Quantum correlations: Where, how and why

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Quantum Engineering of States and Devices, NORDITA, Stockholm, August 23, 2014

Bell's inequality



$S_{ ext{CHSH}} \stackrel{\scriptscriptstyle ext{\tiny LR}}{\leq} 3$

J. S. Bell, Physics 1, 195 (1964).

Tsirelson's bound



$S_{\mathrm{CHSH}} \stackrel{\scriptscriptstyle\mathrm{LR}}{\leq} 3 \stackrel{\scriptscriptstyle\mathrm{Q}}{\leq} 2 + \sqrt{2}$

B. S. Cirel'son [Tsirelson], Lett. Math. Phys. 4, 93 (1980).

Popescu-Rohrlich non-local non-signalling boxes





$S_{\mathrm{CHSH}} \stackrel{\mathrm{\tiny LR}}{\leq} 3 \stackrel{\mathrm{\tiny Q}}{\leq} 2 + \sqrt{2} \stackrel{\mathrm{\tiny NS}}{\leq} 4$

S. Popescu and D. Rohrlich, Found. Phys. 24, 379 (1994).



 $S_{\text{CHSH}} \stackrel{\text{\tiny LR}}{\leq} 3 \stackrel{\text{\tiny Q}}{\leq} 2 + \sqrt{2} \stackrel{\text{\tiny NS}}{\leq} 4$

Quantum correlations

 "Quantum correlations": Correlations between outcomes of joint measurements of sharp quantum observables (i.e., von Neumann's "quantum observables") predicted by quantum theory.

NC inequalities

- "Non-contextuality (NC) inequality": Bound satisfied by noncontextual hidden variable (NCHV) theories for a linear combination of correlations between the outcomes of compatible observables.
- "NCHV theories": Those in which outcomes are determined and are independent of which other compatible measurements are performed.
- Every Bell inequality is a NC inequality, but most NC inequalities cannot be expressed as Bell inequalities, since they require compatible observables that cannot be decomposed into observables on different subsystems.

Purpose of this talk

- Explain quantum correlations
 - Where? = Which Bell and NC inequalities are violated?
 - **How?** = Which are the limits of the quantum violations?
 - **Why?** = Which principle enforces these limits?

NC inequalities and exclusivity graphs

- Every **NC inequality** can be associated to a **graph**.
- Each vertex represents an event (e.g., "A and B have been measured with results 0 and 1, respectively").
- There is an edge iff the events are mutually exclusive (e.g., "A and B have been measured with results 0 and 1" and "A and B' have been measured with results 1 and 1" are mutually exclusive).

A. Cabello, S. Severini, and A. Winter, arXiv:1010.2163.
A. Cabello, S. Severini, and A. Winter, Phys. Rev. Lett. 112, 040401 (2014).

Exclusivity graph of the CHSH Bell inequality

$$\beta = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \stackrel{\text{\tiny LR}}{\leq} 2$$

 $\pm \langle A_i B_j \rangle = 2[P(1, \pm 1 \mid i, j) + P(-1, \pm 1 \mid i, j)] - 1$

S = P(1, 1 | 0, 0) + P(-1, -1 | 0, 0) + P(1, 1 | 0, 1) + P(-1, -1 | 0, 1) + P(1, 1 | 1, 0) + P(-1, -1 | 1, 0) + P(1, -1 | 1, 1) + P(-1, 1 | 1, 1)

$$S \stackrel{\mathrm{LR}}{\leq} 3$$

 $S = \frac{\beta}{2} + 2$

Exclusivity graph of the CHSH Bell inequality

S = P(1, 1 | 0, 0) + P(-1, -1 | 0, 0) + P(1, 1 | 0, 1) + P(-1, -1 | 0, 1) + P(1, 1 | 1, 0) + P(-1, -1 | 1, 0) + P(1, -1 | 1, 1) + P(-1, 1 | 1, 1)



Exclusivity graphs of the chained Bell inequalities



circulant graphs $Ci_{2n}(1,n)$ $|\langle a_1b_2 \rangle + \langle b_2a_3 \rangle + \dots + \langle a_{2n-1}b_{2n} \rangle - \langle b_{2n}a_1 \rangle| \le 2n-2$

P. M. Pearle, Phys. Rev. D 2, 1418 (1970).
S. L. Braunstein and C. M. Caves, Nucl. Phys. B, Proc. Suppl. 6, 211 (1989).

Exclusivity graph of the Mermin Bell inequality



Exclusivity graphs of the CGLMP Bell inequalities



Chen J L et al 2014 (in preparation)

D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002).

