Photonic Multipartite Entanglement

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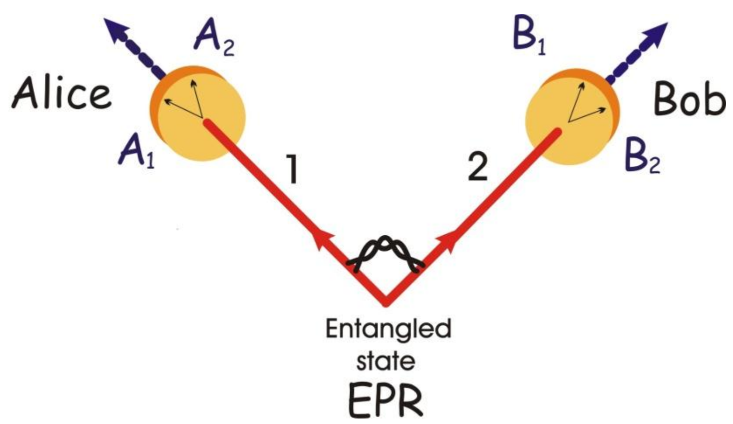
Introduction

Bipartite state: EPR State $|\psi^-\rangle_{ab} = (1/\sqrt{2})(|01\rangle - |10\rangle)_{ab}$

Tests on Foundations of Quantum mechanics

Applications in Quantum Information Processing:

Quantum Cryptography: Quantum key Distribution
Quantum Teleportation
etc



Werner state

$$\rho_W = p|\Psi^-\rangle\langle\Psi^-| + (1-p)\frac{\mathbb{1}}{4}$$

$$\rho_W = \begin{pmatrix} \frac{1-p}{4} & 0 & 0 & 0 \\ 0 & \frac{1+p}{4} & \frac{p}{2} & 0 \\ 0 & \frac{p}{2} & \frac{1+p}{2} & 0 \\ 0 & 0 & 0 & \frac{1-p}{4} \end{pmatrix}$$

Violation of CHSH inequality: $\langle B_{CHSH}\rangle_{\rho_W} = 2\sqrt{2p^2}$

$p = 0$  $p = 1$

$p > 1/\sqrt{2}$

Peres-Horodecki criterion: $\frac{1}{4}(1-3p), \frac{1+p}{4}, \frac{1+p}{4}, \frac{1+p}{4}$

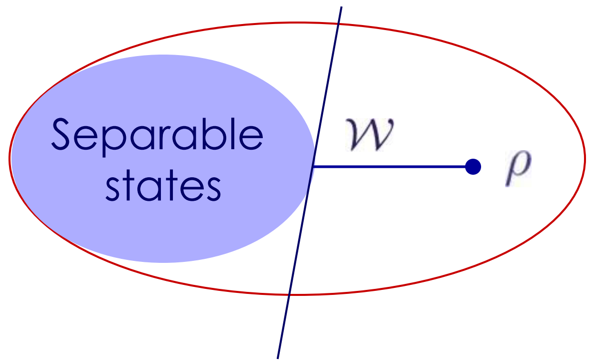
$p = 0$  $p = 1$

$p > 1/3$

$p = 0$  $p = 1$

Witnessing Entanglement

Entangled states vs Separable states



A operator $\mathcal{W}$ is called Entanglement witness detecting the entangled state $|\psi\rangle$ iff $\mathrm{Tr}(\mathcal{W}|\psi\rangle\langle\psi|) < 0$

$$\mathcal{W} = \alpha 1 - |\psi\rangle\langle\psi|$$

$$\alpha = \max_{|\phi\rangle \in B} |\langle\phi|\psi\rangle|^2$$

Horodecki^3, PLA 223
Terhal, PLA 271

Local decomposition

Decomposition into local von Neumann (or projective) measurement

$$\mathcal{W} = \sum_{k=1}^{K} M_k$$

$$M_k = \sum_{l_1,\ldots,l_n} d^{(k)}_{l_1,\ldots,l_n} |a^{(k,1)}_{l_1}\rangle\langle a^{(k,1)}_{l_1}| \otimes \ldots \otimes |a^{(k,n)}_{l_n}\rangle\langle a^{(k,n)}_{l_n}|$$

