Holographic Quantum Hall Ferromagnet

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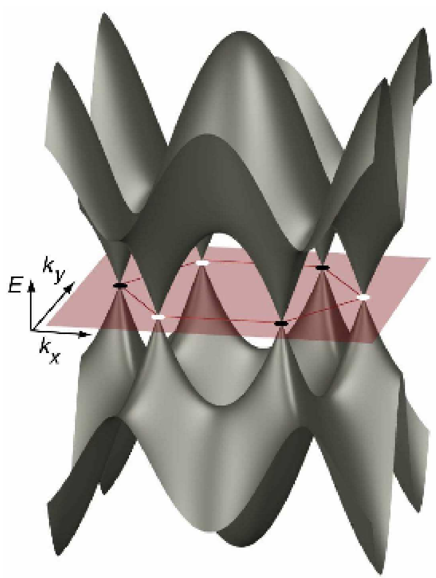
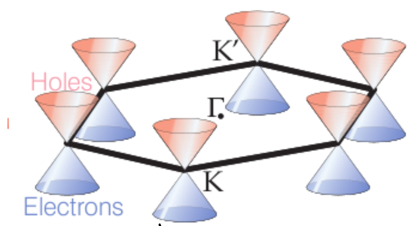
Graphene has relativstic 2+1-D fermions with SU(4) symmetry



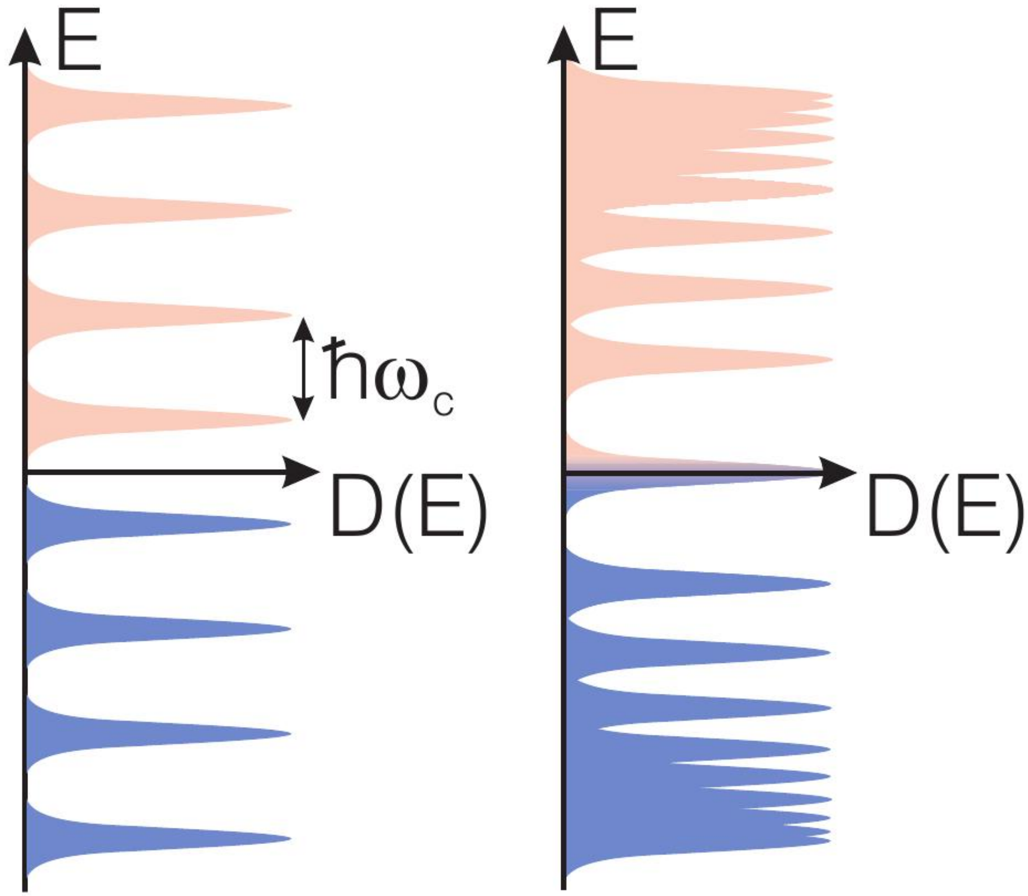


Graphene: $E(k) = \hbar v_F |\vec{k}|$ $v_F \sim c/300$ good up to $\sim 1ev$

2 valleys×2 spins

$$S = \int d^3x \sum_{\sigma=1}^{4} \bar{\psi}^\sigma \, i\gamma^\mu \partial_\mu \, \psi^\sigma + \text{interactions}$$

