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Flat bands and long range Coulomb interactions: conducting or insulating ?

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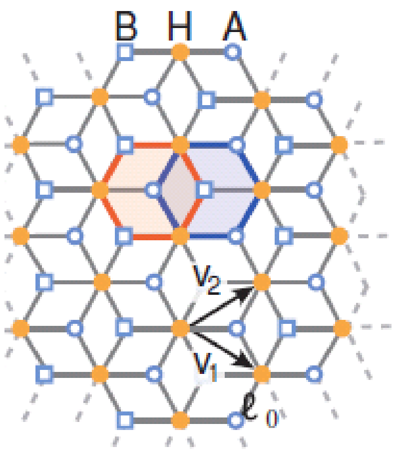
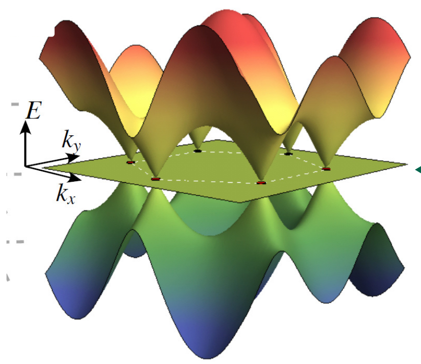
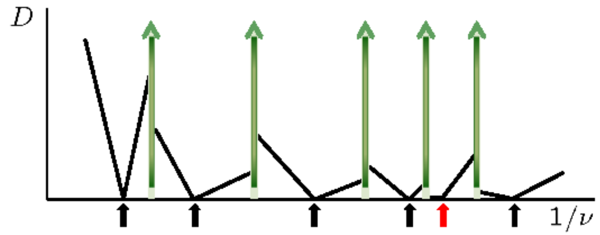
τ_3 -lattice



k - dispersion



flat band conductivity



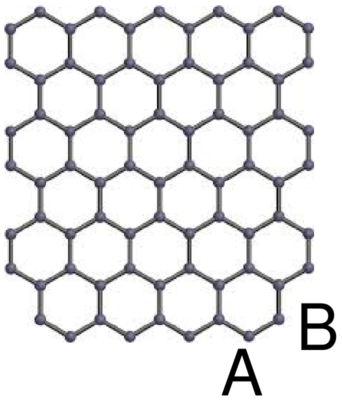
discussions with Dario Bercioux (Freiburg, Berlin)
Daniel Urban (Freiburg)
Reinhold Egger (Düsseldorf)

flat !

