Quantum Simulations & Quantum Devices with Ultracold Atoms in Optical Lattices

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Outline

- **Ultracold atoms for quantum engineering of states and devices: a brief introduction**

- **An example of quantum engineering of states with ultracold atoms in optical lattices → quantum simulation of the XXZ chain**

- **An example of quantum engineering of devices with ultracold atoms in optical lattices → implementation of a fermionic Josephson junction**

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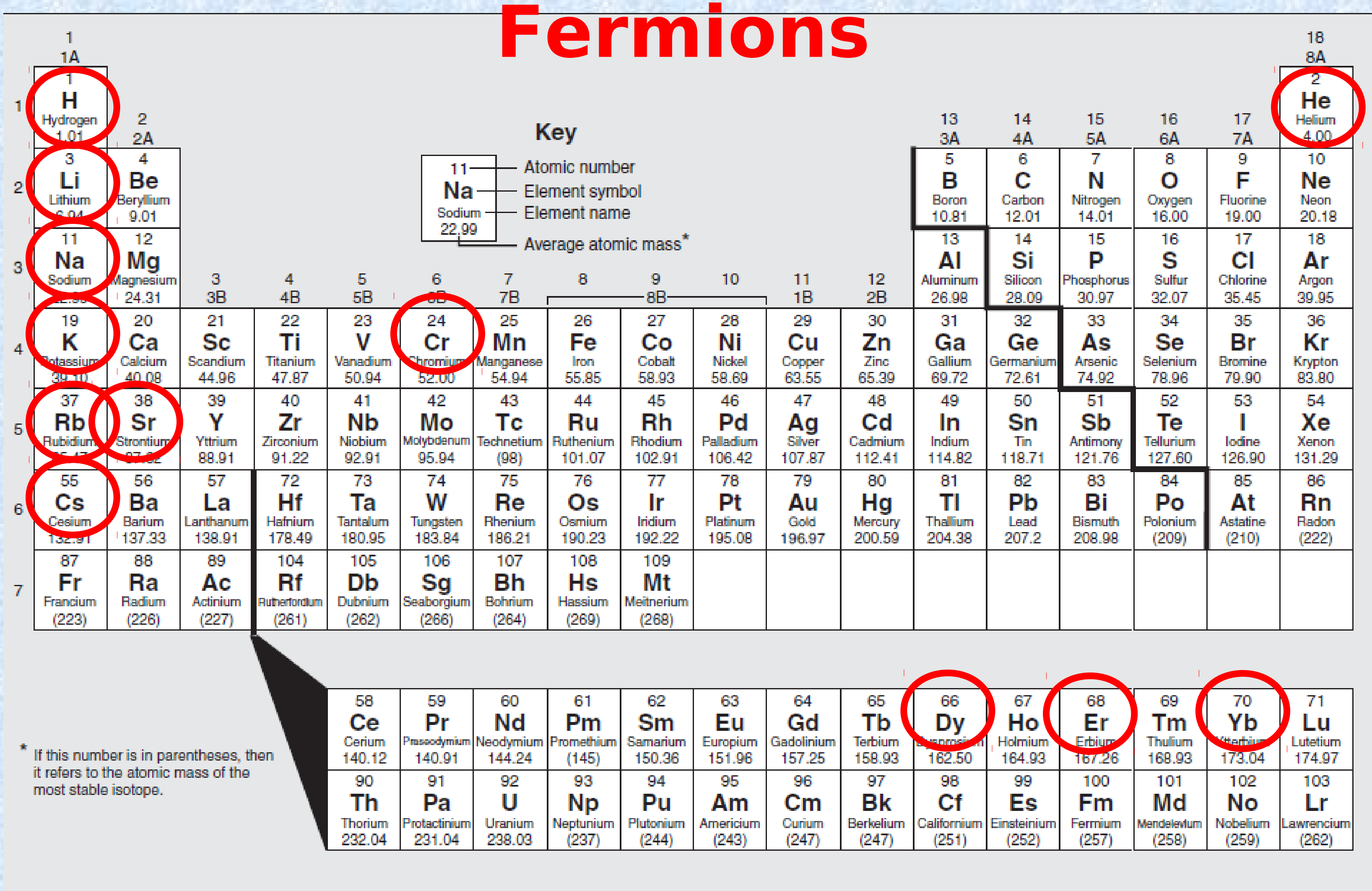
The ultracold family: Bosons & Fermions



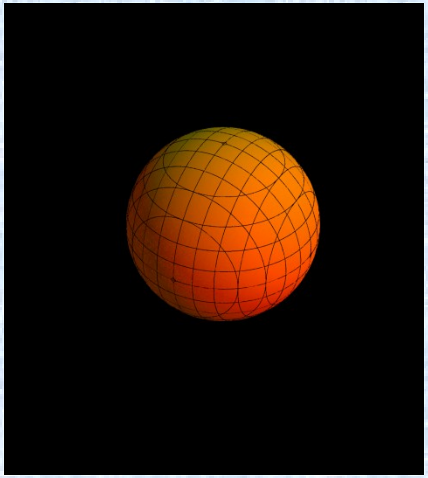
Typical values:

$$\begin{cases} numbers:\ 10^3 - 10^6\ atoms \\ temperatures: 10 - 100\ nanoKelvin \\ sizes:\ 1 - 50\ micrometers \end{cases}$$

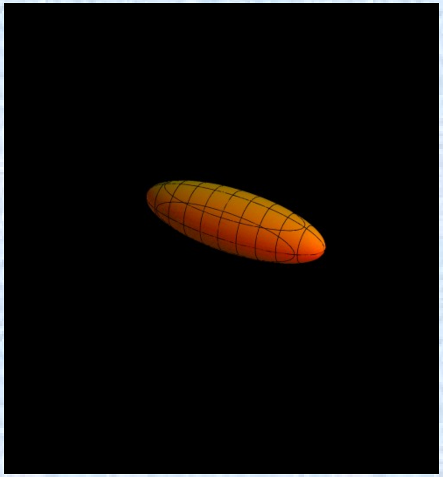
Geometry control for ultracold atoms

Magnetic harmonic potential: $V(x,y,z)=\frac{1}{2}m\left(\omega_x^2 x^2+\omega_y^2 y^2+\omega_z^2 z^2\right)$

