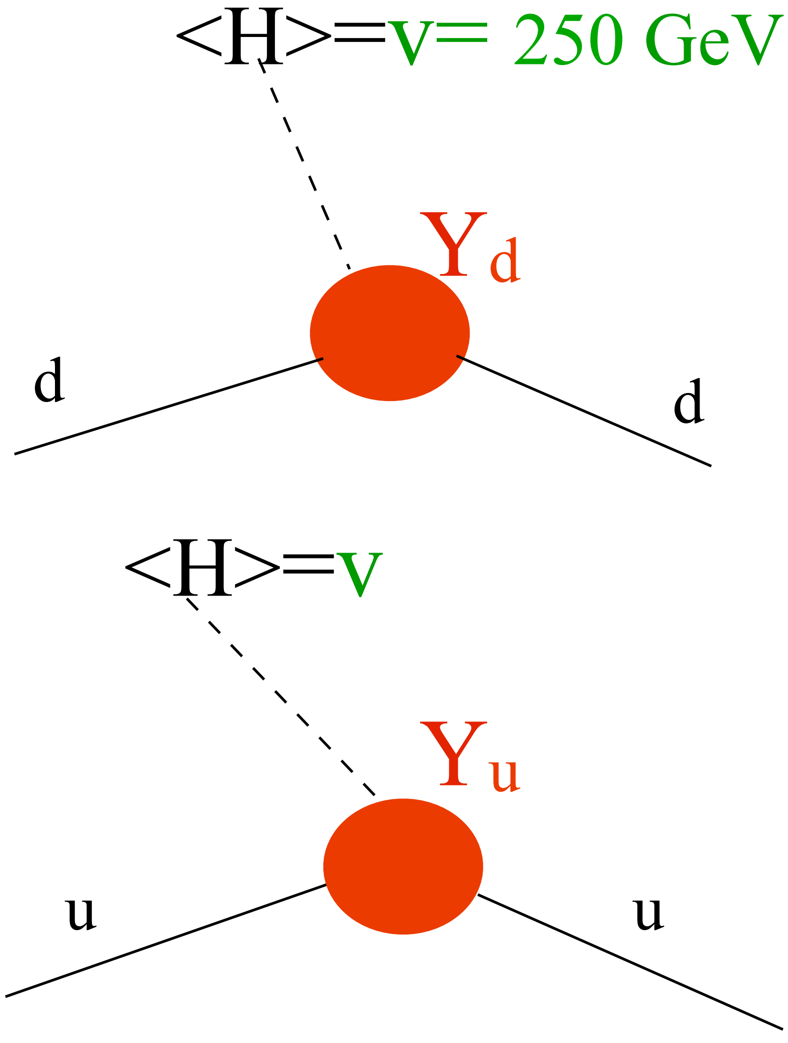

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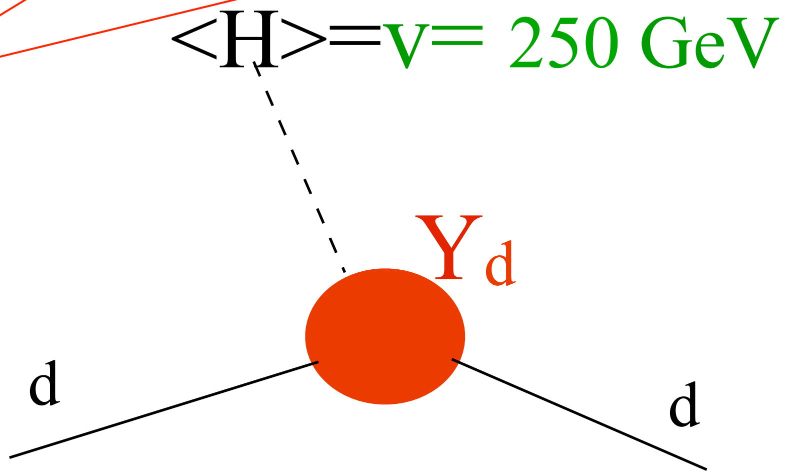
$$\delta\mathcal{L}_m = Q\, Y_d\, H\, d + Q\, Y_u\, \tilde{H}\, u + \text{h.c.}$$

YUKAWAs

are numbers



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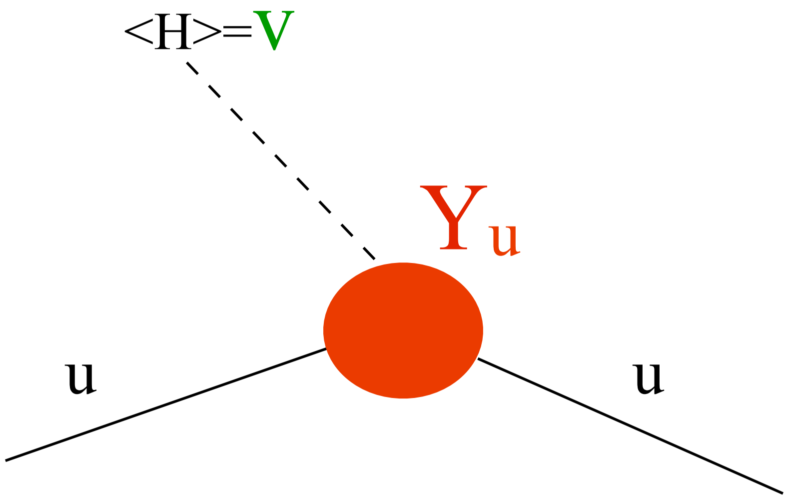

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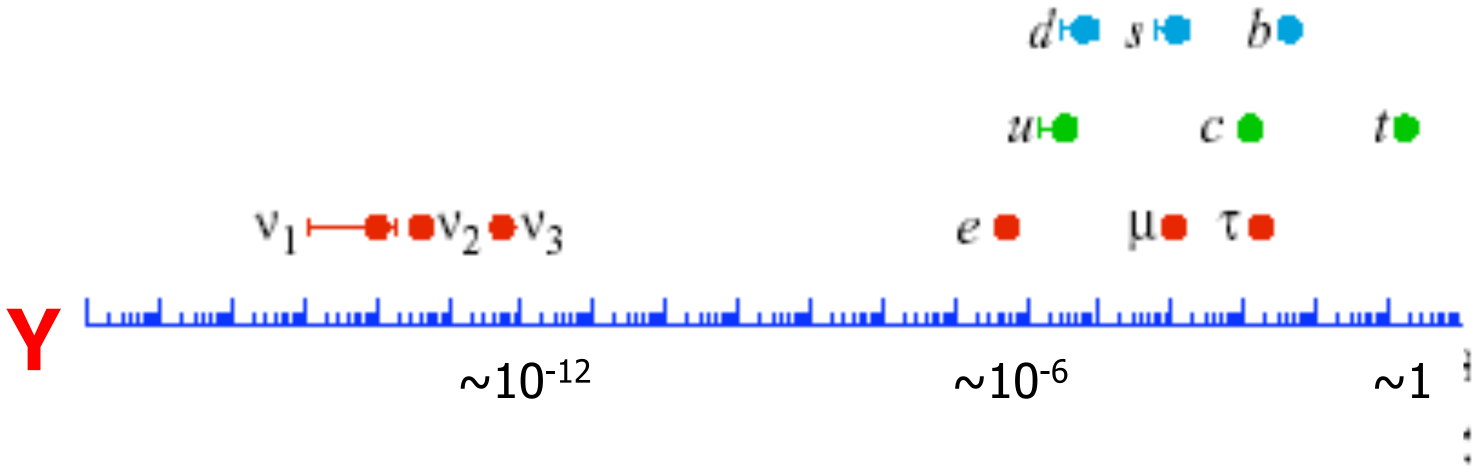
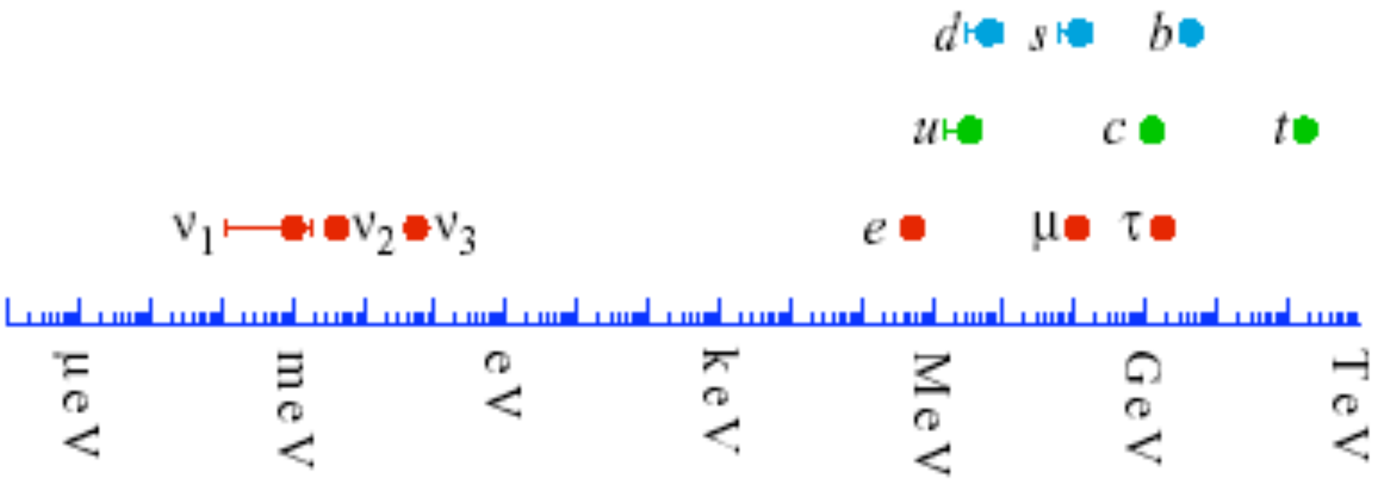
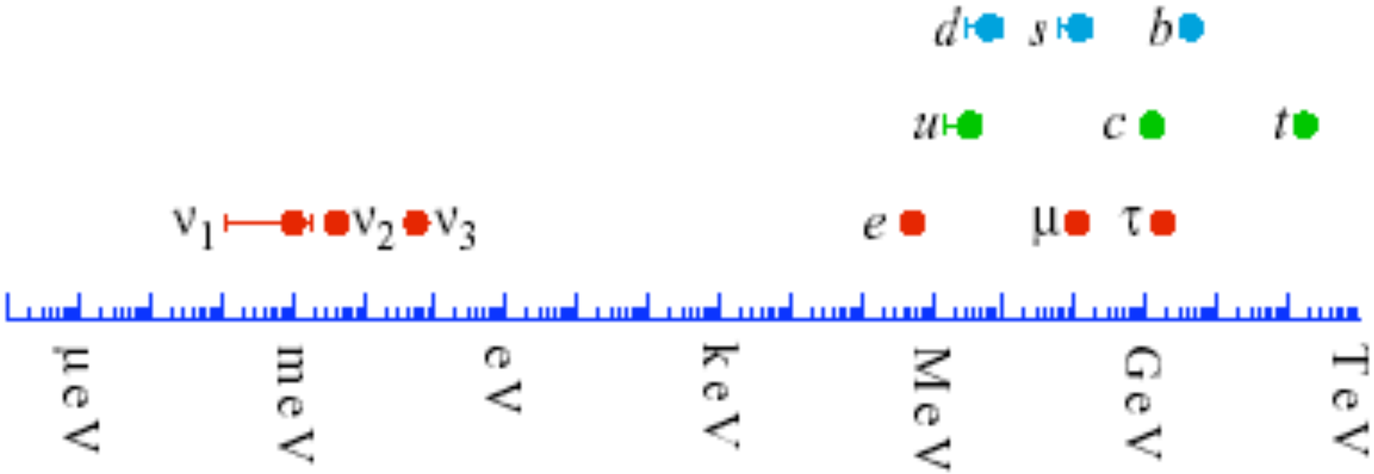
The mass spectrum in terms of YUKAWA couplings



Neutrino light on flavour ?



Neutrinos lighter because Majorana?



Neutrinos lighter because Majorana?

$$\nu = \bar{\nu}$$

* To have a Majorana mass

* $\nu = \bar{\nu}$ (that is, Majorana neutrino)

* L non conserved

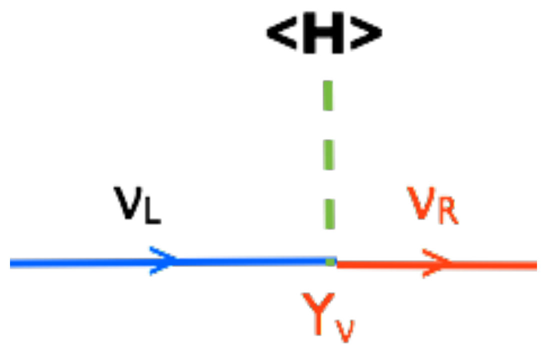
are all equivalent concepts

Any of them implies the other two

Kayser

Simple case: add right-handed neutrino to SM

$$\delta\mathcal{L}_m = Y_\nu \, \overline{L} \, H \, \nu_R + h.c.$$



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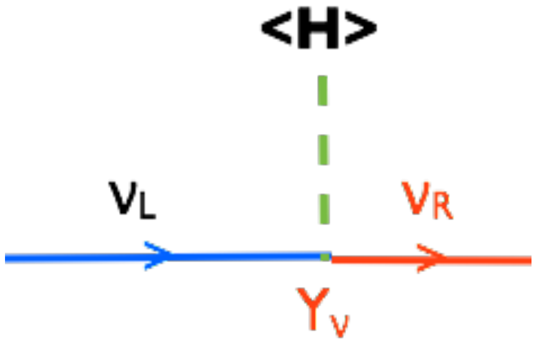
Simple case: add right-handed neutrino to SM

$$\delta\mathcal{L}_m = Y_\nu\,\overline{L}\,H\,\nu_R + h.c. + M\,\overline{\nu_R{}^C}\,\nu_R$$

allowed by SM gauge invariance



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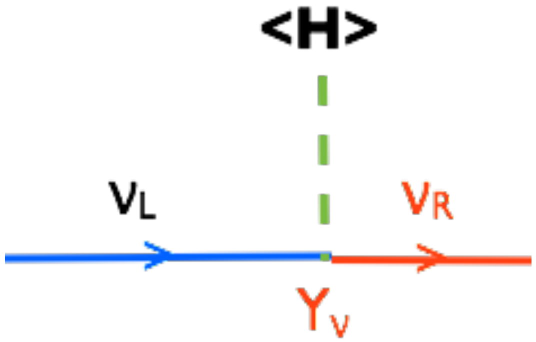

$$\mathcal{L}_{M_\nu} = \begin{pmatrix} 0 & \mathbf{Y}^T v \\ \mathbf{Y} v & \mathbf{M} \end{pmatrix}$$

* For **M** > v :

$m_1 \sim M$

$m_2 \sim \mathbf{Y}^2 v^2 / M$

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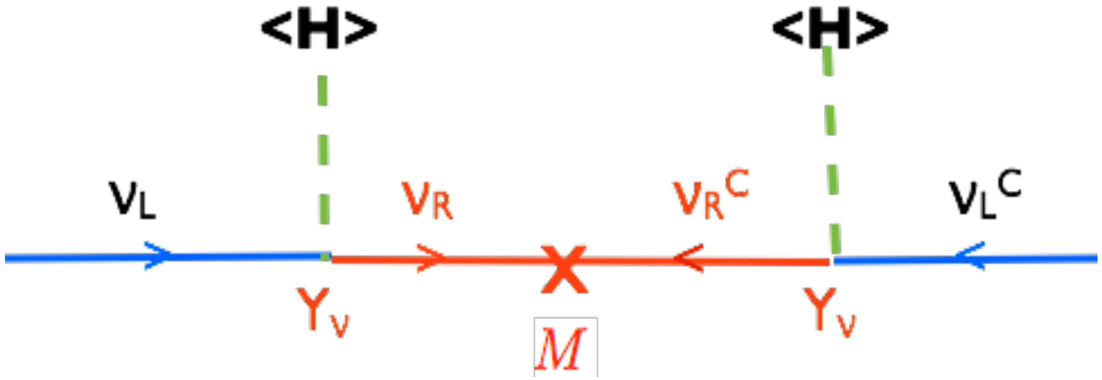

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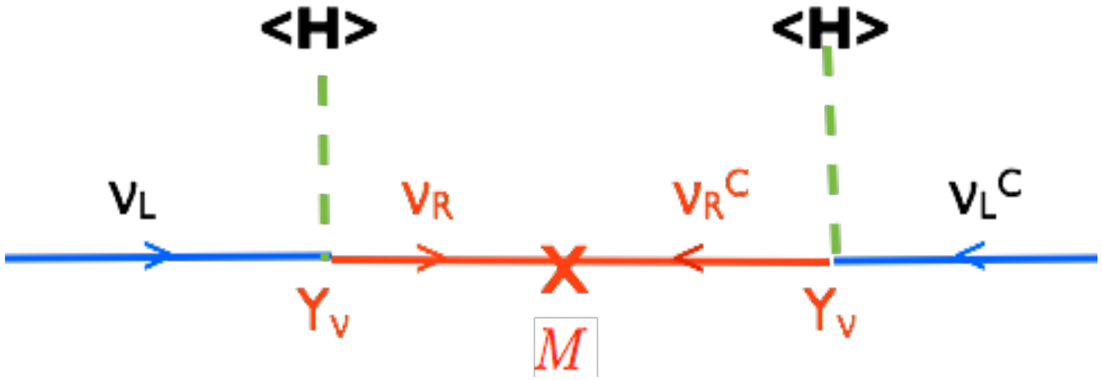
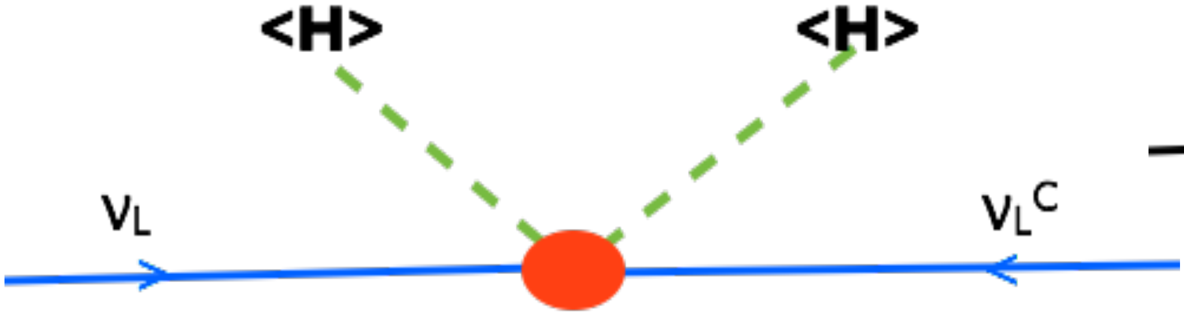


At energies E<M:

