On neutrinos in E6 GUT

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Outline

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Grand Unified Theories

Simplicity, minimality:

- Instead of 3 gauge couplings $g_{1,2,3}$ in SM $\to$ only g_{GUT}
- 5 representations per generation Q, L, u^c, d^c, e^c $\to$ less

$$\begin{aligned} SU(5): &\qquad 10=(Q,u^c,e^c)\ ;\quad \bar{5}=(d^c,L) \\ SO(10): &\qquad 16=(Q,u^c,e^c,d^c,L,\nu^c) \\ E_6: &\qquad 27=(Q,u^c,e^c,d^c,L,\nu^c,d',L'^c,d'^c,L',s) \end{aligned}$$

- instead of 4 Yukawa's ($Y_{U,D,E,N}$) in SM$\to$ less

  $\to$ neutrino mass matrix connected with charged lepton ones

- The gauge structure explains electric charge quantization

  $\bar{5} = (d_1^c, d_2^c, d_3^c, \nu, e) \to 3q_{d^c} + q_\nu + q_e = 0 \to q_{d^c} = -q_e/3$

  $\to$ existence of magnetic monopoles predicted

- Theory of proton decay:

$$\mathcal{L} = c_{ijkl} q_i q_j q_k l_l / \Lambda$$

c_{ijkl}, Λ arbitrary in SM

in GUTs $\Lambda = M_{GUT}$ and c_{ijkl} predicted (model dependent)

In principle operators with $\Lambda = M_{Planck}$ could be present:

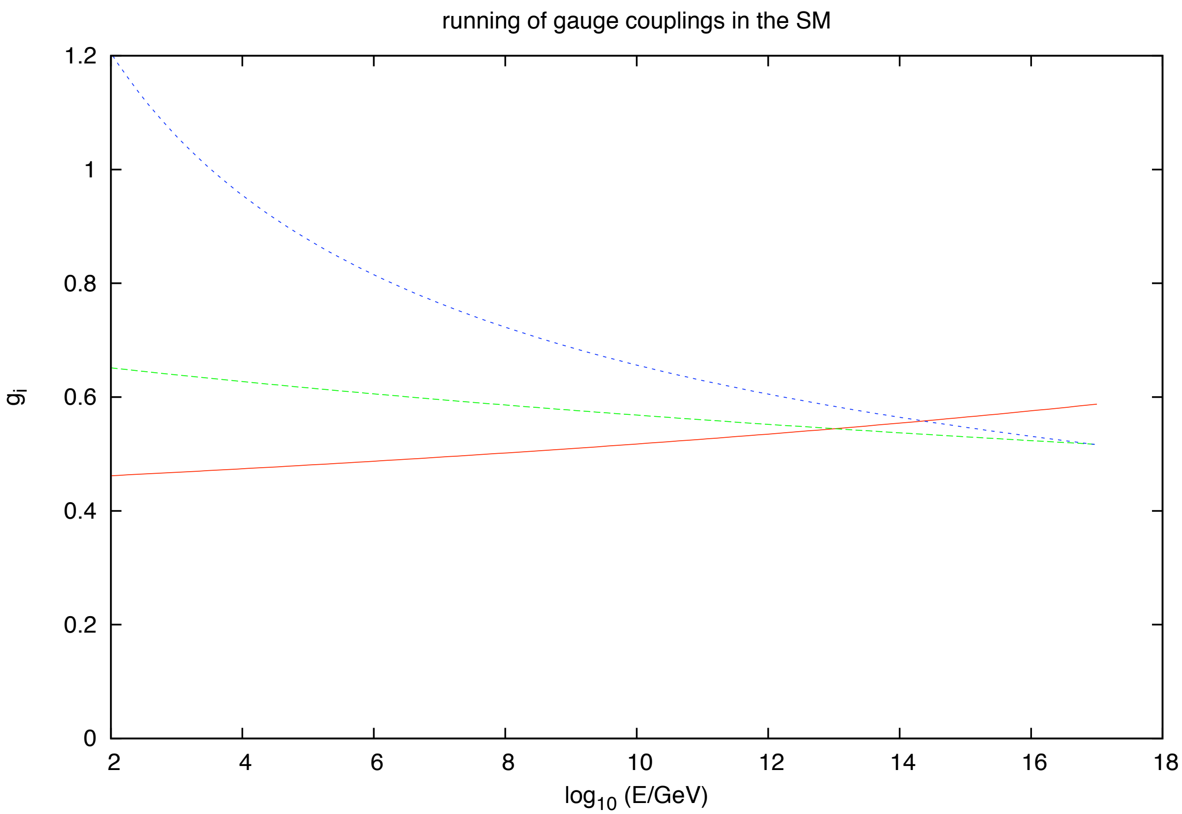
$$SU(5): \quad c\, 10\, 10\, 10\, \bar{5}/M_{Planck}$$