Optimal local decomposition when K minimal

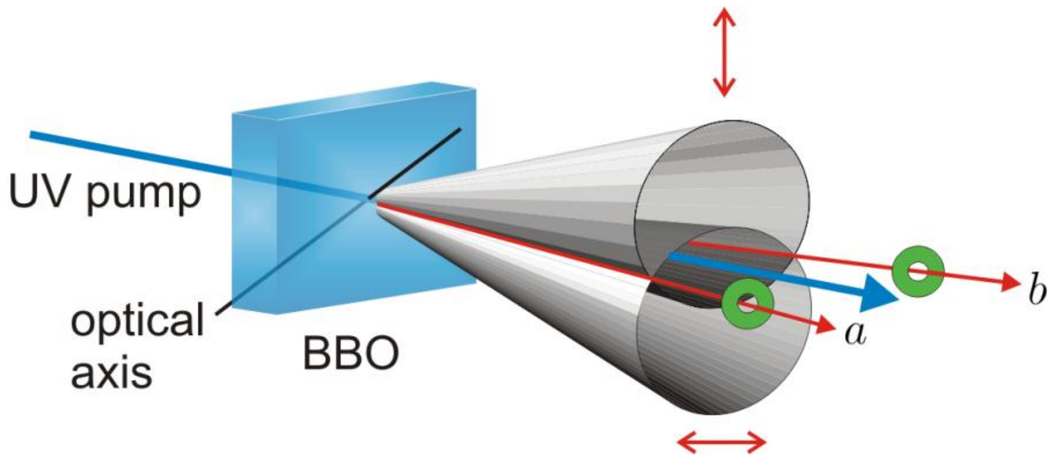
Gühne et al, PRA 66

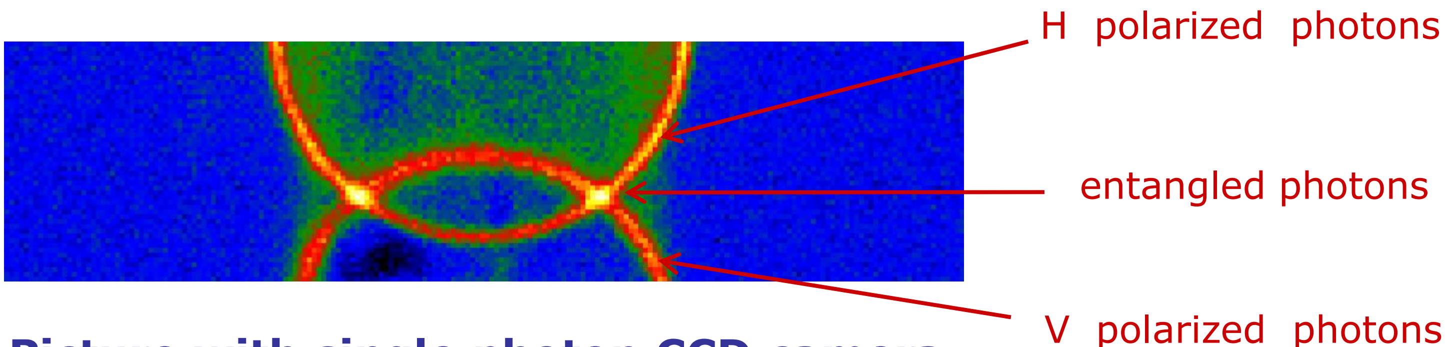
Two photon polarization entanglement

Bell State: Two photon polarization entangled state

$$|\Psi^{+}\rangle = \frac{1}{\sqrt{2}}(|HV\rangle_{ab} + |VH\rangle_{ab})$$

Spontaneous Parametric Down-Conversion



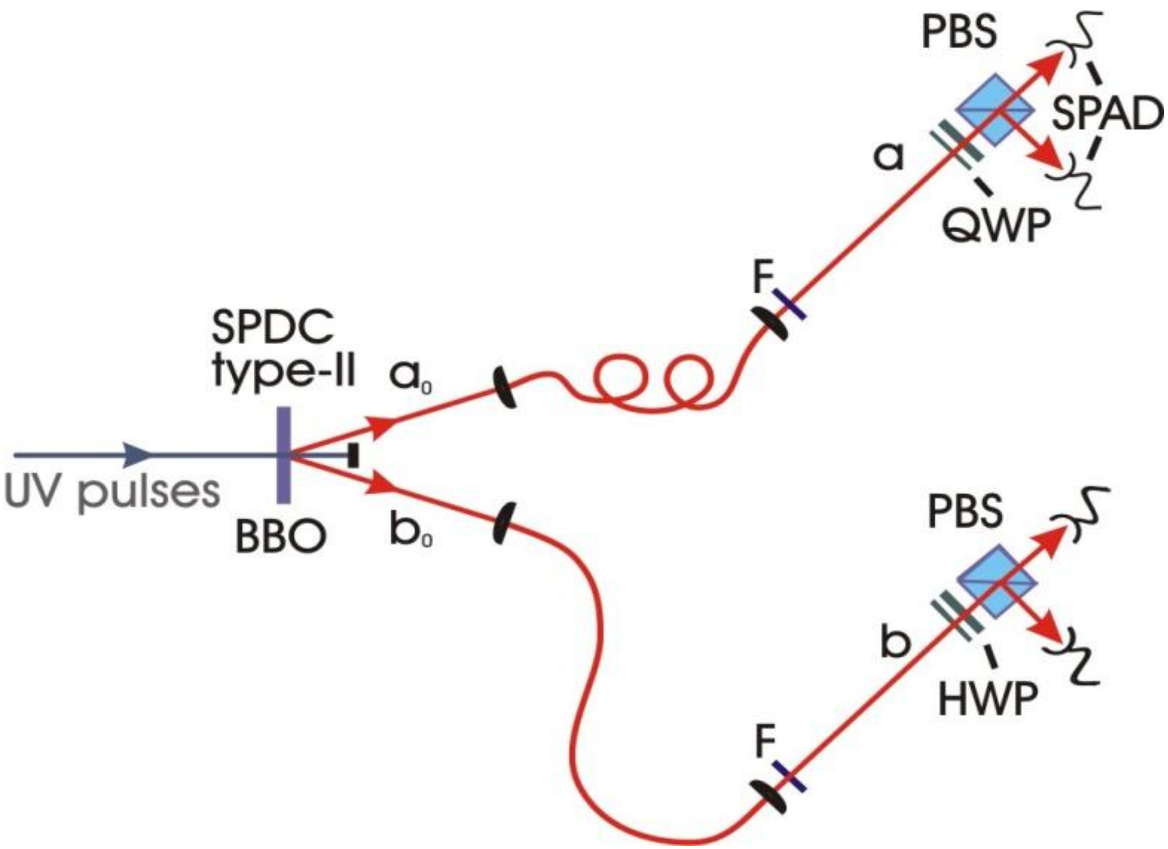


Picture with single photon CCD camera

Two photon polarization entanglement

Bell State: Two photon polarization entangled state

$$|\Psi^{+}\rangle = \frac{1}{\sqrt{2}}(|HV\rangle_{ab} + |VH\rangle_{ab})$$



Bipartite Entanglement (via witness)

$$\rho_{ac}^{\Psi^{(4)}} = Tr_{bd}[\rho_{abcd}^{\Psi^{(4)}}] = \frac{2}{3}|\Psi^-\rangle_{ac}\langle\Psi^-| + \frac{1}{3}\frac{\mathbb{1}_{ac}}{4}$$

$$\mathcal{W}(\rho_{ac}^{\Psi^{(4)}}) = \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} + \sigma_x\otimes\sigma_x + \sigma_y\otimes\sigma_y + \sigma_z\otimes\sigma_z)$$

