Many-Body Cavity QED

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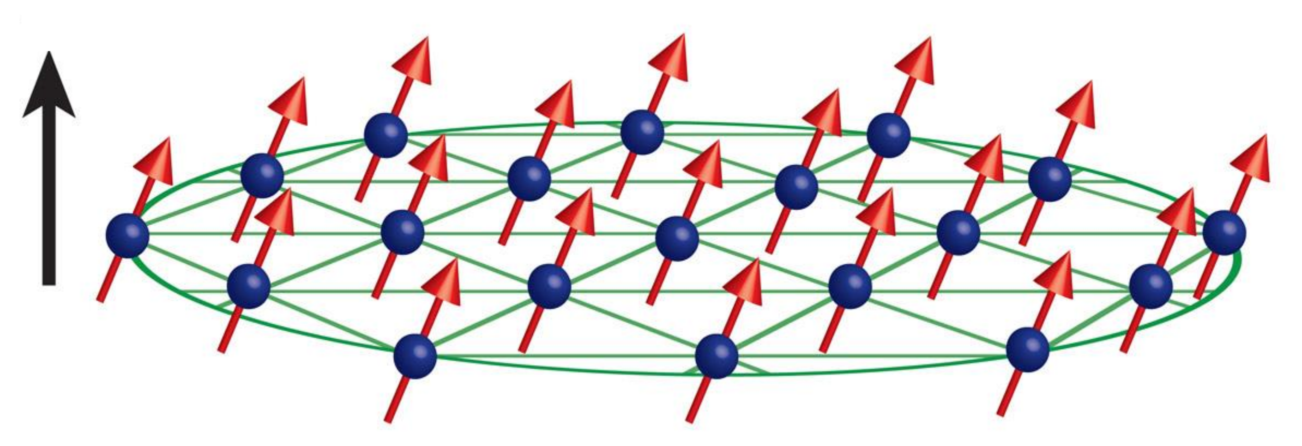
Motivation

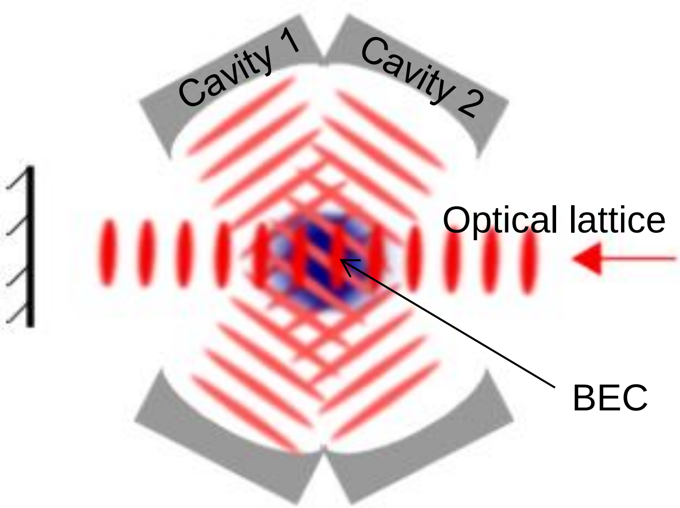
- Atom physics + (quantum) optics + condensed matter physics = hybrid systems and quantum many-body systems.
- Optical lattices + control → quantum simulators.

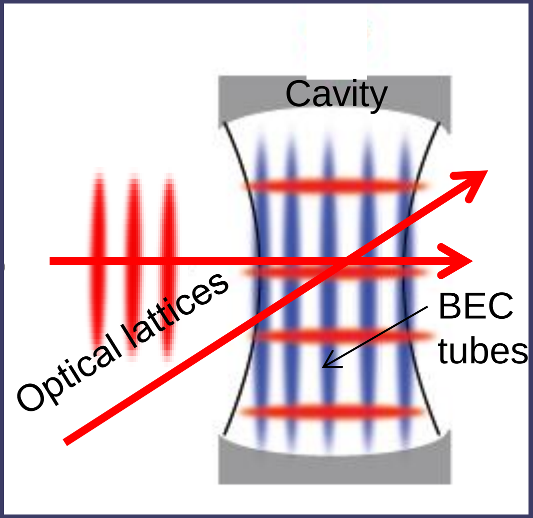


- Hubbard models, spin models,…
- Ask for novel many-body systems beyond paradigm condensed matter models.

Motivation

- Light and particles (atoms) on the same footing → cavity QED.
- Two new experimental setups in the *ETH* Esslinger group:





Setup 1. Two overlapping cavities, one optical lattice drive, atomic condensate.

Setup 2. One cavity, tubes of atomic condensates (external optical lattice).

- Interesting physics?

Outline

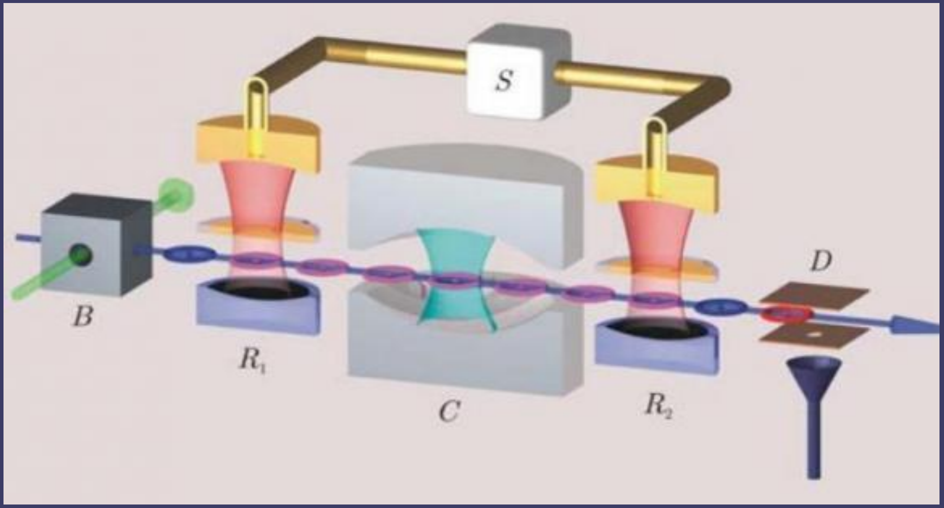
1. Cavity QED in five or so minutes.
2. Many-body cQED: (quantum) optical bistability
3. Collective phenomena, Dicke physics.
4. Setup 1 – $SU(3)$ Dicke model.
5. Setup 2 – Dynamical disorder.

Cavity Quantum Electrodynamics

Cavity QED

Jaynes-Cummings physics

- Cavity QED = coupling between few material (atomic) and few electromagnetic degrees of freedom.
- Cavity → atom-field coupling $g \sim {}^{1}/_{\sqrt{V}}$ (V effective mode volume).
- *Strong coupling regime* $g\sqrt{\bar{n}} \gg \gamma, \kappa$ (γ/κ atom/photon decay rates).



