The Zee-Babu revisited in the light of new data

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Introduction
Neutrino oscillations: $\nu$ are massive and mix

We don’t know the absolute $\nu$ mass scale, only upper bounds:

- **Cosmology**: $\sum_i m_i < 0.23$.
- **Tritium $\beta$ decay**: $m_{\nu_e} \equiv \sqrt{\sum_i m_i^2 |U_{ei}|^2} < 2 \text{ eV}$.
- **$0\nu\beta\beta$**: $m_{ee} \equiv |\sum_i m_i U_{ei}^2| < 0.2 \text{ eV}$.

nor the spectrum:

- **NH**: $m_1 \lesssim m_2 < m_3 - \sum_i m_i \gtrsim 0.06 \text{ eV}$.
- **IH**: $m_3 < m_1 \lesssim m_2 - \sum_i m_i \gtrsim 0.1 \text{ eV}$.
- **QD**: $m_1 \approx m_2 \approx m_3$. 
Fits to neutrino oscillations

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<th>Free Fluxes + RSBL</th>
<th>Huber Fluxes, no RSBL</th>
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<tr>
<td>⋁</td>
<td>bfp ±1σ</td>
<td>bfp ±1σ</td>
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<tr>
<td>sin^2 θ_{12}</td>
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<td>0.313^{+0.013}_{-0.012}</td>
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<td>θ_{12}/°</td>
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<td>sin^2 θ_{23}</td>
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<td>θ_{23}/°</td>
<td>41.9^{+0.5}<em>{-0.4} \oplus 50.3^{+1.6}</em>{-2.5}</td>
<td>41.8^{+2.1}<em>{-1.8} \oplus 50.3^{+1.6}</em>{-2.5}</td>
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<td>sin^2 θ_{13}</td>
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<td>δ_{CP}/°</td>
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<td>Δm_{21}^{2}</td>
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<td>+2.249 \rightarrow +2.639</td>
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<td>Δm_{32}^{2}</td>
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<td>-2.422^{+0.063}_{-0.061}</td>
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<td>10^{-3} eV^2</td>
<td>-2.602 \rightarrow -2.226</td>
<td>-2.614 \rightarrow -2.235</td>
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**Figure:** Global fit from Gonzalez-Garcia et al.
Unsolved questions in the neutrino sector

- The nature Dirac or Majorana ($0\nu\beta\beta$), i.e., whether LN is a conserved symmetry or not.
- The existence of leptonic CP violation and its value.
- LFV in the charged sector, like $\mu \rightarrow e\gamma$.
- The absolute mass scale of neutrinos.
- The spectrum.
- The octant of $\theta_{23}$ and more precise mixings/masses.
- If sterile neutrinos exist.
- By which mechanism their masses are generated...
Neutrino masses
Dirac neutrino masses

Add $\nu_R$ to the SM

$$m_D \overline{\nu_R} \nu_L + \text{H.c.}$$

Pros:
- Dirac masses exist for all other fermions.

Cons:
- Impose by hand B-L, which in the SM (without $\nu_R$) is accidental.
- fine-tuned Yukawas, 12 orders of magnitude smaller than the top one!
Majorana neutrino masses

\[ m_L \bar{\nu}_L^c \nu_L + \text{H.c.} \]

- violates LN.
- It can explain why \( m_\nu \) is so smaller than other fermion masses.
- not gauge-invariant in the SM: higher order operator.
The Weinberg operator

The SM is a very good EFT, with NP at higher scales $\Lambda$:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{n=5} \sum_i \left( \frac{C_i^n}{\Lambda^{n-4}} \mathcal{O}_i^n + \text{H.c.} \right).$$

where $n$ is the operator dimension, $i$ labels the different operators of a given $n$, and $C_i^n$ are dimensionless coefficients.

The only $D=5$ op. with the SM fields happens to violate LN:

$$\mathcal{L}_5 = \frac{1}{2} \frac{c_{\alpha\beta}}{\Lambda} (\ell_\alpha \tilde{\phi}) (\phi^\dagger \tilde{\ell}_\beta) + \text{H.c.},$$

where $c_{\alpha\beta}$ are model-dependent coefficients and $\tilde{\ell} \equiv i\sigma_2 \ell^c$. Upon EWSB it gives Majorana masses to neutrinos:

$$m_\nu = c \frac{\nu^2}{\Lambda}.$$
Opening the Weinberg operator at tree level: seesaws

- Rewriting the Weinberg operator:
  \[
  \left(\ell_\alpha \tilde{\phi}\right) \left(\phi^\dagger \tilde{\ell}_\beta\right) = - \left(\ell_\alpha \tilde{\sigma} \tilde{\phi}\right) \left(\phi^\dagger \tilde{\sigma} \tilde{\ell}_\beta\right) = \frac{1}{2} \left(\ell_\alpha \tilde{\sigma} \tilde{\ell}_\beta\right) \left(\phi^\dagger \tilde{\sigma} \tilde{\phi}\right),
  \]
  where \(\alpha\) and \(\beta\) are family indices and \(\tilde{\sigma} \equiv (\sigma_1, \sigma_2, \sigma_3)\).