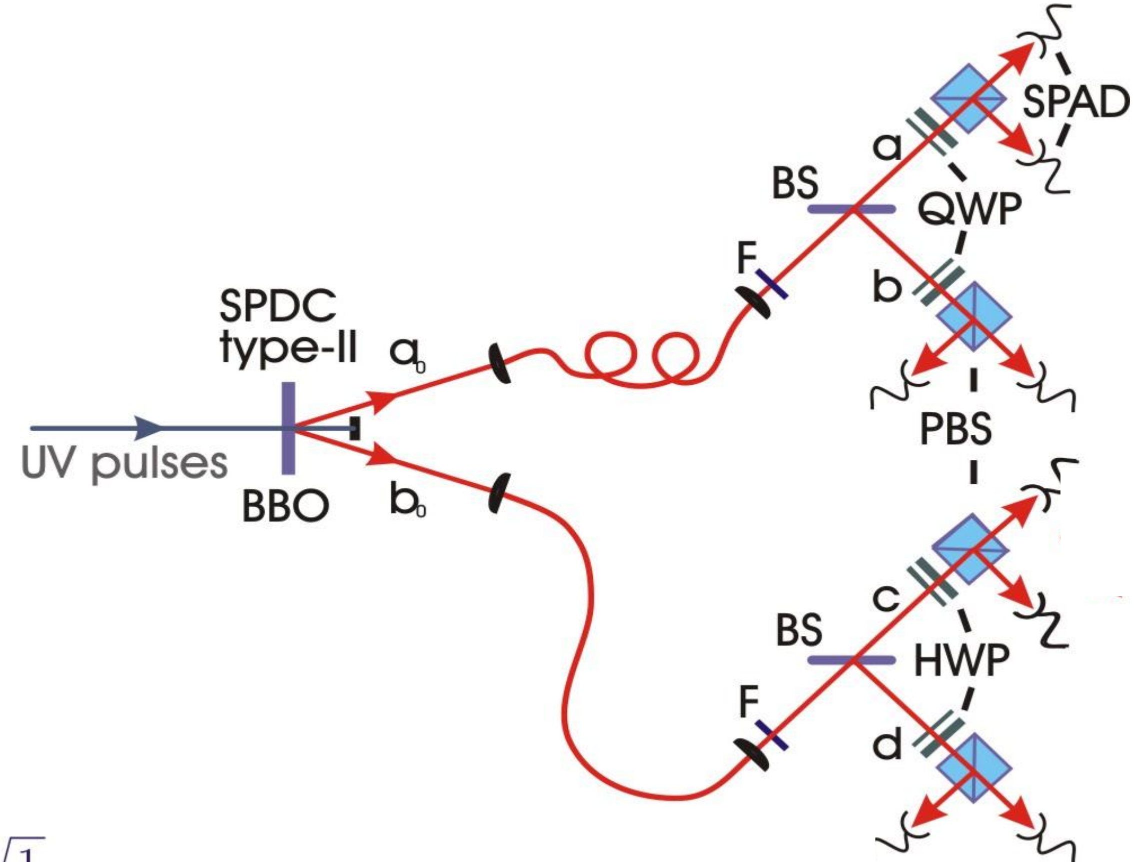
3 local measurements

Theory: -1/4

Experiment: -0.218 ± 0.007

Four Qubits state

Second order process in Spontaneous Parametric Down-Conversion



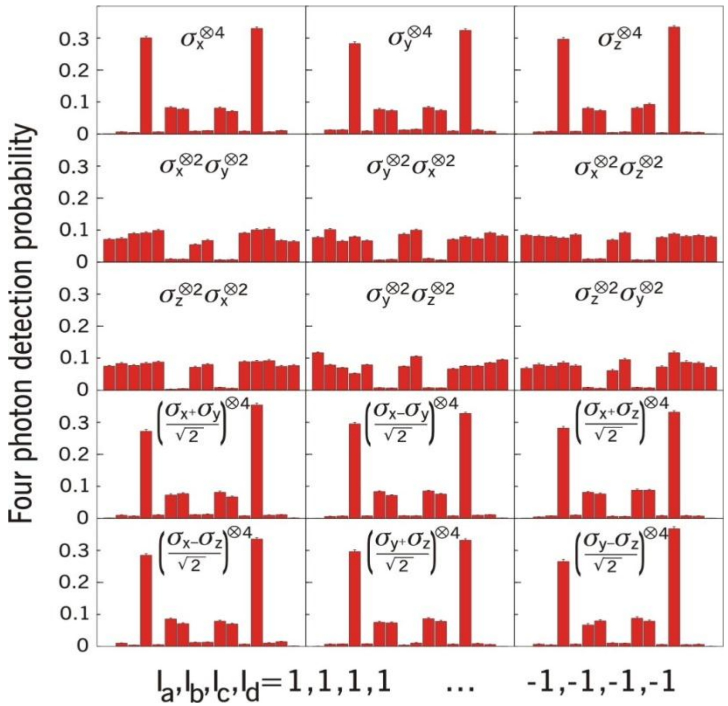
$$\left|\Psi^{(4)}\right\rangle = \sqrt{\frac{1}{3}}\left(|HHVV\rangle + |VVHH\rangle\right.$$
$$\left.-\frac{1}{2}\left(|HVHV\rangle + |HVVH\rangle + |VHHV\rangle + |VHVH\rangle\right)\right)$$

Weinfurter & Zukowski PRA 64

Fourpartite Entanglement Witness

$$
\begin{aligned}
\mathcal{W}_{\Psi^{(4)}} &= \frac{3}{4}1 - |\Psi^{(4)}\rangle\langle\Psi^{(4)}| \\
&= \frac{1}{48}\Big(3\cdot(\sigma_x\sigma_x\sigma_y\sigma_y + \sigma_y\sigma_y\sigma_x\sigma_x + \sigma_x\sigma_x\sigma_z\sigma_z) \\
&+ 3\cdot(\sigma_z\sigma_z\sigma_x\sigma_x + \sigma_y\sigma_y\sigma_z\sigma_z + \sigma_z\sigma_z\sigma_y\sigma_y) \\
&- (\sigma_x+\sigma_y)^{\otimes 4} - (\sigma_x-\sigma_y)^{\otimes 4} - (\sigma_x+\sigma_z)^{\otimes 4} \\
&- (\sigma_x-\sigma_z)^{\otimes 4} - (\sigma_x+\sigma_z)^{\otimes 4} - (\sigma_y-\sigma_z)^{\otimes 4} \\
&+ 33\cdot 1^{\otimes 4} - \sum_{i=x,y,z}\big[\sigma_i\sigma_i 11 + 11\sigma_i\sigma_i - \sigma_i^{\otimes 4} \\
&- 2\cdot(\sigma_i 1\sigma_i 1 + \sigma_i 11\sigma_i + 1\sigma_i\sigma_i 1 + 1\sigma_i 1\sigma_i)\big]\Big)
\end{aligned}
$$

$$\mathrm{Tr}(\mathcal{W}_{\Psi^{(4)}}\rho_{\Psi^{(4)}}) = -1/4$$

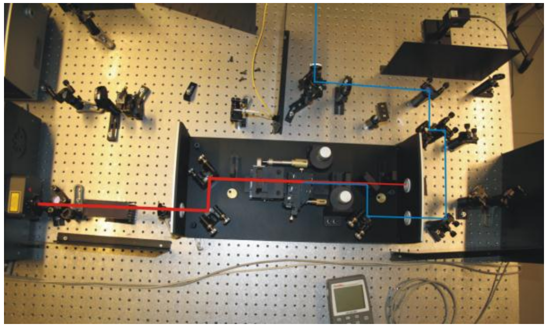


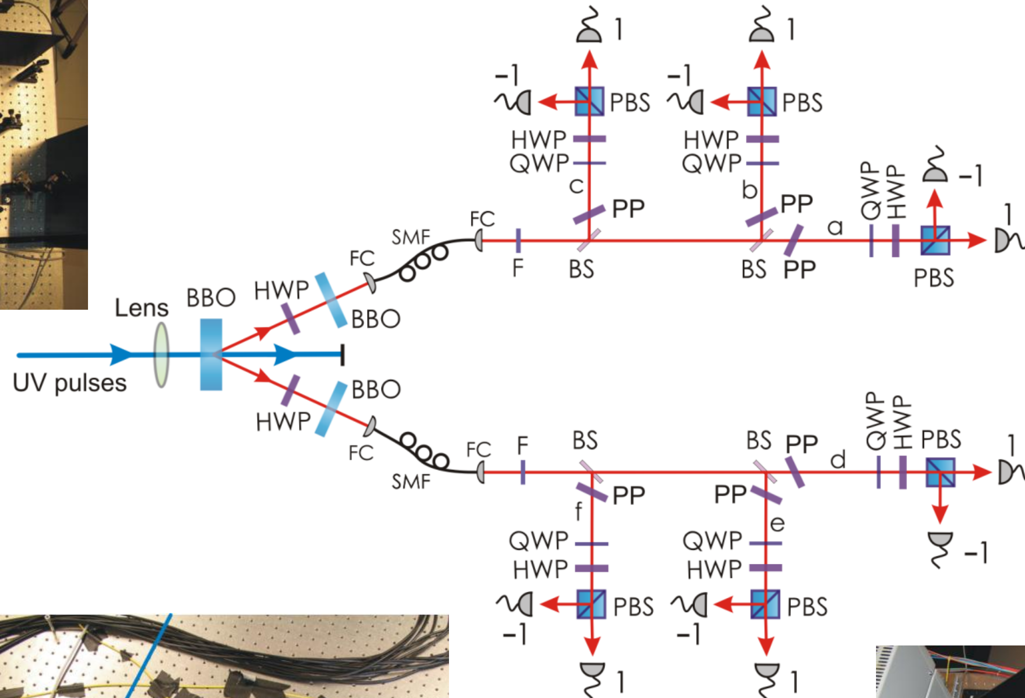
$$\mathrm{Tr}(\mathcal{W}_{\Psi^{(4)}}\rho_{\Psi^{(4)}})_{exp} = -0.151 \pm 0.01$$

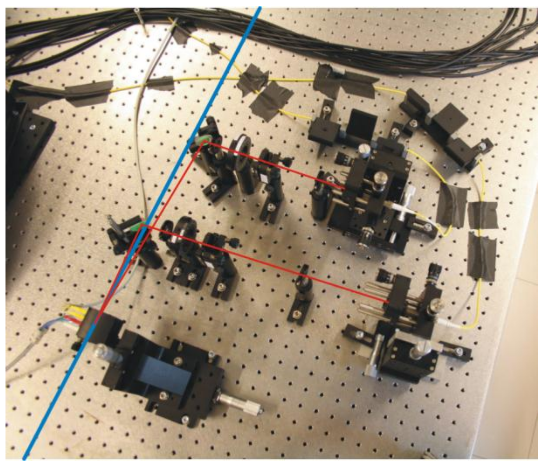
Experimental detection of genuine fourpartite entanglement