Exclusivity graph of the KCBS NC inequality



A. A. Klyachko, M. A. Can, S. Binicioğlu, and A. S. Shumovsky, Phys. Rev. Lett. **101**, 020403 (2008).

Where?

Quantum correlations. Where?

Which NC inequalities are violated?

Quantum correlations. Where?

 QT violates a given NC inequality *iff* the exclusivity graph contains at least one induced pentagon, or heptagon, or nonagon, etc. (i.e., a "hole") or their complements (i.e., an "antihole")



Example: CHSH Bell inequality



A. Cabello, Phys. Rev. Lett. **110**, 060402 (2013).

How?

Quantum correlations. How?

- **1.** Which is the **non-contextual (local) limit**?
- **2.** Which is the **quantum limit**?

Quantum correlations. How?

- **1.** Which is the **non-contextual (local) limit**?
- **2.** Which is the **quantum limit**?
- Answer to 1: The independence number of the exclusivity graph.
- Answer to 2: The Lovász number of the exclusivity graph.

A. Cabello, S. Severini, and A. Winter, arXiv:1010.2163.A. Cabello, S. Severini, and A. Winter, Phys. Rev. Lett. 112, 040401 (2014).

Basic NC inequalities (1)



$$\sum_{i=1}^{n} P(1,0|i,i+\lfloor\frac{n}{2}\rfloor) \stackrel{\text{NCHV}}{\leq} \frac{n-1}{2} \stackrel{\text{Q}}{\leq} \frac{n\cos\left(\frac{\pi}{n}\right)}{1+\cos\left(\frac{\pi}{n}\right)}$$

Y.-C. Liang, R. W. Spekkens, and H. M. Wiseman, Phys. Rep. **506**, 1 (2011).

A. Cabello, S. Severini, and A. Winter, arXiv:1010.2163.

A. Cabello, S. Severini, and A. Winter, Phys. Rev. Lett. **112**, 040401 (2014).

Basic NC inequalities (2)



$$\sum_{i=1}^{n} P(1,0,\ldots,0|i,i+2,\ldots,i+n-3) \stackrel{\text{NCHV}}{\leq} 2 \stackrel{\text{\tiny Q}}{\leq} \frac{1+\cos\left(\frac{\pi}{n}\right)}{\cos\left(\frac{\pi}{n}\right)}$$

A. Cabello, L. E. Danielsen, A. J. López-Tarrida, and J. R. Portillo, Phys. Rev. A 88, 032104 (2013).

CHSH Bell inequality



 $S \stackrel{\text{\tiny LR}}{\leq} 3 \stackrel{\text{\tiny Q}}{\leq} 2 + \sqrt{2}$



 $S_{\mathrm{CHSH}} \stackrel{\mathrm{\tiny LR}}{\leq} 3 \stackrel{\mathrm{\tiny Q}}{\leq} 2 + \sqrt{2}$ ≤ 4



$S_{\rm KCBS} \stackrel{\rm NCHV}{\leq} 2 \stackrel{\rm Q}{\leq} \sqrt{5} \stackrel{\rm GPT}{\leq} \frac{5}{2}$

Forgot all you know about QT



Let's try to understand QT



 Let's try to understand the limits of quantum correlations without knowing QT



• Maximize S=P(1,0|1,2)+P(1,0|2,3)+P(1,0|3,1)





• Maximize S=P(1,0|1,2)+P(1,0|2,3)+P(1,0|3,1)



 Classical state: 3 boxes, one ball in one of them (the other 2 boxes empty). S=1.



• Maximize S=P(1,0|1,2)+P(1,0|2,3)+P(1,0|3,1)



- Classical state: 3 boxes, one ball in one of them (the other 2 boxes empty). S=1.
- Specker's triangle state: Each of the probabilities is $\frac{1}{2}$, S=3/2.

Classical and Specker triangle states



Classical state: S=1

Specker triangle state: S=3/2

E. P. Specker, Dialectica 14, 239 (1960). English translation: arXiv:1103.4537.

Where is quantum theory?



Where is quantum theory?

Not in the classical world (we know that).

