On the interacting Majorana chain

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with

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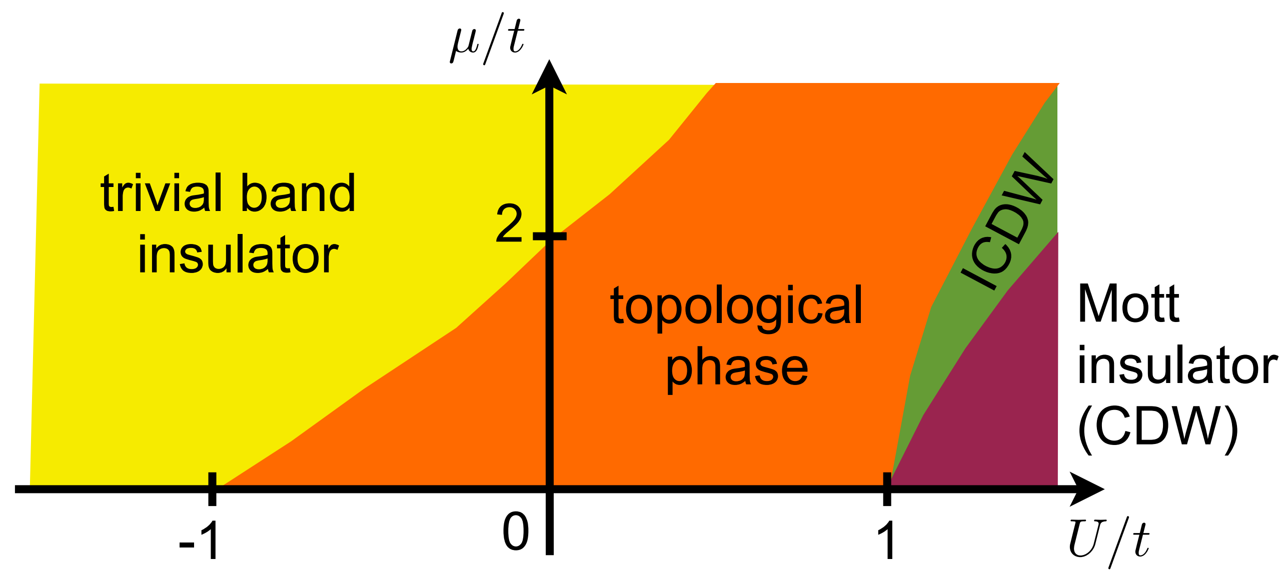
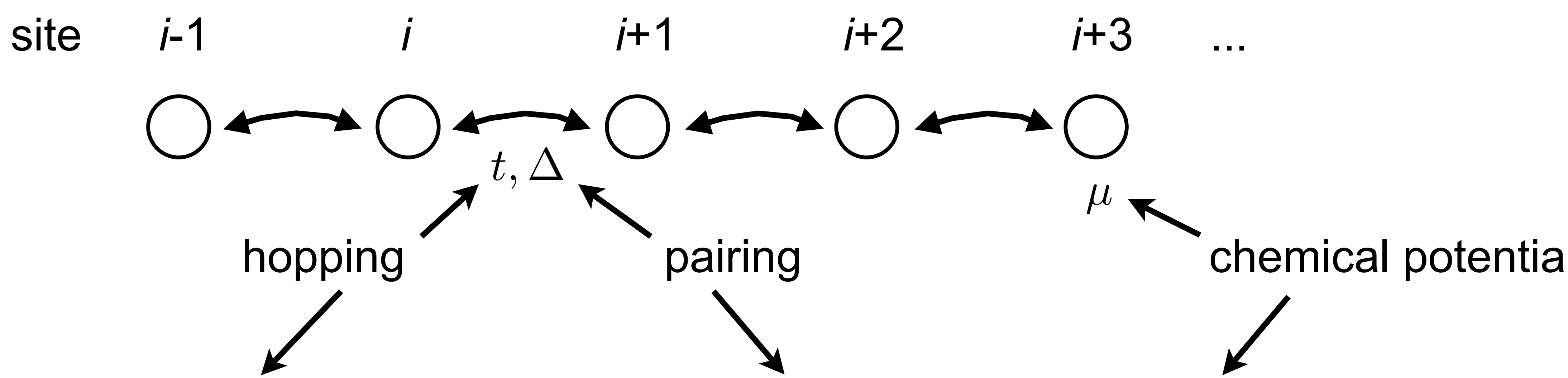


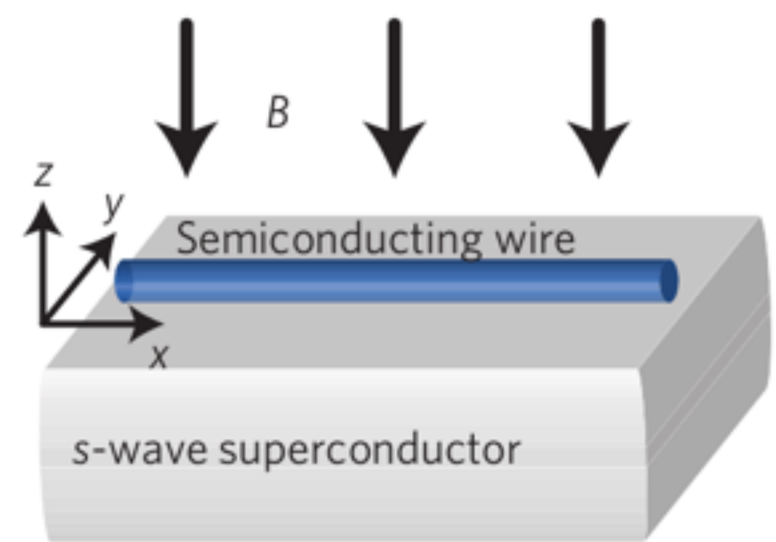
Kitaev chain

consider a chain of spinless fermions
(Kitaev, Phys Usp 2001)



$$H = -t \sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + \Delta \sum_i (c_i c_{i+1} + c_{i+1}^\dagger c_i^\dagger) - \mu \sum_i (c_i^\dagger c_i - \tfrac{1}{2})$$

proposal: semiconductor nanowires with
spin-orbit interaction in a magnetic
field and proximity to superconductor
(Lutchyn et al., PRL 2010; Oreg et al., PRL 2010)

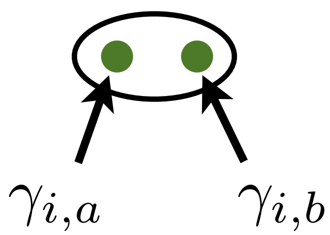


Alicea et al., Nat Phys 2011

Kitaev chain

introduce two Majorana fermions per site

site *i*

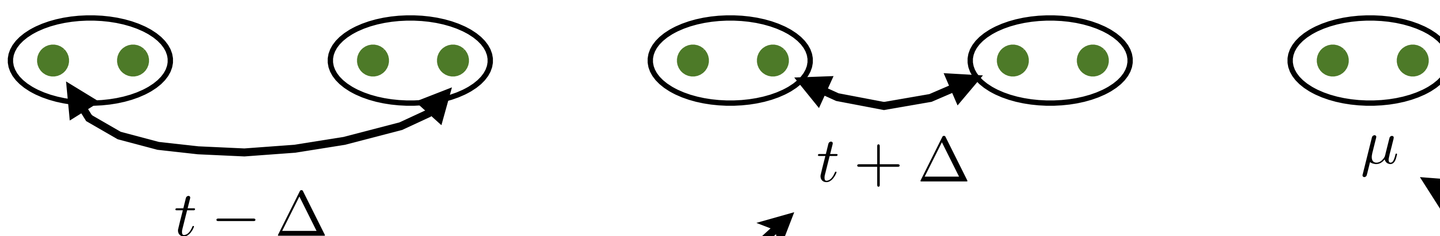


$$c_i = \frac{1}{2}\left(\gamma_{i,a} + \mathrm{i}\gamma_{i,b}\right), \quad c_i^\dagger = \frac{1}{2}\left(\gamma_{i,a} - \mathrm{i}\gamma_{i,b}\right)$$

$$\gamma_{i,a}^\dagger = \gamma_{i,a}, \quad \gamma_{i,b}^\dagger = \gamma_{i,b}$$

$$\left\{\gamma_{i,a}, \gamma_{j,a}\right\} = \left\{\gamma_{i,b}, \gamma_{j,b}\right\} = 2\delta_{ij}$$

$$\left\{\gamma_{i,a}, \gamma_{j,b}\right\} = 0, \quad \gamma_{i,a}^2 = \gamma_{i,b}^2 = 1$$



long-range hopping

hopping

chemical potential

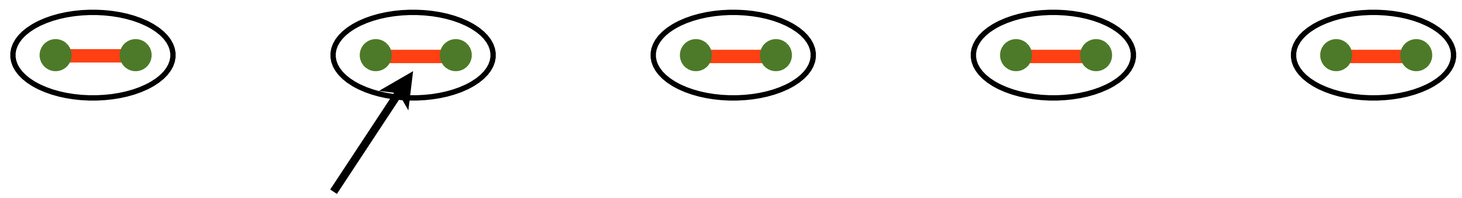
$$H = -\frac{\mathrm{i}}{2}\sum_i (t-\Delta)\gamma_{i,a}\gamma_{i+1,b} + \frac{\mathrm{i}}{2}\sum_i (t+\Delta)\gamma_{i,b}\gamma_{i+1,a} - \frac{\mathrm{i}}{2}\mu\sum_i \gamma_{i,a}\gamma_{i,b}$$

long range ➔ assume $t=\Delta$ in the following

Phases in the Kitaev chain

- trivial phase, e.g. t=Δ=0

$$H = -\mu \sum_i \left(c_i^\dagger c_i - \tfrac{1}{2}\right) = -\frac{\mathrm{i}}{2}\mu \sum_i \gamma_{i,a}\gamma_{i,b}$$



on-site pairing

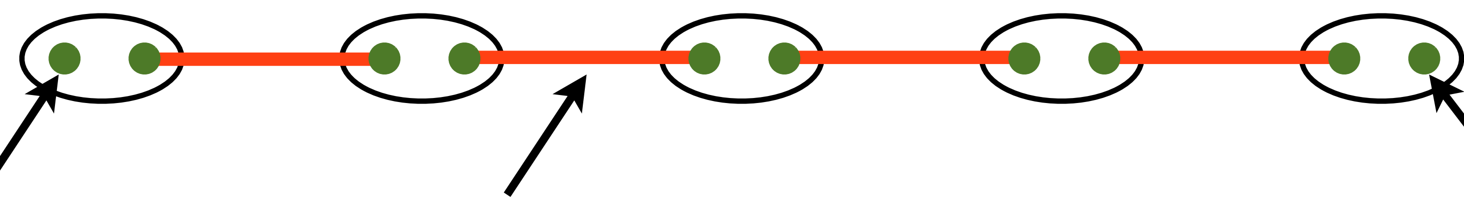
→ filled or empty fermionic sites

→ band insulator

- topological phase, e.g. t=Δ, μ=0

$$H = \mathrm{i}t \sum_i \gamma_{i,b}\gamma_{i+1,a}$$

$\gamma_{1,a}$



$\gamma_{N,b}$

unpaired Majorana

pairing between sites

unpaired Majorana

two-fold degenerate ground state

$|\pm\rangle$ with $(-1)^{\mathrm{F}}\,|\pm\rangle = \prod_i \mathrm{i}\gamma_{i,a}\gamma_{i,b} = \pm\,|\pm\rangle$