arXiv: 1407.6286

Pseudo-spin description near Dirac points

graphene



2 sublattices A , B

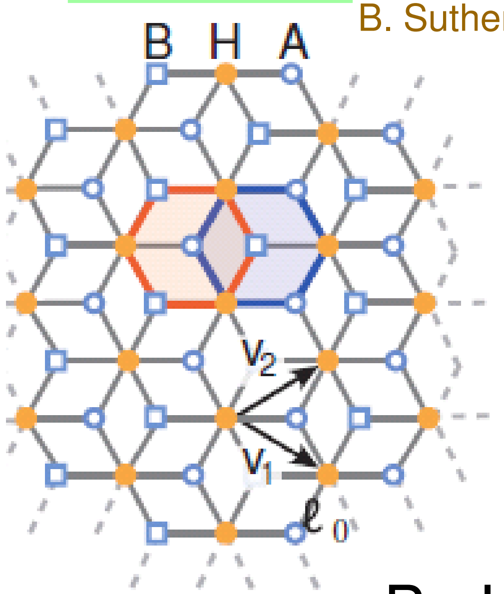
$$H_0 = v_F \vec{S} \cdot \vec{p}$$

pseudo-spin-½

Pauli matrices

T_3 -lattice

B. Sutherland (1986)



3 sublattices B □ , H ● , A ○

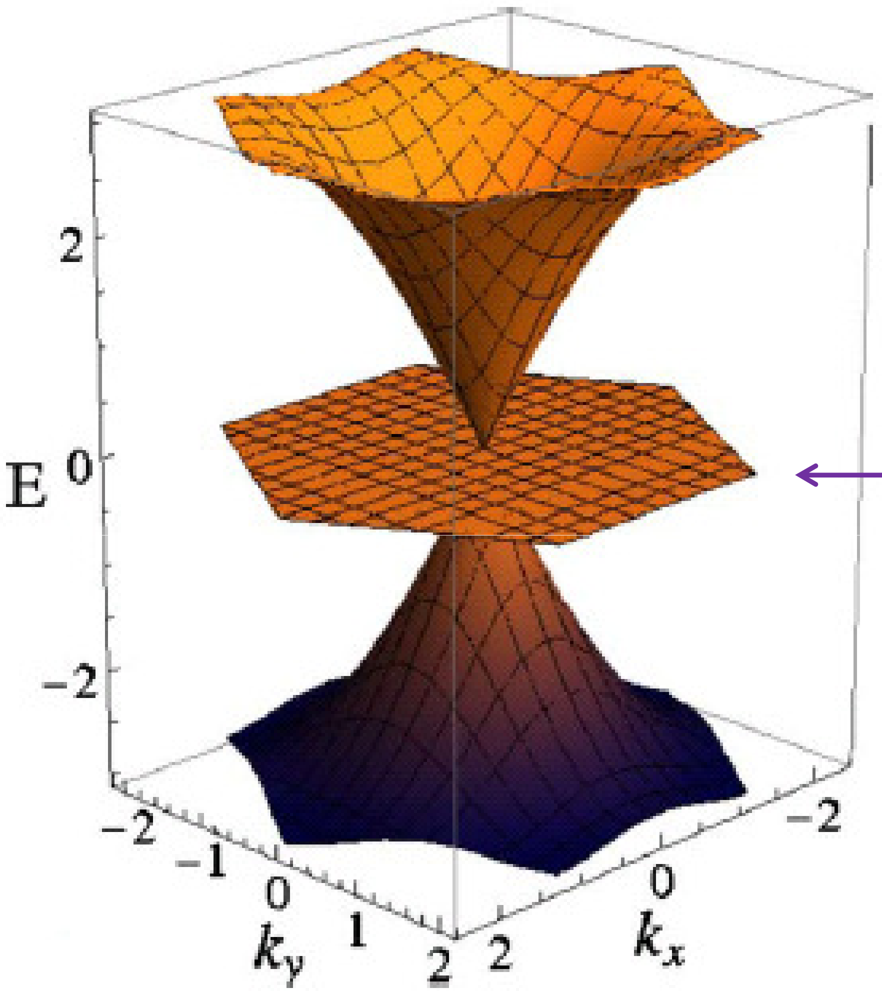
$$H_0 = v_F \vec{S} \cdot \vec{p}$$

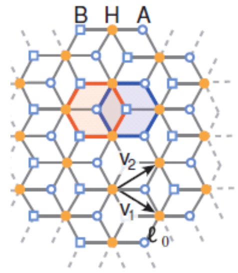
pseudo-spin-1

D. Bercioux, D. F. Urban, H. Grabert, WH (2009)

D.F. Urban, D. Bercioux, M. Wimmer, WH (2011)

