The Zee-Babu revisited in the light of new data

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NuNews, Stockholm, 11th April 2014

Outline

Introduction

Neutrino oscillations: ν are massive and mix

We don't know the absolute ν mass scale, only upper bounds:

- Cosmology: $\sum_i m_i < 0.23$.
- Tritium β decay: $m_{\nu_e} \equiv \sqrt{\sum_i m_i^2 |U_{ei}|^2} < 2\,\mathrm{eV}$.
- $0\nu\beta\beta$: $m_{ee} \equiv |\sum_i m_i U_{ei}^2| < 0.2\,\mathrm{eV}$.

nor the spectrum:

- NH: $m_1 \lesssim m_2 < m_3$ - $\sum_i m_i \gtrsim 0.06$ eV.
- IH: $m_3 < m_1 \lesssim m_2$ - $\sum_i m_i \gtrsim 0.1$ eV.
- QD: $m_1 \approx m_2 \approx m_3$.

Fits to neutrino oscillations

NuFIT 1.2 (2013)

	Free Fluxes + RSBL		Huber Fluxes, no RSBL	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.346$	$0.313^{+0.013}_{-0.012}$	$0.277 \to 0.355$
$\theta_{12}/^\circ$	$33.57^{+0.77}_{-0.75}$	$31.37 \to 36.01$	$34.02^{+0.79}_{-0.76}$	$31.78 \to 36.55$
$\sin^2\theta_{23}$	$0.446^{+0.008}_{-0.008} \oplus 0.593^{+0.027}_{-0.043}$	$0.366 \to 0.663$	$0.444^{+0.037}_{-0.031} \oplus 0.592^{+0.028}_{-0.042}$	$0.361 \to 0.665$
$\theta_{23}/^\circ$	$41.9^{+0.5}_{-0.4} \oplus 50.3^{+1.6}_{-2.5}$	$37.2 \to 54.5$	$41.8^{+2.1}_{-1.8} \oplus 50.3^{+1.6}_{-2.5}$	$36.9 \to 54.6$
$\sin^2\theta_{13}$	$0.0231^{+0.0019}_{-0.0019}$	$0.0173 \to 0.0288$	$0.0244^{+0.0019}_{-0.0019}$	$0.0187 \to 0.0303$
$\theta_{13}/^\circ$	$8.73^{+0.35}_{-0.36}$	$7.56 \to 9.77$	$9.00^{+0.35}_{-0.36}$	$7.85 \to 10.02$
$\delta_{\rm CP}/^\circ$	266^{+55}_{-63}	$0 \to 360$	270^{+77}_{-67}	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.45^{+0.19}_{-0.16}$	$6.98 \to 8.05$	$7.50^{+0.18}_{-0.17}$	$7.03 \to 8.08$
$\frac{\Delta m^2_{31}}{10^{-3}\ \text{eV}^2}$ (N)	$+2.417^{+0.014}_{-0.014}$	$+2.247 \to +2.623$	$+2.429^{+0.055}_{-0.054}$	$+2.249 \to +2.639$
$\frac{\Delta m^2_{32}}{10^{-3}\ \text{eV}^2}$ (I)	$-2.411^{+0.062}_{-0.062}$	$-2.602 \to -2.226$	$-2.422^{+0.063}_{-0.061}$	$-2.614 \to -2.235$

Figure: Global fit from Gonzalez-Garcia et al.

Unsolved questions in the neutrino sector

- The nature Dirac or Majorana ($0\nu\beta\beta$), i.e., whether LN is a conserved symmetry or not.
- The existence of leptonic CP violation and its value.
- LFV in the charged sector, like $\mu \to e\gamma$.
- The absolute mass scale of neutrinos.
- The spectrum.
- The octant of θ_{23} and more precise mixings/masses.
- If sterile neutrinos exist.
- By which mechanism their masses are generated...

Neutrino masses

Dirac neutrino masses

Add ν_{R} to the SM

$$m_{\mathrm{D}}\,\overline{\nu_{\mathrm{R}}}\,\nu_{\mathrm{L}} + \text{H.c.}$$

Pros:

- Dirac masses *exist* for all other fermions.

Cons:

- Impose *by hand* B-L, which in the SM (without ν_R) is accidental.
- fine-tuned Yukawas, 12 orders of magnitude smaller than the top one!

Majorana neutrino masses

$$m_{\rm L}\,\overline{\nu_{\rm L}^c}\,\nu_{\rm L}\;+\text{H.c.}$$

- violates LN.
- It can explain why m_ν is so smaller than other fermion masses.
- not gauge-invariant in the SM: higher order operator.

The Weinberg operator

The SM is a very good EFT, with NP at higher scales Λ:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{n=5} \sum_{i} \left(\frac{C_i^n}{\Lambda^{n-4}} \, \mathcal{O}_i^n + {\rm H.c.} \right) .$$

where n is the operator dimension, i labels the different operators of a given n, and C_i^n are dimensionless coefficients.
The only D=5 op. with the SM fields happens to violate LN:

$$\mathcal{L}_5 = \frac{1}{2} \frac{c_{\alpha\beta}}{\Lambda} (\overline{\ell_\alpha} \tilde{\phi}) \, (\phi^\dagger \tilde{\ell}_\beta) + {\rm H.c.} \, ,$$

where $c_{\alpha\beta}$ are model-dependent coefficients and $\tilde{\ell} \equiv i\sigma_2 \ell^c$. Upon EWSB it gives Majorana masses to neutrinos:

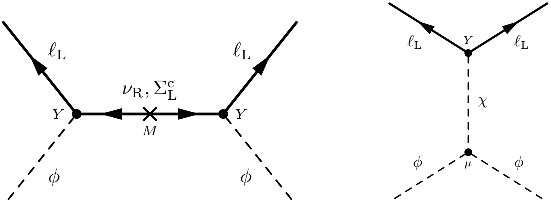
$$m_\nu = c \, \frac{v^2}{\Lambda} \, .$$

Opening the Weinberg operator at tree level: seesaws

- Rewriting the Weinberg operator:

$$\left(\overline{\ell_\alpha}\tilde{\phi}\right)\left(\phi^\dagger\tilde{\ell}_\beta\right) = -\left(\overline{\ell_\alpha}\,\vec{\sigma}\tilde{\phi}\right)\left(\phi^\dagger\,\vec{\sigma}\tilde{\ell}_\beta\right) = \frac{1}{2}\left(\overline{\ell_\alpha}\,\vec{\sigma}\,\tilde{\ell}_\beta\right)\left(\phi^\dagger\,\vec{\sigma}\,\tilde{\phi}\right),$$

where α and β are family indices and $\vec{\sigma} \equiv (\sigma_1, \sigma_2, \sigma_3)$.