Classical atomic motion

Haroche & Raimond, *Exploting the Quantum* (OUP).

Cavity QED

Jaynes-Cummings physics

- Jaynes-Cummings Hamiltonian

$$\hat{H}_{jc} = \underbrace{\omega \hat{a}^{+}\hat{a}}_{\text{Field energy}} + \underbrace{\frac{\Omega}{2}\hat{\sigma}_z}_{\text{Atomic energy}} + \underbrace{g(\hat{a}^{+}\hat{\sigma}^{-} + \hat{\sigma}^{+}\hat{a})}_{\text{Interaction energy}}$$

- *Dressed states* (polaritons)

$$|\psi_{n+}\rangle = \cos\theta\,|e,n\rangle + \sin\theta\,|g,n+1\rangle,$$
$$|\psi_{n-}\rangle = \cos\theta\,|e,n\rangle - \sin\theta\,|g,n+1\rangle,$$

$$E_{n\pm} = \omega n \pm \sqrt{\frac{\Delta^2}{4} + g^2(n+1)}, \qquad \tan 2\theta = 2g\sqrt{n+1}/\Delta,$$

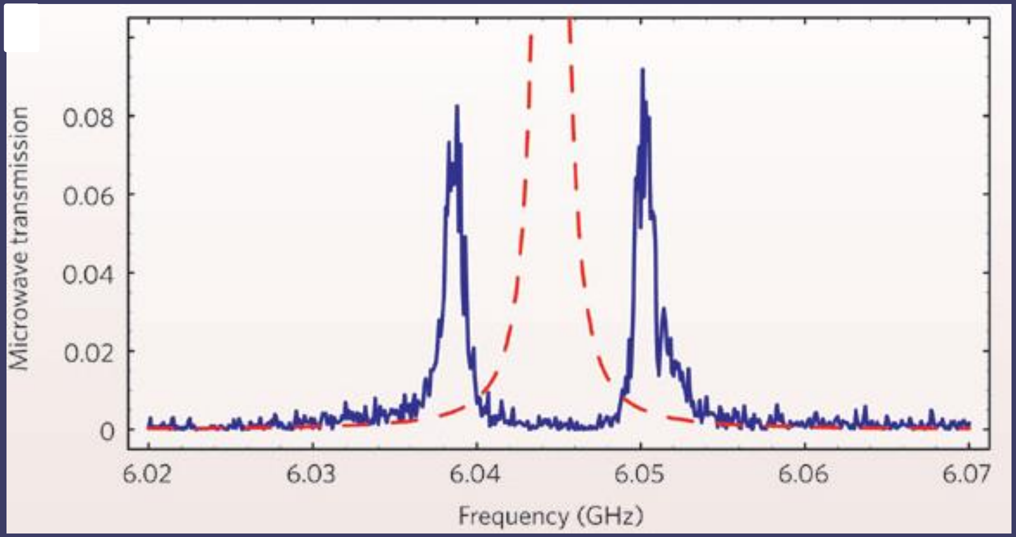
$$\Delta = \Omega - \omega.$$

Haroche & Raimond, *Exploting the Quantum* (OUP).

Cavity QED

Jaynes-Cummings physics

- *Vacuum Rabi splitting*, $E_{0\pm} = \pm g \quad (\Delta = 0)$

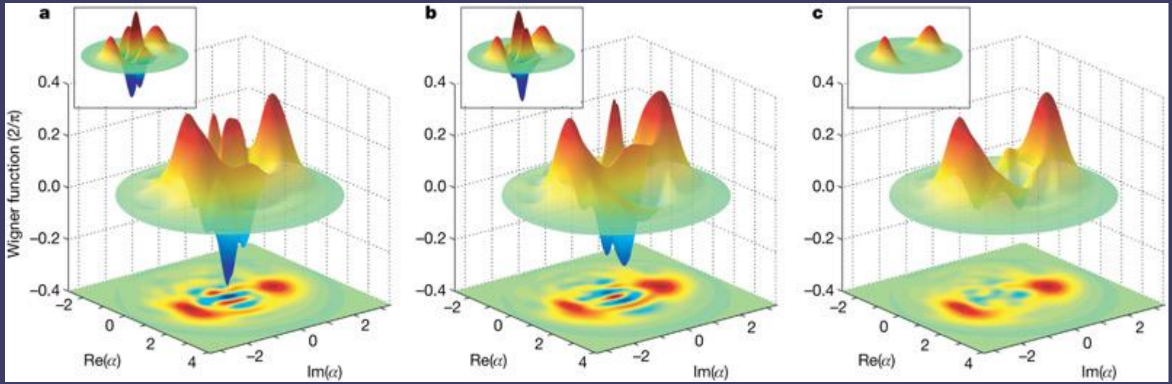


Schoelkopf & Girvin, *Nature* **451** (2008).

Cavity QED

Jaynes-Cummings physics

- Atom-atom, atom-field, field-field entanglement.
- Logic gates.
- ”Cats”.
- Tomography.



- Quantum-classical correspondence.
- Zeno and measurement phenomena.
- Field quantization.
- …

Haroche & Raimond, *Exploting the Quantum* (OUP).

Cavity QED

Dicke physics

- Dicke 1954: How do *N* atoms radiate?

$$\hat{H}_d = \omega \hat{a}^+ \hat{a} + \sum_{i=1}^{N} \frac{\Omega}{2} \hat{\sigma}_i^z + g(\hat{a}^+ + \hat{a}) \hat{\sigma}_i^x \equiv \omega \hat{a}^+ \hat{a} + \frac{\Omega}{2} \hat{S}_z + g(\hat{a}^+ + \hat{a}) \hat{S}_x$$

- *Dicke states* $\hat{S}_z |S, m\rangle = m |S, m\rangle$ (maximal spin sector $S = N/2$).

$$\sum_\psi \left| \langle \psi | \hat{H}_d | S, S, 0 \rangle \right|^2 \sim N^2$$

Superradiance!

- Enhanced radiation by a factor N!

Dicke, *Phys. Rev.* **93** (1954).

Cavity QED

Dicke physics

- Thermodynamic limit ($g \to g/\sqrt{N}$). Z_2-parity symmetry breaking.

$$g_c = \sqrt{\omega\Omega}/2 \qquad \begin{cases} Normal\,phase: & \langle \hat{a} \rangle = 0, \; \langle S_z \rangle = -N/2 \\ Superradiant\,phase: & \langle \hat{a} \rangle \neq 0, \; \langle S_z \rangle > -N/2 \end{cases}$$

- Classical PT (*Ising* type), survives also at $T = 0$ ('quantum' PT).
- *No-go theorem.* Minimal coupling Hamiltonian, assume ground state atoms, no dipole-dipole interaction → Dicke PT forbidden (Rzaźewski, *Phys. Rev. Lett*. **35**, (1975)).
- Large atom-field coupling by "driving" → openness → PT?