- 3 different particles can generate Weinberg op. at tree level:
  - a \(Y = 0\) heavy fermion singlet (triplet), type I (III) seesaw.
  - a \(Y = 1\) heavy scalar triplet, type II seesaw.

- Explains why \(\nu\)'s are light: they couple to high scale fields.
- Drawbacks: typically difficult to test, problem of hierarchies.
at loop level: radiative models. $\nu$'s are light because they are massless at tree level, with their masses generated at $i$ loops

$$m_\nu \propto \frac{1}{(4\pi)^{2i}}$$

Typically there are additional suppressions due to couplings (so LN is violated) and/or ratios of masses.

More than three loops typically yield too light $m_\nu$, so $i < 4$.

So the scale can be not too far away from EWS and can be tested.
The Zee-Babu model
The Zee-Babu model [Cheng and Li, Zee, Babu, Aristizábal, Nebot, Ohlsson, Schmidt…]

Consider the $D=9$ $\Delta L = 2$ eff. operator $\ell \ell \ell \ell e^c e^c$. It generates the Weinberg op. at some loop level (and therefore $m_\nu$). By NDA:

$$m_\nu \sim \frac{1}{(4\pi)^4} \frac{y_e^2 v^2}{\Lambda}$$

One can open this op. by adding a singly- and a doubly-charged scalar $h^\pm, k^{\pm\pm}$ with $Y_h = \pm 1$ and $Y_k = \pm 2$ resp.
The interactions in the Zee-Babu model

\[ \mathcal{L}_Y = \bar{\ell} Y e \phi + \bar{\ell} f \ell h^+ + \bar{e} \bar{c} g e k^{++} + \text{H.c.} \]

Due to Fermi statistics, $f_{ab}$ is AS while $g_{ab}$ is S.

\[ V = m_\phi^2 \phi^\dagger \phi + m_h^2 |h|^2 + m_k^2 |k|^2 + \lambda_\phi (\phi^\dagger \phi)^2 + \lambda_h |h|^4 + \lambda_k |k|^4 \\
+ \lambda_{hk} |h|^2 |k|^2 + \lambda_{h\phi} |h|^2 \phi^\dagger \phi + \lambda_{k\phi} |k|^2 \phi^\dagger \phi + (\mu h^2 k^{++} + \text{H.c.}) \]

LNV requires simultaneous presence of $Y$, $f$, $g$, $\mu$. We choose:

- $Y$ to be diag. with $> 0$ elements, the charged lepton masses.
- We use fermion field rephasings to remove 3 phases from $g$.
- By scalar rephasings we set $\mu > 0$ and remove one phase from $f$. 
Generating the Weinberg Operator at two loops

\[(\mathcal{M}_\nu)_{ij}^{\text{exp}} = (UD_\nu U^T)_{ij} = 16\mu f_{ia} m_a g_{ab}^* l_{ab} m_b f_{jb} = (\mathcal{M}_\nu)_{ij}^{\text{ZB}}\]

\[l_{ab} \simeq l = \frac{1}{(16\pi^2)^2} \frac{1}{M^2} \frac{\pi^2}{3} \tilde{l}(r), \quad r \equiv \frac{m_k^2}{m_h^2}, \quad M \equiv \max(m_h, m_k).\]

\[\tilde{l}(r) = 1 \quad \text{for} \quad r \to 0, \quad 1 + \frac{3}{\pi^2} (\log^2 r - 1) \quad \text{for} \quad r \gg 1\]

\[\mathcal{M}_\nu = \frac{\nu^2 \mu}{48\pi^2 M^2} \tilde{l} f Y g^\dagger Y^T f^T\]

\(f\) is AS \(\rightarrow\) \(\det f = 0 \rightarrow \det \mathcal{M}_\nu = 0\), so one massless \(\nu\) (no QD).
Since one-loop corrections to Yukawa couplings are order

$$\delta f \sim \frac{f^3}{(4\pi)^2}, \quad \delta g \sim \frac{g^3}{(4\pi)^2},$$

one expects $f, g < 4\pi$.

Also from perturbativity, $\lambda_{h,k,k\phi,h\phi,hk} < 4\pi$. 
μ induces radiative corrections to the masses of the scalars

\[ \delta m_k^2, \delta m_h^2 \sim \frac{\mu^2}{(4\pi)^2}. \]

so $\mu < 4\pi \min(m_h, m_k)$. We take:

$\mu < \kappa \min(m_h, m_k),$

and discuss our results for different values of $\kappa = 1, 5, 4\pi$.

Also Higgs receives corrections; we will assume $m_{h,k} \lesssim 2 \text{ TeV}$. 

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A large value of $\mu$ could lead to a deeper minimum of the scalar potential for non-vanishing values of the charged fields, therefore breaking charge conservation.