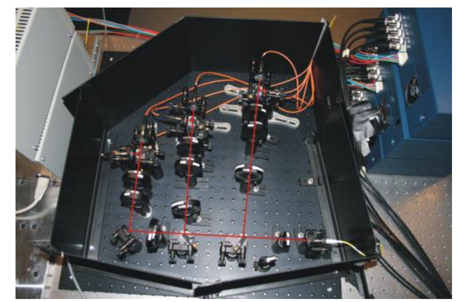
Bourennane et al, PRL 92

Six photon polarization entanglement



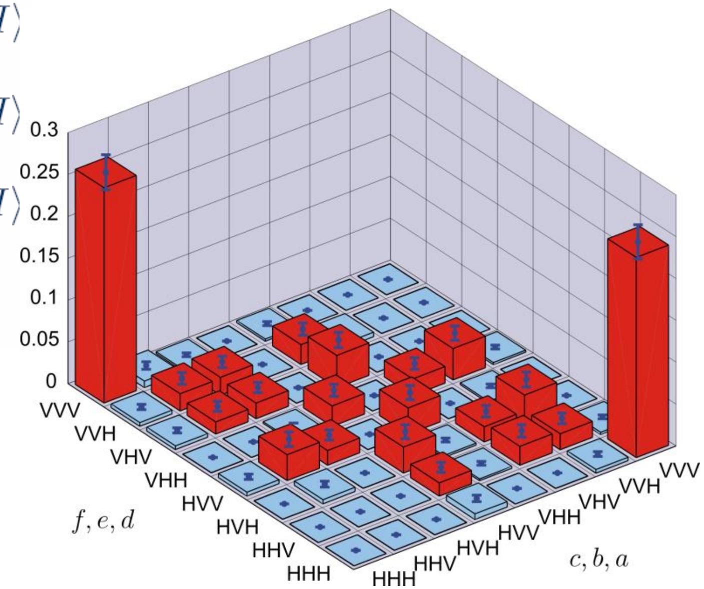






Six photon polarization entanglement

$$
\begin{aligned}
|\Psi_6^-\rangle = &\frac{1}{2}|HHHVVV\rangle - \frac{1}{6}|HHVHVV\rangle - \frac{1}{6}|HVHHVV\rangle \\
&- \frac{1}{6}|VHHHVV\rangle - \frac{1}{6}|HHVVHV\rangle - \frac{1}{6}|HVHVHV\rangle \\
&- \frac{1}{6}|VHHVHV\rangle + \frac{1}{6}|HVVHHV\rangle + \frac{1}{6}|VHVHHV\rangle \\
&+ \frac{1}{6}|VVHHHV\rangle - \frac{1}{6}|HHVVVH\rangle - \frac{1}{6}|HVHVVH\rangle \\
&- \frac{1}{6}|VHHVVH\rangle + \frac{1}{6}|HVVHVH\rangle + \frac{1}{6}|VHVHVH\rangle \\
&+ \frac{1}{6}|VVHHVH\rangle + \frac{1}{6}|HVVVHH\rangle + \frac{1}{6}|VHVVHH\rangle \\
&+ \frac{1}{6}|VVHVHH\rangle - \frac{1}{2}|VVVHHH\rangle
\end{aligned}
$$



$$|\Psi_6^-\rangle = \frac{1}{\sqrt{2}}|GHZ_6^-\rangle + \frac{1}{2}(|\overline{W}_3\rangle|W_3\rangle - |W_3\rangle|\overline{W}_3\rangle)$$

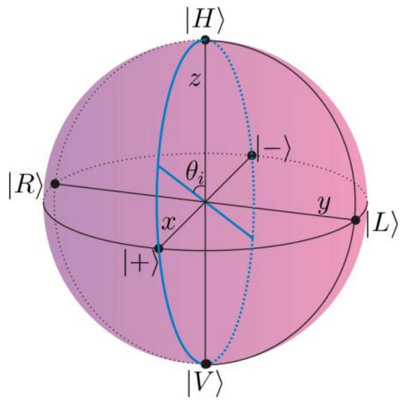
$$|GHZ_6^-\rangle = \frac{1}{\sqrt{2}}(|HHHVVV\rangle - |VVVHHH\rangle)$$

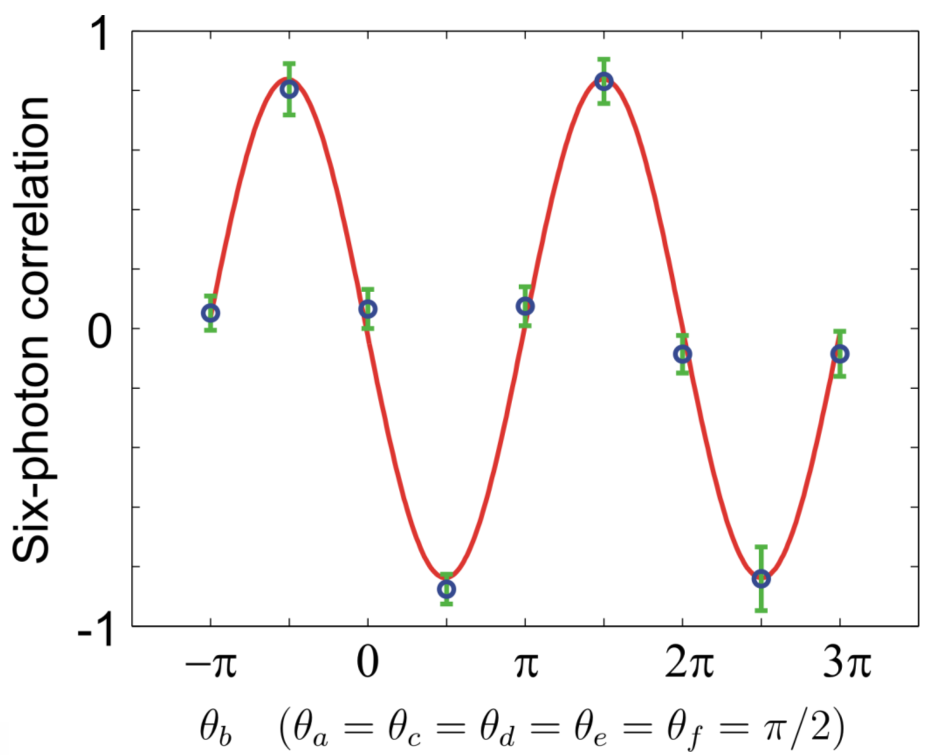
$$|W_3\rangle = \frac{1}{\sqrt{3}}(|HHV\rangle + |HVH\rangle + |VHH\rangle)$$

$$|\overline{W}_3\rangle = \frac{1}{\sqrt{3}}(|VVH\rangle + |VHV\rangle + |HVV\rangle)$$

Six-photon quantum correlation

$$E\left(\frac{\pi}{2},\theta_b,\frac{\pi}{2},\frac{\pi}{2},\frac{\pi}{2},\frac{\pi}{2}\right) = -\sin\theta_b$$





$$\tilde{\rho}_{exp} = \frac{1-V}{2^6}\mathbb{1}^{\otimes 6} + V\left|\Psi_6^-\right\rangle\left\langle\Psi_6^-\right|$$

$$F = \langle\Psi_6^-|\tilde{\rho}_{exp}|\Psi_6^-\rangle = 0.840 \pm 0.029$$

$$V = 0.838 \pm 0.030$$

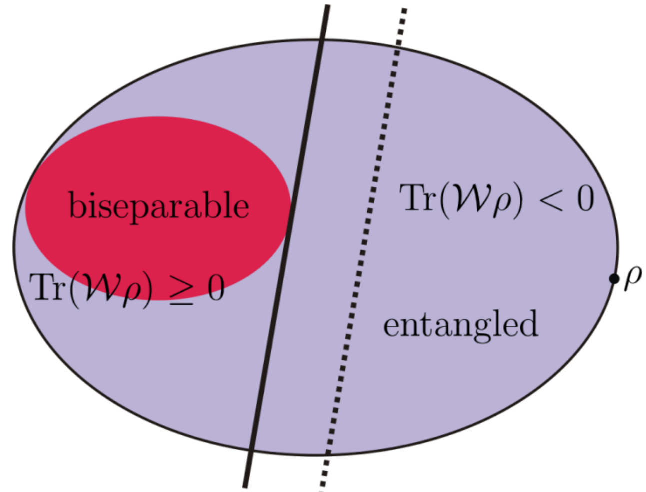
Six photon entanglement: witness

Derived from the maximum overlap witness

+: few measurements (3)

-: lower noise tolerance (15%)

$$\mathrm{Tr}(\mathcal{W}^{\pm}\,|\Psi_6^{\pm}\rangle\langle\Psi_6^{\pm}|) = -\frac{1}{18} \approx -0.056$$



