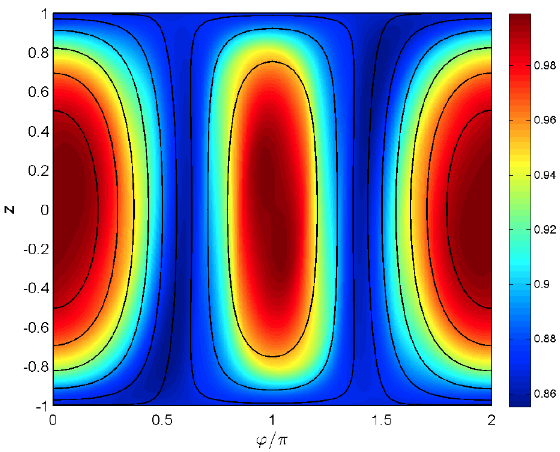
Global Phase Space Study of Coherence and Entanglement in a Double-Well BEC

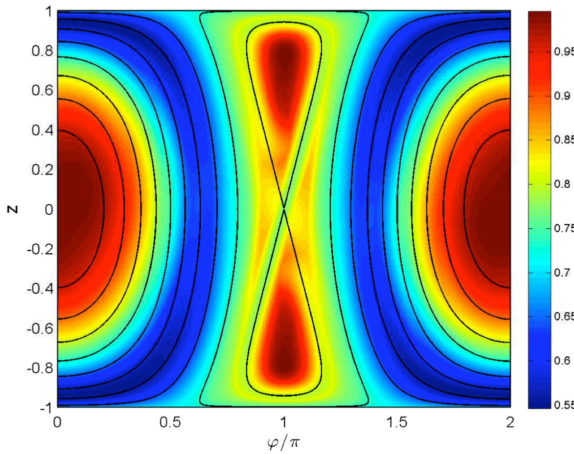
David K. Campbell*, Boston University

Conference on Quantum Engineering of States and Devices

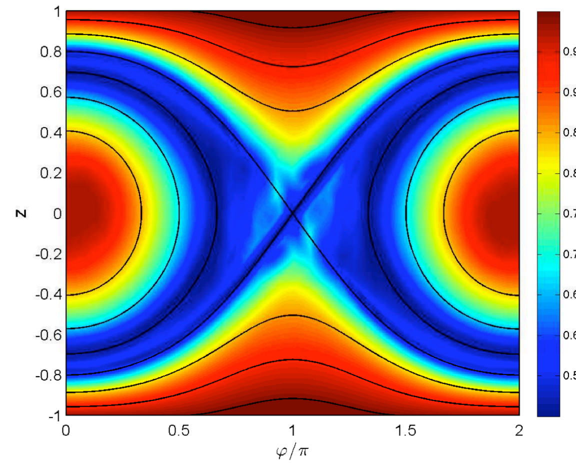
NORDITA, August 19, 2014



Λ = 0.5



Λ=1.5



Λ=5

* With Holger Hennig (Harvard), **Ted Pudlik (BU)** and Dirk Witthaut (MPIDS-Göttingen)

Collaborators

Dirk Witthaut

Holger Hennig

Ted Pudlik

Bottom Line

- The issue of temporal decoherence of BEC is critical for potential applications. For a simple system (*BEC dimer ~ Bosonic Josephson Junction=BJJ*) we show that *coherence is enhanced near fixed points of classical analog system, including self-trapped regions but is supressed near separatrix*
- We do this by introducing "*global phase space*" (*GPS*) portraits of quantum observables including 1) the "*condensate fraction*"; and 2) *entanglement* as functions of time for different values of the key parameters of the system. We show that much of the observed behavior can be understood by studying the phase space of a related classical nonlinear dynamical system, *BUT*
- We predict some *novel quantum effects*, including *tunneling between self-trapped states*, which should be observable in near-term experiments.
- Further, introducing *dissipation* (atom loss), we find surprising result that coherence and entanglement can be *enhanced* by dissipation.

Outline

- Background (cf. Krüger, Eisert, and Trombettoni talks this morning)
  - Bose-Einstein Condensates
  - Optical Lattices
  - Physical Context: BEC dimer= Bose Josephson Junction (BJJ)
  - Some relevant experiments
- Models
  - Bose-Hubbard Model
  - Liouville Dynamics (skip today)
  - Gross-Pitaevski Equation: Discrete NLSE
- Results: "Global Phase space" study of quantum vs. classical
  - Without dissipation
  - With dissipation
- Summary and Conclusions
- References: *Phys. Rev. A* **86** 051604(R) (2012); *Phys. Rev. A* **88** 063606 (2013); in preparation

Background: BEC

- Bose Einstein condensation
  - Macroscopic occupancy of single quantum state by large number of identical bosons
  - Predicted in 1925 by Einstein for non-interacting bosons
  - Observed in 1995 by two groups, Wieman/Cornell and Ketterle: Nobel Prize in 2001

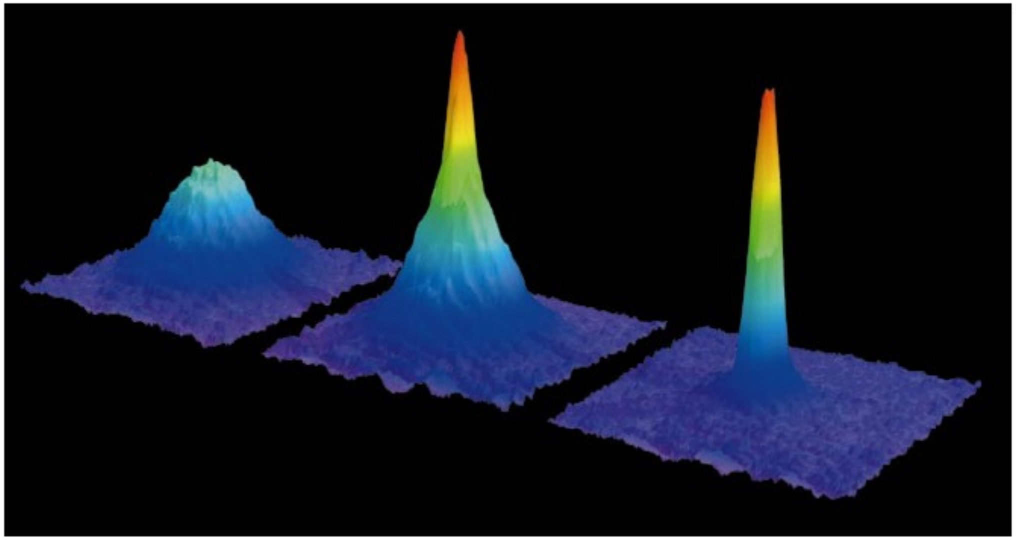
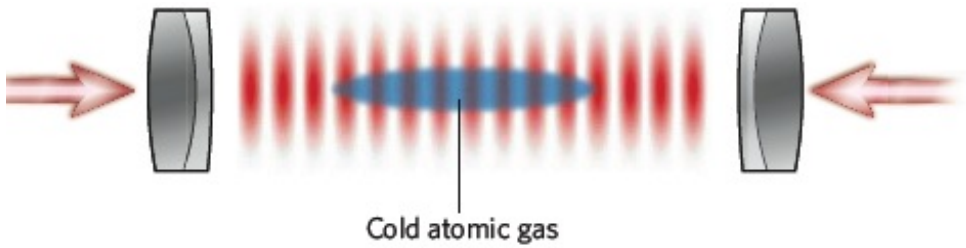


FIG. 7. Observation of Bose-Einstein condensation by absorption imaging. Shown is absorption vs two spatial dimensions. The Bose-Einstein condensate is characterized by its slow expansion observed after 6 ms time of flight. The left picture shows an expanding cloud cooled to just above the transition point; middle: just after the condensate appeared; right: after further evaporative cooling has left an almost pure condensate. The total number of atoms at the phase transition is about 7×10^5, the temperature at the transition point is 2 μK [Color].

Background: BECs in Optical Lattice



Counter-propagating laser pulses creating standing wave that interacts with atoms, so that the BEC experiences a periodic potential of the form:

$$V_{\text{ext}}(\vec{r}) = U_L(x, y)\sin^2[2\pi z/\lambda]$$

$U_L(x,y)$ is transverse confining potential, λ is the laser wavelength (typically 850 nm) and "z" is the direction of motion.

Image from I. Bloch *Nature* **453** 1016-1022 (2008)

Physical Context and Models

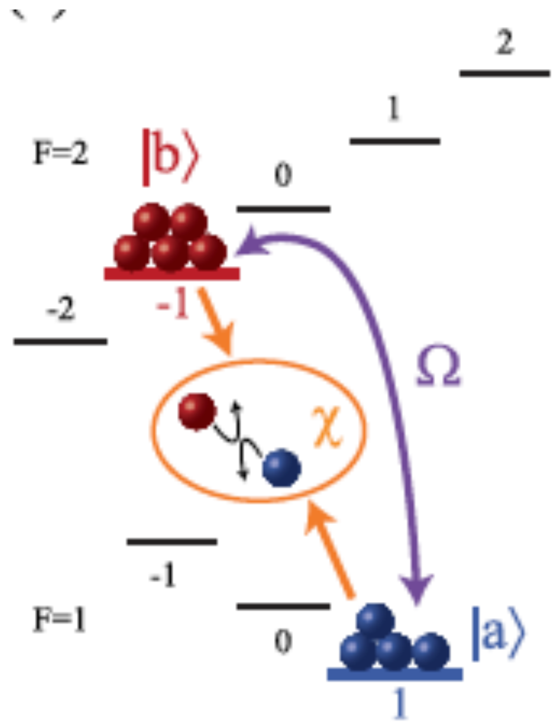
- Single coherent BEC in a “double well” potential or two internal (hyperfine) states with resonant interactions in single well.
  - Analogous to two-site “dimer” system and therefore called “BEC dimer” but also to Josephson Junction therefore called Bosonic Josephson Junction (BJJ).
  - Parameters: number of particles in BEC (N) and in each of two “wells”/states (N_1 and N_2), $z = N_1 - N_2$, phase difference between two condensates ($\phi = \phi_1 - \phi_2$), resonant coupling/tunneling between two wells (J), interaction of Bosons within condensate (U); single parameter in simple models $\Lambda = (NU)/J$
- Three Models:
  - Bose-Hubbard Hamiltonian (*BHH*) : fully quantum *
  - Liouville Dynamics (*LD*) : semi-classical in this system (not today)
  - Gross-Pitaevskii equation (*GPE*): corresponds to integrable classical dynamical system* in (z, ϕ) for this case

* NB: Quantum dimer also intergrable by Bethe Ansatz: solution not useful for calculating physical observables

Different realizations of Dimer

- Two-well optical lattice

- Two internal atomic states



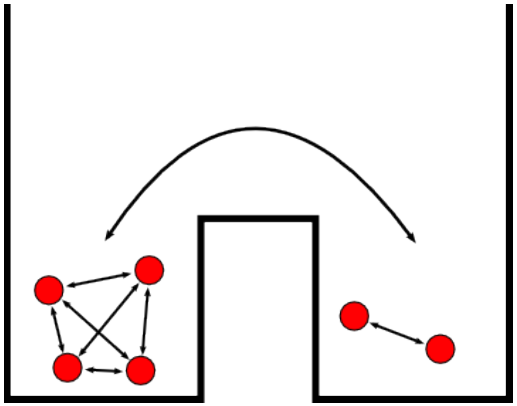
T. Zibold et al. Phys. Rev. Lett. 105, 204101 (2010)

Bose-Hubbard dimer

$$H = -J(a_1^\dagger a_2 + a_2^\dagger a_1) + \frac{U}{2}(a_1^\dagger a_1^\dagger a_1 a_1 + a_2^\dagger a_2^\dagger a_2 a_2)$$