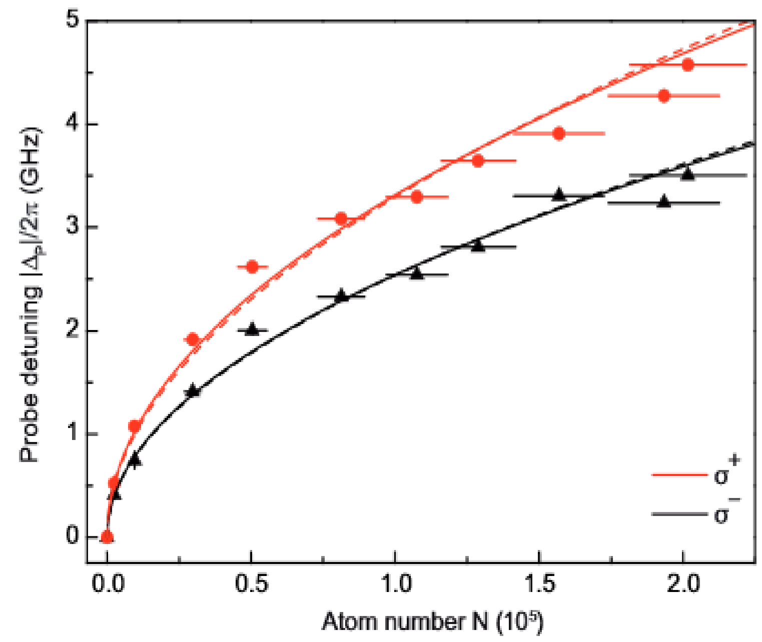
Wang & Hioe, *Phys. Rev. A* **7** (1973).

Many-body cQED

Many-body cQED

Optical bistability

- Thermal atomic gases had been loaded into resonators, next step a BEC.
- 2007; Stamper-Kurn, Reichel, Esslinger. (no strict proof for BEC at this stage).
- Vacuum Rabi splitting $\sim\sqrt{N}$.



Brennecke, *Nature* **450** (2007) .

Many-body cQED

Optical bistability

- Low temperature → coupling to atomic motion.
- Hamiltonian in dispersive regime ($|\Delta| \gg g\sqrt{\bar{n}}$, $U_0 = g^2/\Delta$)

$$\hat{H} = \int dx\, \hat{\psi}^{+}(x)\left[-\frac{d^2}{dx^2} + U_0 \cos^2(x)\hat{a}^{+}\hat{a}\right]\hat{\psi}(x) - \Delta_c \hat{a}^{+}\hat{a} - i\eta(\hat{a} - \hat{a}^{+}).$$

- Rotating frame ($\Delta_c = \omega - \omega_p$), standing wave of resonator, longitudinal cavity pumping (amplitude η, frequency ω_p), neglecting atom-atom interaction.
- Steady state photon number

$$n_s = \frac{\eta^2}{\kappa^2 + \left(\Delta_c - U_0 \int dx |\psi(x)|^2 \cos^2(kx)\right)^2}.$$

Ritsch, *Rev. Mod. Phys*. **85** (2013) .

Many-body cQED

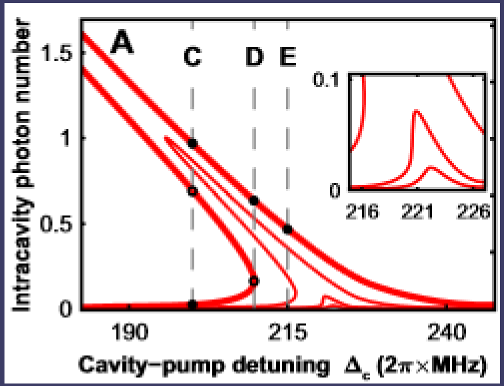
Optical bistability

- Lowest vibrational modes of the BEC:

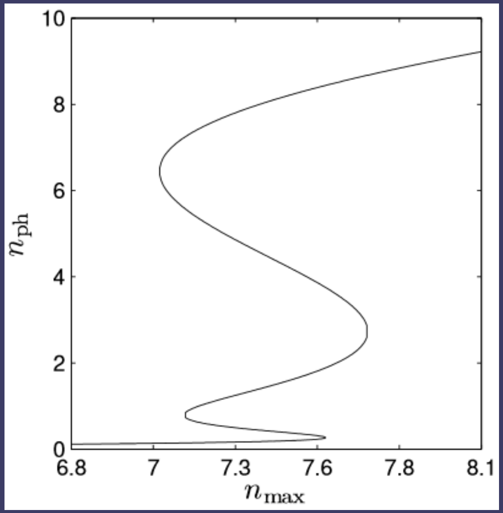
$$\hat{H} = \omega_{rec}\hat{c}^+\hat{c} + (-\Delta_c + g(\hat{c}+\hat{c}^+))\hat{a}^+\hat{a} - i\eta(\hat{a}-\hat{a}^+).$$

- Paradigm optomechanical Hamiltonian.

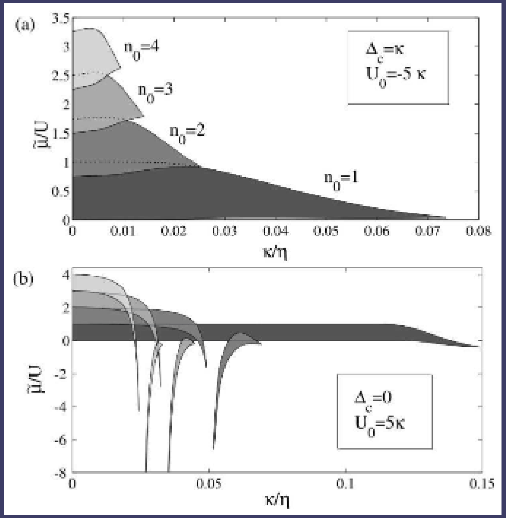
- Multi-stability and strong coupling regime



Brennecke, *Science* **322** (2008) .



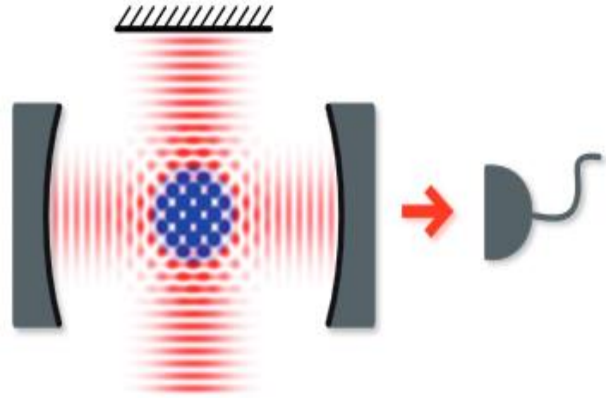
Venkatesh, *Phys. Rev. A* **83** (2011) .