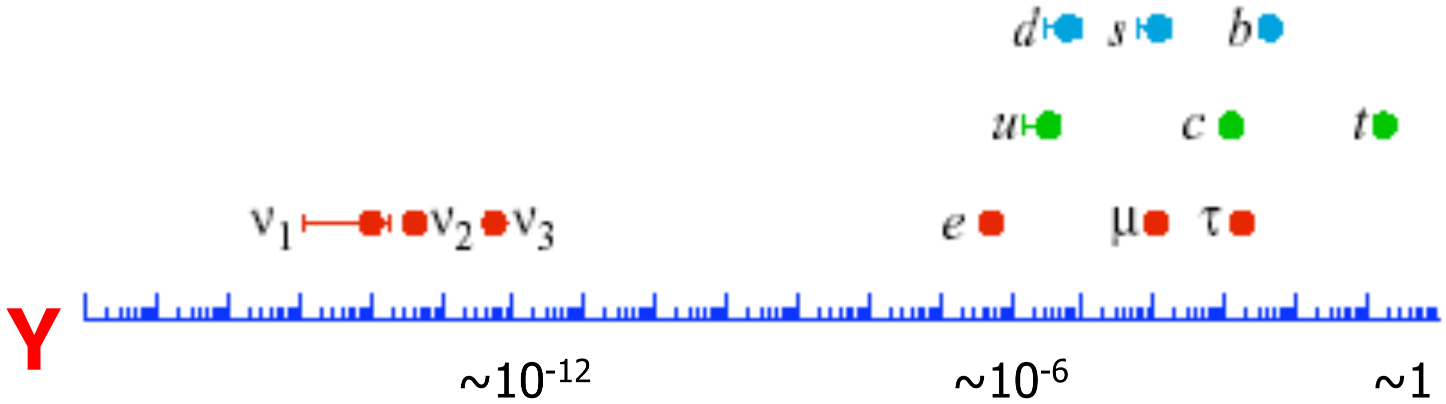


$$\longrightarrow m_\nu \sim \frac{Y_\nu^2 v^2}{2M}$$

Which allows $Y_\nu \sim 1$ --> $M \sim M_{Gut}$

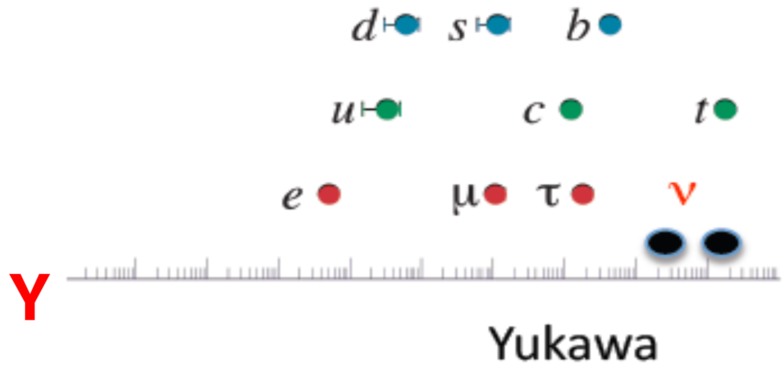
$Y_\nu \sim 10^{-6}$ --> $M \sim \mathrm{TeV}$

In pure SM, the mass spectrum in terms of YUKAWA couplings:

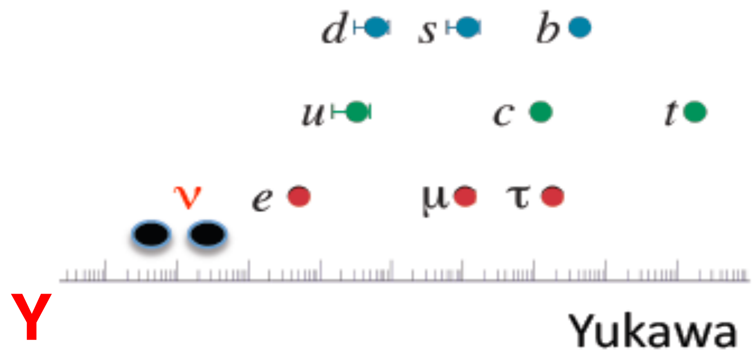


Within seesaw, the size of neutrino Yukawa couplings is similar to that for other fermions:

M ≤ GUT



M = TeV

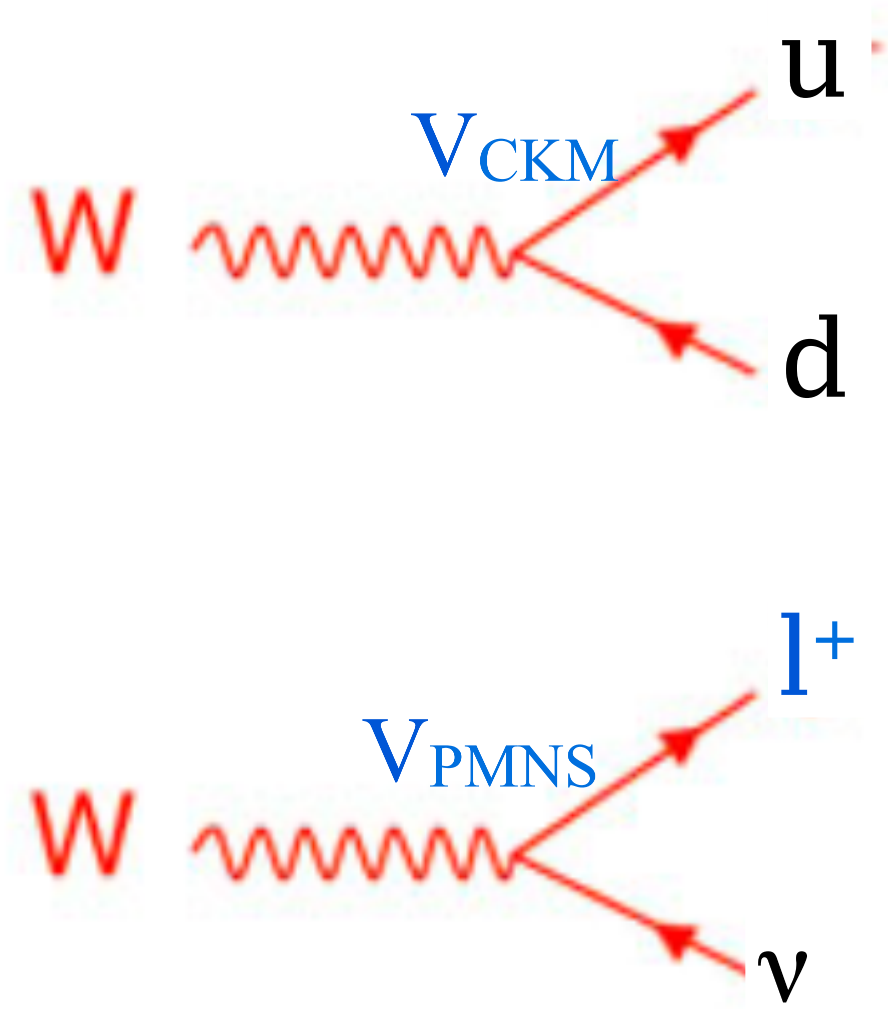


Pilar Hernandez drawings

Minkowski; Gell-Mann, Ramond Slansky; Yanagida, Glashow...

Neutrino oscillations demonstrated

leptonic flavour violation in nature



W
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ν

$$U=\begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23}-s_{23}s_{13}c_{12}e^{-i\delta} & c_{12}c_{23}-s_{23}s_{13}s_{12}e^{-i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-c_{23}s_{13}c_{12}e^{-i\delta} & -s_{23}c_{12}-c_{23}s_{13}s_{12}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}$$

DIRAC O MAJORANA

IF MAJORANA

Leptons

$$V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 9^o \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & +0.7 \end{pmatrix}$$

Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

Why so different?

Leptons

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Perhaps also because ν_s may be Majorana?

Neutrino are optimal windows into the exotic -dark- sectors

* Can mix with new neutral fermions, heavy or light

* Interactions not obscured by strong and e.m. ones

Dark portals

Only three singlet combinations in SM with d < 4:

$H^+ H$ **Scalar**

$B_{\mu\nu}$ **Vector**

$\bar{L}\ H$ **Fermionic**

Any hidden sector, singlet under SM, can couple to the dark portals

Dark portals

$H^{+} H \mathbf{S}$ Scalar

$B_{\mu\nu} \mathbf{V}_{\mu\nu}$ Vector

$\bar{L} H \mathbf{\Psi}$ Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

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Dark portals

$H^{+} H\, S^{2}$ **Scalar**

$B_{\mu\nu}\, V_{\mu\nu}$ **Vector**

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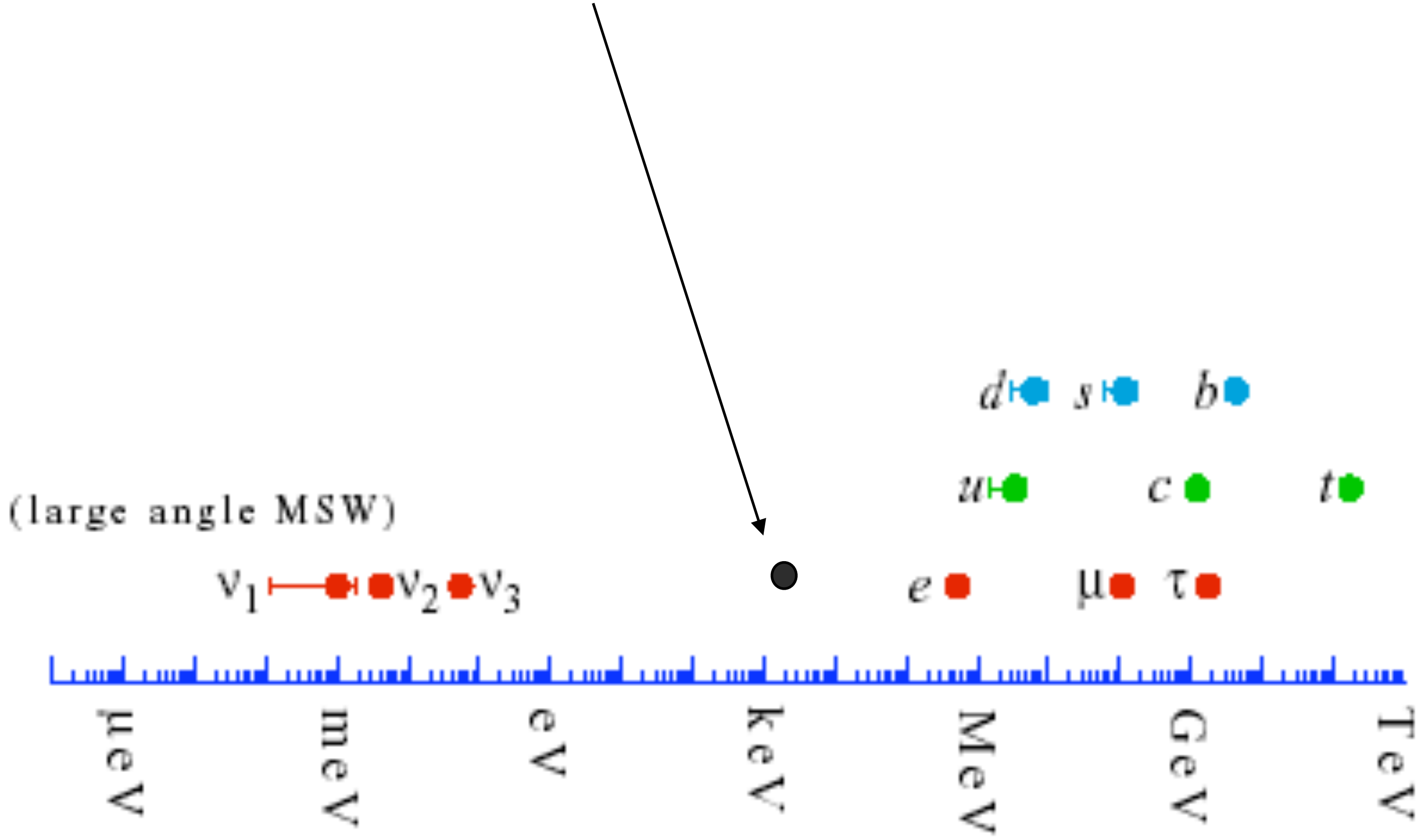
Yukawa coupling

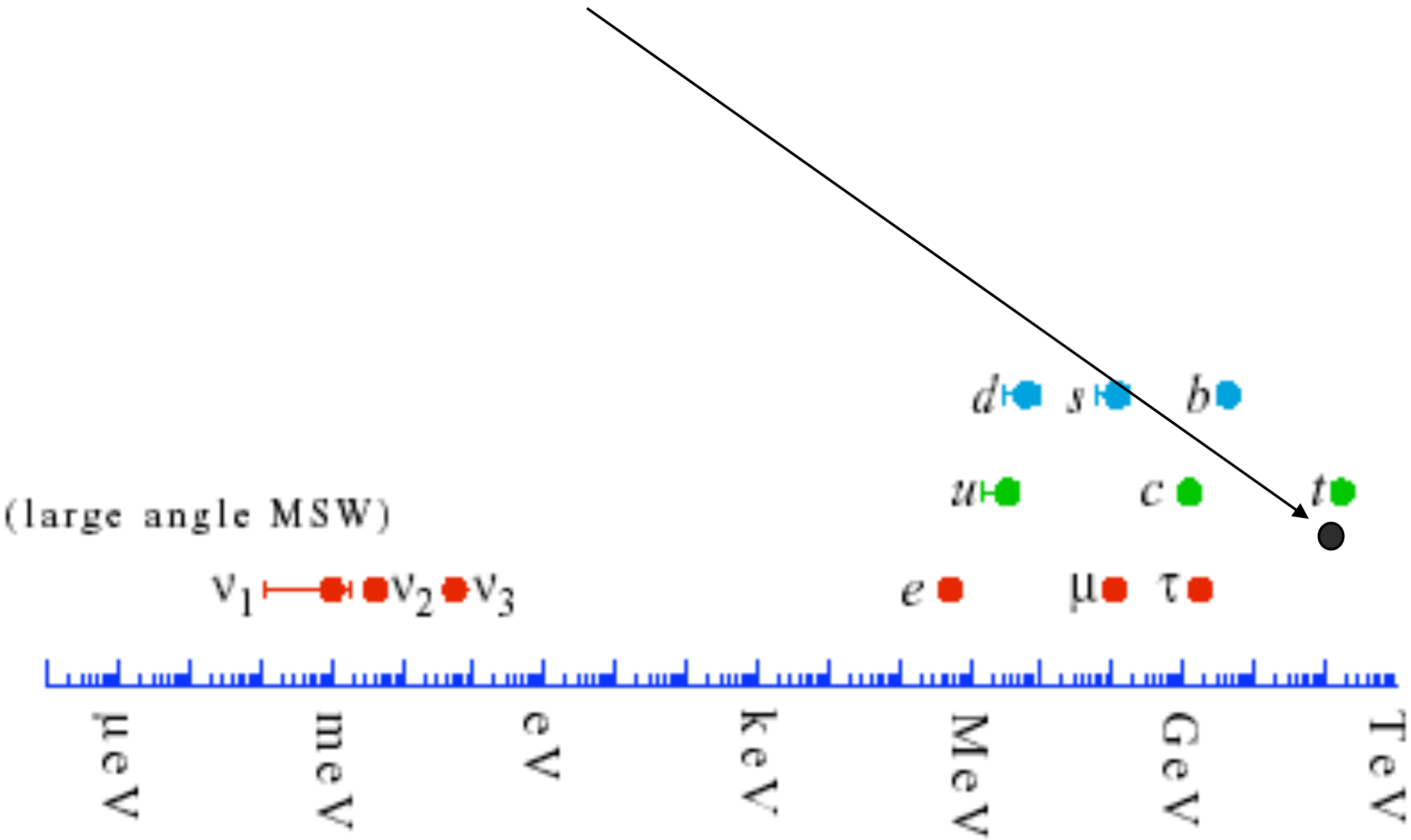
fermion singlets Ψ = "right-handed" neutrino

DARK FLAVOURS ?



.... they can be fermions

DARK FLAVOURS ?



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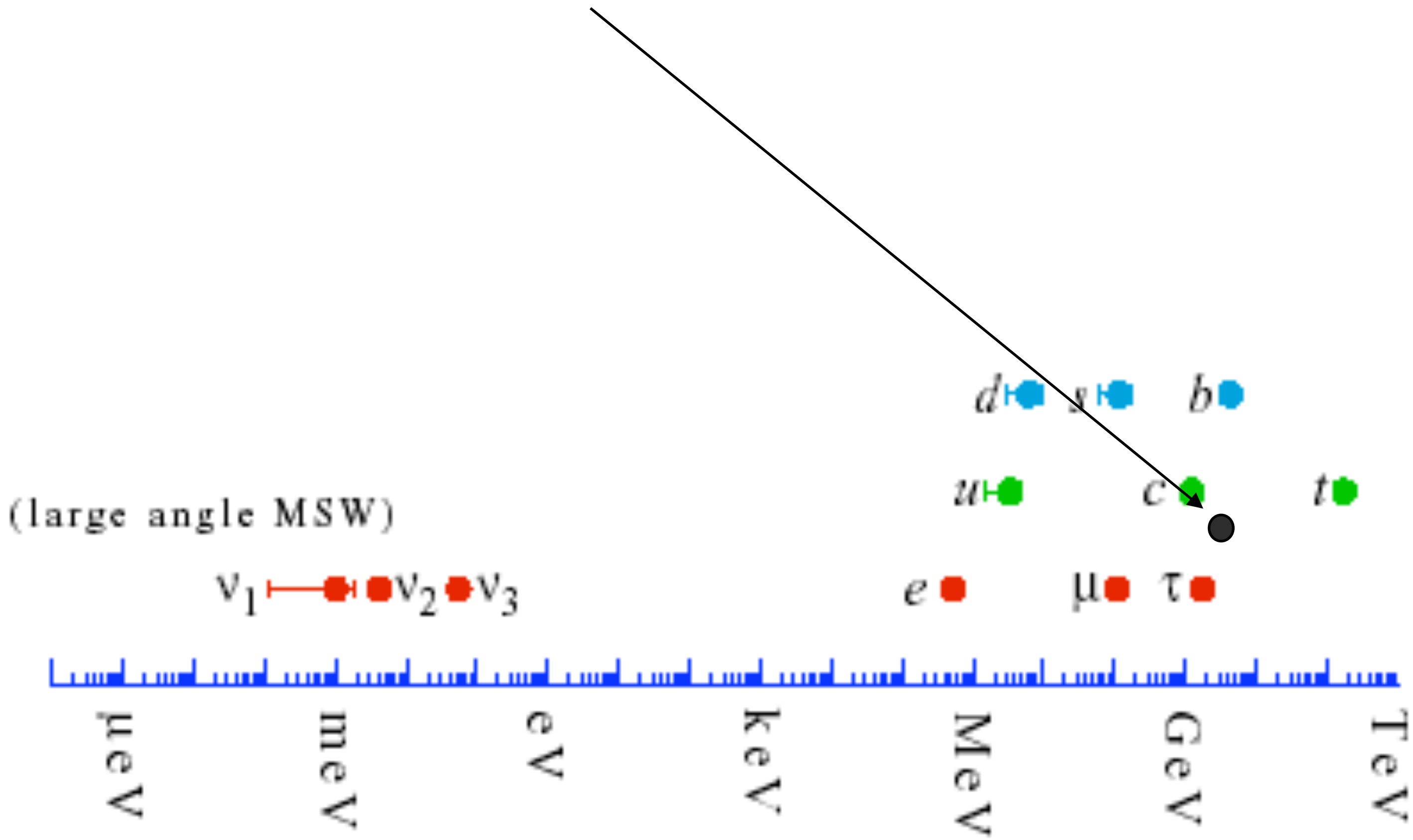