Non-Relativistic and Relativistic Landau Levels

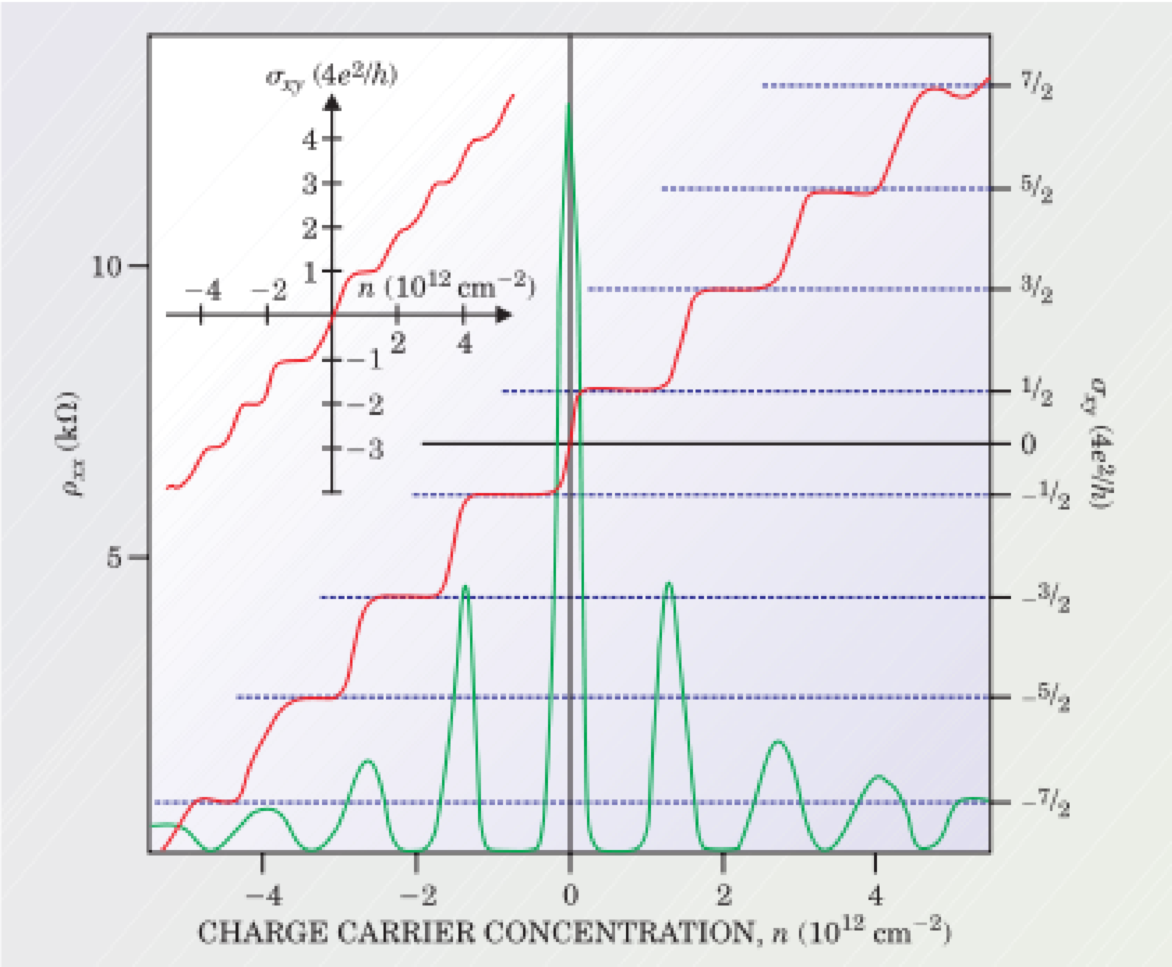


Non-relativistic: $E = \hbar\omega_C \left(n + \frac{1}{2}\right)$, $n = 0, 1, 2, ...$

Relativistic $E = \pm\hbar v_F \sqrt{2|B|n}$ degeneracy $= \frac{e|B|}{2\pi}$

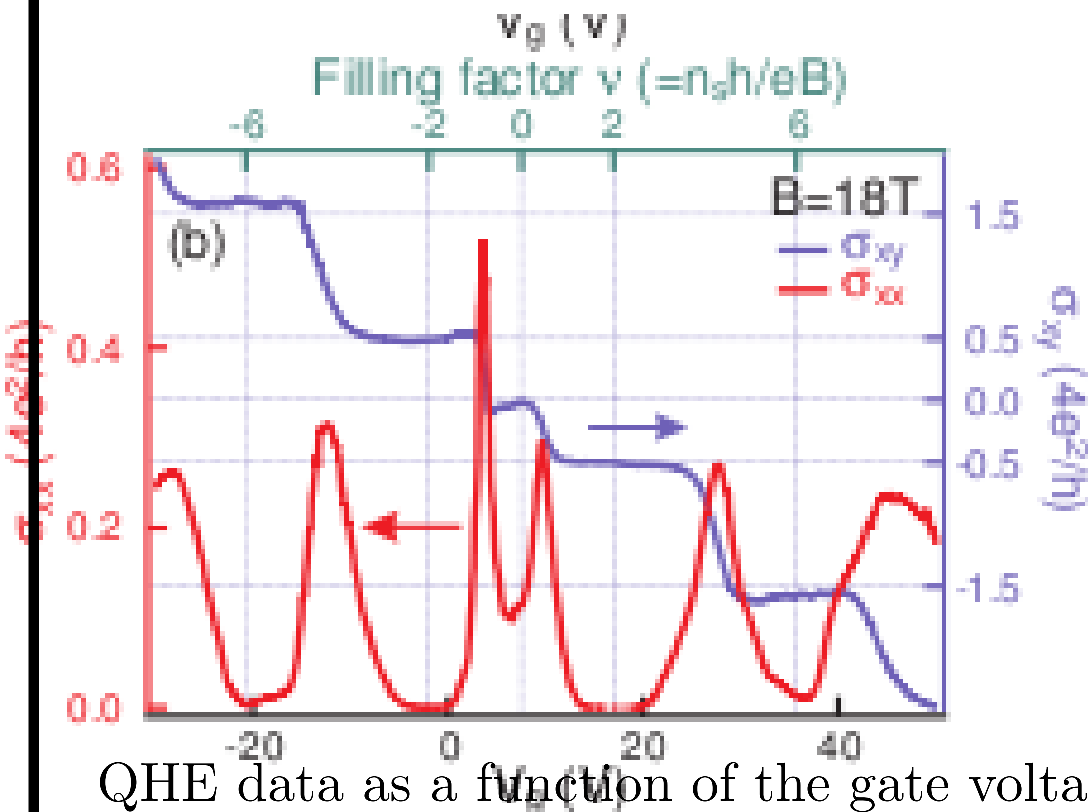
K. Novoselov et. al. *Nature 438, 197 (2005)*

Y. Zhang et. al. *Nature 438, 201 (2005)*



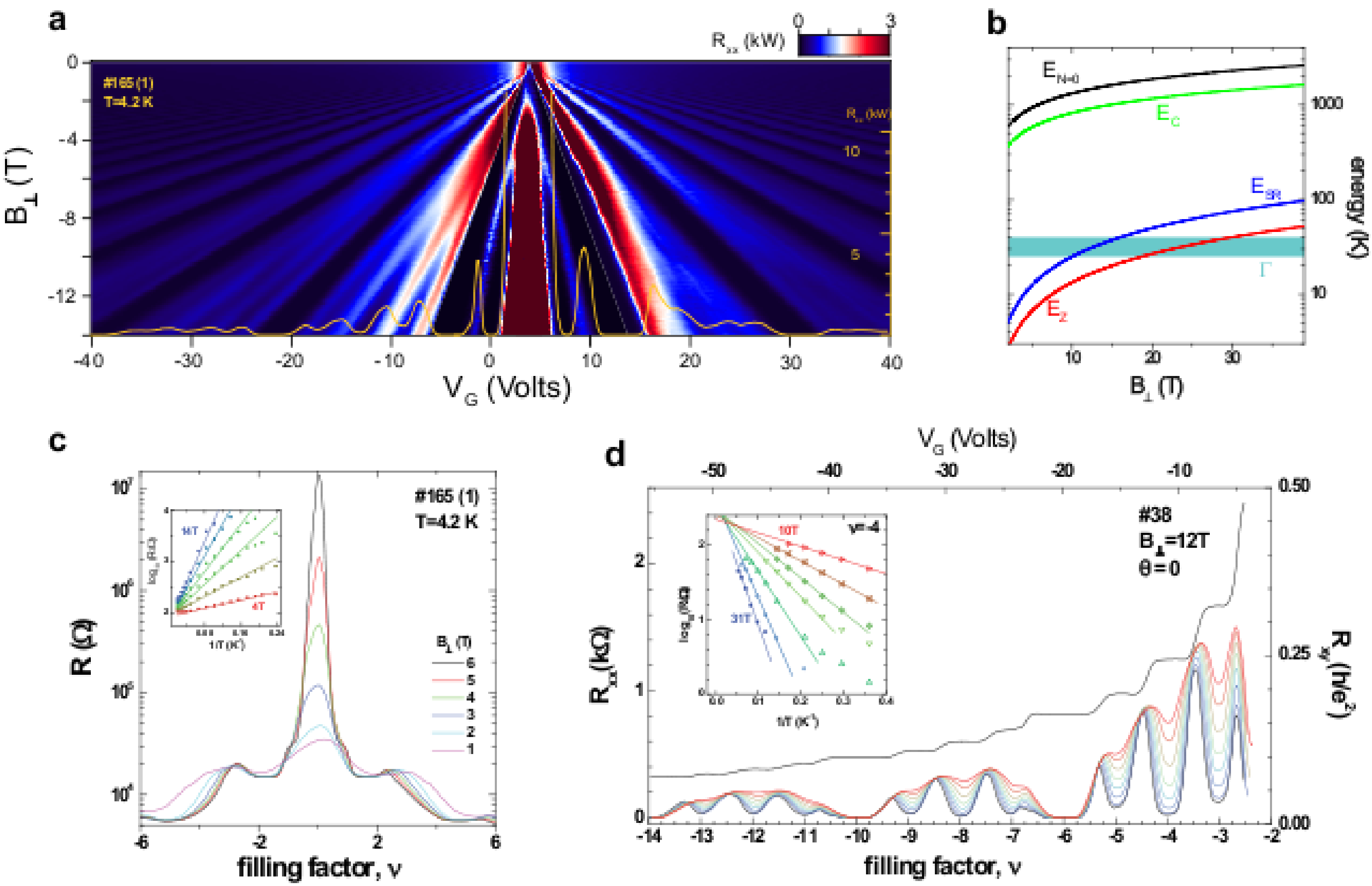
$$\sigma_{xy} = 4\frac{e^2}{h}\left(n + \frac{1}{2}\right)$$

