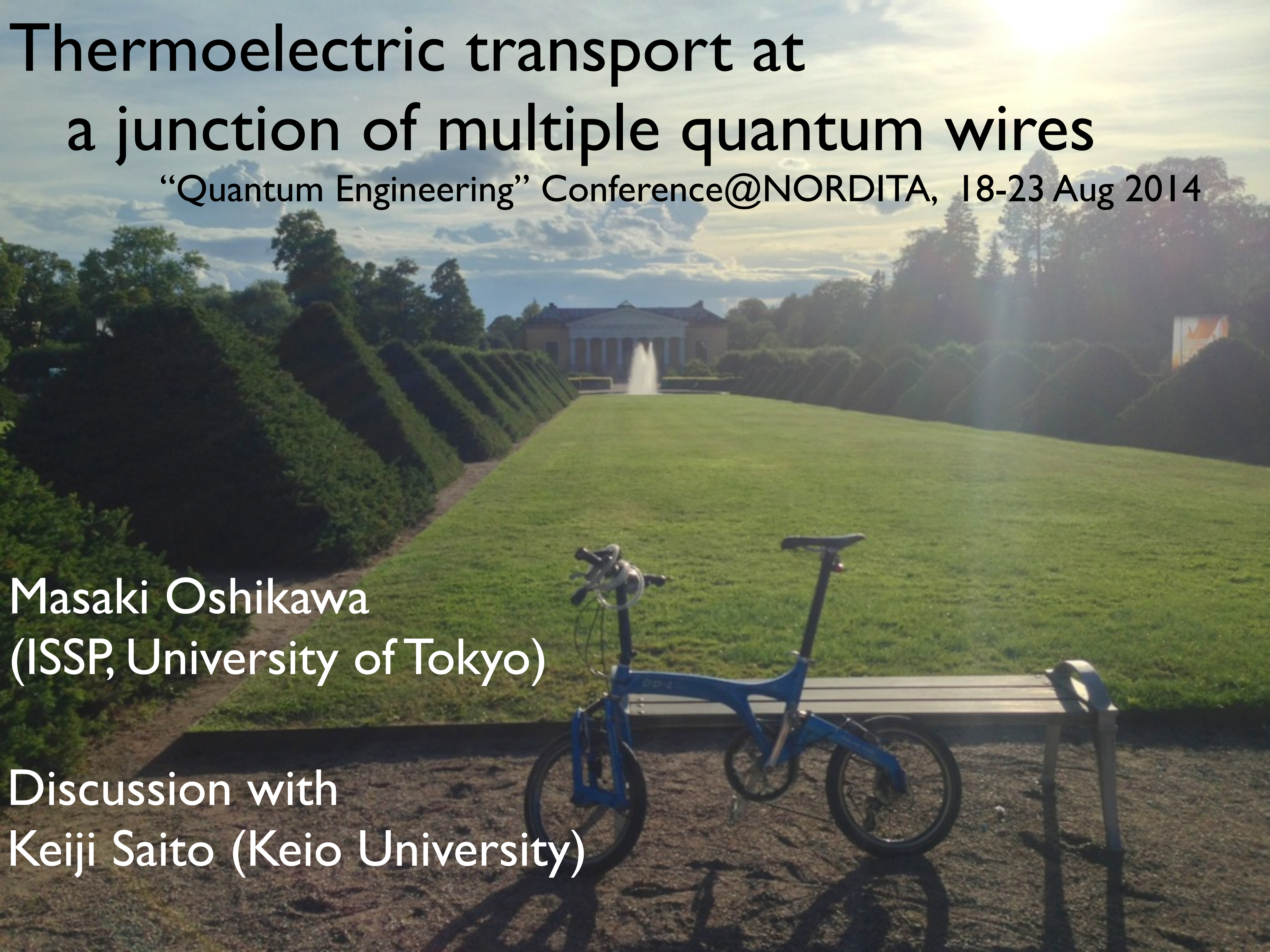
Thermoelectric transport at a junction of multiple quantum wires

"Quantum Engineering" Conference@NORDITA, 18-23 Aug 2014



Masaki Oshikawa
(ISSP, University of Tokyo)

Discussion with
Keiji Saito (Keio University)