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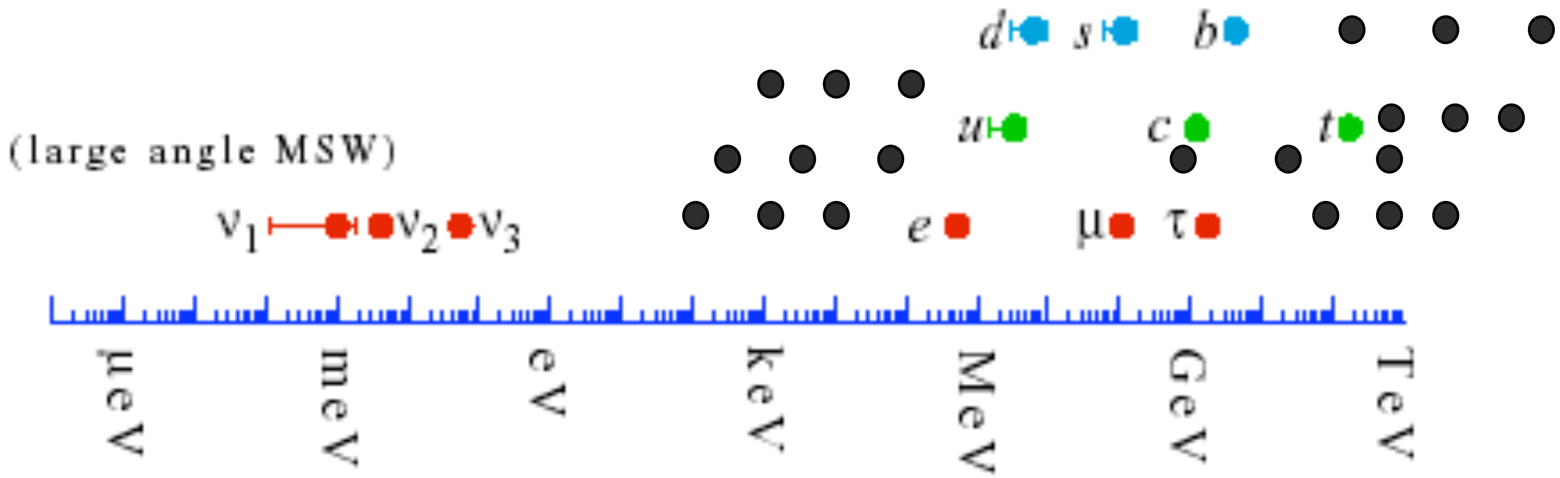

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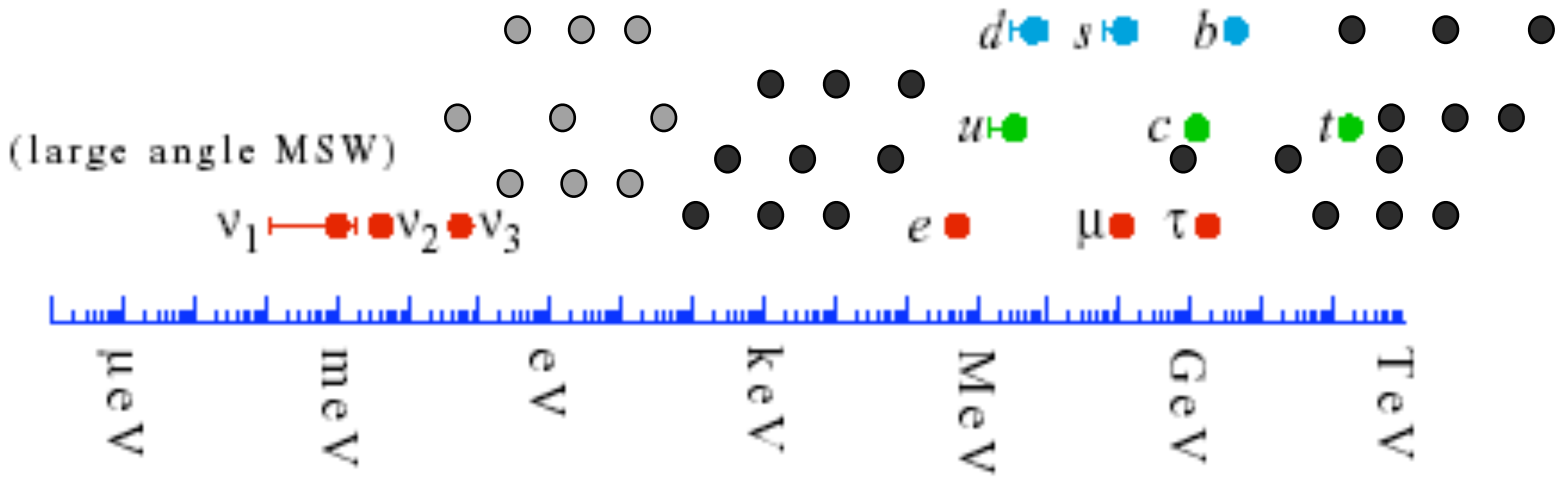


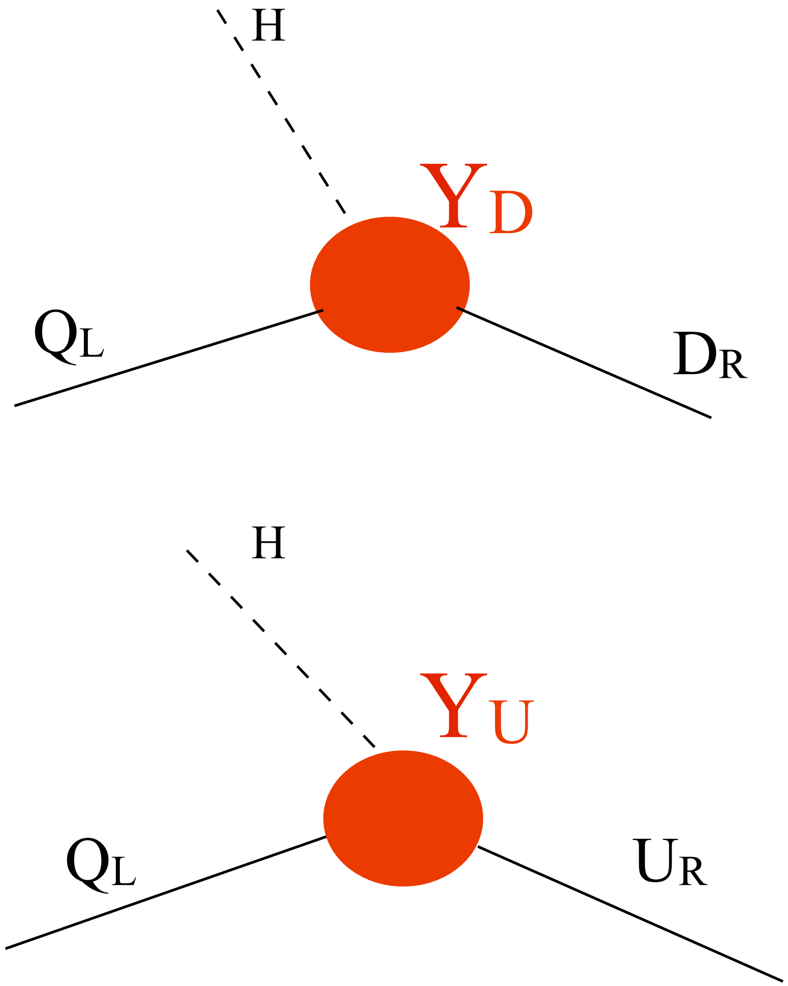
For the rest of the talk:

3 light families of quarks and leptons

- Dynamical Yukawas

Yukawa couplings are the source of flavour in the SM

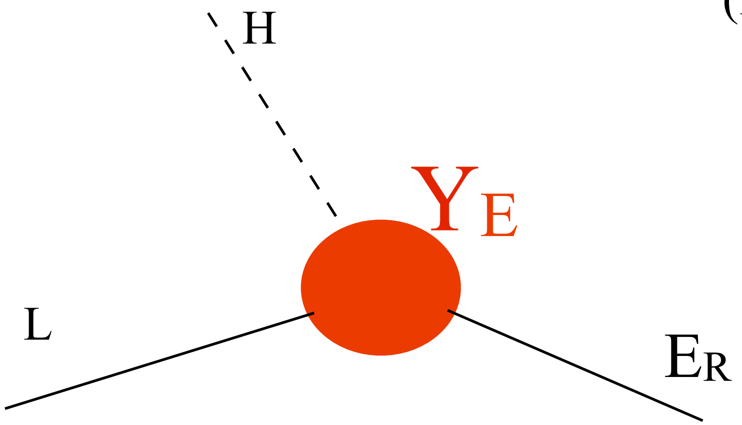
$$\delta\mathcal{L}_m = Q\, {\color{red}Y_d}\, H\, d \;+ Q\, {\color{red}Y_u}\, \tilde{H}\, u + \text{h.c.}$$

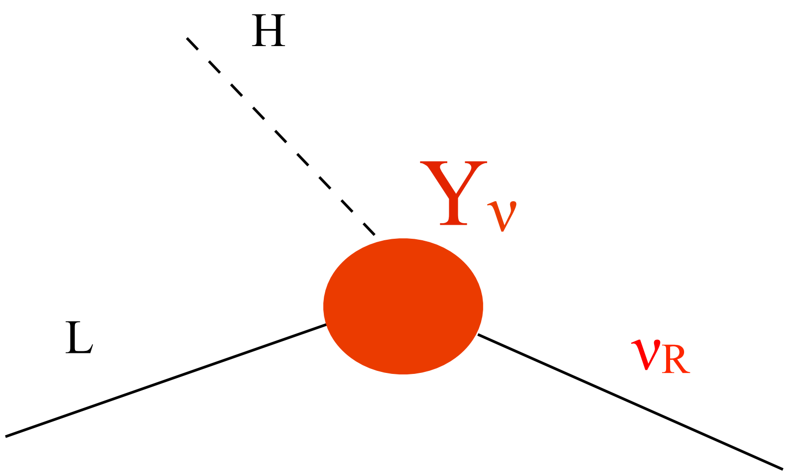


Yukawa couplings are the source of flavour in the νSM

$$\delta\mathcal{L}_m = Y_\nu \, \overline{L} \, H \, \nu_R + h.c. + M \, \overline{\nu_R{}^C} \, \nu_R$$

(i.e. Seesaw type-I)





May they correspond to dynamical fields (e.g. vev of fields that carry flavor) ?

Many attempts: discrete symmetries, continuous symmetries…

Instead of inventing an ad-hoc symmetry group,

why not use the continuous flavour group

suggested by the SM itself?

We have realized that the different pattern for

quarks versus leptons

may be a simple consequence of the

continuous flavour group of the SM (+ seesaw)

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)
2013
(Alonso, Gavela, Isidori, Maiani)

We have realized that the different pattern for

quarks versus leptons

may be a simple consequence of the

continuous flavour group of the SM (+ seesaw)

Our guideline is to use:

- maximal symmetry
- minimal field content

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

2013

(Alonso, Gavela, Isidori, Maiani)

Global flavour symmetry of the SM

* QCD has a global -chiral- symmetry in the limit of massless quarks.
  For n generations:

$$\mathcal{L}_{QCD}^{\text{fermions}} = \bar{\Psi}(i\not{D} - m)\Psi \to \bar{\Psi} i\not{D}\Psi = \overline{\Psi_L} i\not{D}\Psi_L + \overline{\Psi_R} i\not{D}\Psi_R$$

$$SU(n)_L \times SU(n)_R \times U(1)'s$$

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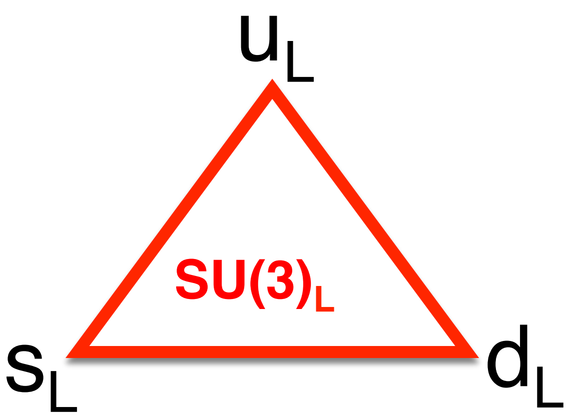
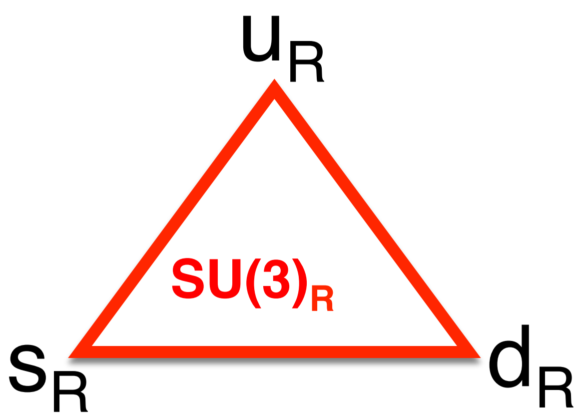
$$SU(n)_L \times SU(n)_R \times U(1)'s$$

e.g., for n=3 : u, d, s . The massless QCD Lag. is invariant interchanging:



or



Global flavour symmetry of the SM

* QCD has a global -chiral- symmetry in the limit of massless quarks. For n generations:

$$\mathcal{L}_{QCD}^{\text{fermions}} = \bar{\Psi}(i\not{D} - m)\Psi \;\rightarrow\; \bar{\Psi} i\not{D}\Psi \;=\; \overline{\Psi_L} i\not{D}\Psi_L + \overline{\Psi_R} i\not{D}\Psi_R$$

$$SU(n)_L \times SU(n)_R \times U(1)'s$$

* In the SM, fermion masses and mixings result from Yukawa couplings. For massless quarks, the SM has a global flavour symmetry:

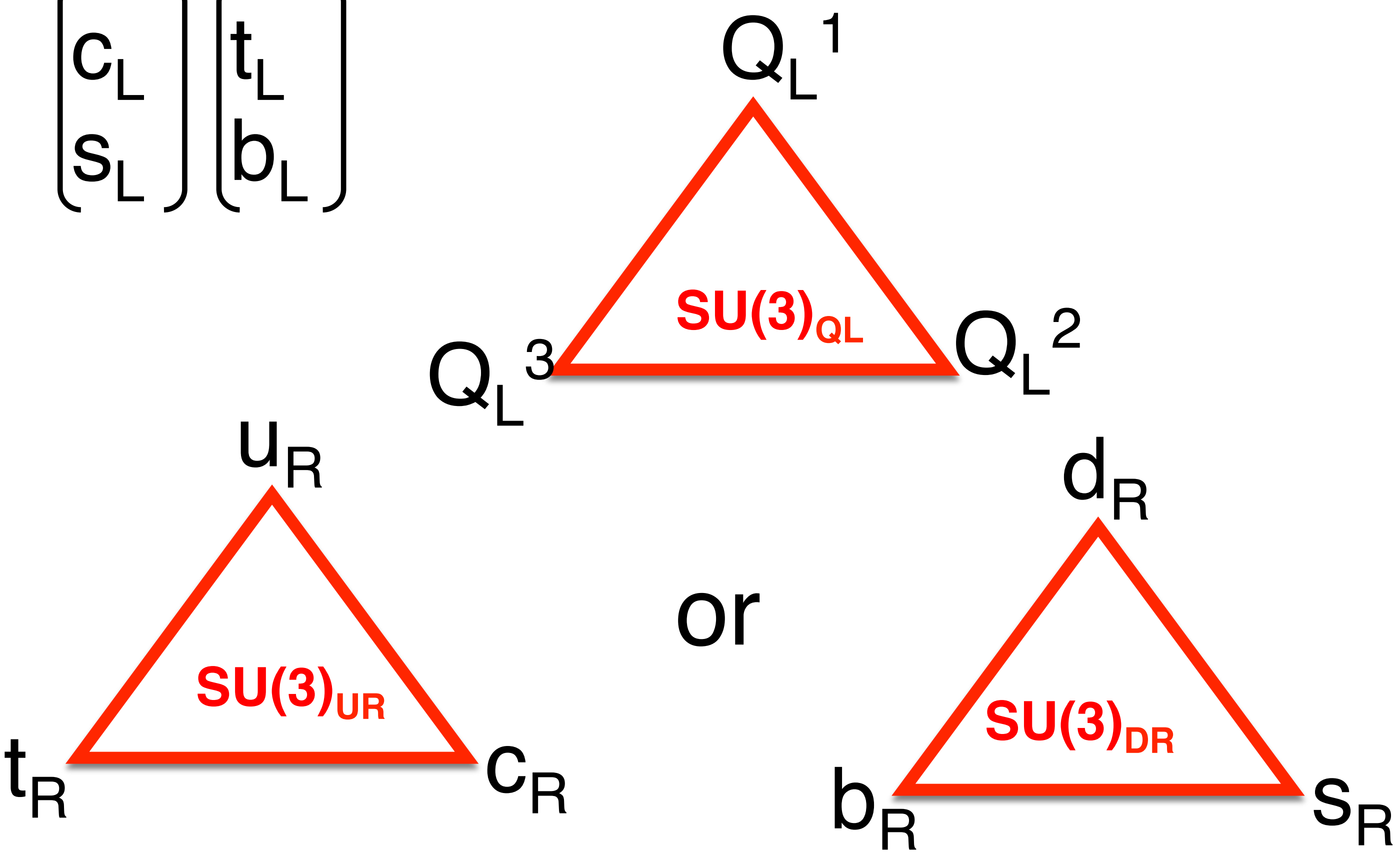
Quarks

$$\mathcal{L}_{SM}^{\text{fermions}} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi}\not{D}\psi \qquad G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

[Georgi, Chivukula, 1987]

There are n=3 quark families. **With null Yukawa couplings,**
the SM Lag. is invariant under $SU(3)_{QL} \times SU(3)_{UR} \times SU(3)_{DR}$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$



