Beyond tree-level Majorana neutrino masses: the two-loop case

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WORK IN PROGRESS

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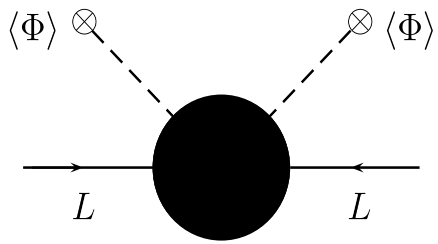
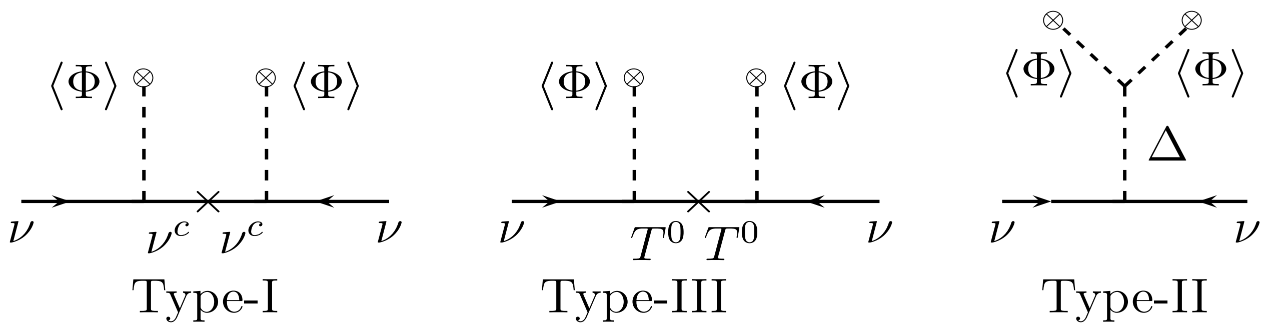
Some remarks on neutrino masses...

Model independent approach: induced by $\mathcal{O}_5 \sim LL\Phi\Phi \Rightarrow \Delta L = 2$

S. Weinberg, Phys. Rev. D **22**, 1694 (1980)



Tree-level UV completions



Minkowski, 1977

Mohapatra & Senjanovic, 1980

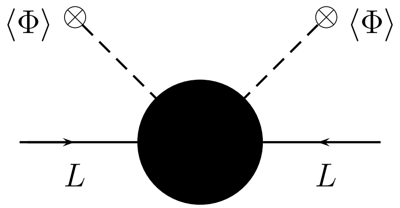
Schechter & JWFV, 1980 ...

Foot, Lew, He & Joshi, 1989

Schechter, JWFV, 1980 ...

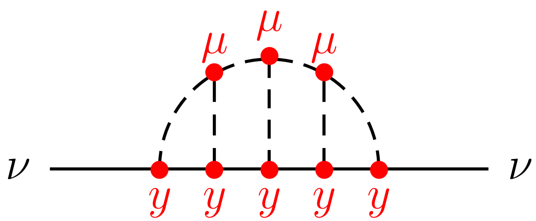
Higher order

Insisting on only $d = 5$ and not slightly broken $U(1)_L$:



$$= \sum_i D^{(i)}$$

Phenomenological constraints however rule out $D^{(i)} > D^{(4)} \ldots$ and perhaps even $D^{(i)} > D^{(3)} \ldots$



$$m_\nu \sim \left(\frac{1}{16\pi^2}\right)^4 m_F^4 y^5 \mu^3 \int d^{16}k \left(\frac{1}{k^2 - m_S^2}\right)^7 \left(\frac{1}{k^2 - m_F^2}\right)^4$$

$$\sim \left(\frac{1}{16\pi^2}\right)^4 \frac{m_F^4}{m_S^6} y^5 \mu^3 \sim 10^3 y^5\,\text{eV} \;\Rightarrow \boldsymbol{\mathcal{O}(y) \sim 0.1}$$

$$\boxed{\text{BR}_{\text{LFV}} > \text{BR}_{\text{LFV}}^{\text{Exp}}}$$

For $D^{(3)}$ one can calculate $\mathcal{O}(y) \sim \mathbf{0.05}$. Some three-loop models analyzed at about ~ 2000-2003:

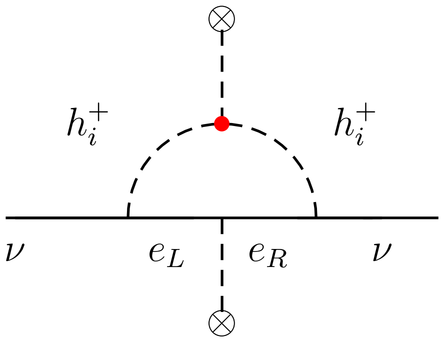
Until 2011 MEGA bound: $\text{BR}_{e\gamma}^{\mu} \lesssim 2.1 \times 10^{-11}$

MEG bound as 2013: $\text{BR}_{e\gamma}^{\mu} \lesssim 5.7 \times 10^{-13}$

3−loop models might be already ruled out (!?)