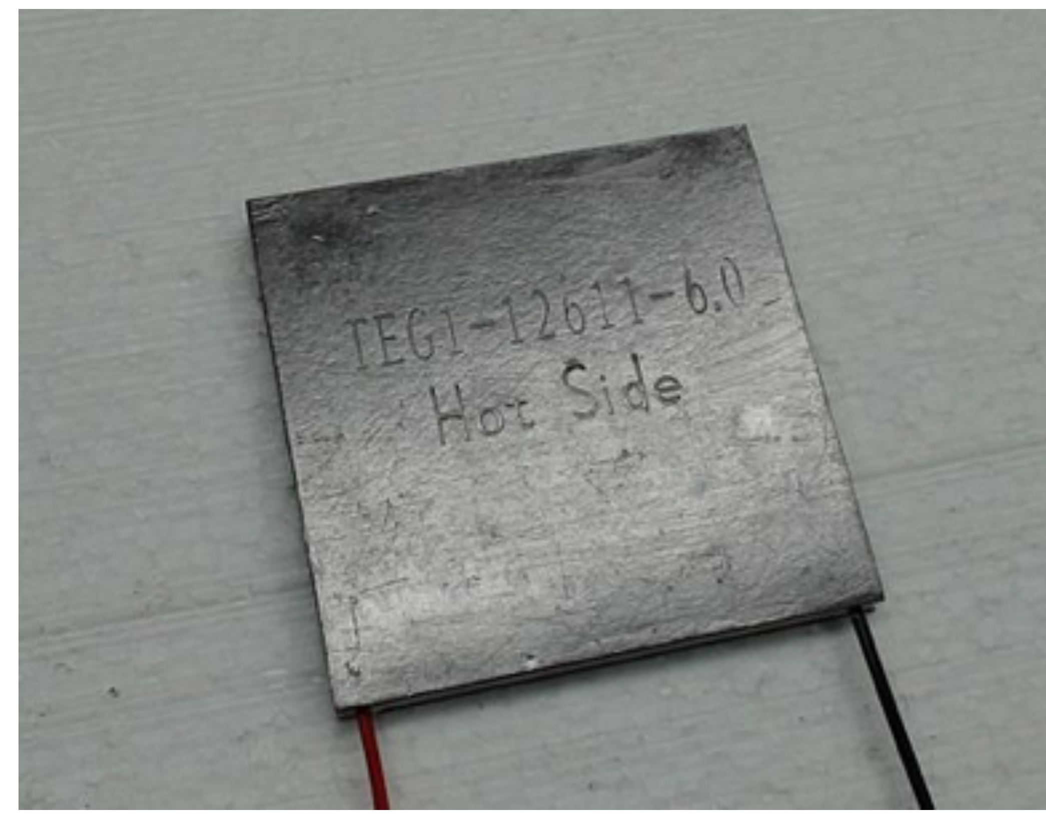
Motivation

Thermoelectric effects

e.g. Seebeck effect:

Temperature difference → electric power

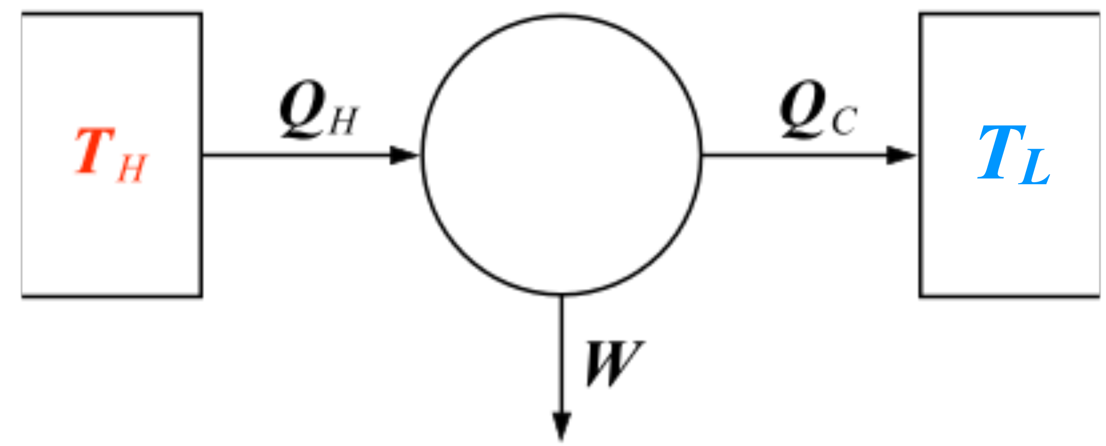
potentially very useful



Actually applications already exist, but still limited (low efficiency)

Thermoelectric Efficiency

Efficiency $\eta = \dfrac{W}{Q_H}$



e.g. Linear Response Regime

$$\eta_{\max} = \eta_C \frac{\sqrt{ZT+1}-1}{\sqrt{ZT+1}+1}$$

$$\eta_C = 1 - \frac{T_L}{T_H}$$

Carnot efficiency

"figure of merit"