$$SO(10): \quad c\, 16\, 16\, 16\, 16/M_{Planck}$$

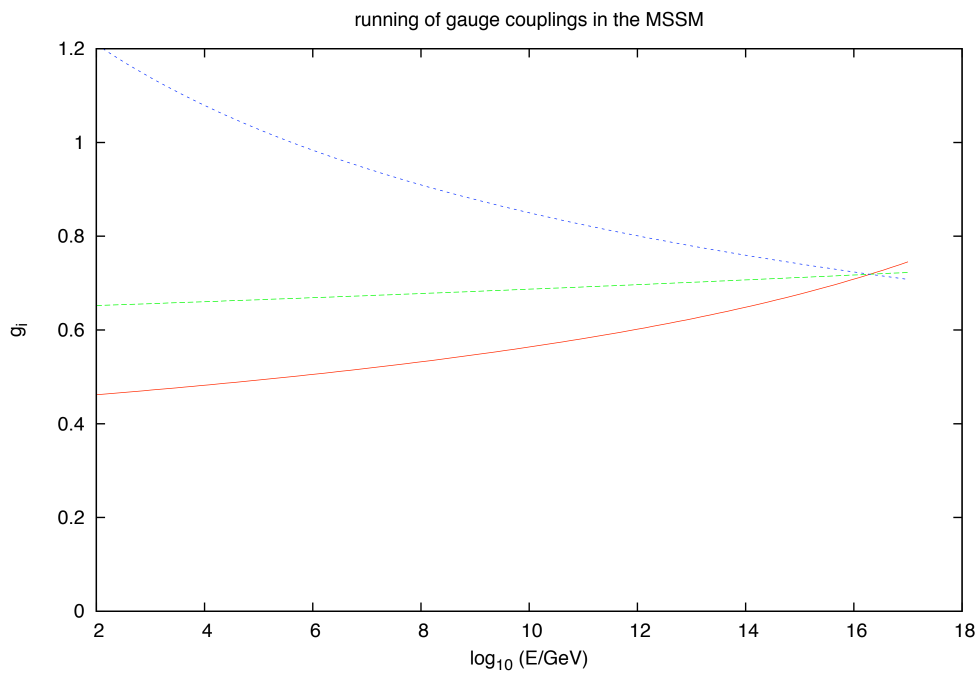
E_6 : no such term 27^4 not invariant

$c \lesssim 10^{-7}$! No explanation.

But what does it mean $g_{1,2,3} \to g_5$ ($g_1 \neq g_2 \neq g_3$)? What if we run g_i, do they unify at some scale? Not completely:



New states needed. If you put MSSM at ≈ 1 TeV: unification at $M_{GUT} \approx 10^{16}$ GeV



Not unique solution, but enough to motivate supersymmetry

A motivation for E_6

The minimal supersymmetric GUT is the Georgi-Glashow SU(5):

- 3 copies of matter $10_F + \bar{5}_F$
- Higgs sector 24_H and $5_H + \bar{5}_H$

If we stick to the TeV susy scale $\rightarrow$

non-renormalizable terms needed to

- correct bad relation $M_D = M_E$
- increase proton lifetime
  (to get unification color triplet M_T too low)

What is the minimal prize to pay to maintain renormalizability?

Add a vectorlike matter-type pair $5_F + \bar{5}_F$:

- of the four $\bar{5}_F$ only 3 combinations are light (chiral):

$$\bar{5}^a_F \left(\eta_a 24_H + \mu_a\right) 5_F$$

  the choice of these combinations breaks SU(5) by $\langle 24_H \rangle$: this corrects the bad relation $M_D = M_E$

- The combination of heavy triplets can account for the heavy color triplet that corrects RGE's. Since this is matter-type, it does not contribute to proton decay

This elegant solution has two drawbacks:

- no theoretical (only phenomenological) motivation for the extra vectorlike $5+\bar{5}$
- what about neutrinos? SU(5) not a good theory of neutrinos

Incorporating all this into SO(10) does not help: ok for neutrinos, still no reason for extra $5+\bar{5}$

E_6 seems a good choice: similarly as SO(10) can put on a same footing neutrinos, but also extra vectorlike $5+\bar{5}$ in 27!