new, non-local fermionic mode

$$d = \frac{1}{2}\left(\gamma_{1,a} + \mathrm{i}\gamma_{N,b}\right)$$

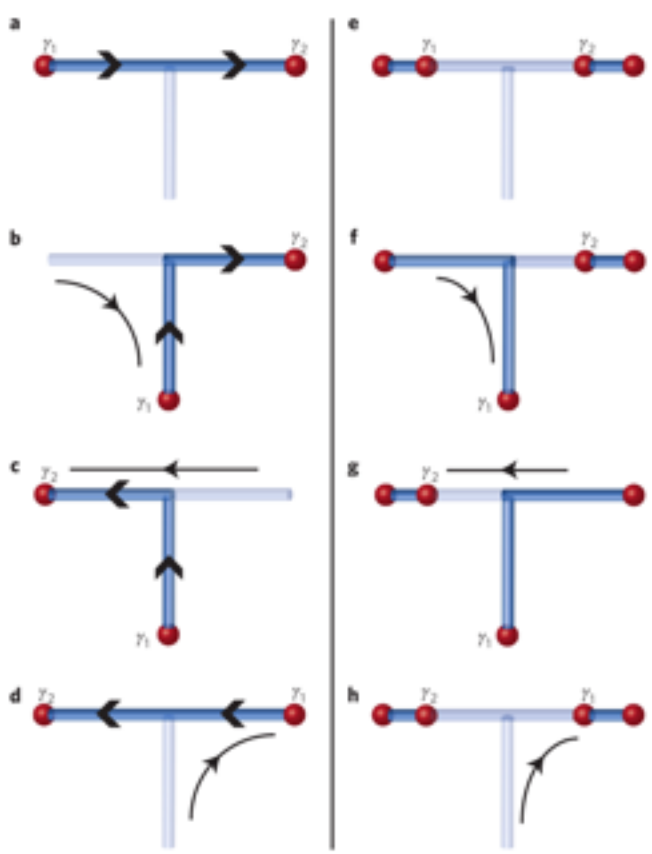
Majorana fermions

- non-local fermion mode can serve as topologically protected q-bit

$$|0\rangle\ ,\ |1\rangle = d^{\dagger}\,|0\rangle$$

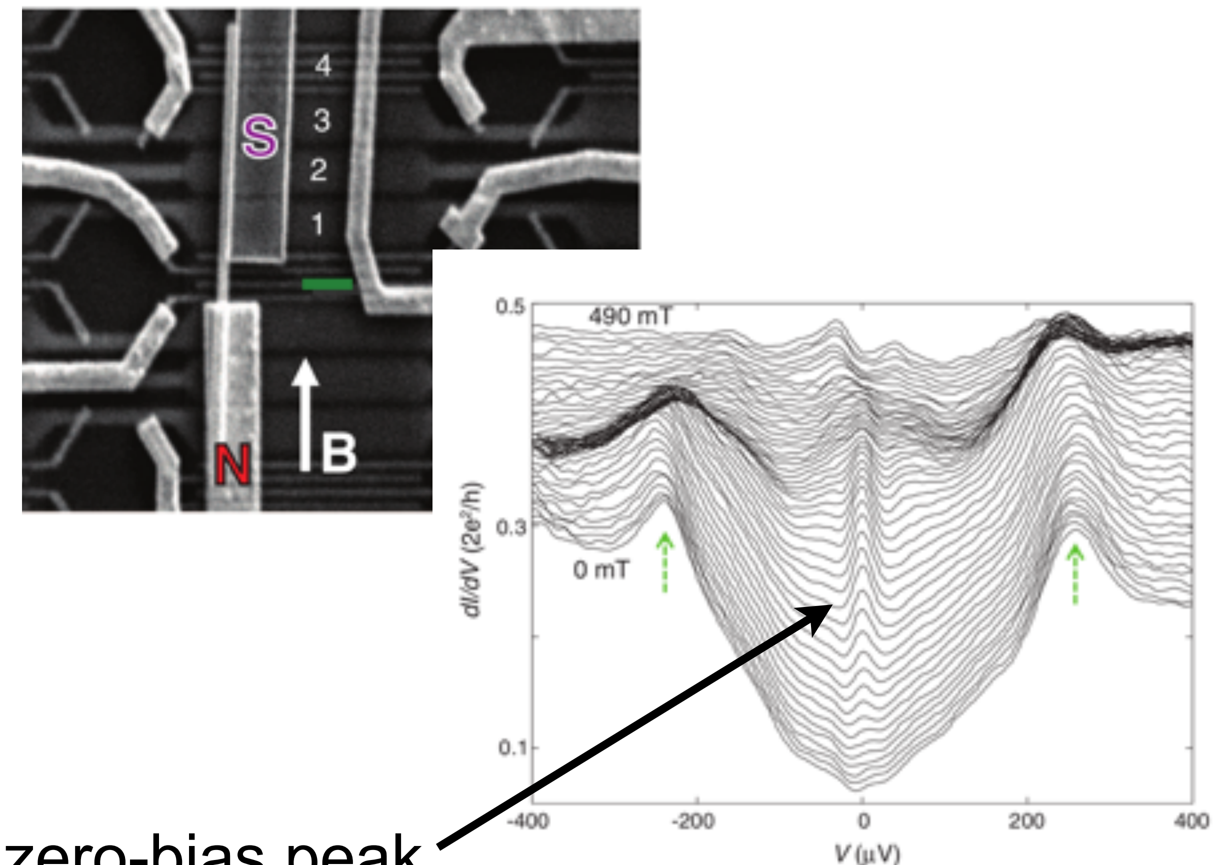
- Majorana fermions exhibit non-Abelian braiding statistics

(Alicea et al., Nat Phys 2011)



- can be seen in experiments?

(Mourik et al., Science 2012)



zero-bias peak

→ Majorana fermion?

we will not discuss these topics, but ...

Question

What is effect of interaction on topological phase and phase diagram?

$$H_U = U\sum_i \big(2c_i^\dagger c_i - 1\big)\big(2c_{i+1}^\dagger c_{i+1} - 1\big)$$

Outline

What is effect of interaction on topological phase and phase diagram?

$$H_U = U \sum_i \big(2c_i^\dagger c_i - 1\big)\big(2c_{i+1}^\dagger c_{i+1} - 1\big)$$

- rewrite interaction term in Majorana fermions
- possible realisation in Josephson junction arrays
- mapping to effective spin model
- discuss phase diagram

→ repulsive interactions stabilise topological phase

- signatures in tunneling conductance

Interacting Majorana chain

chain of interacting spinless fermions with $t=\Delta$

$$H = -t\sum_i \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i - c_i c_{i+1} - c_{i+1}^\dagger c_i^\dagger\right) - \mu \sum_i \left(c_i^\dagger c_i - \tfrac{1}{2}\right)$$
$$+U\sum_i \left(2c_i^\dagger c_i - 1\right)\left(2c_{i+1}^\dagger c_{i+1} - 1\right)$$

mapping to Majorana fermions as before

$$c_i = \frac{1}{2}\left(\gamma_{i,a} + \mathrm{i}\gamma_{i,b}\right), \quad c_i^\dagger = \frac{1}{2}\left(\gamma_{i,a} - \mathrm{i}\gamma_{i,b}\right)$$

interacting Majorana chain with $t=\Delta$

$$H = \mathrm{i}t\sum_i \gamma_{i,b}\gamma_{i+1,a} - \frac{\mathrm{i}}{2}\mu\sum_i \gamma_{i,a}\gamma_{i,b} - U\sum_i \gamma_{i,a}\gamma_{i,b}\gamma_{i+1,a}\gamma_{i+1,b}$$

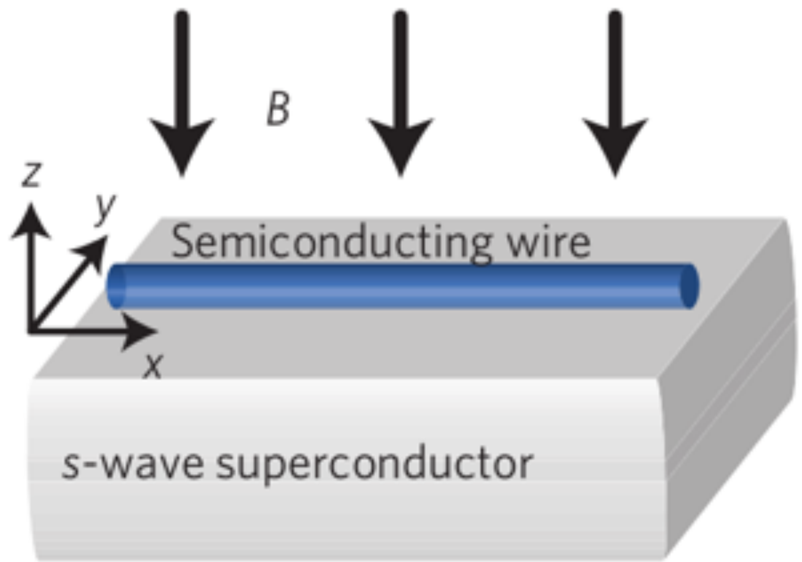
interaction

Possible realisation?

Interacting nanowires

semiconductor nanowire with spin-orbit interaction in a magnetic field and proximity to superconductor (Lutchyn et al., PRL 2010; Oreg et al., PRL 2010)

→ in principle there is a short-range interaction between the electrons



Alicea et al., Nat Phys 2011

$$H_{\mathrm{NW}} = -\int \mathrm{d}x\, \Psi^\dagger \left(\frac{\partial_x^2}{2m} + \mu + \mathrm{i}\alpha\sigma^y\partial_x + B\sigma^z \right)\Psi + \int \mathrm{d}x \left(\Delta\Psi_\uparrow\Psi_\downarrow + \mathrm{h.c.} + U_0|\Psi_\uparrow(x)|^2|\Psi_\downarrow(x)|^2 \right)$$

spin orbit | Zeeman | $\Psi = (\Psi_\uparrow, \Psi_\downarrow)$ | pairing | interaction

but mapping to single-band model reduces interaction

$$\Psi_\downarrow = \frac{\alpha}{2B}(\vec{e}_y + \mathrm{i}\vec{e}_x)\partial_x\Psi_\uparrow$$

$$\frac{U}{t} = \frac{mU_0\alpha^2}{LB^2} \ll 1$$

nanowires are in weak-coupling regime

(cf. Stoudenmire et al., PRB 2011)

Josephson junction array

$$H = \mathrm{i}t\sum_i \gamma_{i,b}\gamma_{i+1,a} - \frac{\mathrm{i}}{2}\mu\sum_i \gamma_{i,a}\gamma_{i,b} - U\sum_i \gamma_{i,a}\gamma_{i,b}\gamma_{i+1,a}\gamma_{i+1,b}$$