- 3 different particles can generate Weinberg op. at tree level:
  - a $Y = 0$ heavy fermion singlet (triplet), type I (III) seesaw.
  - a $Y = 1$ heavy scalar triplet, type II seesaw.
- Explains why ν's are light: they couple to high scale fields.
- Drawbacks: typically difficult to test, problem of hierarchies.

more

Opening the Weinberg Operator at loop level

- at loop level: radiative models. ν's are light because they are massless at tree level, with their masses generated at i loops

$$m_\nu \propto \frac{1}{(4\pi)^{2i}}$$

- Typically there are additional suppressions due to couplings (so LN is violated) and/or ratios of masses.
- More than three loops typically yield too light m_ν, so $i < 4$.
- So the scale can be not too far away from EWS and can be tested.

more

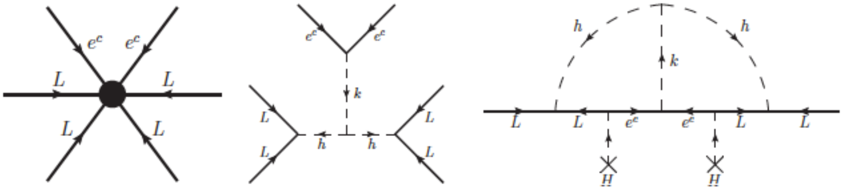
The Zee-Babu model

The Zee-Babu model [Cheng and Li, Zee, Babu, Aristizábal, Nebot, Ohlsson, Schmidt...]

Consider the D= 9 $\Delta L = 2$ eff. operator $\ell\ell\ell\ell e^c e^c$. It generates the Weinberg op. at some loop level (and therefore m_ν). By NDA:

$$m_\nu \sim \frac{1}{(4\pi)^4} \frac{y_e^2 v^2}{\Lambda}$$

One can open this op. by adding a singly- and a doubly-charged scalar $h^\pm, k^{\pm\pm}$ with $Y_h = \pm 1$ and $Y_k = \pm 2$ resp.



The interactions in the Zee-Babu model

$$\mathcal{L}_Y = \overline{\ell}\, Y e \phi + \overline{\tilde{\ell}} f \ell h^+ + \overline{e^c} g\, e\, k^{++} + \mathrm{H.c.}$$

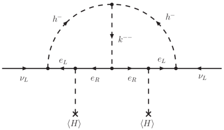
Due to Fermi statistics, f_{ab} is AS while g_{ab} is S.

$$\begin{aligned} V &= m_\phi'^2 \phi^\dagger \phi + m_h'^2 |h|^2 + m_k'^2 |k|^2 + \lambda_\phi (\phi^\dagger \phi)^2 + \lambda_h |h|^4 + \lambda_k |k|^4 \\ &+ \lambda_{hk} |h|^2 |k|^2 + \lambda_{h\phi} |h|^2 \phi^\dagger \phi + \lambda_{k\phi} |k|^2 \phi^\dagger \phi + \left(\mu h^2 k^{++} + \mathrm{H.c.} \right) \end{aligned}$$

LNV requires simultaneous presence of Y, f, g, μ. We choose:

- Y to be diag. with > 0 elements, the charged lepton masses.
- We use fermion field rephasings to remove 3 phases from g.
- By scalar rephasings we set $\mu > 0$ and remove one phase from f.

Generating the Weinberg Operator at two loops



$$(\mathcal{M}_\nu)^{\text{exp}}_{ij} = (U D_\nu U^T)_{ij} = 16 \mu f_{ia} m_a g^*_{ab} I_{ab} m_b f_{jb} = (\mathcal{M}_\nu)^{\text{ZB}}_{ij}$$

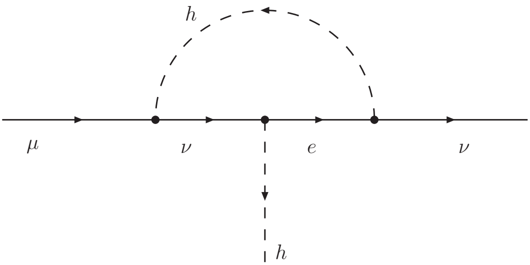
$$I_{ab} \simeq I = \frac{1}{(16\pi^2)^2} \frac{1}{M^2} \frac{\pi^2}{3} \tilde{I}(r), \quad r \equiv m_k^2 / m_h^2, \quad M \equiv \max(m_h, m_k).$$

$$\tilde{I}(r) = 1 \quad \text{for} \quad r \to 0, \quad 1 + \frac{3}{\pi^2}(\log^2 r - 1) \quad \text{for} \quad r \gg 1$$

$$\mathcal{M}_\nu = \frac{v^2 \mu}{48 \pi^2 M^2} \tilde{I}\, f\, Y\, g^\dagger Y^T f^T$$

f is AS $\to$ $\det f = 0 \to \det \mathcal{M}_\nu = 0$, so one massless ν (no QD).

Perturbativity



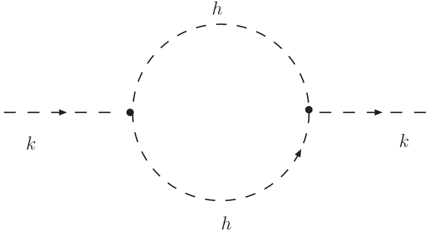
- Since one-loop corrections to Yukawa couplings are order

$$\delta f \sim \frac{f^3}{(4\pi)^2}\,, \qquad \delta g \sim \frac{g^3}{(4\pi)^2}\,,$$

one expects $f, g < 4\pi$.

- Also from perturbativity, $\lambda_{h,k,k\phi,h\phi,hk} < 4\pi$.

Naturality



- μ induces radiative corrections to the masses of the scalars

$$\delta m_k^2, \delta m_h^2 \sim \frac{\mu^2}{(4\pi)^2} \, .$$

so $\mu < 4\pi \min(m_h, m_k)$. We take:

$$\mu < \kappa \min(m_h, m_k) \, ,$$

and discuss our results for different values of $\kappa = 1, 5, 4\pi$.

- Also Higgs receives corrections; we will assume $m_{h,k} \lesssim 2$ TeV.

Charge breaking

- A large value of μ could lead to a deeper minimum of the scalar potential for non-vanishing values of the charged fields, therefore breaking charge conservation.
- Looking at $|H| = |h| = |k| = r$, and requiring $V(r \neq 0) > 0$:

$$\mu^2 < (\lambda_H + \lambda_h + \lambda_k + \lambda_{hH} + \lambda_{kH} + \lambda_{hk}) \left(m_H'^2 + m_h'^2 + m_k'^2\right) .$$