3 energy bands

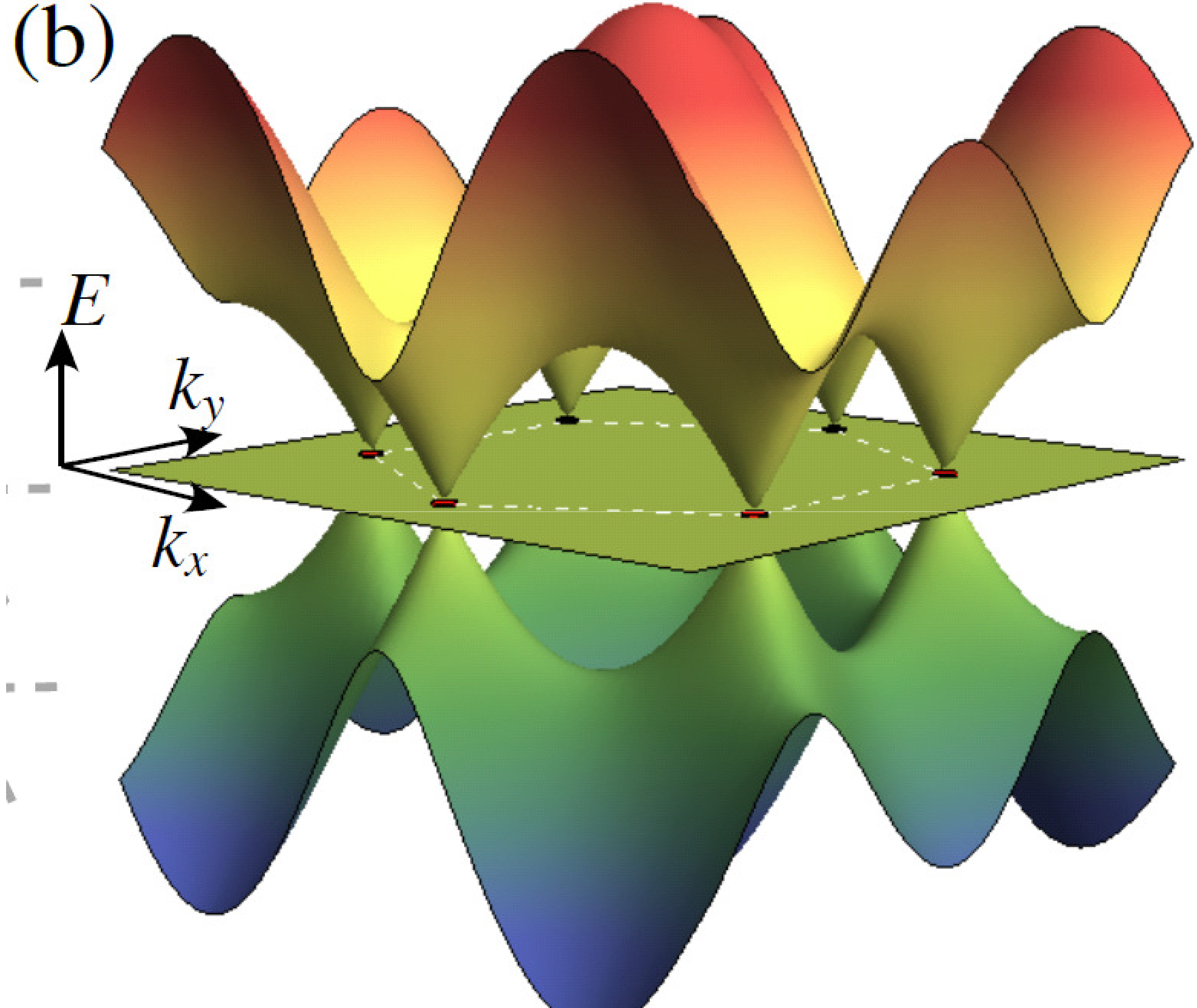




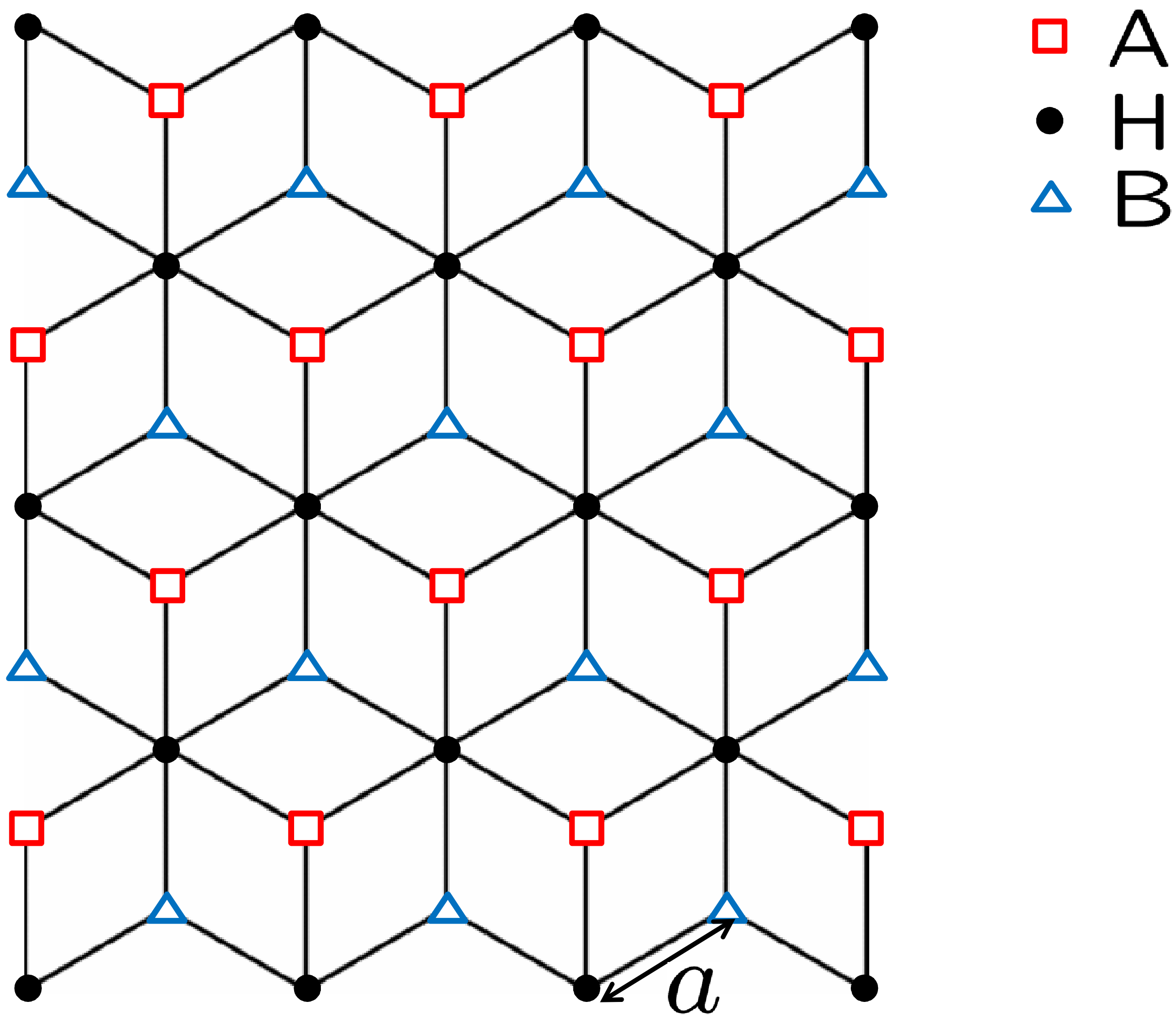
$S_z = 0$

dispersionless

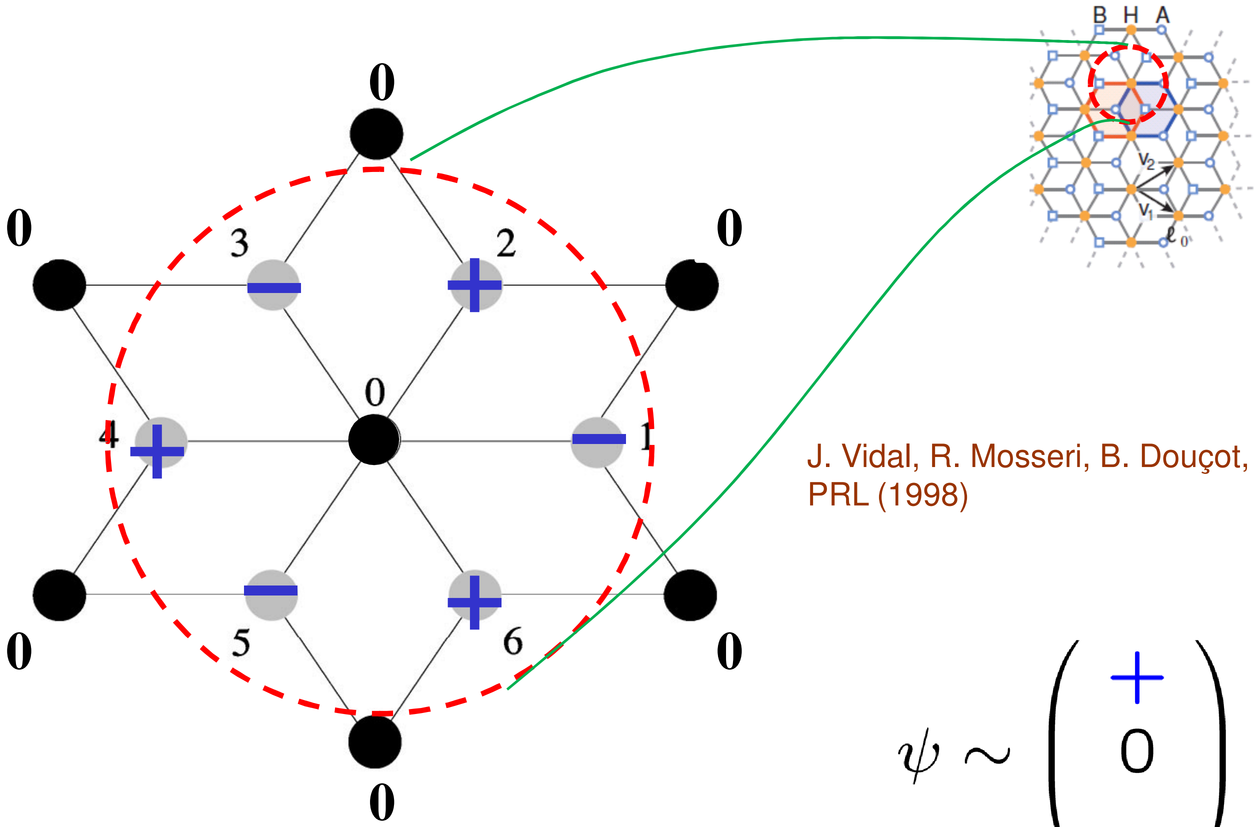
D. Green, L. Santos, C. Chamon (2010)