Looking at $|H| = |h| = |k| = r$, and requiring $V(r \neq 0) > 0$:

$$\mu^2 < (\lambda_H + \lambda_h + \lambda_k + \lambda_{hh} + \lambda_{kh} + \lambda_{hk}) \left( m_H^2 + m_h^2 + m_k^2 \right).$$

Assuming no cancellations and using conservatively $\lambda_i \leq 4\pi$:

$$\mu \lesssim \sqrt{20\pi} \max(m_k, m_h) \sim 8 \max(m_k, m_h).$$
Stability of the potential

\[ \lambda_H > 0, \quad \lambda_h > 0, \quad \lambda_k > 0. \]

For negative mixed couplings, \( \lambda_x H, \lambda_{hk} \ (x = h, k) \), whenever one of the scalar fields \( H, h, k \) is zero stability implies

\[ \alpha, \beta, \gamma > -1, \]

where we have defined

\[ \alpha = \frac{\lambda_{hH}}{2 \sqrt{\lambda_H \lambda_h}}, \quad \beta = \frac{\lambda_{kH}}{2 \sqrt{\lambda_H \lambda_k}}, \quad \gamma = \frac{\lambda_{hk}}{2 \sqrt{\lambda_h \lambda_k}}. \]

If \( \geq 2 \) mixed couplings are \( < 0 \), there is an extra constraint:

\[ 1 - \alpha^2 - \beta^2 - \gamma^2 + 2\alpha\beta\gamma > 0 \quad \vee \quad \alpha + \beta + \gamma > -1. \]
\( \lambda_h, \lambda_k \) always contribute positively to the running of \( \lambda_H \), compensating for the top. Therefore, vacuum stability is alleviated and \( \lambda_H > 0 \) up to \( M_P \) for the present central values of \( m_t \) and \( m_H \).

![Running of the Higgs coupling in the ZB versus SM](image)
\[ R_{\gamma\gamma} = \frac{\Gamma(H \rightarrow \gamma\gamma)_{ZB}}{\Gamma(H \rightarrow \gamma\gamma)_{SM}} = |1 + \delta R(m_h, \lambda_{hH}) + 4 \delta R(m_k, \lambda_{kH})|^2, \]

where (with \( \tau_i \equiv \frac{4m_i^2}{m_H^2} \)):

\[ \delta R(m_x, \lambda_{xH}) \equiv \frac{\lambda_{xH} \nu^2}{2m_x^2} \frac{A_0(\tau_x)}{A_1(\tau_W) + \frac{4}{3}A_{1/2}(\tau_t)}, \]

ATLAS: \( R_{\gamma\gamma} = 1.55^{+0.33}_{-0.28} \),

CMS: \( R_{\gamma\gamma} = 0.78^{+0.28}_{-0.26} \), MVA analysis

CMS: \( R_{\gamma\gamma} = 1.11^{+0.32}_{-0.31} \), cut based analysis
For $R_{\gamma\gamma} \sim 1.5$, $m_h \lesssim 250$ GeV and/or $m_k \lesssim 350$ GeV.

From stability $2\sqrt{\lambda_H \lambda_x} + \lambda_x H > 0$, for $x = h, k$, and $\lambda_H \sim 0.13$, so large and negative $-3 \lesssim \lambda_x H$ push $\lambda_x \to \sim 4\pi$.

For such values RGEs lead to vacuum instability ($2\sqrt{\lambda_H \lambda_x} + \lambda_x H < 0$, $x = h, k$) and/or non-perturbativity ($\lambda_x > 4\pi$) at $\mathcal{O}(1-100$ TeV).
$\ell_a \to \ell_b \nu \bar{\nu}$ bounds the $f_{ab}$ couplings

\[
\left( \frac{G_\mu}{G_\mu^{SM}} \right)^2 \approx 1 + \frac{\sqrt{2}}{G_F m_h^2} |f_{e\mu}|^2 + \frac{1}{2 G_F^2 m_h^4} \left( |f_{e\mu}|^2 + |f_{e\tau}|^2 \right) \left( |f_{e\mu}|^2 + |f_{\mu\tau}|^2 \right)
\]

Since the extraction of CKM $V_{ij}^{\text{exp}}$ assumes the SM:

$$V_{ij}^{\text{exp}} = \frac{G_\beta}{G_\mu} V_{ij}$$
\[ |V_{ud}^{\exp}|^2 + |V_{us}^{\exp}|^2 + |V_{ub}^{\exp}|^2 = \frac{G_\beta^2}{G_\mu^2} = \frac{G_{\mu SM}^2}{G_\mu^2} \approx 1 - \frac{\sqrt{2}}{G_F m_h^2} |f_{e\mu}|^2 \]