- Assuming no cancellations and using conservatively $\lambda_i \leq 4\pi$:

$$\mu \lesssim \sqrt{20\pi} \max(m_k, m_h) \sim 8 \max(m_k, m_h).$$

Stability of the potential

$$\lambda_H > 0, \qquad \lambda_h > 0, \qquad \lambda_k > 0\,.$$

For negative mixed couplings, $\lambda_{xH}, \lambda_{hk}$ $(x = h, k)$, whenever one of the scalar fields H, h, k is zero stability implies

$$\alpha, \beta, \gamma > -1\,,$$

where we have defined

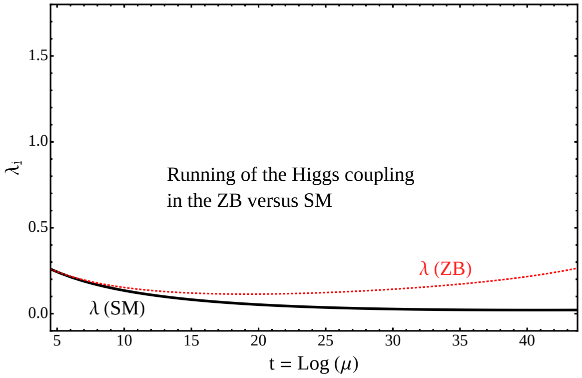
$$\alpha = \lambda_{hH}/(2\sqrt{\lambda_H \lambda_h})\,,\ \beta = \lambda_{kH}/(2\sqrt{\lambda_H \lambda_k})\,,\ \gamma = \lambda_{hk}/(2\sqrt{\lambda_h \lambda_k})\,.$$

If ≥ 2 mixed couplings are < 0, there is an extra constraint:

$$1 - \alpha^2 - \beta^2 - \gamma^2 + 2\alpha\beta\gamma > 0 \qquad \vee \qquad \alpha + \beta + \gamma > -1\,.$$

Running of λ_H (shown at one-loop)

$\lambda_{hH}, \lambda_{kH}$ always contribute positively to the running of λ_H, compensating for the top. Therefore, vacuum stability is alleviated and $\lambda_H > 0$ up to M_P for the present central values of m_t and m_H.



$H \to \gamma\gamma$

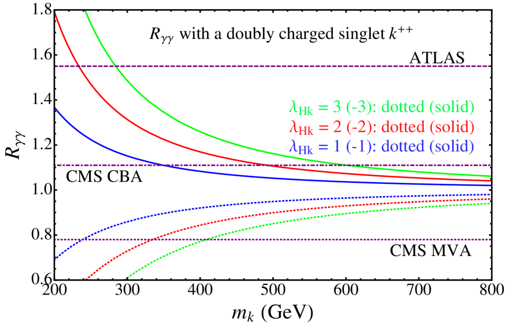
$$R_{\gamma\gamma} = \frac{\Gamma(H \to \gamma\gamma)_{ZB}}{\Gamma(H \to \gamma\gamma)_{SM}} = \left|1 + \delta R(m_h, \lambda_{hH}) + 4\,\delta R(m_k, \lambda_{kH})\right|^2 \ ,$$

where (with $\tau_i \equiv \frac{4 m_i^2}{m_H^2}$):

$$\delta R(m_x, \lambda_{xH}) \equiv \frac{\lambda_{xH}\, v^2}{2 m_x^2} \frac{A_0(\tau_x)}{A_1(\tau_W) + \frac{4}{3} A_{1/2}(\tau_t)} \ ,$$

$$\begin{aligned} \text{ATLAS}: \quad & R_{\gamma\gamma} = 1.55^{+0.33}_{-0.28} \ , \\ \text{CMS}: \quad & R_{\gamma\gamma} = 0.78^{+0.28}_{-0.26} \ , \quad \text{MVA analysis} \\ \text{CMS}: \quad & R_{\gamma\gamma} = 1.11^{+0.32}_{-0.31} \ , \quad \text{cut based analysis} \end{aligned}$$

$H \to \gamma\gamma$ ($H \to Z\gamma$ is anti-correlated)



- For $R_{\gamma\gamma} \sim 1.5$, $m_h \lesssim 250\,\mathrm{GeV}$ and/or $m_k \lesssim 350\,\mathrm{GeV}$.
- From stability $2\sqrt{\lambda_H \lambda_x} + \lambda_{xH} > 0$, for $x = h, k$, and $\lambda_H \sim 0.13$, so large and negative $-3 \lesssim \lambda_{xH}$ push $\lambda_x \to\sim 4\pi$.
- For such values RGEs lead to vacuum instability ($2\sqrt{\lambda_H \lambda_x} + \lambda_{xH} < 0$, $x = h, k$) and/or non-perturbativity ($\lambda_x > 4\pi$) at $\mathcal{O}$(1-100 TeV).

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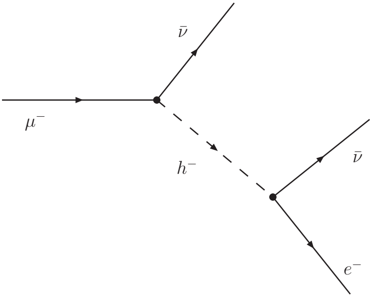
$\ell_a \to \ell_b \nu\bar{\nu}$ bounds the f_{ab} couplings



$$\left(\frac{G_\mu}{G_{\mu SM}}\right)^2 \approx 1 + \frac{\sqrt{2}}{G_F m_h^2}\,|f_{e\mu}|^2 + \frac{1}{2G_F^2 m_h^4}\left(|f_{e\mu}|^2 + |f_{e\tau}|^2\right)\left(|f_{e\mu}|^2 + |f_{\mu\tau}|^2\right)$$

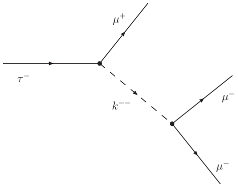
Since the extraction of CKM V_{ij}^{exp} assumes the SM:

$$V_{ij}^{exp} = \frac{G_\beta}{G_\mu} V_{ij}$$

$$|V_{ud}^{exp}|^2 + |V_{us}^{exp}|^2 + |V_{ub}^{exp}|^2 = \frac{G_\beta^2}{G_\mu^2} = \frac{G_{\mu SM}^2}{G_\mu^2} \approx 1 - \frac{\sqrt{2}}{G_F m_h^2} |f_{e\mu}|^2$$

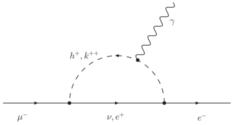
Univ.	Experiment	Bound (90%CL)
l/h	$\sum_q \lvert V_{uq}\rvert^2 = 0.9999 \pm 0.0006$	$\lvert f_{e\mu}\rvert^2 < 0.007 \left(\frac{m_h}{\mathrm{TeV}}\right)^2$
μ/e	$\frac{G_\mu^{exp}}{G_e^{exp}} = 1.0010 \pm 0.0009$	$\lvert\lvert f_{\mu\tau}\rvert^2 - \lvert f_{e\tau}\rvert^2\rvert < 0.024 \left(\frac{m_h}{\mathrm{TeV}}\right)^2$
τ/μ	$\frac{G_\tau^{exp}}{G_\mu^{exp}} = 0.9998 \pm 0.0013$	$\lvert\lvert f_{e\tau}\rvert^2 - \lvert f_{e\mu}\rvert^2\rvert < 0.035 \left(\frac{m_h}{\mathrm{TeV}}\right)^2$
τ/e	$\frac{G_\tau^{exp}}{G_e^{exp}} = 1.0034 \pm 0.0015$	$\lvert\lvert f_{\mu\tau}\rvert^2 - \lvert f_{e\mu}\rvert^2\rvert < 0.04 \left(\frac{m_h}{\mathrm{TeV}}\right)^2$