Larson, *Phys. Rev. Lett*. **100** (2008) .

Many-body cQED

Dicke physics



- Transverse pumping: atoms mediate scattering of photons from pump to cavity.
- Expand in lowest vibrational modes → Effective model = Dicke Hamiltonian in the *Schwinger representation*.
- Effective potential

$$V_{eff}(x,z) = V_0 \cos^2(z) + U_0 |\alpha|^2 \cos^2(x) + W_0(\alpha + \alpha^*) \cos(x) \cos(z)$$

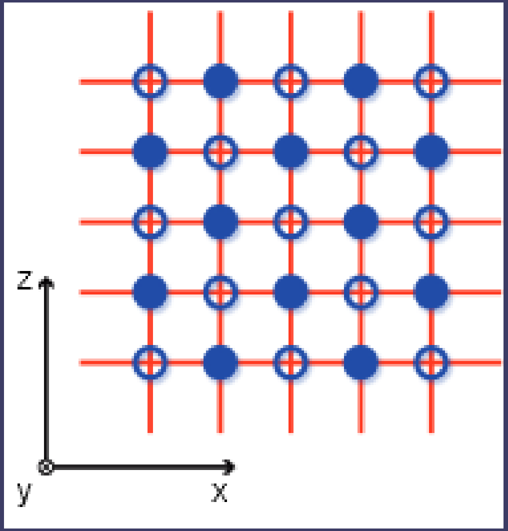
- Field phase $\mathrm{angle}(\alpha + \alpha^*) = 0,\ \pi\ \rightarrow Z_2$ symmetry breaking of Dicke.
- Condensates in a ”checkerboard” supersolid phase: populates every second site.

Nagy, *Phys. Rev. Lett*. **104** (2010).

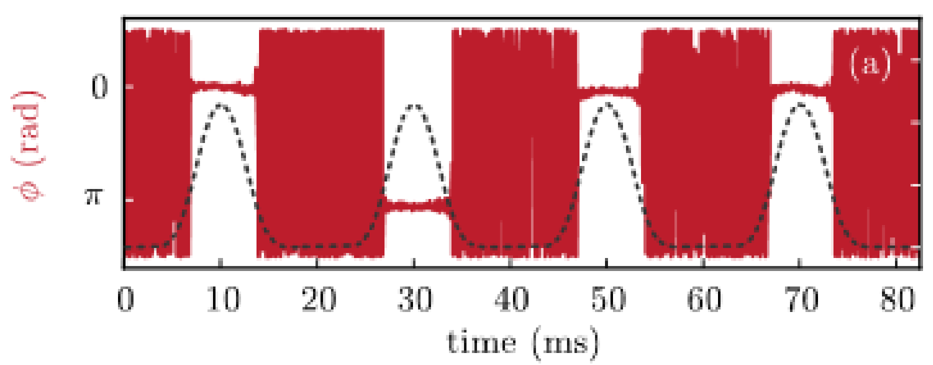
Many-body cQED

Dicke physics

- Experimental realizations:



Schematic picture of supersolid states (*self-organization* transition).



Experimental measurement of field phase.

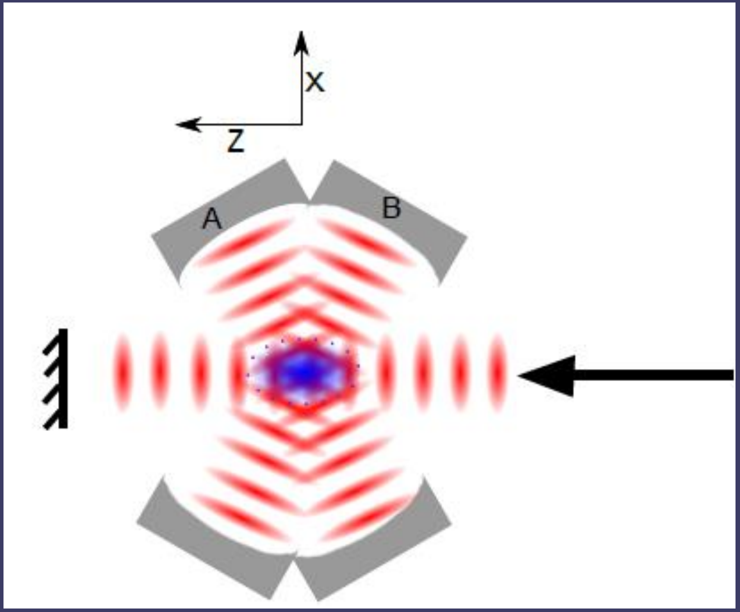
Braumann, *Nature* **464** (2010) , *Phys. Rev.L ett*. **107** (2011).

$SU(3)$ Dicke model

Many-body cQED

Beyond regular Dicke physics

- Esslinger group, new setup 1:



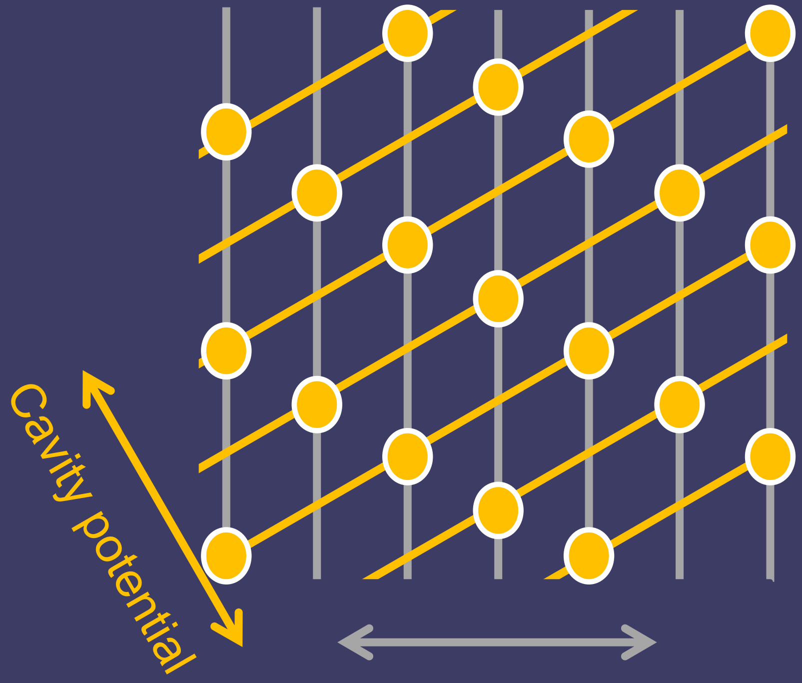
Schematic picture of the 'double-cavity setup'.