Splitting of $\nu = 0$ Landau level Zhang et.al. arXiv:1003.2738



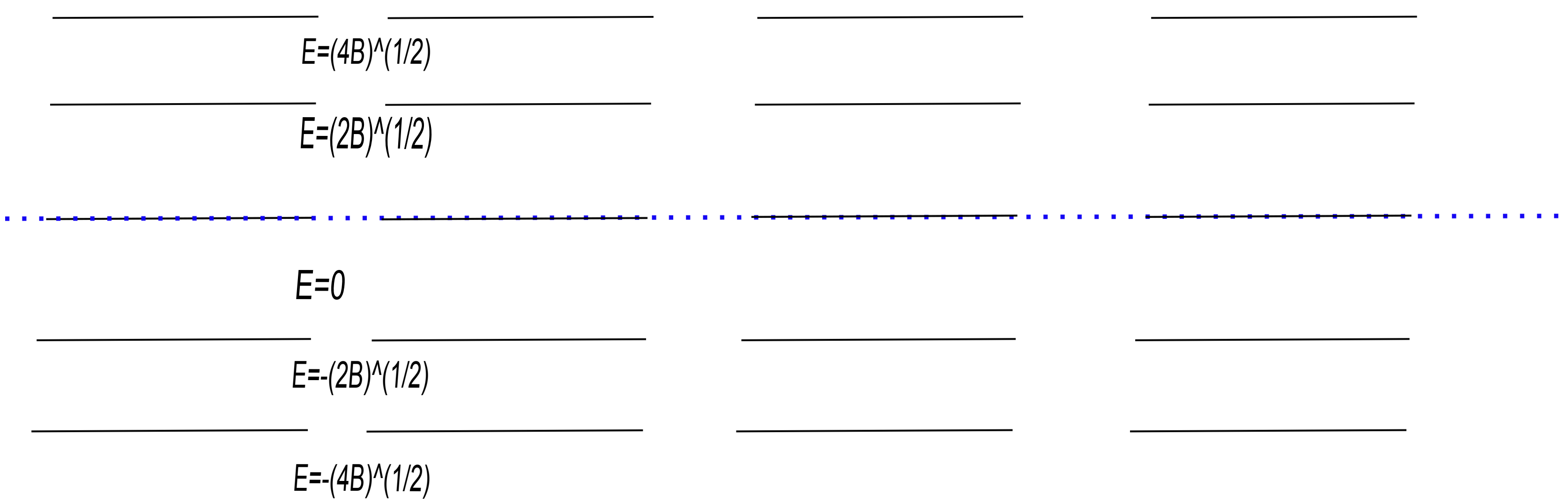
QHE data as a function of the gate voltage V_g, for B = 18 T at T = 0.25 K

Splitting of $\nu = 0$ Landau level A.F.Young et.al., Nat. Phys. 2012



Quantum Hall Ferromagnetism at Weak Coupling

Consider a 4-fold degenerate spectrum of relativistic Landau levels



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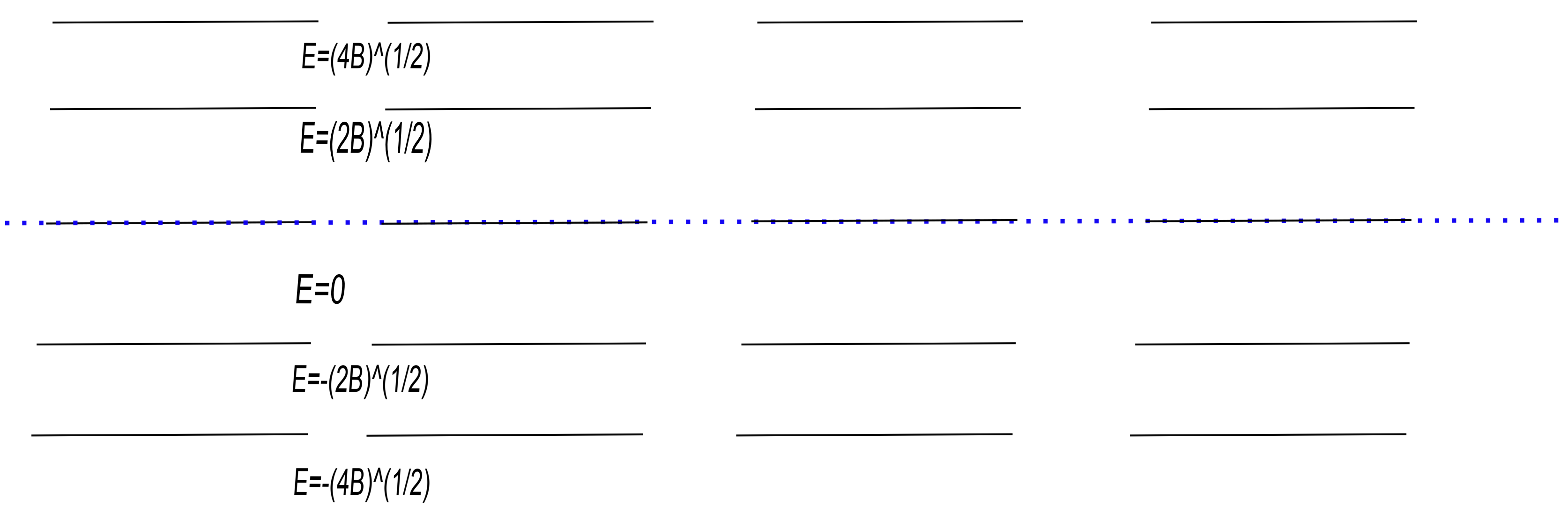
Ground state has negative energy levels filled