$\ell_a^- \to \ell_b^+ \ell_c^- \ell_d^-$ bounds the g_{ab} couplings



Process	Exp. (90% CL)	Bound (90% CL)
$\mu^- \to e^+ e^- e^-$	BR$< 1.0 \times 10^{-12}$	$\lvert g_{e\mu} g_{ee}^* \rvert < 2.3 \times 10^{-5} \left(\frac{m_k}{\mathrm{TeV}}\right)^2$
$\tau^- \to e^+ e^- e^-$	BR$< 2.7 \times 10^{-8}$	$\lvert g_{e\tau} g_{ee}^* \rvert < 0.009 \left(\frac{m_k}{\mathrm{TeV}}\right)^2$
$\tau^- \to e^+ e^- \mu^-$	BR$< 1.8 \times 10^{-8}$	$\lvert g_{e\tau} g_{e\mu}^* \rvert < 0.005 \left(\frac{m_k}{\mathrm{TeV}}\right)^2$
$\tau^- \to e^+ \mu^- \mu^-$	BR$< 1.7 \times 10^{-8}$	$\lvert g_{e\tau} g_{\mu\mu}^* \rvert < 0.007 \left(\frac{m_k}{\mathrm{TeV}}\right)^2$
$\tau^- \to \mu^+ e^- e^-$	BR$< 1.5 \times 10^{-8}$	$\lvert g_{\mu\tau} g_{ee}^* \rvert < 0.007 \left(\frac{m_k}{\mathrm{TeV}}\right)^2$
$\tau^- \to \mu^+ e^- \mu^-$	BR$< 2.7 \times 10^{-8}$	$\lvert g_{\mu\tau} g_{e\mu}^* \rvert < 0.007 \left(\frac{m_k}{\mathrm{TeV}}\right)^2$
$\tau^- \to \mu^+ \mu^- \mu^-$	BR$< 2.1 \times 10^{-8}$	$\lvert g_{\mu\tau} g_{\mu\mu}^* \rvert < 0.008 \left(\frac{m_k}{\mathrm{TeV}}\right)^2$
$\mu^+ e^- \to \mu^- e^+$	$G_{M\bar{M}} < 0.003 G_F$	$\lvert g_{ee} g_{\mu\mu}^* \rvert < 0.2 \left(\frac{m_k}{\mathrm{TeV}}\right)^2$

$\ell_a^- \to \ell_b^- \gamma$ bounds the f_{ab} and the g_{ab} couplings



$$\delta a_\mu = (21 \pm 10) \times 10^{-10}$$
$$r\left(|f_{e\mu}|^2 + |f_{\mu\tau}|^2\right) + 4\left(|g_{e\mu}|^2 + |g_{\mu\mu}|^2 + |g_{\mu\tau}|^2\right) < 7.9\,(m_k/\mathrm{TeV})^2$$

$$BR(\mu \to e\gamma) < 5.7 \times 10^{-13}$$
$$r^2|f_{e\tau}^* f_{\mu\tau}|^2 + 16|g_{ee}^* g_{e\mu} + g_{e\mu}^* g_{\mu\mu} + g_{e\tau}^* g_{\mu\tau}|^2 < 1.6 \times 10^{-6}\,(m_k/\mathrm{TeV})^4$$

$$BR(\tau \to e\gamma) < 3.3 \times 10^{-8}$$
$$r^2|f_{e\mu}^* f_{\mu\tau}|^2 + 16|g_{ee}^* g_{e\tau} + g_{e\mu}^* g_{\mu\tau} + g_{e\tau}^* g_{\tau\tau}|^2 < 0.52\,(m_k/\mathrm{TeV})^4$$

$$BR(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$$
$$r^2|f_{e\mu}^* f_{e\tau}|^2 + 16|g_{e\mu}^* g_{e\tau} + g_{\mu\mu}^* g_{\mu\tau} + g_{\mu\tau}^* g_{\tau\tau}|^2 < 0.7\,(m_k/\mathrm{TeV})^4$$

$0\nu\beta\beta$

Just the light neutrino contribution, with $m_{\rm lightest} \sim 0$:

- In the NH case,

$$(\mathcal{M}_\nu^{NH})_{ee} = \sqrt{\Delta_S} c_{13}^2 s_{12}^2 e^{i\phi} + \sqrt{\Delta_A} s_{13}^2 \,.$$

$0.001 \lesssim \, \mathrm{eV} |(\mathcal{M}_\nu^{NH})_{ee}| \lesssim 0.004\,\mathrm{eV}$, outside reach...

- In the IH case,

$$(\mathcal{M}_\nu^{IH})_{ee} = \sqrt{\Delta_A + \Delta_S} c_{13}^2 s_{12}^2 e^{i\phi} + \sqrt{\Delta_A} c_{13}^2 c_{12}^2 \,.$$

$0.01\,\mathrm{eV} \lesssim |(\mathcal{M}_\nu^{NH})_{ee}| \lesssim 0.05\,\mathrm{eV}$, observable in planned exps.

Non-standard interactions [Ohlsson et al.]

By integrating out h^+ (where ℓ here refer to the charged leptons):

$$\mathcal{L}_{d=6}^{NSI} = 2\sqrt{2}G_F\epsilon_{\alpha\beta}^{\rho\sigma}(\overline{\nu_\alpha}\gamma^\mu P_L\nu_\beta)(\overline{\ell_\rho}\gamma_\mu P_L\ell_\sigma), \quad \epsilon_{\alpha\beta}^{\rho\sigma} = \frac{f_{\sigma\beta}f_{\rho\alpha}^*}{\sqrt{2}G_F m_h^2}.$$

In matter only $\epsilon_{\mu\tau}^m, \epsilon_{\mu\mu}^m, \epsilon_{\tau\tau}^m \neq 0$, with $\epsilon_{\alpha\beta}^m = \epsilon_{\alpha\beta}^{ee}$.
NSI affect also $\mu \to e\overline{\nu_\beta}\nu_\alpha$ (ν factory). Source effects in $\nu_\mu \to \nu_\tau$ ($\nu_e \to \nu_\tau$) produced respectively by:

$$\epsilon_{\mu\tau}^s = \epsilon_{\tau e}^{e\mu} = \frac{f_{\mu e}f_{e\tau}^*}{\sqrt{2}G_F m_h^2}, \quad \epsilon_{e\tau}^s = \epsilon_{\mu\tau}^{e\mu} = \frac{f_{\mu\tau}f_{e\mu}^*}{\sqrt{2}G_F m_h^2}.$$

As $BR(\mu \to e\gamma) \sim |f_{e\tau}^* f_{\mu\tau}|^2$ limit is now ~ 0.05 smaller, NSI reduced by $\sim 1/4$, so in IH $\epsilon_{e\tau}^s$ and $\epsilon_{\mu\tau}^s \sim 3 \times (10^{-5} - 10^{-4})$, difficult to probe (maybe in a ν factory with a ν_τ near detector?).