chain of superconducting islands with nanowires

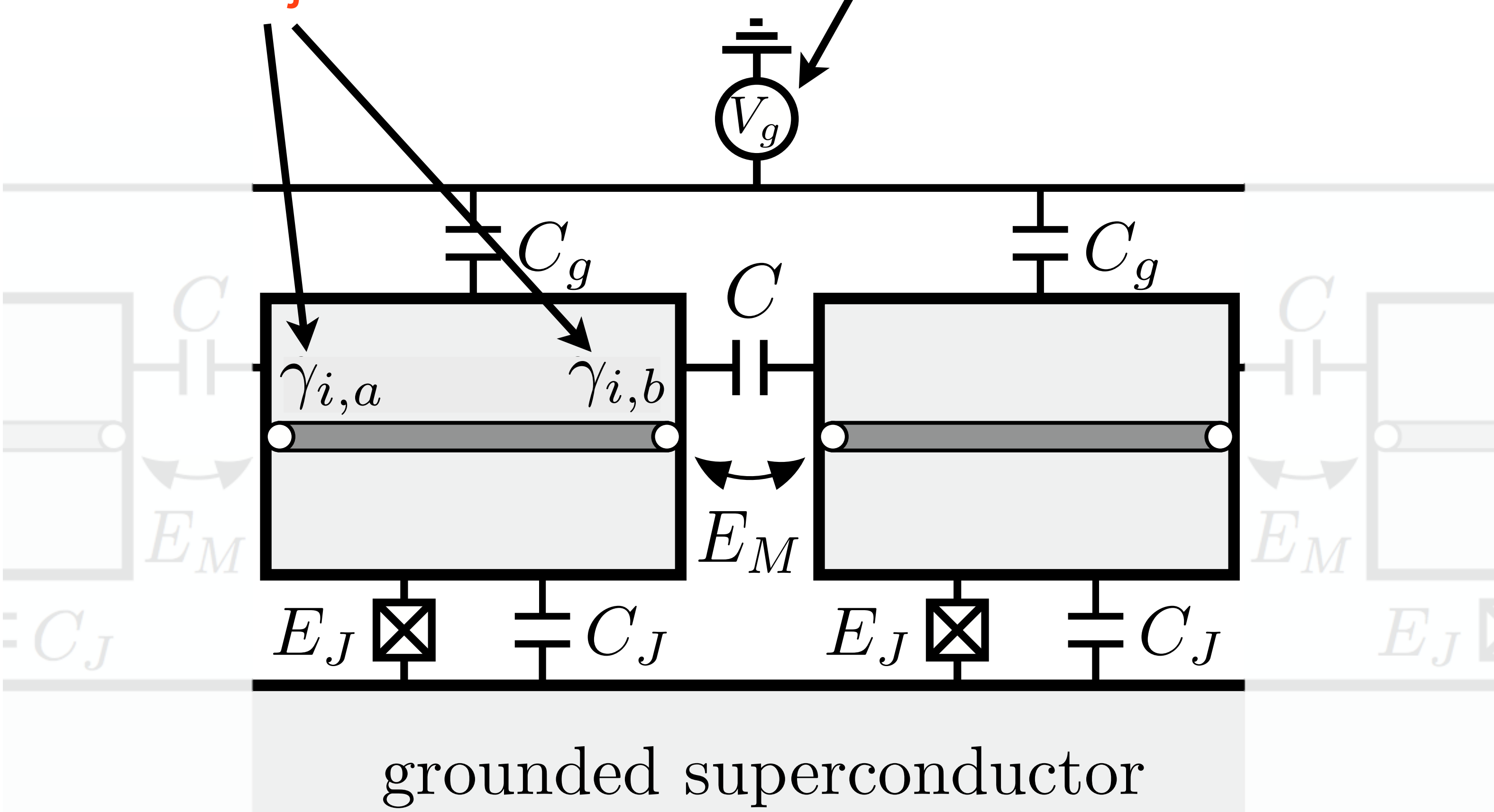
→ each island has two Majorana modes

induced charge on island

$q = C_g V_g$

$E_J \gg E_M, E_C$

→ superconducting phases pinned



relation of parameters

$t = E_M$

$\mu = 2\Gamma_\mu \cos(\pi q/e)$

$U = \Gamma_U \cos(2\pi q/e)$

non-trivial functions of

$E_J,\ C_J,\ C_g,\ C$

for C=0 (U=0): van Heck et al., PRB 2011; NJP 2012 cf. Sela et al., PRB 2013

Mapping to spin chain

interacting Majorana chain $H = \mathrm{i}t \sum_i \gamma_{i,b}\gamma_{i+1,a} - \frac{\mathrm{i}}{2}\mu \sum_i \gamma_{i,a}\gamma_{i,b} - U \sum_i \gamma_{i,a}\gamma_{i,b}\gamma_{i+1,a}\gamma_{i+1,b}$

Jordan-Wigner transformation

$\sigma_i^z = 2c_i^\dagger c_i - 1 = \mathrm{i}\gamma_{i,a}\gamma_{i,b}$ local transformation

$\sigma_i^x = \prod_{j<i} \left(1 - 2c_j^\dagger c_j\right)\left(c_i^\dagger + c_i\right)$ non-local transformation

→ $\mathrm{i}\gamma_{i,b}\gamma_{i+1,a} = \sigma_i^x \sigma_{i+1}^x$

effective spin model $H = t \sum_i \sigma_i^x \sigma_{i+1}^x - \frac{\mu}{2} \sum_i \sigma_i^z + U \sum_i \sigma_i^z \sigma_{i+1}^z$

invariant under $t \to -t$ and $\mu \to -\mu$ via $\sigma_i^{x,y} \to (-1)^i \sigma_i^{x,y},\ \sigma_i^{x,z} \to -\sigma_i^{x,z}$

→ consider $t \to -t$ with $t,\mu>0$ in the following

Question: What is phase diagram?

Non-interacting Majorana chain

Kitaev chain
(Kitaev, Phys Usp 2001)

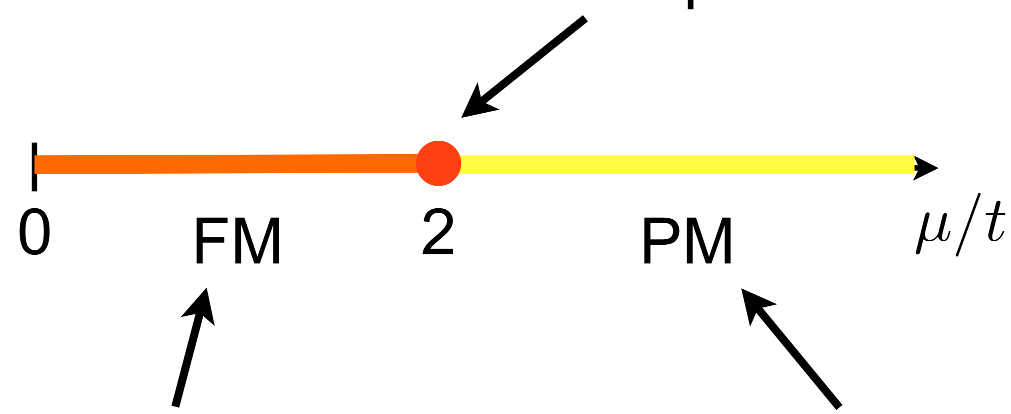
non-interacting Majorana chain $$H = \mathrm{i}t \sum_i \gamma_{i,b}\gamma_{i+1,a} - \frac{\mathrm{i}}{2}\mu \sum_i \gamma_{i,a}\gamma_{i,b}$$

transverse field Ising chain $$H = -t \sum_i \sigma_i^x \sigma_{i+1}^x - \frac{\mu}{2} \sum_i \sigma_i^z$$

phase diagram

quantum critical point

order parameter $\langle \sigma_i^x \rangle$



two-fold degenerate ground state with $\langle \sigma_i^x \rangle \neq 0$

unique ground state with $\langle \sigma_i^x \rangle = 0,\ \langle \sigma_i^z \rangle > 0$

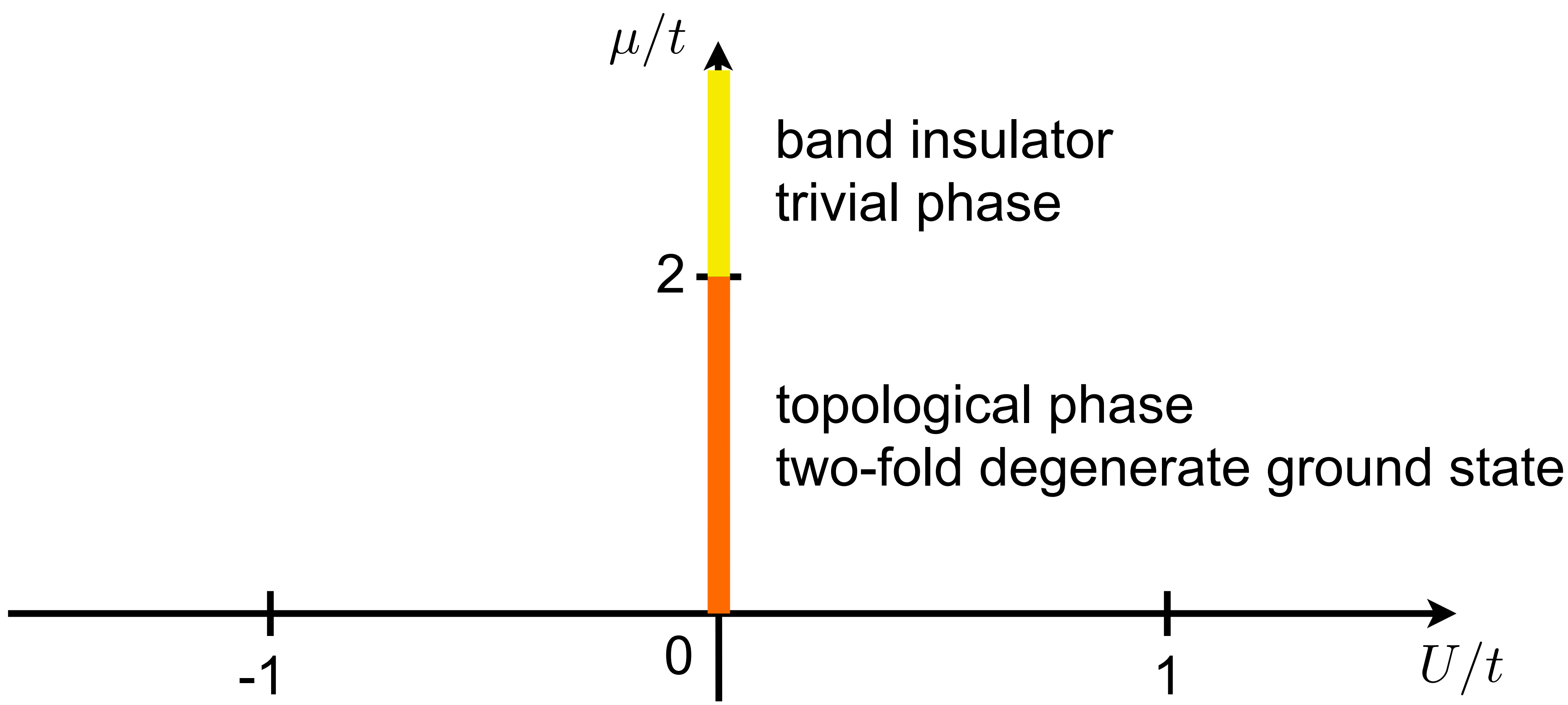
(and zero-energy boundary modes)

Non-interacting Majorana chain

Kitaev chain
(Kitaev, Phys Usp 2001)

non-interacting Majorana chain
$$H = \mathrm{i}t \sum_i \gamma_{i,b}\gamma_{i+1,a} - \frac{\mathrm{i}}{2}\mu \sum_i \gamma_{i,a}\gamma_{i,b}$$

transverse field Ising chain
$$H = -t \sum_i \sigma_i^x \sigma_{i+1}^x - \frac{\mu}{2} \sum_i \sigma_i^z$$



Interacting Majorana chain (μ=0)

interacting Majorana chain $$H = \mathrm{i}t \sum_i \gamma_{i,b}\gamma_{i+1,a} - U \sum_i \gamma_{i,a}\gamma_{i,b}\gamma_{i+1,a}\gamma_{i+1,b}$$

effective spin model $$H = -t \sum_i \sigma_i^x \sigma_{i+1}^x + U \sum_i \sigma_i^z \sigma_{i+1}^z$$

duality transformation $$\tau_i^z = \sigma_i^x \sigma_{i+1}^x , \; \tau_i^x = \prod_{j<i} \sigma_j^z$$ non-local transformation

→ $$H = -t \sum_i \tau_i^z + U \sum_i \tau_i^x \tau_{i+2}^x$$ two decoupled Ising chains

→ phase transitions at $U/t = \pm 1$

Interacting Majorana chain (μ=0)

reminder
$$\sigma_i^z = 2c_i^\dagger c_i - 1$$

interacting Majorana chain
$$H = \mathrm{i}t \sum_i \gamma_{i,b}\gamma_{i+1,a} - U \sum_i \gamma_{i,a}\gamma_{i,b}\gamma_{i+1,a}\gamma_{i+1,b}$$

effective spin model
$$H = -t \sum_i \sigma_i^x \sigma_{i+1}^x + U \sum_i \sigma_i^z \sigma_{i+1}^z$$