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<th>Bound (90%CL)</th>
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<td>V_{uq}</td>
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<td>$\mu/e$</td>
<td>$\frac{G_{\mu}^{\exp}}{G_e^{\exp}} = 1.0010 \pm 0.0009$</td>
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<tr>
<td>$\tau/\mu$</td>
<td>$\frac{G_{\tau}^{\exp}}{G_{\mu}^{\exp}} = 0.9998 \pm 0.0013$</td>
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<tr>
<td>$\tau/e$</td>
<td>$\frac{G_{\tau}^{\exp}}{G_e^{\exp}} = 1.0034 \pm 0.0015$</td>
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The ZB model revisited

Stockholm, 11th April 2014

\( \ell_a^- \rightarrow \ell_b^+ \ell_c^- \ell_d^- \) bounds the \( g_{ab} \) couplings

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<th>Bound (90% CL)</th>
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<td>( \mu^- \rightarrow e^+ e^- e^- )</td>
<td>( \text{BR} &lt; 1.0 \times 10^{-12} )</td>
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<tr>
<td>( \tau^- \rightarrow e^+ e^- e^- )</td>
<td>( \text{BR} &lt; 2.7 \times 10^{-8} )</td>
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<td>( \tau^- \rightarrow e^+ e^- \mu^- )</td>
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<td>( \tau^- \rightarrow e^+ \mu^- \mu^- )</td>
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<td>( \tau^- \rightarrow \mu^+ e^- e^- )</td>
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<td>( \tau^- \rightarrow \mu^+ e^- \mu^- )</td>
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<td>( \mu^+ e^- \rightarrow \mu^- e^+ )</td>
<td>( G_{M\tilde{M}} &lt; 0.003 G_F )</td>
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\( \ell^-_a \rightarrow \ell^-_b \gamma \) bounds the \( f_{ab} \) and the \( g_{ab} \) couplings

\[
\delta a_\mu = (21 \pm 10) \times 10^{-10}
\]

\[
r (|f_{e\mu}|^2 + |f_{\mu\tau}|^2) + 4 (|g_{e\mu}|^2 + |g_{\mu\mu}|^2 + |g_{\mu\tau}|^2) < 7.9 (m_k/\text{TeV})^2
\]

\[
BR(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}
\]

\[
r^2 |f_{e\tau}^* f_{\mu\tau}|^2 + 16 |g_{ee}^* g_{e\mu} + g_{e\mu}^* g_{\mu\mu} + g_{e\tau}^* g_{\mu\tau}|^2 < 1.6 \times 10^{-6} (m_k/\text{TeV})^4
\]

\[
BR(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}
\]

\[
r^2 |f_{e\mu}^* f_{e\tau}|^2 + 16 |g_{ee}^* g_{e\tau} + g_{e\mu}^* g_{\mu\tau} + g_{e\tau}^* g_{\tau\tau}|^2 < 0.52 (m_k/\text{TeV})^4
\]

\[
BR(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8}
\]

\[
r^2 |f_{e\mu}^* f_{e\tau}|^2 + 16 |g_{e\mu}^* g_{e\tau} + g_{\mu\mu}^* g_{\mu\tau} + g_{\mu\tau}^* g_{\tau\tau}|^2 < 0.7 (m_k/\text{TeV})^4
\]
Just the light neutrino contribution, with $m_{\text{lightest}} \sim 0$:

- In the NH case,

$$ (\mathcal{M}_{\nu}^{\text{NH}})_{ee} = \sqrt{\Delta_S c_{13}^2 s_{12}^2} e^{i\phi} + \sqrt{\Delta_A} s_{13}^2. $$

$$ 0.001 \lesssim \text{eV} |(\mathcal{M}_{\nu}^{\text{NH}})_{ee}| \lesssim 0.004 \text{eV}, \text{ outside reach...} $$

- In the IH case,

$$ (\mathcal{M}_{\nu}^{\text{IH}})_{ee} = \sqrt{\Delta_A + \Delta_S c_{13}^2 s_{12}^2} e^{i\phi} + \sqrt{\Delta_A} c_{13}^2 c_{12}^2. $$

$$ 0.01 \text{eV} \lesssim |(\mathcal{M}_{\nu}^{\text{NH}})_{ee}| \lesssim 0.05 \text{eV}, \text{ observable in planned exps.} $$
By integrating out $h^+$ (where $\ell$ here refer to the charged leptons):

$$\mathcal{L}_{d=6}^{\text{NSI}} = 2\sqrt{2}G_F\epsilon^\rho_\alpha^\sigma_\beta (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{\ell}_\rho \gamma^\mu P_L \ell_\sigma), \quad \epsilon^\rho_\alpha^\sigma_\beta = \frac{f_\sigma_\beta f^*_\rho_\alpha}{\sqrt{2}G_F m_h^2}.$$ 