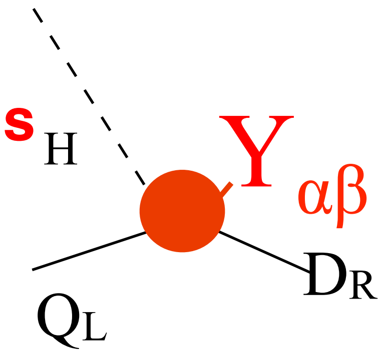
This continuous global symmetry of the SM

$$G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

is phenomenologically very successful and

at the basis of *Minimal Flavour Violation*

in which the Yukawa couplings are only **spurions**



D'Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise

This continuous global symmetry of the SM

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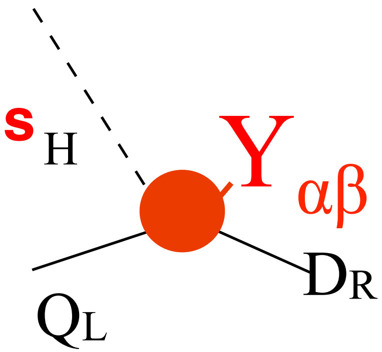
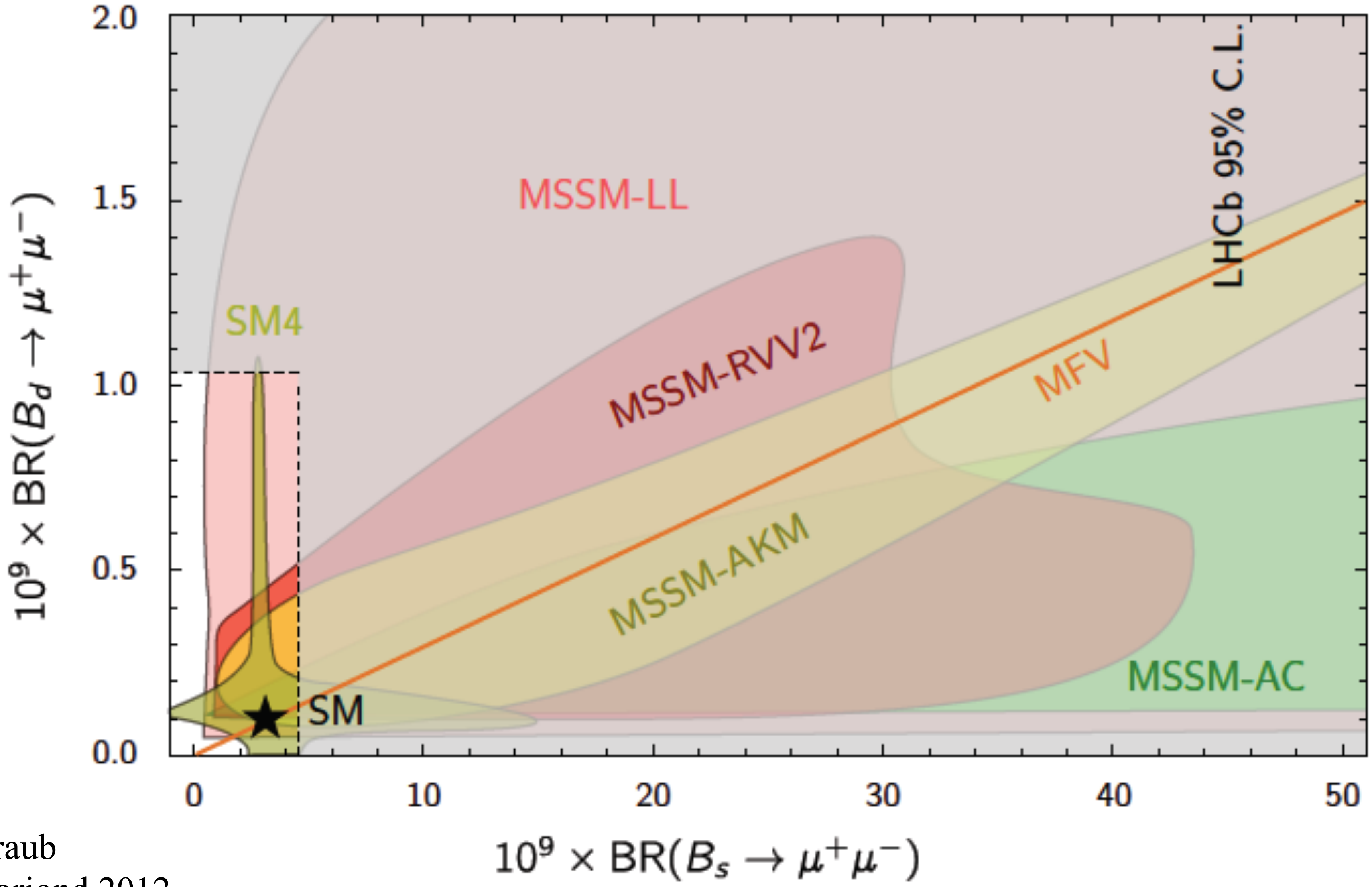


$$\frac{Y_{\alpha\beta}^{+}\, Y_{\delta\gamma}}{\Lambda_f^2}\, \overline{Q}_\alpha \gamma_\mu Q_\beta\, \overline{Q}_\gamma \gamma^\mu Q_\delta$$

D'Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise

AFTER LHCb 2012



Straub
Moriond 2012

One step further :

dynamical Ys

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin, 2011-2013)

(Alonso, Gavela, Isidori, Maiani, 2013)

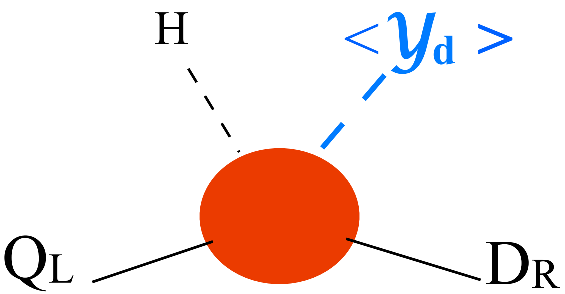
Quarks

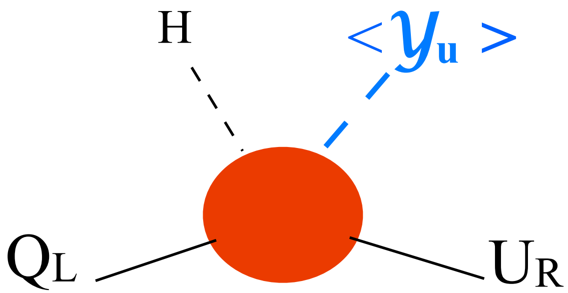
For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{<\mathcal{Y}>}{\Lambda_f}$$

quarks:





Anselm+Berezhiani 96; Berezhiani+Rossi 01... Alonso+Gavela+Merlo+Rigolin 11...

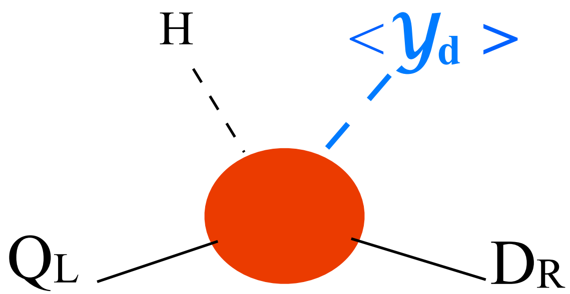
$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$ ....

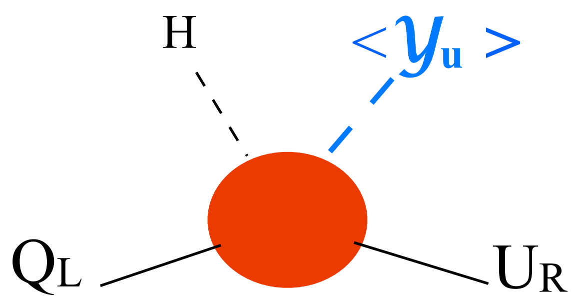
For this talk:

each Y_{SM} -- > one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{<\mathcal{Y}>}{\Lambda_f}$$

quarks:





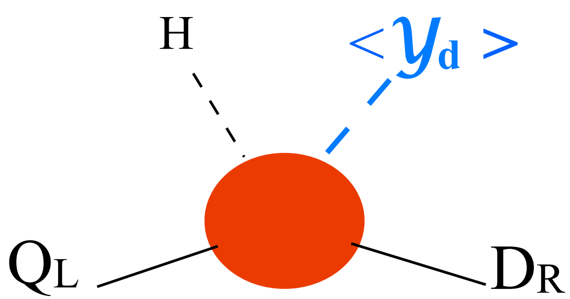
$\mathcal{Y}_d \sim (3,1,\overline{3})$ "bifundamentals" $\mathcal{Y}_u \sim (3,\overline{3},1)$

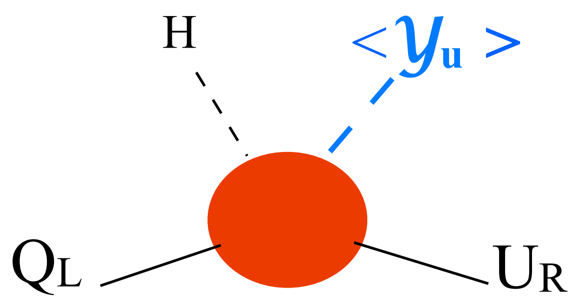
$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \; ....$$

$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \; \ldots.$$

$\mathcal{Y}_d \sim (3,1,\bar{3})$ $\qquad$ $\mathcal{Y}_u \sim (3,\bar{3},1)$

That is, two dynamical scalars





$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \; ....$$

$\mathcal{Y}_d \sim (3,1,\bar{3})$ $\mathcal{Y}_u \sim (3,\bar{3},1)$

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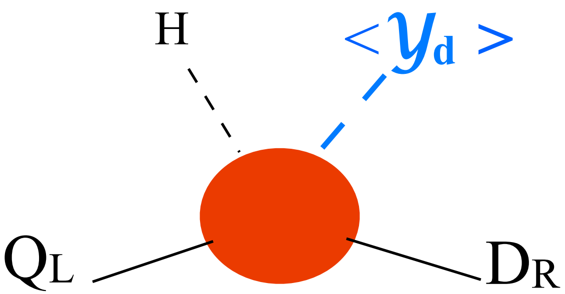
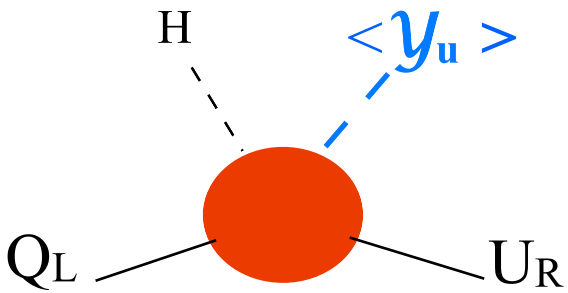


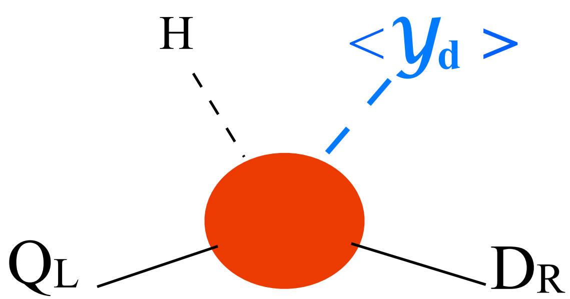


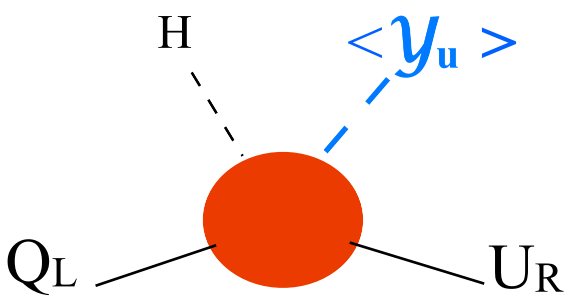
¿$V(\mathcal{Y}_d, \mathcal{Y}_u)$?

$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$ ....

$\mathcal{Y}_d \sim (3,1,\bar{3})$ $\mathcal{Y}_u \sim (3,\bar{3},1)$

That is, two dynamical scalars





*** Does the minimum of the scalar potential justify the observed masses and mixings?**

$$\mathcal{Y}_d \sim (3, \bar{3}, 1) \qquad \mathcal{Y}_u \sim (3, 1, \bar{3})$$

$$\boxed{\frac{<\mathcal{Y}_d>}{\Lambda_f} = Y_D = V_{CKM} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}}, \quad \boxed{\frac{<\mathcal{Y}_u>}{\Lambda_f} = Y_U = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}}$$

$$V(\mathcal{Y}_d,\ \mathcal{Y}_u)$$

* Invariant under the SM gauge symmetry

* Invariant under its global flavour symmetry $G_{flavour}$

$$G_{flavour} = U(3)_{QL} \times U(3)_{UR} \times U(3)_{DR}$$

The basis of the game is to find the minima of the invariants that you can construct out of Yukawa couplings

L. Michel+Radicati 70, Cabibbo+Maiani71 for the spectrum of masses

List of possible invariants for quarks: Hanani, Jenkins, Manohar 2010

The basis of the game is to find the minima of the invariants that you can construct out of Yukawa couplings

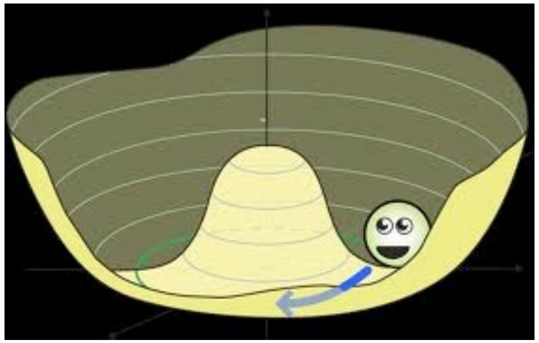
L. Michel+Radicati 70, Cabibbo+Maiani71 for the spectrum of masses

Cabibbo's dream

Flavour Symmetry Breaking

Spontaneous breaking of flavour symmetry dangerous

To prevent Goldstone Bosons the symmetry can be Gauged



[Grinstein, Redi, Villadoro
Guadagnoli, Mohapatra, Sung
Feldman]

$$\mathbf{V}(\mathcal{Y}_d, \mathcal{Y}_u)$$

* Invariant under the SM gauge symmetry

* Invariant under its global flavour symmetry $G_{flavour}$

$$G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

There are as many independent invariants I as physical variables

$$\mathbf{V}(\mathcal{Y}_d, \mathcal{Y}_u) = \mathbf{V}(\, \mathbf{I}(\mathcal{Y}_d, \mathcal{Y}_u))$$

Minimization

a variational principle fixes the vevs of Fields

$$\delta V = 0$$

$$\sum_j \frac{\partial I_j}{\partial y_i} \frac{\partial V}{\partial I_j} \equiv J_{ij} \frac{\partial V}{\partial I_j} = 0 \,,$$

masses, mixing angles etc.

This is an homogenous linear equation;

if the rank of the Jacobian $J_{ij} = \partial I_j / \partial y_i,$ is:

Maximum:

then the only solution is: $\frac{\partial V}{\partial I_j} = 0 \,,$

Less than Maximum:

then the number of equations reduces to a number equal to the rank

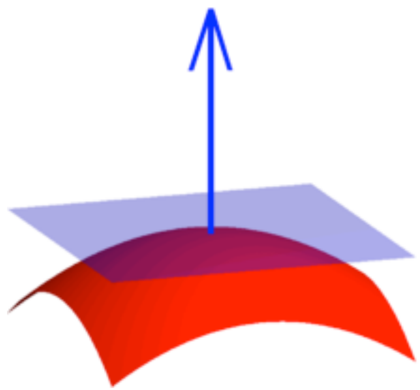
Boundaries

for a reduced rank of the Jacobian,

$$\det(J) = 0$$

there exists (at least) a direction δy_i for which a variation of the field variables does not vary the invariants

$$\delta I_j = \sum_i \frac{\partial I_j}{\partial y_i} \delta y_i = 0$$



that is a Boundary of the *I-manifold*

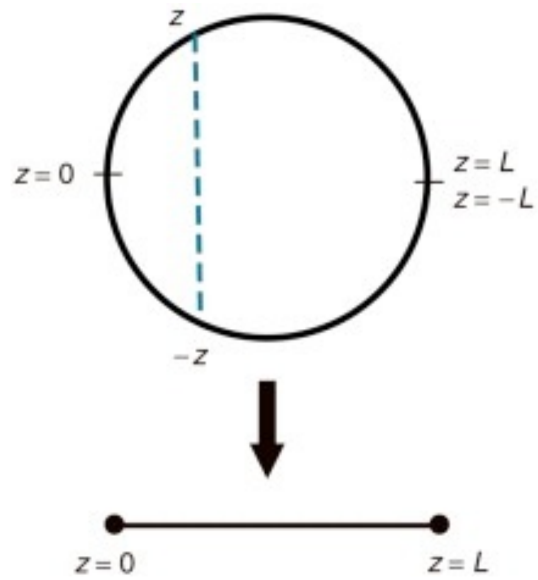
[Cabibbo, Maiani,1969]

$\boxed{\text{Boundaries Exhibit Unbroken Symmetry}}$ [Michel, Radicati, 1969]

(maximal subgroups)

Boundaries Exhibit Unbroken Symmetry

Extra-dimensions Example



The smallest boundaries are extremal points of any function

[Michel, Radicati, 1969]

ancestors of dynamical Yukawas decades ago (only to explain the mass spectrum) in

Cabibbo

Michel,+Radicati, Cabibbo+Maiani ...