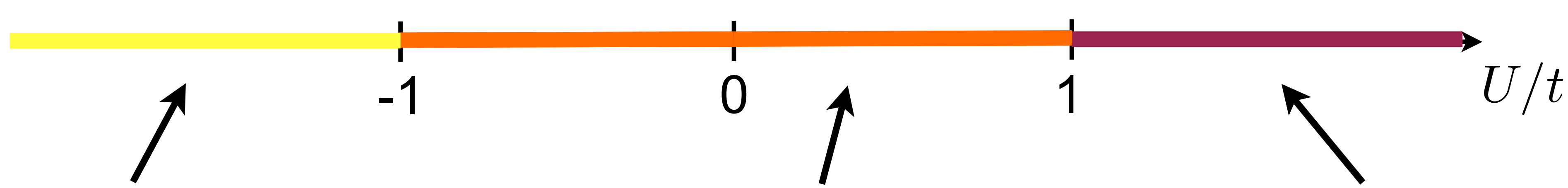
duality transformation
$$\tau_i^z = \sigma_i^x \sigma_{i+1}^x, \; \tau_i^x = \prod_{j<i} \sigma_j^z$$

non-local transformation

→
$$H = -t \sum_i \tau_i^z + U \sum_i \tau_i^x \tau_{i+2}^x$$

two decoupled Ising chains

phase diagram



$\langle \sigma_i^z \rangle \neq 0$ → band insulator

$\langle \sigma_i^x \rangle \neq 0$ → two-fold degenerate ground state

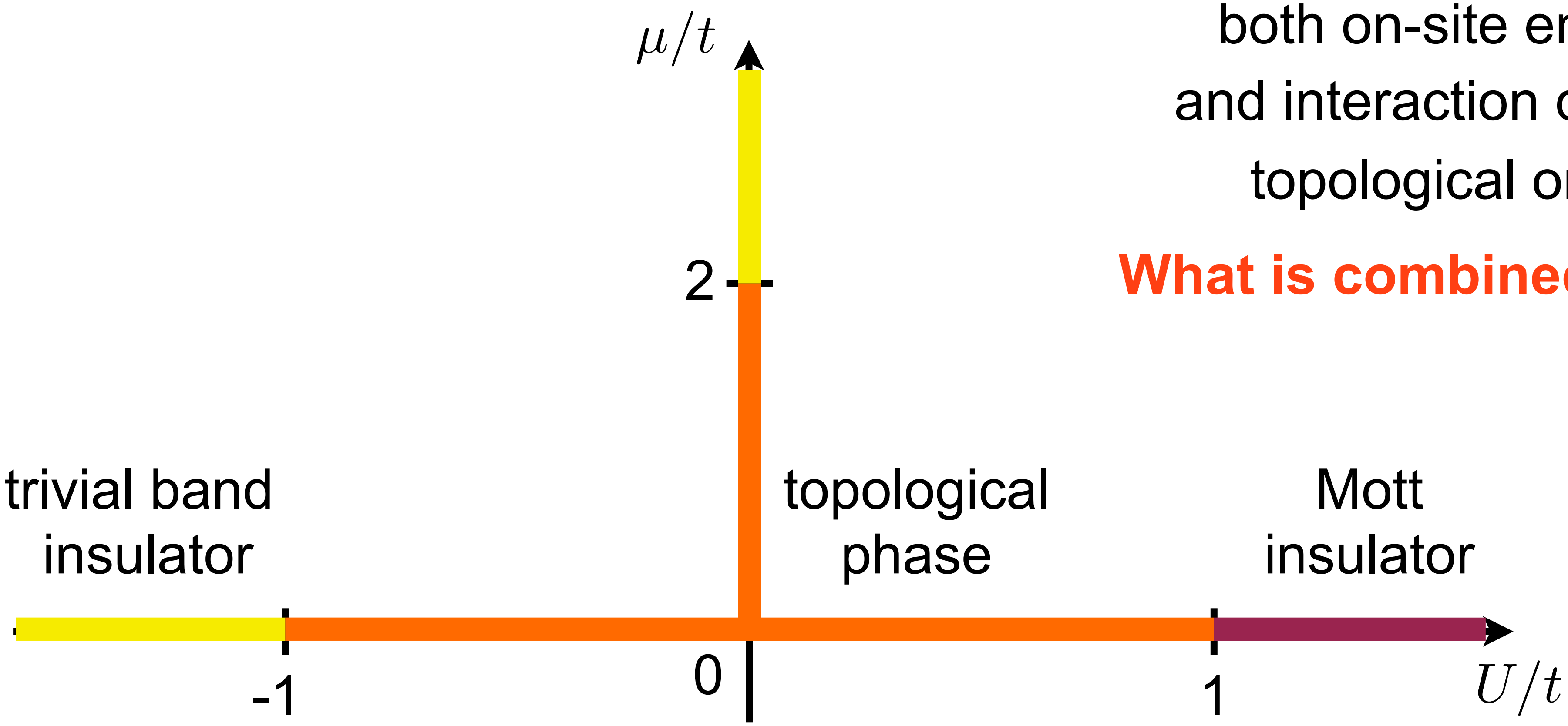
$\langle \sigma_i^z \rangle \propto (-1)^i$ → Mott insulator (half-filled)

Interacting Majorana chain

interacting Majorana chain $$H = \mathrm{i}t\sum_i \gamma_{i,b}\gamma_{i+1,a} - \frac{\mathrm{i}}{2}\mu\sum_i \gamma_{i,a}\gamma_{i,b} - U\sum_i \gamma_{i,a}\gamma_{i,b}\gamma_{i+1,a}\gamma_{i+1,b}$$

effective spin model $$H = -t\sum_i \sigma_i^x\sigma_{i+1}^x - \frac{\mu}{2}\sum_i \sigma_i^z + U\sum_i \sigma_i^z\sigma_{i+1}^z$$

duality transformation $$H = -t\sum_i \tau_i^z - \frac{\mu}{2}\sum_i \tau_i^x\tau_{i+1}^x + U\sum_i \tau_i^x\tau_{i+2}^x$$



both on-site energy and interaction destroy topological order

What is combined effect?

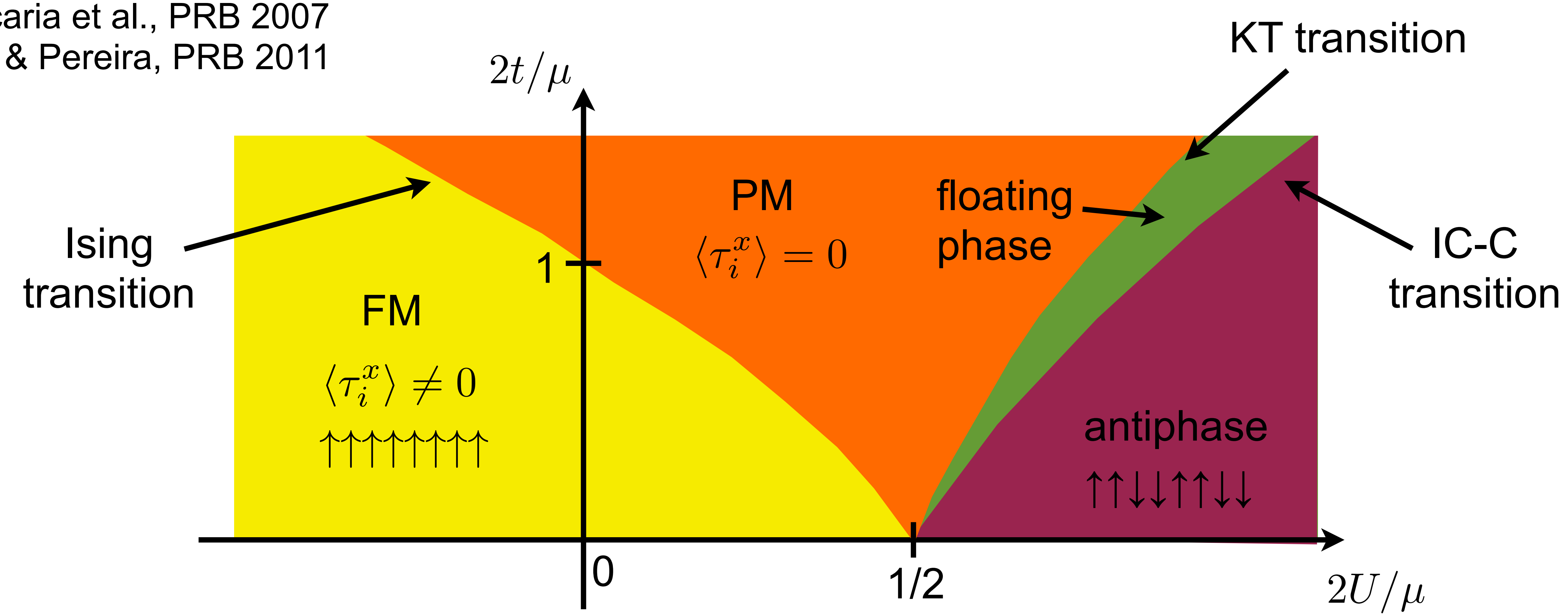
ANNNI model

axial next-nearest-neighbour Ising chain

$$H_{\mathrm{ANNNI}} = -t\sum_i \tau_i^z - \frac{\mu}{2}\sum_i \tau_i^x \tau_{i+1}^x + U\sum_i \tau_i^x \tau_{i+2}^x$$
$$= -\frac{\mu}{2}\sum_i \left(\tau_i^x \tau_{i+1}^x - \frac{2U}{\mu}\tau_i^x \tau_{i+2}^x + \frac{2t}{\mu}\tau_i^z\right)$$

many studies in statistical mechanics (Selke, Phys Rep 1988)

phase diagram ANNNI model
Beccaria et al., PRB 2007
Sela & Pereira, PRB 2011

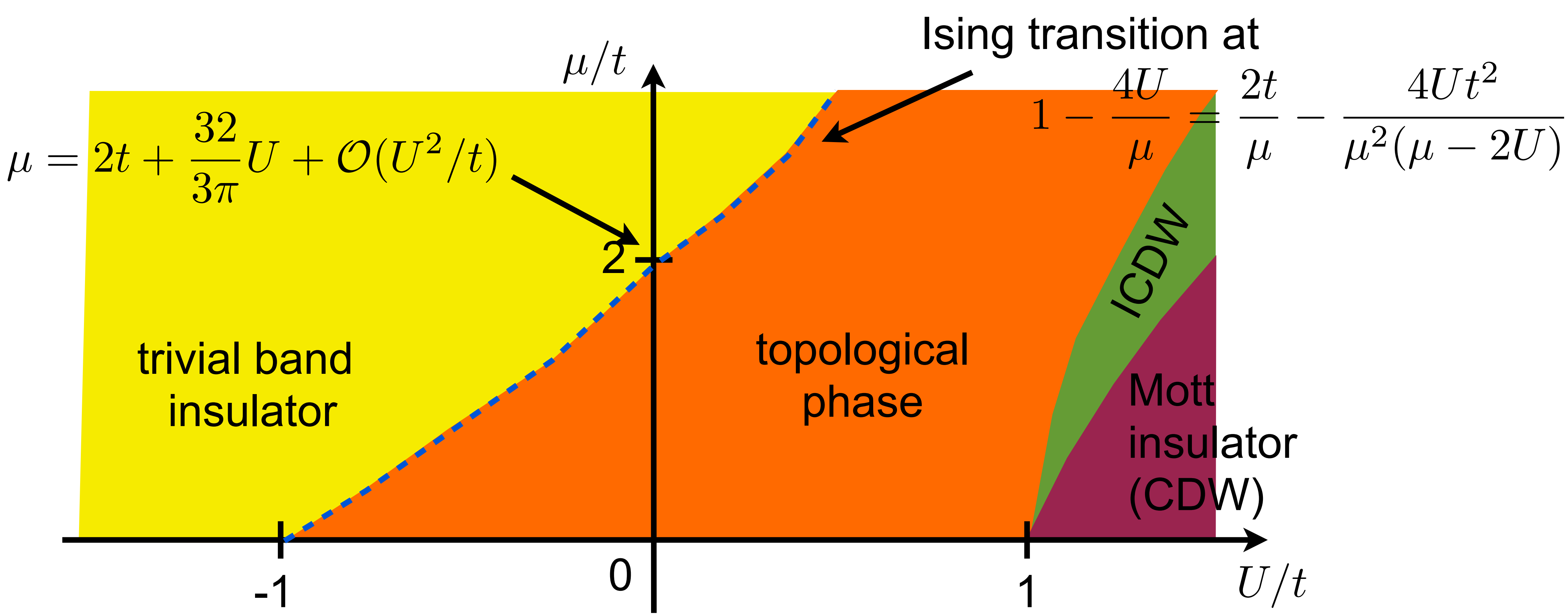


Phase diagram

interacting Majorana chain $H = \mathrm{i}t\sum_i \gamma_{i,b}\gamma_{i+1,a} - \frac{\mathrm{i}}{2}\mu\sum_i \gamma_{i,a}\gamma_{i,b} - U\sum_i \gamma_{i,a}\gamma_{i,b}\gamma_{i+1,a}\gamma_{i+1,b}$

effective spin model $H = -t\sum_i \sigma_i^x\sigma_{i+1}^x - \frac{\mu}{2}\sum_i \sigma_i^z + U\sum_i \sigma_i^z\sigma_{i+1}^z$

duality transformation $H = -t\sum_i \tau_i^z - \frac{\mu}{2}\sum_i \tau_i^x\tau_{i+1}^x + U\sum_i \tau_i^x\tau_{i+2}^x$



Phase diagram

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effective spin model $H = -t \sum_i \sigma_i^x \sigma_{i+1}^x - \frac{\mu}{2} \sum_i \sigma_i^z + U \sum_i \sigma_i^z \sigma_{i+1}^z$