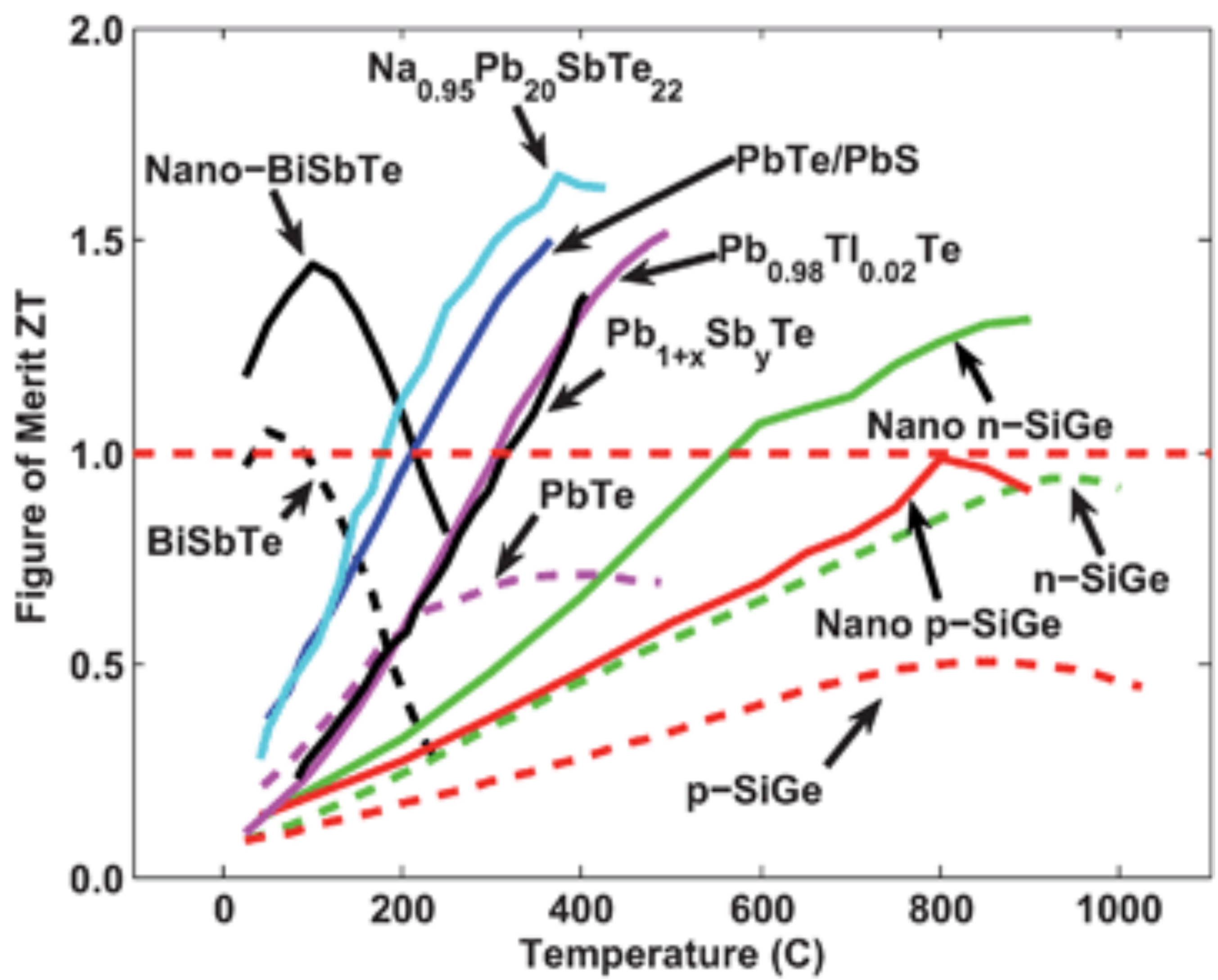
$$ZT = \frac{\sigma S^2}{\kappa} T,$$

σ: electric conductivity

S: Seebeck coefficient

κ: thermal conductivity

current state-of-the-art thermoelectric materials



Minnich et al. *Energy Environ. Sci.* 2009

Is the "efficiency" practical?

$\eta \leq \eta_C$ thermodynamic constraint

The "maximum efficiency" is usually achieved in the quasi-static limit (infinitesimal power output)

In the applications, we want a higher power output…

Discuss the efficiency at the maximum power

"Curzon-Ahlborn upper bound"

$$\eta(\omega_{\max}) \leq \eta_{\mathrm{CA}} = 1 - \sqrt{\frac{T_L}{T_H}} \sim \frac{\eta_C}{2}$$

PRL **106,** 230602 (2011) PHYSICAL REVIEW LETTERS week ending 10 JUNE 2011

Thermodynamic Bounds on Efficiency for Systems with Broken Time-Reversal Symmetry

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- Curzon-Ahlborn bound is based on Onsager reciprocal relations
- Curzon-Ahlborn bound can be overcome by breaking time-reversal (magnetic field etc.)
- Efficiency at maximum power depends on "figure of merit" and "asymmetry"