• Not in Specker's world.
Where is quantum theory?

Not in the classical world (we know that).

• Not in Specker's world.

Observation: In Specker's world the E principle is not satisfied.

The E principle

- A theory satisfies the E principle when any set of <u>pairwise</u> mutually exclusive events is jointly (i.e., n-wise mutually) exclusive.
- Therefore, from Kolmogorov's axioms of probability, the sum of the probabilities of any set of pairwise mutually exclusive events cannot be higher than 1.
- Important: The E principle cannot be derived from Kolmogorov's axioms of probability.

Classical (Specker) triangle states satisfy (do not) E



Classical state: S=1

Specker triangle state: S=3/2

Where is quantum theory?



5-box game

• Maximize S=P(1,0|1,2)+P(1,0|2,3)+P(1,0|3,4)+P(1,0|4,5)+P(1,0|5,1)



5-box game

• Maximize S=P(1,0|1,2)+P(1,0|2,3)+P(1,0|3,4)+P(1,0|4,5)+P(1,0|5,1)



- Classical state: 5 boxes, two balls in non adjacent boxes (the other 3 boxes empty). S=2.
- Wright's pentagon state: Each of the probabilities is ¹/₂, S=5/2.

R. Wright, in *Mathematical Foundations of Quantum Mechanics*, edited by A. R. Marlow (Academic, San Diego, 1978), p. 255.

Classical and Wright's pentagon states





Classical state: S=2

Wright's pentagon state: S=5/2

Classical, quantum (KCBS's) and Wright's states



The KCBS NC inequality

$S_{\text{KCBS}} \stackrel{\text{NCHV}}{\leq} 2 \stackrel{\text{Q}}{\leq} \sqrt{5} \stackrel{\text{GPT}}{\leq} \frac{5}{2}$

A. A. Klyachko, M. A. Can, S. Binicioğlu, and A. S. Shumovsky, Phys. Rev. Lett. **101**, 020403 (2008).



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Vienna-Stockholm events



Joint probability of two independent events: $P(a, b, c, d|i_V, i+1_V, j_S, j+1_S) = P(a, b|i_V, i+1_V)P(c, d|j_S, j+1_S)$

 $P(10|12)P(10|12) + P(10|23)P(10|45) + P(10|34)P(10|23) + P(10|45)P(10|51) + P(10|51)P(10|34) \stackrel{\mathrm{e}}{\leq} 1$



 $P(10|12)P(10|34) + P(10|23)P(10|12) + P(10|34)P(10|45) + P(10|45)P(10|23) + P(10|51)P(10|51) \stackrel{\mathrm{E}}{\leq} 10^{-10} + P(10|12)P(10|12) + P(10|12)P(10|12)P(10|12) + P(10|12)P(10|12) + P(10|12)P(10|12)P(10|12) + P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|1$



 $P(10|12)P(10|51) + P(10|23)P(10|34) + P(10|34)P(10|12) + P(10|45)P(10|45) + P(10|51)P(10|23) \stackrel{\mathrm{E}}{\leq} 1$



E inequality #4



 $P(10|12)P(10|23) + P(10|23)P(10|51) + P(10|34)P(10|34) + P(10|45)P(10|12) + P(10|51)P(10|45) \stackrel{\mathrm{E}}{\leq} 1 + P(10|12)P(10|12) + P(10|12)P(10|12)P(10|12) + P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(10|12)P(1$

 $P(10|12)P(10|45) + P(10|23)P(10|23) + P(10|34)P(10|51) + P(10|45)P(10|34) + P(10|51)P(10|12) \stackrel{\mathrm{E}}{\leq} 1$



$$\begin{array}{l} P(10|12)P(10|51) + P(10|23)P(10|34) + P(10|34)P(10|12) + P(10|45)P(10|45) + P(10|51)P(10|23) \stackrel{\text{E}}{\leq} 1\\ P(10|12)P(10|23) + P(10|23)P(10|51) + P(10|34)P(10|34) + P(10|51)P(10|12) \stackrel{\text{E}}{\leq} 1\\ P(10|12)P(10|45) + P(10|23)P(10|23) + P(10|34)P(10|51) + P(10|45)P(10|34) + P(10|51)P(10|12) \stackrel{\text{E}}{\leq} 1\\ S^2 \stackrel{\text{E}}{\leq} 5\\ S \stackrel{\text{KCBS}}{\leq} S \stackrel{\text{NCHV}}{\leq} 2 \stackrel{\text{Q}}{\leq} \sqrt{5}\\ \text{A. Cabello, Phys. Rev. Lett. 110, 060402 (2013)} \end{array}$$