- hopping
- repulsive interaction

- Atoms in a double-well optical trap
- Two spin states of atoms trapped in one well



- Observables of interest:

Single-particle density matrix (Condensate Fraction/ purity)

$$\rho = \frac{1}{N}\begin{pmatrix} \langle a_1^\dagger a_1 \rangle & \langle a_1^\dagger a_2 \rangle \\ \langle a_2^\dagger a_1 \rangle & \langle a_2^\dagger a_2 \rangle \end{pmatrix}$$

Entanglement measure

$$\mathrm{EPR} = \langle a_1^\dagger a_2 \rangle \langle a_2^\dagger a_1 \rangle - \langle a_1^\dagger a_1 a_2^\dagger a_2 \rangle$$

Semiclassical approximation

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle, \quad \langle\Psi|a_i|\Psi\rangle = \psi_i$$

$$\langle\Psi|H|\Psi\rangle = -J(\psi_1^*\psi_2 + \psi_2^*\psi_1) + \frac{U}{2}\left(|\psi_1|^2(|\psi_1|^2 - 1) + |\psi_2|^2(|\psi_2|^2 - 1)\right)$$

- Operators replaced with c-numbers

$$\psi_i = \sqrt{N(1 \pm z)/2}\, \mathrm{e}^{\imath\theta \pm \imath\phi/2} \qquad \begin{cases} z & \text{population imbalance} \\ \phi & \text{phase difference} \end{cases}$$

$$\mathcal{H} = \frac{\Lambda}{2} z^2 - \sqrt{1 - z^2} \cos\phi$$

$$\Lambda = \frac{U(N-1)}{2J}$$

Equivalent Nonlinear Dynamical System

- Let $\Psi_j = b_j\, e^{i\theta j}$, where b_j and θ_j are real. Define

$$z = [b_1^2 - b_2^2]/N = [N_1 - N_2]/N, \ \varphi = \theta_2 - \theta_1$$

one can show that

$$\dot{z} = -\sqrt{1 - z^2}\sin\varphi$$

$$\dot{\varphi} = \Lambda z + \frac{z\cos\varphi}{\sqrt{1 - z^2}}$$

a simple one degree of freedom (=>integrable) dynamical system, like a pendulum with length dependent on its momentum

- Analysis of FPs shows for

  $\Lambda < 1$, two stable FPs, $(z,\varphi) = (0,0)$ and $(0,\pi)$ and for

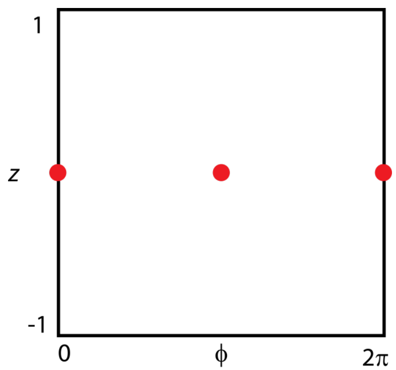
  $\Lambda > 1$, three stable FPs, $(z,\varphi)=(0,0)$, (z_+,π), and (z_-,π)

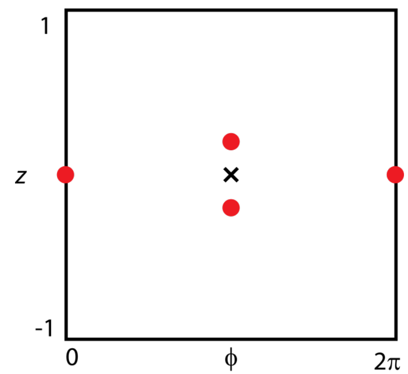
  $z_{+/-} = +/- (\Lambda^2 - 1)^{1/2}/\Lambda \approx +/- 1$ for $\Lambda \Rightarrow \infty$

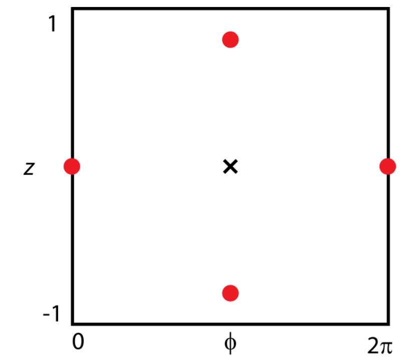
- Next slides show how quantum dynamics of the dimer/BJJ follows (or does not follow) classical phase portraits of this system: first plot $c(z,\phi;T)$ and then $p(z,T)$ at fixed ϕ.