Selected for a Viewpoint in *Physics*
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Strong Bounds on Onsager Coefficients and Efficiency for Three-Terminal Thermoelectric Transport in a Magnetic Field

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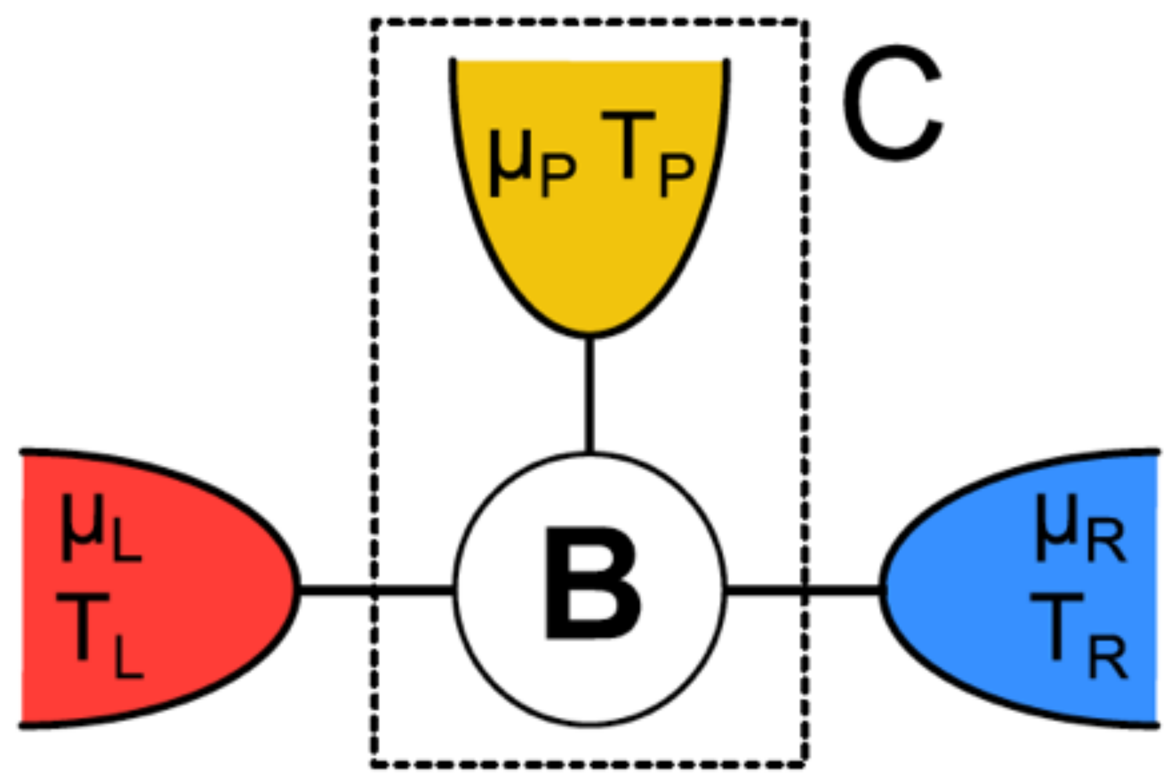
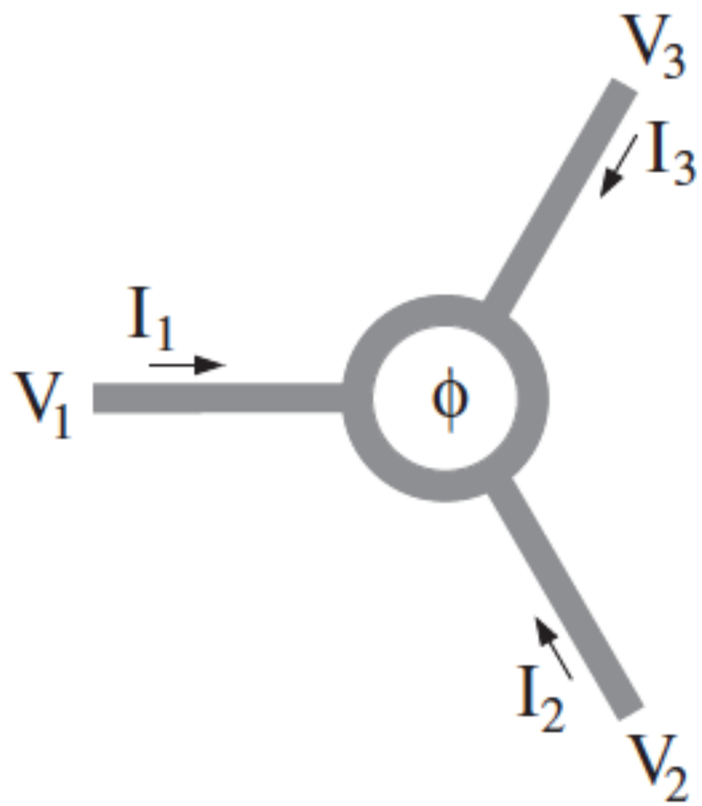
Construction of an example with a maximum power exceeding the Curzon-Ahlborn bound

Three-terminal device of free electrons,
with a magnetic field
(one terminal: "probe")



With Interactions?