(b)
E
k_y
k_x

τ_3 -lattice (“dice lattice”)



why flat ?

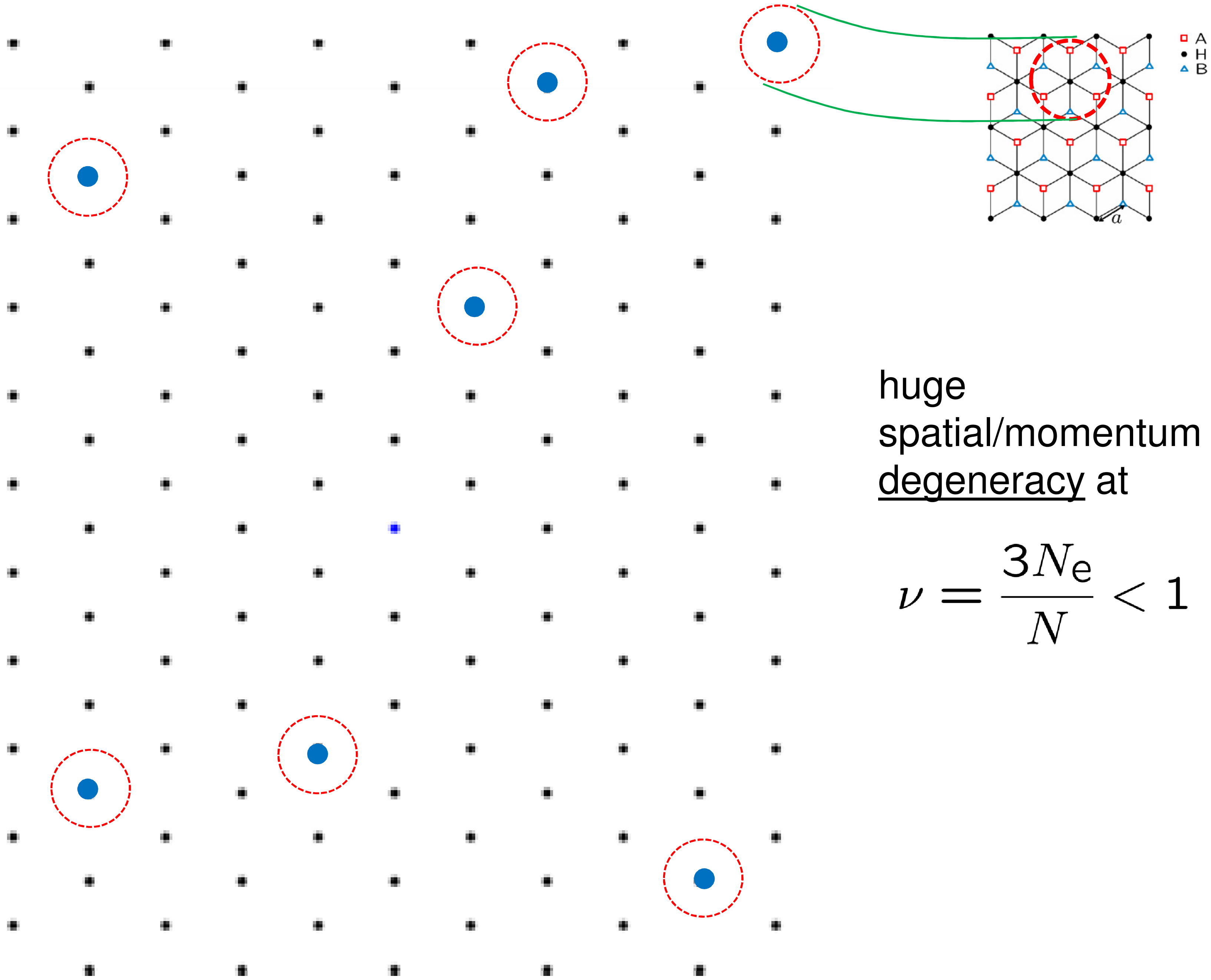


J. Vidal, R. Mosseri, B. Douçot, PRL (1998)

$$\psi \sim \begin{pmatrix} + \\ 0 \\ - \end{pmatrix}$$

➡ **caging !**

B. Sutherland, PRB (1986)



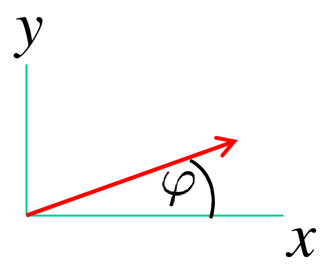
huge spatial/momentum <u>degeneracy</u> at

$$\nu = \frac{3N_\mathrm{e}}{N} < 1$$

current in flat band

current matrix elements $\vec{j}_{kk'} \sim e\, v_{\mathsf{F}}\, \psi_k^* \begin{pmatrix} 0 & \mathrm{e}^{-\mathrm{i}\varphi} & 0 \\ \mathrm{e}^{\mathrm{i}\varphi} & 0 & \mathrm{e}^{-\mathrm{i}\varphi} \\ 0 & \mathrm{e}^{\mathrm{i}\varphi} & 0 \end{pmatrix} \psi_{k'}$

with $\psi \sim \begin{pmatrix} + \\ 0 \\ - \end{pmatrix}$ $= 0$



➡ **insulator**

Coulomb interaction

J. Vidal, P. Butaud, B. Douçot, R. Mosseri, PRB (2001)
Z. Gulácsi, A. Kampf, D. Vollhardt, PRL (2007)
T. Liu, C. Repellin, B.A. Bernevig, N. Regnault, PRB (2013)
L.M. Läuchli, Z. Liu, E.J. Bergholz, R. Moessner, PRL (2013)