Basically, viable realizations are reduced to one and two loops:

Zee model



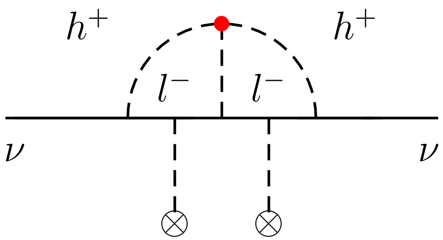
Scalar sector: $h^\pm,\ H_{1,2}$: $\mathcal{L} = f\bar{L}^c L h^+ + \underbrace{\mu H_1 H_2 h^+}_{\Delta L = 2}$

<u>Restricted version</u>: Type-I 2HDM $\Rightarrow$ Maximal $\theta_\odot$ **Ruled out**

<u>General version</u>: Type-III 2HDM **Viable!!**

At the light of LHC data **worth exploring!!**

Cheng-Li-Babu-Zee model



Scalar sector: $h^+,\ k^{++}$: $\mathcal{L} = f\bar{L}^c L h^+ + h\bar{e}^c e k^{++} + \underbrace{\mu h^+ h^+ k^{--}}_{\Delta L = 2}$

Rich LFV and collider phenomenology

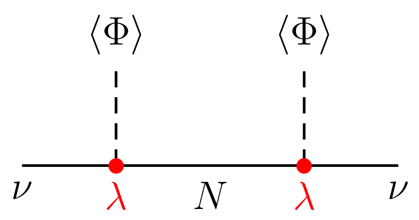
$\mathrm{Br}(\mu \to e\gamma)$ can place stringent constraints

Recently reanalyzed by Herrero et. al./Schwetz et. al.

Worth exploring at the LHC and/or ILC!

High scale approaches

"Conventional wisdom": Neutrino acquire their masses via the type-I seesaw (standard seesaw):



- $M_N \gg \Lambda_{\rm EW} \quad \Rightarrow \quad \mathcal{O}(\lambda) \sim 1$ $\boxed{m_\nu \sim 0.1 \text{ eV}}$
- $N \to LH$ addresses $n_{\Delta B} \sim 10^{-10}$

Lacking experimental prove

- No direct prove possible given the large scale involved $M_N \sim \Lambda_{\rm GUT}$
- No indirect test possible:

$$\{9|\lambda_{ij}|, 6\,\text{CP phases}, 3M_N\} \quad \textbf{VS} \quad \{3|\theta_{ij}|, 3\,\text{CP phases}, 3m_{\nu_i}, n_{\Delta B}\}$$

The Lagrangian parameters can not be reconstructed

A "novel" path can be followed to "test" these approaches

Underpinning the mechanism?

Models involving LHC physics are based in the following possibilities:

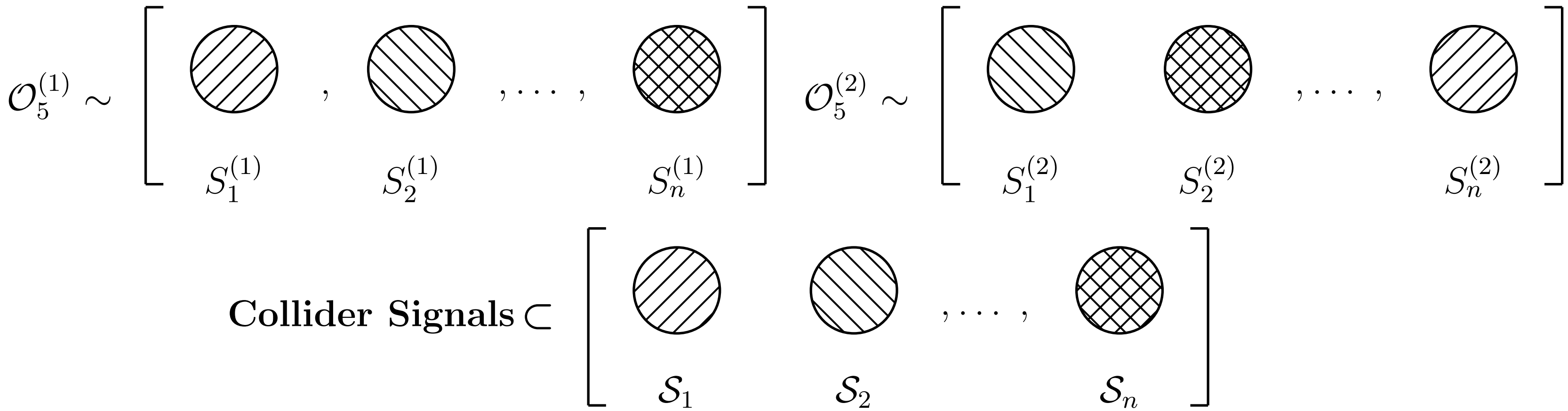
Bonnet, Hernandez, Ota and Winter [arXiv:0907.3143]

1. $\mathscr{O}_5$ arising at the one or two loop order.
2. $\mathscr{O}_5 = 0$ and so Majorana neutrino masses generated from $d = 7$ effective operators.
3. $\mathscr{O}_5$ involving small parameters related with slightly broken L.