Three-terminal junction
of Tomonaga-Luttinger Liquid,
with magnetic flux?

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Junctions of Three Quantum Wires and the Dissipative Hofstadter Model

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Junctions of three quantum wires

Masaki Oshikawa[1], Claudio Chamon[2] and Ian Affleck[3]

The formulation developed in these papers can be applied to thermal conduction/thermoelectric effect.
At this point, I only have results on thermal conductance, which I will report in this talk.

Tomonaga-Luttinger Liquid

Spinless fermions → 1-component free boson field theory

$$\mathcal{L} = \frac{g}{4\pi}(\partial_\mu \varphi)^2 = \frac{1}{4\pi g}(\partial_\mu \theta)^2$$

g : Luttinger parameter

(repulsive→ $g<1$, free→ $g=1$, attractive→ $g>1$)

Right/Left-mover currents

$$J^R = \sqrt{g}\partial_z \varphi \qquad z = i\tau + x$$

$$J^L = \sqrt{g}\partial_{\bar{z}} \varphi \qquad \bar{z} = -i\tau + x$$

Thermal conduction of TLLs well-studied, but I give a reformulation based on boundary CFT to study junctions

Currents

Particle (charge) density: $\rho_c \propto J_R + J_L$

Particle (charge) current: $J_c \propto J_R - J_L$

Energy density: $\mathcal{E} = \frac{g}{2\pi}\left[(\partial_t\varphi)^2 + (\partial_x\varphi)^2\right] = \frac{1}{2\pi}\left[(J^R)^2 + (J^L)^2\right]$

Energy current: $J_{\mathcal{E}} = \frac{g}{2\pi}(\partial_t\varphi)(\partial_x\varphi) = \frac{1}{2\pi}\left[(J^R)^2 - (J^L)^2\right]$

Discuss the thermal conduction in linear response theory, in analogy to current conduction (a la Wong-Affleck etc.)

Setup

Nakano-Kubo 1953
Fisher-Lee 1982

- Equilibrium (with temperature T) at $t = 0$
- Turn on an (infinitesimal) voltage/temperature difference over a section of length L
- Measure current at one point, after infinite time
- Result independent of L and the point of measurement
- May not give what is measured in experiments (yields a "wrong" result ge^2/h for conductance)

The unrenormalized conductance e^2/h, independent of g, was observed in experiments: can be understood theoretically by attaching Fermi liquid leads. Maslov-Stone 1995, Safi-Schulz 1995
Applied to Y-junction in Chamon-MO-Affleck 2003, 2006 but I will not discuss the leads in the present talk.

Kubo formula

Let us verify first if the present approach reproduces the known results for the bulk (single TLL)

Conductance

$$
\begin{aligned}
G &= -\frac{e^2}{h}\lim_{\bar{\omega}\to+0}\frac{g}{\pi\bar{\omega}L}\int d\tau\, e^{i\bar{\omega}\tau}\int_{x_0}^{x_0+L}\langle\left[J^R(y,\tau)-J^L(y,\tau)\right]\left[J^R(x,0)-J^L(x,0)\right]\rangle \\
&= g\frac{e^2}{h}
\end{aligned}
$$

Kane-Fisher 1991

Thermal conductance

$$
\begin{aligned}
\kappa &= -\lim_{\bar{\omega}\to+0}\frac{1}{\bar{\omega}LT}\int d\tau\, e^{i\bar{\omega}\tau}\int_{x_0}^{x_0+L}dx\,\frac{1}{4\pi^2}\langle\left[(J^R)^2-(J^L)^2\right](y,\tau)\left[(J^R)^2-(J^L)^2\right](x,0)\rangle \\
&= \frac{\pi T}{6}
\end{aligned}
$$