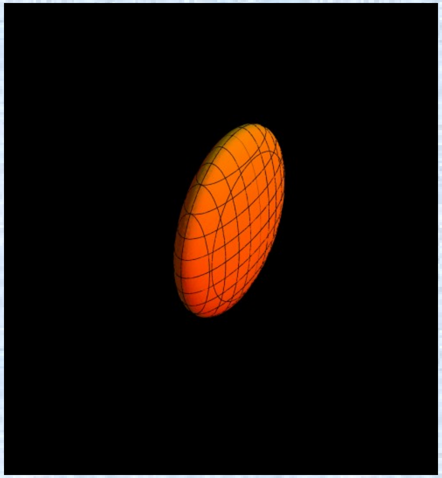
$\omega_x=\omega_y=\omega_z$



$\omega_x \ll \omega_y=\omega_z$



$\omega_x \gg \omega_y=\omega_z$



Achievements & Perspectives

1995	Bose-Einstein condensation of bosonic gases
1995-…	Feshbach resonances, superfluidity and rotating condensates, spinorial condensates
1998	Polarized (=ideal) Fermi gases at T_F
2001	Optical lattices
2005	Dipolar gases
2005-2006	Interacting fermions
2008	Anderson localization
2009	Mott insulators for fermions
...	...

quantum devices **quantum simulators**

Quantum Simulations

Realization by purpose of a model or *quantum states* of interest (microscopic or phenomenological) in an experimental setup with highly tunable parameters

Need to:

- compute from the microscopics the parameters of the model
- check how reliable is the “simulation” of the desired Hamiltonian (e.g., how the observable quantities match)

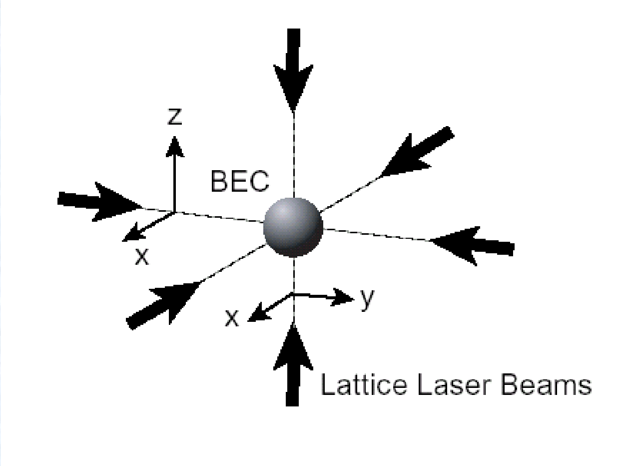
Example

Desired Hamiltonian: Hubbard model (i.e., interacting particles on a lattice)

$$H = -t \sum_i b_i^+ b_{i+1} + \frac{U}{2} \sum_i n_i (n_i - 1) + V \sum_i n_i n_{i+1}$$

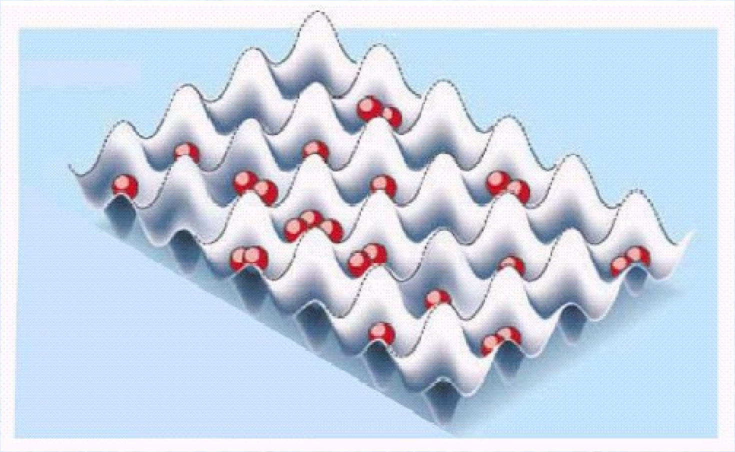
→ use (suitable) ultracold atoms in a (suitable) optical lattice...

Ultracold bosons in an optical lattice



$$V_{opt}(x) = V_0 \sin^2(kx)$$

e.g., a 1D lattice



It is possible to control:
- barrier height
- interaction term
- the shape of the network
- the dimensionality (1D, 2D, …)
- the tunneling among planes or among tubes (in order to have a layered structure)
…

Effective Hamiltonian for ultracold bosons in optical lattices

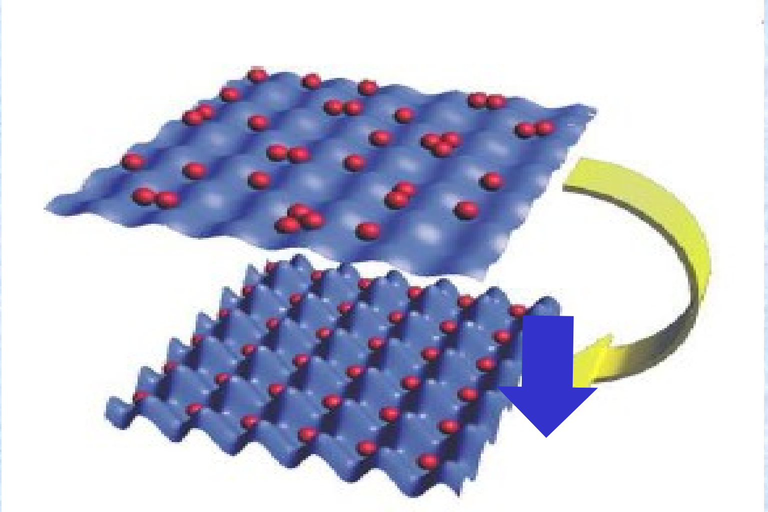
$$\hat{H} = -t \sum_{<i,j>} \left(\hat{b}_i^+ \hat{b}_j + h.c. \right) + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right)$$

$\hat{n}_i \equiv \hat{b}_i^+ \hat{b}_i$ $\quad N_T$ *number of particles on N sites* $\quad$ *filling* $f = \frac{N_T}{N}$

Bose-Hubbard Hamiltonian
[D. Jaksch et al., PRL (1999)]

t/U>>1 → **Superfluid**
dynamics described by the classical Bose-Hubbard [A. Trombettoni and A. Smerzi, PRL (2001)]

t/U<<1 → **Mott insulator** quantum fluctuations dominate



Main available "ingredients"

- **Bosons and/or fermions**
- **Geometry (1D / 2D)**
- **Long-range interactions**
- **Add disorder**
- **Time-dependence (and to a certain extent space-dependence) of the parameters of the Hamiltonian**
- **Simulate a magnetic field through a rotation or with optical tools**
- **Explicit tuning of the interactions via Feshbach resonances**
- **Optical lattices (i.e., periodic potentials and minima of the potential located on a lattice)**