Classical Nonlinear Dynamics

$$\mathcal{H} = \frac{\Lambda}{2} z^2 - \sqrt{1 - z^2}\cos\phi$$



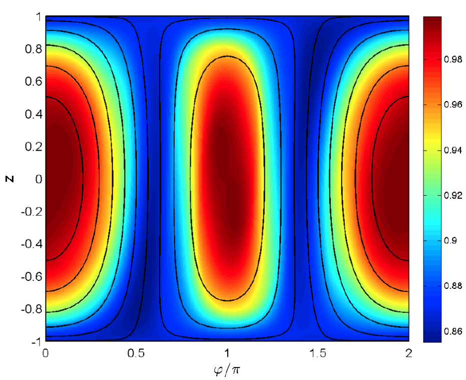


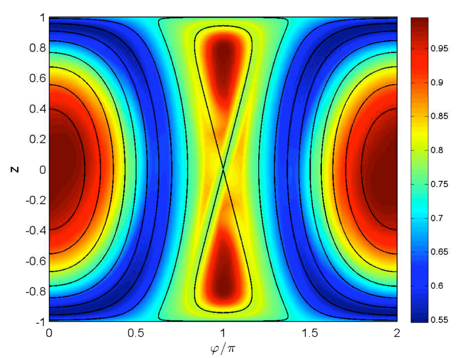


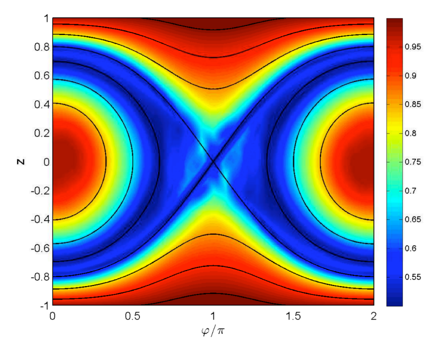
Λ = 0.5

Λ= 1.5

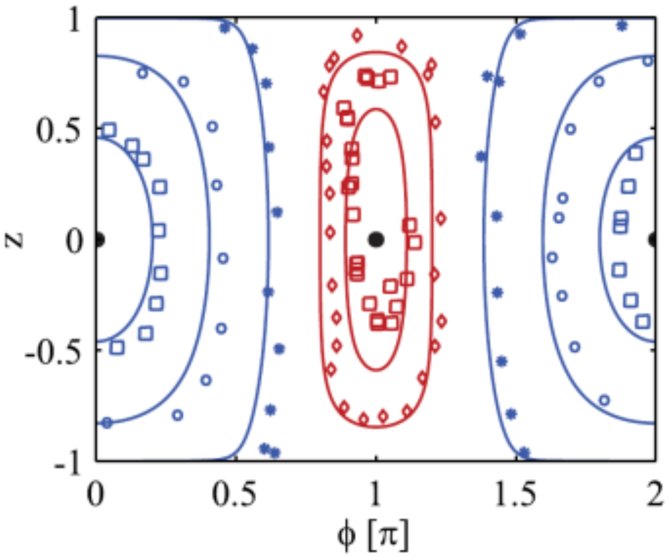
Λ = 5



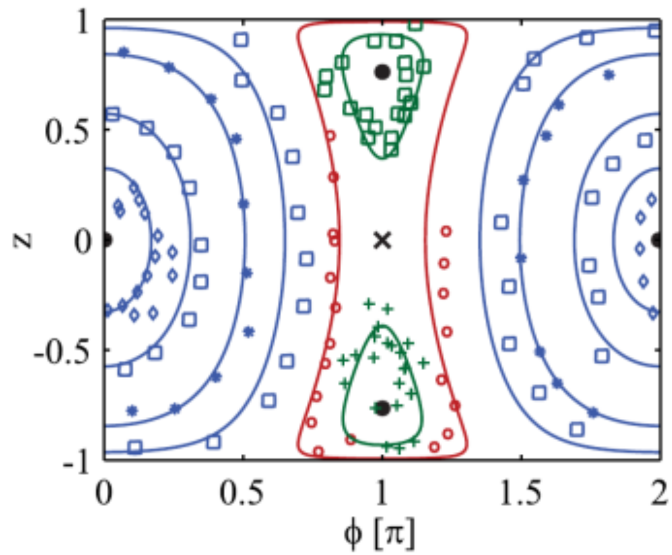




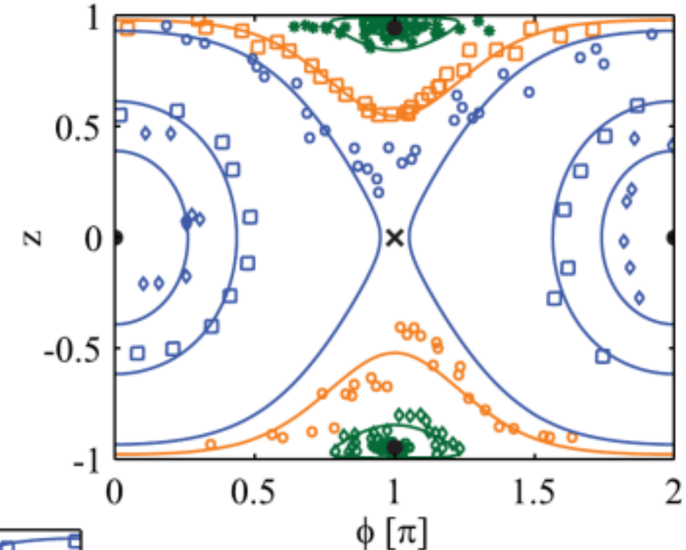
Relevant experiments



$\Lambda = 0.78$



$\Lambda = 1.55$



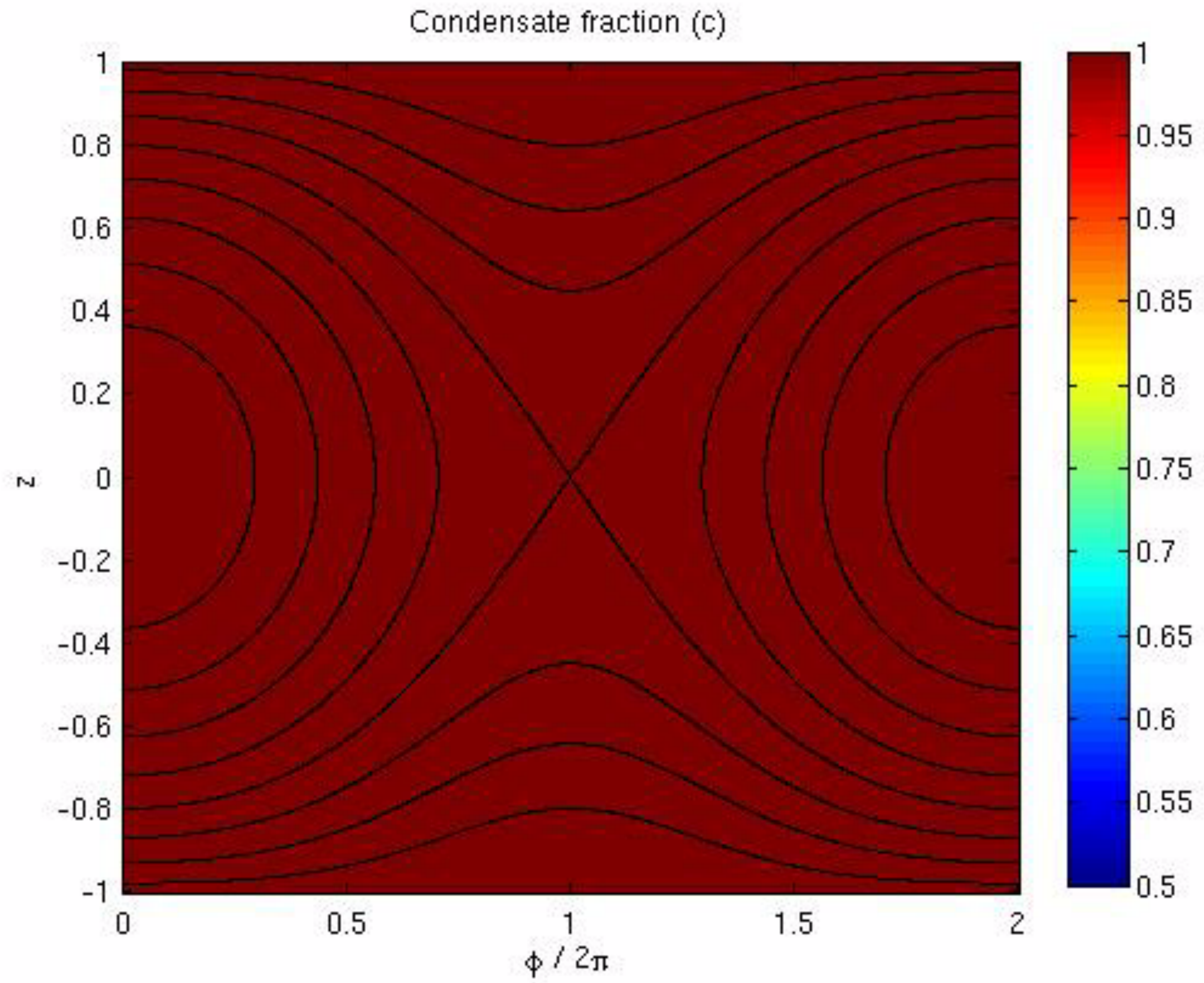
$\Lambda = 3.1$

- Zibold et al., PRL **105** 204101 (2010)

Observables and Results

- We study the time evolution of quantum observables: discuss two today
  1) "condensate fraction"—$C(z,\phi) \cong \lambda_{max}/N \leq 1$, λ_{max} = largest eigenvalue of single particle density matrix
  2) "entanglement"— $E \cong |\langle a_1^* a_2 \rangle|^2 - \langle a_1^* a1 a_2^* a_2 \rangle$, $E > 0$ for entangled state
- Results
  - Both $C(z,\phi)$ and E generally "track" classical orbits in GPS
  - $C(z,\phi)$ decreases dramatically near classical separatrix, remains large at classical fixed point and especially in self-trapped region=> regular motion enhances coherence, chaotic motion destroys coherence
  - Symmetry $z \Rightarrow -z$ of classical equations is broken by quantum dynamics
  - "Ridge" of enhanced coherence exists in quantum but not in LD or GPE (classical) or dynamics
  - Overlap of initial coherent state with eigenfunctions of BHH is minimal along separatrix, high near fixed points

Classical phase space, quantum dynamics



$\Lambda = 5$

• Hennig et al., PRA **86**, 051604(R) (2012)

Condensate Fraction at T=1 sec Λ = 1.5

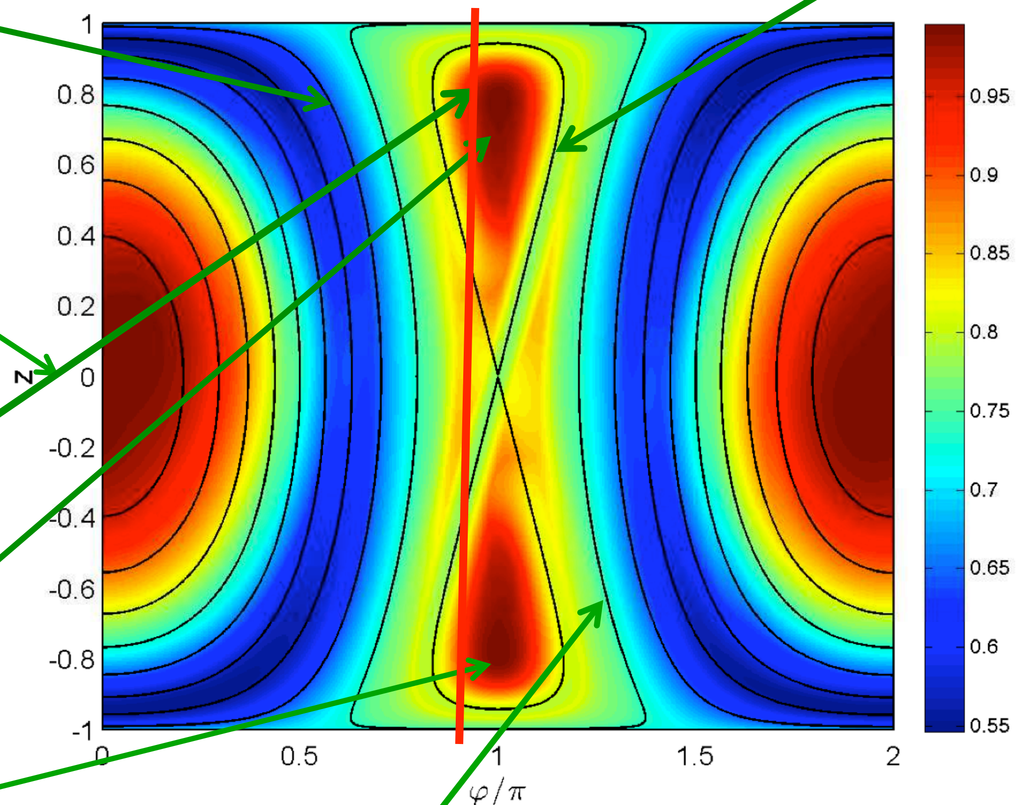
Note considerable variation in $C(z,\phi,T=1)$ and how it generally follows contours of classical dynamics

Constant energy contours

Separatrix

Orbits around FP (0,0): have $\langle z\rangle=\langle\phi\rangle=0$

$z \Rightarrow -z$ symmetry broken



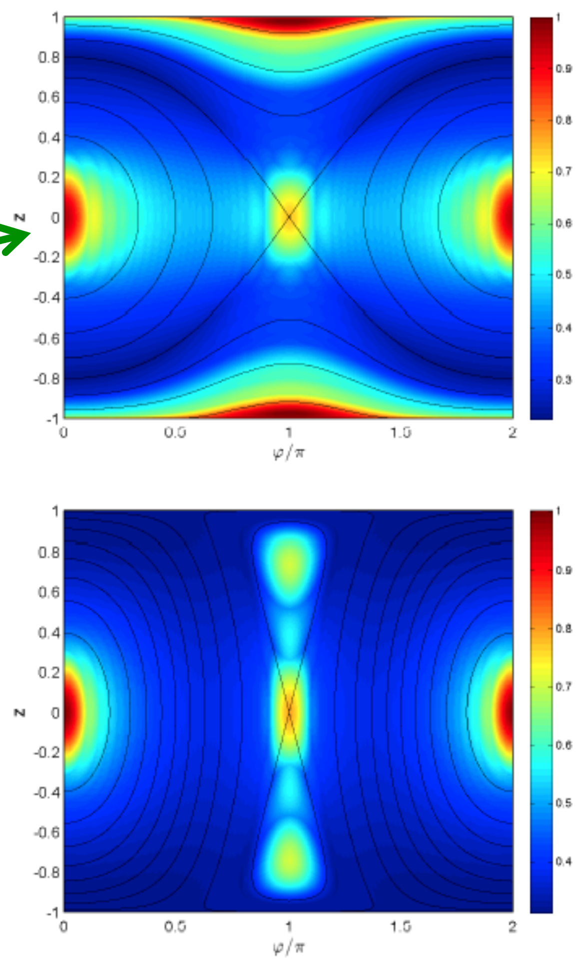
Orbits around FP_+ and FP_- are macroscopic self-trapped states, $\langle z\rangle\neq 0$, $\langle\phi\rangle=\pi$

"π-phase" orbits still exist, $\langle z\rangle=0$, $\langle\phi\rangle=\pi$

Initial State overlap with Eigenfunctions

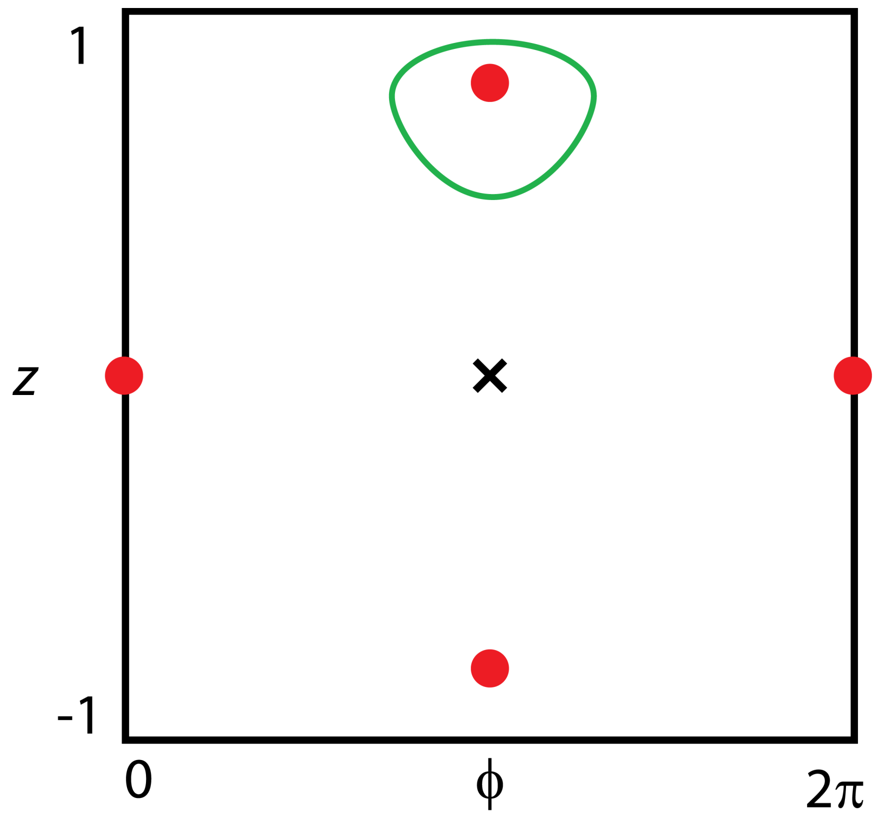
Color code shows $A(\phi,z) \cong \max_n \langle \phi,z|E_n \rangle$, where E_n is an eigenstate of the BH dimer