Venkatesh & Larson, underway

Many-body cQED

Beyond regular Dicke physics

Only cavity A:



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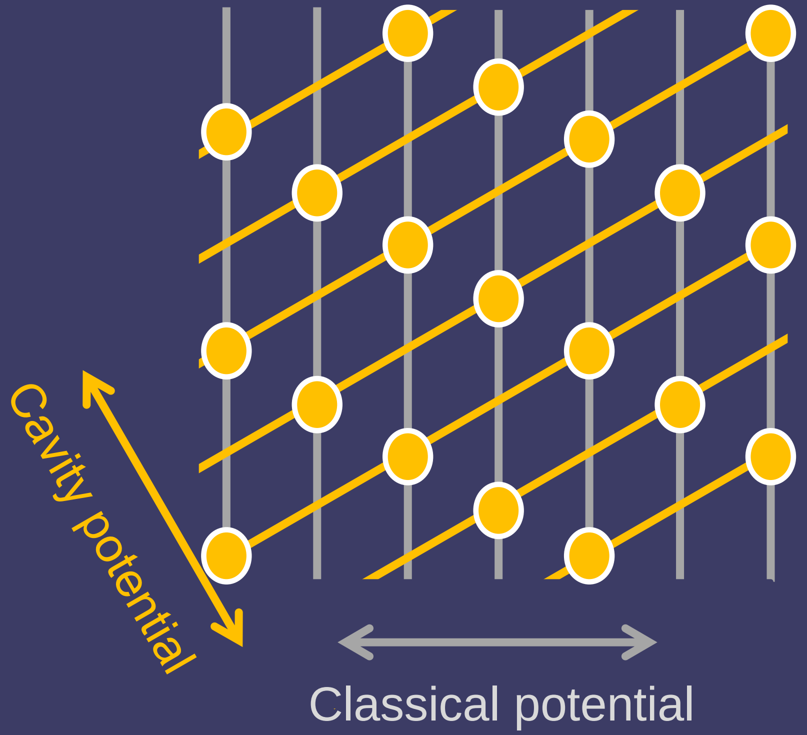
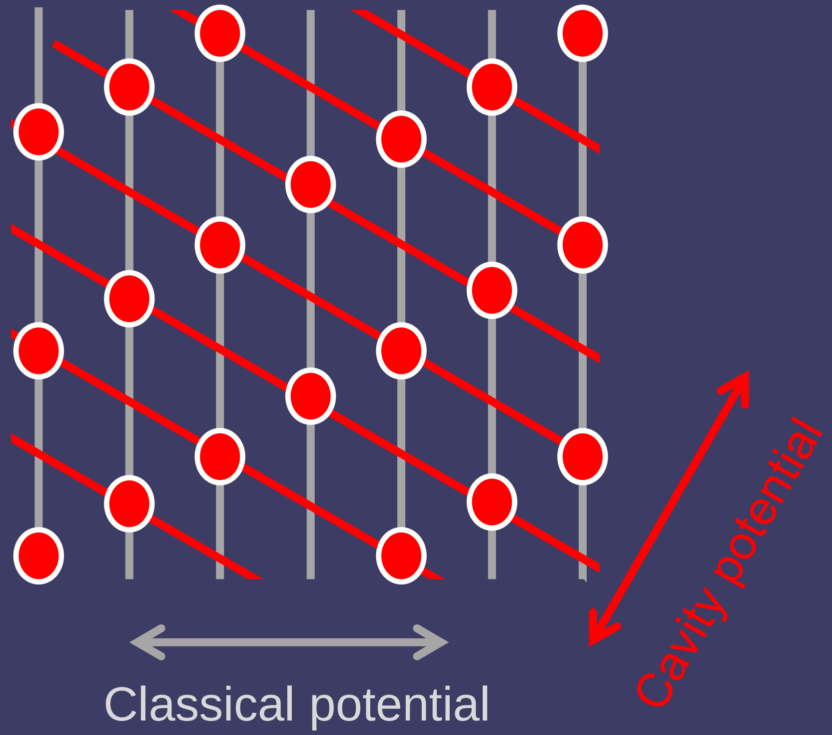
Many-body cQED

Beyond regular Dicke physics

Only cavity A:



Only cavity B:



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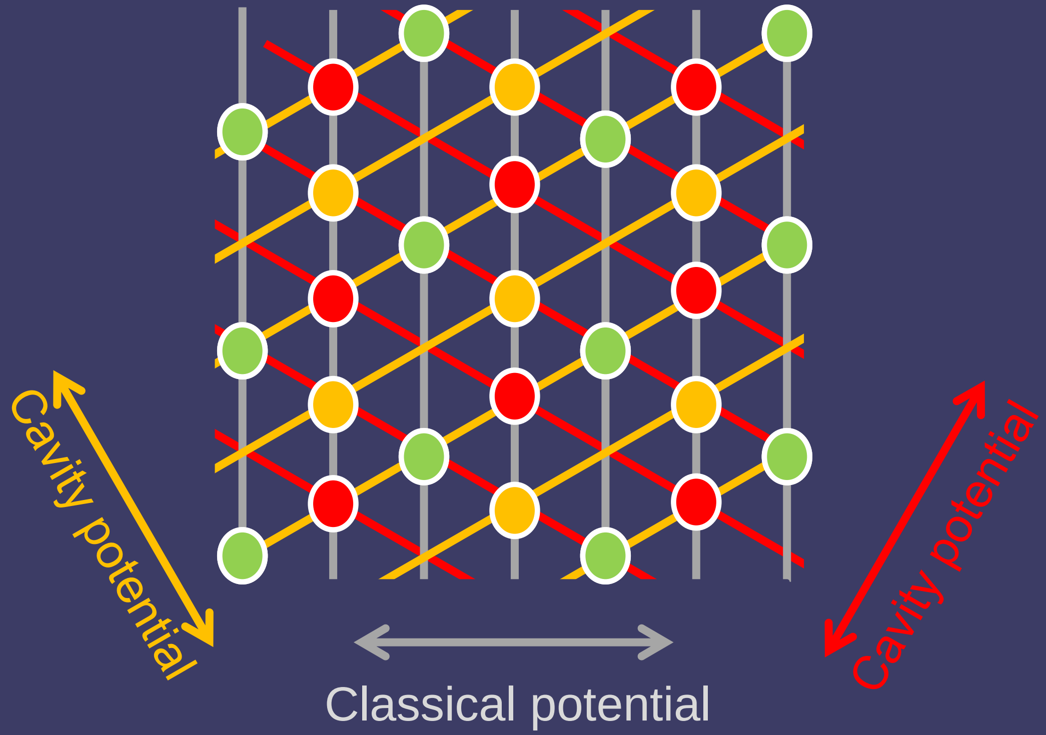
Many-body cQED