IDEAL/NAIVE PROGRAM

I. Systematically classify the viable $\mathscr{O}_5$ one and two loop realizations.

II. Classify the different possibilities in sets, according to their collider signals.



Addressing item I.

A systematic classification of the possible realizations is feasible through the following **"recipe"**

Bonnet, Hirsch, Ota and Winter [arXiv:arXiv:1204.5862]

Algorithm

1. Identify possible topologies.
2. For all possible external legs configurations $(2\Phi + 2L)$ insert internal lines (fermion or boson) subject to renormalizability conditions.
3. Calculate loop integrals
4. Assuming the internal fermion/bosons are $SU(3)_C$ singlets fix the $SU(2)_L \times U(1)_Y$ quantum numbers.

Following different approach, partially done at the 1-loop level by E. Ma [hep-ph/9805219]

Items 1 & 2 can be done by using `FeynArts` cleverly

Following "algorithm", task completed by Bonnet, Hirsch, Ota, Winter for 1-loop. arXiv:1204.5862

Farzan et. al. arXiv:1208.2732

Volkas et. al. arXiv:1212.6111

First step: topologies

Two-loop case: topologies (I)

Ask FeynArts to calculate $2 \leftrightarrow 2$ “scattering”

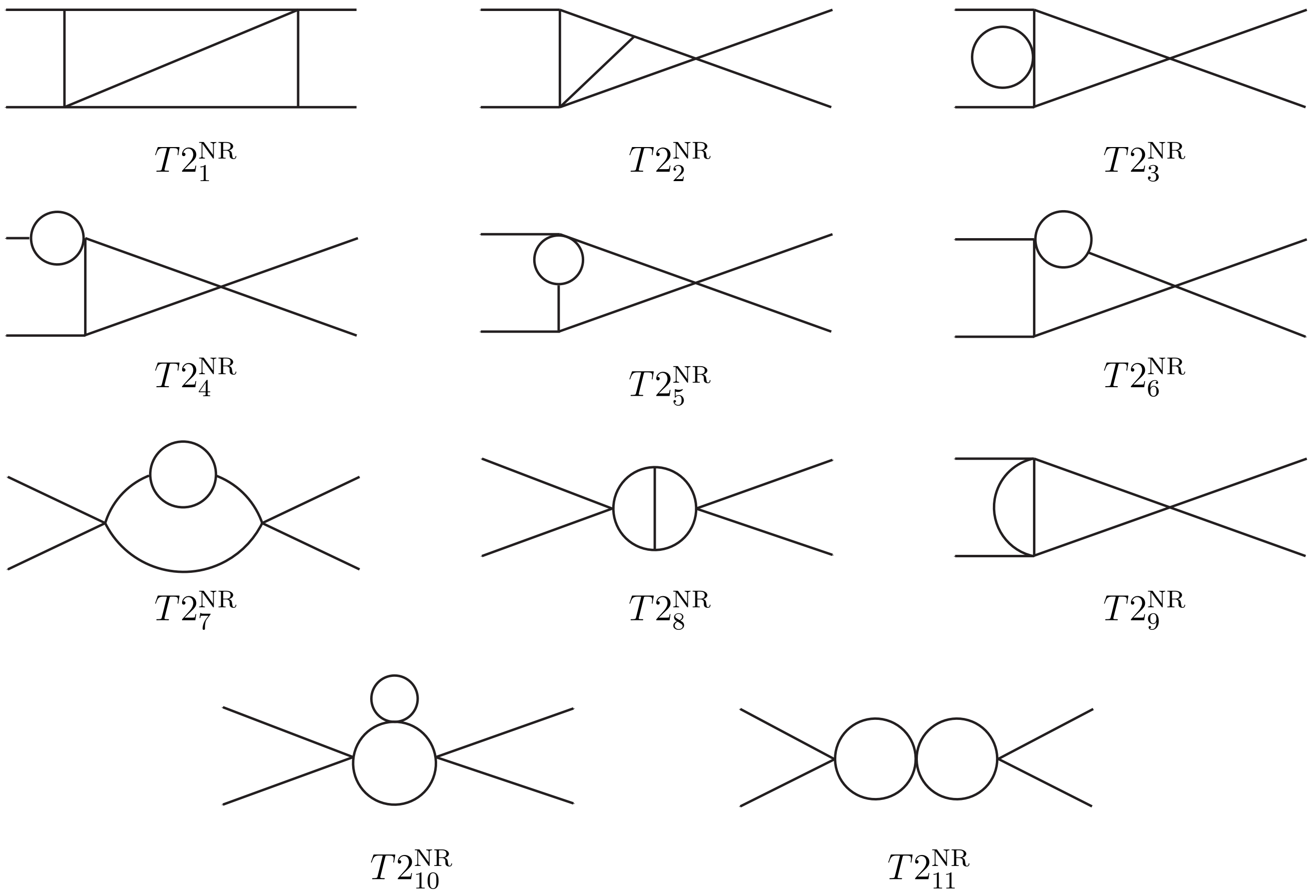
for **only** ID and without self-energies and tadpoles

$\mathcal{O} \sim$ **200** diagrams

HOPELESS?