$\Lambda = 5.0$: $\Lambda = 1.5$



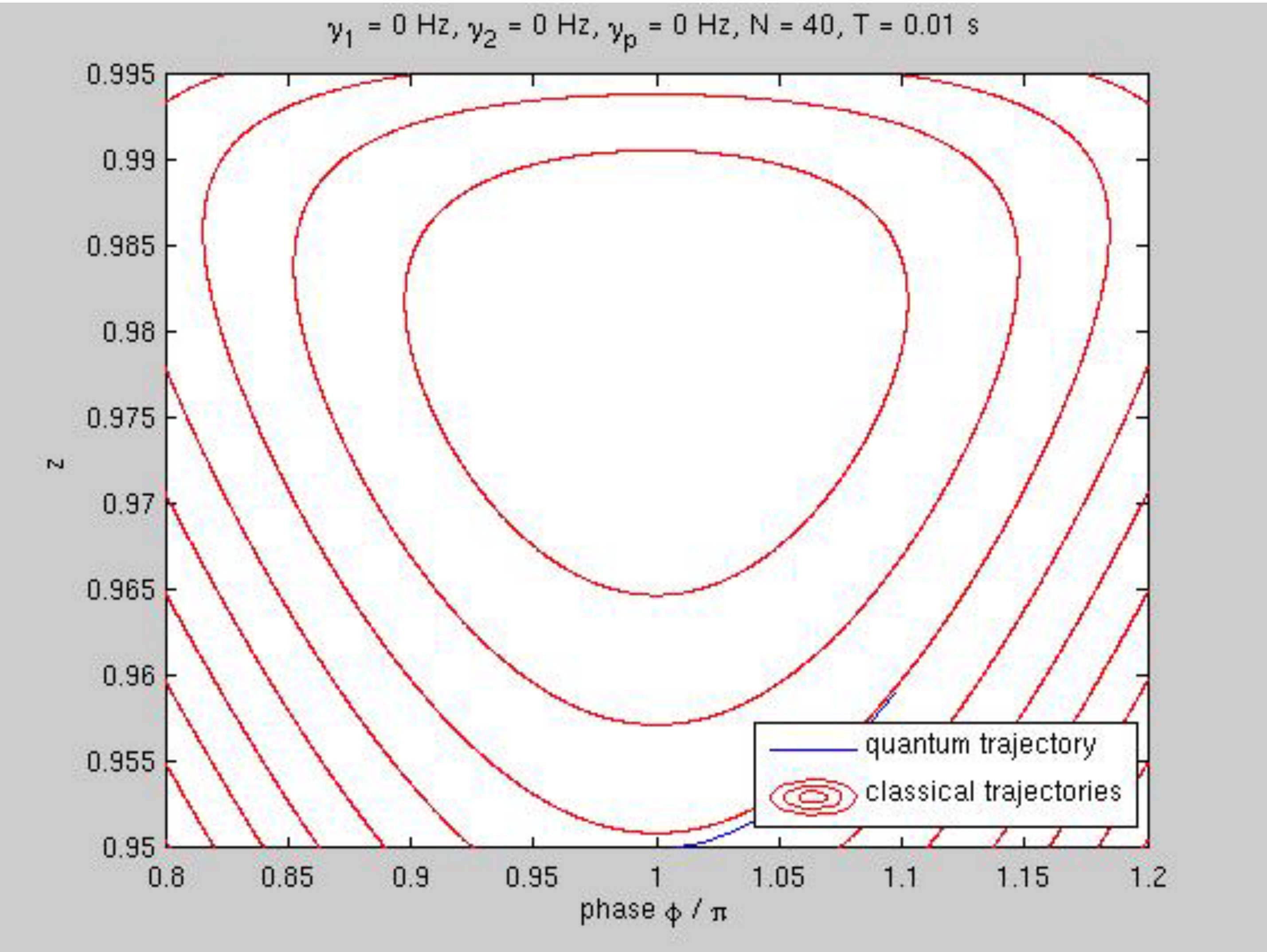
Conclusion: localization in phase space (FPs) maintains coherence, delocalization (separatrix) induces decoherence.

What do quantum *orbits* really look like?



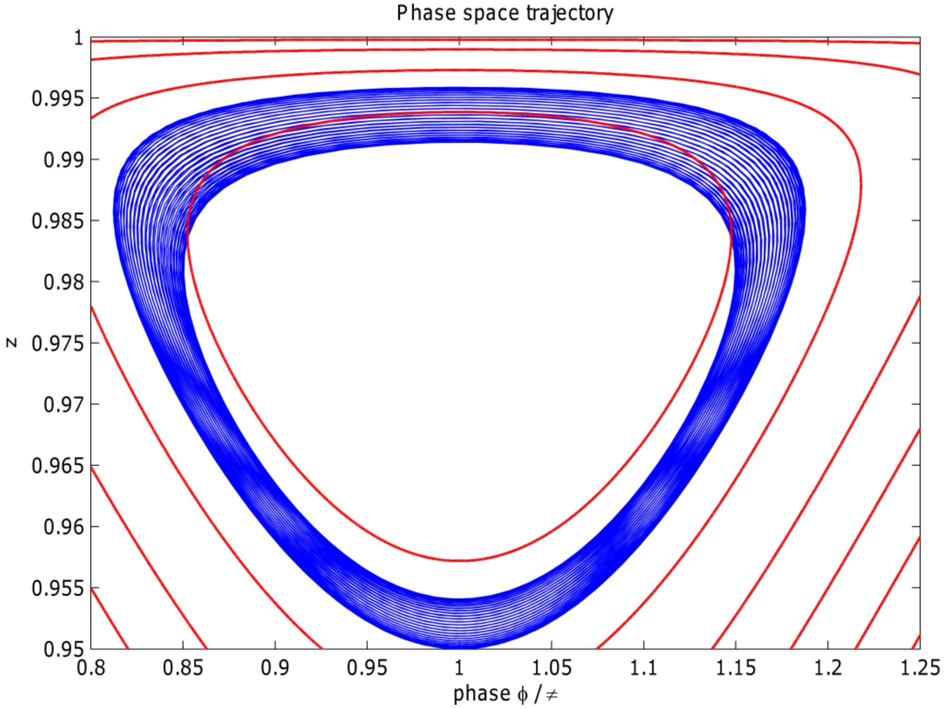
- Large Λ, near fixed point, green is classical orbit

Dissipationless Quantum Dynamics



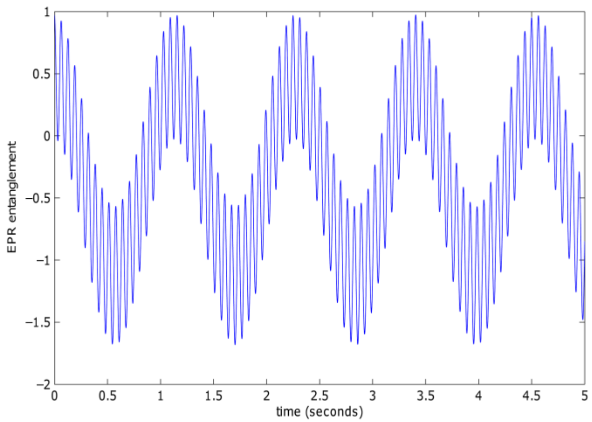
Quantum „orbits," interesting features

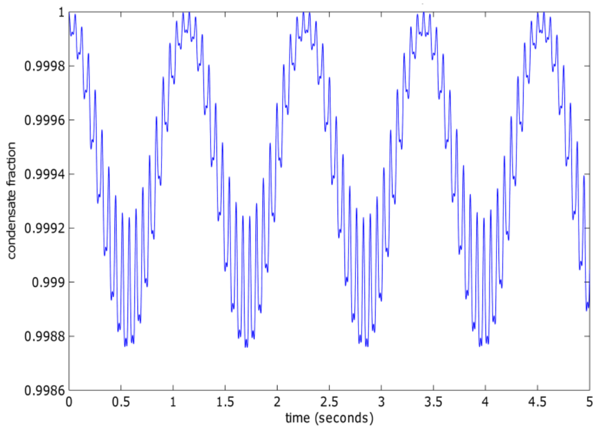
- „thick" orbits



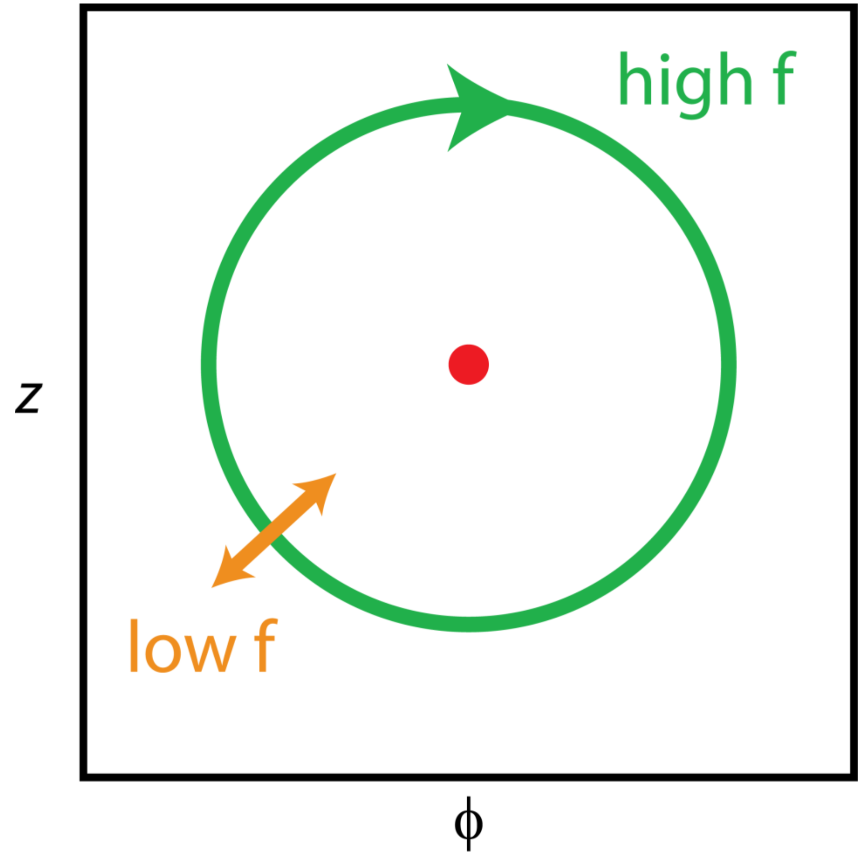
$$\Lambda = 5$$

- *two frequencies* in EPR, condensate fraction





Explain two frequencies: High frequency



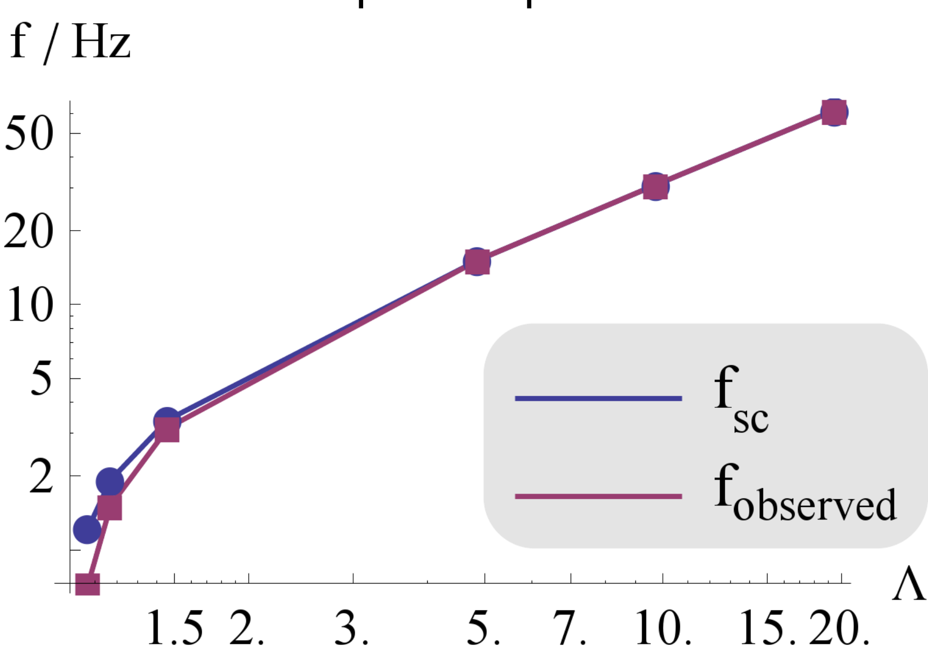
$$\Lambda = \frac{U(N-1)}{2J}$$

- High frequency is mean-field (semiclassical)