Beyond regular Dicke physics

Cavity A + B:

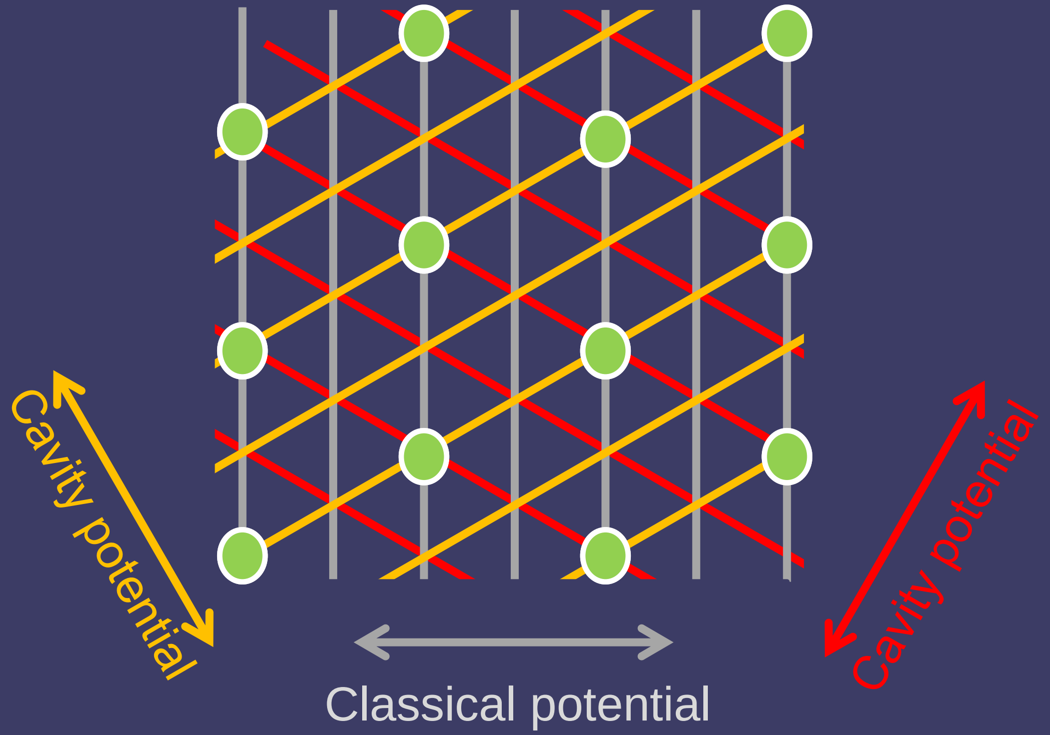


Venkatesh & Larson, underway

Many-body cQED

Beyond regular Dicke physics

Cavity A + B:



No classical frustration!

Z_2 symmetries.

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Many-body cQED

Beyond regular Dicke physics

- Physics beyond mean-field.
- Identify the low energy vibrational states

$$\widehat{\Psi}(x,z) = \hat{c}_0\psi_0(x,z) + \hat{c}_1\psi_1(x,z) + \hat{c}_2\psi_2(x,z),$$

$$\psi_0(x,z) = \frac{1}{\sqrt{N}}, \quad \psi_{1,2}(x,z) = \frac{2}{\sqrt{N}}\cos\left(\frac{\sqrt{3}}{2}x \pm \frac{1}{2}z\right)\cos(z),$$

- Generalized Schwinger representation

$$\widehat{H}_{su3} = \omega_A \hat{a}^+\hat{a} + \omega_B \hat{b}^+\hat{b} + \Omega\widehat{\Lambda}_8 + \frac{g_A}{\sqrt{N}}(\hat{a}+\hat{a}^+)\widehat{\Lambda}_4 + \frac{g_B}{\sqrt{N}}\left(\hat{b}+\hat{b}^+\right)\widehat{\Lambda}_6.$$

- $\widehat{\Lambda}_i$, $i = 1, \dots, 8$ $SU(3)$ algebra.
- $SU(2)$ subgroups; $\{\widehat{\Lambda}_1, \widehat{\Lambda}_2, \widehat{\Lambda}_3\}$, $\{\widehat{\Lambda}_4, \widehat{\Lambda}_5, \hat{\alpha}\}$, and $\{\widehat{\Lambda}_6, \widehat{\Lambda}_7, \hat{\beta}\}$. $\widehat{\Lambda}_4$ and $\widehat{\Lambda}_6$ belong to different subgroups → true $SU(3)$ algebra.

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Many-body cQED

Beyond regular Dicke physics

- Symmetries:

1. Z_2 generalized Dicke parity symmeties

$$\hat{\Pi}_A = \exp\left[i\pi\left(\hat{n}_A + \hat{\Lambda}_3/2 + \sqrt{3}\,\hat{\Lambda}_8/2\right)\right],$$

$$\hat{\Pi}_B = \exp\left[i\pi\left(\hat{n}_B - \hat{\Lambda}_3/2 + \sqrt{3}\,\hat{\Lambda}_8/6\right)\right].$$

2. If parameters $X_A = X_B$ an 'emergent' continuous $U(1)$ symmetry

$$\hat{U}(\theta) = \exp\left[-i\theta\left(\hat{a}^+\hat{b} - \hat{b}^+\hat{a}\right) - i\theta\hat{\Lambda}_2\right].$$

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Many-body cQED

Beyond regular Dicke physics

- Mean-field of $SU(3)$ Dicke

$$\dot{\Lambda}_\alpha = \cdots, \quad \alpha = 1,2,\dots,8,$$

$$\dot{a} = \cdots, \qquad \dot{b} = \cdots.$$

- Steady state solutions ($U(1)$ symmetry $X_A = X_B$):

$$\frac{\Lambda_4}{N} = \cos\theta/2\sqrt{1-{g_c}^4/g} \qquad \text{("Dipole 01")},$$

$$\frac{\Lambda_6}{N} = \sin\theta/2\sqrt{1-{g_c}^4/g} \qquad \text{("Dipole 02")},$$

$$\frac{\Lambda_1}{N} = \frac{\sin\theta}{2}(1-{g_c}^2/g) \qquad \text{("Dipole 12")},$$

$$\frac{\Lambda_3}{N} = \frac{\cos\theta}{2}(1-{g_c}^2/g) \qquad \text{("Inversion 12")},$$

$$\frac{a+a^*}{\sqrt{N}} = -\frac{2\omega g}{\kappa^2+\omega^2}\cos\theta/2\sqrt{1-{g_c}^4/g^4} \qquad \text{(Quadrature A)},$$

$$\frac{b+b^*}{\sqrt{N}} = -\frac{2\omega g}{\kappa^2+\omega^2}\sin\theta/2\sqrt{1-{g_c}^4/g^4} \qquad \text{(Quadrature B)}.$$

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Many-body cQED

Beyond regular Dicke physics

- Critical coupling $g_c = \frac{1}{2}\sqrt{\frac{\Omega(\kappa^2+\omega^2)}{\omega}}$, cavity losses κ.
- $x_A, x_B \sim |1-\frac{g_c}{g}|^{1/2} \rightarrow \beta = -1/2$ as in ”open” $SU(2)$ Dicke model.
- For the Z_2’s identical critical coupling g_c and exponent β.
- Breaking of Z_2’s → one cavity empty in superradiant phase.
- Breaking of $U(1)$ → both civities populated, amplitude set by θ.