C. D. Froggat, H. B. Nielsen

Well before the electroweak SM: masses

Jacobian Analysis: [40 years ago…]

Breaking of $SU(3) \times SU(3)$ [Cabibbo, Maiani]

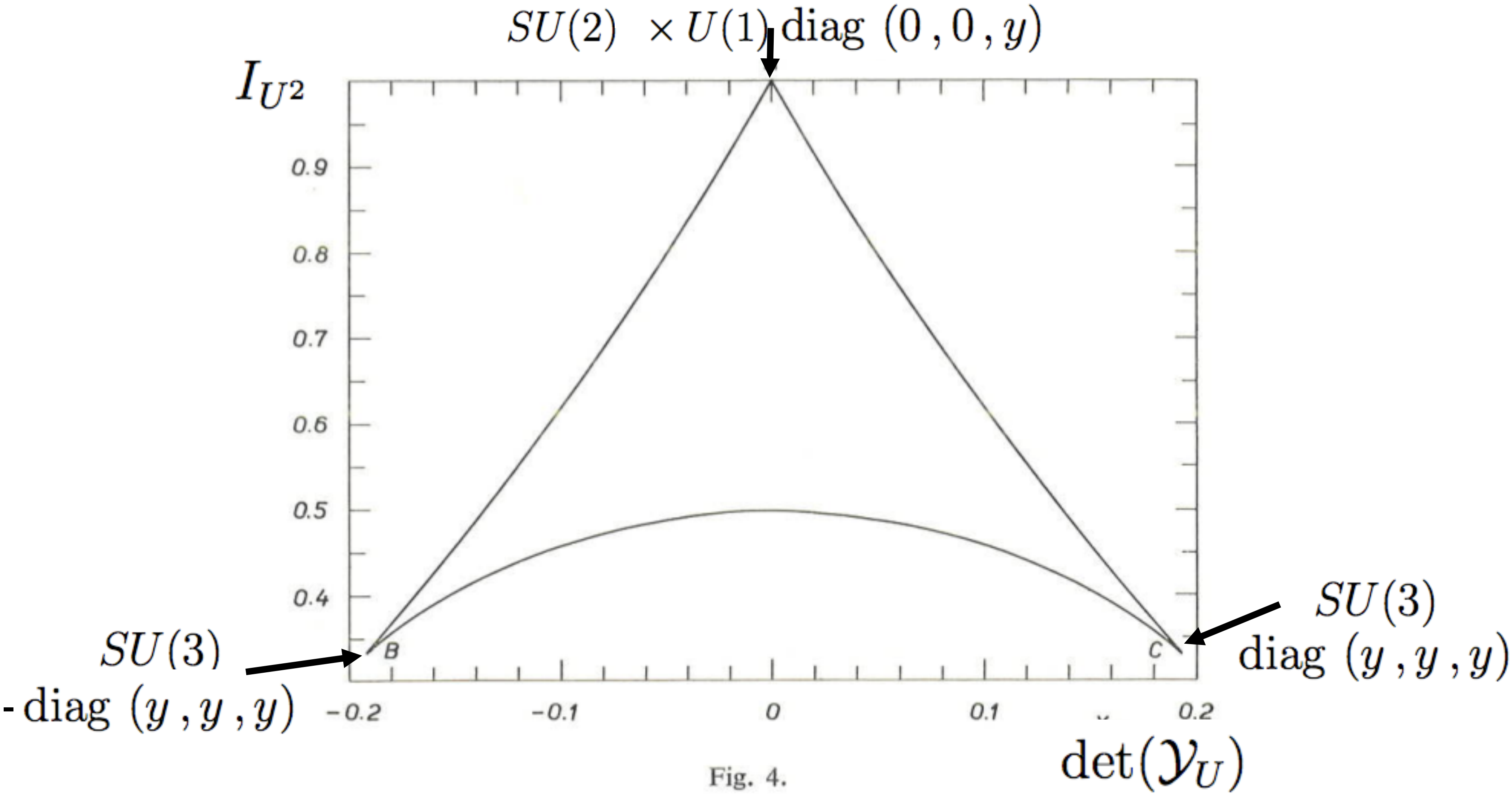


Fig. 4.

quark case

Bi-fundamental Flavour Fields

For quarks: 10 independent invariants (because 6 masses+ 3 angles + 1 phase) that we may choose as

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\right]\,, \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D\mathcal{Y}_D^\dagger\right]\,,$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^2\right]\,, \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right]\,,$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^3\right]\,, \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^3\right]\,,$$

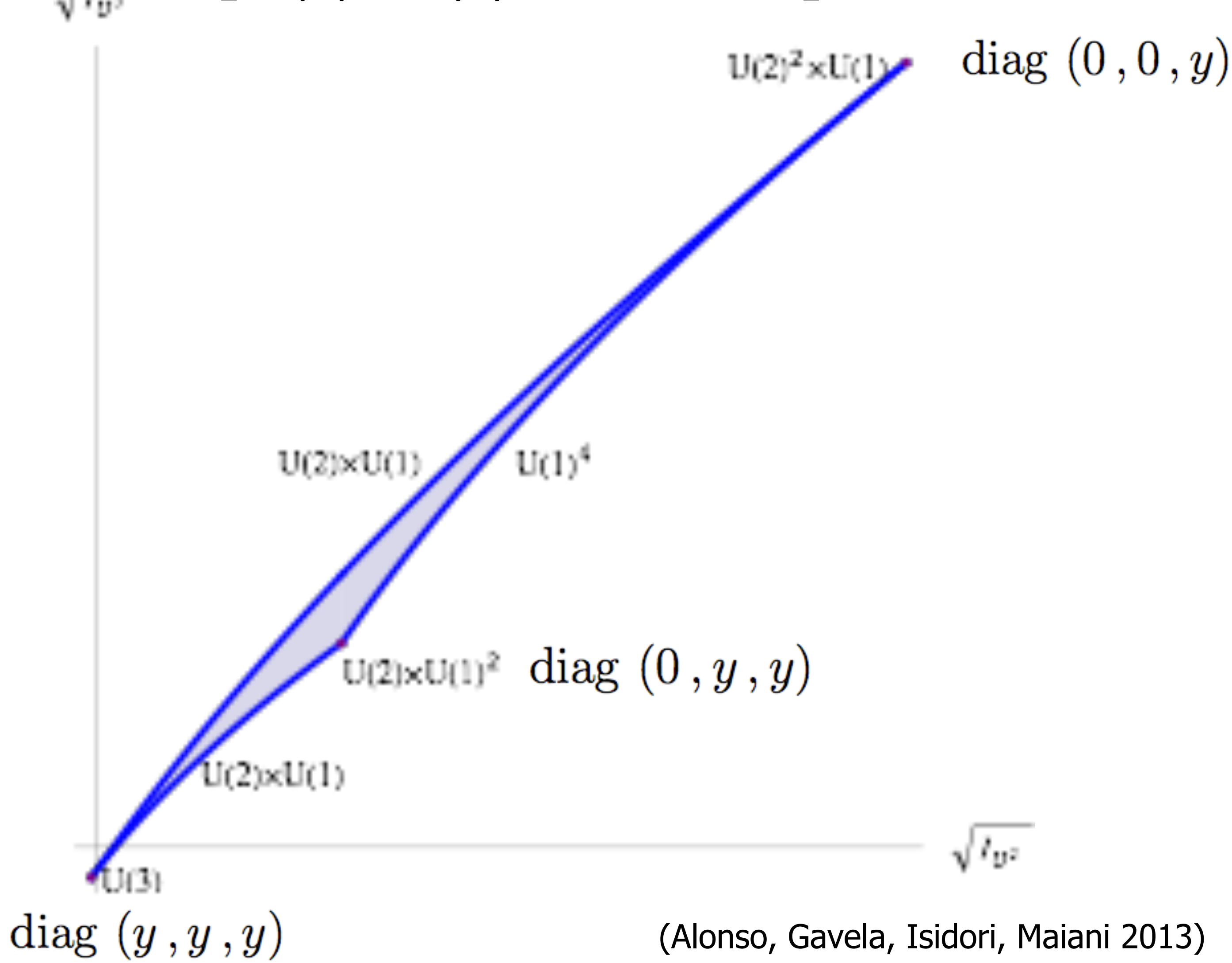
$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right]\,, \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right]\,,$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right]\,, \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right]\,.$$

[Feldmann, Jung, Mannel; Jenkins, Manohar]

Jacobian Analysis: Masses

[U(3)xU(3) broken to]



(Alonso, Gavela, Isidori, Maiani 2013)

quark case

Bi-fundamental Flavour Fields

$$\mathrm{Tr}[\mathbf{y}_U \mathbf{y}_U] = \sum y_\alpha^2$$

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right], \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right],$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right], \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right], \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right],$$

only masses

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right], \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right], \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right].$$

masses and mixings

[Feldmann, Jung, Mannel; Jenkins, Manohar
Alonso, Gavela, Isidori, Maiani 2013]

Jacobian Analysis: Mixing

$$\det\left(J_{UD}\right) = \left(y_u^2 - y_t^2\right)\left(y_t^2 - y_c^2\right)\left(y_c^2 - y_u^2\right)$$
$$\left(y_d^2 - y_b^2\right)\left(y_b^2 - y_s^2\right)\left(y_s^2 - y_d^2\right)$$
$$\times\ |V_{ud}||V_{us}||V_{cd}||V_{cs}|$$

the rank is reduced the most for:

$\boxed{V_{CKM} = \text{PERMUTATION}}$

no mixing: reordering of states

(Alonso, Gavela, Isidori, Maiani 2013)

Quark Natural Flavour Pattern

Summarizing, a possible and natural breaking pattern arises:

$$G_{\text{flavour (quarks)}} \quad : \quad U(3)^3 \to U(2)^3 \times U(1)$$

giving a hierarchical mass spectrum **without mixing**

$$< y_D > = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad < y_U > = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

a good approximation to the observed Yukawas to order $(\lambda_C)^2$

And what happens for leptons ?

Any difference with Majorana neutrinos?

Leptons

Global flavour symmetry of the SM + seesaw

* In the SM, for quarks the maximal global symmetry in the limit of massless quarks was:

$$\mathcal{L}^{\text{quarks}}_{\text{SM}} = i \sum_{\psi = Q_L}^{D_R} \overline{\psi} \not{D} \psi \qquad G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

* In SM +type I seesaw, for leptons:

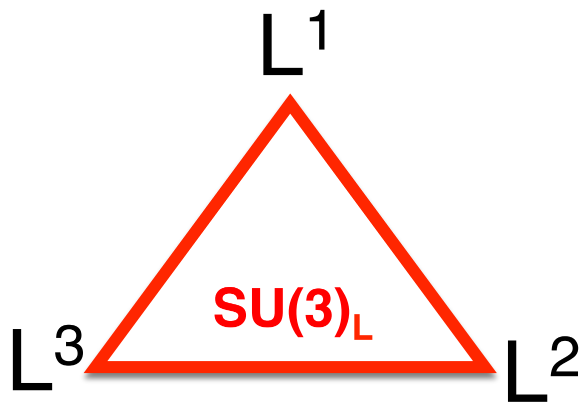
$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\not{\partial} N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2} \overline{N_R} M N_R{}^c + h.c. \right]$$

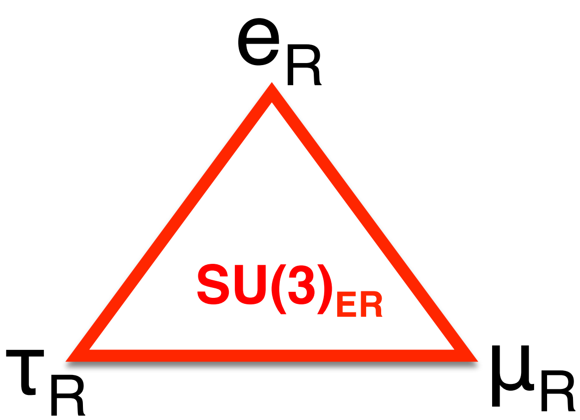
the maximal leptonic global symmetry in the limit of massless light leptons is $U(n)_L \times U(n)_{E_R} \times O(n)_{N_R}$

-> degenerate heavy neutrinos

There are n=3 lepton families. **With null Yukawa couplings,** the maximal symmetry of the Lag. is $SU(3)_L \times SU(3)_{ER} \times O(3)_{NR}$

$$L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$





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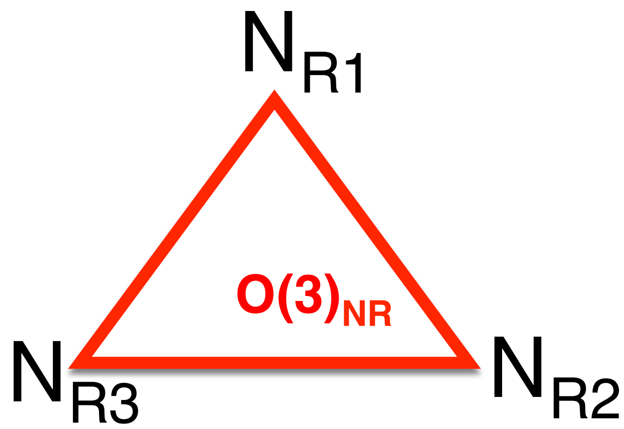


Ilustration: 2 families (Casas-Ibarra parametrization)

for 2 generations, the mixing terms in $\mathbf{V}(\mathcal{Y}_E, \mathcal{Y}_\nu)$ is :

Leptons

$$Tr(\mathcal{Y}_E\, \mathcal{Y}_E^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+) \propto$$

$$(m_\mu^2 - m_e^2)\left[\cos 2\omega (m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2 \sin 2\omega \sqrt{m_{\nu_2} m_{\nu_1}}\, sin2\alpha \sin 2\theta\right]$$

Casas-Ibarra variable in R

where $U_{PMNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$

Quarks

$$Tr(\mathcal{Y}_u\, \mathcal{Y}_u^+\, \mathcal{Y}_d\, \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_{\mathbf{E}}, \mathcal{Y}_{\mathbf{\nu}})$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+) \propto$$

$$(m_\mu^2 - m_e^2)\left[\cos 2\omega (m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2 \sin 2\omega \sqrt{m_{\nu_2} m_{\nu_1}}\, sin2\alpha \sin 2\theta\right]$$

This mixing term unphysical if either “up” or “down” fermions degenerate

Mixing physical even with degenerate neutrino masses, if Majorana phase non-trivial

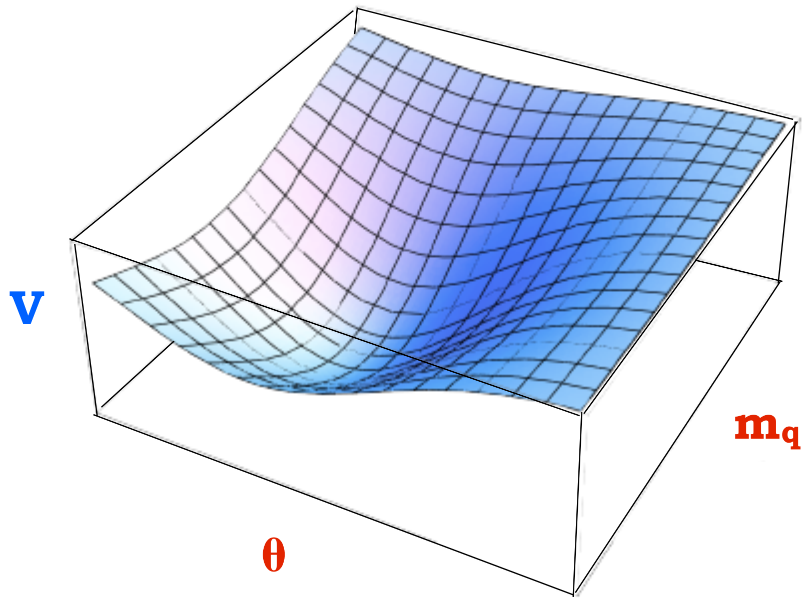
Quarks

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

e.g. for the case of two families:

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

at the minimum: $\boxed{(m_c^2 - m_u^2)(m_s^2 - m_d^2)\sin 2\theta = 0}$



-> NO MIXING

same conclusion for 3 families

Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_\mathbf{E}, \mathcal{Y}_\mathbf{\nu})$ is :

Minimisation (for non trivial $\sin 2\omega$)

$\mathrm{Tr}(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+)$

* $\sin 2\omega \sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0 \longrightarrow \boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$

Maximal Majorana phase

* $\mathrm{tg} 2\theta = \sin 2\alpha \dfrac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}$ **tgh 2ω**

Large angles correlated with degenerate masses

*** For instance for two generations:** $O(2)_{NR}$

e.g. two families

$$m_\nu \sim Y_\nu \frac{v^2}{M} Y_\nu^T = y_1\, y_2\, \frac{v^2}{M} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$U_{PMNS} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}$$

Degenerate neutrino masses

Generically, O(2) allows :

- **one mixing angle maximal**
- **one relative Majorana phase of $\pi/2$**
- **two degenerate light neutrinos**

Now for three generations and considering all possible independent invariants

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$\mathbf{Y}_\nu = \langle \frac{\mathcal{Y}_\nu}{\Lambda_f} \rangle = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R\,, \qquad \mathbf{Y}_E = \langle \frac{\mathcal{Y}_E}{\Lambda_f} \rangle = \mathrm{y}_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1\,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1\,,$$

* $m_{e,\,\mu,\,\tau} = v\, y_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

$$m_\nu = \mathbf{Y} \frac{v^2}{M} \mathbf{Y}^T$$

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$Y_\nu = < \frac{\mathcal{Y}_\nu}{\Lambda_f} > = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R\,, \qquad Y_E = < \frac{\mathcal{Y}_E}{\Lambda_f} > = y_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1\,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1\,,$$

→ * $m_{e,\mu,\tau} = v\, y_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

→ $$U_{PMNS}\, \mathbf{m}_\nu\, U_{PMNS}^T = \frac{v^2}{2M} \mathcal{U}_L\, \mathbf{y}_\nu\, \mathcal{U}_R \mathcal{U}_R^T\, \mathbf{y}_\nu\, \mathcal{U}_L^T\,,$$

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$Y_\nu = \left\langle \frac{\mathcal{Y}_\nu}{\Lambda_f} \right\rangle = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R \,, \qquad Y_E = \left\langle \frac{\mathcal{Y}_E}{\Lambda_f} \right\rangle = y_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1 \,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1 \,,$$

$\Rightarrow$ * $m_{e,\,\mu,\,\tau} = v\, y_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

$\mathcal{U}_R$ is relevant for leptons

$$\Rightarrow \quad U_{PMNS}\, \mathbf{m}_\nu\, U_{PMNS}^T = \frac{v^2}{2M} \mathcal{U}_L\, \mathbf{y}_\nu\, \mathcal{U}_R \mathcal{U}_R^T\, \mathbf{y}_\nu\, \mathcal{U}_L^T \,,$$

Number of Physical parameters = number of Independent Invariants

15 invariants for $G_{\text{flavour (leptons)}} = U(3)_L \times U(3)_{E_R} \times O(3)_{N_R}$

Leptons

$$I_E = \text{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_\nu = \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right],$$

$$I_{E^2} = \text{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right], \qquad I_{\nu^2} = \text{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{E^3} = \text{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^3\right], \qquad I_{\nu^3} = \text{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^3\right],$$

$$I_L = \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

$$I_{L^2} = \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

$$I_{L^3} = \text{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{L^4} = \text{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

U_L and eigenvalues

$$I_R = \text{Tr}\left[\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^2} = \text{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^3} = \text{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right],$$

U_R and eigenvalues

$$I_{LR} = \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_{RL} = \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

New Invariants wrt Quarks

(Alonso, Gavela, Isidori, Maiani 2013)

Number of Physical parameters = number of Independent Invariants

15 invariants for $G_{\text{flavour (leptons)}} = U(3)_L \times U(3)_{E_R} \times O(3)_{N_R}$

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$$I_L = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

$$I_{L^2} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

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$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

U_L and eigenvalues

$$I_R = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)^{\mathrm{T}}\right]$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right],$$

U_R and eigenvalues

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

New Invariants wrt Quarks

(Alonso, Gavela, Isidori, Maiani 2013)

Number of Physical parameters = number of Independent Invariants

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$$I_{E^3} = \text{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^3\right], \qquad I_{\nu^3} = \text{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^3\right],$$

$$I_L = \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

$$I_{L^2} = \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

$$I_{L^3} = \text{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{L^4} = \text{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

UL and eigenvalues

$$\text{Tr}(\mathbf{y}_\nu^2 \mathcal{U}_R \mathcal{U}_R^T \mathbf{y}_\nu^2 \mathcal{U}_R^* \mathcal{U}_R^\dagger)$$

$$I_{R^2} = \text{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^3} = \text{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right],$$

UR and eigenvalues

New Invariants wrt Quarks

$$I_{LR} = \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_{RL} = \text{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

(Alonso, Gavela, Isidori, Maiani 2013)