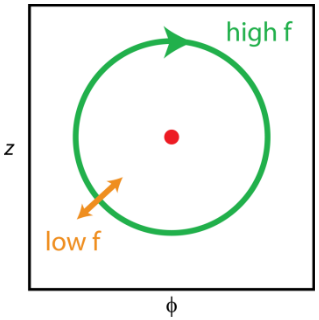
$$f_{sc} = \frac{\sqrt{\Lambda^2 - 1}}{\pi} \frac{J}{\hbar}$$

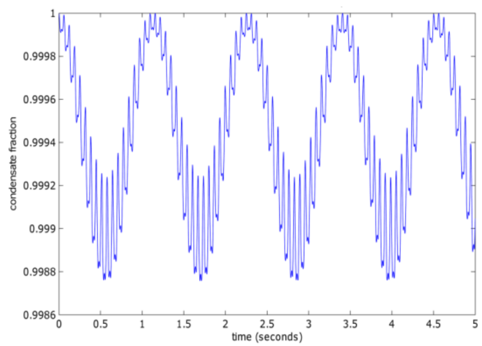
- Conversion to seconds

- Data from power spectrum of EPR:



Low-frequency oscillations





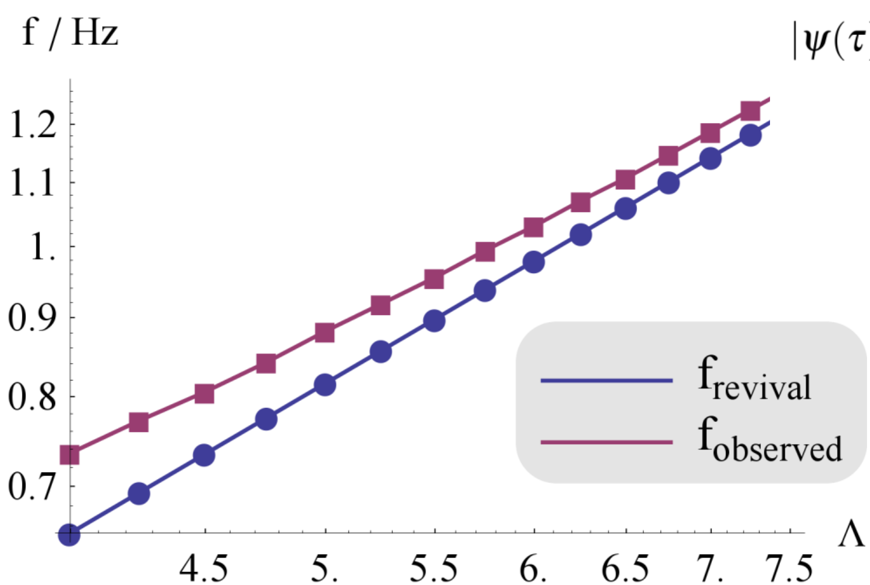
- Low frequency quantum revival

- In the limit J/U = 0, for any state $\tau \equiv \frac{\pi\hbar}{U}$

and $\mathrm{EPR}(t=\tau) = \mathrm{EPR}(t=0)$

$$\max \operatorname{eig} \rho\,(t=\tau) = \max \operatorname{eig} \rho\,(t=0)$$

$$|\psi(\tau)\rangle = \begin{cases} |\psi(0)\rangle & \text{for } N = 1+4p \text{ with } p \in \mathbb{Z}_{\geq 0}, \\ -\sum_{n=0}^{N}(-1)^n a_n\,|E_n\rangle & \text{for } N = 2+4p \text{ with } p \in \mathbb{Z}_{\geq 0}, \\ -|\psi(0)\rangle & \text{for } N = 3+4p \text{ with } p \in \mathbb{Z}_{\geq 0}, \\ \sum_{n=0}^{N}(-1)^n a_n\,|E_n\rangle & \text{for } N = 4+4p \text{ with } p \in \mathbb{Z}_{\geq 0}. \end{cases}$$

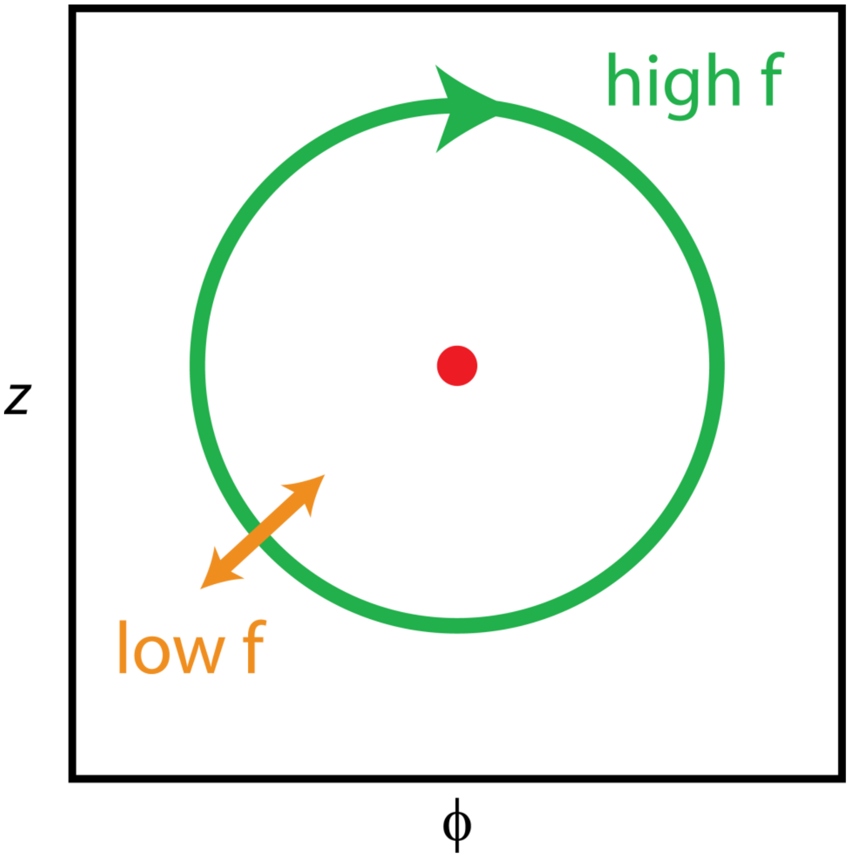


- But revivals seen also for J/U > 0 (or, Λ < ∞)

$$\Lambda = \frac{U\,(N-1)}{2J}$$

Greiner et al., Nature **419** 51 (2002)

Summary: Dynamics near the FP



$$\Lambda = \frac{U(N-1)}{2J}$$

- High frequency is mean-field (semiclassical)

$$f_{sc} = \frac{\sqrt{\Lambda^2 - 1}}{\pi} \frac{J}{\hbar}$$

- Low frequency is a quantum revival:

$$f_G = \frac{U}{\pi\hbar}$$

But how far from the fixed point is this a good picture?

Projections as Interpretation of Two Frequencies

- Energy eigenstate expansion: $|\psi\rangle = \sum_{n=0}^{N} a_n \,|E_n\rangle\,, \quad |a_n|^2 < |a_{n-1}|^2$
- Projection onto „most important" states:

$$|\psi'\rangle = \sum_{n=0}^{2} a_n \,|E_n\rangle\,, \quad |a_n|^2 < |a_{n-1}|^2$$

$$|\psi'(t)\rangle = \sum_{n=0}^{2} a_n \,|E_n\rangle\, \mathrm{e}^{\imath E_n t/\hbar}$$

- Three eigenstates produce beats in observables:

$$\frac{|E_i - E_j|}{2\pi} \sim f_{\mathrm{fast}}$$

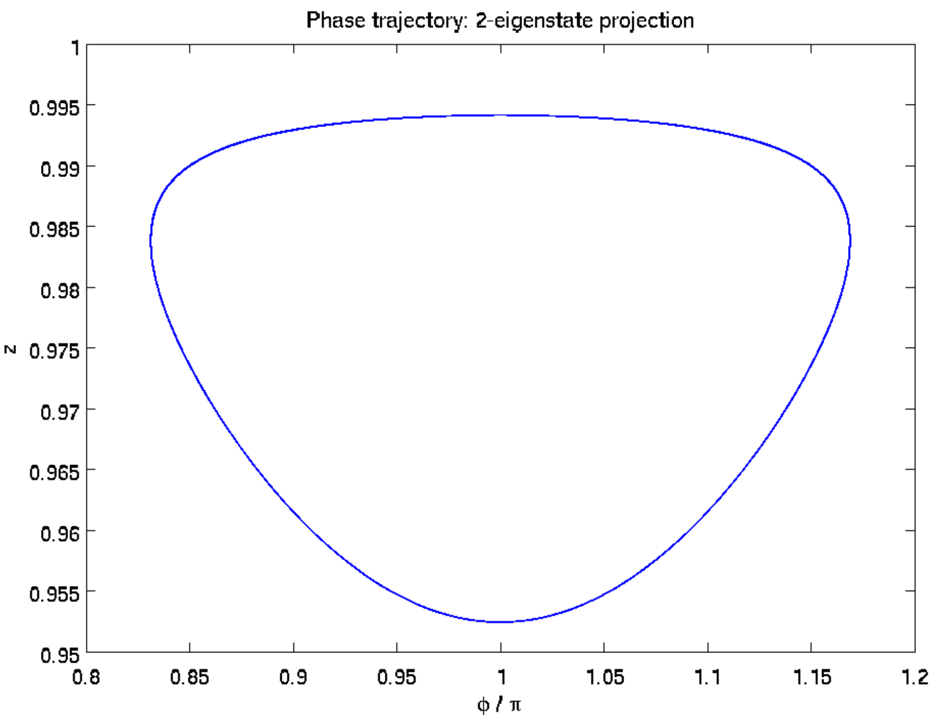
$$\frac{|E_i - E_j|}{2\pi} - \frac{|E_j - E_k|}{2\pi} \sim f_{\mathrm{slow}}$$