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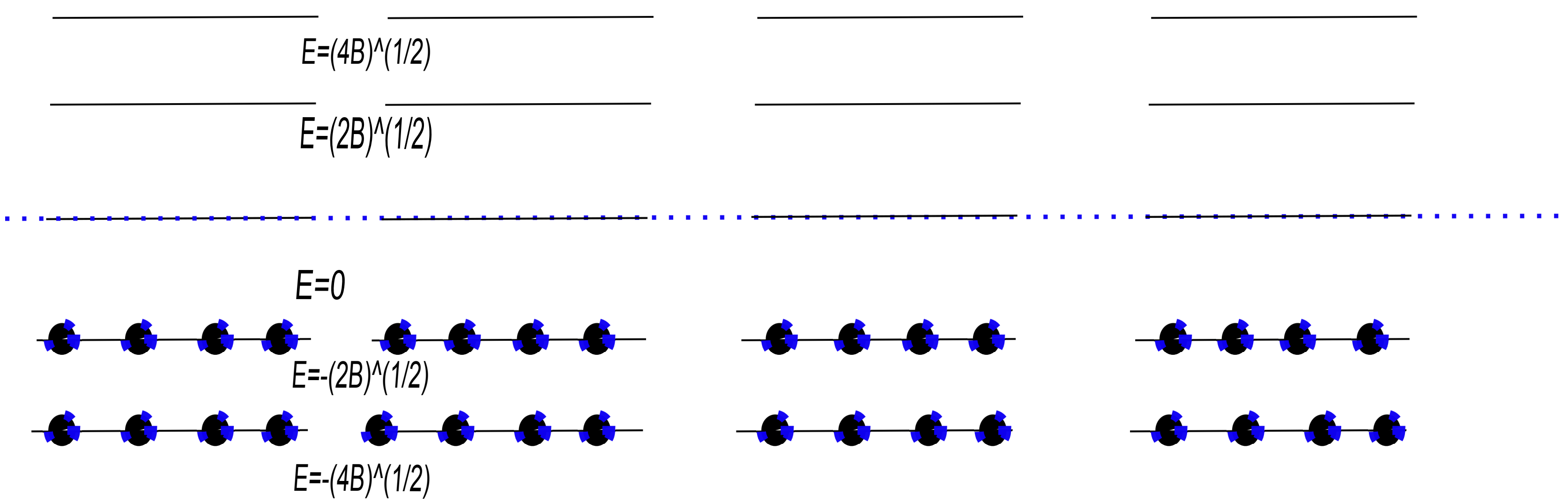

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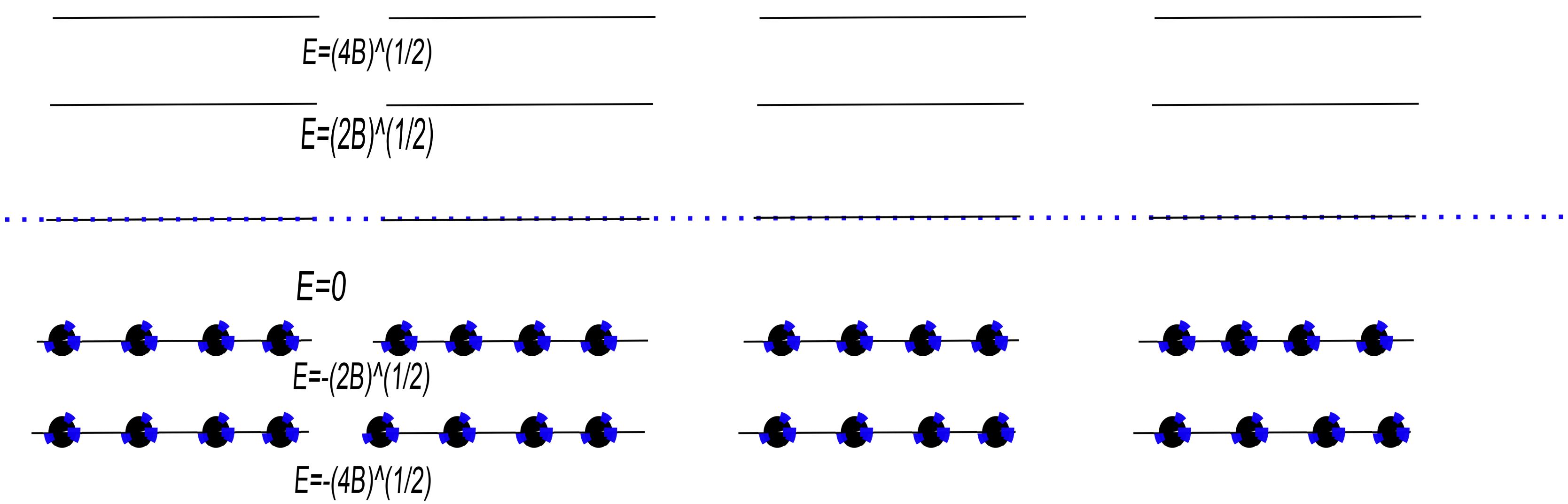
The zero energy states should be half-filled



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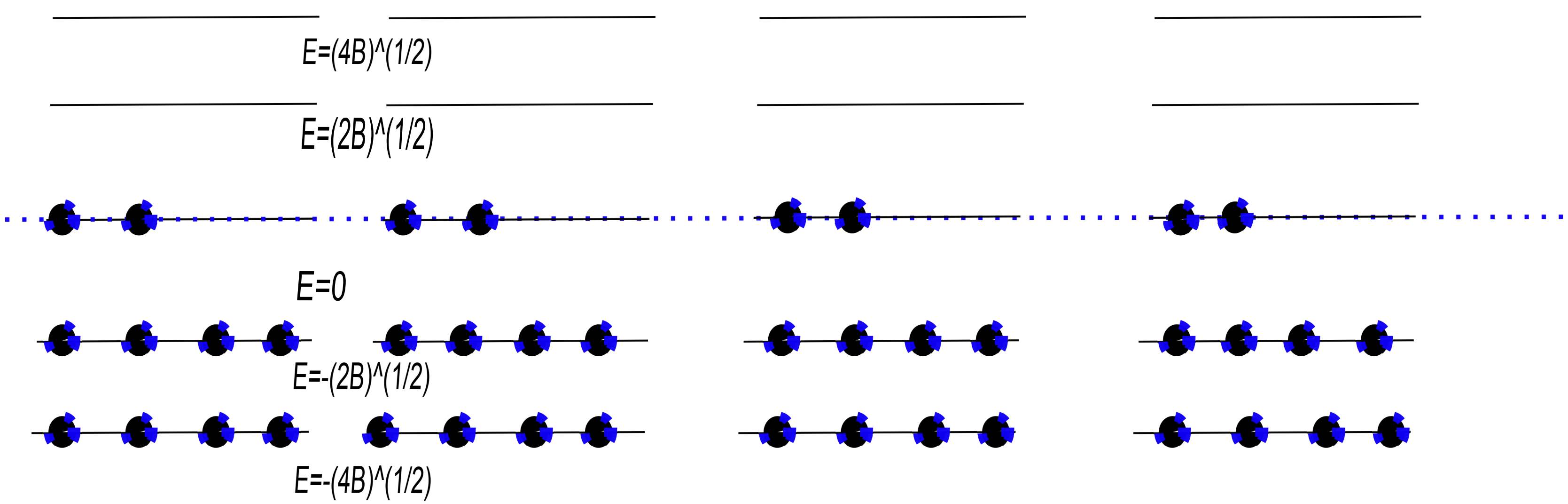