Kane-Fisher 1996

Half-Infinite Wire

Low-energy limit ↔ conformally invariant boundary conditio

TLL: “Neumann” or “Dirichlet”

$$J^L = J^R \qquad\qquad J^L = -J^R$$

“unfold” the system to infinite line w/o boundary, by

$$J^R(-x,\tau) \equiv \pm J^L(x,\tau)$$

“cross term” between original J^R and J^L contribute

Neumann: $G = \kappa = 0$ cf.) $J_{\mathcal{E}} \propto (J^R)^2 - (J^L)^2$

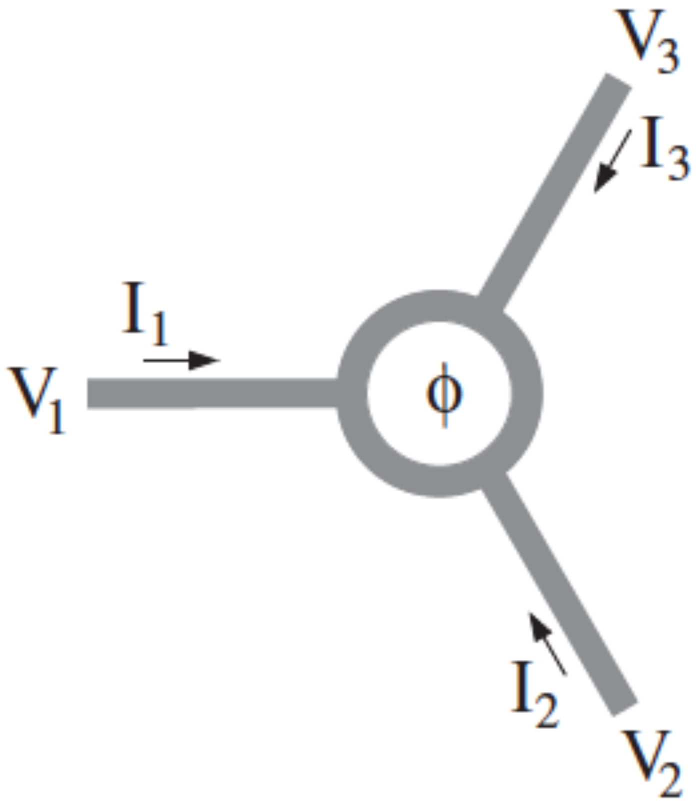
Dirichlet: $G = 2g\dfrac{e^2}{h}$ (doubled) $\kappa = 0$

What do they mean?

Neumann: $J^L = J^R$ current conserved at the boundary
"open end" → no conduction

Dirichlet: $J^L = -J^R$ current NOT conserved at the boundary
represents the TLL attached to a gapped superconductor

Andreev reflection → conductance is doubled
but thermal conduction vanishes



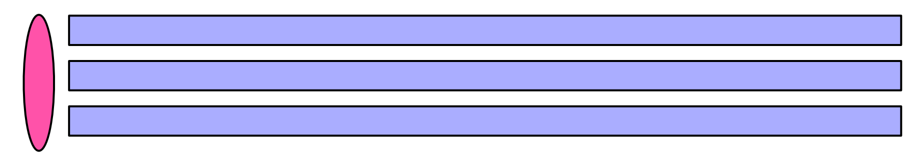
Y-junction

$$I_j = \sum_k G_{jk} V_k$$

Define thermal conductance tensor

κ_{jk} in a similar manner

Low-energy limit of the junction↔



conformally invariant b.c. of 3-component boson

Disconnected fixed point: Neumann for each component

$$J_i^R = J_i^L \quad \Longrightarrow \quad G_{ij} = \kappa_{ij} = 0$$

Other fixed points

New basis for the boson fields

$$\Phi_0 = \frac{1}{\sqrt{3}}(\varphi_1 + \varphi_2 + \varphi_3)$$
$$\Phi_1 = \frac{1}{\sqrt{2}}(\varphi_1 - \varphi_2)$$
$$\Phi_2 = \frac{1}{\sqrt{6}}(\varphi_1 + \varphi_2 - 2\varphi_3)$$

Current conservation at the junction
(no reservoir/superconductor attachment)
→ Φ_0 always obeys Neumann,
but there is a freedom in choosing the b.c. for the other two components Φ_1, Φ_2

D_P/D_N fixed points

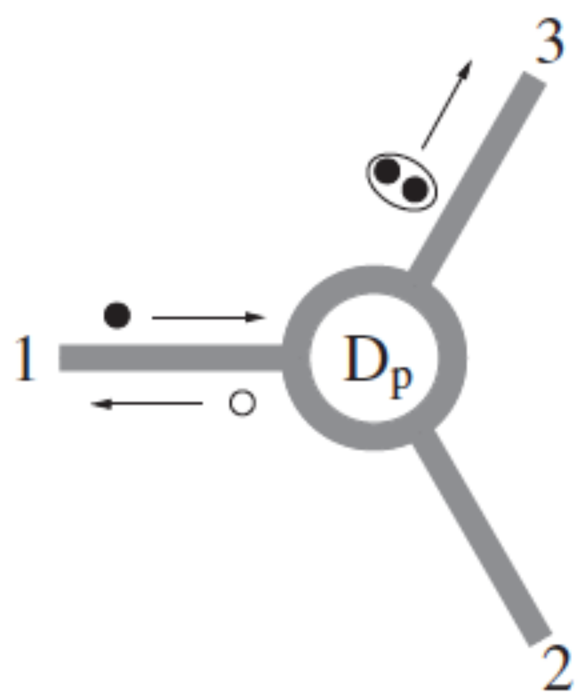
Φ_0: Neumann
Φ_1, Φ_2: Dirichlet

Stable for $g>3$ (strongly attractive)