Analytical estimates

f completely fixed by mixing angles and δ

Since $\det f = 0$, $f\mathbf{a} = 0$ for $\mathbf{a} = (f_{\mu\tau}, -f_{e\tau}, f_{e\mu})$ and:

$$D_\nu U^T \mathbf{a} = 0,$$

which leads to 3 eqs., one trivially satisfied because one element of D_ν is zero: $m_{\rm lightest} \sim 0$. The other 2 allow one to write, for NH:

$$\begin{aligned}
\frac{f_{e\tau}}{f_{\mu\tau}} &= \tan\theta_{12}\frac{\cos\theta_{23}}{\cos\theta_{13}} + \tan\theta_{13}\sin\theta_{23}e^{-i\delta}\,,\\
\frac{f_{e\mu}}{f_{\mu\tau}} &= \tan\theta_{12}\frac{\sin\theta_{23}}{\cos\theta_{13}} - \tan\theta_{13}\cos\theta_{23}e^{-i\delta}\,,
\end{aligned}$$

and for IH:

$$\frac{f_{e\tau}}{f_{\mu\tau}} = -\frac{\sin\theta_{23}}{\tan\theta_{13}}e^{-i\delta}\,, \qquad \frac{f_{e\mu}}{f_{\mu\tau}} = \frac{\cos\theta_{23}}{\tan\theta_{13}}e^{-i\delta}\,,$$

f estimation

- As $s_{12}^2 \sim 0.3$, $s_{23}^2 \sim 0.4$ and $s_{13}^2 \sim 0.02$:
  - NH: the first term dominates: $f_{e\mu} \sim \frac{f_{\mu\tau}}{2} \sim f_{e\tau}$.
  - IH: $\frac{f_{e\tau}}{f_{e\mu}} = -\tan\theta_{23} \sim -1$ and $|\frac{f_{e\mu}}{f_{\mu\tau}}| \sim |\frac{f_{e\tau}}{f_{\mu\tau}}| \sim 4$.
- we leave $f_{\mu\tau}$ free and real, and obtain (complex) $f_{e\mu}$ and $f_{e\tau}$.
- Regarding g, we keep:
  - $g_{ee}, g_{e\mu}, g_{e\tau}$ as free complex parameters.
  - $g_{\mu\mu}, g_{\mu\tau}, g_{\tau\tau}$ fixed by:

$$m_{ij} = (UD_\nu U^T)_{ij} = \zeta f_{ia}\omega_{ab} f_{jb},$$

where $\omega_{ab} \equiv m_a g_{ab}^* m_b$, and $\zeta = \frac{\mu}{48\pi^2 M^2}\tilde{I}(r)$.

Estimation and cancellation for IH

If we approximate $\omega_{ea} = 0$:

$$m_{22} \simeq \zeta f_{\mu\tau}^2 \omega_{\tau\tau}, \;\; m_{23} \simeq -\zeta f_{\mu\tau}^2 \omega_{\mu\tau}, \;\; m_{33} \simeq \zeta f_{\mu\tau}^2 \omega_{\mu\mu}.$$

From the large atmospheric angle we expect

$$|\omega_{\tau\tau}| \simeq |\omega_{\mu\tau}| \simeq |\omega_{\mu\mu}| \;\rightarrow\; g_{\tau\tau} : g_{\mu\tau} : g_{\mu\mu} \sim \frac{m_\mu^2}{m_\tau^2} : \frac{m_\mu}{m_\tau} : 1.$$

From oscillation parameters, when $e^{i\phi} \sim e^{i\delta} \sim 1$:

$$\zeta f_{\mu\tau}^2 |\omega_{ab}| \simeq 0.025 \text{ eV}\,, \qquad a, b = \mu, \tau,$$

however for IH if $\phi \sim \delta \sim \pi$ a cancellation occurs:

$$\zeta f_{\mu\tau}^2 |\omega_{\mu\mu}| \simeq 0.003 \,\text{eV},$$

which allows for smaller $g_{\mu\mu}$ and so lighter m_k.

The lowest scalar masses

$$\frac{m_{33}}{0.05\,\text{eV}} \simeq 500|g_{\mu\mu}||f_{\mu\tau}|^2 \frac{\mu}{M}\frac{\text{TeV}}{M}\tilde{I}(r) \lesssim 0.26\,\frac{\mu\, m_k}{\epsilon\, M^2}\left(\frac{m_h}{\text{TeV}}\right)^2 \tilde{I}(r)$$

where we used:

- $\epsilon \equiv |f_{e\tau}/f_{\mu\tau}| \sim 1/2$ (4) in NH (IH)
- $|g_{\mu\mu}| \lesssim 0.4(m_k/\text{TeV})$ $(\tau \to 3\mu)$
- $\epsilon |f_{\mu\tau}|^2 \lesssim 1.3 \cdot 10^{-3}(m_h/\text{TeV})^2$ $(\mu \to e\gamma)$

For $m_{33} \sim 0.025$ eV:

- $m_h > m_k \gtrsim \frac{1\,(3)\,\text{TeV}}{\sqrt{\kappa}}$ NH (IH)
- $m_k > m_h \gtrsim \sqrt{\frac{m_k}{m_h\,\kappa\,\tilde{I}(r)}}\; 1\,(3)$ TeV NH (IH)

In IH if $\phi \simeq \delta \simeq \pi$ ($m_{33} \sim 0.003$ eV), much lower masses allowed.

Numerical analysis

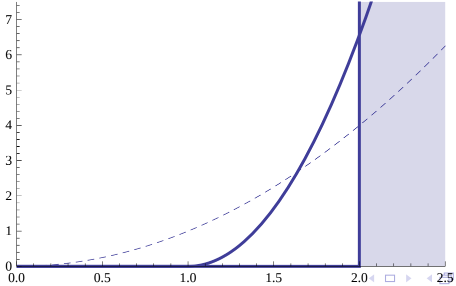
Parameters of the numerical scan

- 9 moduli: 3 from f (2 fixed), and 6 from g (3 fixed).
- 5 phases: 3 from g and 2 from f (fixed).
- the real and positive parameter μ, and m_h, m_k.