 $P(10|12)P(10|34) + P(10|23)P(10|12) + P(10|34)P(10|45) + P(10|45)P(10|23) + P(10|51)P(10|51) \stackrel{\mathrm{E}}{\leq} 1$

 $P(10|12)P(10|12) + P(10|23)P(10|45) + P(10|34)P(10|23) + P(10|45)P(10|51) + P(10|51)P(10|34) \stackrel{\mathrm{e}}{\leq} 1$

Summing the 5 E inequalities

Result 2: If G is a self-complementary graph, the E principle, without any further assumptions, excludes any set of probability distributions strictly larger than the quantum set.

B. Amaral, M. Terra Cunha, and A. Cabello, Phys. Rev. A **89**, 030101(R) (2014).

A. B. Sainz, T. Fritz, R. Augusiak, J. Bohr Brask, R. Chaves, A. Leverrier, and A. Acín, Phys. Rev. A **89**, 032117 (2014).

Quantum physicists obsessed by Tsirelson's bound



Why?



$$\begin{split} S &= P(1,1 \mid 0,0) + P(-1,-1 \mid 0,0) + P(1,1 \mid 0,1) + P(-1,-1 \mid 0,1) \\ &+ P(1,1 \mid 1,0) + P(-1,-1 \mid 1,0) + P(1,-1 \mid 1,1) + P(-1,1 \mid 1,1) \end{split}$$

Why Tsirelson's bound?



Let's assume that each of the 8 events has prob. p



Consider a second copy





Then



Now, in addition to the Bell-inequality measurements





One can measure





Alternatively



So one also has other type of events



Consider the following 9 pairwise exclusive events

Event	Probability
$e_1 = (A_0 +, B_0 +, A'_0 +, B'_1 +, A_0 A'_0 +)$	p^2
$e_2 = (A_0 -, B_0 -, A'_0 -, B'_1 -, A_0 A'_0 +)$	p^2
$e_3 = (A_0 +, B_0 -, A'_0 +, B'_1 -, A_0 A'_0 +)$	$\left(\frac{1}{2}-p\right)^2$
$e_4 = (A_0 -, B_0 +, A'_0 -, B'_1 +, A_0 A'_0 +)$	$\left(\frac{1}{2}-p\right)^2$
$e_5 = (A_1 +, B_0 +, A_1' +, B_1' -, A_1 A_1' +)$	p^2
$e_6 = (A_1 -, B_0 -, A_1' -, B_1' +, A_1 A_1' +)$	p^2
$e_7 = (A_1 +, B_0 -, A_1' +, B_1' +, A_1 A_1' +)$	$\left(\frac{1}{2}-p\right)^2$
$e_8 = (A_1 -, B_0 +, A_1' -, B_1' -, A_1 A_1' +)$	$\left(\frac{1}{2}-p\right)^2$
$e_9 = (A_0 A_0' -, A_1 A_1' -)$	$P(e_9)$