This is what we will assume.

E_6 : few technicalities

In some sense it is similar as SU(N):

$$T^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n}$$

SU(N): $\alpha_i, \beta_j = 1, \ldots, N$

E6: $\alpha_i, \beta_j = 1, \ldots, 27$ (fundamental)

SU(N): $\epsilon_{\alpha_1 \ldots \alpha_N}$ or $\epsilon^{\alpha_1 \ldots \alpha_N}$, made of $0, \pm 1$

completely antisymmetric Levi-Civita tensor

E6: $d^{\mu\nu\lambda}$, $d_{\mu\nu\lambda}$, made of $0, \pm 1$

completely symmetric tensor (but 0 if two indices equal)

The lowest dimensional representations:

$$
\begin{array}{rcl}
27^{\mu} & \ldots & \text{fundamental} \\
\overline{27}_{\mu} & \ldots & \text{anti-fundamental} \\
78^{\mu}{}_{\nu} & \ldots & \text{adjoint } (= (t^A)^{\mu}{}_{\nu} 78^A) \\
351^{\mu\nu} = -351^{\nu\mu} & \ldots & \text{two indices antisymmetric} \\
\overline{351}_{\mu\nu} = -\overline{351}_{\nu\mu} & \ldots & \text{two indices antisymmetric} \\
351'^{\mu\nu} = +351'^{\nu\mu} & \ldots & \text{two indices symmetric } (d_{\lambda\mu\nu} 351'^{\mu\nu} = 0) \\
\overline{351'}_{\mu\nu} = +\overline{351'}_{\nu\mu} & \ldots & \text{two indices symmetric } (d^{\lambda\mu\nu} \overline{351'}_{\mu\nu} = 0) \\
650^{\mu}{}_{\nu} & \ldots & (650^{\mu}{}_{\mu} = (t^A)^{\nu}{}_{\mu} 650^{\mu}{}_{\nu} = 0)
\end{array}
$$

Invariants constructed in both SU(N) and E_6 cases by products of tensors with all lower indices contracted (summed) with all upper indices.

SU(5):

$$\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} A^{\alpha_1\alpha_2}_{\alpha_6} B^{\alpha_3\alpha_6} C^{\alpha_4} D^{\alpha_5\alpha_7} E_{\alpha_7} \quad , \quad \alpha_i = 1, \ldots, 5$$

E6:

$$d_{\alpha_1\alpha_2\alpha_3} d_{\alpha_4\alpha_5\alpha_6} A^{\alpha_1\alpha_2} B^{\alpha_3} C^{\alpha_4} D^{\alpha_5\alpha_6\alpha_7} E_{\alpha_7} \quad , \quad \alpha_i = 1, \ldots, 27$$

Generic Yukawa sector in E_6

In all generality three types of Yukawas

$$W = 27_i \left(Y_{27}^{ij}\, 27 + Y_{\overline{351}'}^{ij}\, \overline{351}' + Y_{\overline{351}}^{ij}\, \overline{351} \right) 27_j$$

$$Y_{27,\overline{351}'} = Y_{27,\overline{351}'}^T \qquad \text{symmetric}$$

$$Y_{\overline{351}} = -Y_{\overline{351}}^T \qquad \text{antisymmetric}$$

Completely analogous to SO(10):

$$W = 16_i \left(Y_{10}^{ij}\, 10 + Y_{\overline{126}}^{ij}\, \overline{126} + Y_{120}^{ij}\, 120 \right) 16_j$$

$$Y_{10,\overline{126}} = Y_{10,\overline{126}}^T \qquad \text{symmetric}$$

$$Y_{120} = -Y_{120}^T \qquad \text{antisymmetric}$$

In fact

$$
\begin{aligned}
27 &= 1 + 10 + 16 \\
\overline{351}' &= 1 + 10 + \overline{16} + 54 + \overline{126} + 144 \\
\overline{351} &= 10 + \overline{16} + 16 + 45 + 120 + 144
\end{aligned}
$$

But now also extra Higgs doublets in 10, 16 and 144

$\rightarrow$ mixing between 16_i, 10_i and 1_i in matter 27_i.