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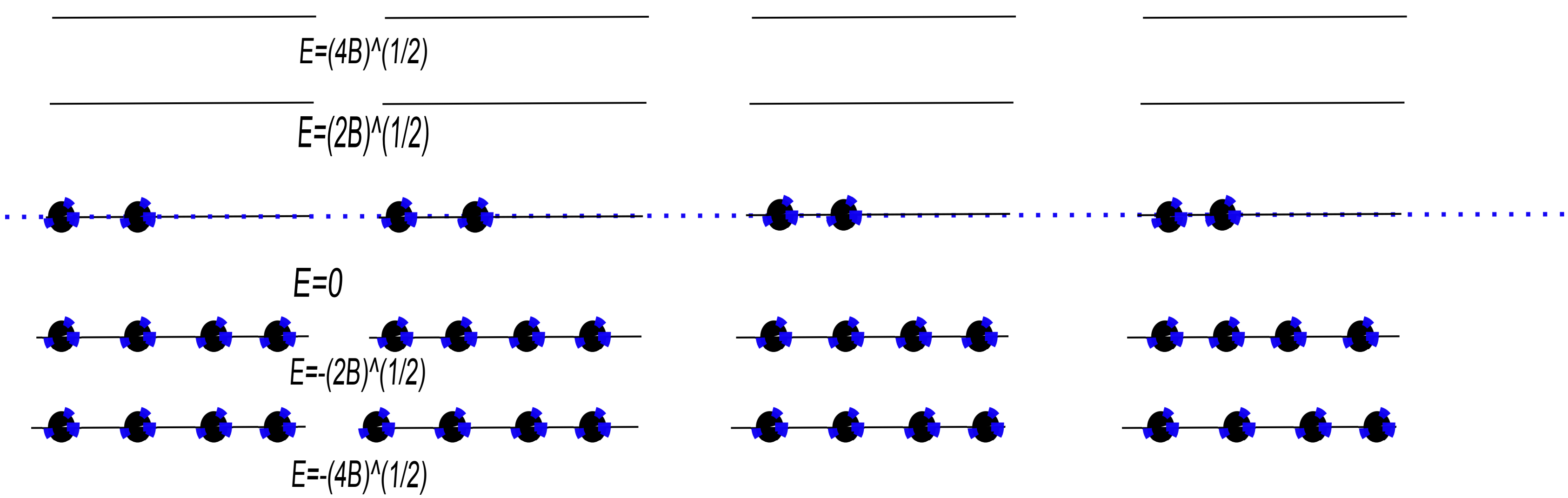

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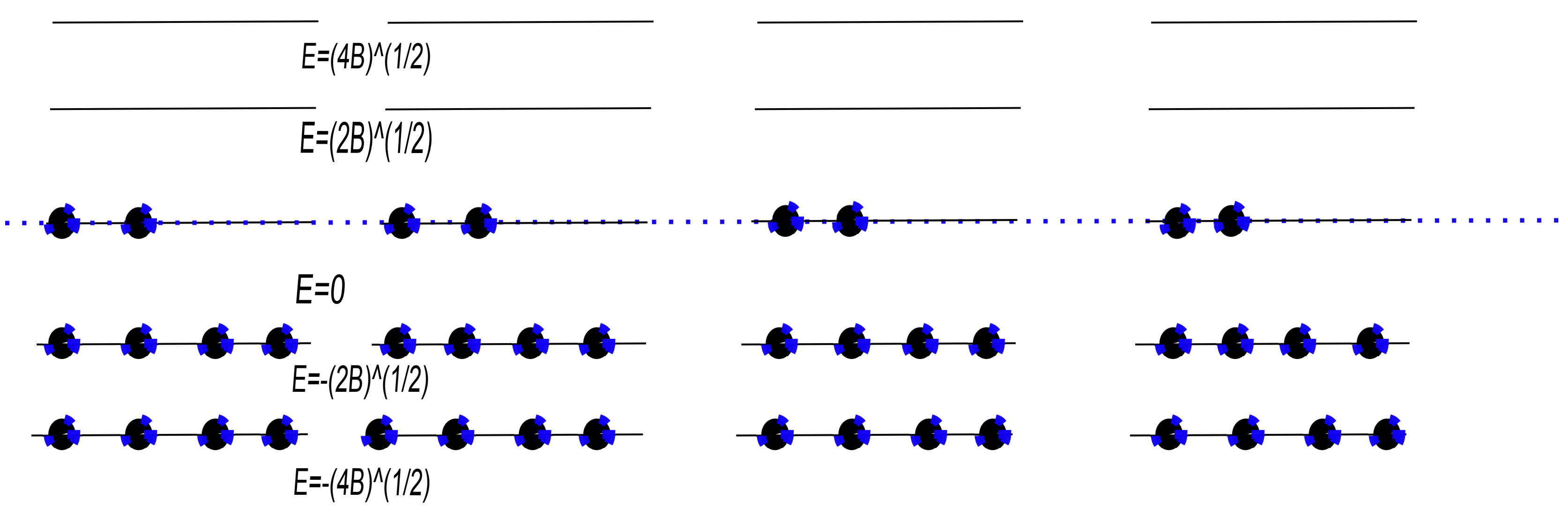