$$G_{11} = \frac{4g}{3}\frac{e^2}{h} \qquad G_{12} = G_{13} = -\frac{2g}{3}\frac{e^2}{h}$$

MO-Chamon-Affleck 2006

Conductance enhanced by (partial) Andreev reflection

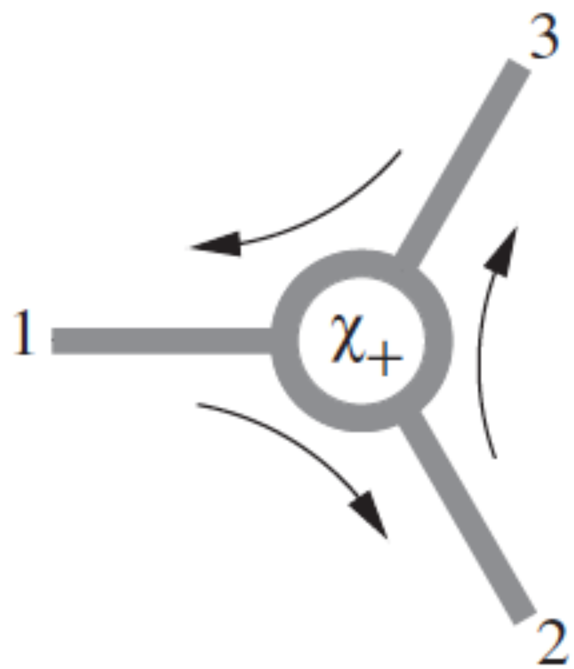


$$\kappa_{11} = \frac{8}{9}\frac{\pi T}{6} \qquad \kappa_{12} = \kappa_{13} = -\frac{4}{9}\frac{\pi T}{6}$$

NEW!

Thermal conductance suppressed by partial Andreev reflection!

Chiral Fixed Points



T-reversal broken (by magnetic flux)

$$G_{11} = \frac{4g}{3+g^2}\frac{e^2}{h}$$

$$G_{12} = \frac{4g(1+g)}{3+g^2}\frac{e^2}{h} \qquad G_{13} = \frac{4g(1-g)}{3+g^2}\frac{e^2}{h}$$

Chamon-MO-Affleck 2003, 2006

Boundary condition in the new basis:

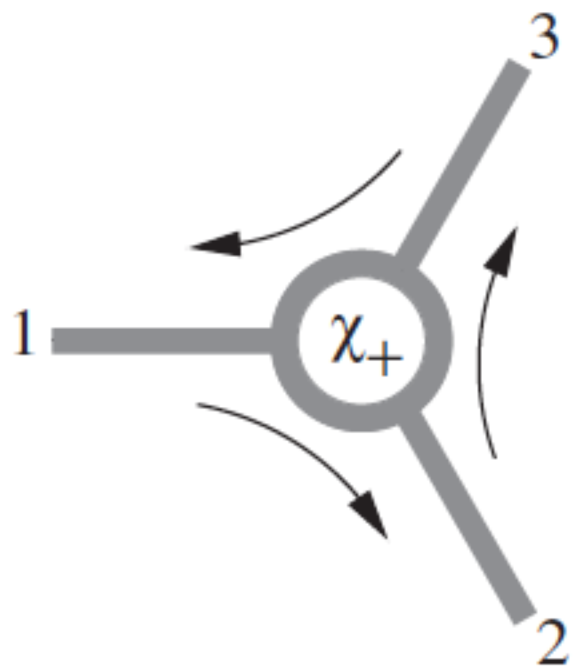
$$J_0^L = J_0^R$$

$$\begin{pmatrix} J_1^L \\ J_2^L \end{pmatrix} = \mathcal{R}_\xi \begin{pmatrix} J_1^R \\ J_2^R \end{pmatrix}$$