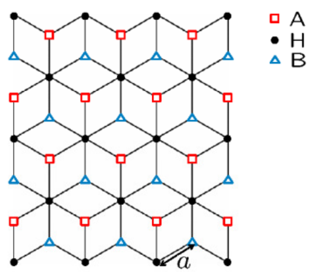
$$+\frac{e^2}{2\kappa}\sum_{i,j}\frac{1}{|\vec{r}_i-\vec{r}_j|}$$

➡ **maxiumum inter-particle distances**

WC of cages

O. Poplavskyy, M.O. Goerbig, C. Morais Smith, PRB (2009)

HOWEVER : cages are bound to H-sites



➡

commensurate WC

if $\frac{1}{\nu}=n_1^2+n_2^2-n_1n_2\,,\quad n_1,n_2\in\mathbb{Z}$

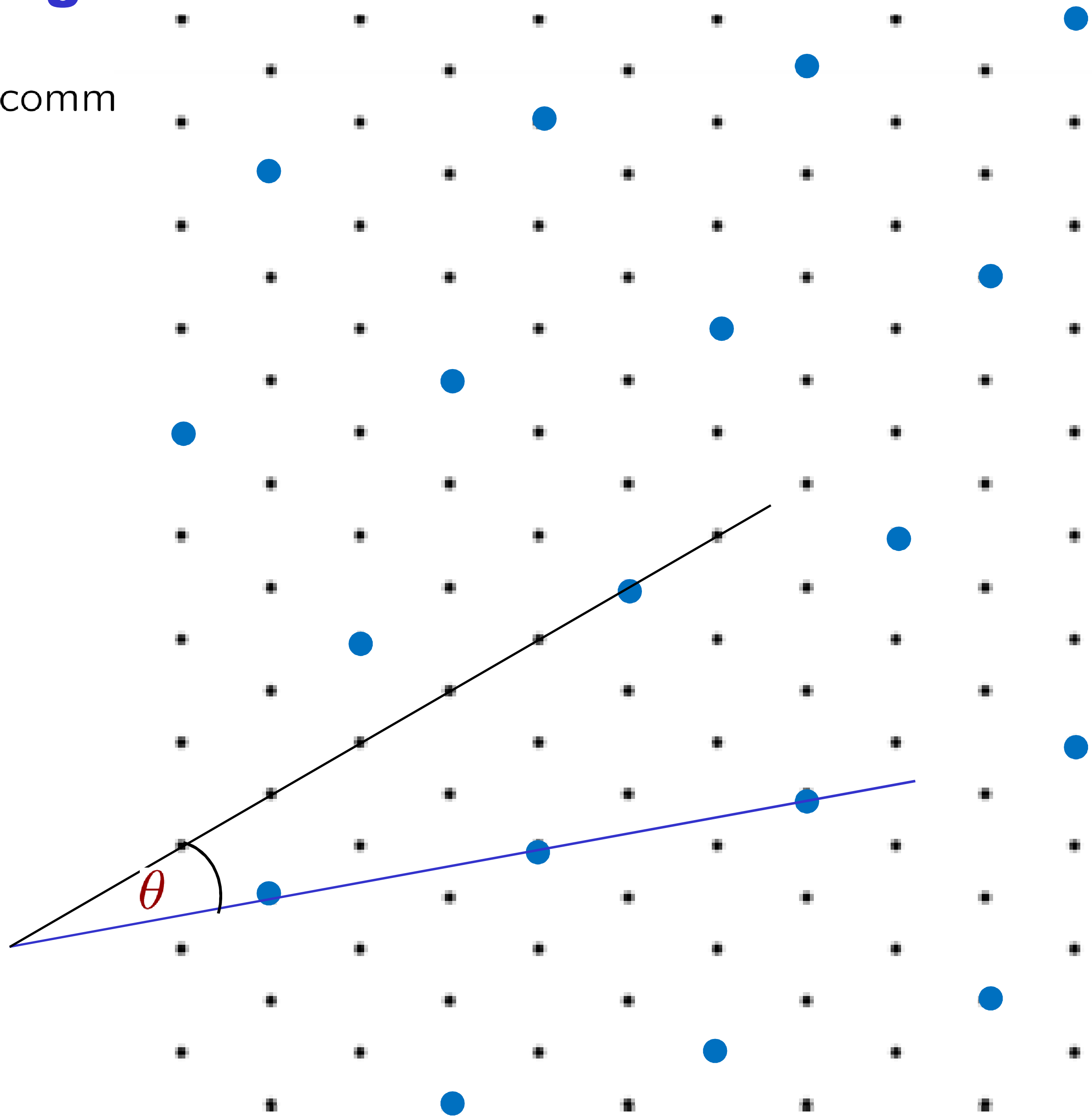
still insulating !