Negative by 2.0 standard deviations

$$\mathrm{Tr}(\mathcal{W}^{-}\rho_{\Psi_6^-}) = -0.023 \pm 0.012$$

PRL 103, 150501 (2009) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 9 OCTOBER 2009

Experimental Test of Fidelity Limits in Six-Photon Interferometry and of Rotational Invariance Properties of the Photonic Six-Qubit Entanglement Singlet State

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Four qubit bound entanglement

Bound entanglement:
Non distillable noise entangled state

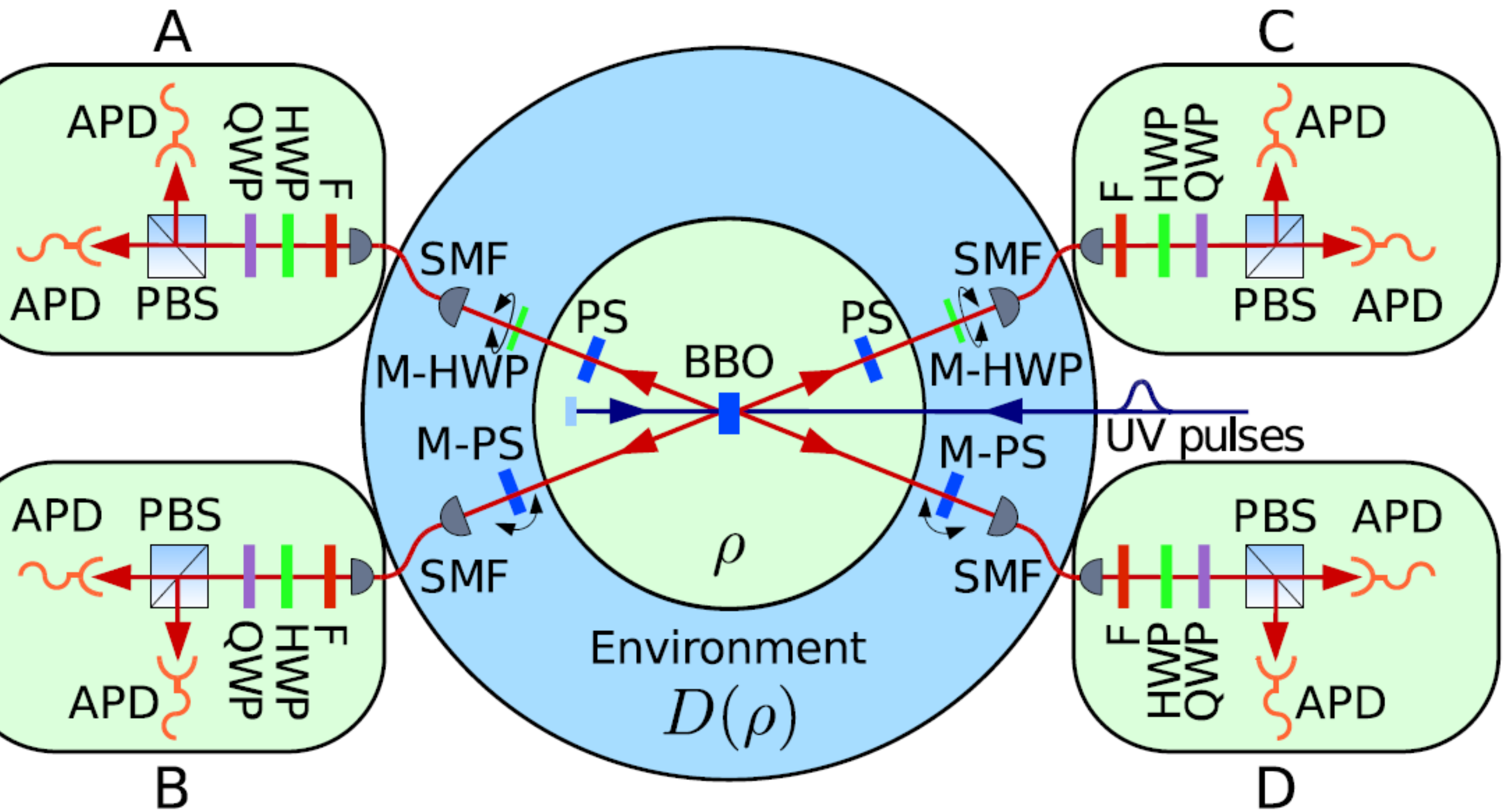
Smolin state, John Smolin, PRA 63

$$\rho_S = \sum_{i\in\{1,2,3,4\}} \frac{1}{4} |\,\psi_{AB}^{(i)}\,\rangle\langle\,\psi_{AB}^{(i)}\,| \otimes |\,\psi_{CD}^{(i)}\,\rangle\langle\,\psi_{CD}^{(i)}\,|$$

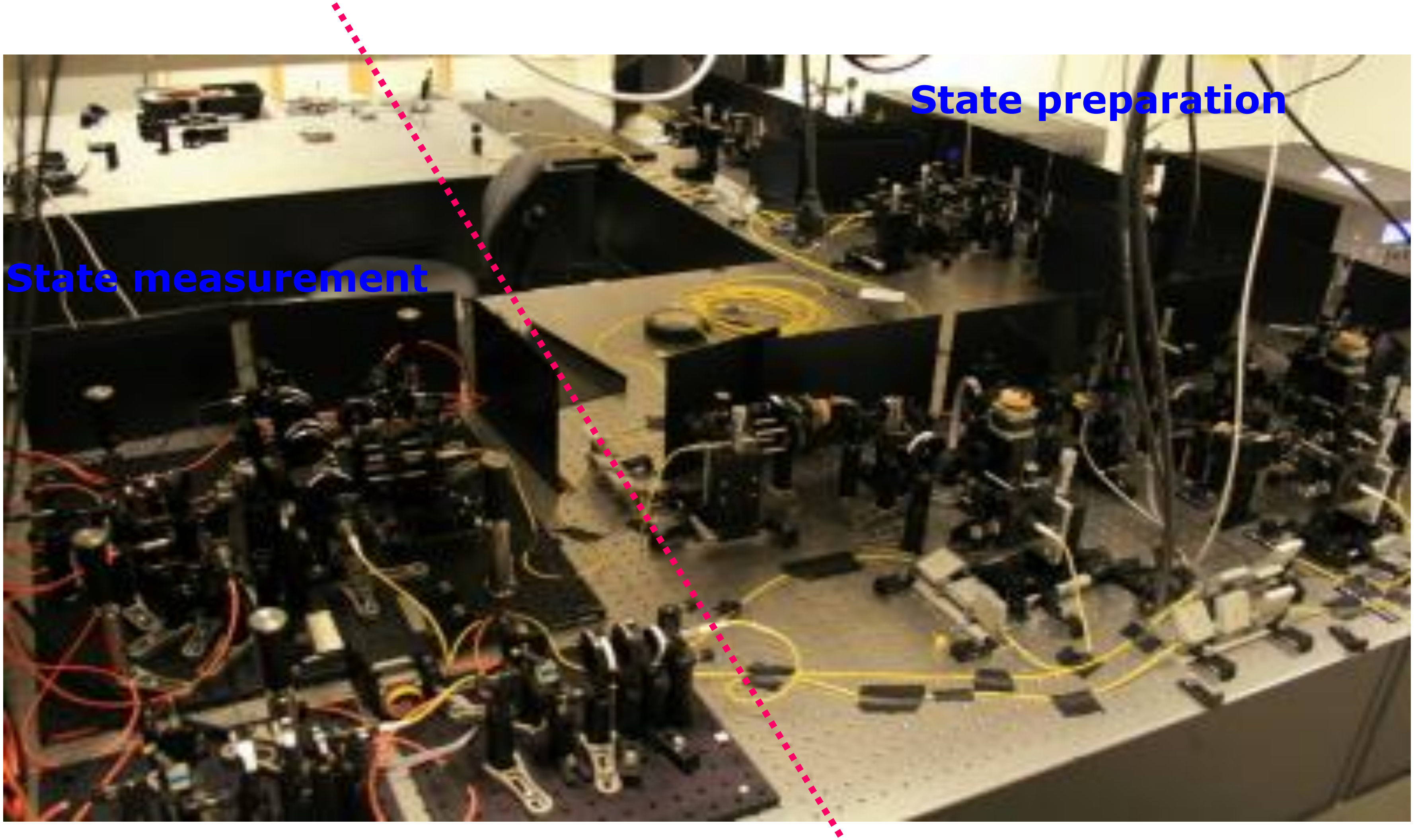
$$\psi^i \in \{|\,\Psi^{\pm}\,\rangle, |\,\Phi^{\pm}\,\rangle\} \qquad \begin{aligned} |\,\Psi^{\pm}\,\rangle &= \frac{1}{\sqrt{2}}(|\,01\,\rangle \pm |\,10\,\rangle) \\ |\,\Phi^{\pm}\,\rangle &= \frac{1}{\sqrt{2}}(|\,00\,\rangle \pm |\,11\,\rangle) \end{aligned}$$