The antisymmetric $\overline{351}$ contribution (similar as 120 in SO(10)) seems less promising so we will concentrate on the symmetric 27 and $\overline{351}'$ from now on.

But we will leave the possibility of multiple 27.

$$W = \begin{pmatrix} 16 & 10 & 1 \end{pmatrix} Y_{27} \begin{pmatrix} 10 & 16 & 0 \\ 16 & 1 & 10 \\ 0 & 10 & 0 \end{pmatrix} \begin{pmatrix} 16 \\ 10 \\ 1 \end{pmatrix}$$

$$+ \begin{pmatrix} 16 & 10 & 1 \end{pmatrix} Y_{\overline{351}'} \begin{pmatrix} \overline{126} + 10 & 144 & \overline{16} \\ 144 & 54 & 10 \\ \overline{16} & 10 & 1 \end{pmatrix} \begin{pmatrix} 16 \\ 10 \\ 1 \end{pmatrix}$$

- several new Higgs doublets (not only in 10 and $\overline{126}$)
- some fields have large $\mathcal{O}(M_{GUT})$ vevs $\rightarrow$
  - mixing between $\bar{5} \in 16$ and $\bar{5} \in 10$ (d^c, L)
  - mixing between $1 \in 1$ and $1 \in 16$ (ν^c)
- $M^U_{3\times 3}$, $M^D_{6\times 6}$, $M^E_{6\times 6}$, $M^N_{15\times 15} \rightarrow$ light $(M_{U,D,E,N})_{3\times 3}$

Higgs sector with $351' + \overline{351}' + 27 + \overline{27}$

- What are the large vevs that produce family mixings with vectorlike extra matter?
- Where are the MSSM Higgs doublets?

The full model needed.

The minimal Higgs sector with $E_6 \to$ SM composed of $351' + \overline{351}' + 27 + \overline{27}$.

$$
\begin{aligned}
W &= m_{351'}\, \overline{351}'\, 351' + \lambda_1\, 351'^3 + \lambda_2\, \overline{351'}^3 \\
&+ m_{27}\, \overline{27}\, 27 + \lambda_3\, 27\, 27\, \overline{351}' + \lambda_4\, \overline{27}\, \overline{27}\, 351' \\
&+ \lambda_5\, 27^3 + \lambda_6\, \overline{27}^3
\end{aligned}
$$

The SM singlets:

$$\begin{aligned} 27 &: \quad c_1, c_2 \\ \overline{27} &: \quad d_1, d_2 \\ 351' &: \quad e_1, e_2, e_3, e_4, e_5 \\ \overline{351'} &: \quad f_1, f_2, f_3, f_4, f_5 \end{aligned}$$

More than one solution. For example:

$$c_2 = e_2 = e_4 = 0, \qquad d_2 = f_2 = f_4 = 0$$

$$d_1 = \frac{m_{351'} m_{27}}{2\lambda_3 \lambda_4 c_1}$$

$$e_1 = -\frac{m_{351'}}{6\lambda_1^{2/3}\lambda_2^{1/3}}, \qquad f_1 = -\frac{m_{351'}}{6\lambda_1^{1/3}\lambda_2^{2/3}}$$

$$e_3 = -\lambda_3 {c_1}^2 / m_{351'}, \qquad f_3 = -\frac{m_{351'} m_{27}^2}{4\lambda_3^2 \lambda_4 {c_1}^2}$$

$$e_5 = \frac{m_{351'}}{3\sqrt{2}\lambda_1^{2/3}\lambda_2^{1/3}}, \qquad f_5 = \frac{m_{351'}}{3\sqrt{2}\lambda_1^{1/3}\lambda_2^{2/3}}$$

with

$$0 = |m_{351'}|^4 |m_{27}|^4 + 2|m_{351'}|^4 |m_{27}|^2 |\lambda_3|^2 |c_1|^2 - 8|m_{351'}|^2 |\lambda_3|^4 |\lambda_4|^2 |c_1|^6 - 16|\lambda_3|^6 |\lambda_4|^2 |c_1|^8$$

This case seems really minimal: 27 and $\overline{351}'$ that participate to symmetry breaking could contribute to Yukawa terms!

Can the weak doublets with $Y = \pm 1$ in 27 and $\overline{351'}$ be the Higgses $H, \bar{H}$ of the MSSM?