otherwise

- two phases with phase boundary
- ➡ **doped commensurate WC**

WC of cages

at $\nu = \nu_{\mathsf{comm}}$



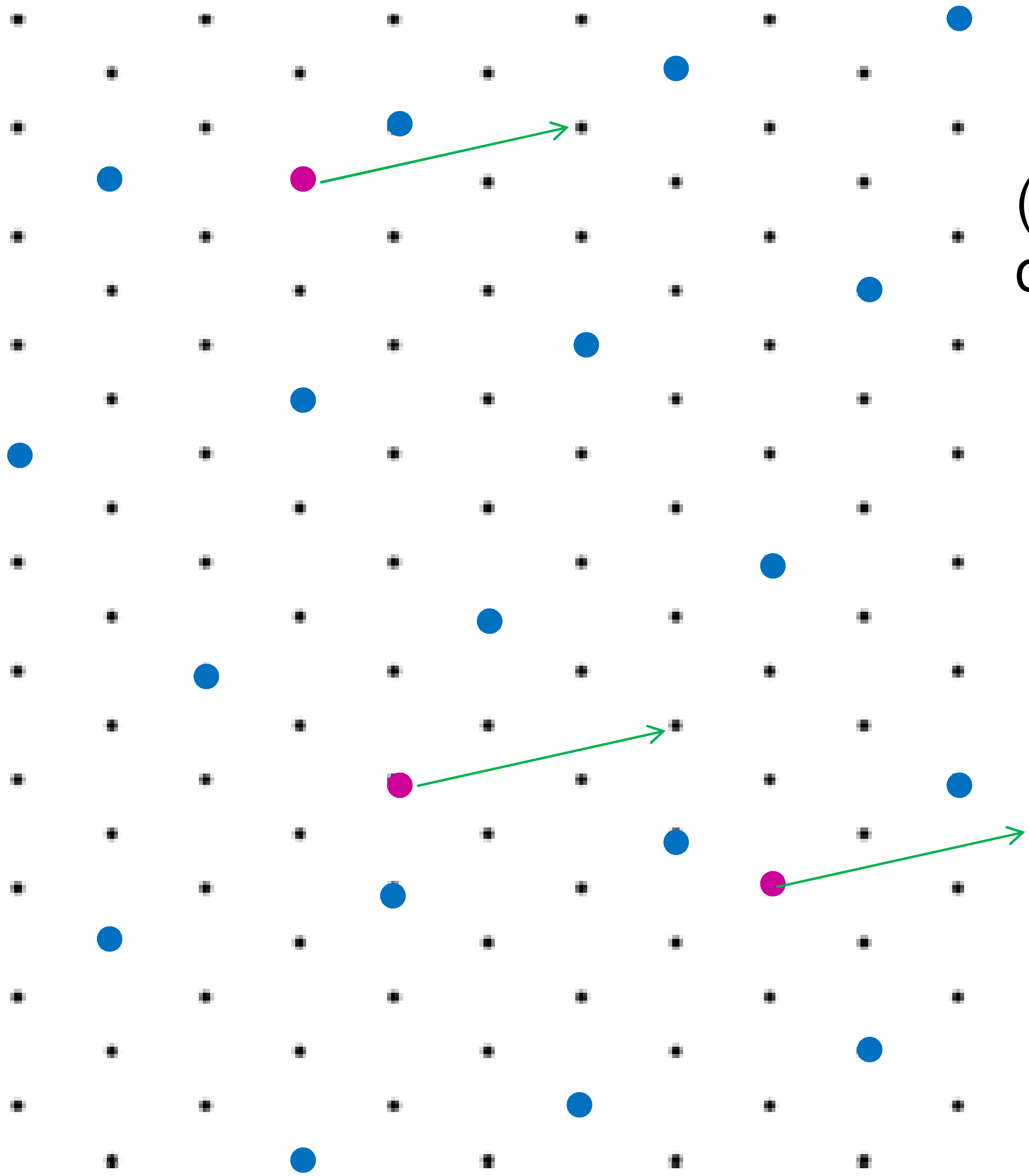
commensurate fillings

$1/\nu_{n_1 n_2}$	n_1	n_2	$\theta_{n_1 n_2}$	$\Delta_{n_1 n_2}$
3	1	1	30.	1
4	2	0	0.	3
7	2	1	19.1066	2
9	3	0	0.	3
12	2	2	30.	1
13	3	1	13.8979	3
16	4	0	0.	3
19	3	2	23.4132	2
21	4	1	10.8934	4
25	5	0	0.	2
⋮				
1477	31	12	15.6886	6
1483	38	1	1.2886	5
1488	28	16	21.0517	1
1489	37	3	3.8606	3
1492	34	8	10.3327	5
1497	32	11	14.2536	4
1501	36	5	6.4171	15
1516	30	14	18.1432	3
1519	35	7	8.9483	2
1521	39	0	0.	3
⋮				