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Available tools

- Bosons and/or fermions
- Geometry (1D)
- Long-range interactions
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- Time-dependence (and to a certain extent space-dependence) of the parameters of the Hamiltonian
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- Optical lattices (i.e., periodic potentials having minima located on a lattice)
- …

Low-energy Hamiltonian for ultracold bosons in optical lattices

$$H_{BH} = -t \sum_i b_i^+ b_{i+1} + \frac{U}{2} \sum_i n_i (n_i - 1) + V \sum_i n_i n_{i+1}$$

$$n_i \equiv b_i^+ b_i \quad N_T \, \textit{number of particles on L sites} \quad \textit{filling } f = \frac{N_T}{L}$$

(Bose-Hubbard Hamiltonian)

For U infinite and filling f half-integer only two states are relevant:

$$\left| f - \frac{1}{2} \right\rangle, \; \left| f + \frac{1}{2} \right\rangle$$

Putting

$$s_i^z = n_i - f$$

one gets..

… the XXZ Hamiltonian:

$$H_{XXZ} = -J\sum_i \left(s_i^x s_{i+1}^x + s_i^y s_{i+1}^y - \Delta s_i^z s_{i+1}^z\right)$$

integrable in 1D

$$J = 2t\left(f + \frac{1}{2}\right) \quad J\Delta = V$$

A lot is known also about correlation functions in the thermodynamic limit:

$$\langle\Psi_0|s_i^z s_j^z|\Psi_0\rangle = (-1)^{i-j}\frac{A_z}{|i-j|^{1/\eta}} - \frac{1}{4\pi^2\eta\,(i-j)^2},$$

$$\langle\Psi_0|s_i^x s_j^x|\Psi_0\rangle = (-1)^{i-j}\frac{A_x}{|i-j|^{\eta}} - \frac{\tilde{A}_x}{|i-j|^{\eta+1/\eta}},$$

$$\eta = 1 - \frac{1}{\pi}\arccos\Delta$$

(Luther and Peschel, PRB 1975)

Also the correlation amplitudes are known (Lukyanov and Zamolodchikov, Nucl. Phys. B 1998, Lukyanov, PRB 1999):

$$A^x = \frac{\mathcal{A}^\eta}{8(1-\eta)^2} \exp\left[-\int_0^\infty \frac{dt}{t}\left(\frac{\sinh(\eta t)}{\sinh(t)\cosh[(1-\eta)t]} - \eta e^{-2t}\right)\right]$$

$$\tilde{A}^x = \frac{\mathcal{A}^{\eta+1/\eta}}{2\eta(1-\eta)} \exp\left[-\int_0^\infty \frac{dt}{t}\left(\frac{\cosh(\eta t)e^{-2t}-1}{2\sinh(\eta t)\sinh(t)\cosh[(1-\eta)t]} + \frac{1}{\sinh(\eta t)} - \frac{\eta^2+1}{\eta}e^{-2t}\right)\right]$$

$$A^z = \frac{2\mathcal{A}^{1/\eta}}{\pi^2} \exp\left[\int_0^\infty \frac{dt}{t}\left(\frac{\sinh[(2\eta-1)t]}{\sinh(\eta t)\cosh[(1-\eta)t]} - \frac{2\eta-1}{\eta}e^{-2t}\right)\right]$$

$$\mathcal{A} = \frac{\Gamma\left(\frac{\eta}{2(1-\eta)}\right)}{2\sqrt{\pi}\,\Gamma\left(\frac{1}{2(1-\eta)}\right)}$$

(results available also for the XXZ *finite* chain)

Finite U effective Hamiltonian:

What happens at finite U? The effective Hamiltonian is not an XXZ chain, but...

Similarity RG procedure
(Glazek and Wilson, PRB 1993)
+
Luttinger representation

↓

an effective XXZ Hamiltonian with renormalized parameters is found, giving correlation functions in excellent agreement with numerical results from the Bose-Hubbard model

[D. Giuliano, D. Rossini, P. Sodano, and A. Trombettoni, PRB (2013)]

Collecting all together:

$$H^{(1)} = H^{(1)}_{\text{diag}} + H^{(1)}_{\text{offd}}$$

$$H^{(1)}_{\text{Diag}} = -\frac{4(\bar{n}+1)t^2}{U}\sum_i s_i^z - \frac{t^2}{U}\left(3\bar{n}^2+6\bar{n}+4\right)\sum_i s_i^z s_{i+1}^z$$

$$H^{(1)}_{\text{Offd}} = -\frac{t^2(\bar{n}+1)^2}{U}\sum_i \left(s_{i+1}^+ s_{i-1}^- + s_{i+1}^- s_{i-1}^+\right) - \frac{2t^2(\bar{n}+1)}{U}\sum_i \left(s_{i+1}^+ s_{i-1}^- + s_{i+1}^- s_{i-1}^+\right) s_i^z$$

Final step: **passing to Jordan-Wigner fermions and bosonizing via the Luttinger representation**

Final result

$$H_{XXZ}^{\text{eff}} = -J_{\text{eff}} \sum_j \left(s_j^x s_{j+1}^x + s_j^y s_{j+1}^y - \Delta_{\text{eff}} s_j^z s_{j+1}^z \right)$$