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Many-body cQED

Beyond regular Dicke physics - dynamics

- Write the model in quadratures $\hat{x} = \frac{i(\hat{a}-\hat{a}^{+})}{\sqrt{2}}$, $\hat{p}_x = \frac{(\hat{a}+\hat{a}^{+})}{\sqrt{2}}$, …

$$\widehat{H}_{su3} = \omega\left(\frac{\hat{p}_x{}^2}{2} + \frac{\hat{p}_y{}^2}{2}\right) + \omega\left(\frac{\hat{x}^2}{2} + \frac{\hat{y}^2}{2}\right) + \omega_0\widehat{\Lambda}_8 + \frac{g}{\sqrt{N/2}}\left(\hat{p}_x{}^2\widehat{\Lambda}_4 + \hat{p}_y{}^2\widehat{\Lambda}_6\right)$$

- Semi-classical equations of motion

$$\begin{aligned}
\dot{x} &= \cdots, \\
\dot{y} &= \cdots, \\
\dot{p}_x &= \cdots, \\
\dot{p}_y &= \cdots, \\
\dot{\Lambda}_1 &= \cdots, \\
&\vdots \\
\dot{\Lambda}_8 &= \cdots.
\end{aligned}$$

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Many-body cQED

Beyond regular Dicke physics - dynamics

- Lowest dispersion in an adiabatic *Born-Oppenheimer* picture has the familiar sombrero shape

- Polar symmetric!

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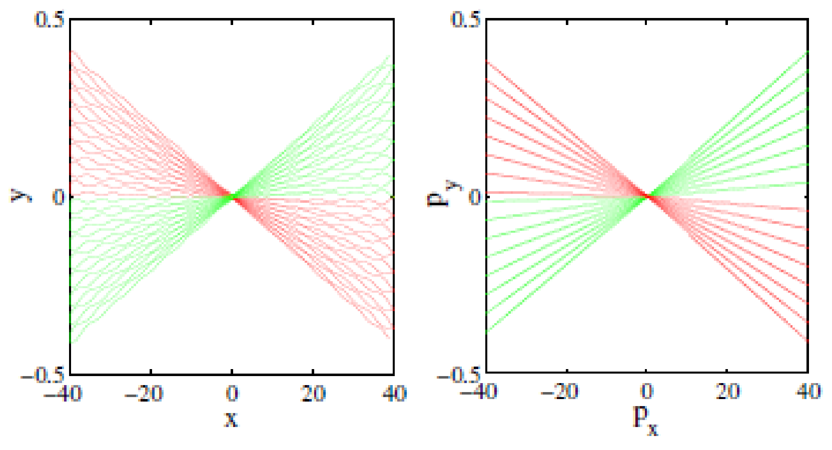
Many-body cQED

Beyond regular Dicke physics - dynamics

- Classical evolution



$(x(t), y(t))$ and $\left(p_x(t), p_y(t)\right)$,

$\left(x(0), y(0), p_x(0), p_y(0)\right) = (40{,}0{,}0{,}0)$,

$\Lambda_2(0) > 0$ (red), $\Lambda_2(0) < 0$ (green),

$\omega_0 \equiv 0$.

- Emergent gauge structure:

$$\widehat{H}_{su3} = \omega\left[\frac{(\hat{p}_x - \hat{A}_x)^2}{2} + \frac{\left(\hat{p}_y - \hat{A}_y\right)^2}{2}\right] + \omega\left(\frac{\hat{x}^2}{2} + \frac{\hat{y}^2}{2}\right),$$

$$\hat{A}_x = \lambda\widehat{\Lambda}_4, \quad \hat{A}_y = \lambda\widehat{\Lambda}_6, \qquad \lambda = \frac{g}{\sqrt{N/2\omega}}.$$

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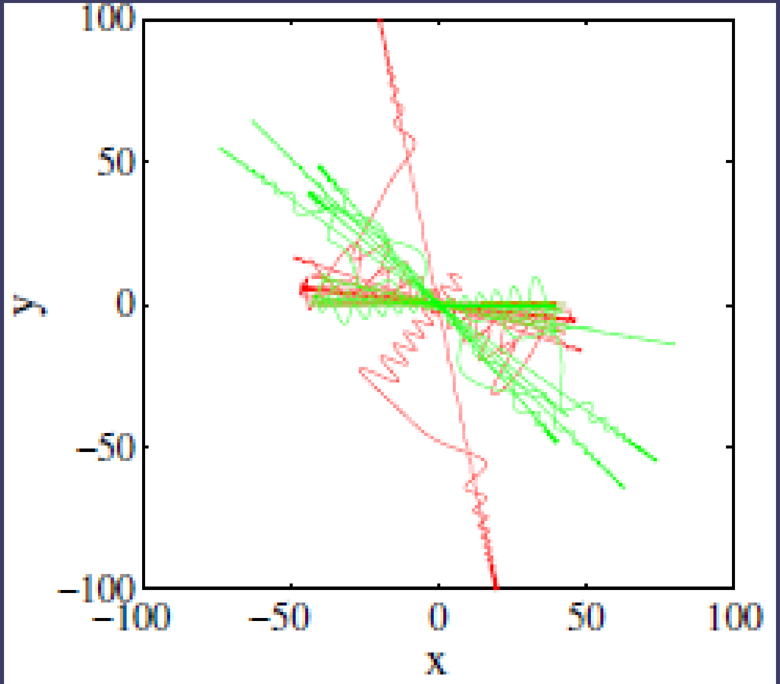
Many-body cQED

Beyond regular Dicke physics - dynamics

- Non-zero ω_0



$\left(x(t), y(t)\right)$ and $\left(p_x(t), p_y(t)\right)$,

$\left(x(0), y(0), p_x(0), p_y(0)\right) = (40,0,0,0)$,

$\Lambda_2(0) > 0$ (red), $\Lambda_2(0) < 0$ (green),

$\omega_0 \neq 0$.