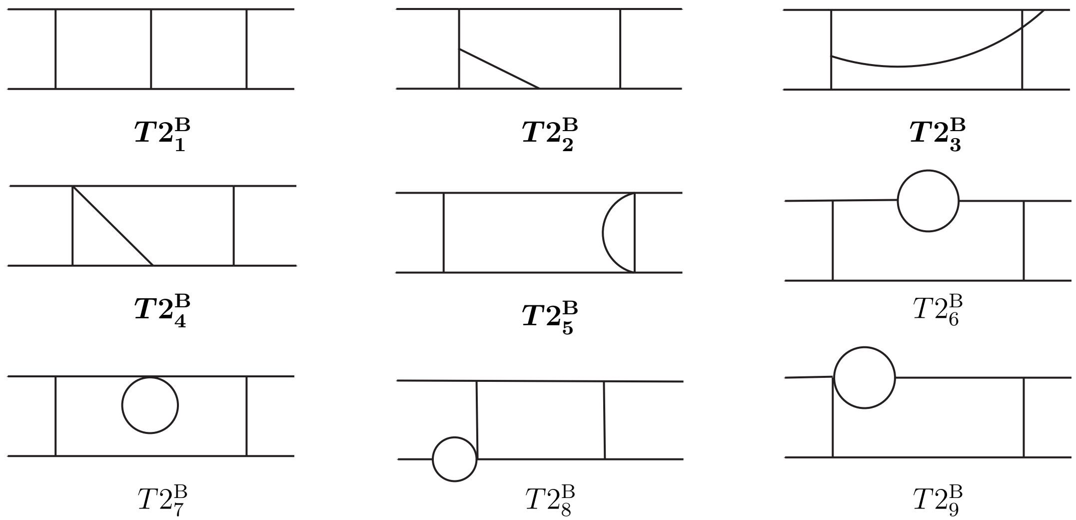
Topological equivalence ⇔ 29

Non-renormalizable

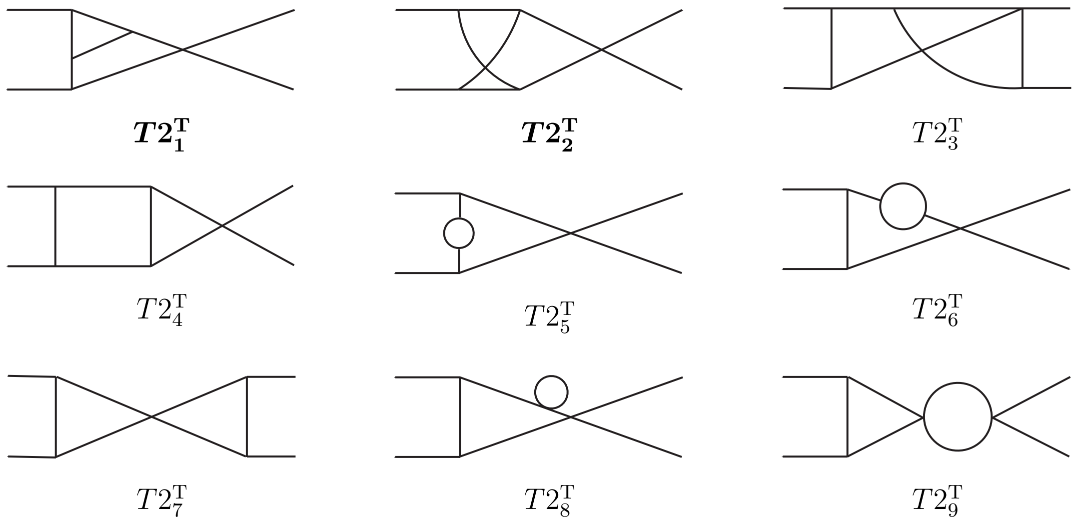


Two-loop case: topologies (II)

Box-based



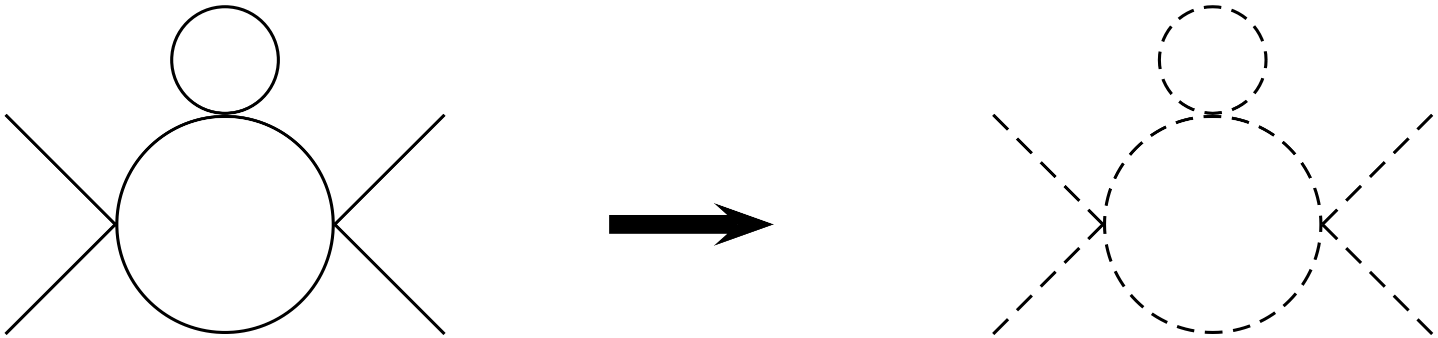
Triangular-based



Selecting criteria

Selecting relevant topologies should be done systematically as well, and this requires a "tasty recipe".