In matter only $\epsilon^m_\mu_\tau, \epsilon^m_\mu_\mu, \epsilon^m_\tau_\tau \neq 0$, with $\epsilon^m_\alpha_\beta = \epsilon^{ee}_\alpha_\beta$. NSI affect also $\mu \rightarrow e \bar{\nu}_\beta \nu_\alpha$ ($\nu$ factory). Source effects in $\nu_\mu \rightarrow \nu_\tau$ ($\nu_e \rightarrow \nu_\tau$) produced respectively by:

$$\epsilon^s_\mu_\tau = \epsilon^{e\mu}_\tau_e = \frac{f_\mu_\epsilon f^*_\epsilon_\tau}{\sqrt{2}G_F m_h^2}, \quad \epsilon^s_\epsilon_\tau = \epsilon^{e\mu}_\mu_\tau = \frac{f_\mu_\tau f^*_\epsilon_\mu}{\sqrt{2}G_F m_h^2}.$$ 

As $BR(\mu \rightarrow e\gamma) \sim |f^*_\epsilon_\tau f_\mu_\tau|^2$ limit is now $\sim 0.05$ smaller, NSI reduced by $\sim 1/4$, so in IH $\epsilon^s_\epsilon_\tau$ and $\epsilon^s_\mu_\tau \sim 3 \times (10^{-5} - 10^{-4})$, difficult to probe (maybe in a $\nu$ factory with a $\nu_\tau$ near detector?).
Analytical estimates
f completely fixed by mixing angles and δ

Since \( \det f = 0 \), \( fa = 0 \) for \( a = (f_{\mu\tau}, -f_{e\tau}, f_{e\mu}) \) and:

\[
D_\nu U^T a = 0,
\]

which leads to 3 eqs., one trivially satisfied because one element of \( D_\nu \) is zero: \( m_{\text{lightest}} \sim 0 \). The other 2 allow one to write, for NH:

\[
\frac{f_{e\tau}}{f_{\mu\tau}} = \tan \theta_{12} \frac{\cos \theta_{23}}{\cos \theta_{13}} + \tan \theta_{13} \sin \theta_{23} e^{-i\delta},
\]

\[
\frac{f_{e\mu}}{f_{\mu\tau}} = \tan \theta_{12} \frac{\sin \theta_{23}}{\cos \theta_{13}} - \tan \theta_{13} \cos \theta_{23} e^{-i\delta},
\]

and for IH:

\[
\frac{f_{e\tau}}{f_{\mu\tau}} = -\frac{\sin \theta_{23}}{\tan \theta_{13}} e^{-i\delta}, \quad \frac{f_{e\mu}}{f_{\mu\tau}} = \frac{\cos \theta_{23}}{\tan \theta_{13}} e^{-i\delta},
\]
As $s_{12}^2 \sim 0.3$, $s_{23}^2 \sim 0.4$ and $s_{13}^2 \sim 0.02$:
- NH: the first term dominates: $f_{e\mu} \sim \frac{f_{\mu\tau}}{2} \sim f_{e\tau}$.
- IH: $\frac{f_{e\tau}}{f_{e\mu}} = - \tan \theta_{23} \sim -1$ and $\left| \frac{f_{e\mu}}{f_{\mu\tau}} \right| \sim \left| \frac{f_{e\tau}}{f_{\mu\tau}} \right| \sim 4$.

- we leave $f_{\mu\tau}$ free and real, and obtain (complex) $f_{e\mu}$ and $f_{e\tau}$.

Regarding $g$, we keep:
- $g_{ee}, g_{e\mu}, g_{e\tau}$ as free complex parameters.
- $g_{\mu\mu}, g_{\mu\tau}, g_{\tau\tau}$ fixed by:

$$m_{ij} = (UD_\nu U^T)_{ij} = \zeta f_{ia} \omega_{ab} f_{jb},$$

where $\omega_{ab} \equiv m_a g_{ab}^* m_b$, and $\zeta = \frac{\mu}{48\pi^2 M^2} \tilde{l}(r)$. 


Estimation and cancellation for IH

If we approximate $\omega_{ea} = 0$:

$$m_{22} \simeq \zeta f_{\mu\tau}^2 \omega_{\tau\tau}, \quad m_{23} \simeq -\zeta f_{\mu\tau}^2 \omega_{\mu\tau}, \quad m_{33} \simeq \zeta f_{\mu\tau}^2 \omega_{\mu\mu}.$$  

From the large atmospheric angle we expect

$$|\omega_{\tau\tau}| \simeq |\omega_{\mu\tau}| \simeq |\omega_{\mu\mu}| \rightarrow g_{\tau\tau} : g_{\mu\tau} : g_{\mu\mu} \sim \frac{m_{\mu}^2}{m_{\tau}^2} : \frac{m_{\mu}}{m_{\tau}} : 1.$$  