Jacobian

$$J = \begin{pmatrix} \partial_{\mathbf{y}_E} I_{E^n} & 0 & 0 & \partial_{\mathbf{y}_E} I_{L^n} & \partial_{\mathbf{y}_E} I_{LR} \\ 0 & \partial_{\mathbf{y}_\nu} I_{\nu^n} & \partial_{\mathbf{y}_\nu} I_{R^n} & \partial_{\mathbf{y}_\nu} I_{L^n} & \partial_{\mathbf{y}_\nu} I_{LR} \\ 0 & 0 & \partial_{\mathcal{U}_R} I_{R^n} & 0 & \partial_{\mathcal{U}_R} I_{LR} \\ 0 & 0 & 0 & \partial_{\mathcal{U}_L} I_{L^n} & \partial_{\mathcal{U}_L} I_{LR} \\ 0 & 0 & 0 & 0 & \partial_{\mathcal{U}_L \mathcal{U}_R} I_{LR} \end{pmatrix},$$

$$\mathrm{Diag}(J) \equiv (J_E\,,\, J_\nu\,,\, J_{\mathcal{U}_R}\,,\, J_{\mathcal{U}_L}\,,\, J_{LR})$$

Jacobian Analysis: Mixing

$$\det\left(J_{\mathcal{U}_L}\right) = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)$$
$$\left(y_e^2 - y_\mu^2\right)\left(y_\mu^2 - y_\tau^2\right)\left(y_\tau^2 - y_e^2\right)|\mathcal{U}_L^{e1}||\mathcal{U}_L^{e2}||\mathcal{U}_L^{\mu 1}||\mathcal{U}_L^{\mu 2}|\,.$$

same as for V_{CKM}

O(3) vs U(3)

$$\det J_{\mathcal{U}_R} = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)^3\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)^3\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)^3$$
$$\times\,|\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{11}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{22}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{12}|$$

the rank is reduced the most for $\boxed{\mathcal{U}_R\mathcal{U}_R^T \textit{ being a permutation}}$

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T\,,$$

...in fact it allows maximal mixing:

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$- \frac{v^2}{M} \begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2} y_{\nu_3} \\ 0 & y_{\nu_2} y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS} \begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T \,,$$

...in fact it allows maximal mixing:

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix} \,, \quad m_{\nu 2} = m_{\nu 3} = \frac{v^2}{M} y_{\nu_2} y_{\nu_3} \,, \quad m_{\nu_1} = \frac{v^2}{M} y_{\nu_1}^2 \,.$$

and maximal Majorana phase

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$
-\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T\,,
$$

...in fact it leads to one maximal mixing angle:

$\theta_{23} = 45°$;

Majorana Phase Pattern (-i,-i,1)

& at this level mass degeneracy: $m_{\nu 2} = m_{\nu 3}$

related to the O(2) substructure

[Alonso, Gavela, D. Hernández, L. Merlo;

[Alonso, Gavela, D. Hernández, L. Merlo, S. Rigolin]

If the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS} \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_{PMNS}^T = -\frac{y_\nu v^2}{M} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

This very simple structure is signaled by the extrema of the potential and

has eigenvalues (1,1,-1) → 3 degenerate light neutrinos + a maximal Majorana phase

and is diagonalized by a maximal θ = 45°

What is the symmetry in this boundary?

a very intriguing

$$\boxed{U(1)_{diag}}$$

With hierarchical charged leptons, always (either two or three neutrinos degenerate), the symmetry pattern at this stage is

$SU(3)_L \times SU(3)_E \times O(3)_N \rightarrow SU(2)_E \times U(1)_{diag}$

Generalization to any seesaw model

the effective Weinberg Operator

$$\bar{\ell}_L \tilde{H} \frac{C^{d=5}}{M} \tilde{H}^T \ell_L^c$$

shall have a flavour structure that breaks $U(3)_L$ to $O(3)$

$$\frac{v^2 \; C^{d=5}}{M} = - m_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

then the results apply to any seesaw model

First conclusion:

* at the same order in which the minimum of the potential

  does NOT allow quark mixing,

  it allows:

    - hierarchical charged leptons

    - quasi-degenerate neutrino masses

    - one angle of ~45 degrees

    - one maximal Majorana phase

The conclusions hold irrespective of the

renormalizability of the potential,

and are thus stable under radiative corrections

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix}U^T_{PMNS} = - \frac{y_\nu v^2}{M}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$: U_{PMNS} \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_{PMNS}^T = - \frac{y_\nu v^2}{M} \begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \ \ , \ \ \theta_{13} \simeq \epsilon$$

Majorana phases: 1 maximal, 1 large

Dirac-like CP phase generically large

~ degenerate spectrum

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS} \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U^T_{PMNS} = - \frac{y_\nu v^2}{M} \begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \ , \ \theta_{13} \simeq \epsilon$$

only this vanishes with the perturbations

Majorana phases: 1 maximal, 1 large

Dirac-like CP phase generically large

this angle does not vanish with vanishing perturbations

~ degenerate spectrum

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U^T_{PMNS} = - \frac{y_\nu v^2}{M}\begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \quad , \quad \theta_{13} \simeq \epsilon$$

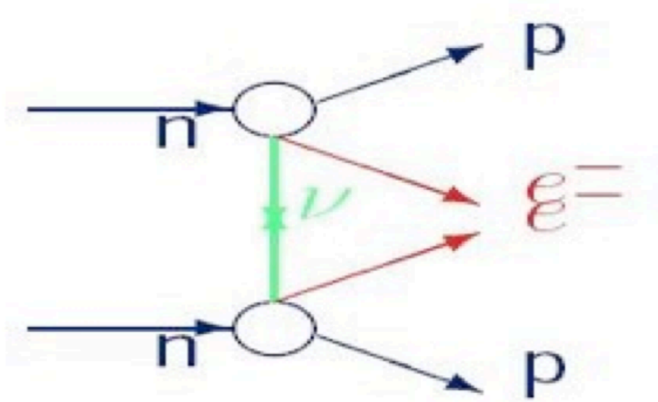
Majorana phases: 1 maximal, 1 large

Dirac-like CP phase generically large

~ degenerate spectrum

Neutrinoless double beta decay

Neutrinoless double beta decay, (A, Z) → (A, Z+2) + 2 e, will test the nature of neutrinos. It violates L by 2 units.



The half-life time depends on neutrino

$$\left[T^{1/2}_{0\nu}(0^+ \to 0^+)\right]^{-1} \propto |M_F - g_A^2 M_{GT}|^2 \, |<m>|^2$$

$$\boxed{|<m>| \equiv \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}} \right| ,}$$

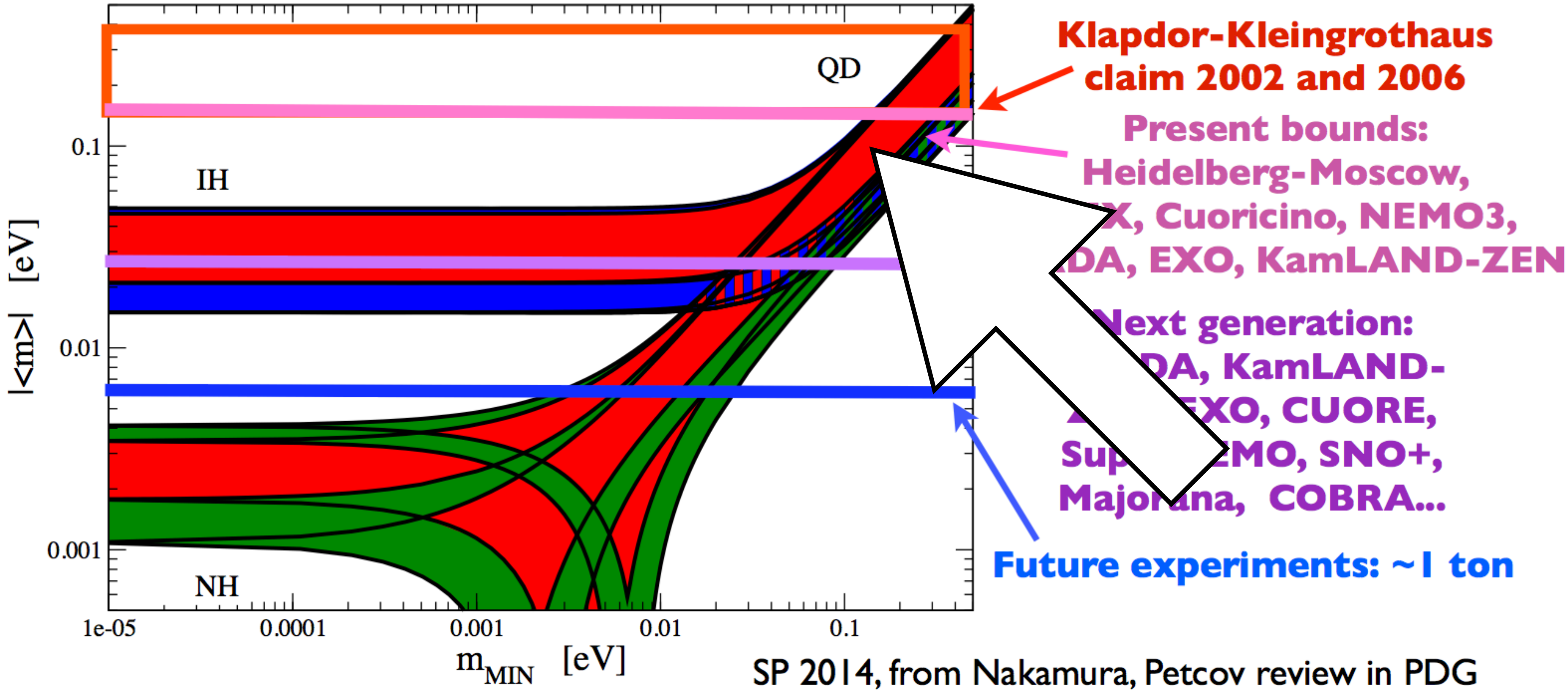
Mixing angles (mostly known)

CPV phases (unknown)

courtesy S. Pascoli

accommodation of angles requires degenerate spectrum

at reach in future neutrinoless double β exps.!

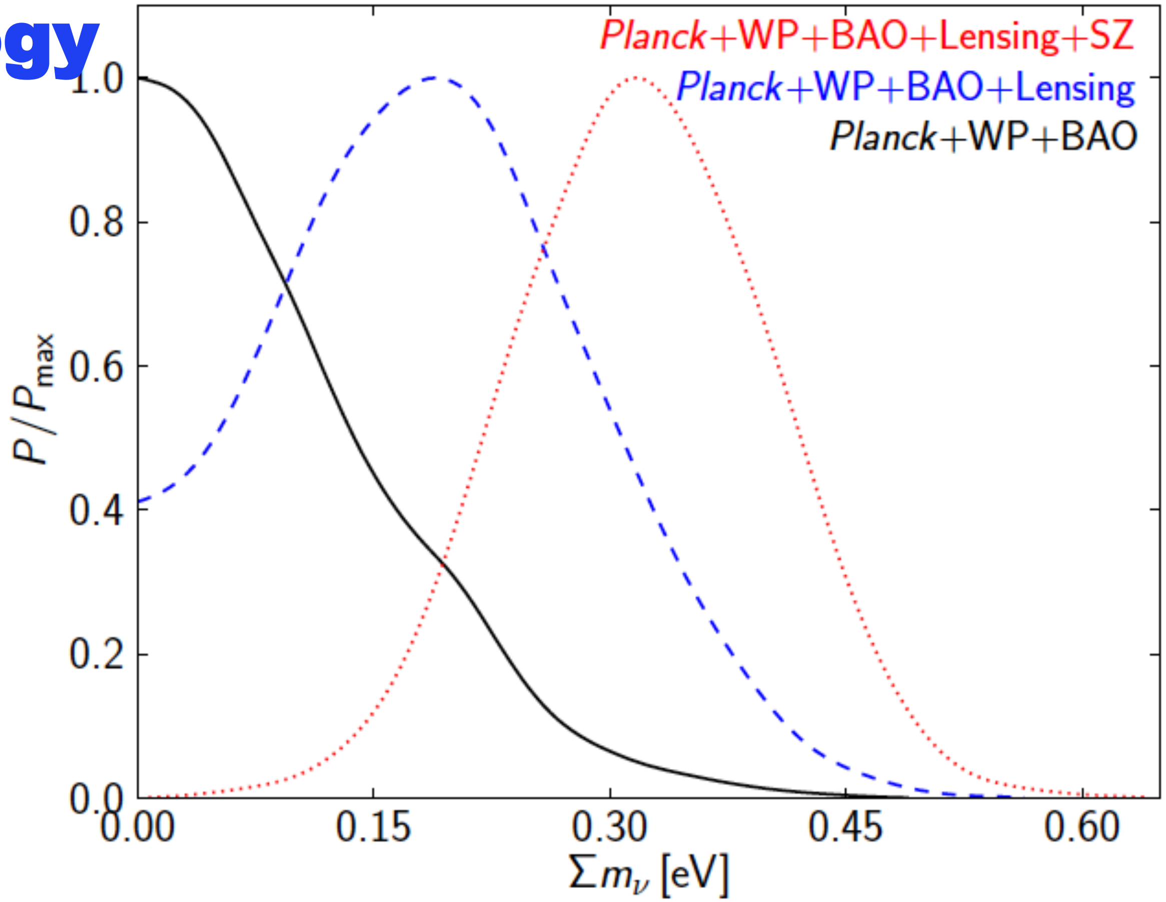


SP 2014, from Nakamura, Petcov review in PDG

Wide experimental program for the future: **a positive signal would indicate that L is violated!**

courtesy S. Pascoli

Cosmology



Battye and Moss

$$\sum m_\nu = (0.320 \pm 0.081)\,\mathrm{eV}$$

FIG. 2: Marginalized likelihoods for $\sum m_\nu$. The datasets are colour coded in the legend, but the solid line is for (I), the dashed line is for (II) and the dotted line is for (III). It is clear that inclusion of lensing leads to a preference for $\sum m_\nu > 0$ which is compatible with that coming from the SZ cluster counts and that there is a strong preference ($\approx 4\sigma$) in the case of dataset (III).

Where do the differences in Mixing originated?

in the symmetries of the

Quark and Lepton sectors

$\mathcal{G}_{\mathcal{F}}^{q} \sim U(3)^3$

$\mathcal{G}_{\mathcal{F}}^{l} \sim U(3)^2 \times O(3)$

for the type I seesaw employed here;

Where do the differences in Mixing originated?

in the symmetries of the

Quark and Lepton sectors

$\mathcal{G}^q_{\mathcal{F}} \sim U(3)^3$

$\mathcal{G}^l_{\mathcal{F}} \sim U(3)^2 \times O(3)$

for the type I seesaw employed here;

in general $U(n_g)$ vs $O(n_g)$

Where do the differences in Mixing originated?

From the

MAJORANA vs DIRAC nature of fermions

Conclusions

- Spontaneous Flavour Symmetry Breaking is a predictive dynamical scenario
- Simple solutions arise that resemble nature in first approximation
- The differences in the mixing pattern of Quarks and Leptons arise from their Dirac vs Majorana nature (U vs. O symmetries). O(2) singled out -> O(3).
- A correlation between large angles and degenerate spectrum emerges. Explicitly, for neutrinos we find: one maximal Majorana phases $(\alpha,1,i)$, $\theta_{23}=45^\circ$, θ_{12} large, θ_{13} small and deg. ν's
- This scenario will be tested in the near future by $0\nu2\beta$ experiments (~.1eV).... or cosmology!!!

The prediction:

large mixing angles $\Leftrightarrow$ Majorana degenerate neutrinos

leads to neutrinoless double beta decay and CMB signals that could be observed in a not too distant future !!

We set the perturbations by hand.
Can we predict them also dynamically?

Fundamental Fields

May provide dynamically the perturbations

In the case of quarks they can give the right corrections:

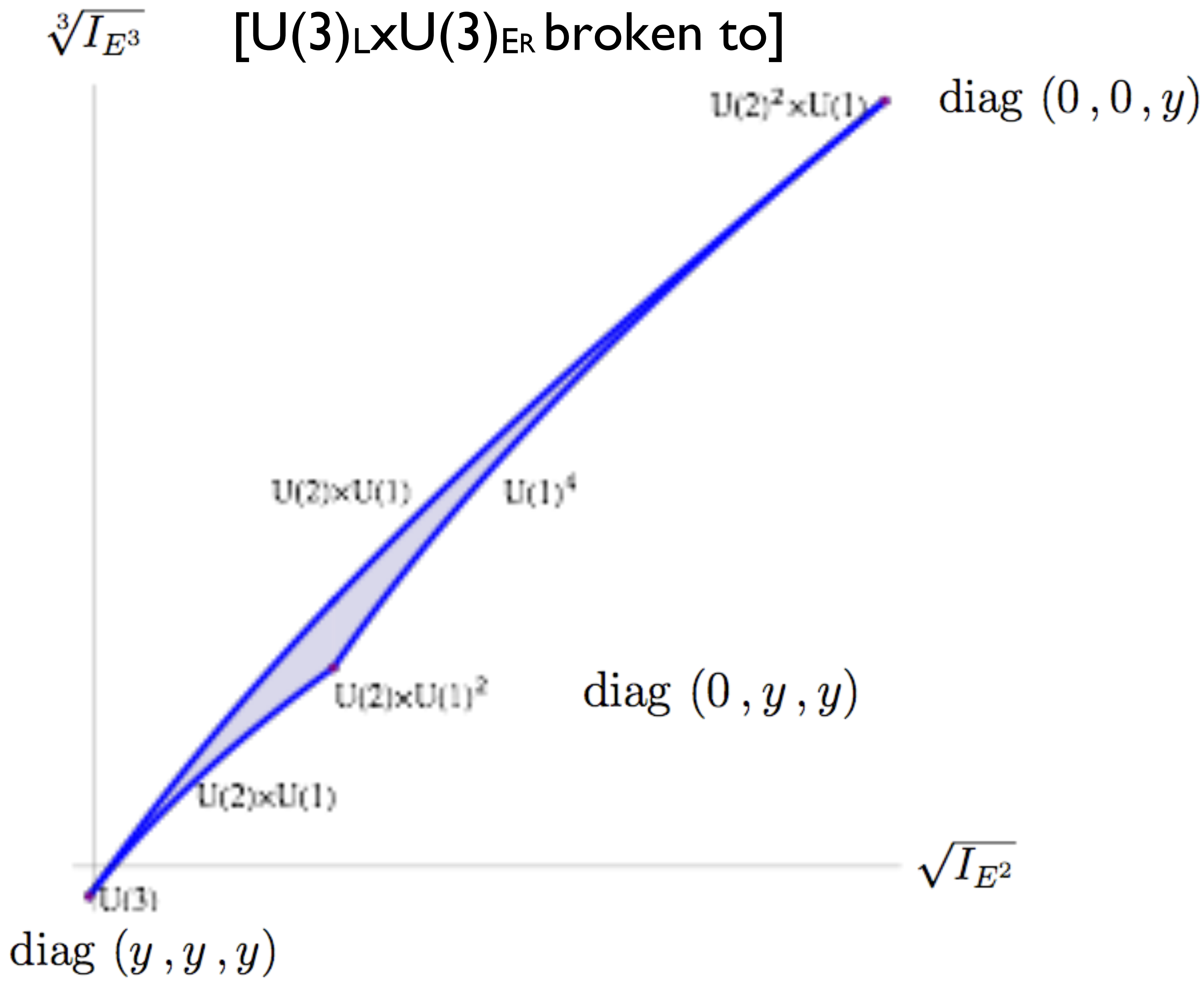
$$\frac{\mathcal{Y}_U}{\Lambda_f} + \frac{\chi_U^L \chi_U^{R\dagger}}{\Lambda_f^2} \sim \begin{pmatrix} 0 & \sin\theta_c\, y_c & 0 \\ 0 & \cos\theta_c\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