$$\rho_S = \frac{1}{16}(\mathbb{1}^{\otimes 4} + \sum_{i\in\{1,2,3\}} \sigma_i^A \otimes \sigma_i^B \otimes \sigma_i^C \otimes \sigma_i^D)$$

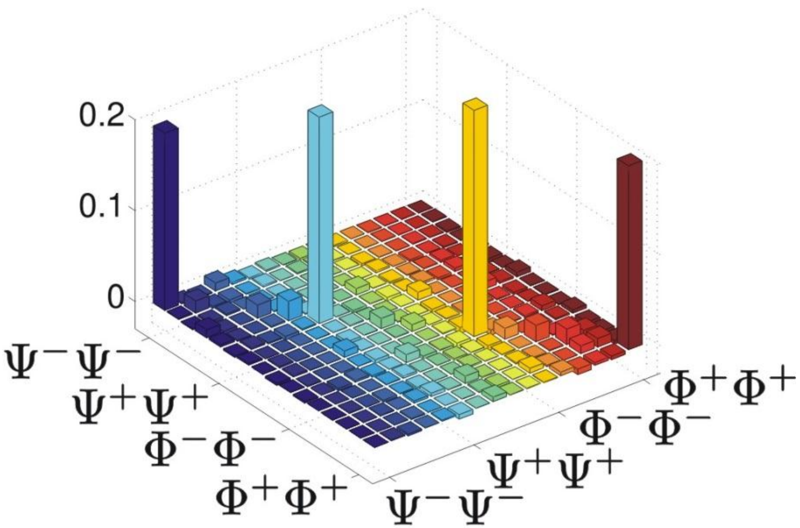
Bound entanglement

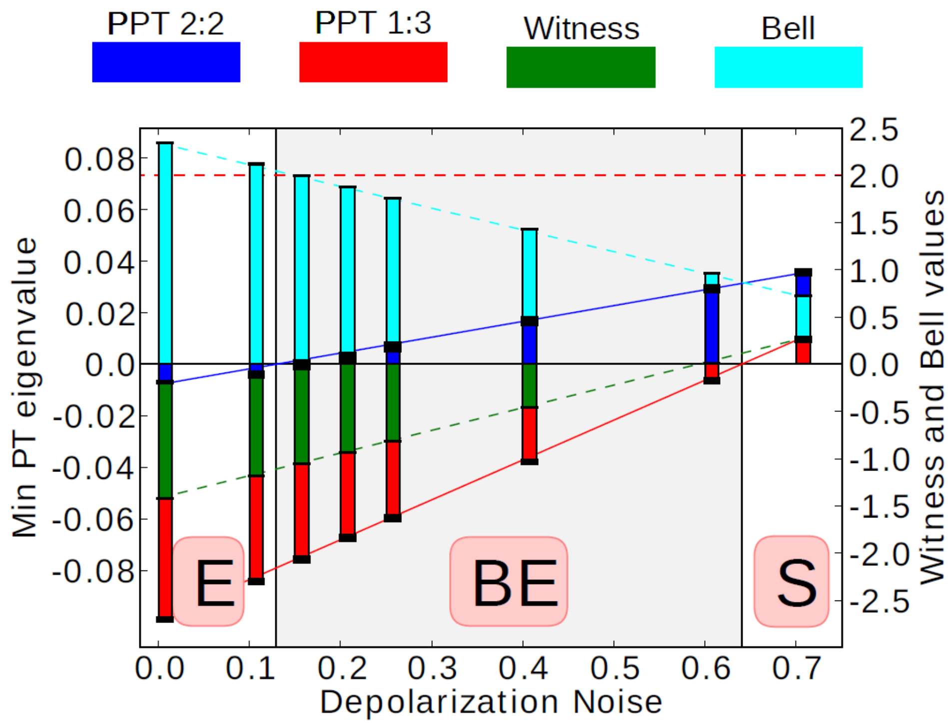


Experimental setup



Bound entanglement





ARTICLES
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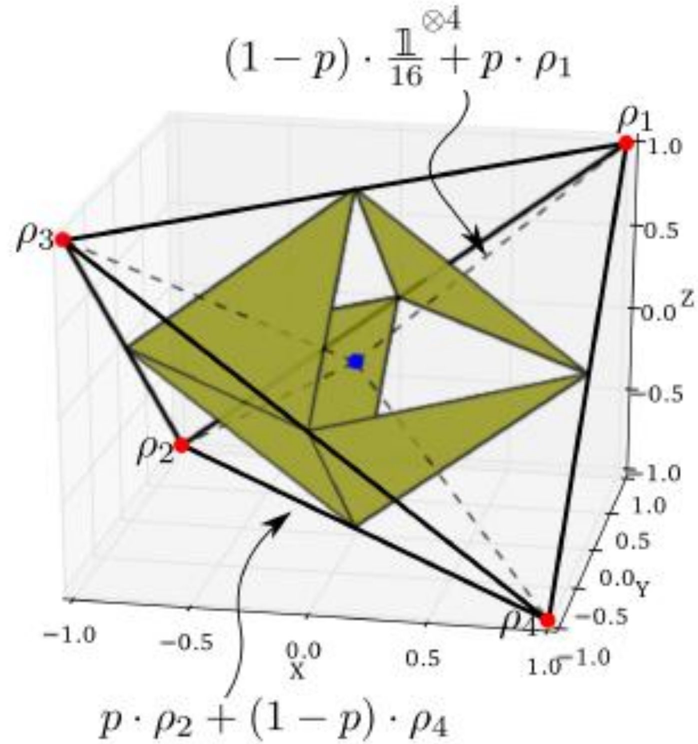
nature physics

Experimental four-qubit bound entanglement

Elias Amselem and Mohamed Bourennane*

Bound entanglement

$$\rho_1 = \frac{1}{16}(\sigma_{\mathbb{1}}^{\otimes 4} + \sigma_z^{\otimes 4} + \sigma_x^{\otimes 4} + \sigma_y^{\otimes 4})$$
$$\rho_2 = \frac{1}{16}(\sigma_{\mathbb{1}}^{\otimes 4} - \sigma_z^{\otimes 4} - \sigma_x^{\otimes 4} + \sigma_y^{\otimes 4})$$
$$\rho_3 = \frac{1}{16}(\sigma_{\mathbb{1}}^{\otimes 4} + \sigma_z^{\otimes 4} - \sigma_x^{\otimes 4} - \sigma_y^{\otimes 4})$$
$$\rho_4 = \frac{1}{16}(\sigma_{\mathbb{1}}^{\otimes 4} - \sigma_z^{\otimes 4} + \sigma_x^{\otimes 4} - \sigma_y^{\otimes 4})$$