Since E_6 is a GUT, this means:

Can we make the doublet-triplet splitting with the massless eigenvector living in both 27 and $\overline{351'}$?

The doublet-triplet splitting

Problem present in all minimal GUTs. The prototype example in SU(5):

$$5_H = \begin{pmatrix} T \\ H \end{pmatrix} \quad , \quad \bar{5}_H = \begin{pmatrix} \bar{T} \\ \bar{H} \end{pmatrix}$$

$$\begin{aligned} W_{Yukawa} &= Y_{\bar{5}}^{ij}\ \bar{5}_i\ 10_j\ \bar{5}_H + Y_{10}^{ij}\ 10_i\ 10_j\ 5_H \\ &\to Y_{\bar{5}}^{ij} \left(d_i^c\ Q_j + L_i\ e_j^c \right) \bar{H} + Y_{10}^{ij}\ u_i^c\ Q_j\ H \\ &+ Y_{\bar{5}}^{ij} \left(L_i\ Q_j + d_i^c\ u_j^c \right) \bar{T} + Y_{10}^{ij} \left(Q_i\ Q_j + u_i^c\ e_j^c \right) T \end{aligned}$$

$H, \bar{H} \ldots$ Higgses of MSSM $\to M_H \approx m_Z$

$T, \bar{T}$ mediate proton decay $\tau \propto M_T^2 \to M_T \approx M_{GUT} \gg m_Z$

How to get such a large splitting from components of same multiplet?

$$W = \mu \, \bar{5}_H \, 5_H + \eta \, \bar{5}_H \, 24_H \, 5_H$$

Since

$$\langle 24_H \rangle = M_{GUT} \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}$$

$$W = \bar{H}\,(\mu - 3\eta\, M_{GUT})\, H + \bar{T}\,(\mu + 2\eta\, M_{GUT})\, T$$

$$\begin{aligned} M_H &= \mu - 3\eta\, M_{GUT} \approx 0 \\ M_T &= \mu + 2\eta\, M_{GUT} \approx M_{GUT} \end{aligned}$$

$$\to \mu = 3\eta M_{GUT} \approx M_{GUT}$$

Fine-tuning unavoidable in minimal models

In our E_6 case doublets and triplets live in $351'$, $\overline{351'}$, 27, $\overline{27}$.

$351'$ has 8 doublets (9 triplets)

$\overline{351}'$ has 8 doublets (9 triplets)

27 has 3 doublets (3 triplets)

$\overline{27}$ has 3 doublets (3 triplets)

All together 22 doublets (11 with $Y = +1$ and 11 with $Y = -1$):
doublet matrix M_D is 11×11

All together 24 triplets (12 with $Y = +2/3$ and 12 with $Y = -2/3$):
triplet matrix M_T is 12×12

analysis complicated by presence of would-be-Goldstones in
$16 + \overline{16} \in 78$

$\rightarrow$ $M_{T,D}$ have automatically one zero eignevalue

We need the determinant without the zero-modes:

$$Det(M) \equiv \prod_{i=2}^{n} m_i$$

We would like to get

$$Det(M_D) = 0 \quad , \quad Det(M_T) \neq 0$$

But after long calculation the result is:

$$Det(M_T) = \# Det(M_D)$$

i.e doublet-triplet splitting impossible !

Bizarre situation: all was ok, we seems to fail on doublet-triplet splitting. And not because we don't like fine-tuning, we cannot even fine-tune!

Simplest solutions:

- add another $27 + \overline{27}$ pair with coupling

$$W_{DT} = m_{27}\, 27\, \overline{27} + \kappa_1\, 27\, 27\, \overline{351'} + \kappa_2\, \overline{27}\, \overline{27}\, 351' + \kappa_3\, 27\, 27\, 27 + \kappa_4\, \overline{27}\, \overline{27}\, \overline{27}$$

with $\langle 27 \rangle\, ,\, \langle \overline{27} \rangle = \mathcal{O}(m_Z)$

DT splitting now possible: MSSM Higgs live only in $27\, ,\, \overline{27}$

In spite of this 3 Yukawa matrices involved.

- add another 78: although it does not contribute to Yukawas, it changes the symmetry breaking pattern (not being needed) thus relaxing constraints on DT.

DT now possible in the old sector: MSSM Higgses live also in $\overline{351}'$ and 27!

This possibility more minimal, only 2 Yukawas.