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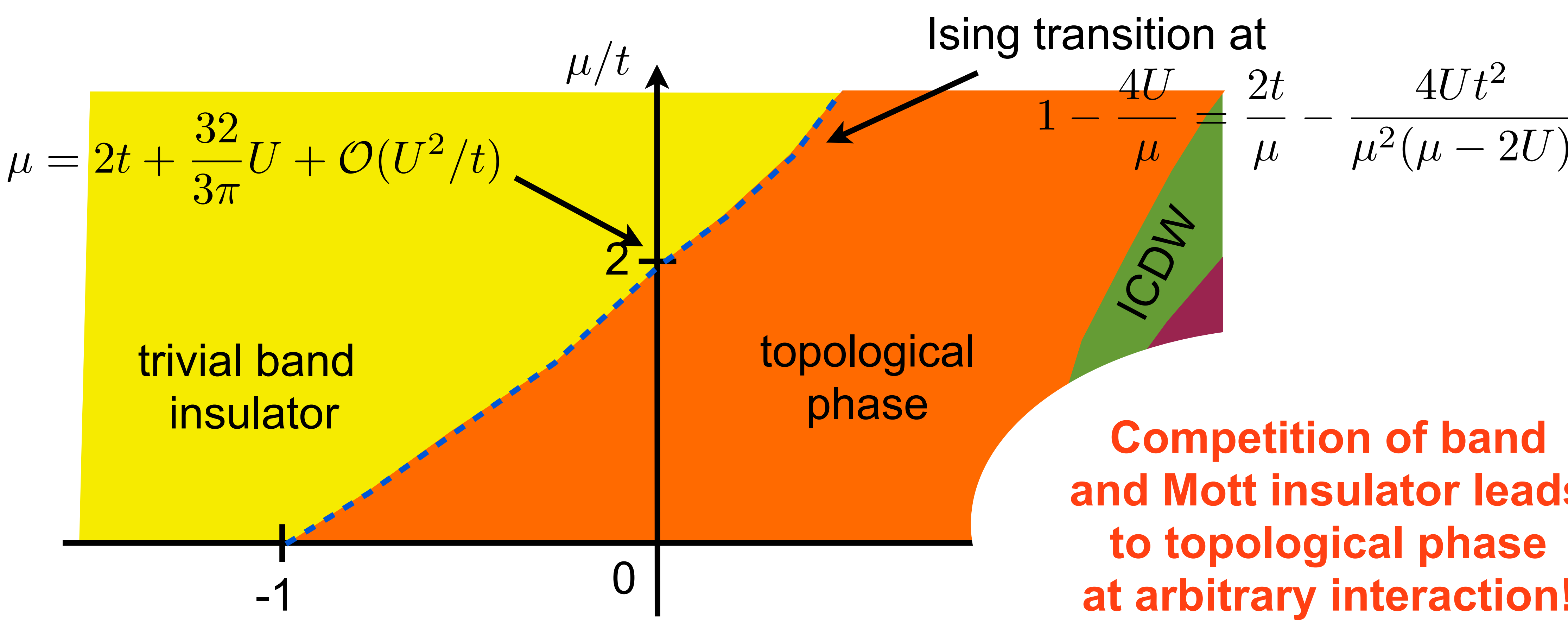


Competition of band and Mott insulator leads to topological phase at arbitrary interaction!

Peschel & Emery,
Z Phys B 1981

Exactly solvable line

along $\mu = 4\sqrt{U(t+U)}$ two-fold degenerate ground state

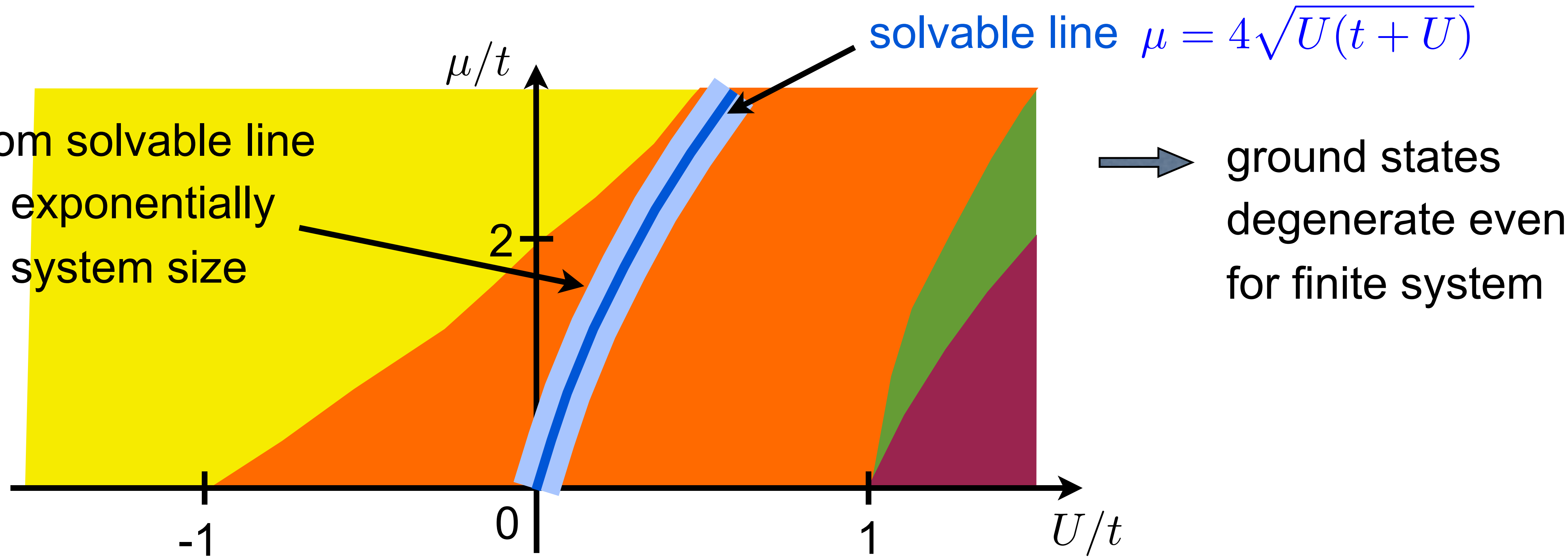
$$|\Psi_\pm\rangle = \prod_i \left(\cos\frac{\theta}{2}\, |+\rangle_i \pm \sin\frac{\theta}{2}\, |-\rangle_i \right), \quad \cos\theta = \frac{\mu}{4(t+U)}$$

fermion parity eigenstates

$$|\Phi_\pm\rangle = \frac{|\Psi_+\rangle \pm |\Psi_-\rangle}{\sqrt{2(1 \pm \cos^N \theta)}}, \quad (-1)^{\mathrm{F}}\, |\Phi_\pm\rangle = \prod_i \sigma_i^z\, |\Phi_\pm\rangle = \pm\, |\Phi_\pm\rangle$$

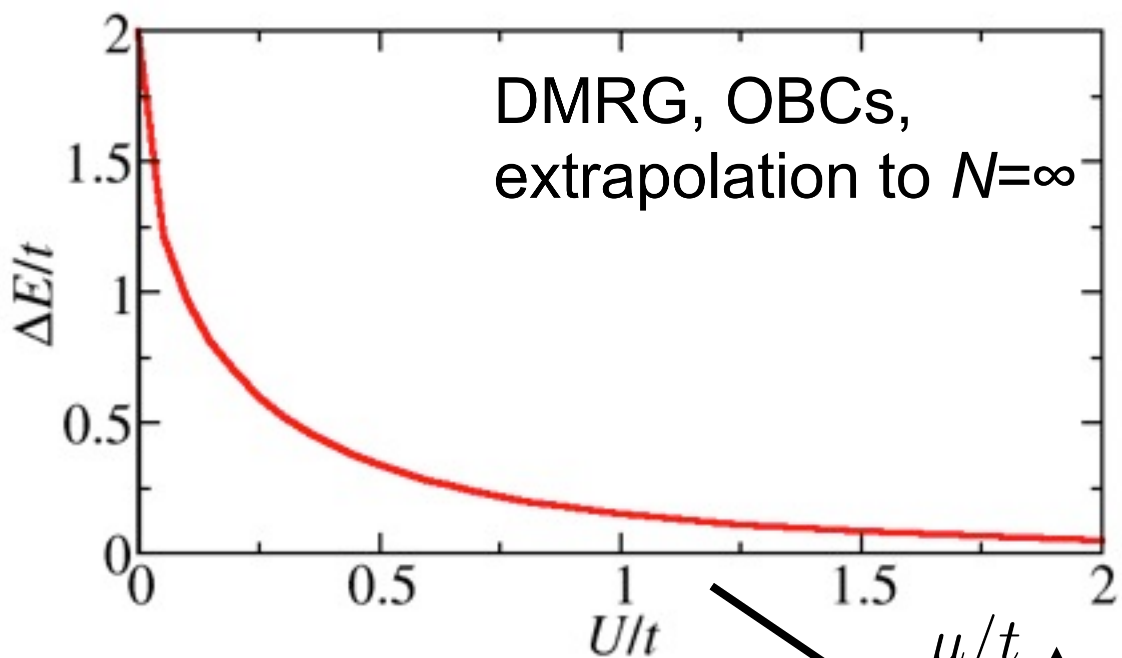
ground states locally (in fermions) indistinguishable, e.g.

$$\langle \Phi_+|\, c_i^\dagger c_i\, |\Phi_+\rangle = \langle \Phi_-|\, c_i^\dagger c_i\, |\Phi_-\rangle\,, \quad \langle \Phi_+|\, c_i c_{i+1}^\dagger\, |\Phi_+\rangle = \langle \Phi_-|\, c_i c_{i+1}^\dagger\, |\Phi_-\rangle$$



Exactly solvable line

on solvable line: gap ΔE above $|\Phi_\pm\rangle$

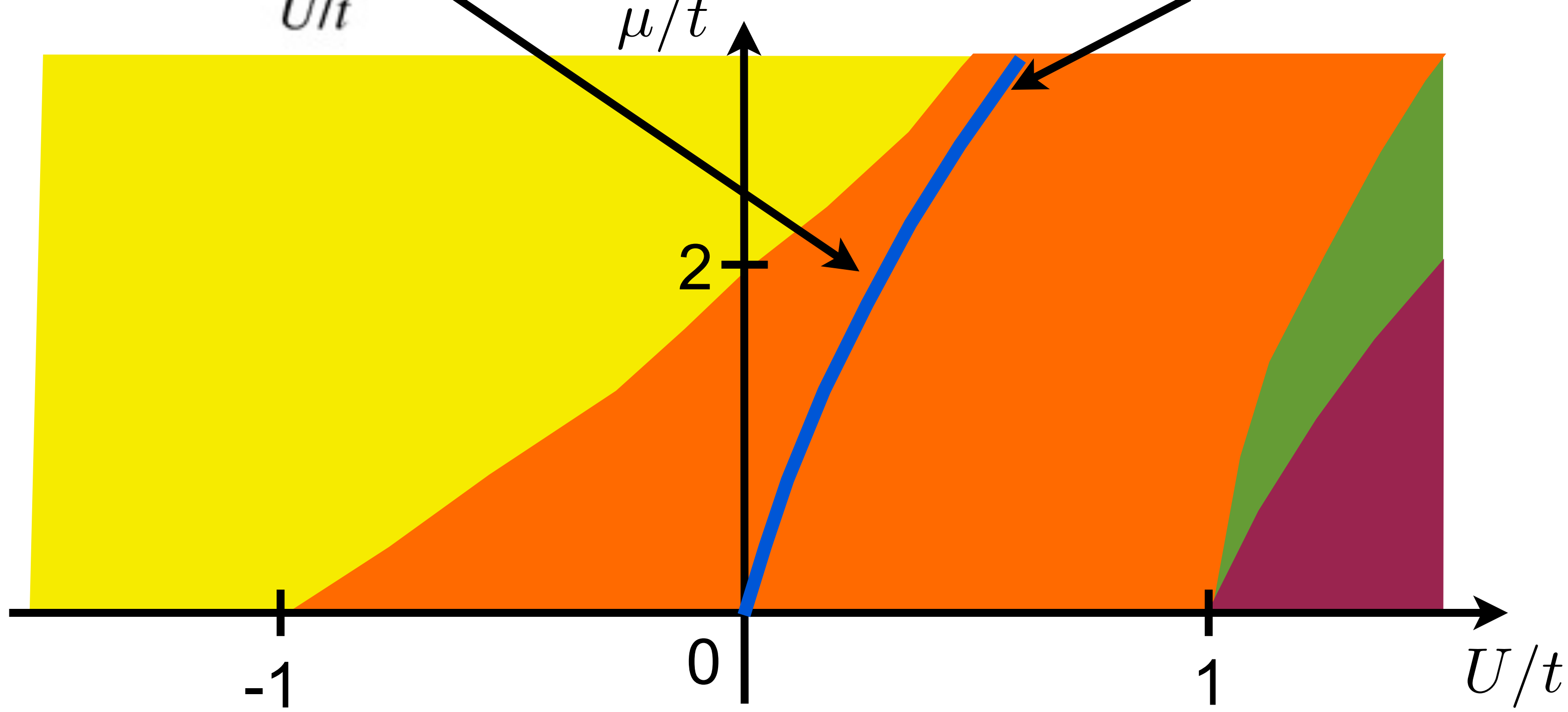


e.g. use Knabe's method
(Knabe, J Stat Phys 1988)