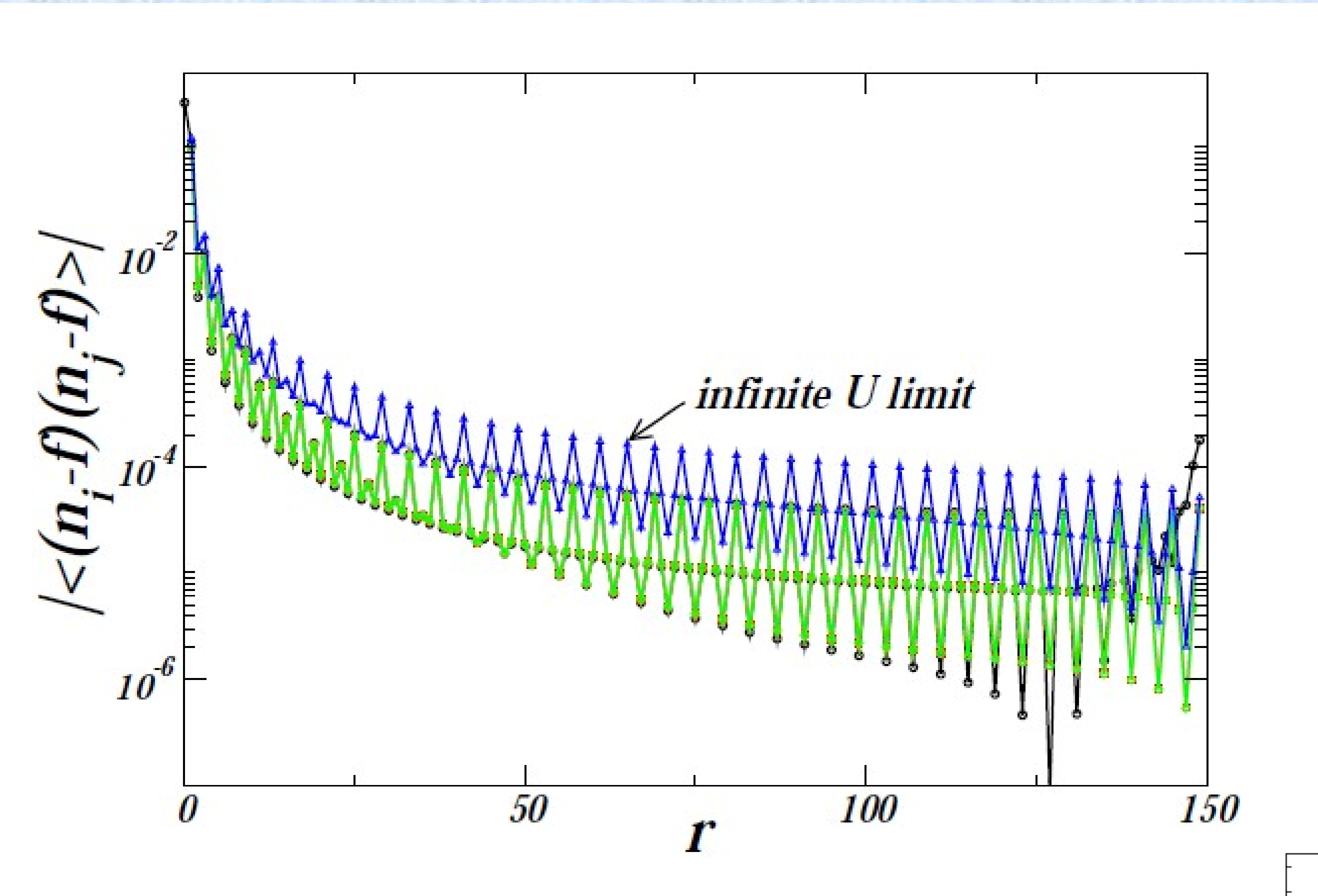
$$\Delta_{\text{eff}} = \left[\frac{1}{1 - \frac{2J}{U} \cos(k_F)} \right] \Delta$$

$$\Delta = \frac{V}{J} - \frac{t^2(3\bar{n}^2 + 6\bar{n} + 4)}{JU} - \frac{4t^2(\bar{n}+1)^2}{JU}$$

$$J_{\text{eff}} = J \left(1 - \frac{2J}{U} \cos k_F \right)$$

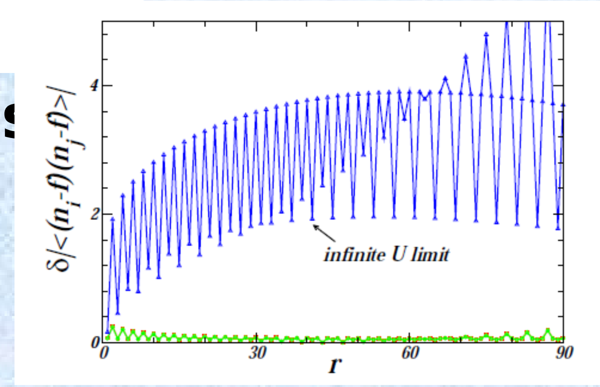
$$\cos k_F = \frac{U(\bar{n}+1)}{2J} - \sqrt{\left(\frac{U(\bar{n}+1)}{2J} \right)^2 + \bar{n} + 2}$$

Comparison between (XXZ) analytical and (Bose-Hubbard) numerical results (I):