Higgs sector with $351' + \overline{351}' + 27 + \overline{27} + 78$

$$
\begin{aligned}
W &= m_{351'}\,\overline{351}'\,351' + \lambda_1\,351'^3 + \lambda_2\,\overline{351'}^3 \\
&+ m_{27}\,\overline{27}\,27 + \lambda_3\,27^2\,\overline{351}' + \lambda_4\,\overline{27}^2\,351' \\
&+ \lambda_5\,27^3 + \lambda_6\,\overline{27}^3 \\
&+ m_{78} 78^2 + \lambda_7\,27\,78\,\overline{27} + \lambda_8\,351'\,78\,\overline{351}'
\end{aligned}
$$

Other SM singlets:

$$78 \quad : \quad a_1, a_2, a_3, a_4, a_5$$

Solution with $a_i \neq 0$ shown explicitly to be possible. Disconnected with the previous one (no limit gives the previous solution with $a_i \to 0$).

Yukawa sector in the minimal E_6 model

As an example of what happens let's see the down sector:

$$\begin{pmatrix} d^{cT} & d'^{cT} \end{pmatrix} \begin{pmatrix} \bar{v}_2 Y_{27} + \left(\frac{1}{2\sqrt{10}} \bar{v}_4 + \frac{1}{2\sqrt{6}} \bar{v}_8 \right) Y_{\overline{351}'} & c_2 Y_{27} \\ -\bar{v}_3 Y_{27} - \left(\frac{1}{2\sqrt{10}} \bar{v}_9 + \frac{1}{2\sqrt{6}} \bar{v}_{11} \right) Y_{\overline{351}'} & \frac{1}{\sqrt{15}} f_4 Y_{\overline{351}'} \end{pmatrix} \begin{pmatrix} d \\ d' \end{pmatrix}$$

$\bar{v}_{2,3,4,8,9,11} = \mathcal{O}(m_Z)$; $c_2, f_4 = \mathcal{O}(M_{GUT})$

$$\left. \begin{array}{l} d^c \in \bar{5}_{SU(5)} \in 16_{SO(10)} \\ d'^c \in \bar{5}_{SU(5)} \in 10_{SO(10)} \end{array} \right\} \text{mix}$$

$$d \in 10_{SU(5)} \in 16_{SO(10)}$$

$$d' \in 5_{SU(5)} \in 10_{SO(10)} \ldots \text{ heavy}$$

The matrix above has the form

$$\mathcal{M} = \begin{pmatrix} m_1 & M_1 \\ m_2 & M_2 \end{pmatrix}$$

with $m_{1,2} = \mathcal{O}(m_Z)$ and $M_{1,2} = \mathcal{O}(M_{GUT})$

All are 3×3 matrices.

the idea is to find a 6×6 unitary matrix $\mathcal{U}$ that

$$\mathcal{U} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \text{something} \end{pmatrix}$$

The solution is

$$\mathcal{U} = \begin{pmatrix} \left(1 + X X^\dagger\right)^{-1/2} & -\left(1 + X X^\dagger\right)^{-1/2} X \\ X^\dagger \left(1 + X X^\dagger\right)^{-1/2} & \left(1 + X^\dagger X\right)^{-1/2} \end{pmatrix}$$

with

$$X = M_1 \, M_2^{-1}$$

so that

$$\mathcal{U}\mathcal{M} = \begin{pmatrix} \underbrace{\mathcal{O}(m_Z)}_{\text{light sector}} & 0 \\ \mathcal{O}(m_Z) & \mathcal{O}(M_{GUT}) \end{pmatrix}$$

For charged fermions they turn out to be

$$M_U = -v_1 Y_{27} + \left(\frac{1}{2\sqrt{10}} v_5 - \frac{1}{2\sqrt{6}} v_7\right) Y_{\overline{351}'},$$

$$M_D^T = \left(1 + X X^\dagger\right)^{-1/2} \left(\left(\bar{v}_2 - \bar{v}_3 X\right) Y_{27} + \left(\frac{1}{2\sqrt{10}}\left(\bar{v}_4 - \bar{v}_9 X\right) + \frac{1}{2\sqrt{6}}\left(\bar{v}_8 - \bar{v}_{11} X\right)\right) Y_{\overline{351}'}\right)$$

$$M_E = \left(1 + \frac{4}{9} X X^\dagger\right)^{-1/2} \left(\left(-\bar{v}_2 - \frac{2}{3}\bar{v}_3 X\right) Y_{27} + \left(-\frac{1}{2\sqrt{10}}\left(\bar{v}_4 + \frac{2}{3}\bar{v}_9 X\right) + \sqrt{\frac{3}{8}}\left(\bar{v}_8 + \frac{2}{3}\bar{v}_{11} X\right)\right) Y_{\overline{351}'}\right)$$

with

$$X = -3\sqrt{\frac{5}{3}}\,\frac{c_2}{f_4}\, Y_{27}\, Y^{-1}_{\overline{351}'},$$

$X \to 0$ gives minimal SO(10), but here not available ($c_2 \neq 0$) !