→ $|\Phi_\pm\rangle$ adiabatically connected to ground states of non-interacting system in topological phase

→ topological order

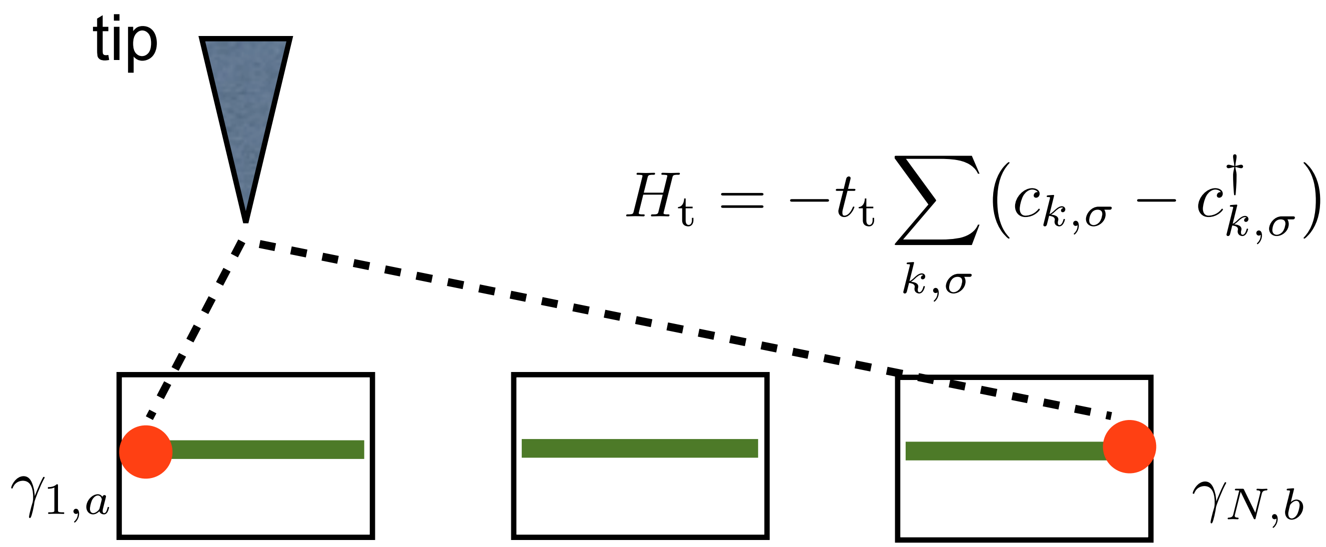
solvable line $\mu = 4\sqrt{U(t+U)}$



can be generalised to $t \neq \Delta$

Tunneling conductance

Law et al., PRL 2009
Flensberg, PRB 2010



$$H_\mathrm{t} = -t_\mathrm{t} \sum_{k,\sigma} \left(c_{k,\sigma} - c_{k,\sigma}^\dagger\right)\left(\gamma_{1,a} + \gamma_{N,b}\right)$$

tunneling rate in wide band limit $\quad \Gamma_0 = 2\pi t_\mathrm{t}^2 \rho_0$

consider effective two-level system

$$H_\mathrm{eff} = -\frac{\mathrm{i}}{2}\Delta E\, \gamma_{1,a}\gamma_{N,b}$$

energy splitting

tunneling rates into Majorana modes

$$\Gamma_{ij} = \Gamma_0\, \psi_i(x=1)\, \psi_j(x=1)$$

Majorana wave functions

tunneling current

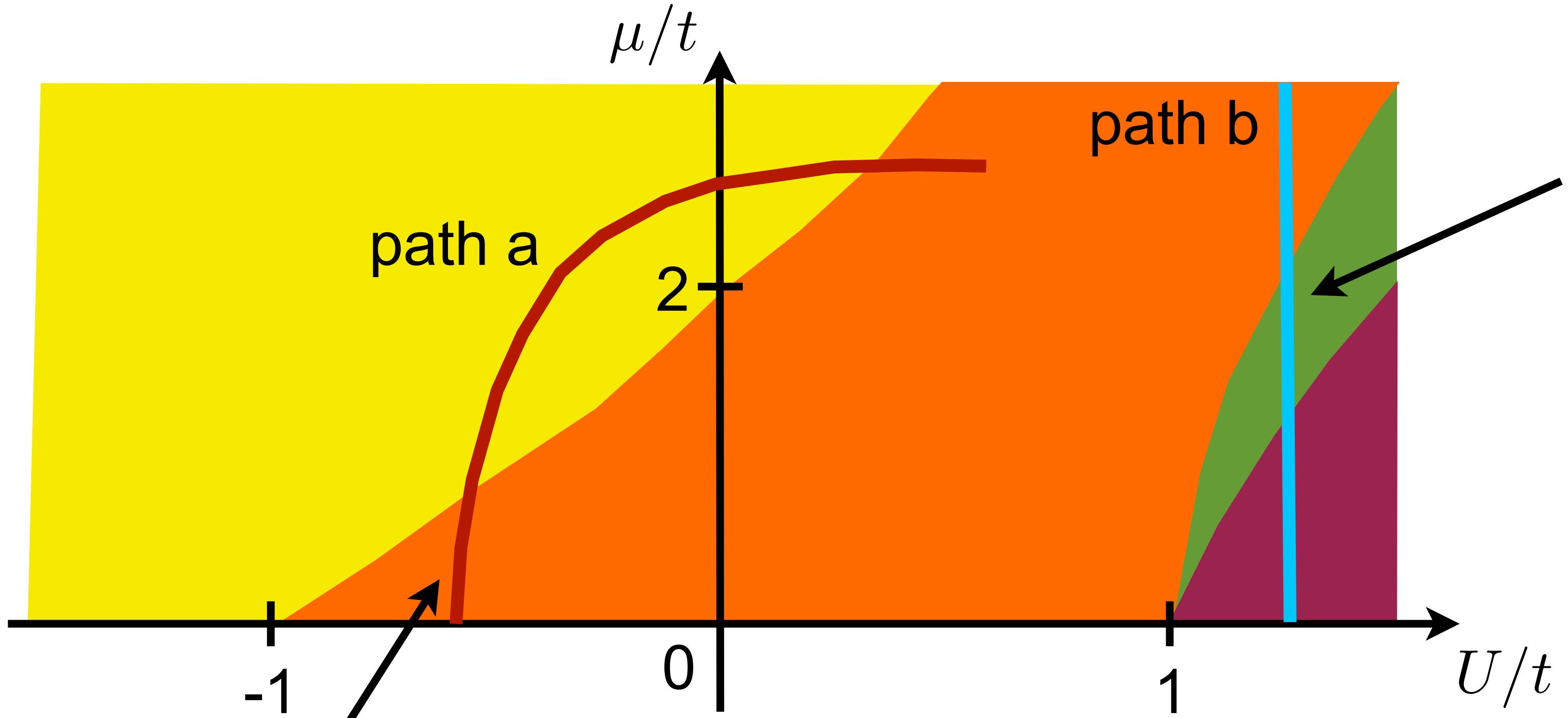
Majorana Green functions

$$I = \frac{e}{h}\int \mathrm{d}\omega\, \mathrm{tr}\left[G^\mathrm{R}(\omega)\Gamma^*(-\omega)G^\mathrm{A}(\omega)\Gamma(\omega)\right]\left[f(eV-\omega) - f(\omega - eV)\right]$$

broadened conductance

$$\bar{G} = \int_{-\Delta\omega/2}^{\Delta\omega/2} \frac{\mathrm{d}(eV)}{\Delta\omega} G(V) \qquad \Delta\omega \sim V, T, \ldots$$