- Replacing the full state with the projection should work well where,

$$|\langle\psi'|\psi'\rangle|^2 \approx |\langle\psi|\psi\rangle|^2 = 1$$

How informative is the projection? GPS view

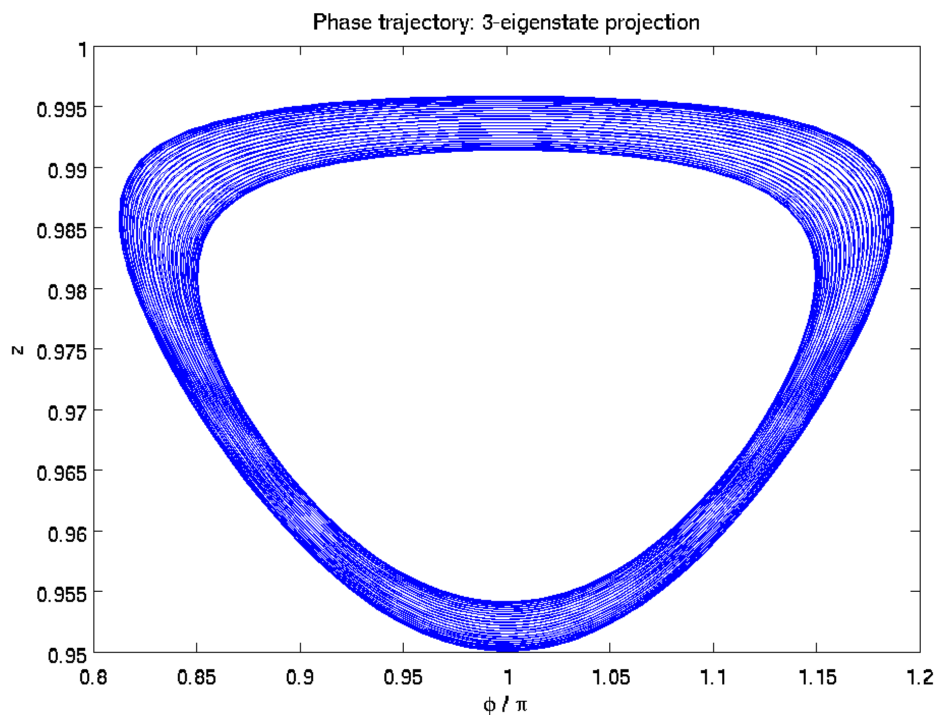
2 states: only fast motion



$$\frac{|E_0 - E_1|}{2\pi} = f_{\text{fast}}$$

$$\Lambda = 5$$

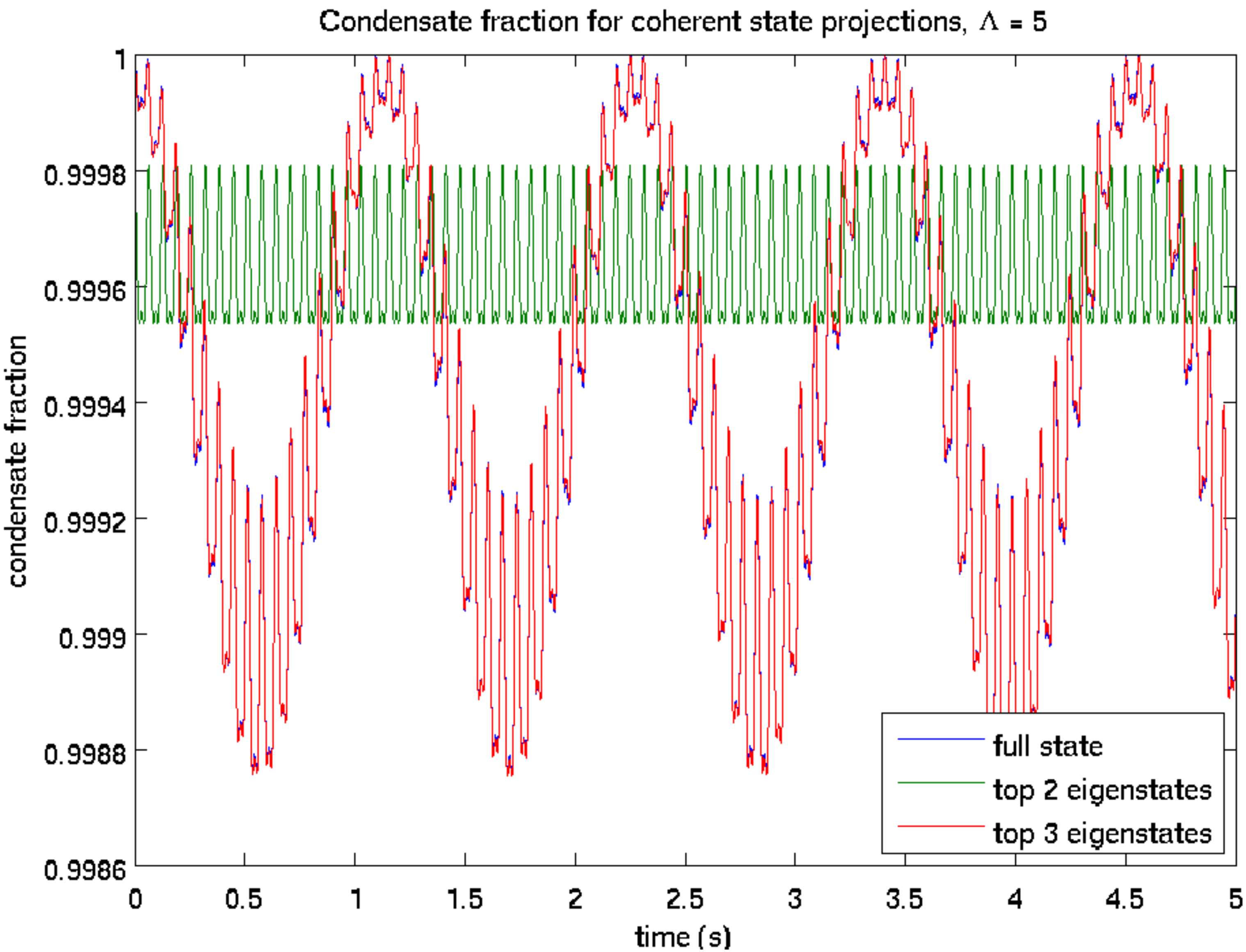
3 states: fast and slow motion



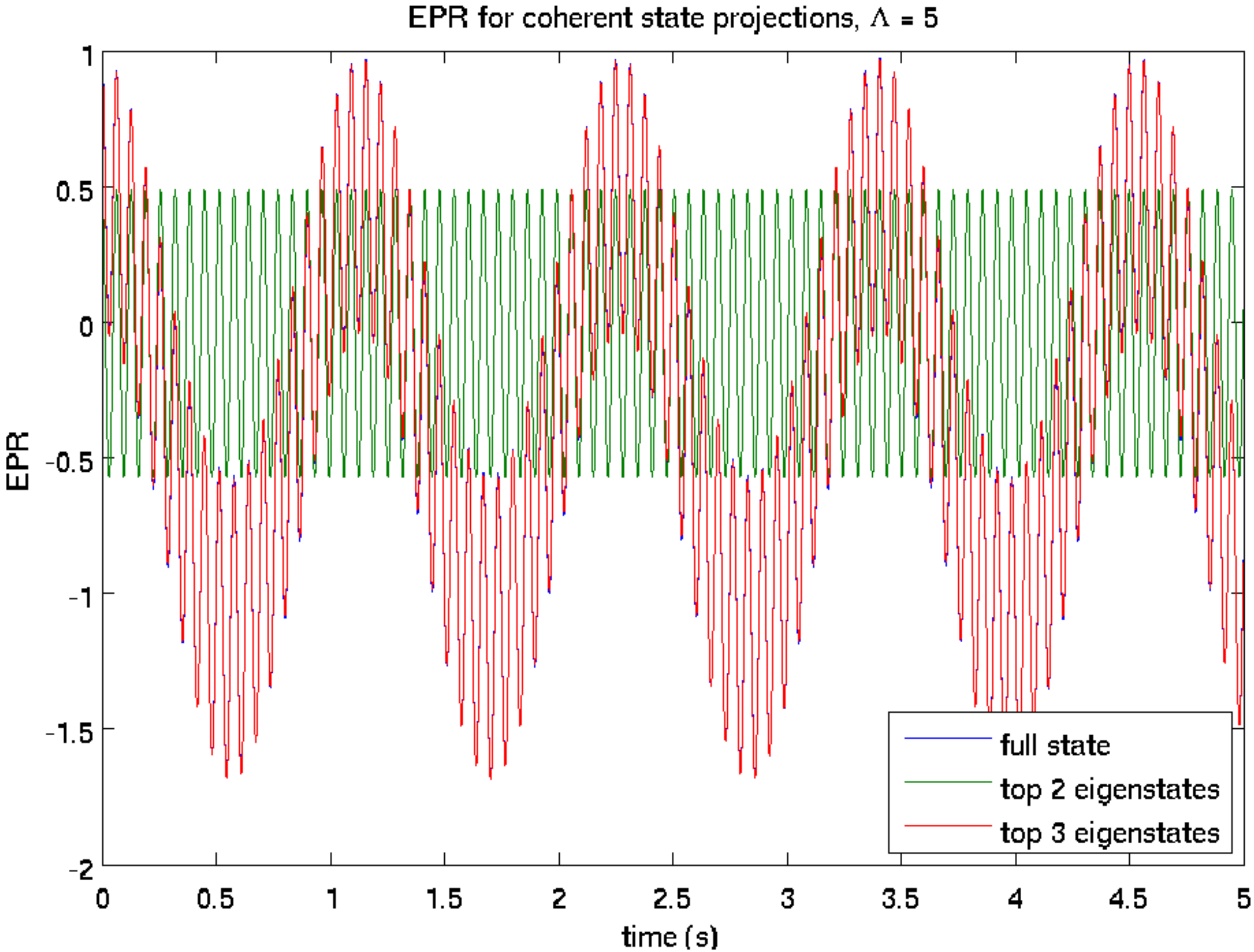
$$\frac{|E_0 - E_1|}{2\pi} = f_{\text{fast}}$$

$$\frac{|E_0 - E_1|}{2\pi} - \frac{|E_1 - E_2|}{2\pi} = f_{\text{slow}}$$

How informative is the projection? Cond. Fraction



How informative is the projection? Entanglement



How informative is the projection? Quantitative

- For $z = 0.95$, $\Phi = \pi$ the contributions of three highest eigenstates are

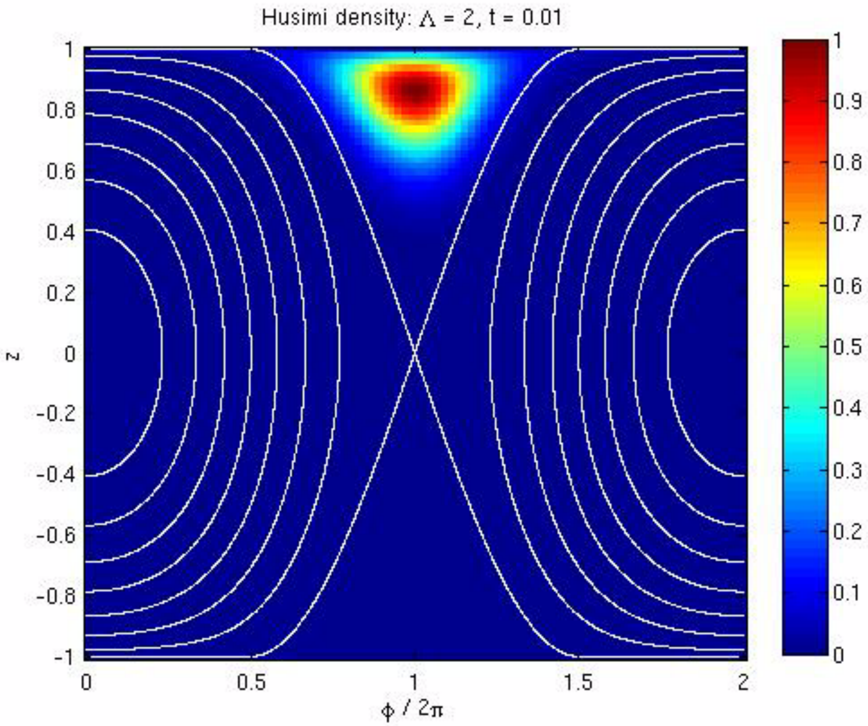
$$a_0 = 0.9353,$$
$$a_1 = 0.3474,$$
$$a_2 = 0.0653 \text{ so that}$$