9 $\sum P(e_i) \stackrel{\mathrm{E}}{\leq} 1$ i=1

Similarly

Event	Probability	Event	Probability
$e_1 = (A_0 +, B_0 +, A'_0 +, B'_1 +, A_0 A'_0 +)$	p^2	$f_1 = (A_0 +, B_0 +, A'_0 +, B'_0 +, A_0 A'_0 +)$	p^2
$e_2 = (A_0 -, B_0 -, A'_0 -, B'_1 -, A_0 A'_0 +)$	p^2	$f_2 = (A_0 -, B_0 -, A'_0 -, B'_0 -, A_0 A'_0 +)$	p^2
$e_3 = (A_0 +, B_0 -, A'_0 +, B'_1 -, A_0 A'_0 +)$	$\left(\frac{1}{2}-p\right)^2$	$f_3 = (A_0 +, B_0 -, A'_0 +, B'_0 -, A_0 A'_0 +)$	$\left(\frac{1}{2}-p\right)^2$
$e_4 = (A_0 -, B_0 +, A'_0 -, B'_1 +, A_0 A'_0 +)$	$\left(\frac{1}{2}-p\right)^2$	$f_4 = (A_0 -, B_0 +, A'_0 -, B'_0 +, A_0 A'_0 +)$	$\left(\frac{1}{2}-p\right)^2$
$e_5 = (A_1 +, B_0 +, A_1' +, B_1' -, A_1 A_1' +)$	p^2	$f_5 = (A_1 +, B_0 +, A'_1 -, B'_0 -, A_1 A'_1 -)$	p^2
$e_6 = (A_1 -, B_0 -, A_1' -, B_1' +, A_1 A_1' +)$	p^2	$f_6 = (A_1 -, B_0 -, A'_1 +, B'_0 +, A_1 A'_1 -)$	p^2
$e_7 = (A_1 +, B_0 -, A_1' +, B_1' +, A_1 A_1' +)$	$\left(\frac{1}{2}-p\right)^2$	$f_7 = (A_1 +, B_0 -, A_1' -, B_0' +, A_1 A_1' -)$	$\left(\frac{1}{2}-p\right)^2$
$e_8 = (A_1 -, B_0 +, A_1' -, B_1' -, A_1 A_1' +)$	$\left(\frac{1}{2}-p\right)^2$	$f_8 = (A_1 -, B_0 +, A'_1 +, B'_0 -, A_1 A'_1 -)$	$\left(\frac{1}{2}-p\right)^2$
$e_9 = (A_0 A_0' - , A_1 A_1' -)$	$P(e_9)$	$f_9 = (A_0 A_0' -, A_1 A_1' +)$	$\frac{1}{2} - P(e_9)$

 $\sum^{9} P(e_i) \stackrel{\mathrm{E}}{\leq} 1$ i=1

9 $\sum^{g} P(f_i) \stackrel{\mathrm{E}}{\leq} 1$ i=1
Summing the 2 E inequalities

9 $\sum P(e_i) + P(f_i) \stackrel{\mathrm{E}}{\leq} 2$ i=1

$$p \stackrel{\mathrm{E}}{\leq} \frac{2 + \sqrt{2}}{8}$$

$$S \stackrel{\mathrm{E}}{\leq} 2 + \sqrt{2}$$

Assuming arbitrary probabilities for the 8 vertices

There are 16 tables. Summing them all, we get

$$S^{2} + (4 - S)^{2} + 4 \stackrel{\text{E}}{\leq} 16$$

 $S \stackrel{\text{E}}{\leq} 2 + \sqrt{2}$

A. Cabello, arXiv:1406.5656.

Principles (2009)

	Information causality	Macroscopic locality	Jocal orthogonality	Е
Q maximum of KCBS	No	No	No	Yes (the whole Q set!)
Q maximum of the basic				
NC inequalities	No	No	No	"Strong evidence" that yes
Q maximum of Bell CHSH	Yes	Yes	?	Yes
Lack of extremal				
tripartite correlations	No	No	Yes	Yes
Lack of irreducible				
third-order interference implies	?	?	Yes	Yes

M. Pawłowski, T. Paterek, D. Kaszlikowski, V. Scarani, A. Winter, and M. Żukowski, Nature (London) **461**, 1101 (2009). M. Navascués and H. Wunderlich, Proc. Royal Soc. A **466**, 881 (2009).

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T. Fritz, A. B. Sainz, R. Augusiak, J. Bohr Brask, R. Chaves, A. Leverrier, and A. Acín, Nat. Comm. 4, 2263 (2013).A. Cabello, Phys. Rev. Lett. 110, 060402 (2013).

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A. Cabello, L. E. Danielsen, A. J. López-Tarrida, and J. R. Portillo, Phys. Rev. A 88, 032104 (2013).

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R. Gallego, L. E. Würflinger, A. Acín, and N. Navascués, Phys.
Rev. Lett. 107, 210403 (2011).
T. H. Yang, D. Cavalcanti, M. L. Almeida, C. Teo, and V. Scarani, New J. Phys. 14, 013061 (2012).

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J. Henson, arXiv:1406.3281.

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Lack of irreducible				\backslash /
third-order interference implies	?	?	Yes	Yes

Conclusions and open question

- The E principle explains more about quantum correlations than any other proposed principle.
- The E principle is not an "information principle", but a "logic principle".
- Is the E principle "the" principle of quantum correlations and therefore a key principle of QT?