Renormalizability criteria: 3PVs: $F^2 S$, S^3 and 4PV: S^4 $\Rightarrow$ Topologies involving two external 4PVs are in general NR.



Only Box-based and triangular-based topologies are relevant in the general problem

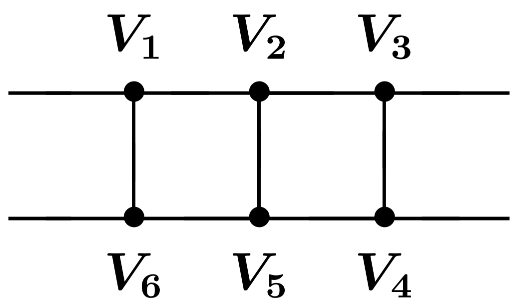
Second step: Field insertions

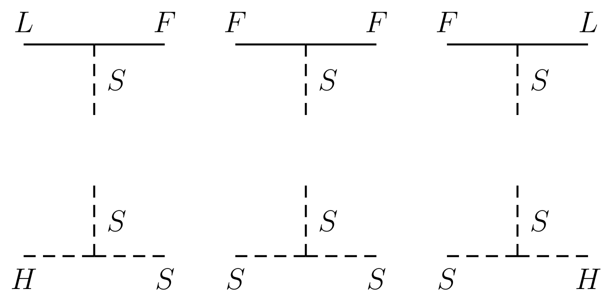
Approach

Focusing only on fermions and scalar bosons [Not considering gauge bosons]:

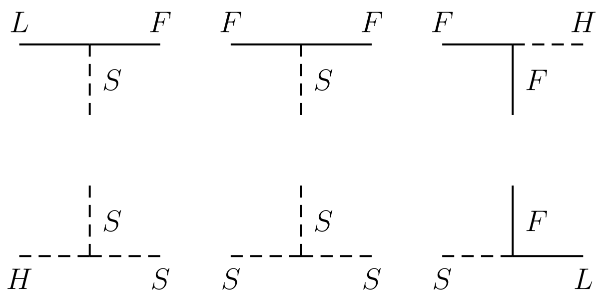
Ask `FeynArts` to insert fermions and bosons

Double check by hand using tree-like structures and sequential vertex insertions

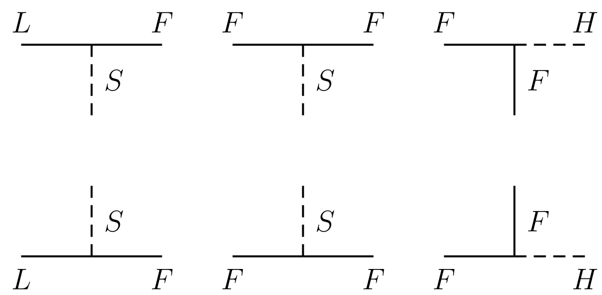




$= D_1^{T2_1^B}$



$= D_2^{T2_1^B}$



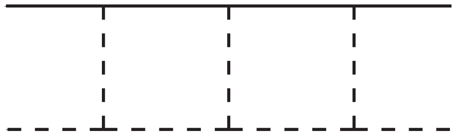
$= D_3^{T2_1^B}$

Full sequential insertion

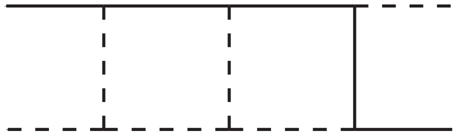
By following that procedure one can find the diagrams associated to each of the relevant topologies. For $T2_1^B$:

V_1	V_2	V_3	V_4	V_5	V_6	Diagram
LSF	FSF	FSL	HSS	SSS	SSH	$D_1^{T2_1^B}$
		FFH	LFS	SSS	SSH	$D_2^{T2_1^B}$
			HFF	FSF	FSL	$D_3^{T2_1^B}$
	FFS	SFL	HFF	FFS	SSH	$D_4^{T2_1^B}$
			LFS	SFF	FSL	✗
		SSH	HSS	SFF	FSL	$D_5^{T2_1^B}$
			LSF	FFS	SSH	$D_6^{T2_1^B}$
LFS	SFF	FFH	LFS	SFF	FFH	$D_7^{T2_1^B}$
			HFF	FFS	SFL	$D_8^{T2_1^B}$
		FSL	LSF	FFS	SFL	✗
			HSS	SFF	FFH	$D_4^{T2_1^B}$
	SSS	SFL	LFS	SSS	SFL	✗
			HFF	FSF	FFH	$D_9^{T2_1^B}$
		SSH	LSF	FSF	FFH	$D_2^{T2_1^B}$
			HSS	SSS	SFL	$D_{10}^{T2_1^B}$