Tunneling conductance

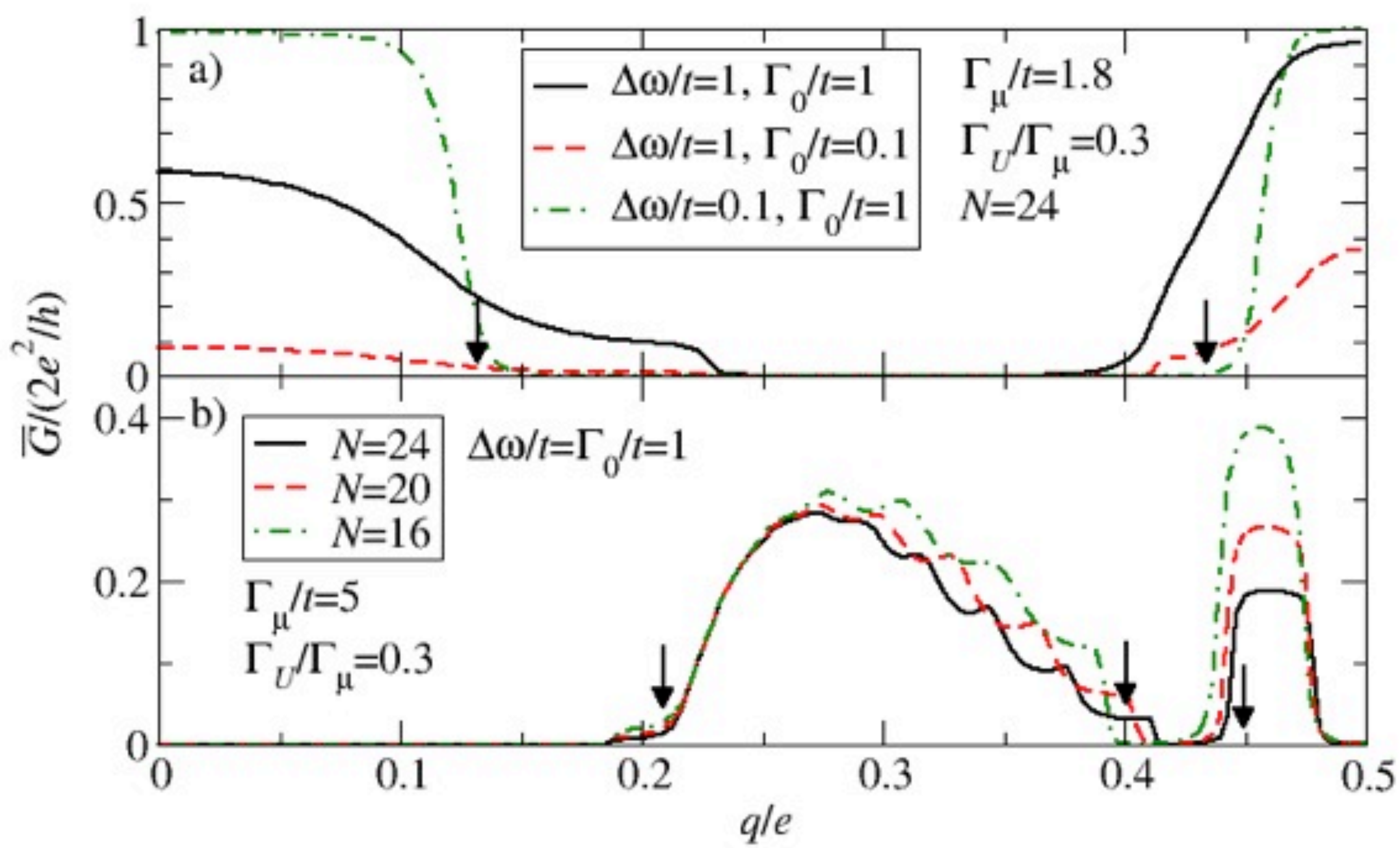


individually tuneable gate voltages necessary

$$q_i = (-1)^i\, q$$
$$\mu = 2\Gamma_\mu \cos(\pi q/e)$$
$$U = \Gamma_U$$

move around in phase diagram by varying induced charge *q*

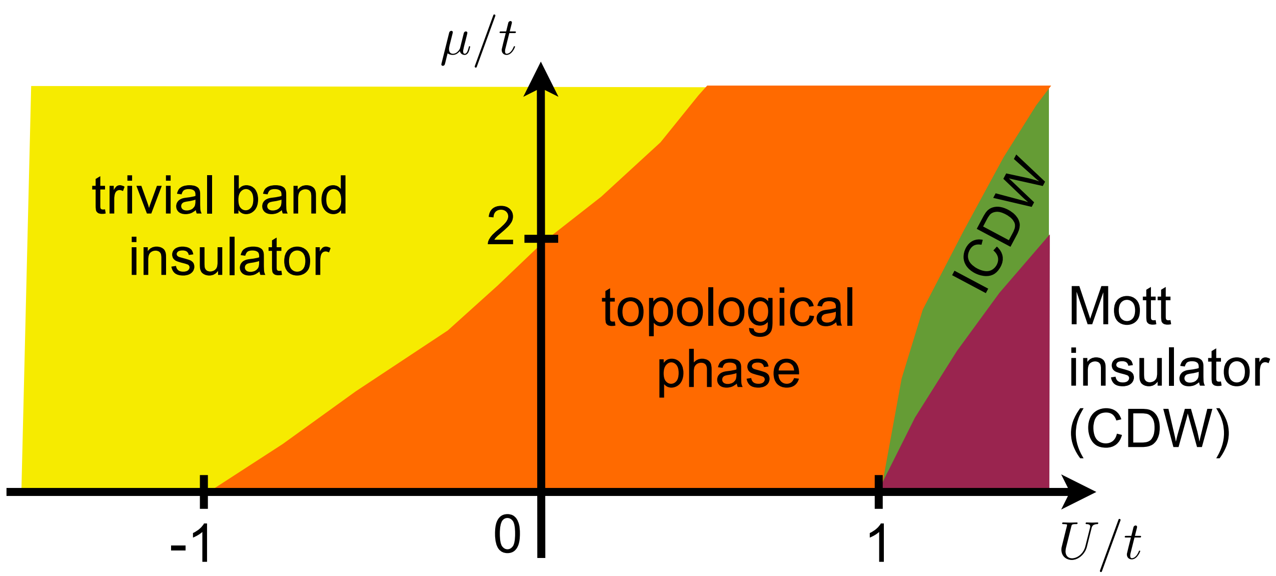
$$\mu = 2\Gamma_\mu \cos(\pi q/e)$$
$$U = \Gamma_U \cos(2\pi q/e)$$



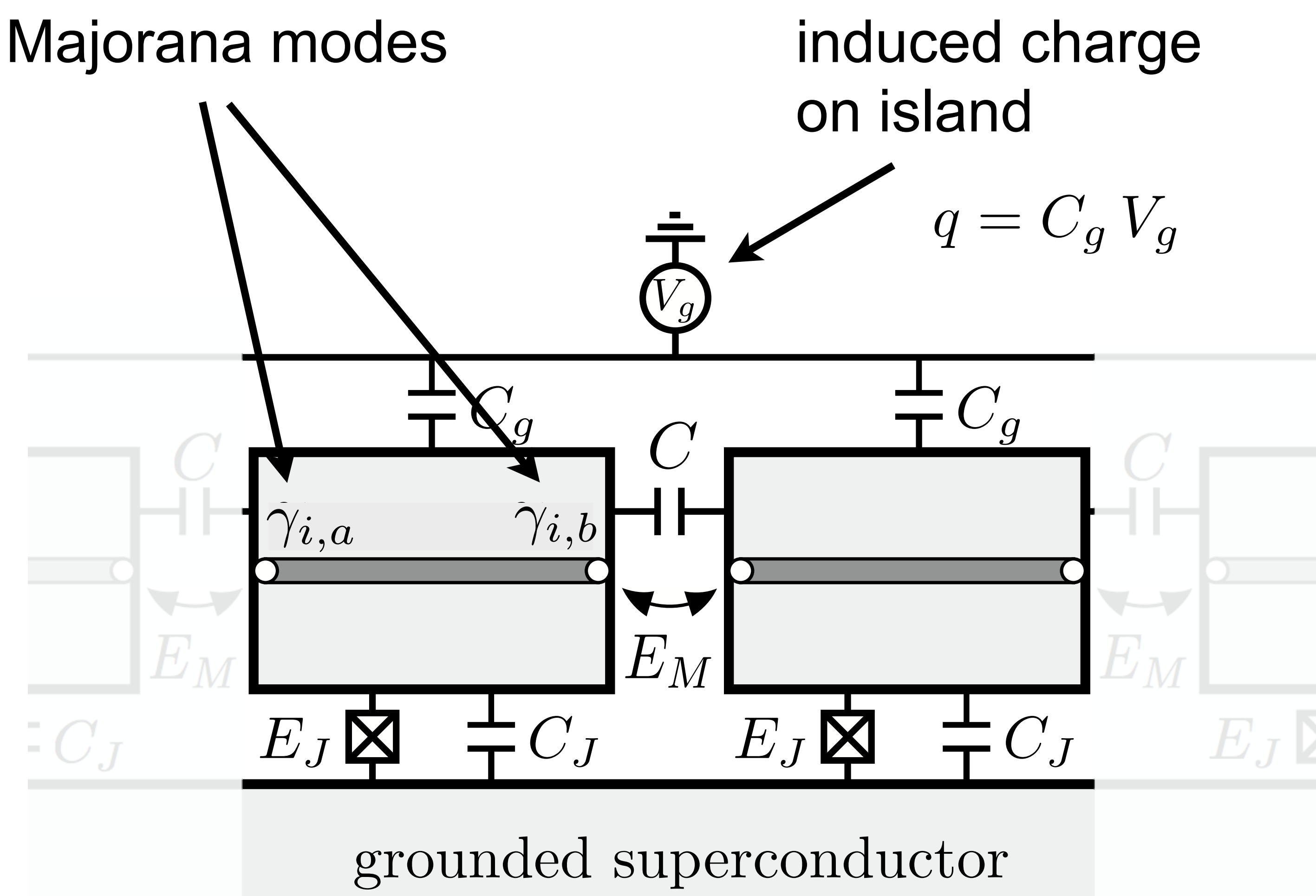
→ phase boundaries clearly visible

Conclusions

- studied effect of interactions on Majorana chain
- chemical potential and strong interactions individually destroy topological phase
- competition between band and Mott insulator leads to topological phase at arbitrary interactions
- Josephson junction array with Majorana fermions implements ANNNI model
- explicit results along Peschel-Emery line
- signatures of phases in tunneling conductance
- reference (for parts of this talk): Hassler & Schuricht, New J Phys 14, 125018 (2012)



Appendix: Josephson junction array



$$H_M = \mathrm{i}\, t \sum_i \gamma_{i,b}\gamma_{i+1,a} \cos\frac{\phi_i - \phi_{i+1}}{2}$$

$$H_J = E_J \sum_i (1 - \cos\phi_i)$$

$$E_J \gg E_M, E_C$$

→ superconducting phases pinned

relation of parameters

$$t = E_M$$

$$\mu = 2\Gamma_\mu \cos(\pi q/e)$$

$$U = \Gamma_U \cos(2\pi q/e)$$

quasi-classical calculation of quantum phase slip rates yields

$$\Gamma_\mu \simeq E_C^{1/4} E_J^{3/4} e^{-\sqrt{8E_J/E_C}[1+(\pi^2-12)\eta^2/96]} + \ldots$$

$$\Gamma_U \simeq E_C^{1/4} E_J^{3/4} e^{-\sqrt{16(2-\eta)E_J/E_C}} + \ldots$$

$$\eta = \frac{2C}{2C + C_g + C_J}$$

Xu & Fu, PRB 2010

Appendix: Exactly solvable line

open boundary conditions

$$H = \sum_{i=1}^{N-1} h_i, \quad h_i = -t\sigma_i^x \sigma_{i+1}^x + U\sigma_i^z \sigma_{i+1}^z - \frac{\mu}{4}\left(\sigma_i^z + \sigma_{i+1}^z\right)$$

chemical potential halved at boundary

local Green function

$$\langle \Phi_\pm | \, c_i c_{i+1}^\dagger \, | \Phi_\pm \rangle = -\frac{1}{4} \frac{\sin^2\theta}{1 \pm \cos^N\theta}\left(1 \mp \cos^{N-2}\theta\right)$$

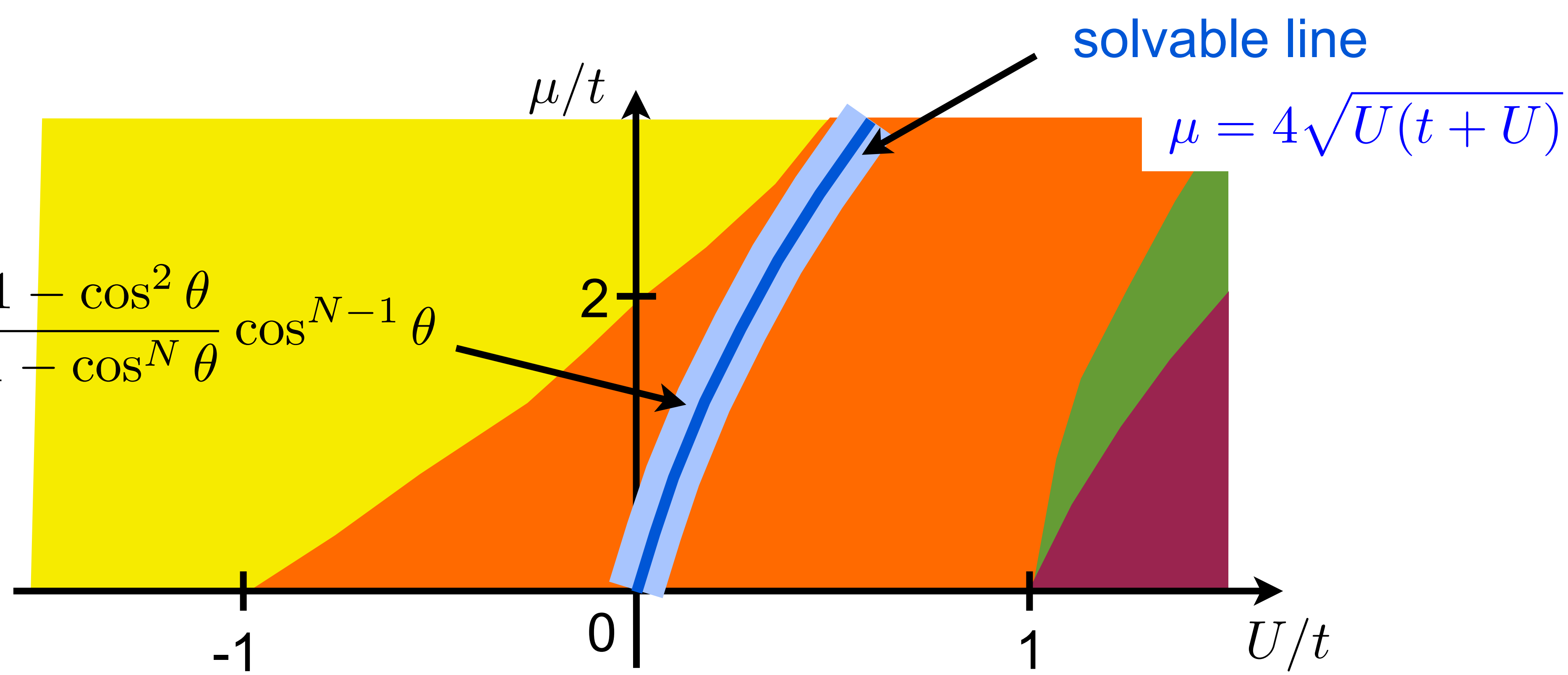
exponentially small in *N*

$$\cos\theta = \frac{\mu}{4(t+U)} = \frac{\sqrt{U(t+U)}}{t+U} < 1$$

perturbation

$$V = -\frac{\delta\mu}{2}\sum_i \sigma_i^z$$

$$\frac{\Delta E}{N} = -\frac{\delta\mu}{1+\cos^N\theta}\,\frac{1-\cos^2\theta}{1-\cos^N\theta}\cos^{N-1}\theta$$



Knabe,
J Stat Phys 1988

Appendix: Knabe's method

rescale Hamiltonian $$\tilde{H} = \sum_{i=1}^{N-1} \tilde{h}_i, \quad \tilde{h}_i = \frac{1}{2(t+2U)}(h_i + t + U)$$