From oscillation parameters, when $e^{i\phi} \sim e^{i\delta} \sim 1$:

$$\zeta f_{\mu\tau}^2 |\omega_{ab}| \simeq 0.025 \text{ eV}, \quad a, b = \mu, \tau,$$

however for IH if $\phi \sim \delta \sim \pi$ a cancellation occurs:

$$\zeta f_{\mu\tau}^2 |\omega_{\mu\mu}| \simeq 0.003 \text{ eV},$$

which allows for smaller $g_{\mu\mu}$ and so lighter $m_k$. 
The lowest scalar masses

\[
\frac{m_{33}}{0.05 \text{ eV}} \simeq 500 |g_{\mu\mu}| |f_{\mu\tau}|^2 \frac{\mu}{M} \frac{\text{TeV}}{M} \tilde{f}(r) \lesssim 0.26 \frac{\mu m_k}{\epsilon M^2} \left( \frac{m_h}{\text{TeV}} \right)^2 \tilde{f}(r)
\]

where we used:

- \(\epsilon \equiv \frac{|f_{e\tau}|}{|f_{\mu\tau}|} \sim \frac{1}{2} (4)\) in NH (IH)
- \(|g_{\mu\mu}| \lesssim 0.4 (m_k/\text{TeV}) (\tau \to 3\mu)\)
- \(\epsilon |f_{\mu\tau}|^2 \lesssim 1.3 \cdot 10^{-3} (m_h/\text{TeV})^2 (\mu \to e\gamma)\)

For \(m_{33} \sim 0.025\) eV:

- \(m_h > m_k \gtrsim \frac{1 (3) \text{ TeV}}{\sqrt{\kappa}}\) NH (IH)
- \(m_k > m_h \gtrsim \sqrt{\frac{m_k}{m_h \kappa}} \tilde{f}(r)\) (3) TeV NH (IH)

In IH if \(\phi \simeq \delta \simeq \pi\) \((m_{33} \sim 0.003\) eV), much lower masses allowed.
Numerical analysis
Parameters of the numerical scan

- 9 moduli: 3 from $f$ (2 fixed), and 6 from $g$ (3 fixed).
- 5 phases: 3 from $g$ and 2 from $f$ (fixed).
- the real and positive parameter $\mu$, and $m_h, m_k$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Allowed range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_S$</td>
<td>$(7.50 \pm 0.19) \times 10^{-5} \text{eV}^2$</td>
</tr>
<tr>
<td>$\Delta_A$</td>
<td>$(2.45 \pm 0.07) \times 10^{-3} \text{eV}^2$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.30 \pm 0.13$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>$(0.42 \pm 0.04) \cup (0.60 \pm 0.04)$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$0.023 \pm 0.002$</td>
</tr>
<tr>
<td>$\delta, \phi$</td>
<td>$[0, 2\pi]$</td>
</tr>
<tr>
<td>arg($g_{ee}$), arg($g_{e\mu}$), arg($g_{e\tau}$)</td>
<td>$[0, 2\pi]$</td>
</tr>
<tr>
<td>$f_{\mu\tau},</td>
<td>g_{ee}</td>
</tr>
<tr>
<td>$m_h$</td>
<td>$[100, 2 \times 10^3] \text{GeV}$</td>
</tr>
<tr>
<td>$m_k$</td>
<td>$[200, 2 \times 10^3] \text{GeV}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$[1, 2\kappa \times 10^3] \text{GeV}$</td>
</tr>
</tbody>
</table>
With a constant and a Gaussian part. It avoids imposing stepwise bounds or half-Gaussian with best value at zero that penalize deviating from null when this might not be supported.

For $B^{O}_{[90\% CL]}$ at 90% CL (1.64$\sigma$), the $\chi^2$ contribution of $O_{th}$ is

$$\chi^2(O_{th}) = \begin{cases} 
0, & O_{th} < B^{O}_{[90\% CL]}/1.64, \\
\left(\frac{1.64O_{th}}{B^{O}_{[90\% CL]}} - 1\right)^2 \left(\frac{1.64}{0.64}\right)^2, & O_{th} \geq B^{O}_{[90\% CL]}/1.64.
\end{cases}$$
$m_h$ vs. $m_k$ for NH (left) and IH (right), $\kappa = 1, 5, 4\pi$
Correlation between $\delta$ and $m_k$ in NH (left) and IH (right)

- In IH, $m_k \lesssim 1$ TeV only allowed if $\delta \neq 0$ ($\kappa = 5$).
- The correlation of $\delta$ with $m_h$ is entirely analogous.
- A similar correlation with the phase $\phi$ for IH is present.
Lower bounds for the scalar masses, for $\delta = \pi \ (\delta = 0)$

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\mathbf{1}$</th>
<th>$\mathbf{5}$</th>
<th>$4\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_h$ (GeV)</td>
<td>700 (1000)</td>
<td>300 (400)</td>
<td>200 (250)</td>
</tr>
<tr>
<td>$m_k$ (GeV)</td>
<td>700 (1100)</td>
<td>300 (450)</td>
<td>200 (250)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\mathbf{1}$</th>
<th>$\mathbf{5}$</th>
<th>$4\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_h$ (GeV)</td>
<td>220 (&gt; 2000)</td>
<td>100 (1000)</td>
<td>100 (650)</td>
</tr>
<tr>
<td>$m_k$ (GeV)</td>
<td>200 (&gt; 2000)</td>
<td>200 (1000)</td>
<td>200 (550)</td>
</tr>
</tbody>
</table>
LFV strongly constrain $g_{e\tau}$ and $g_{e\mu}$ to be $\lesssim \mathcal{O}(0.01)$, while $g_{ee}$ can be larger, $\mathcal{O}(1)$. From $\mu \to 3e$, $|g_{ee}g_{e\mu}| < 2.3 \times 10^{-5} \left( m_k / \text{TeV} \right)^2$. 
log \(|g_{\mu\mu}/g_{\mu\tau}|\) and log \(|g_{\tau\tau}/g_{\mu\tau}|\) vs \(\delta\) for NH