U=10t
V=0.5t
filling=0.5
L=150
J/U=0.2

relative error



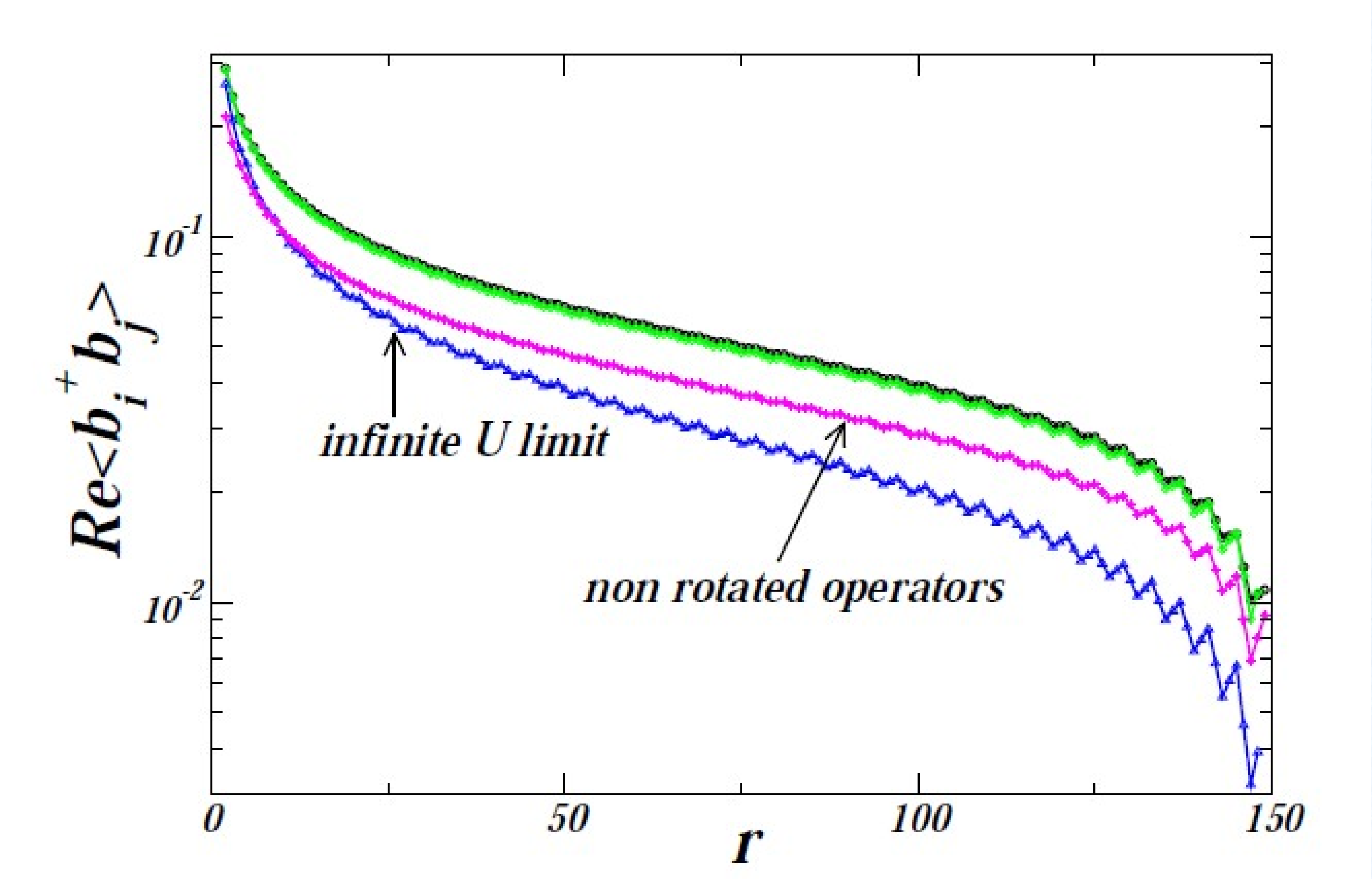
Black: numerical Bose-Hubbard res

Red: XXZ numerical results

Green: XXZ analytical results

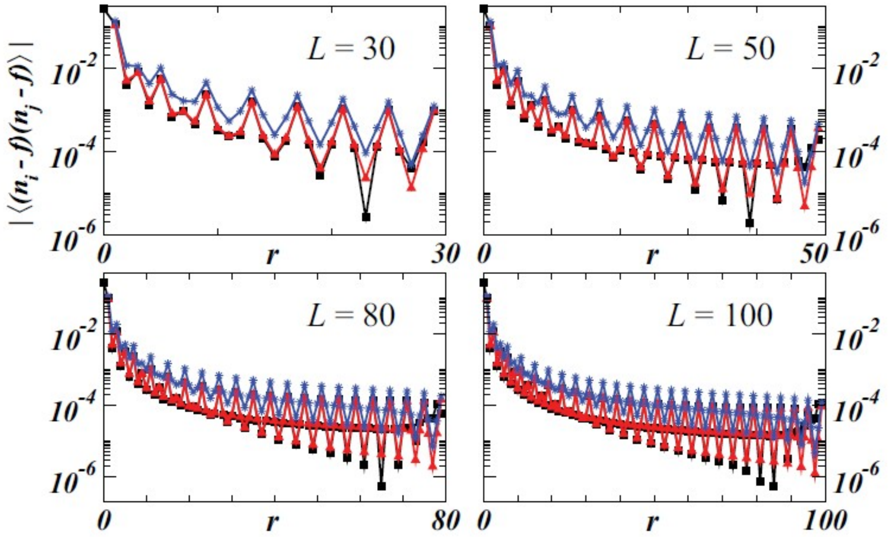
Blue: infinite U result

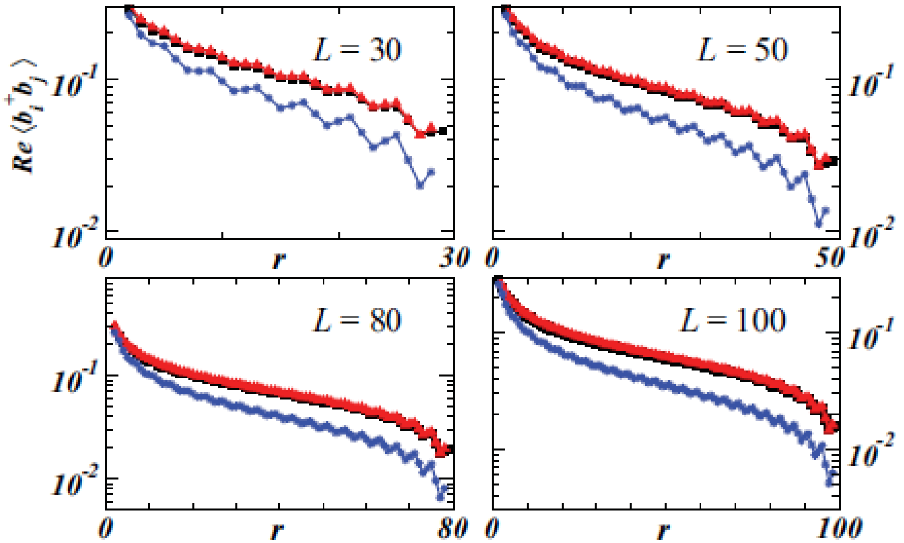
Comparison between (XXZ) analytical and (Bose-Hubbard) numerical results (II):



Black: numerical Bose-Hubbard results
Red: XXZ numerical results
Green: XXZ analytical results
Blue: infinite U result
Magenta: XXZ results with non GW rotated operators

Stability of results varying the size...