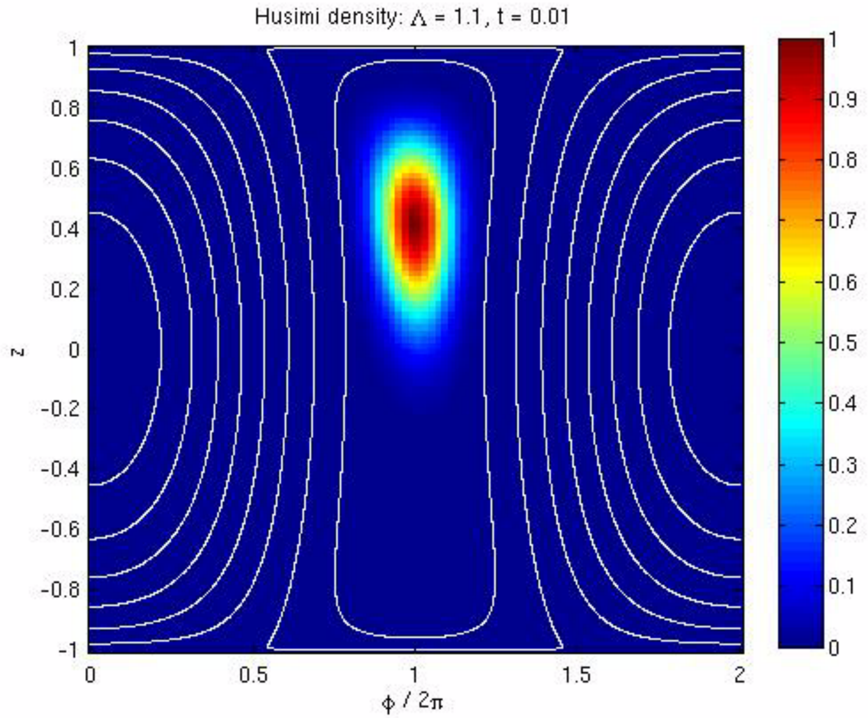
$$|a_0|^2 + |a_1|^2 + |a_2|^2 = 0.9997$$

- And eigenfrequency differences fit with observed frequencies in numerics almost to within 3 %

Low-Λ anomaly: Husimi density

$$Q(z, \phi, t) = |\langle z,\, \phi | \psi(t) \rangle|^2$$





Semi-classical approximation poor for Λ close to 1; observe quantum tunneling : should be seen in experiments !!

Dissipation in a BEC dimer

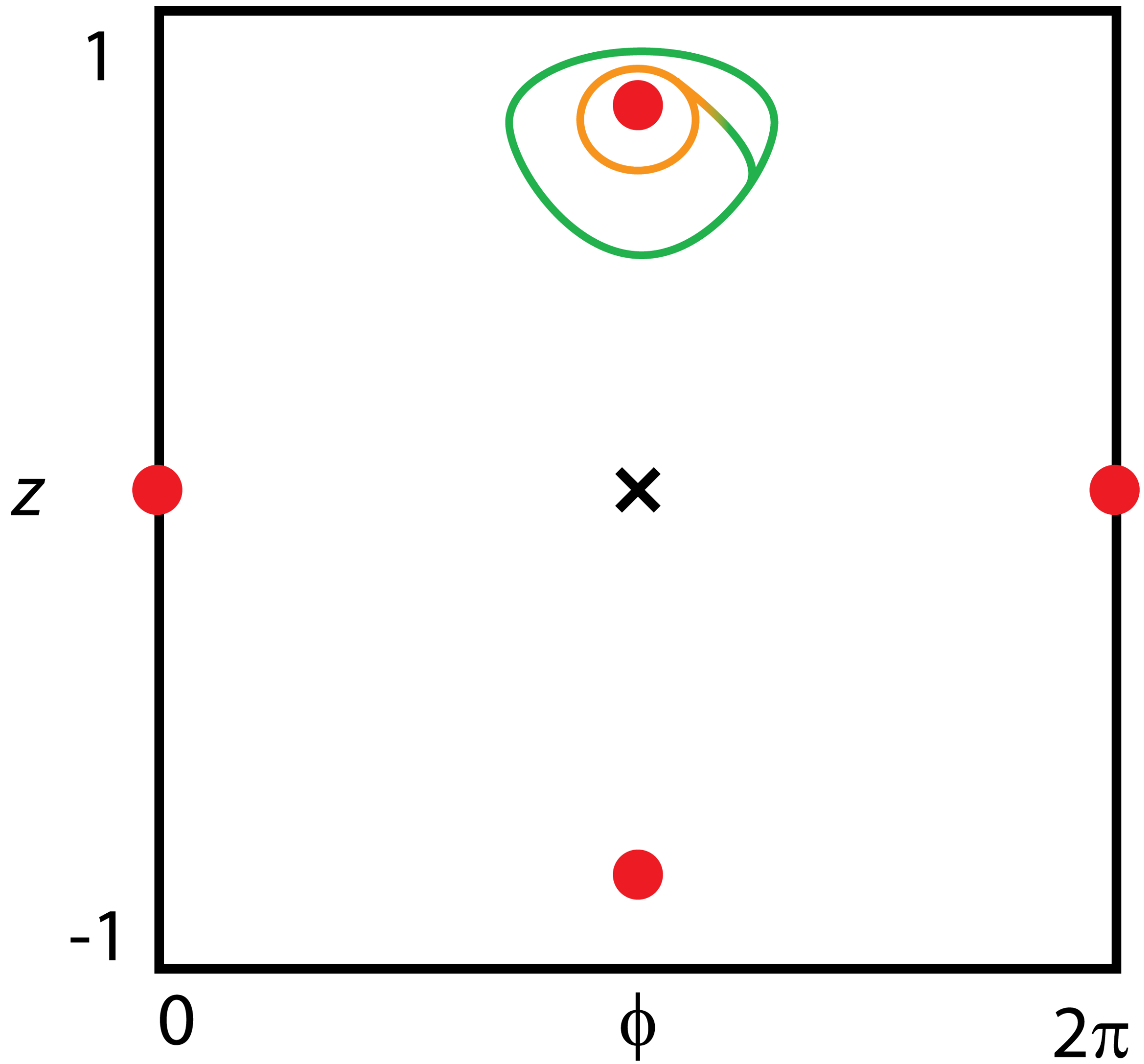
- Focused electron beam removes atoms from one of the wells

  Gericke et al., Nature Phys. **4**, 949 (2008)

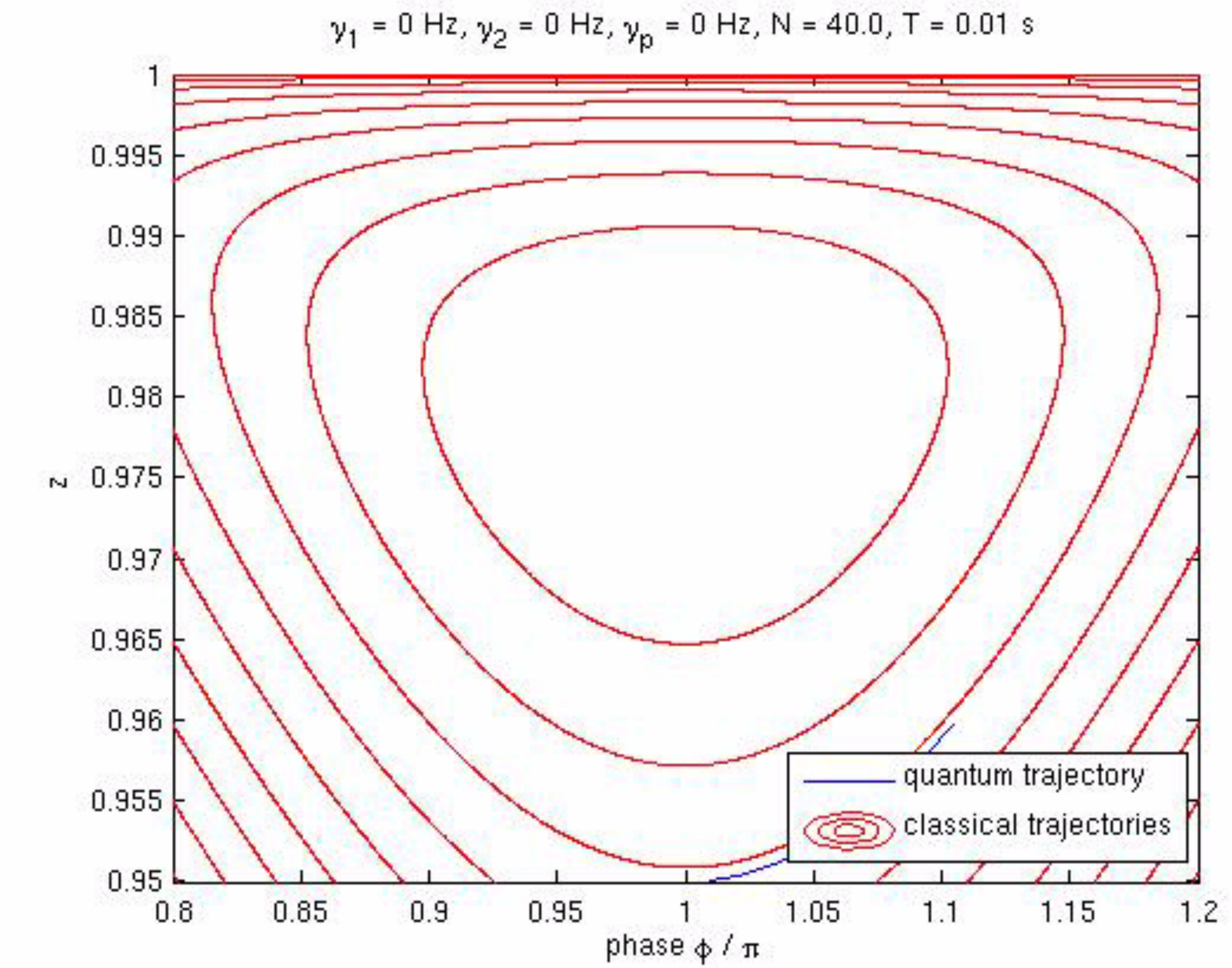
- Open quantum system: quantum jump method

  Dalibard et al., PRL **68**(5) 580 (1992)
  Garraway and Knight, PRA **49**(2) 1266 (1994)
  Witthaut et al., PRA **83**(6) 1 (2011)