- Always \(g_{\tau\tau} \ll g_{\mu\tau}\).
- \(g_{\mu\mu}/g_{\mu\tau} \sim m_\tau/m_\mu\) is only fulfilled for NH.
- The horizontal red lines are the naive approximation.
\log |g_{\mu\mu}/g_{\mu\tau}| \text{ and } \log |g_{\tau\tau}/g_{\mu\tau}| \text{ vs } \delta \text{ for IH}
IH: $g_{\mu\mu}$ is smaller for $\delta \sim \pi$ and $g_{\tau\tau}$ for $\delta \sim 0$

\[ m_{33} \propto f_{\mu\tau} g_{\mu\mu} m_{\mu}^2 \]
\[ m_{22} \propto f_{\mu\tau} g_{\tau\tau} m_{\tau}^2 \]

![Graph showing $m_{ij}$ vs $\delta$]
Largest couplings vs $m_k$ ($g_{\mu\mu}$ in NH and $g_{\mu\tau}$ for IH)

They tend to dominate the decays of the $k^{++}$. 
Production and decays of the $k$

- **NH**: for $m_k \lesssim 400$ GeV, $\text{BR}(k \rightarrow ee) + \text{BR}(k \rightarrow \mu\mu) \sim 1$, since $k \rightarrow hh$ is closed, so $m_k > 310$ GeV.
- **IH**: $\text{BR}(k \rightarrow \mu\tau)$ can also be significant and $k \rightarrow hh$ is open unless $\kappa = 1$ (for $m_k > 440$ GeV), so bound is $m_k > 200$ GeV.
- $k \rightarrow hh$ is open for $m_k \gtrsim 400$ GeV, and can be dominant, so in general LHC-14 limits will not apply.
- If $k$ is detected, $\text{BR}(k \rightarrow e\mu, e\tau, \tau\tau)$ negligible, while large $\text{BR}(k \rightarrow \mu\tau)$ implies IH.
Difficult to test, SM background like $W \rightarrow e\nu$.

- $\theta_{23} < 45^\circ$ left, $\theta_{23} > 45^\circ$ (right).
- $e\nu$ is the best option to discriminate between hierarchies.
- NH: dependence on $\delta$ in the $\mu\nu$ and $\tau\nu$ channels.
- IH: $\mu\nu$ and $\tau\nu$ are interchanged.
Distinguishing ZB from triplet model [Garayoa, Schmidt]

- at a like-sign electron linear collider $k$ is produced at tree level: $e^- e^- \rightarrow a^- b^-$, with cross section ($S$ - beams polarization):

\[
\sigma(ee \rightarrow ab) = \frac{S|g_{ee}g_{ab}|^2}{4\pi(1 + \delta_{ab})} \frac{s}{(s - m_k)^2 + m_k^2\Gamma_k^2}
\]

- $g_{ee}$ can be zero (can also be $\sim O(1)$), so no lower bound exists.
- in the case of the triplet, the flavour structure is $g_{ab} \propto (m_\nu)_{ab}$, while in the ZB the relation is more complicated.
- Triplet: $m_{\text{lightest}}$ crucial for $\sigma$ (in $m_{ee}$), specially for NH. If degenerate, ZB is ruled-out.
- ZB: $m_k$ lower bounds are stronger, specially in IH $\delta \neq \pi$.
- Triplet $m_k - m_h \lesssim \nu$, while in the ZB they are unrelated.
- the polarization of the beams can be used.
Conclusions
Conclusions

- ZB updated: $\theta_{13}$, $\mu \rightarrow e\gamma$, $m_H$ and LHC results.

- Neutrino data and low energy constraints are still compatible with masses accessible to LHC in both hierarchies.

- If any of the singlets is discovered, the ZB can be falsified using their decay modes and neutrino data.

- Also hierarchy (and even CP) could in principle be tested.

- NH: $m_k < 600$ GeV, $\kappa = 5$, no $k \rightarrow hh$, $k \rightarrow ee, \mu\mu$, $m_k > 310$ GeV.

- IH: for $\delta \sim \phi \sim \pi$ $k \rightarrow hh$ is open, so bound is $\sim 200$ GeV.

- If IH and $\delta$ is quite different from $\sim \pi$, $k, h$ will be outside LHC reach.