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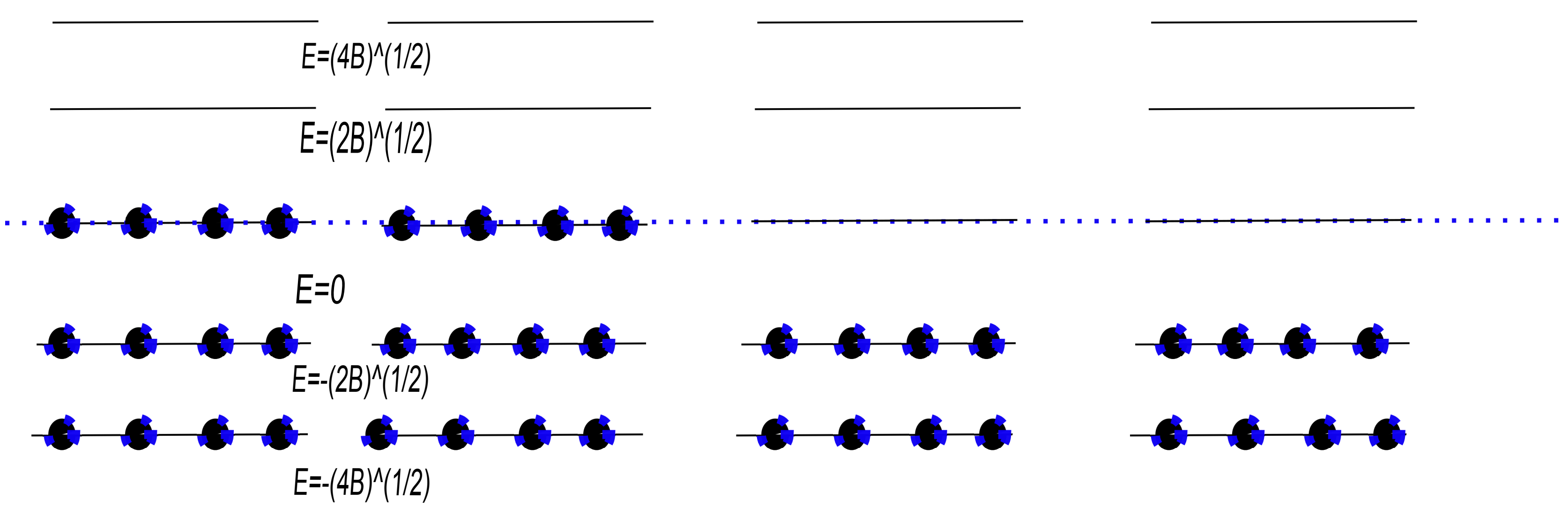


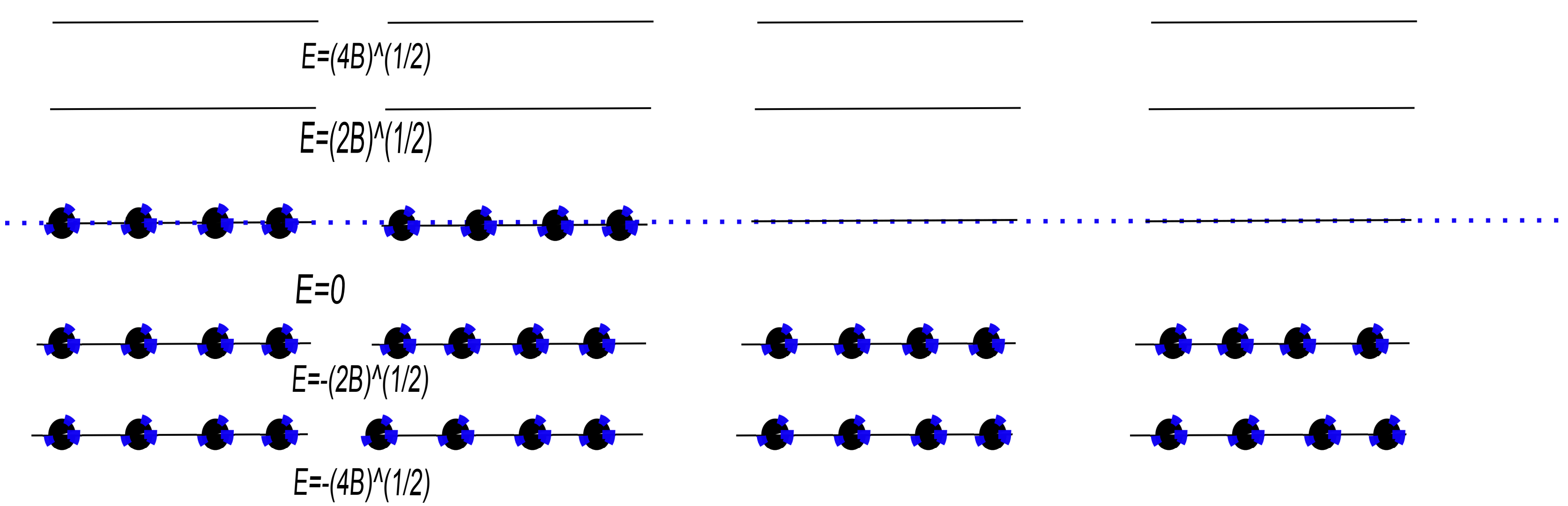
Quantum Hall Ferromagnet at Weak Coupling:

Spontaneous breaking $U(4) \to U(2) \times U(2)$

Coulomb interaction

$H_{\rm Coulomb} = \frac{1}{2}\int \psi^\dagger(r)\psi(r)\frac{e^2}{4\pi|\vec{r}-\vec{r}'|}\psi^\dagger(r')\psi(r')$

$$\rho = \langle \psi^\dagger \psi \rangle = \frac{B}{4\pi}(1,1,-1,-1) \quad , \quad \langle \bar\psi \psi \rangle = \frac{B}{4\pi}(1,1,-1,-1)[1+\ldots]$$



Graphene with Coulomb interaction $V(r) = \frac{e^2}{4\pi r}$

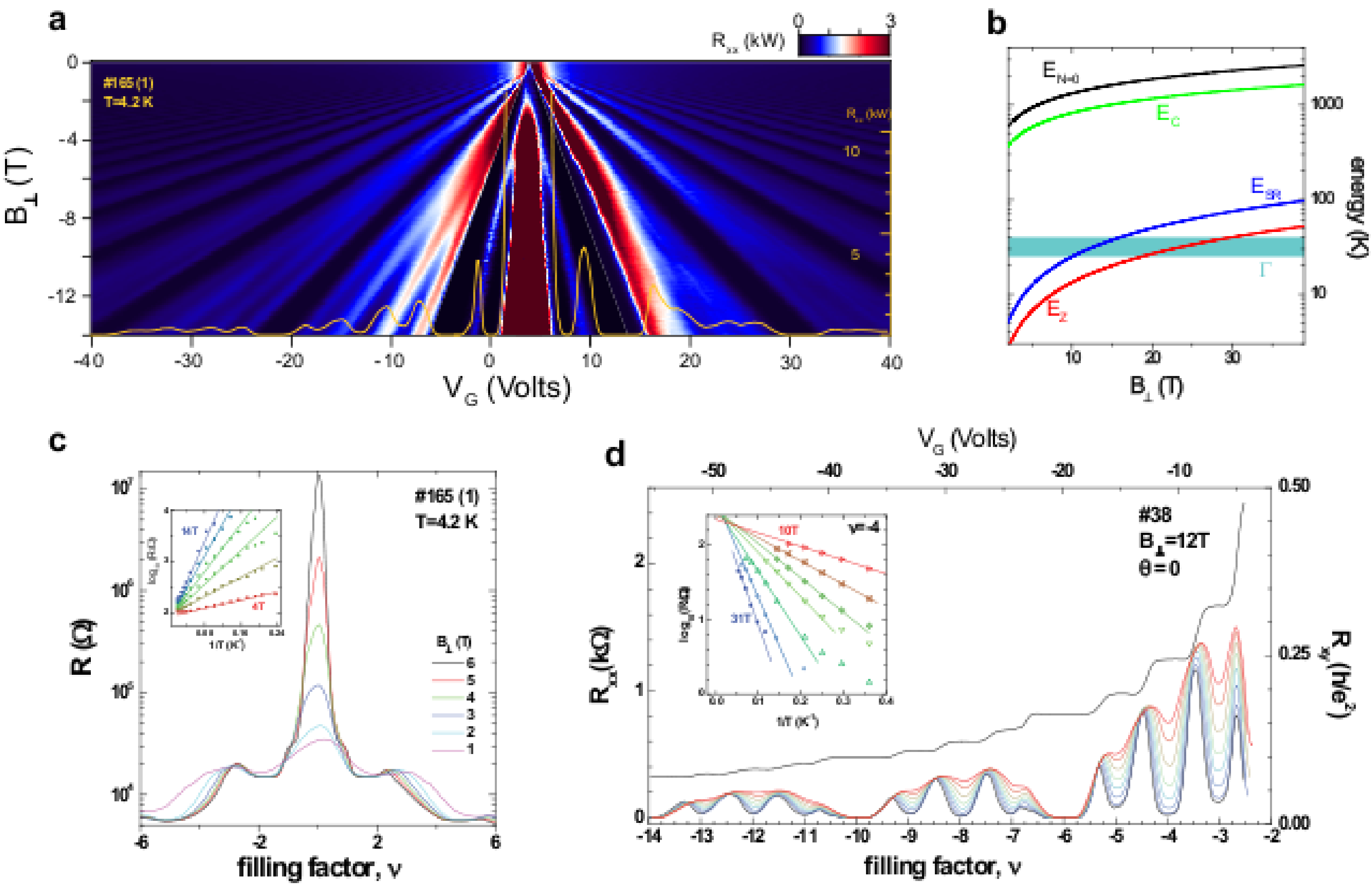
$$S = \int dtdxdy \sum_{k=1}^{4} \bar{\psi}_k \left[\gamma^t (i\partial_t - A_t) + v_F \vec{\gamma} \cdot (i\vec{\nabla} - \vec{A}) \right] \psi_k$$