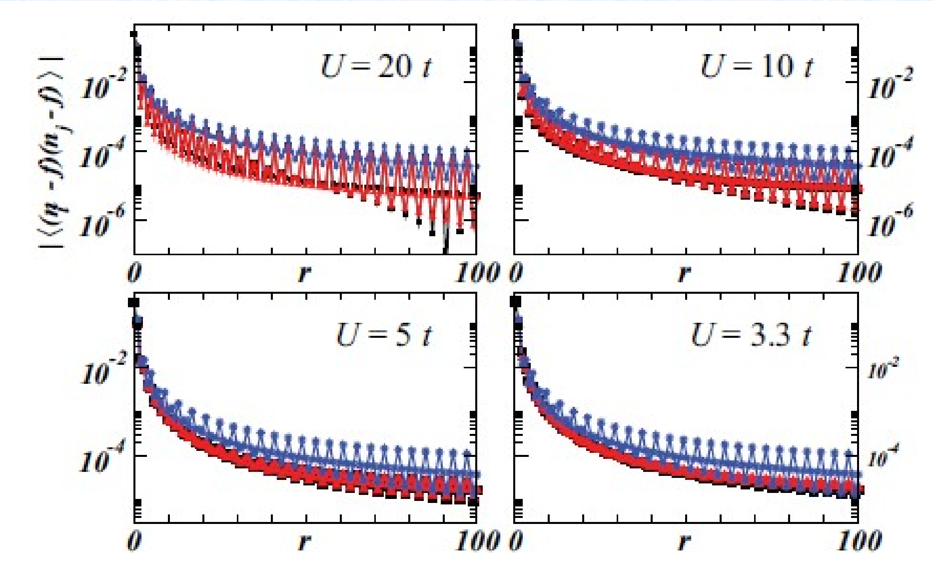


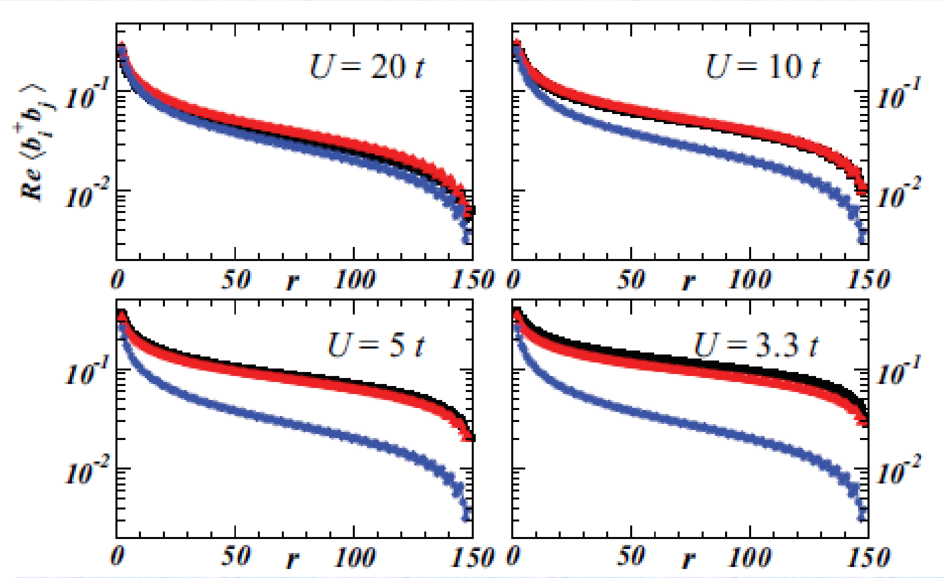


U=10t; V=0.5t; filling=0.5; J/U=0.2

Black: numerical Bose-Hubbard results
Red: XXZ numerical results
Blue: infinite U result

...and varying the interaction





V=0.5t; filling=0.5; L=150 → J/U=0.1, 0.2, 0.4, 0.6

Black: numerical Bose-Hubbard results
Red: XXZ numerical results
Blue: infinite U result

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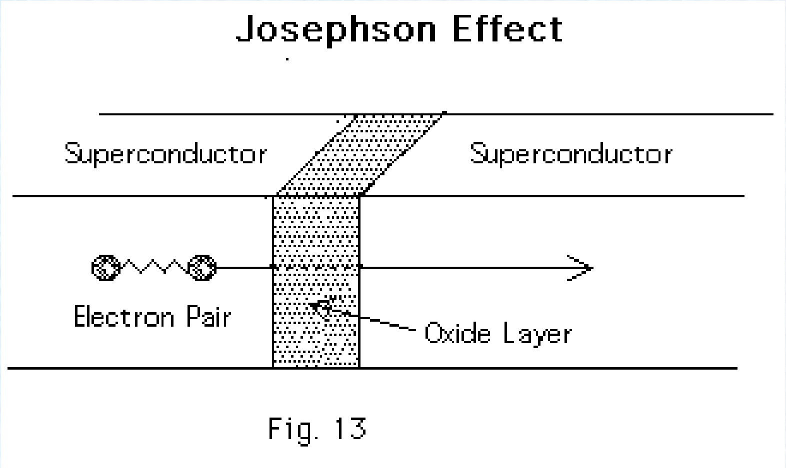
Main available “ingredients”

- Bosons and/or <u>(attractively interacting) fermions</u>
- Geometry (1D / 2D)
- Long-range interactions
- Add disorder
- Time-dependence (and to a certain extent space-dependence) of the parameters of the Hamiltonian
- Simulate a magnetic field through a rotation or with optical tools
- Explicit tuning of the interactions via Feshbach resonances
- <u>Optical lattices or double well potentials</u>

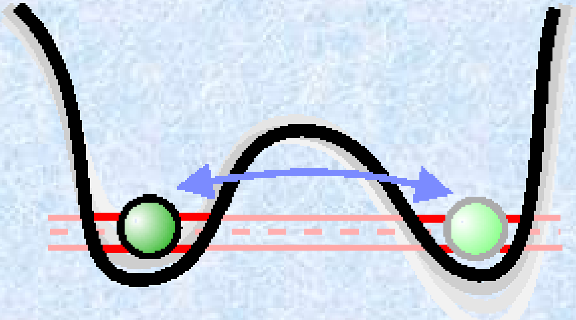
Ultracold atoms as quantum simulators of:

- **Strongly interacting lattice systems (e.g., Fermi and/or Bose Hubbard-like models)**
- **Quantum magnetism**
- **Dirac and relativistic field theories**
- **Low-dimensional systems**
- **Quantum Hall physics**
- **(BCS) Superconductors with Cooper pairs**
- **Josephson junctions**
- **…**

Ultracold weak link: a Josephson junction

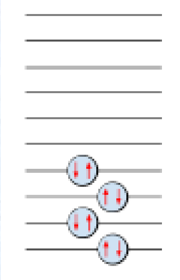


A two-component Fermi mixture in a double well ($T<T_c$) is a fermionic Josephson junction *[bosonic junctions → F.S. Cataliotti et al., Science (2001) – M. Albiez et al. PRL (2005)]:*



Single system (I)

We describe a single, uncoupled, system by the so-called Richardson model: N levels, M pairs



$$H_R = \sum_{\alpha=1}^{N} \varepsilon_\alpha \left(c^\dagger_{\alpha\uparrow} c_{\alpha\uparrow} + c^\dagger_{\alpha\downarrow} c_{\alpha\downarrow} \right) - 2g \sum_{\alpha\beta=1}^{N} c^\dagger_{\alpha\uparrow} c^\dagger_{\alpha\downarrow} c_{\beta\downarrow} c_{\beta\uparrow}$$