Y_{27} and $Y_{\overline{351}'}$ symmetric $\to M_U$ symmetric

Not true for X and so not for $M_{D,E}$

Choose system with $M_U = M_U^d$ (diagonal). Then we can always parametrize

$$X = M_U^d Y$$

with

$$Y = Y^T \quad \text{symmetric}$$

$$
\begin{aligned}
M_D^T &= \left(1 + M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(a + b\,(M_U^d Y) + c\,(M_U^d Y)^2\right)\left(d + (M_U^d Y)\right)^{-1} M_U^d \\
M_E &= \left(1 + (4/9) M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(a' + b'\,(M_U^d Y) + c'\,(M_U^d Y)^2\right)\left(d + (M_U^d Y)\right)^{-1} M_U^d \\
M_N &= \left(1 + (4/9)\, M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(a'' + b''\,(M_U^d Y) + c''\,(M_U^d Y)^2 + d''\,(M_U^d Y)^3\right. \\
&\qquad \left. + e''\,(M_U^d Y)^4\right)\left(d + (M_U^d Y)\right)^{-1} M_U^d \\
&\times \left(1 + (4/9)\, M_U^d Y^* Y M_U^d\right)^{-1/2}
\end{aligned}
$$

- Neutrino mass sum of type I and type II contributions
- a, b, c, d, a', b', c', a'', b'', c'', d'', e'' are $f(c_a, f_b, v_i, \bar{v}_j, m_i, \lambda_j)$
- Highly nonlinear, seems hopeless (unless numerically)

But remember that (let's simplify our life assuming $N_g = 2$)

- any function of a 2×2 matrix M can be always written as

$$f(M) = \alpha + \beta M$$

  with α, β written with invariants of M.

- Any 2×2 matrix A can be written as (with basis chosen)

$$A = a_1 + a_2 M_U^d + a_3 Y + a_4 M_U^d Y$$

  This simplifies the work and decreases number of unknowns (combinations)

$$
\begin{aligned}
M_D^T &= \left(1 + M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(\alpha + \beta M_U^d Y\right) M_U^d \\
M_E &= \left(1 + (4/9) M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(\alpha' + \beta' M_U^d Y\right) M_U^d \\
M_N &= \left(1 + (4/9)\, M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(\alpha'' + \beta'' M_U^d Y\right) M_U^d \\
&\times \left(1 + (4/9)\, M_U^d Y^* Y M_U^d\right)^{-1/2}
\end{aligned}
$$

$N_g = 2$ case

Unknowns (9):

α, β, α', β', α'', β'',

$Y_1 \equiv Tr(Y)$, $Y_2 \equiv det(Y)$, $Z \equiv Tr(M_U^d Y)$

To fit (7):

m_s, m_b, m_μ, m_τ, V_{cb},

Δm^2_{23}, $\sin^2\theta_{23}$

Apparently possible to fit, but turns out not easy!

$N_g = 3$ case

$$f(M) = \alpha + \beta M + \gamma M^2$$

Unknowns (15):

α, β, γ, α', β', γ', α'', β'', γ'',

$Y_{1,2,3}$, $Z_{1,2,3}$

To fit (14):

m_d, m_s, m_b, m_e, m_μ, m_τ, $\theta^q_{1,2,3}$,

$\theta^l_{1,2,3}$, Δm^2_{23}, Δm^2_{12}

Still looks possible to fit, but even harder

Conclusions

- E_6 a respectable (although complicated) theory
- shown examples of (so far) realistic cases or even possibly (nontrivially) ruled out

Some open questions:

- Neutrino mass scale should be lower than M_{GUT}. To get it the full mass spectrum at that scale should be known and included in gauge couplings RGEs
- Landau pole very close just above M_{GUT}. Any possibility to treat it ?