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Many-body cQED

Beyond regular Dicke physics - dynamics

- Vector potential

$$\left(\hat{A}_0, \hat{A}_x, \hat{A}_y, \hat{A}_z\right) = \left(\omega_0 \widehat{\Lambda}_8, \lambda \widehat{\Lambda}_4, \lambda \widehat{\Lambda}_6, 0\right)$$

- Field tensor $\hat{F}_{\alpha\beta} = \partial_\alpha \hat{A}_\beta - \partial_\beta \hat{A}_\alpha - i\left[\hat{A}_\alpha, \hat{A}_\beta\right], \qquad \alpha, \beta = 0, x, y, z.$

$$\hat{F} = \begin{bmatrix} 0 & \hat{E}_x & \hat{E}_y & \hat{E}_z \\ -\hat{E}_x & 0 & \hat{B}_z & -\hat{B}_y \\ -\hat{E}_y & -\hat{B}_z & 0 & \hat{B}_x \\ -\hat{E}_z & \hat{B}_y & -\hat{B}_x & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_0 \lambda \widehat{\Lambda}_5 & -\omega_0 \lambda \widehat{\Lambda}_5 & 0 \\ \omega_0 \lambda \widehat{\Lambda}_5 & 0 & \lambda^2 \widehat{\Lambda}_2 & 0 \\ \omega_0 \lambda \widehat{\Lambda}_5 & -\lambda^2 \widehat{\Lambda}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

- Spin-orbit coupling → ”magnetic force”, non-zero ω_0 → ”electric force”.

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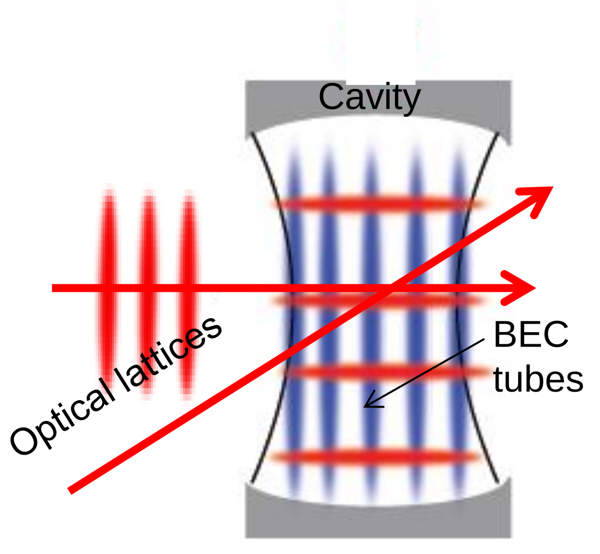
Dynamical disorder

Long range interaction

Localization?

- Setup 2:



Atoms in (classical) 2D optical lattice → 1D tubes. Setup confined in a resonator.

“Tubes” independently interact with the same cavity mode.

~1500 tubes, 10-100 atoms/tube.

Long range interaction

Localization

- Optical lattice + cavity field (standing wave).
- Incommensurate lattices (cavity field weak), mean-field (atomic part)

$$\partial_t \varphi_i = -J(\varphi_{i-1} + \varphi_{i+1}) + \mu_i \hat{a}^+ \hat{a} \varphi_i,$$

$$\partial_t \hat{a} = -i\Delta \hat{a} - i \textstyle\sum_i \mu_i \varphi^*{}_i \varphi_i \, \hat{a} + \eta.$$

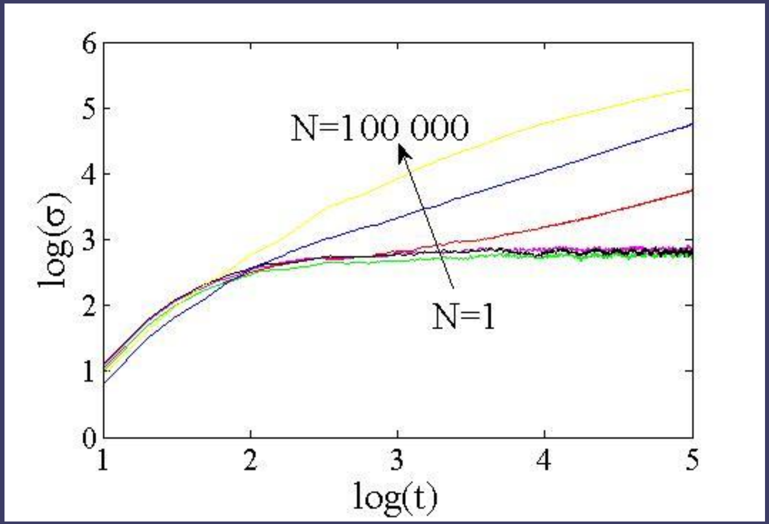
- $\partial_t \varphi_i = -J(\varphi_{i-1} + \varphi_{i+1}) + \mu_i \varphi_i$. Anderson model, onsite disorder → localization (Kramer & MacKinnon, *Rep. Prog. Phys*. **56**, (1993)).
- $\partial_t \varphi_i = -J(\varphi_{i-1} + \varphi_{i+1}) + \mu_i \varphi_i + |\varphi_i|^2 \varphi_i$, weak nonlinearity → localization lost at ’large times’ (Pikovsky & Shepelyansky, *Phys. Rev. Lett*. **100**, (2008)).
- $\partial_t \varphi_i = -J(\varphi_{i-1} + \varphi_{i+1}) + \mu_i f\left(\sum_i \mu_i |\varphi_i|^2\right) \varphi_i$, localization?

Long range interaction

Localization

- $\partial_t \varphi_i = -J(\varphi_{i-1} + \varphi_{i+1}) + \mu_i f\left(\sum_i \mu_i |\varphi_i|^2\right) \varphi_i$. Photons induce effective infinite range interaction.
- Nonlinearity increases with number N of atoms.

Mean-field



$\sigma =$ width of state.

- Beyond mean-field. Atom-photon entanglement → effective dephasing of condensate → breakdown of localization?

Long range interaction

Glassy states?

- *N*-channel Dicke realization

Atom-atom interaction

$$\hat{H}_{Nd} = \omega\hat{a}^{+}\hat{a} + \Omega\sum_{i=1}^{N}\hat{S}^{(i)}{}_{z} + g_i(\hat{a}+\hat{a}^{+})\hat{S}^{(i)}{}_{x} + U\hat{S}^{(i)}{}_{z}\hat{S}^{(i)}{}_{z}.$$

- Trace out boson field

$$\hat{H}_{rLMG} = \sum_{i=1}^{N}\hat{S}^{(i)}{}_{z} + U\left(\hat{S}^{(i)}{}_{z}\right)^{2} + \sum_{i,j} g_{ij}\hat{S}^{(i)}{}_{x}\hat{S}^{(j)}{}_{x}.$$

- *N*-channel (quasi random) *Lipkin-Meshkov-Glick model*.
- g_{ij} Gaussian distributed $\overset{?}{\rightarrow}$ glassy states (Strack & Sachdev, *Phys. Rev. Lett*. **107**, (2011); Buchhold, *Phys. Rev. A*. **87**, (2013)).

Thanks!