- Enhancement in $H \rightarrow \gamma\gamma$ possible in small region of parameter space with light $k$ and/or $h$, but possible instabilities of the potential.
Jag tackar för er uppmärksamhet!
\[16 \pi^2 \beta_H = \frac{3}{8} \left[ (g^2 + g'^2)^2 + 2g^4 \right] - (3g'^2 + 9g^2) \lambda_H + 24 \lambda_H^2 + \lambda_{hH}^2 + \lambda_{kH}^2 - 6y_t^4 + 12 \lambda_H y_t^2\]

\[16 \pi^2 \beta_h = 6g'^4 - 12g'^2 \lambda_h + 20\lambda_h^2 + 2\lambda_{hH}^2 + \lambda_{hk}^2\]

\[16 \pi^2 \beta_k = 96g'^4 - 48g'^2 \lambda_k + 20\lambda_k^2 + 2\lambda_{kH}^2 + \lambda_{hk}^2\]

\[16 \pi^2 \beta_{hH} = 3g'^4 - \frac{1}{2}(15g'^2 + 9g^2) \lambda_{hH} + 12\lambda_H \lambda_{hH} + 8\lambda_h \lambda_{hH} + 2\lambda_{kH} \lambda_{hk} + 4\lambda_{hH}^2 + 6\lambda_{hH} y_t^2\]

\[16 \pi^2 \beta_{kH} = 12g'^4 - \frac{1}{2}(51g'^2 + 9g^2) \lambda_{kH} + 12\lambda_H \lambda_{hk} + 8\lambda_k \lambda_{kH} + 2\lambda_{hH} \lambda_{hk} + 4\lambda_{kH}^2 + 6\lambda_{kH} y_t^2\]

\[16 \pi^2 \beta_{hk} = 48g'^4 - 30g'^2 \lambda_{hk} + 4\lambda_{kH} \lambda_{hH} + 8\lambda_h \lambda_{hk} + 8\lambda_k \lambda_{hk} + 4\lambda_{hk}^2\]

\[16 \pi^2 \beta_g' = \frac{5}{3} \left( \frac{41}{10} + 1 \right) g'^3, \quad 16 \pi^2 \beta_g = -\frac{19}{6} g^3, \quad 16 \pi^2 \beta_{g_3} = -7 g_3^2\]

\[16 \pi^2 \beta_t = y_t \left\{ \frac{9}{2} y_t^2 - \left( \frac{17}{12} g'^2 - \frac{9}{4} g^2 - 8g_3^2 \right) \right\}.\]
Contours of $R_{\gamma\gamma} = 1.55$ (0.78)

\begin{align*}
R_{\gamma\gamma} &= 1.55 \\
\lambda_{kH}, \lambda_{hH} &= -3 \\
\lambda_{kH}, \lambda_{hH} &= -2 \\
\lambda_{kH}, \lambda_{hH} &= -1
\end{align*}
$H \rightarrow Z\gamma$

$R_{Z\gamma}$ with a doubly charged singlet $k^{++}$

- $\lambda_{Hk} = 3 (-3)$: dotted (solid)
- $\lambda_{Hk} = 2 (-2)$: dotted (solid)
- $\lambda_{Hk} = 1 (-1)$: dotted (solid)

$m_k$ (GeV)

$R_{Z\gamma}$
The Zee model [Zee]

New charged singlet, so the term $f \bar{\ell} \ell \chi^+$ exists, with $f$ AS.
Two Higgs doublets (can take that $\phi_2$ does not couple to leptons), so the LNV term $\mu \bar{\phi}_2 \phi_1 \chi^-$ exists.
Masses at one loop, with $m_\ell$ the charged leptons masses:

$$(m_\nu)_{ij} \sim \frac{1}{(4\pi)^2} \frac{\mu}{m_\chi^2} f_{ij} (m_{\ell i}^2 - m_{\ell j}^2)$$

Figure: The Zee model diagram contributing to neutrino masses.
Adding right-handed neutrinos: seesaw type I

\[ \mathcal{L}_{\nu_R} = i \overline{\nu_R} \gamma^\mu \partial_\mu \nu_R - \left( \overline{\ell} \tilde{\phi} Y \nu_R + \frac{1}{2} \nu^c_R m_R \nu_R + \text{H.c.} \right), \]

where \( m_R \) is a \( n \times n \) symmetric matrix. After SSB:

\[ \mathcal{L}_{\nu \text{ mass}} = -\frac{1}{2} \left( \nu_L \quad \nu^c_R \right) \begin{pmatrix} 0 & m_D \\ m^T_D & m_R \end{pmatrix} \begin{pmatrix} \nu^c_L \\ \nu_R \end{pmatrix} + \text{H.c.}, \]

where \( m_D = Y \frac{v}{\sqrt{2}} \).

If \( m_R \gg m_D \), one gets \( n m_R \) leptons (mainly singlets) and

\[ m_\nu \simeq -m_D m_R^{-1} m_D^T. \]