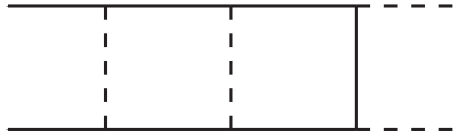
Results for double-box topology



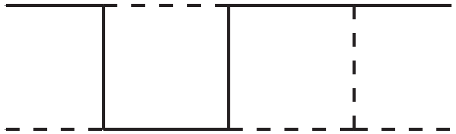
$D_1^{T2_1^{\mathbf{B}}}$



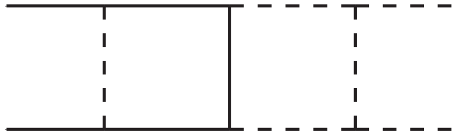
$D_2^{T2_1^{\mathbf{B}}}$



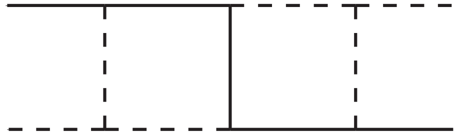
$D_3^{T2_1^{\mathbf{B}}}$



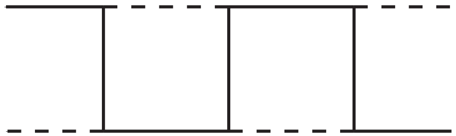
$D_4^{T2_1^{\mathbf{B}}}$



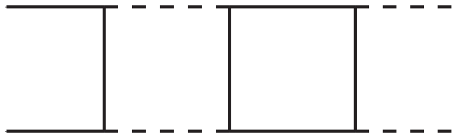
$D_5^{T2_1^{\mathbf{B}}}$



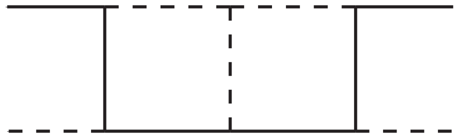
$D_6^{T2_1^{\mathrm{B}}}$



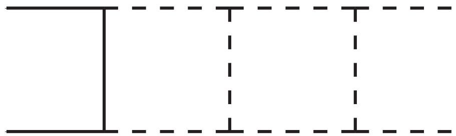
$D_7^{T2_1^{\mathrm{B}}}$



$D_8^{T2_1^{\mathrm{B}}}$



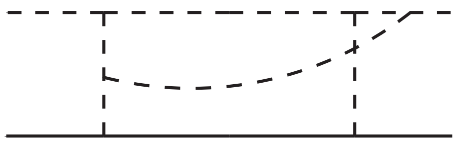
$D_9^{T2_1^{\mathrm{B}}}$



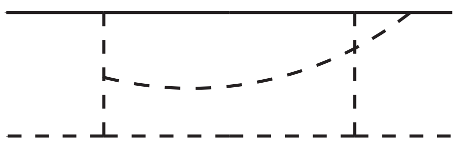
$D_{10}^{T2_1^{\mathrm{B}}}$

Another example: non-coplanar diagrams

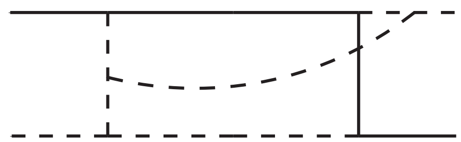
For the non-coplanar box-based topology the tree-like structures and sequential vertex insertion lead to:



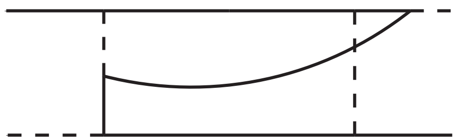
$D_1^{T2_3^{\mathbf{B}}}$



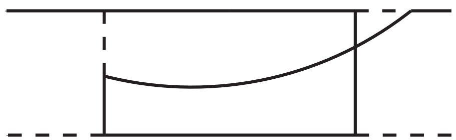
$D_2^{T2_3^{\mathbf{B}}}$



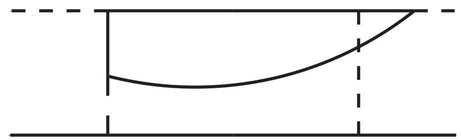
$D_3^{T2_3^{\mathbf{B}}}$



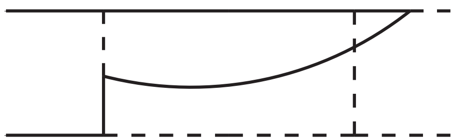
$D_4^{T2_3^{\mathbf{B}}}$



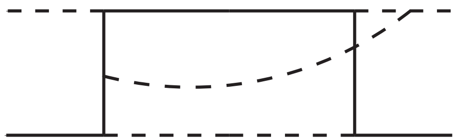
$D_5^{T2_3^{\mathbf{B}}}$



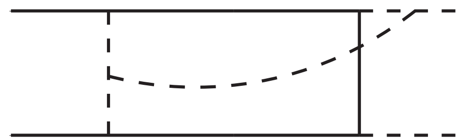
$D_6^{T2_3^{\mathrm{B}}}$



$D_7^{T2_3^{\mathrm{B}}}$



$D_8^{T2_3^{\mathrm{B}}}$



$D_9^{T2_3^{\mathrm{B}}}$

Second step: rèsumè

At this point the number of possible diagrams can be already determined.