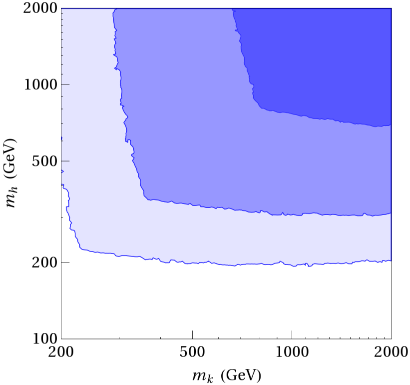
Parameter	Allowed range
Δ_S	$(7.50 \pm 0.19) \times 10^{-5}\,\mathrm{eV}^2$
Δ_A	$(2.45 \pm 0.07) \times 10^{-3}\mathrm{eV}^2$
$\sin^2\theta_{12}$	0.30 ± 0.13
$\sin^2\theta_{23}$	$(0.42 \pm 0.04) \cup (0.60 \pm 0.04)$
$\sin^2\theta_{13}$	0.023 ± 0.002
δ, ϕ	$[0, 2\pi]$
$\arg(g_{ee}), \arg(g_{e\mu}), \arg(g_{e\tau})$	$[0, 2\pi]$
$f_{\mu\tau}, \lvert g_{ee}\rvert, \lvert g_{e\mu}\rvert, \lvert g_{e\tau}\rvert$	$[10^{-7}, 5]$
m_h	$[100, 2 \times 10^3]\,\mathrm{GeV}$
m_k	$[200, 2 \times 10^3]\,\mathrm{GeV}$
μ	$[1, 2\kappa \times 10^3]\,\mathrm{GeV}$

- With a constant and a Gaussian part. It avoids imposing stepwise bounds or half-Gaussian with best value at zero that penalize deviating from null when this might not be supported.
- For $B^{\mathcal{O}}_{[90\%\mathrm{CL}]}$ at 90% CL (1.64σ), the χ^2 contribution of $\mathcal{O}_{\rm th}$ is

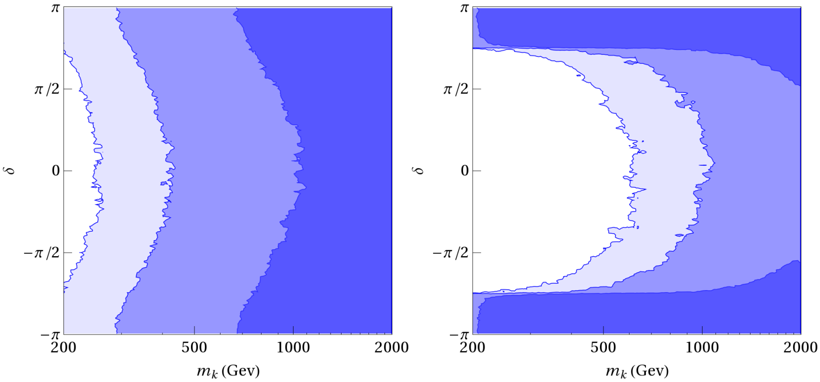
$$\chi^2(\mathcal{O}_{\rm th}) = \begin{cases} 0, \ \mathcal{O}_{\rm th} < B^{\mathcal{O}}_{[90\%\mathrm{CL}]}/1.64, \\ \left(\frac{1.64\mathcal{O}_{\rm th}}{B^{\mathcal{O}}_{[90\%\mathrm{CL}]}} - 1\right)^2 \left(\frac{1.64}{0.64}\right)^2, \ \mathcal{O}_{\rm th} \geq B^{\mathcal{O}}_{[90\%\mathrm{CL}]}/1.64. \end{cases}$$



m_h vs. m_k for NH (left) and IH (right), $\kappa = 1, 5, 4\pi$



Correlation between δ and m_k in NH (left) and IH (right)



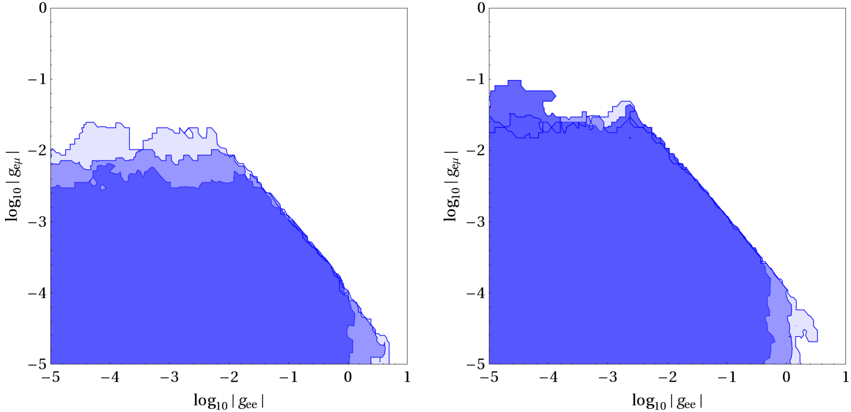
- In IH $m_k \lesssim 1$ TeV only allowed if $\delta \neq 0$ ($\kappa = 5$).
- The correlation of δ with m_h is entirely analogous.
- A similar correlation with the phase ϕ for IH is present.

Lower bounds for the scalar masses, for $\delta = \pi$ ($\delta = 0$)

	NH		
κ	**1**	**5**	**4**π
$\mathbf{m_h}$ (GeV)	700 (1000)	300 (400)	200 (250)
$\mathbf{m_k}$ (GeV)	700 (1100)	300 (450)	200 (250)

	IH		
κ	**1**	**5**	**4**π
$\mathbf{m_h}$ (GeV)	220 ($>$ 2000)	100 (1000)	100 (650)
$\mathbf{m_k}$ (GeV)	200 ($>$ 2000)	200 (1000)	200 (550)

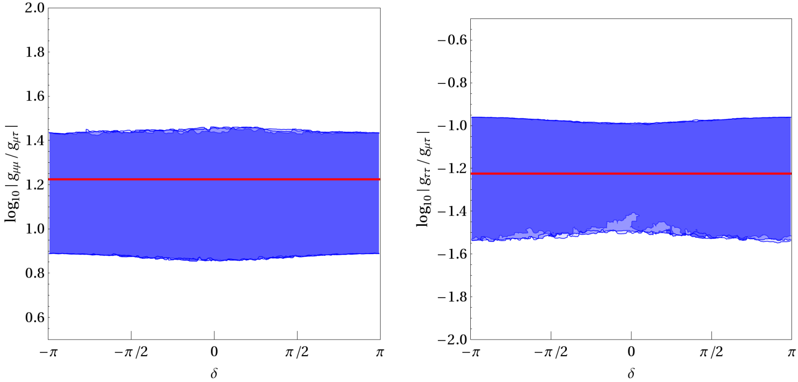
$\log|g_{e\mu}|$ vs $\log|g_{ee}|$ for NH (left) and IH (right)



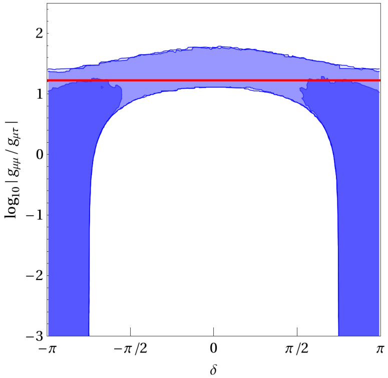
LFV strongly constrain $g_{e\tau}$ and $g_{e\mu}$ to be $\lesssim \mathcal{O}(0.01)$, while g_{ee} can be larger, $\mathcal{O}(1)$. From $\mu \to 3e$, $|g_{ee} g_{e\mu}| < 2.3 \times 10^{-5}\,(m_k/\mathrm{TeV})^2$.