$\mathcal{R}_\xi$ rotation matrix with angle ξ

$$\tan\frac{\xi}{2} = \frac{\sqrt{3}}{g}.$$

Chiral Fixed Points



Thermal conductance

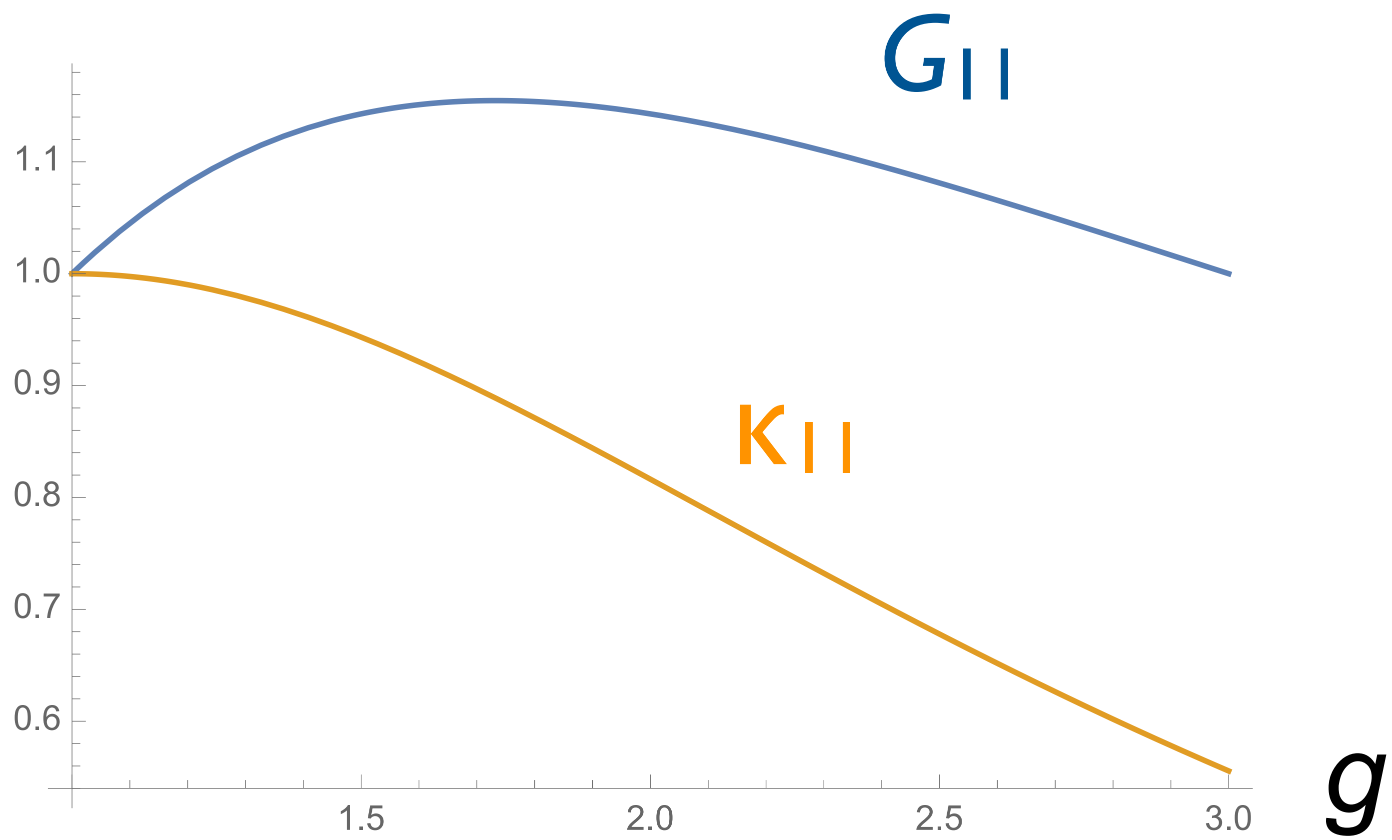
$$\kappa_{11} = \frac{8(1+g^2)}{(3+g^2)^2}$$

$$\kappa_{12} = 4\left(\frac{g+1}{(3+g^2)}\right)^2 \quad \kappa_{13} = 4\left(\frac{g-1}{(3+g^2)}\right)^2$$

NEW!

Asymmetry appears also in the thermal conductance

g-dependence



Diagonal conductance: non-monotonic dependence, enhanced by attractive interaction (but dependence disappears by attaching Fermi-liquid leads)
Diagonal thermal conductance: monotonic dependence, suppressed by attractive interaction

Summary

- Reformulation of linear response theory of thermal conductance in TLLs, in terms of currents and boundary conditions
- Although the "naive" thermal conductance is independent of Luttinger parameter g in the bulk, it can generally depend on g for at junctions
- Andreev reflection enhances conductance but suppresses thermal conductance
- Chiral fixed point of Y-junction exhibits an asymmetry also in thermal conductance; thermal conductance suppressed by the interaction

Open problems

Thermopower: work in progress

Efficiency (at maximum power and in other cases)

… and many others