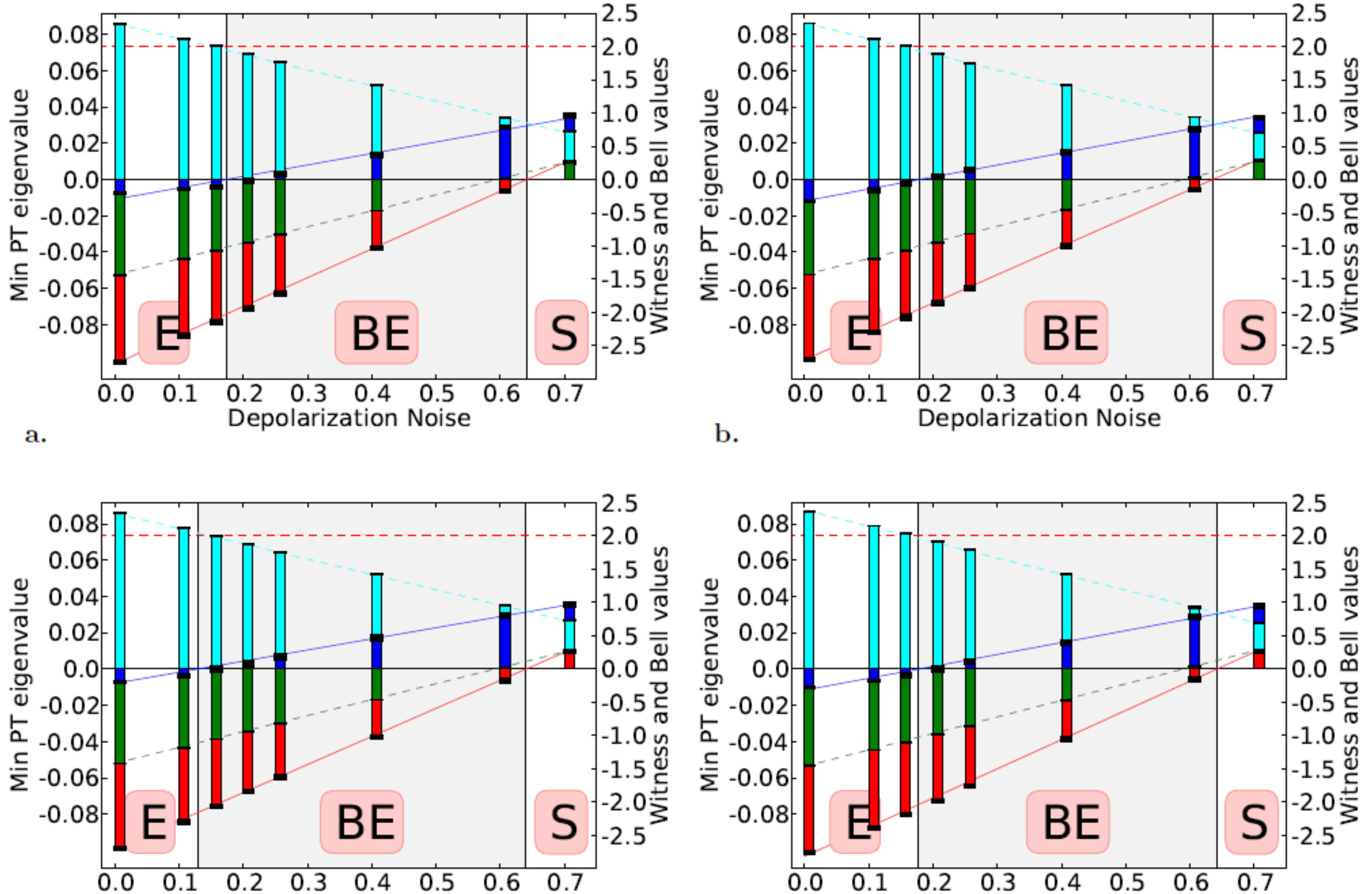
Depolarizing noise

$$(1-p)\cdot\frac{\mathbb{1}}{16}^{\otimes 4} + p\cdot\rho_i$$

$$\rho_3 = \frac{1}{16}(\sigma_{\mathbb{1}}^{\otimes 4} + \sigma_z^{\otimes 4} - \sigma_x^{\otimes 4} - \sigma_y^{\otimes 4})$$

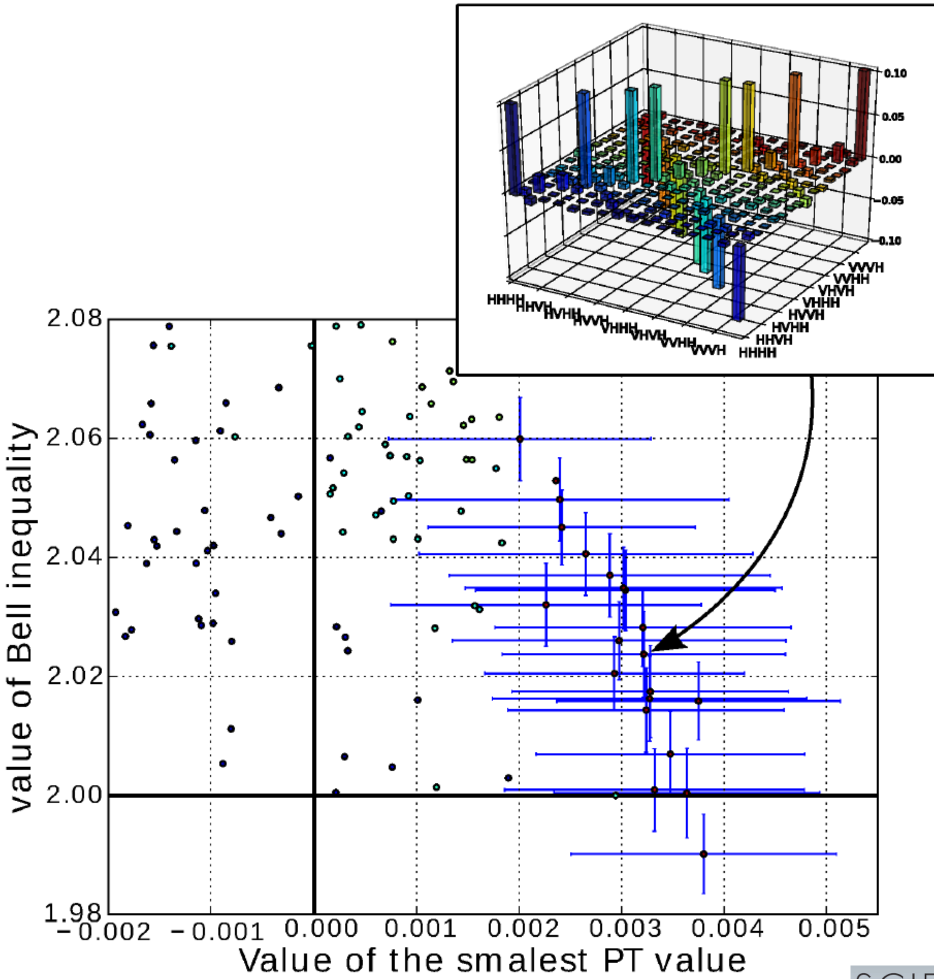
Bound entanglement:
Non distillable noise entangled state

Noise quantum channels



a. b. c. d.