Dissipation-induced coherence

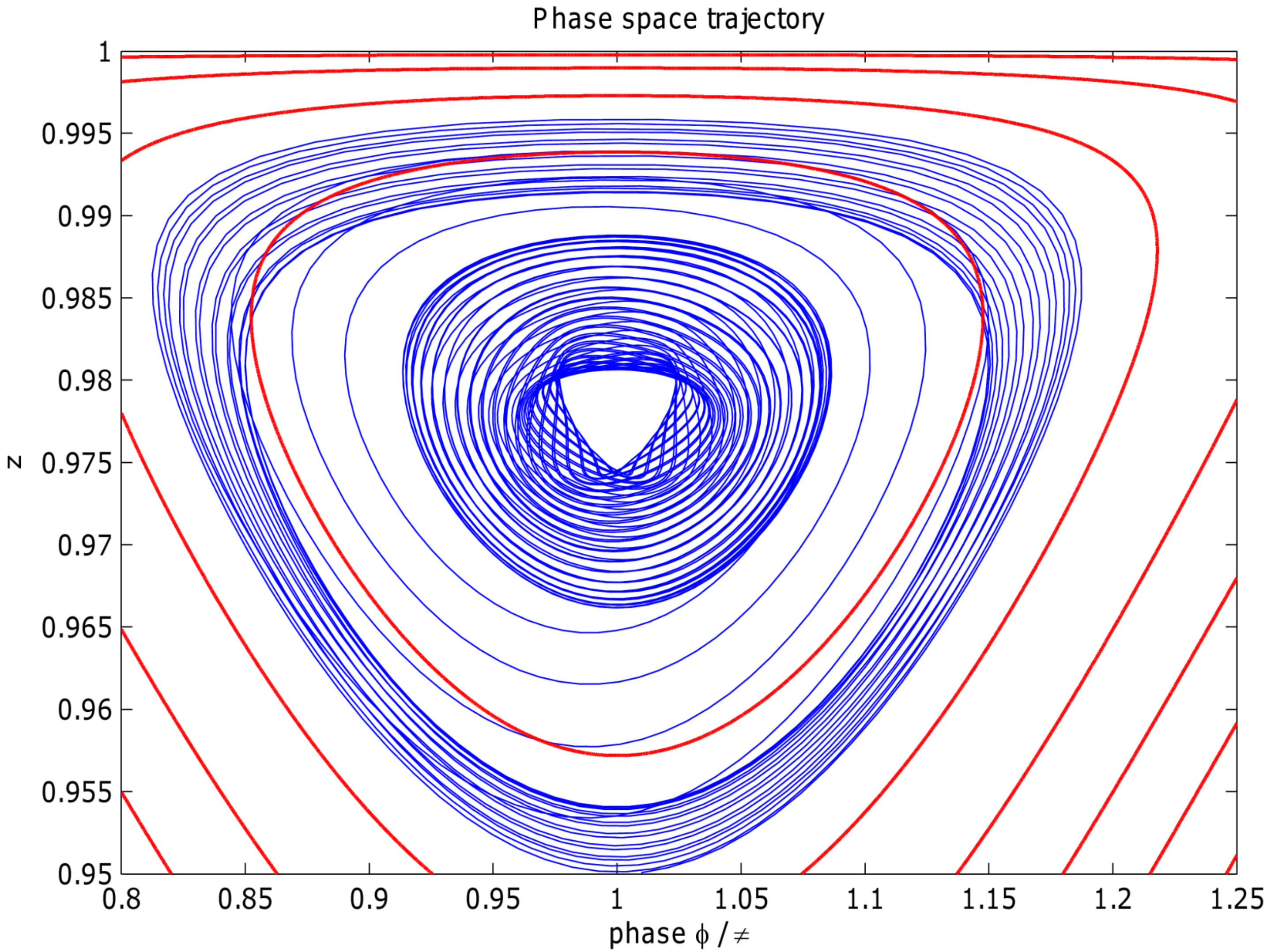


Dissipative Quantum Dynamics



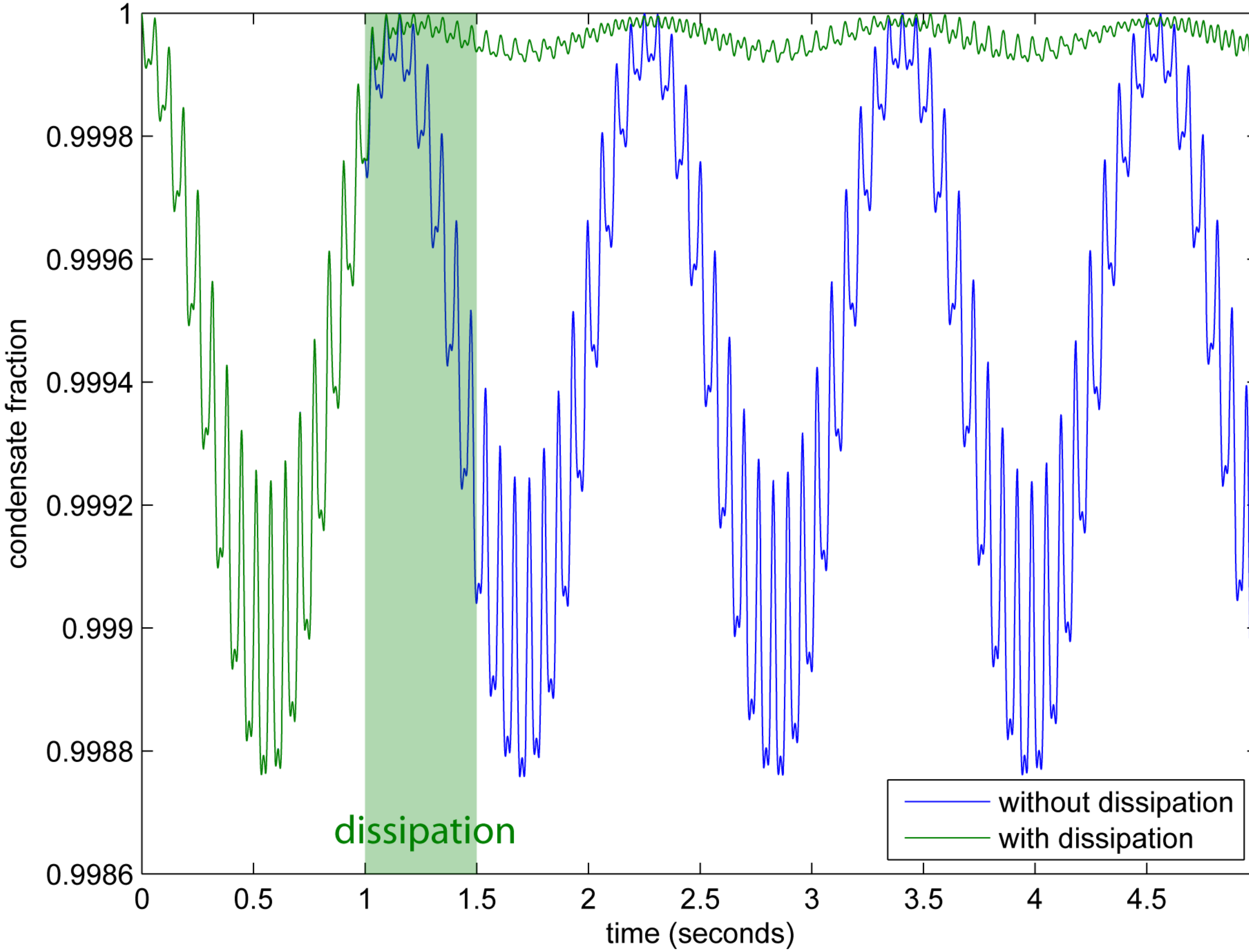
$\Lambda = 5$

Results: phase space



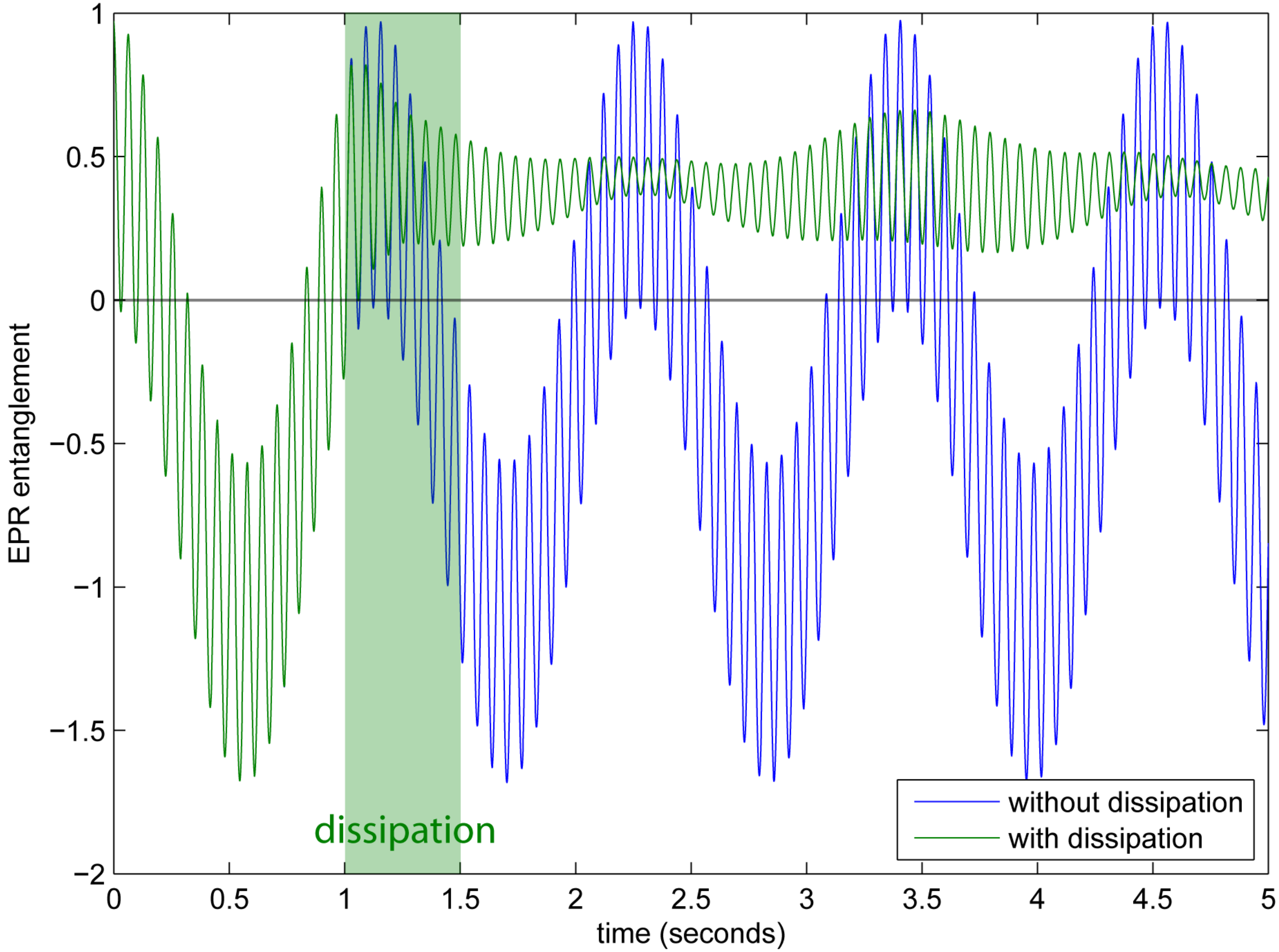
Results: condensate fraction

$$\max \operatorname{eig} \rho = \frac{1}{2}\left(\rho_{11} + \rho_{22} + \sqrt{4\rho_{12}\rho_{21} + (\rho_{11} - \rho_{22})^2}\right)$$



Results: EPR entanglement

$$\mathrm{EPR} = \langle a_1^\dagger a_2 \rangle \langle a_2^\dagger a_1 \rangle - \langle a_1^\dagger a_1 a_2^\dagger a_2 \rangle$$



Summary of Results

- Results for 1) "condensate fraction"—$C(z,\phi) \cong \lambda_{max}/N \le 1$, λ_{max} = largest eigenvalue of single particle density matrix and "entanglement"— $E \cong |\langle a_1^* a_2\rangle|^2 - \langle a_1^* a1 a_2^* a_2\rangle$, $E > 0$ for entangled state
  - Both $C(z,\phi)$ and E generally "track" classical orbits in GPS
  - $C(z,\phi)$ decreases dramatically near classical separatrix, remains large at classical fixed point and especially in self-trapped region=> regular motion enhances coherence, chaotic motion destroys coherence
  - Symmetry $z \Rightarrow -z$ of classical equations is broken by quantum dynamics
  - "Ridge" of enhanced coherence exists in quantum but not in LD or GPE (classical) or dynamics
  - Overlap of initial coherent state with eigenfunctions of BHH is minimal along separatrix, high near fixed points
  - Quantum "orbits" show two frequencies: one semiclassical, the other quantum revival ; interpretation in terms of three leading eigenfrequencies
  - See Josephson tunneling at small Λ, as in experiments
  - Dissipation can enhance coherence for both *C* and *E*

Beyond the Dimer

For spatially *extended* systems, “Intrinsic Localized Modes”/ “Discrete Breathers” are natural generalization of dimer’s self-trapped states: ILMs *may* serve as means of maintaining quantum coherence in large optical lattices or macromolecules. Current experiments being done by the Greiner group at Harvard may answer this question.

References:

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QUESTIONS ?