However with certain caution!

Box-based topologies

TOPOLOGY	$T2_1^B$	$T2_2^B$	$T2_3^B$	$T2_4^B$	$T2_5^B$	$T2_6^B$	$T2_7^B$	$T2_8^B$	$T2_9^B$	TOTAL
# OF DIAG	10	14	9	3	1	12	4	2	3	58

Triangle-based topologies

TOPOLOGY	$T2_1^T$	$T2_2^T$	$T2_3^T$	$T2_4^T$	$T2_5^T$	$T2_6^T$	$T2_7^T$	$T2_8^T$	$T2_9^T$	TOTAL
# OF DIAG	2	1	2	2	1	2	1	1	1	13

Order-2-uniqueness applied to resulting diagrams

Order-2-uniqueness: $D2_i$ present while $D1_i$ absent.

RECIPE

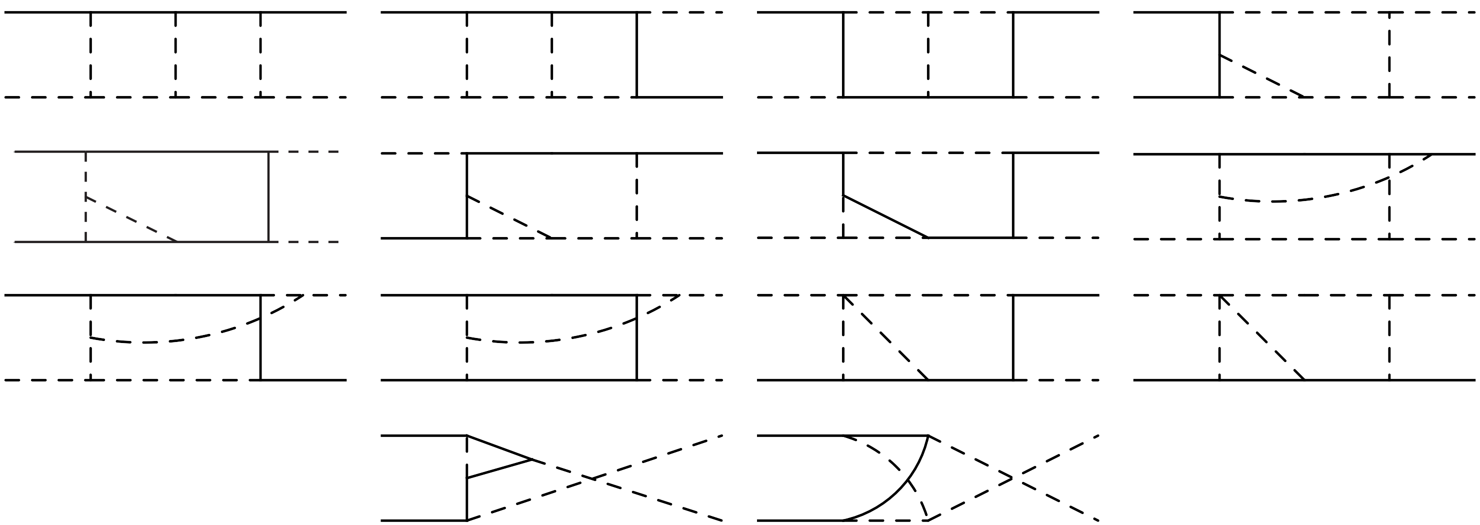
1. Identify the diagram from which $D2_i$ originates (one-loop box or triangle)
2. Assign arbitrary charges q_i to all fields (new symmetry, gauge symmetry itself).
3. Impose q_i conservation vertex by vertex and derive C^{2_i} and C^{1_i}.

Solutions are $C^{1_i} \subset C^{2_i}$ $\Rightarrow$ Non-genuine diagram

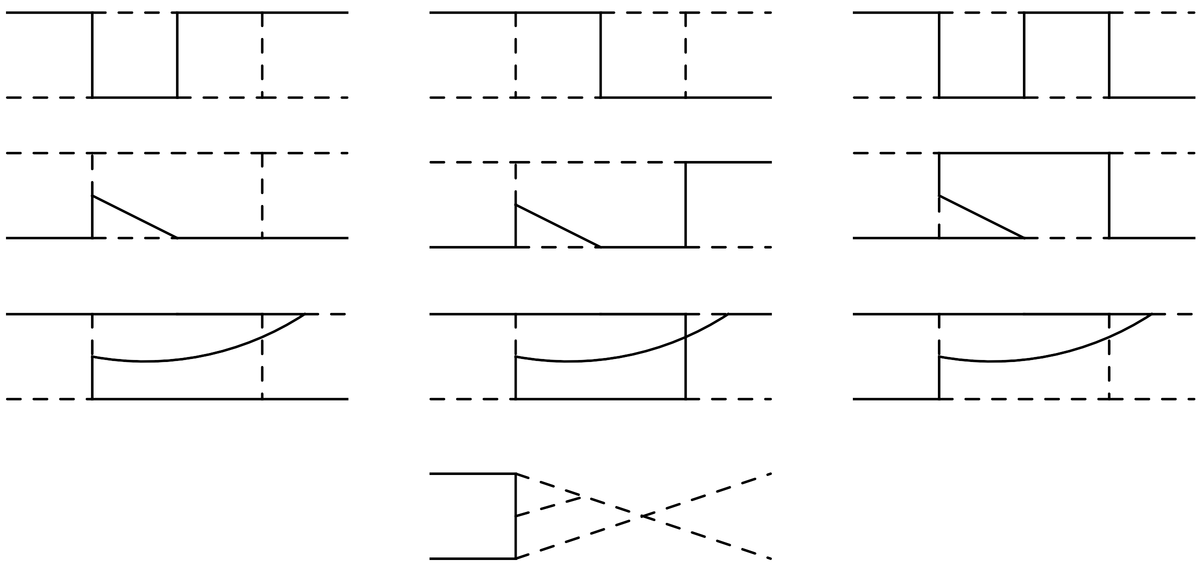
Solutions are such that $C^{1_i} \not\subset C^{2_i}$ $\Rightarrow$ Genuine diagram