[Alonso, Gavela, Merlo, Rigolin]

under study in the lepton sector

Jacobian Analysis: Eigenvalues

[U(3)$_L$xU(3)$_{E_R}$ broken to]



Renormalizable Potential

Invariants at the Renormalizable Level

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\right]\,, \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D\mathcal{Y}_D^\dagger\right]\,,$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^2\right]\,, \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right]\,,$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^3\right]\,, \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^3\right]\,,$$

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right]\,, \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right]\,,$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right]\,, \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right]\,.$$

Renormalizable Potential

with the definition

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)\right)^T ,$$

the potential

$$V^{(4)} = -\,\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\,\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) + h_U \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right) + h_D \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) ,$$

mass spectrum

which contains 8 parameters

Renormalizable Potential

with the definition

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)\right)^T ,$$

the potential

mixing

$$V^{(4)} = -\,\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\,\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) + h_U \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right) + h_D \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) ,$$

which contains 8 parameters

Renormalizable Potential, mixing three families

Von Neumann Trace Inequality

$$y_u^2 y_b^2 + y_s^2 y_c^2 + y_d^2 y_t^2 \leq \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) \leq y_u^2 y_d^2 + y_s^2 y_c^2 + y_b^2 y_t^2 \,.$$

So the Potential selects:

coefficient in the potential

"normal" Hierarchy

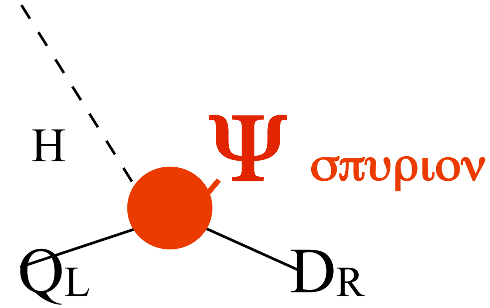
$$g < 0\,, \quad V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

"inverted" Hierarchy

$$g > 0\,, \quad V_{CKM} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

No mixing, independently of the mass spectrum

Some good ideas:



Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (Chivukula+ Georgi)

quarks: $G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$

- Assume that Yukawas are the only source of flavour in the SM and beyond

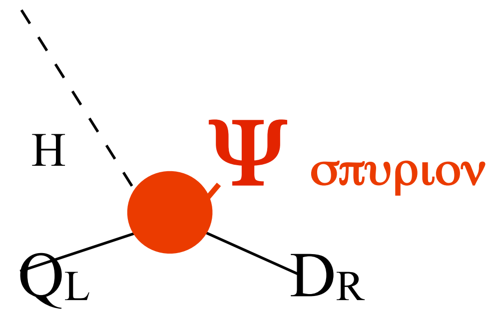
$$\frac{Y_{\alpha\beta}^{+} Y_{\delta\gamma}}{\Lambda_{flavour}^{2}} \bar{Q}_\alpha \gamma_\mu Q_\beta \bar{Q}_\gamma \gamma^\mu Q_\delta$$

... agrees with flavour data being aligned with SM

... allows to bring down $\Lambda_{flavour}$ --> TeV

D'Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise

Some good ideas:



Minimal Flavour Violation:

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- Assume that Yukawas are the only source of flavour in the SM and beyond

$$\frac{Y_{\alpha\beta}^{+}\, Y_{\delta\gamma}}{\Lambda_{flavour}^{2}}\, \bar{Q}_\alpha \gamma_\mu Q_\beta\, \bar{Q}_\gamma \gamma^\mu Q_\delta$$

... agrees with flavour data being aligned with SM

... allows to bring down $\Lambda_{flavour}$ --> TeV

(Chivukula+Georgi 87; Hall+Randall; D'Ambrosio+Giudice+Isidori+Strumia; Cirigliano+Isidori+Grisntein+Wise; Davidson+Pallorini; Kagan+G. Perez + Volanski+Zupan,...)

Lalak, Pokorski, Ross; Fitzpatrick, Perez, Randall; Grinstein, Redi, Villadoro

Bi-fundamental Flavour Fields

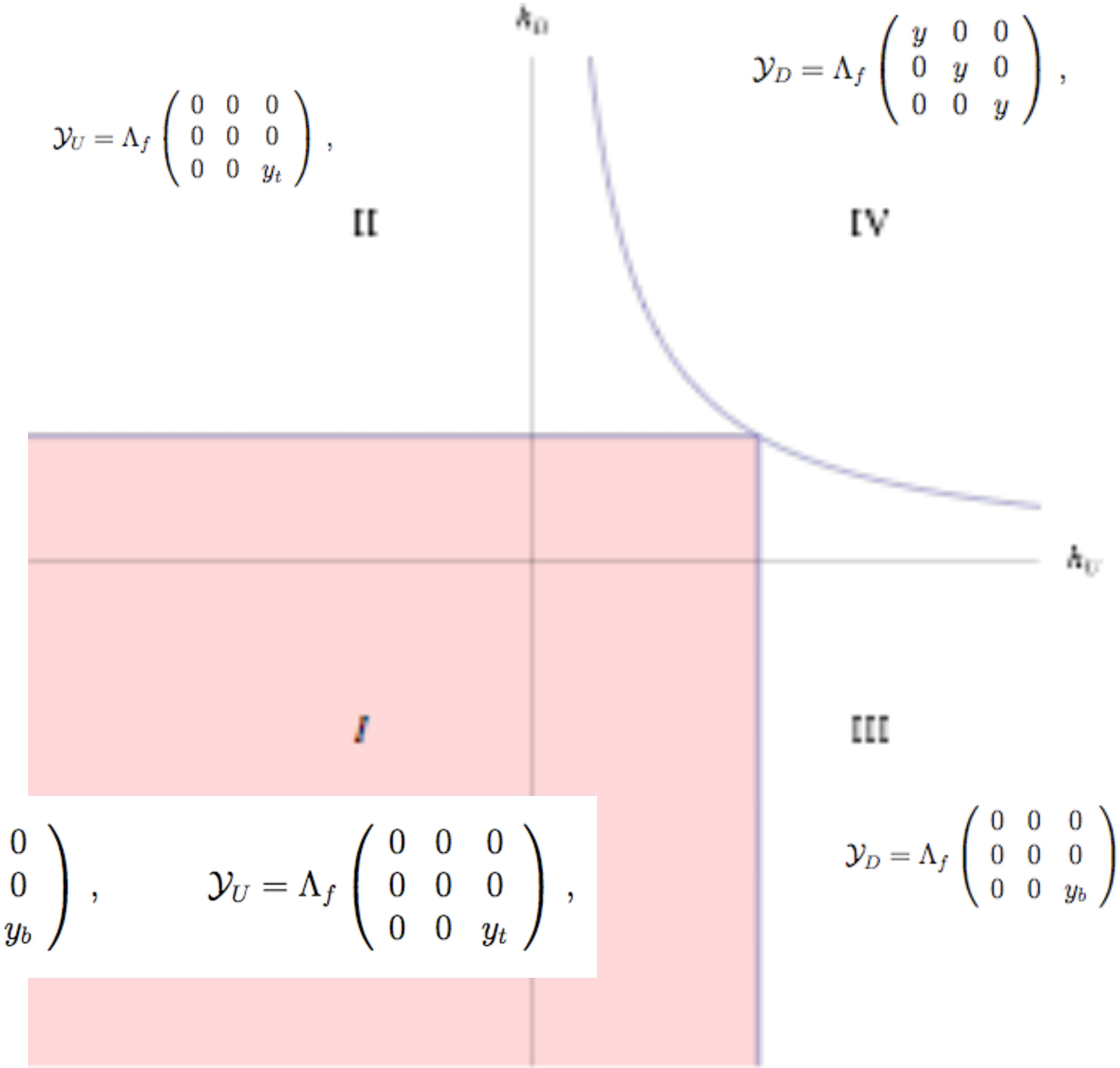
Physical parameters
=Independent Invariants

$$\underset{2\times 18}{\#\,\text{d.o.f. in }\mathcal{Y}_{U,D}} \text{ - } \underset{3\times 9-1}{\left(\dim(\mathcal{G}^q_{\mathcal{F}}) - 1_{U(1)_B}\right)} = 10$$

These are (proportional to):

3 masses in de up sector,
3 masses in de down sector,
4 mixing parameters in V_{CKM}

Renormalizable Potential, masses



$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} y' & 0 & 0 \\ 0 & y' & 0 \\ 0 & 0 & y' \end{pmatrix},$$

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

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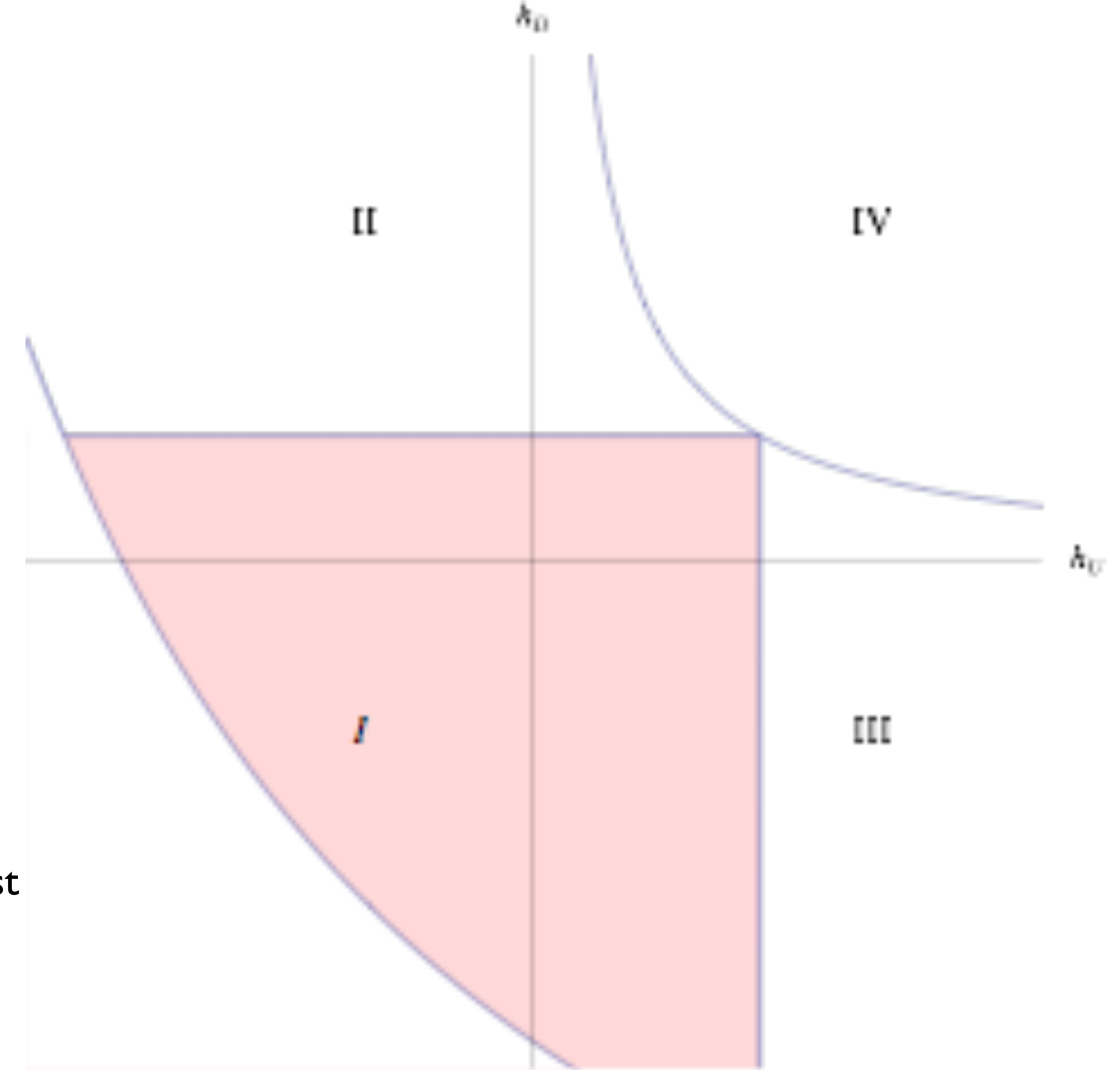
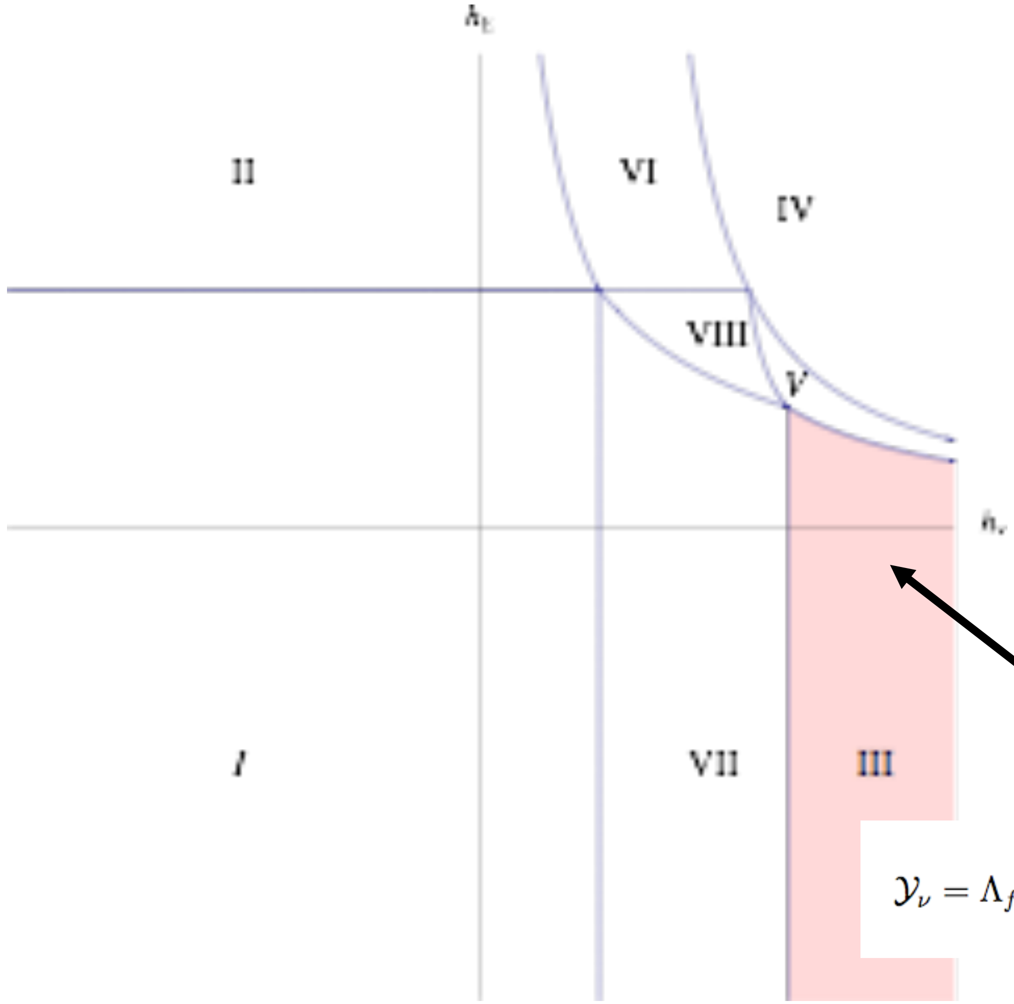
Renormalizable Potential, Stability



This region's size nonetheless depends on the rest of paramters (λ,g)

Renormalizable Potential: Masses



$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix},$$

$$\mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_{\nu_1}/\sqrt{2} & 0 & i y_{\nu_1}/\sqrt{2} \\ 0 & y_{\nu_2} & 0 \\ -y_{\nu_3}/\sqrt{2} & 0 & i y_{\nu_3}/\sqrt{2} \end{pmatrix}$$

* What is the role of the neutrino flavour group?

To analyze this in general, use common parametrization for quarks and leptons:

$$Y = U_L \; y^{diag.} \, U_R$$

* Quarks, for instance: U_R unphysical, $U_L \dashrightarrow U_{CKM}$

$$Y_D = U_{CKM} \, \mathrm{diag}(y_d, y_s, y_b) \quad ; \quad Y_U = \mathrm{diag}(y_u, y_c, y_t)$$

* Leptons:

$$Y_E = \mathrm{diag}(y_e, y_\mu, y_\tau) \quad ; \quad Y_\nu = U_L \; y^{diag.} \, U_R$$

U_{PMNS} diagonalize

$$\boxed{m_\nu \sim Y_\nu \frac{v^2}{M} Y_\nu^{\,T} = U_L \, y_\nu^{diag.} \, U_R \frac{v^2}{M} U_R^{\,T} \, y_\nu^{diag.} \, U_L^{\,T}}$$

* What is the role of the neutrino flavour group?

$U(n)$

* What is the role of the neutrino flavour group?

$U(n)$

i.e.: $U(3)_L \times U(3)_{E_R} \times U(2)_{N_R}$

or: $U(3)_L \times U(3)_{E_R} \times U(3)_{N_R}$

* What is the role of the neutrino flavour group?

e.g. $U(n)_{N_R}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}\,\mathrm{M}\,N_R{}^c + h.c.\right]$$

with M carrying flavour $\longrightarrow$ M spurion

More invariants in this case:

$Tr(\mathcal{Y}_E \mathcal{Y}_E^+)$ $\quad$ $Tr(\mathcal{Y}_E \mathcal{Y}_E^+)^2$ $\quad$ $Tr(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+)$

$Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+)$ $\quad$ $Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+)^2$

$Tr(M_N M_N^+)$ $\quad$ $Tr(M_N M_N^+)^2$ $\quad$ $Tr(M_N M_N^+ \mathcal{Y}_\nu^+ \mathcal{Y}_\nu)$

Result: no mixing for flavour groups U(n)

SU(n)

* What is the role of the neutrino flavour group?

e.g. $SU(n)_{N_R}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R} M N_R{}^c + h.c.\right]$$

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$Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+)$ $\quad Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+)^2$

$Tr(\mathcal{M}_N \mathcal{M}_N^+)$ $\quad Tr(\mathcal{M}_N \mathcal{M}_N^+)^2$ $\quad Tr(\mathcal{M}_N \mathcal{M}_N^+ \mathcal{Y}_\nu^+ \mathcal{Y}_\nu)$

At the minimum:

* $Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+ \mathcal{Y}_E \mathcal{Y}_E^+) = Tr(U_L\, y_\nu^{diag.\,2}\, U_L^+\, y_l^{diag.\,2}) \longrightarrow U_L=1$

* $Tr(\mathcal{M}_N \mathcal{M}_N^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+) = Tr(U_R\, y_\nu^{diag.\,2}\, U_R^+\, M_i^{diag.\,2}) \longrightarrow U_R=1$

same conclusion for 3 families of quarks:

$$Y = U_L \; y^{\text{diag.}} \, U_R$$

* Quarks, for instance: U_R unphysical, $U_L \dashrightarrow U_{CKM}$

$$Y_D = U_{CKM} \, \text{diag}(y_d, y_s, y_b) \quad ; \quad Y_U = \text{diag}(y_u, y_c, y_t)$$

$$\text{Tr} \left(\mathcal{Y}_u \, \mathcal{Y}_u^+ \, \mathcal{Y}_d \, \mathcal{Y}_d^+ \right) = \text{Tr} \left(U_L \, y_u^{\text{diag.}\,2} \, U_L^{\,+} \, y_d^{\text{diag.}\,2} \right)$$

$\longrightarrow$ $U_L = U_{CKM} \sim 1$ at the minimum

NO MIXING

In many BSM the Yukawas do not come from dynamical fields:

Some good ideas:

D.B. Kaplan-Georgi in the 80's proposed a light SM scalar because being a (quasi) goldstone boson: ***composite Higgs***

(D.B. Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison.......Contino, Nomura, Pomarol; Agashe, Contino, Pomarol; Giudice, Pomarol, Ratazzi, Grojean; Contino, Grojean, Moretti; Azatov, Galloway, Contino... Frigerio, Pomarol, Riva, Urbano...)