$$+ \frac{1}{4e^2} \int dtdxdydz \left[\frac{1}{c} F_{0i} F_{0i} - c F_{ij} F_{ij} \right]$$

- The graphene fine structure constant

$$\alpha_{\text{graphene}} = \frac{\frac{e^2}{4\pi\lambda}}{\hbar v_F / \lambda} = \frac{e^2}{4\pi\hbar v_F} = \frac{e^2}{4\pi\hbar c} \frac{c}{v_F} \approx \frac{300}{137}$$

Splitting of $\nu = 0$ Landau level A.F.Young et.al., Nat. Phys. 2012



D3-D7 system

D3-D7 brane system as a holographic model of graphene.

With 4 probe D7 branes (SU(4) symmetry), the dual gauge theory has 4 species of 2-component fermions occupying a 2+1-dimensional defect in 3+1-dimensional spacetime

$\mathcal{N} = 4$ sypersymmetric Yang-Mills theory in 3+1-dimensional bulk

The degrees of freedom on the defect are 4 species of 2-component relativistic fermions á la graphene.

$$S = \int d^3x \sum_{a=1}^{4} \bar{\psi}_a \gamma^\mu (i\partial_\mu + A_\mu)\psi_a + \int d^4x \, \mathrm{Tr}\left[-\frac{1}{4g_{YM}^2} F_{\mu\nu}F^{\mu\nu} + \ldots \right]$$

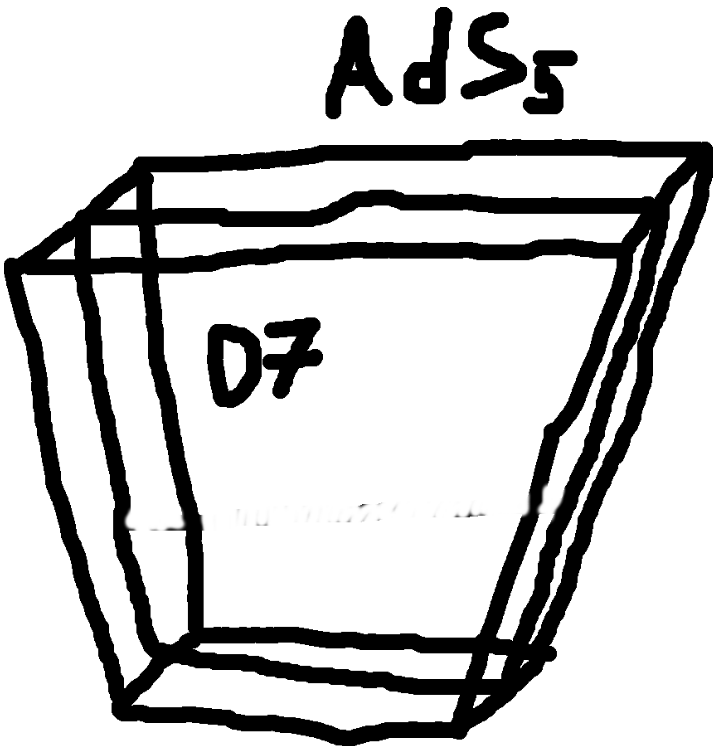
SU(N) gauge symmetry, Solve in large N limit.

Classical embedding of D brane in $AdS_5 \times S^5$ is the strong coupling, planar $\lambda = g^2_{YM} N$ limit of gauge field theory.

Chiral symmetry is broken with condensate $\bar{\psi}\psi \sim \Lambda^2 e^{-1/\sqrt{\lambda - \lambda_c}}$.

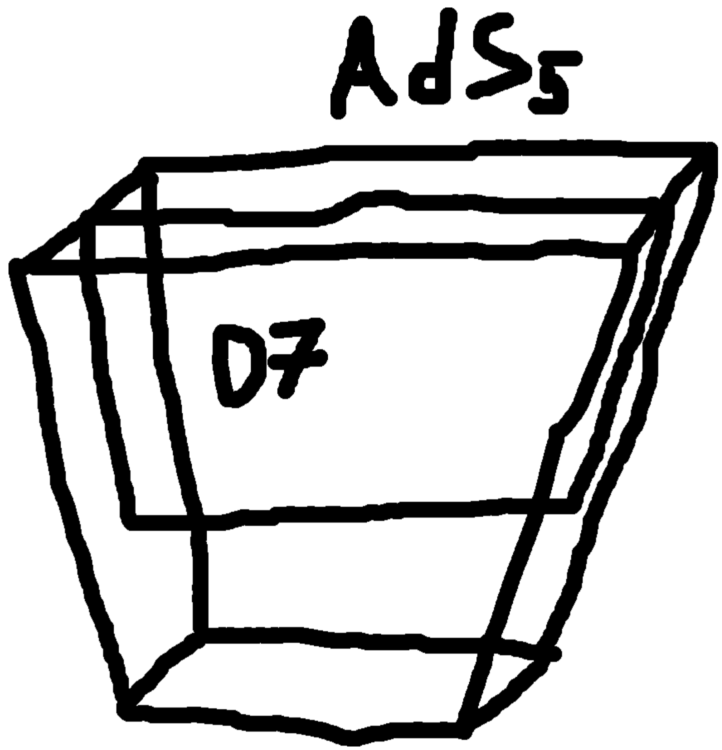
Unless λ is tuned close to λ_c, physics is highly cutoff dependent. D brane is large λ limit.