Genuine diagrams

GROUP 1



GROUP 2

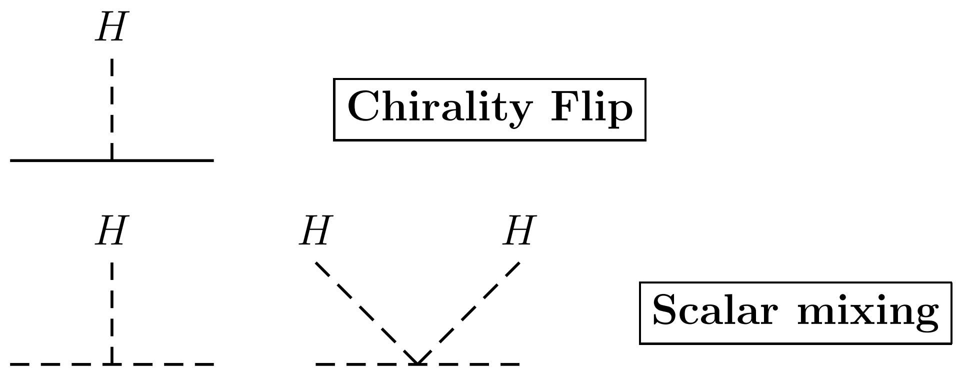


Third step: Two-loop integrals

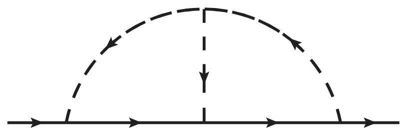
Number of relevant integrals

External Higgs legs determine the type of interactions needed for a certain diagram to be constructed: **essential in the determination of the different realizations.**

For 2-loop calculation scalar external legs are irrelevant

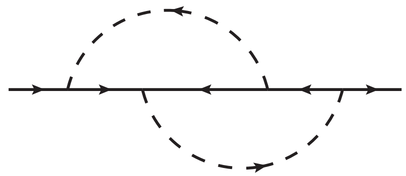


Group 1



McDonald, McKellar [hep-ph/0309270]

Group 2



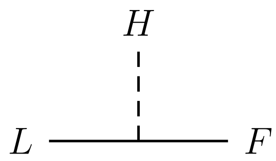
J. Herrero et. al. [arXiv:1104.4068]
P.W.Angel et. al. [arXiv:1308.0463]

Fourth step: Quantum numbers

Approach

The lepton and Higgs G_{SM} quantum numbers can be used to “fix” the quantum numbers of the BSM fields:

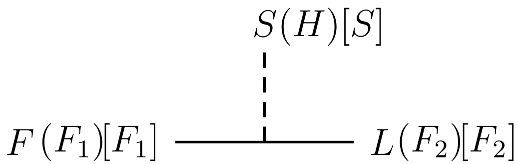
Yukawas



$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3}$$

$$Y_L + Y_H + Y_F = 0$$

Unique

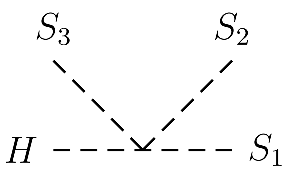


$$\mathbf{n_F} \otimes \mathbf{n_S} \supset \mathbf{2}$$

$$Y_F + Y_S + Y_L = 0$$

Not Unique

Quartic couplings



$$\mathbf{n_{S_1}} \otimes \mathbf{n_{S_2}} \otimes \mathbf{n_{S_3}} \supset \mathbf{2}$$

$$Y_H + \sum_i Y_{S_i} = 0$$

Not Unique

Vertices involving more than one BSM field allow in principle infinite choices

Stick to EW singlets, doublets and triplets and fix $Q < 3$

Summary

Rèsumè

- Loop-induced neutrino masses allow for low-scale (TeV) physics, in some cases testable at LHC.
- Testing the origin of neutrino masses can be done by experimentally studying the signals arising from these realizations.
- Systematic analysis of categories is needed.

At the 2-loop order such a task is possible and worth doing!