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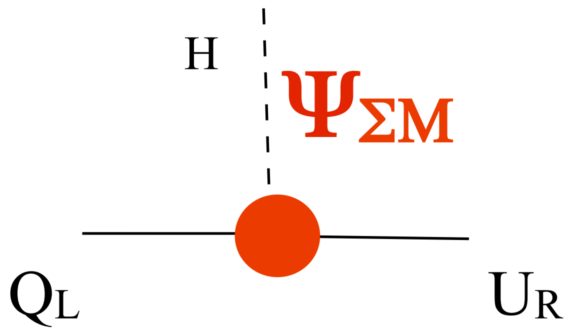
Flavour "Partial compositeness" D.B Kaplan 91:

A sort of "seesaw for quarks"

(nowadays sometimes justified from extra-dim physics)



$m_q = v\, Y_{SM}$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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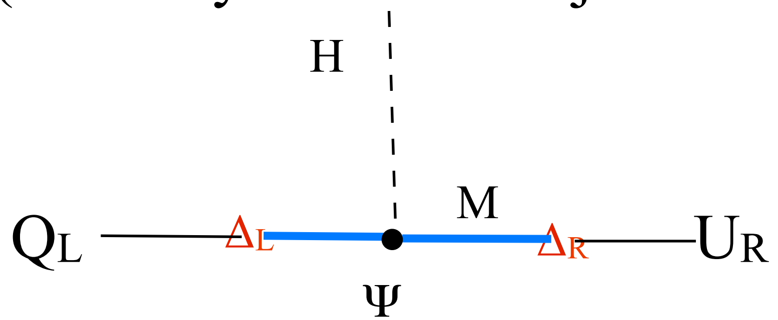

$$\Psi_{\Sigma M} = \Psi \Delta_L \, \Delta_R / M^2$$

$$m_q = v \, Y_{SM}$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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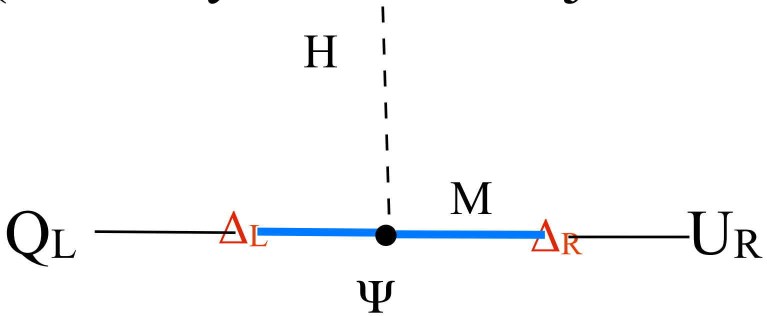
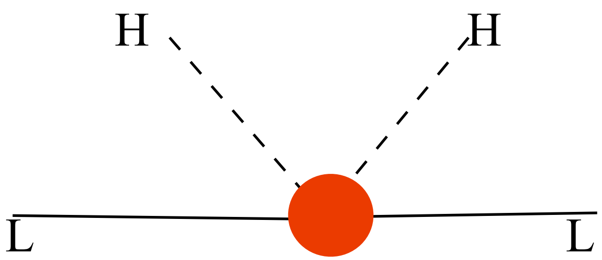


$$\Psi_{\Sigma M} = \Psi \Delta_L \, \Delta_R / M^2$$

$$m_q = v \, Y_{SM}$$

Neutrino masses:



d=5 Weinberg operator

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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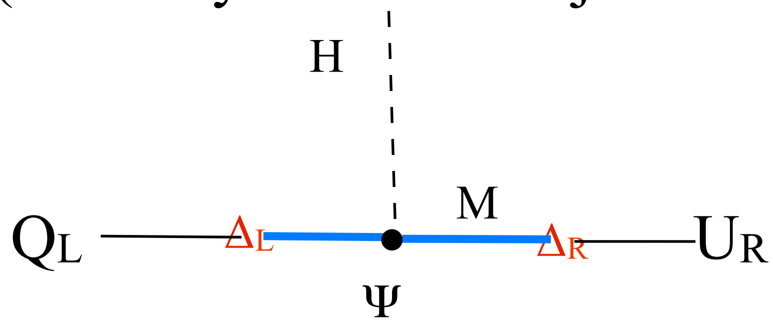

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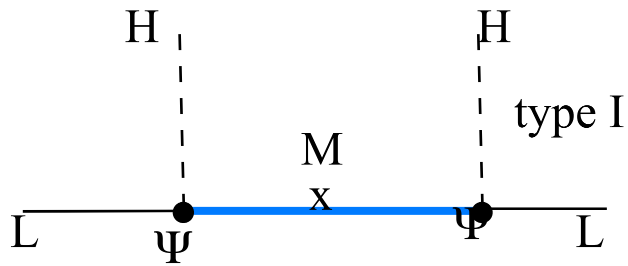

$$\Psi_{\Sigma M} = \Psi \Delta_L \Delta_R / M^2$$

$$m_q = v\, Y_{SM}$$

$$m_\nu = Y\, v^2 / M\, Y^T$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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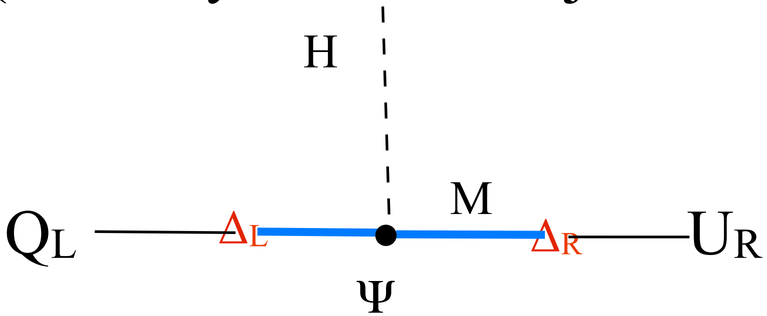

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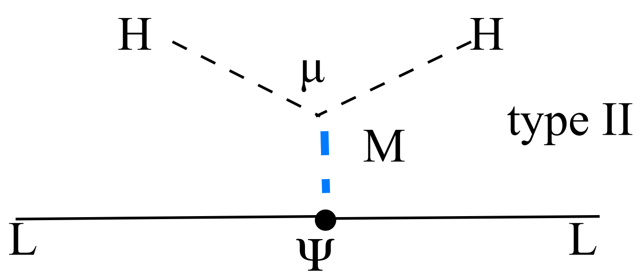


$$m_\nu = Y \mu v^2/M^2$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

For instance, in discrete symmetry ideas:

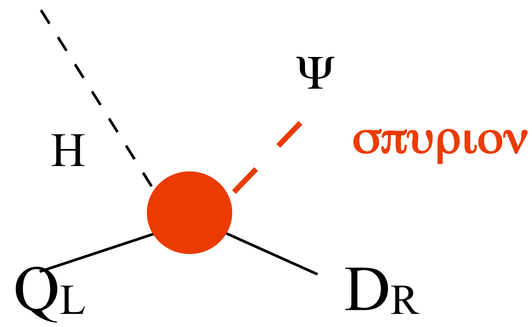
The Yukawas are indeed explained in terms of dynamical fields.
And they do not need to worry about goldstone bosons.

In spite of θ_{13} not very small, there is activity.
For instance, combine generalized CP (Bernabeu, Branco, Gronau 80s) with discrete Z_2 groups in the neutrino sector : maximal θ_{23}, strong constraints on values of CP phases
(Feruglio, Hagedorn and Ziegler 2013; Holthausen, Lindner and Schmidt 2013)

They were popular mainly because they can lead easily to large mixings (tribimaximal, bimaximal...)

But:
- Discrete approaches do not relate mixing to spectrum
- Difficulties to consider both quarks and leptons

Some good ideas:



Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (Chivukula+ Georgi)

quarks: $G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$

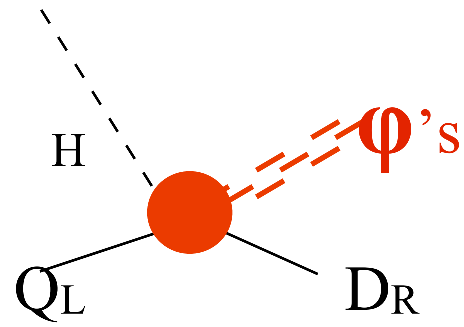
Hybrid dynamical-non-dynamical Yukawas:

U(2) (Pomarol, Tomasini; Barbieri, Dvali, Hall, Romanino...)....

$U(2)^3$ (Craig, Green, Katz; Barbieri, Isidori, Jones-Peres, Lodone, Straub.. ..Sala)

$$\begin{pmatrix} U(2) & & (\ ) \\ 0 & 0 & 1 \end{pmatrix}$$

Sequential ideas (Feldman, Jung, Mannel; Berezhiani+Nesti; Ferretti et al., Calibbi et al. ...)



Some good ideas, based on continuous symmetries:

Frogatt-Nielsen '79: $U(1)_{flavour}$ symmetry

- Yukawa couplings are effective couplings,
- Fermions have $U(1)_{flavour}$ charges

$$\left(\frac{<\varphi>}{\Lambda}\right)^n Q\, H\, q_R \quad , \quad Y \sim \left(\frac{<\varphi>}{\Lambda}\right)^n$$

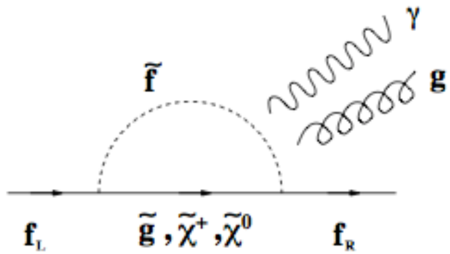
e.g. n=0 for the top, n large for light quarks, etc.

--> FCNC ?

The FLAVOUR WALL for BSM

i) Typically, BSMs have **electric dipole moments** at one loop

i.e susy MSSM:

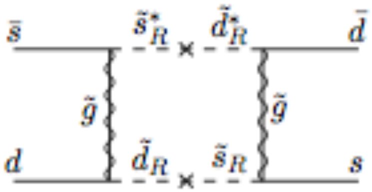


< 1 loop in SM ---> Best (precision) window of new physics

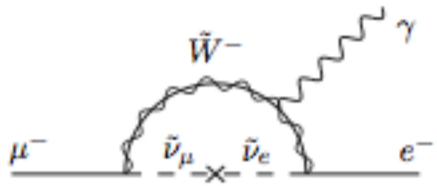
ii) **FCNC**

i.e susy MSSM:

$K^0 - \overline{K}^0$ mixing



$\mu \to e\gamma$



competing with SM at one-loop

Minimal Flavour violation (MFV)

- Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawa

MFV Hypothesis ≡ The Yukawas are the only sources (*irreducible*) of flavour violation. in BSM

R. S. Chivukula and H. Georgi, Phys. Lett. B 188, 99 (1987).

It is very predictive for quarks:

$O^{d=6} \sim \overline{Q}_\alpha Q_\beta \overline{Q}_\gamma Q_\delta$

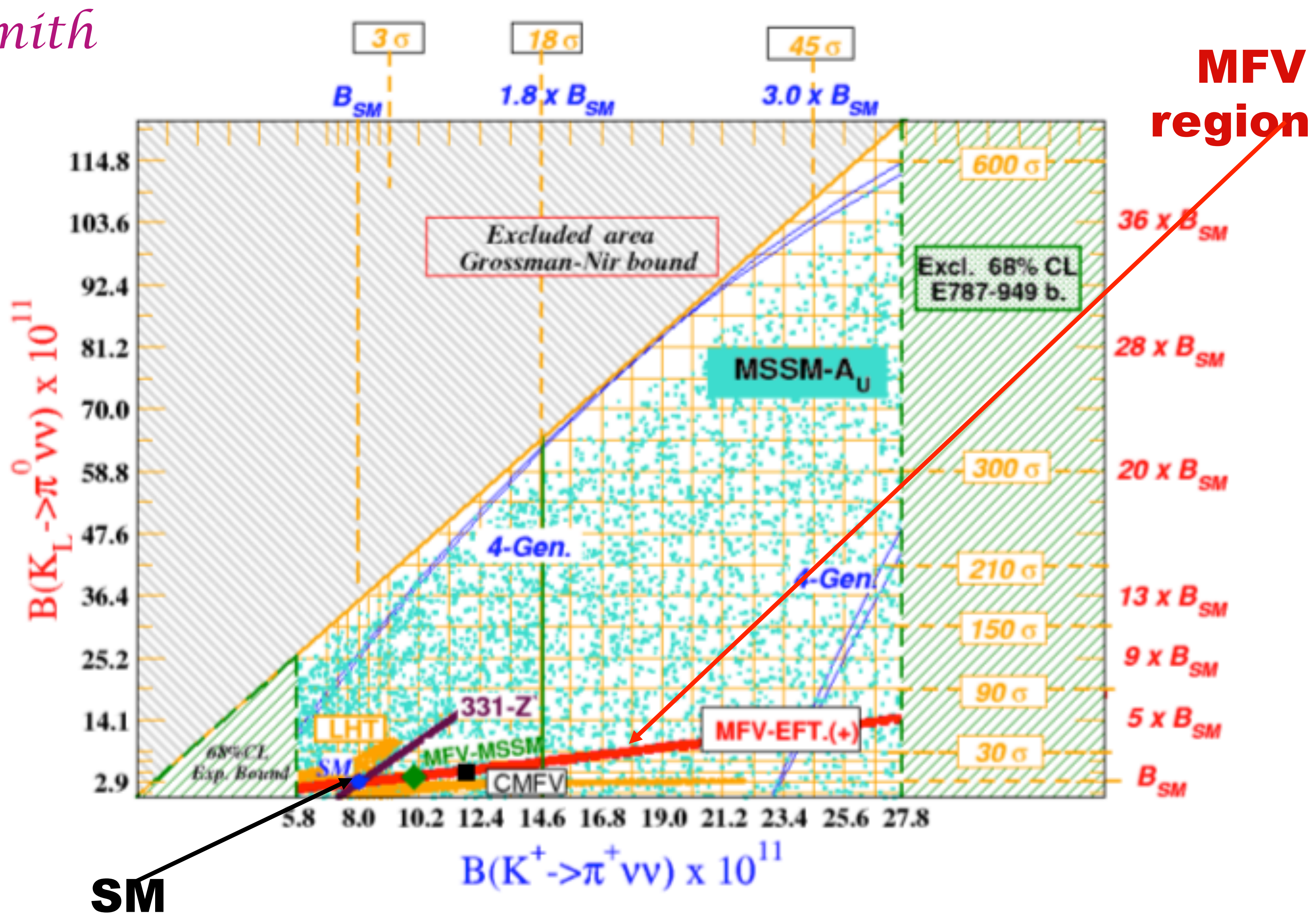
$$\mathcal{L} = \mathcal{L}_{SM} + c^{d=6} \frac{O^{d=6}}{\Lambda^2_{\text{flavour}}} + \ldots\ldots$$

known function of Yukawas

(D'Ambrosio, Cirigliano, Isidori, Grinstein, Wise….Buras….)

I. NA62 main targets are the rare K decays ($Br \lesssim 10^{-11}$), e.g. $K^+ \to \pi^+ \nu\bar{\nu}$ s

Smith



Gonzalez-Alonso

Minimal Flavour violation (MFV)

•Unitarity of CKM first row:

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9999\pm0.0006$$

•*Restrict to flavour blind ops.-> 4 operators

•Correction is only multiplicative to β and μ decay rate

- **The direct experimental limit puts strong constraints on all 4 operators, at the level of the colliders constraints or better.**

$$\Delta_{CKM}=-(0.1\pm0.6)\cdot10^{-3} \rightarrow \Lambda_i^{eff}>11\,\mathrm{TeV}\ (90\%\ \mathrm{CL})$$

Current neutrino parameters

3 sizable mixing angles

NuFIT 1.2 (2013)

	Free Fluxes + RSBL		Huber Fluxes, no RSBL	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.346$	$0.313^{+0.013}_{-0.012}$	$0.277 \to 0.355$
$\theta_{12}/^\circ$	$33.57^{+0.77}_{-0.75}$	$31.37 \to 36.01$	$34.02^{+0.79}_{-0.76}$	$31.78 \to 36.55$
$\sin^2\theta_{23}$	$0.446^{+0.008}_{-0.008} \oplus 0.593^{+0.027}_{-0.043}$	$0.366 \to 0.663$	$0.444^{+0.037}_{-0.031} \oplus 0.592^{+0.028}_{-0.042}$	$0.361 \to 0.665$
$\theta_{23}/^\circ$	$41.9^{+0.5}_{-0.4} \oplus 50.3^{+1.6}_{-2.5}$	$37.2 \to 54.5$	$41.8^{+2.1}_{-1.8} \oplus 50.3^{+1.6}_{-2.5}$	$36.9 \to 54.6$
$\sin^2\theta_{13}$	$0.0231^{+0.0019}_{-0.0019}$	$0.0173 \to 0.0288$	$0.0244^{+0.0019}_{-0.0019}$	$0.0187 \to 0.0303$
$\theta_{13}/^\circ$	$8.73^{+0.35}_{-0.36}$	$7.56 \to 9.77$	$9.00^{+0.35}_{-0.36}$	$7.85 \to 10.02$
$\delta_{\mathrm{CP}}/^\circ$	266^{+55}_{-63}	$0 \to 360$	270^{+77}_{-67}	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.45^{+0.19}_{-0.16}$	$6.98 \to 8.05$	$7.50^{+0.18}_{-0.17}$	$7.03 \to 8.08$
$\frac{\Delta m^2_{31}}{10^{-3}\ \mathrm{eV}^2}$ (N)	$+2.417^{+0.014}_{-0.014}$	$+2.247 \to +2.623$	$+2.429^{+0.055}_{-0.054}$	$+2.249 \to +2.639$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$ (I)	$-2.411^{+0.062}_{-0.062}$	$-2.602 \to -2.226$	$-2.422^{+0.063}_{-0.061}$	$-2.614 \to -2.235$

Gonzalez-Garcia, Maltoni, Salvado, Schwetz, 1209.3023

2 mass squared differences