Introducing the pair operator $b_\alpha = c_{\alpha\downarrow} c_{\alpha\uparrow}$

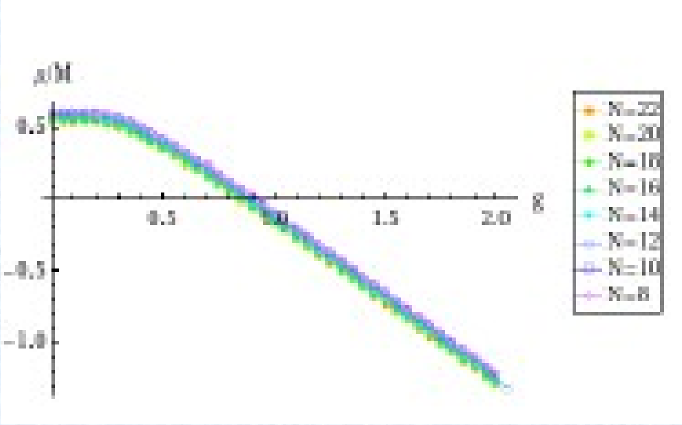
and ignoring single fermions (only pairs) one gets

$$H_R = 2 \sum_{\alpha=1}^{N} \varepsilon_\alpha b^\dagger_\alpha b_\alpha - 2g \sum_{\alpha\beta=1}^{N} b^\dagger_\alpha b_\beta$$

The model is solvable by Bethe ansatz [R.W. Richardson, Phys. Lett. (1963)] and it gives a fair description of the BCS-BEC crossover for systems with few attractively interacting fermions [G. Ortiz and M. Dukelsky, PRA (2005)].

Single system (II)

μ vs g



M=N/2-1

...typical of the BCS/BEC crossover

Weakly coupled systems

The Hamiltonian for two coupled Richardson models is:

$$H = H_R + H_L + H_T$$

$$H_T = -\eta \sum_{\sigma=\uparrow,\downarrow} \sum_{\alpha,\beta=1}^{N} (c^\dagger_{\alpha\sigma,L} c_{\beta\sigma,R} + \text{H.c.})$$

The effective Hamiltonian for weak coupling λ<<1 is [B.D. Josephson, Phys. Lett (1962); D. Gobert, U. Schollwock, and J. von Delft, Eur. Phys. J. B (2004)] reads:

$$H^{(2)} \approx -\lambda \Delta_{\mathrm{BCS}} \sum_{\alpha,\beta} \frac{b^\dagger_{\alpha,L} b_{\beta,R} + b_{\alpha,L} b^\dagger_{\beta,R}}{E_\alpha + E_\beta}$$

$$E_\alpha = \sqrt{\xi_\alpha^2 + \Delta^2_{\mathrm{BCS}}},\ \xi_\alpha = \varepsilon_\alpha - \mu, \text{ and } \lambda = 2\eta^2/\Delta_{\mathrm{BCS}}$$

Tunneling dynamics

We studied the exact dynamics up to 20 total levels (and 15 pairs): we choose an umbalaced initial state

$$|\Psi(t=0)\rangle = \frac{1}{\sqrt{1+\xi^2}} \left(\left|\Phi_{M_0}^{(L)}\right\rangle \otimes \left|\Phi_{M_0-D}^{(R)}\right\rangle + e^{i\phi_0}\xi \left|\Phi_{M_0-D}^{(L)}\right\rangle \otimes \left|\Phi_{M_0}^{(R)}\right\rangle \right)$$

Definition and formation of the relative phase

We considered two different ways of defining a relative phase (when the relative phase is well defined they should agree):

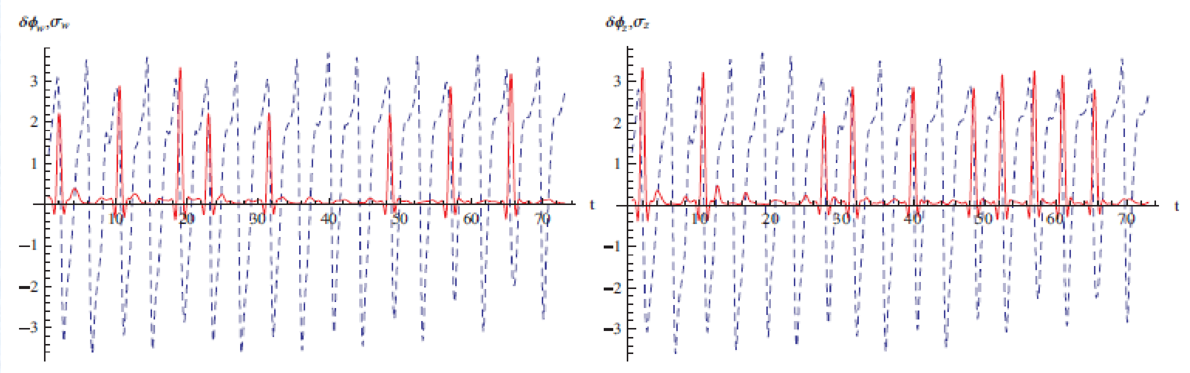
$$w_\alpha(t) = \langle \Psi(t) | \, b^\dagger_{\alpha,L} b_{\alpha,R} \, | \Psi(t) \rangle \equiv |w_\alpha(t)| \, e^{i\delta\phi_w(t;\alpha)}$$

$$z_\alpha(t) = \langle \Psi(t) | \, b^\dagger_{\alpha,L} b_{N/2,R} \, | \Psi(t) \rangle \equiv |z_\alpha(t)| \, e^{i\delta\phi_z(t;\alpha)}$$

$$\delta\phi_{w,z}(t) = \frac{1}{N} \sum_{\alpha=1}^{N} \delta\phi_{w,z}(t;\alpha) \qquad \sigma^2_{w,z}(t) = \frac{1}{N} \sum_{\alpha=1}^{N} \left(\delta\phi_{w,z}(t;\alpha) - \delta\phi_{w,z}(t)\right)^2$$