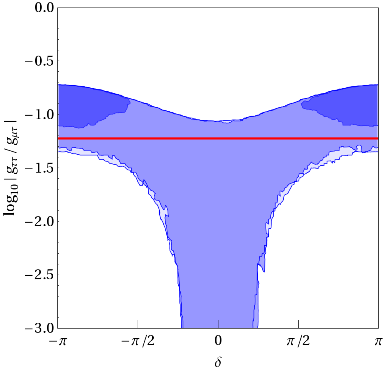
$\log|g_{\mu\mu}/g_{\mu\tau}|$ and $\log|g_{\tau\tau}/g_{\mu\tau}|$ vs δ for NH

- Always $g_{\tau\tau} \ll g_{\mu\tau}$.
- $g_{\mu\mu}/g_{\mu\tau} \sim m_\tau/m_\mu$ is only fulfilled for NH.
- The horizontal red lines are the naive approximation.



$\log|g_{\mu\mu}/g_{\mu\tau}|$ and $\log|g_{\tau\tau}/g_{\mu\tau}|$ vs δ for IH

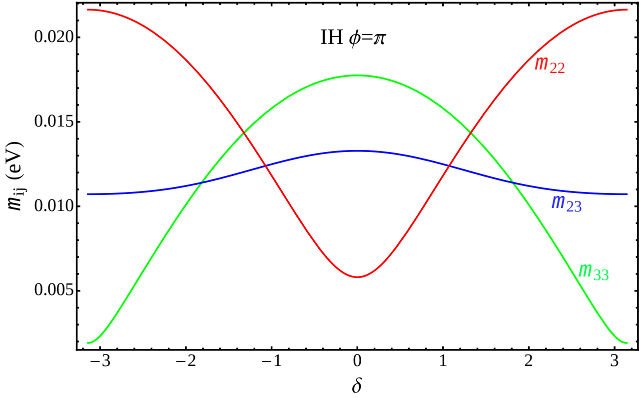




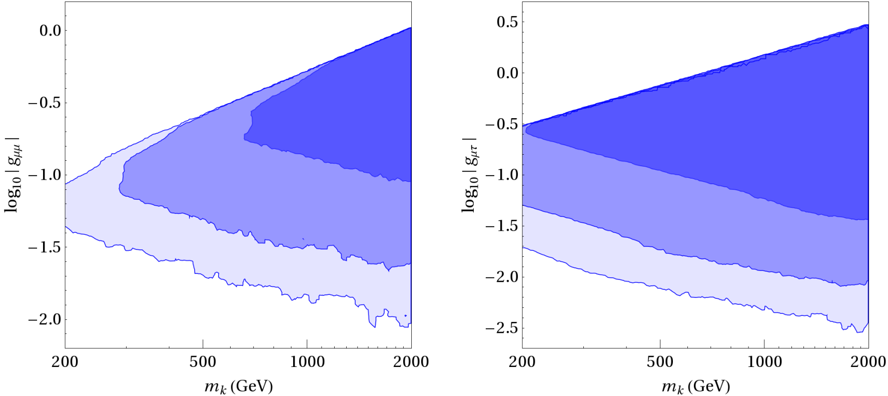
IH: $g_{\mu\mu}$ is smaller for $\delta \sim \pi$ and $g_{\tau\tau}$ for $\delta \sim 0$

$$m_{33} \propto f_{\mu\tau} g_{\mu\mu} m_\mu^2$$

$$m_{22} \propto f_{\mu\tau} g_{\tau\tau} m_\tau^2$$

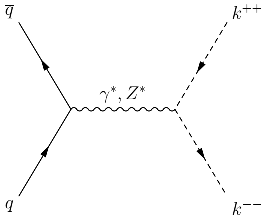


Largest couplings vs m_k ($g_{\mu\mu}$ in NH and $g_{\mu\tau}$ for IH)



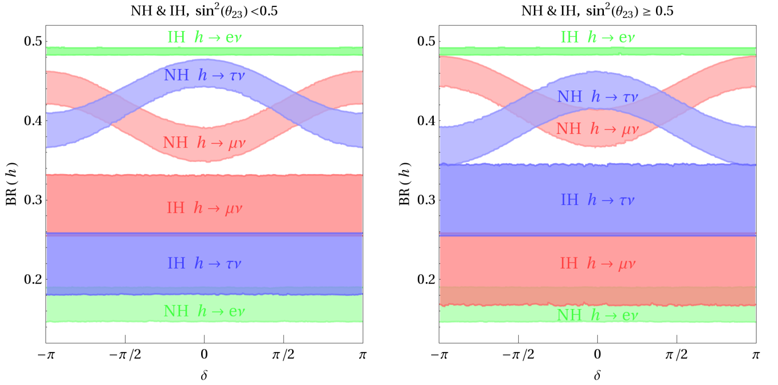
They tend to dominate the decays of the k^{++} .

Production and decays of the k



- NH: for $m_k \lesssim 400$ GeV, $\text{BR}(k \to ee) + \text{BR}(k \to \mu\mu) \sim 1$, since $k \to hh$ is closed, so $m_k > 310$ GeV.
- IH: $\text{BR}(k \to \mu\tau)$ can also be significant and $k \to hh$ is open unless $\kappa = 1$ (for $m_k > 440$ GeV), so bound is $m_k > 200$ GeV.
- $k \to hh$ is open for $m_k \gtrsim 400$ GeV, and can be dominant, so in general LHC-14 limits will not apply.
- If k is detected, $\text{BR}(k \to e\mu, e\tau, \tau\tau)$ negligible, while large $\text{BR}(k \to \mu\tau)$ implies IH.

BR($h \to e\nu, \mu\nu, \tau\nu$) fixed by the f's [Aristizábal, this work]



- Difficult to test, SM background like $W \to e\nu$.
- $\theta_{23} < 45^\circ$ left, $\theta_{23} > 45^\circ$ (right).
- $e\nu$ is the best option to discriminate between hierarchies.
- NH: dependence on δ in the $\mu\nu$ and $\tau\nu$ channels.
- IH: $\mu\nu$ and $\tau\nu$ are interchanged.

Distinguishing ZB from triplet model [Garayoa, Schmidt]

- at a like-sign electron linear collider k is produced at tree level: $e^- \, e^- \to a^- \, b^-$, with cross section (S - beams polarization):

$$\sigma(ee \to ab) = \frac{S|g_{ee}g_{ab}|^2}{4\pi(1+\delta_{ab})} \frac{s}{(s-m_k)^2 + m_k^2\Gamma_k^2}$$

- g_{ee} can be zero (can also be $\sim \mathcal{O}(1)$), so no lower bound exists.
- in the case of the triplet, the flavour structure is $g_{ab} \propto (m_\nu)_{ab}$, while in the ZB the relation is more complicated.
- Triplet: m_{lightest} crucial for σ (in m_{ee}), specially for NH. If degenerate, ZB is ruled-out.
- ZB: m_k lower bounds are stronger, specially in IH $\delta \neq \pi$.
- Triplet $m_k - m_h \lesssim v$, while in the ZB they are unrelated.
- the polarization of the beams can be used.