$$\frac{1}{\nu} = n_1^2 + n_2^2 - n_1 n_2 \; ,$$

$$n_1, n_2 \in \mathbb{Z}$$

incommensurate filling



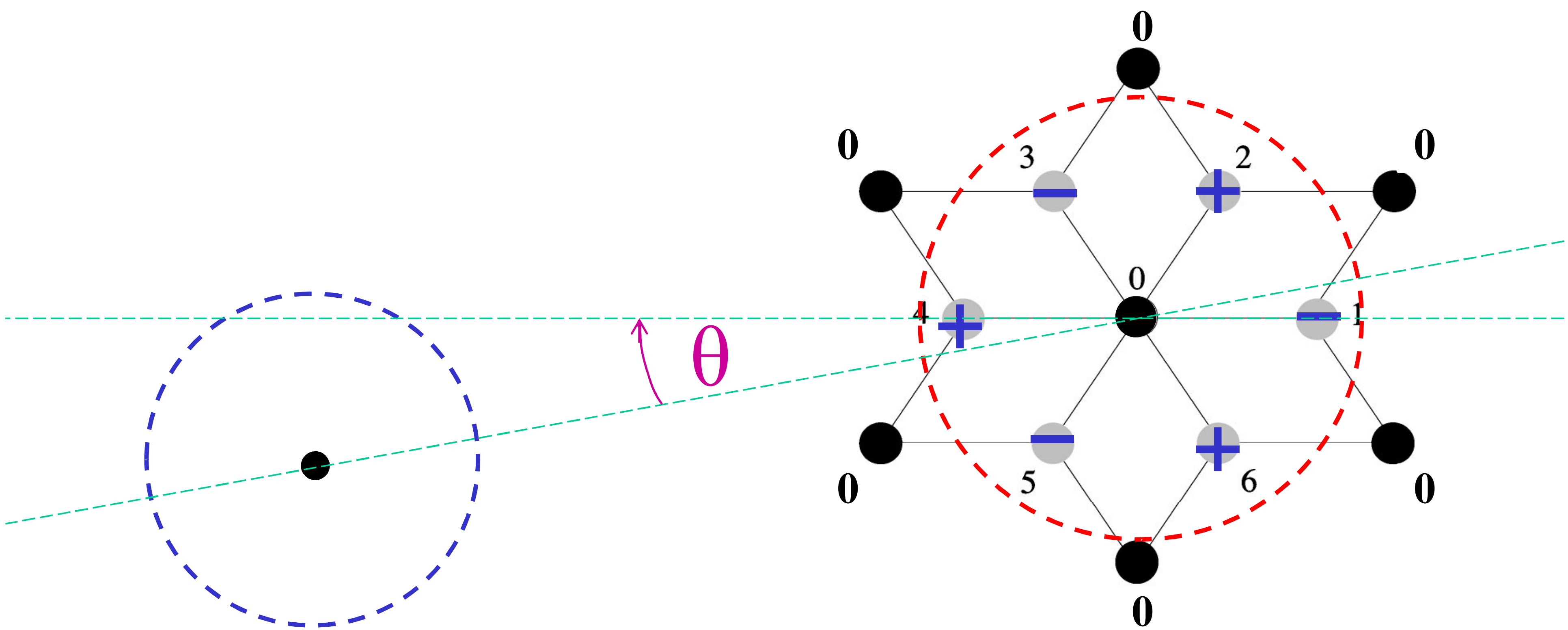
dopants

(may)
carry current

at

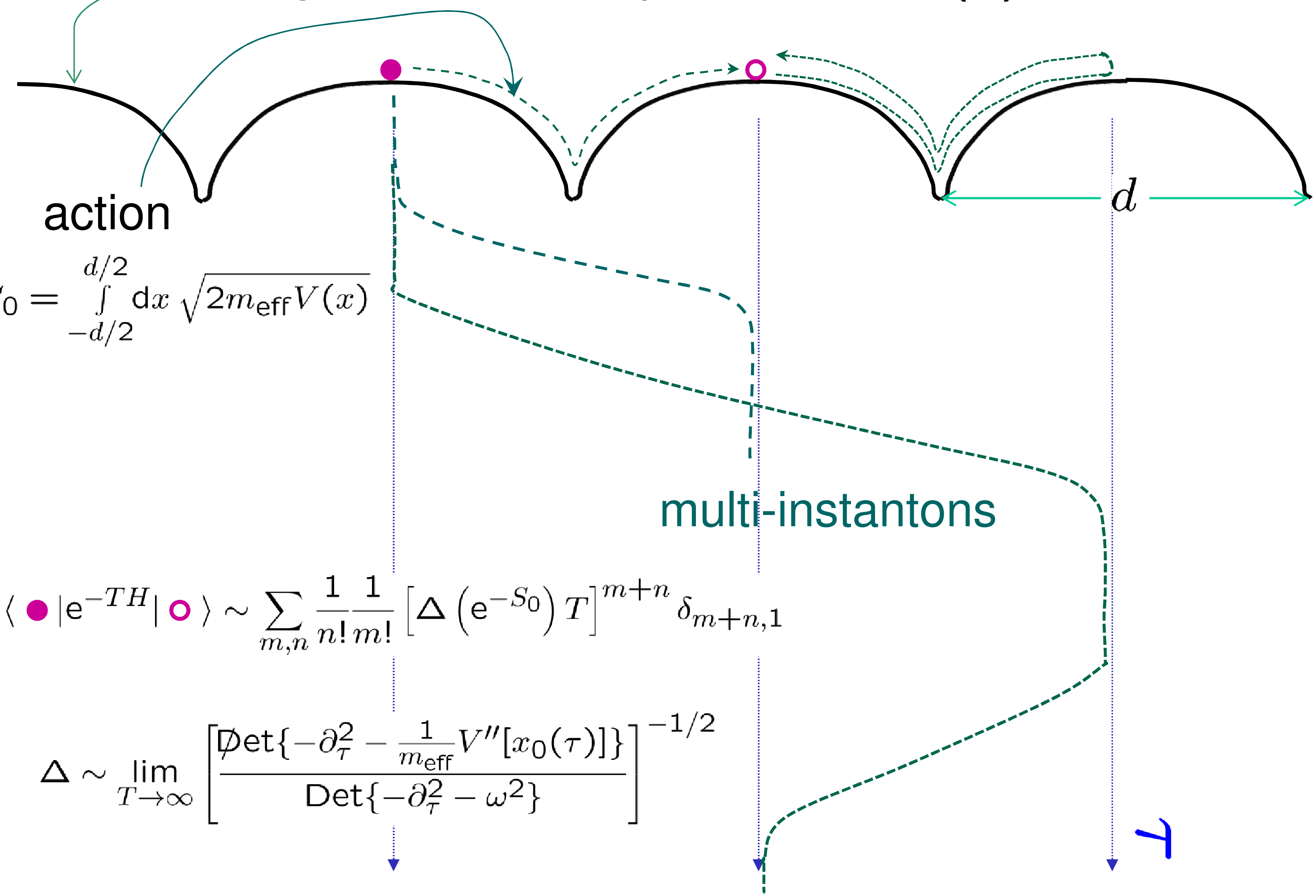
$$\nu = \nu_{\text{comm}} + \delta\nu$$

destruction of caging



➡ dopants acquire kinetic energy and mass m_{eff}

tunneling during imaginary times τ :