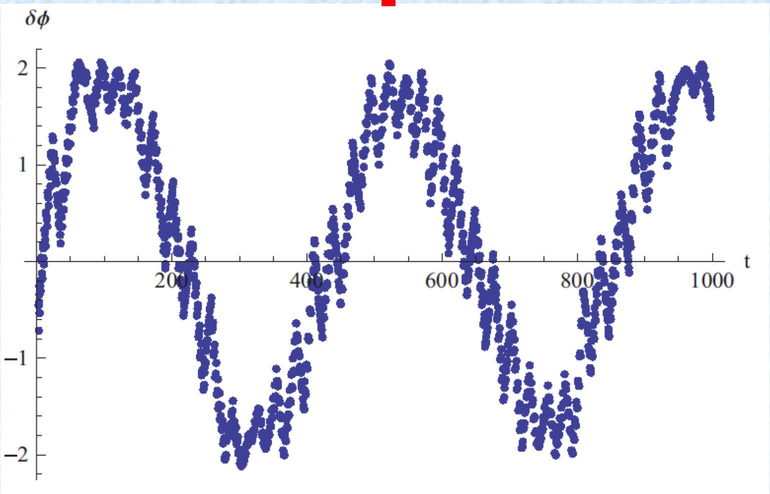
A typical result:

For each system:
N=8, M=4,
g=0.6
Initial state with D=2
(λ=0.1)

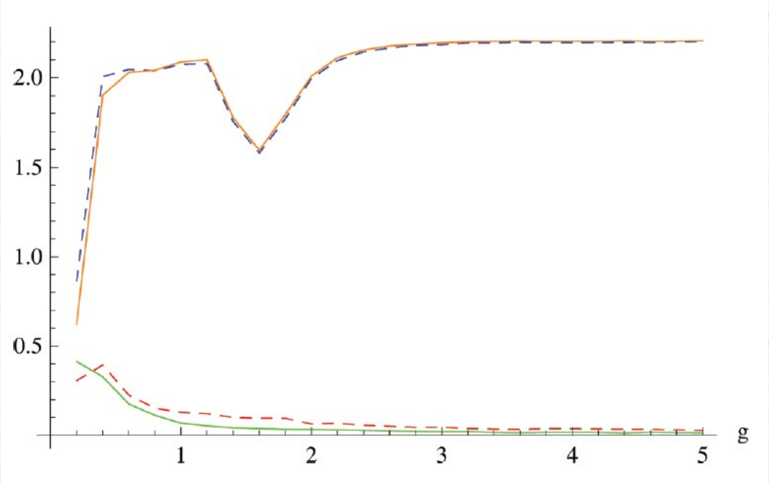


[F. Buccheri and A. Trombettoni, PRB (2013)]

Definition and formation of the relative phase: results

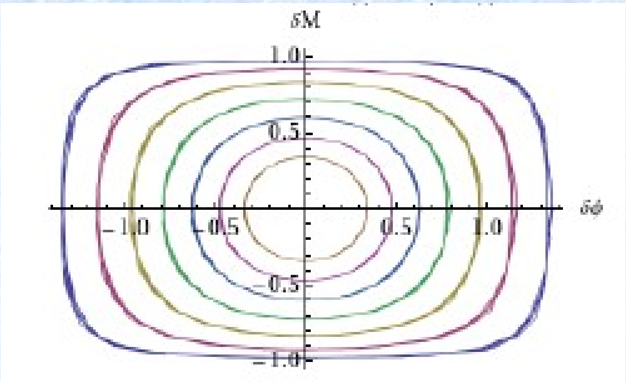


Example of relative phase vs time

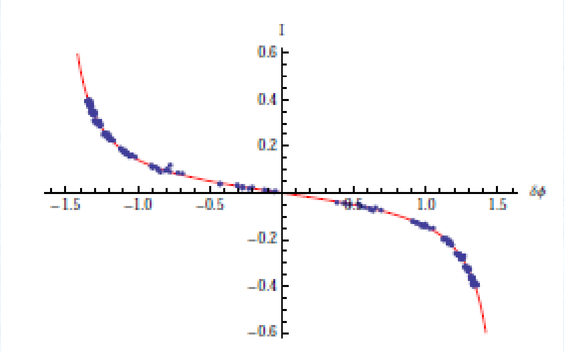


Time average of the variance of the relative phase (bottom) << time average of its mean (top) – <u>notice that this is valid also for numbers of pairs as low 10</u>

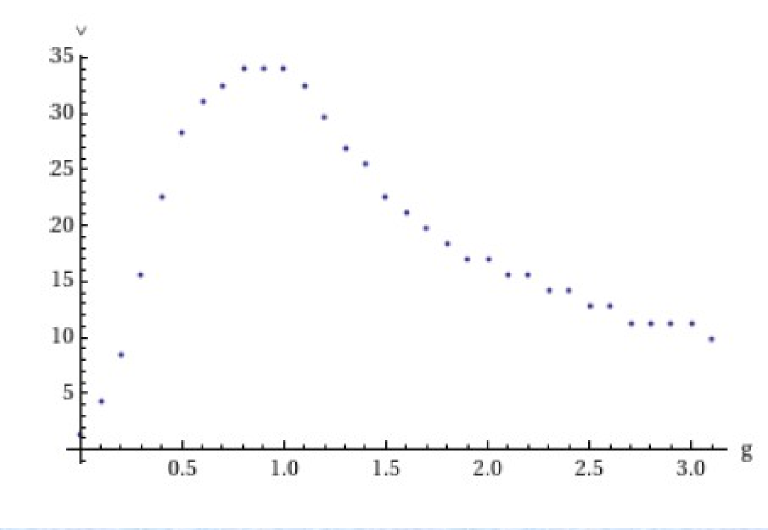
Phase portrait and Josephson coupling



Population imbalance-relative phase portrait
N=10 – M=7
g=0.5



Current vs relative phase for different times



Josephson frequency vs g: maximum at the unitary limit

[in qualitative agreement with results obtained by mean-field / Bogoliubov-De Gennes approaches by the groups of Camerino (PRL 2007, Phys. Rep. 2010), Padova (PRA 2008, PRA 2009) and Trento (PRA 2008, PRA 2009)].

[F. Buccheri and A. Trombettoni, PRB (2013)]

Summary

- Ultracold atoms are routinely used/proposed for <u>quantum engineering of states and devices</u>

- Optical lattices can be added, realizing lattice models and Josephson physics with ultracold atoms

- <u>Quantum simulation</u> of the XXZ chain →
  from one side, ultracold bosonic lattices can be used to perform a quantum simulation of the XXZ chain
  from the other correlation functions of the Bose-Hubbard model at half-filling very well captured by analytical formulas from an effective (integrable) XXZ model

- <u>Josephson junctions</u> realizable with ultracold fermions in double-well or multi-well potentials → a definite relative phase forms also for rather small number of fermions (as low as 10)

Thank you!