Unstable symmetric configuration of D7 brane



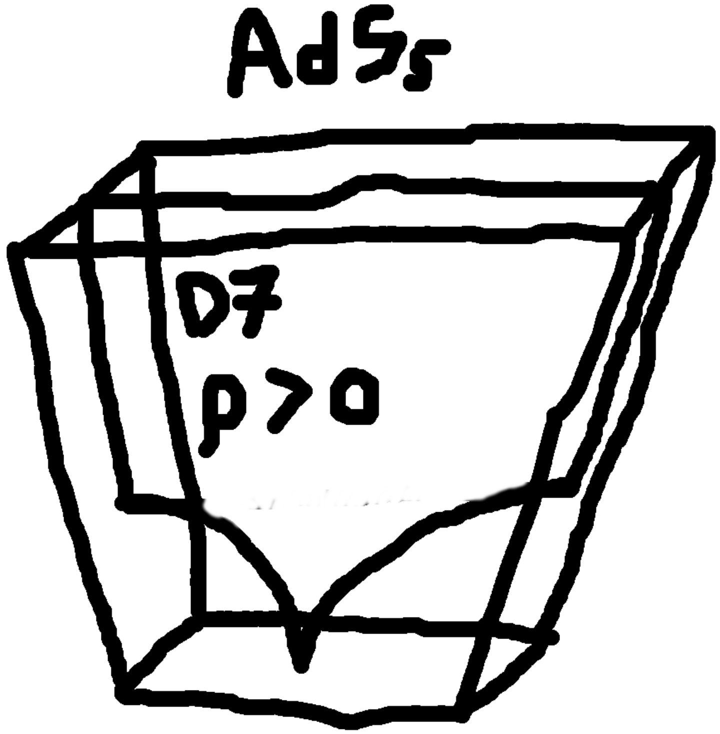
Worldvolume is $AdS_4 \times S^4$ with S^4 on equator of S^5

Chiral symmetry breaking configuration of D7 brane

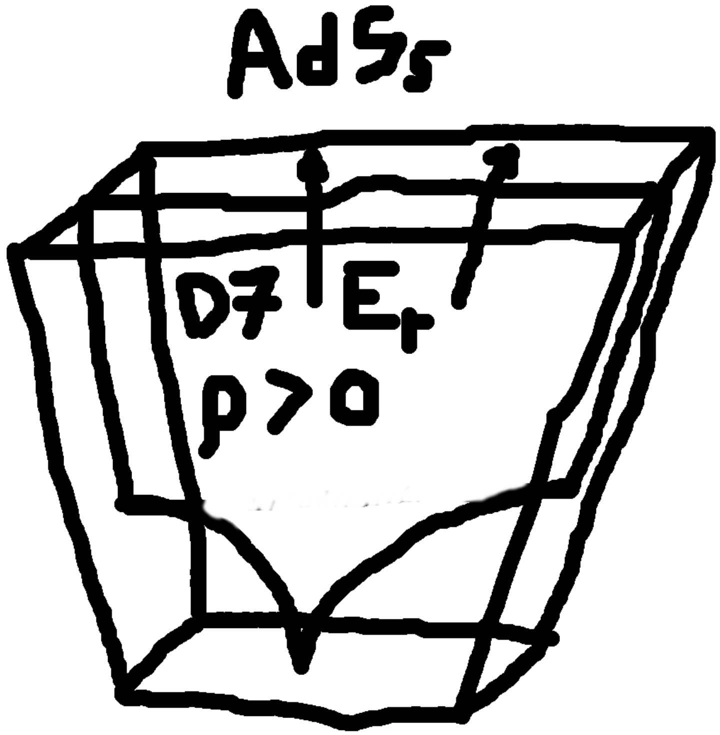


S^4 slips of equator of S^5

D7 brane with electric charge extends to Poincare horizon



D7 brane with electric charge has worldvolume electric field



World Volume Axion

$$S \sim \int \sqrt{\det(g + 2\pi\alpha' F)} + \int (2\pi\alpha')^2 F \wedge F \wedge C^{(4)}$$

Maxwell equations on the brane worldvolume ($\approx AdS_4$) have an axion term

$$\frac{d}{dr}\left[\frac{\partial}{\partial E_r}\sqrt{\det(g + 2\pi\alpha' F)} + \frac{1}{2\pi^2} B\Theta(r)\right] = 0$$

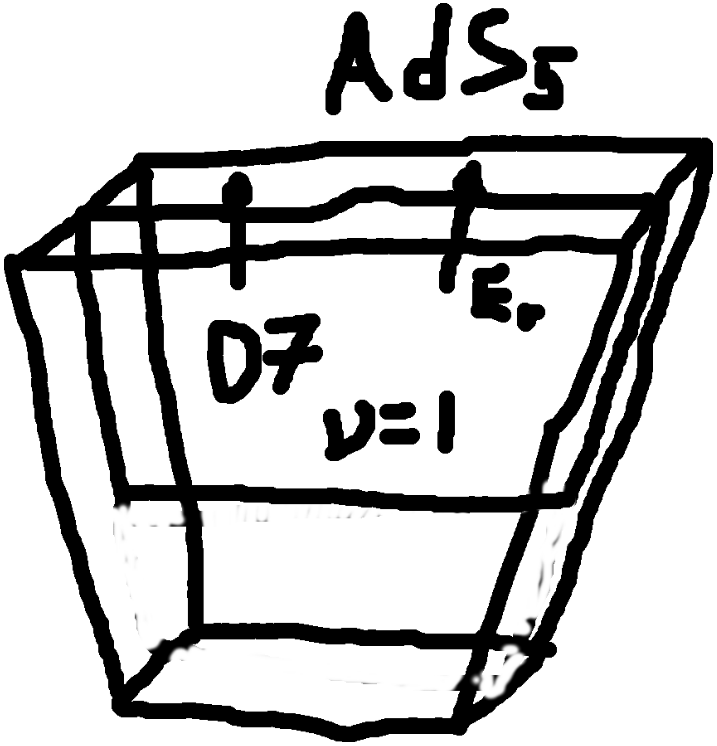
$$\frac{\partial}{\partial E_r}\sqrt{\det(g + 2\pi\alpha' F)} + \frac{1}{2\pi^2} B\Theta(r) = \rho$$

Gapped solution when

$$\nu = \frac{2\pi\rho}{B} \;=\; 1$$

$$\Theta(r) = \int_{S^4 \subset S^5} C^{(4)}$$

D7 brane with $\nu = 1$ has charge gapped incompressible IQHE state



With 4 D7 branes, we find such states with $\nu = -2, -1, 0, 1, 2$

Conclusion

-Holographic version of Landau levels (splitting of degeneracy of zero mode Landau level)

-We find the states with $\nu = -2, 1, 0, 1, 2$

-energy gaps $\sim f(\Lambda, B)$

-readily compute transport properties

-other Landau levels?

-fractional Hall effect?