Conclusions

Conclusions

- ZB updated: θ_{13}, $\mu \to e\gamma$, m_H and LHC results.
- Neutrino data and low energy constraints are still compatible with masses accessible to LHC in both hierarchies.
- If any of the singlets is discovered, the ZB can be falsified using their decay modes and neutrino data.
- Also hierarchy (and even CP) could in principle be tested.
- NH: $m_k < 600$ GeV, $\kappa = 5$, no $k \to hh$, $k \to ee, \mu\mu$, $m_k > 310$ GeV.
- IH: for $\delta \sim \phi \sim \pi$ $k \to hh$ is open, so bound is ~ 200 GeV.
- If IH and δ is quite different from $\sim \pi$, k, h will be outside LHC reach.
- Enhancement in $H \to \gamma\gamma$ possible in small region of parameter space with light k and/or h, but possible instabilities of the potential.

Jag tackar för er uppmärksamhet!

Back-up slides

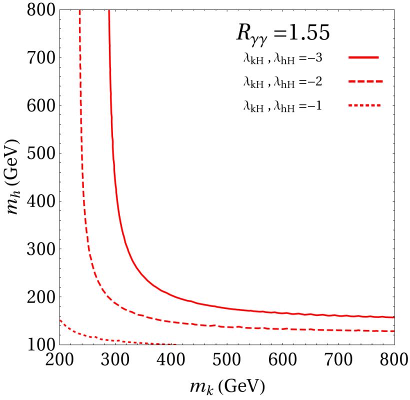
RGEs in the ZB model

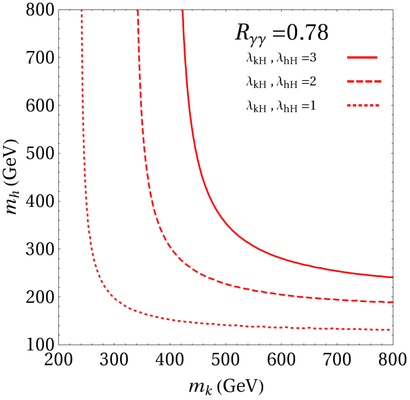
$$
\begin{aligned}
16\pi^2\beta_H &= 3/8\left[(g^2+g'^2)^2+2g^4\right]-(3g'^2+9g^2)\lambda_H+24\lambda_H^2+\lambda_{hH}^2\\
&+ \lambda_{kH}^2-6y_t^4+12\lambda_H y_t^2\\
16\pi^2\beta_h &= 6g'^4-12g'^2\lambda_h+20\lambda_h^2+2\lambda_{hH}^2+\lambda_{hk}^2\\
16\pi^2\beta_k &= 96g'^4-48g'^2\lambda_k+20\lambda_k^2+2\lambda_{kH}^2+\lambda_{hk}^2\\
16\pi^2\beta_{hH} &= 3g'^4-1/2(15g'^2+9g^2)\lambda_{hH}+12\lambda_H\lambda_{hH}+8\lambda_h\lambda_{hH}+\\
&+ 2\lambda_{kH}\lambda_{hk}+4\lambda_{hH}^2+6\lambda_{hH}y_t^2\\
16\pi^2\beta_{kH} &= 12g'^4-1/2(51g'^2+9g^2)\lambda_{kH}+12\lambda_H\lambda_{hk}+8\lambda_k\lambda_{kH}+\\
&+ 2\lambda_{hH}\lambda_{hk}+4\lambda_{kH}^2+6\lambda_{kH}y_t^2\\
16\pi^2\beta_{hk} &= 48g'^4-30g'^2\lambda_{hk}+4\lambda_{kH}\lambda_{hH}+8\lambda_h\lambda_{hk}+8\lambda_k\lambda_{hk}+4\lambda_{hk}^2
\end{aligned}
$$

$$16\pi^2\beta_{g'}=\frac{5}{3}\left(\frac{41}{10}+1\right)g'^3,\quad 16\pi^2\beta_g=-\frac{19}{6}g^3,\quad 16\pi^2\beta_{g_3}=-7g_3^2\ ,$$

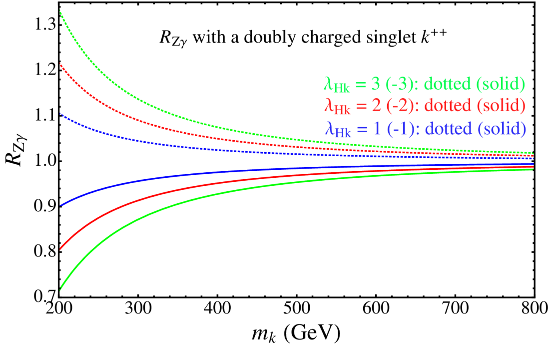
$$16\pi^2\beta_t=y_t\left\{\frac{9}{2}y_t^2-\left(\frac{17}{12}g'^2-\frac{9}{4}g^2-8g_3^2\right)\right\}\ .$$

Contours of $R_{\gamma\gamma} = 1.55$ (0.78)





$H \to Z\gamma$



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The Zee model [Zee]

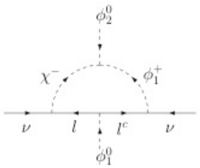

Figure: The Zee model diagram contributing to neutrino masses.

- New charged singlet, so the term $f\bar{\tilde{\ell}}\ell\chi^+$ exists, with f AS.
- Two Higgs doublets (can take that ϕ_2 does not couple to leptons), so the LNV term $\mu\,\overline{\tilde{\phi}_2}\phi_1\,\chi^-$ exists.
- Masses at one loop, with m_ℓ the charged leptons masses:

$$(m_\nu)_{ij} \sim \frac{1}{(4\pi)^2}\frac{\mu}{m_\chi^2}\, f_{ij}(m_{\ell i}^2 - m_{\ell j}^2)$$

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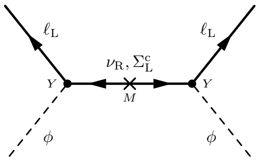
Adding right-handed neutrinos: seesaw type I

$$\mathcal{L}_{\nu_R} = i\,\overline{\nu_R}\gamma^\mu\partial_\mu\nu_R - \left(\bar{\ell}\,\tilde{\phi}\,Y\,\nu_R + \frac{1}{2}\overline{\nu_R^c}m_R\nu_R + \text{H.c.}\right)\,,$$

where m_R is a $n \times n$ symmetric matrix. After SSB:

$$\mathcal{L}_{\nu\,\text{mass}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\begin{pmatrix}0 & m_D\\ m_D^T & m_R\end{pmatrix}\begin{pmatrix}\nu_L^c\\ \nu_R\end{pmatrix} + \text{H.c.}\,,$$

where $m_D = Y\frac{v}{\sqrt{2}}$.



If $m_R \gg m_D$, one gets n m_R leptons (mainly singlets) and

$$m_\nu \simeq -m_D\, m_R^{-1}\, m_D^T.$$

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