$$S_0 = \int_{-d/2}^{d/2} \mathrm{d}x \, \sqrt{2 m_{\mathrm{eff}} V(x)}$$

$$\langle \bullet | \mathrm{e}^{-TH} | \circ \rangle \sim \sum_{m,n} \frac{1}{n!} \frac{1}{m!} \left[\Delta \left(\mathrm{e}^{-S_0} \right) T \right]^{m+n} \delta_{m+n,1}$$

$$\Delta \sim \lim_{T \to \infty} \left[\frac{\not{\mathrm{D}}\mathrm{et}\{ -\partial_\tau^2 - \frac{1}{m_{\mathrm{eff}}} V''[x_0(\tau)] \}}{\mathrm{Det}\{ -\partial_\tau^2 - \omega^2 \}} \right]^{-1/2}$$

Drude weight

$$\sigma(\omega) = D \frac{1}{\mathrm{i}\omega}$$

$$D = \eta \, \delta\nu$$

⬆ dilute instanton approximation:

$$\eta \sim \nu^{-1/8} \, \mathrm{e}^{-S_0} \left| \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} 3\theta \right|^{1/4}$$

J. Zinn-Justin, Nucl. Phys. B, (1981)
" (1983)
Z. Ambrozinski, Acta Phys. Pol. (2013)

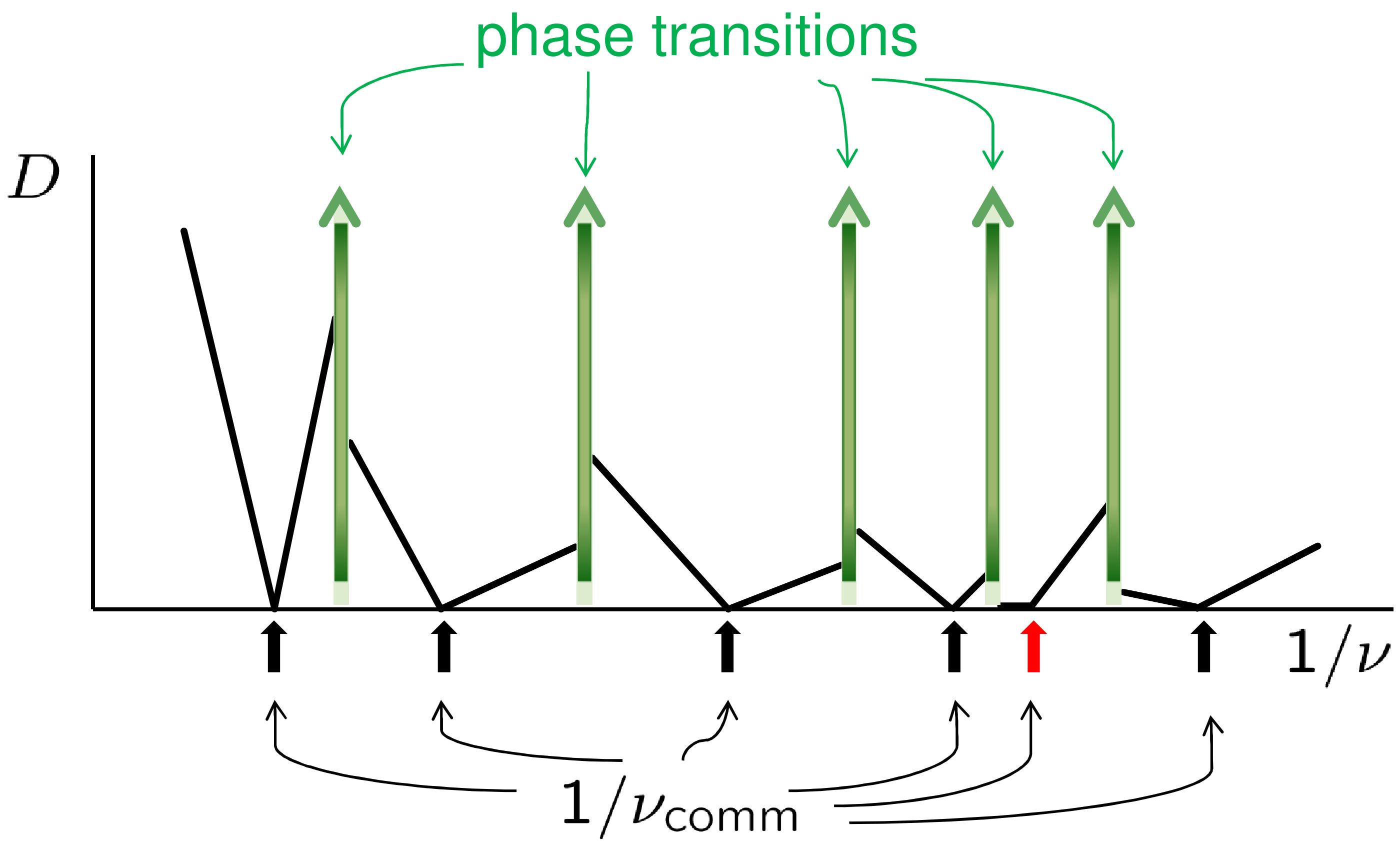
with

$$S_0 \sim \nu^{-3/2} \left| \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} 3\theta \right|^{-1/2}$$

for $\left\{ \begin{matrix} \text{electrons} \\ \text{holes} \end{matrix} \right\}$

θ : angle of WC wrt. $\mathcal{T}_3$ lattice

Drude weight D



$$D = \eta \, \delta\nu$$

↑ can be zero

$\nu = \nu_{\text{comm}} + \delta\nu$

Summary :

- Coulomb interactions can turn a flat band insulator into a conductor

- but not in a simple way as a function of flat band filling ν .