Noise quantum channels



Bell inequality violation by bound entangled states

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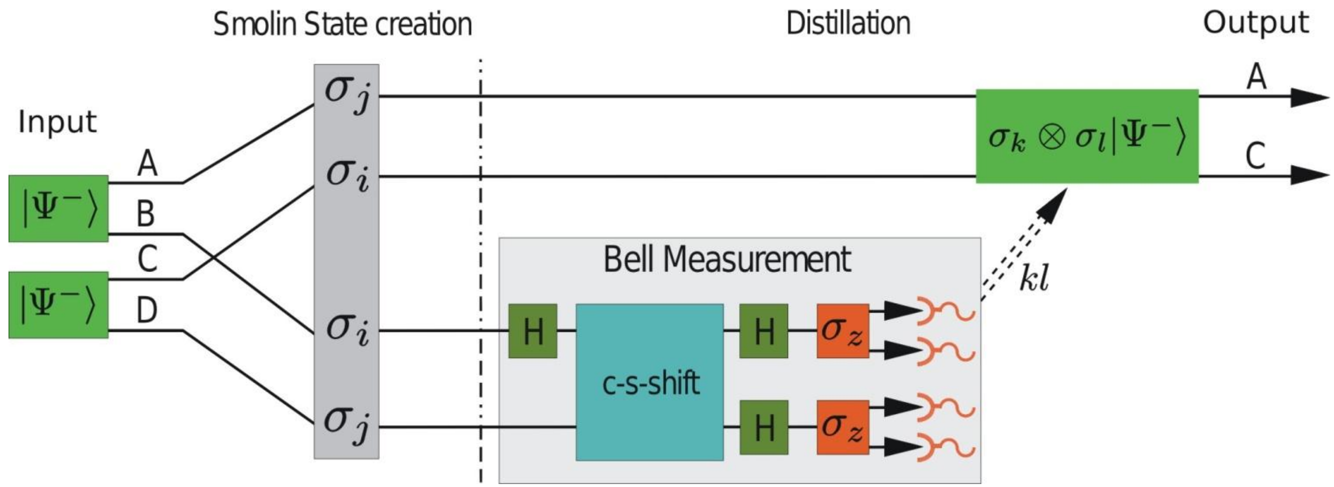
Experimental bound entanglement through a Pauli channel

Elias Amselem, Muhammad Sadiq & Mohamed Bourennane

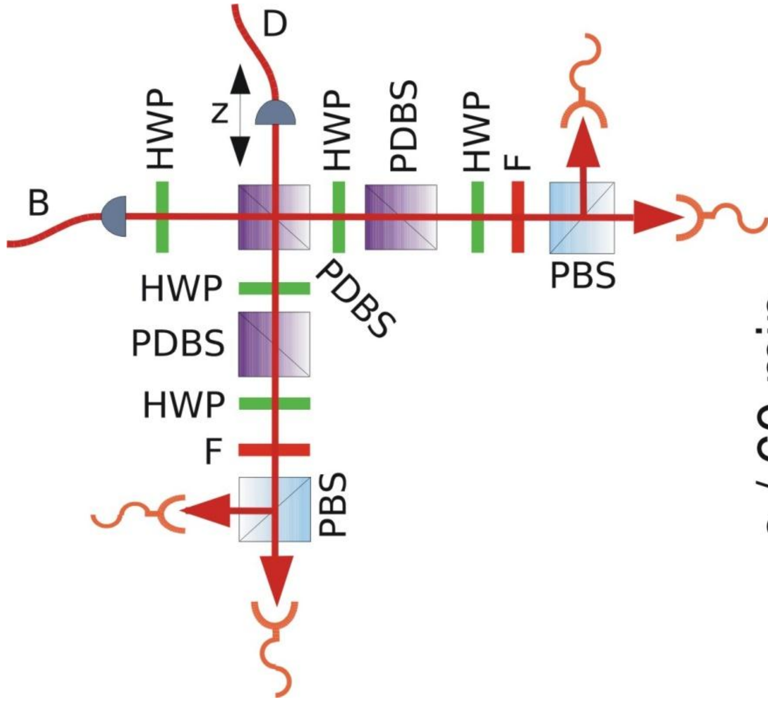
Department of Physics, Stockholm University, S-10691, Stockholm, Sweden.

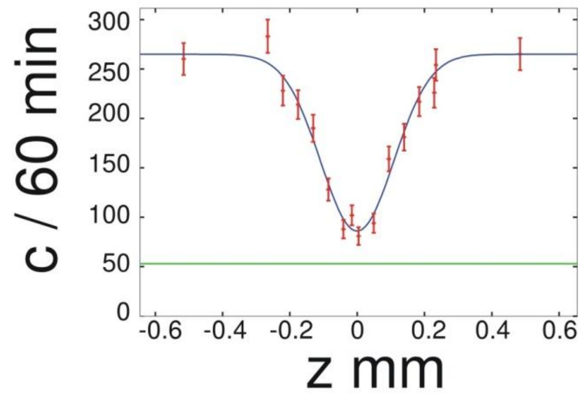
Unlocking of entanglement

If two parties B and D come together in the same laboratory
To perform a joint measurement: Bell measurement



Complete Bell measurement with Linear optics





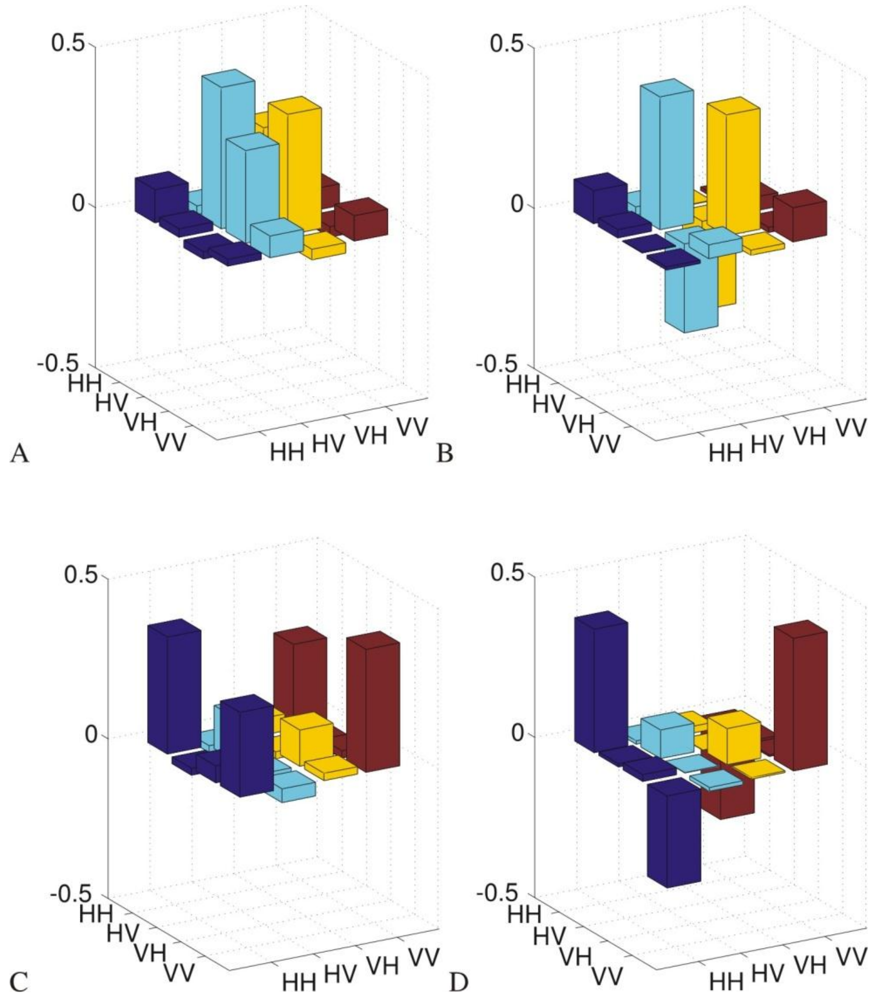
Results: Unlocking of entanglement

$|\Psi^+\rangle$

$F = 0.84 \pm 0.01$

$|\Psi^-\rangle$

$F = 0.82 \pm 0.01$



$|\Phi^+\rangle$

$F = 0.80 \pm 0.01$

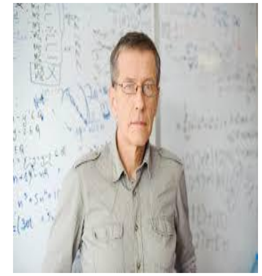
$|\Phi^-\rangle$

$F = 0.83 \pm 0.01$

Conclusion

Six qubit invariant entangled state

four qubit bound entangled state



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