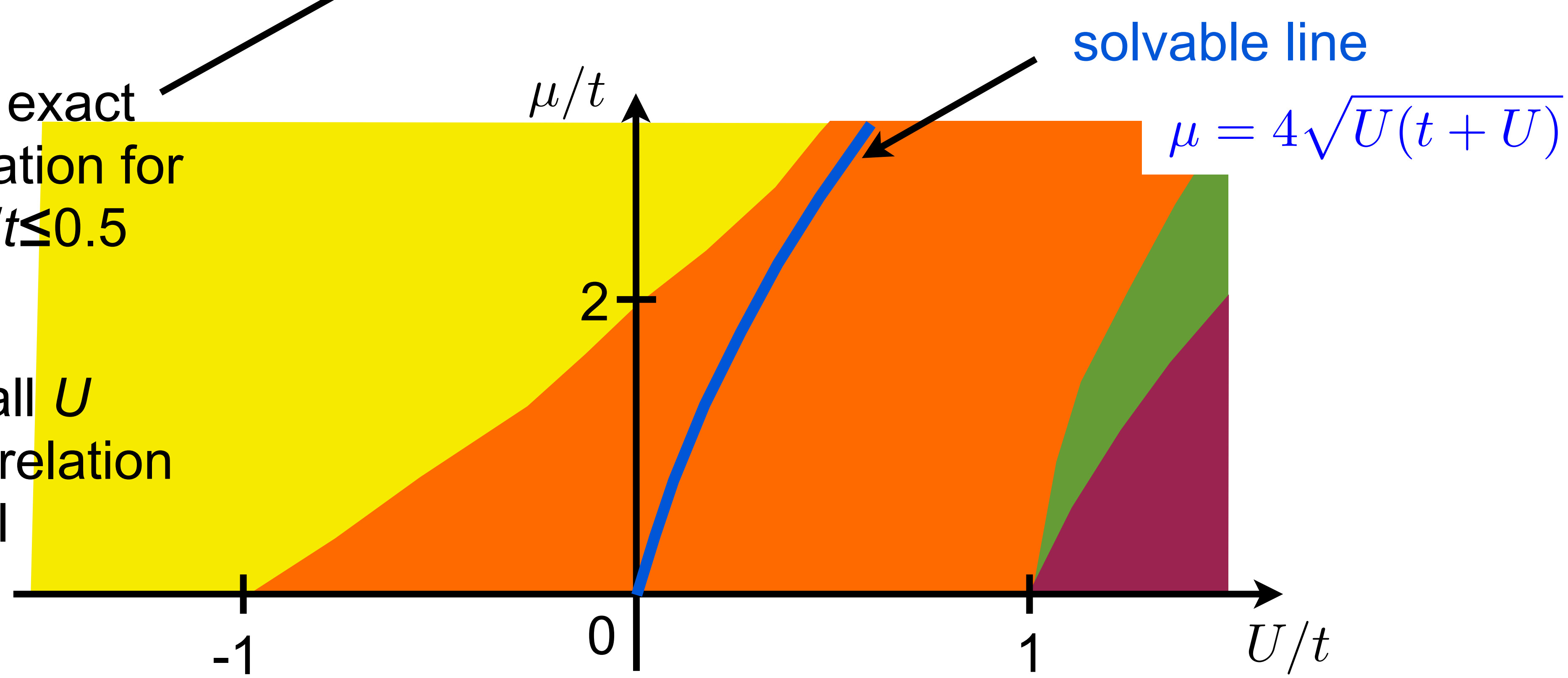
$\Longrightarrow$ $0 \leq \tilde{h}_i \leq 1 \;\Rightarrow\; \tilde{h}_i^2 \leq \tilde{h}_i$ in the sense $\langle\Psi|\,\tilde{h}_i^2\,|\Psi\rangle \leq \langle\Psi|\,\tilde{h}_i\,|\Psi\rangle \leq \langle\Psi|\,\Psi\rangle$

if (m+1)-site Hamiltonian $\tilde{H}_m = \sum_{i=1}^{m} \tilde{h}_i$ satisfies $\tilde{H}_m^2 \geq \epsilon_m \tilde{H}_m, \; \epsilon_m > \frac{1}{m}$

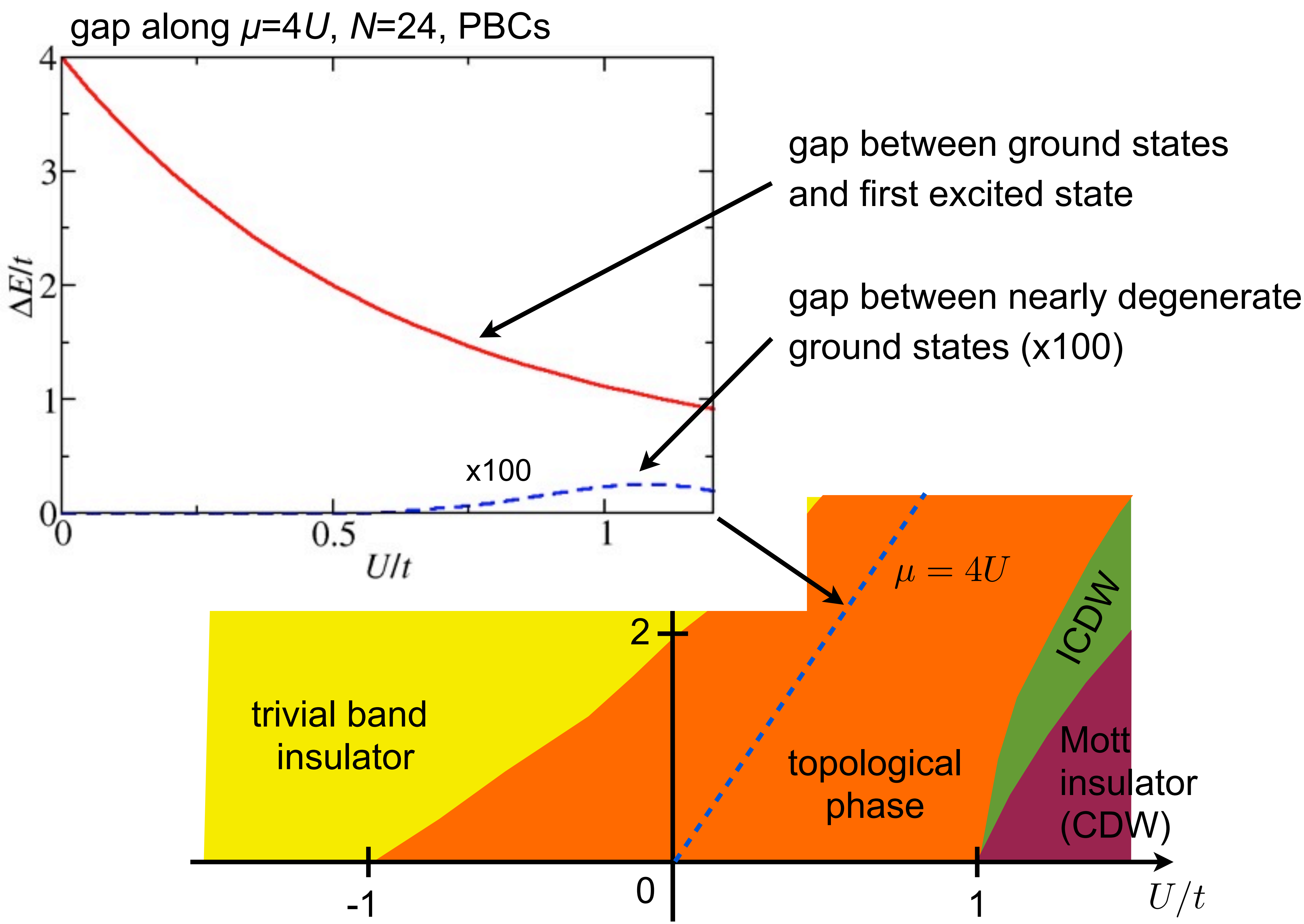
$\Longrightarrow$ $\tilde{H}^2 \geq \epsilon \tilde{H}, \quad \epsilon = \frac{m}{m-1}\left(\epsilon_m - \frac{1}{m}\right) > 0$ $\Longrightarrow$ gap of H $\Delta E > 2(t+2U)\epsilon$

verified by exact diagonalisation for m=9, 0≤U/t≤0.5

existence of gap for all U can be proven using relation to free-fermion model



Appendix: Gap in topological phase



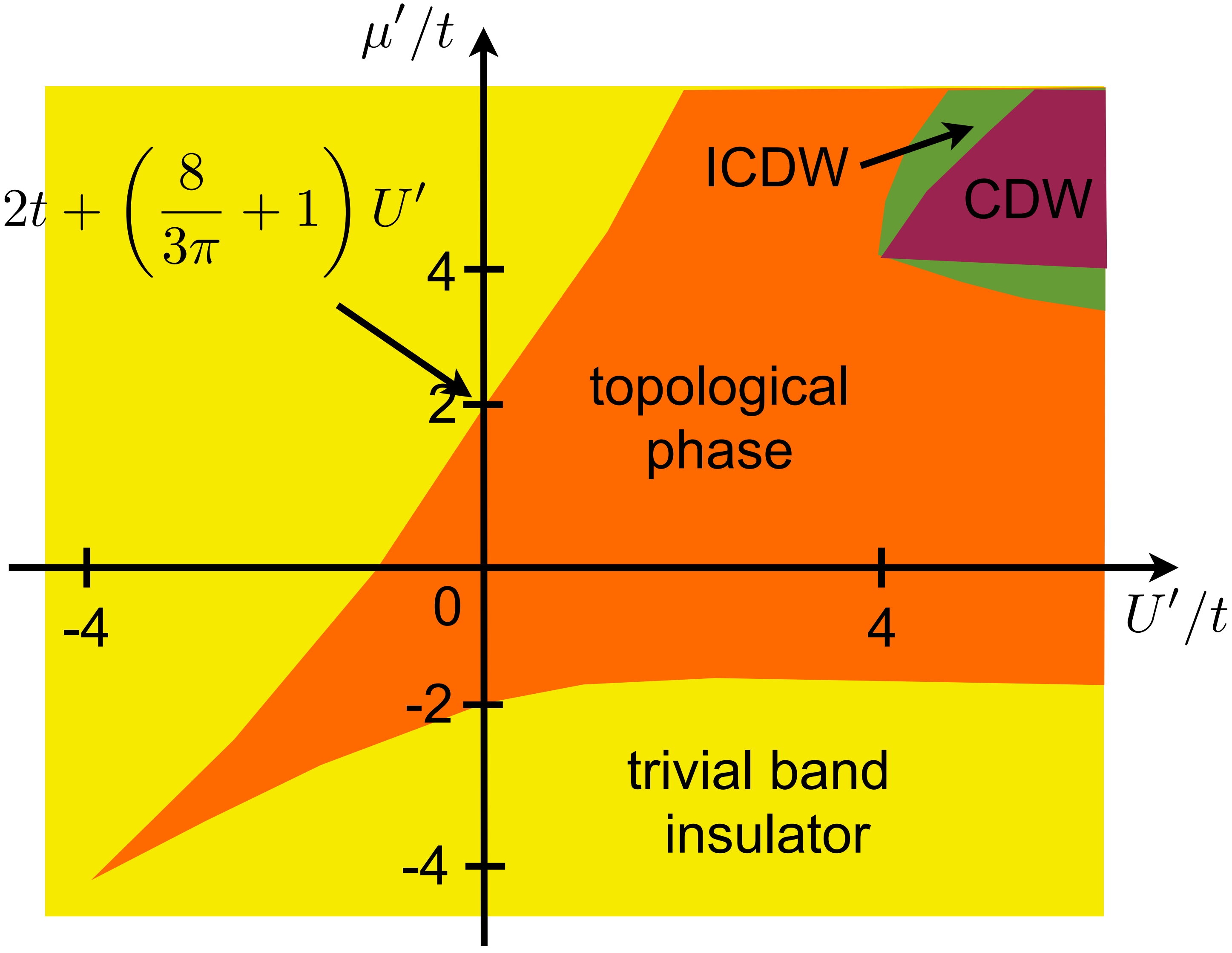
Appendix: Broken particle-hole symmetry

$$H' = -t\sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i - c_i c_{i+1} - c_{i+1}^\dagger c_i^\dagger) - \mu' \sum_i c_i^\dagger c_i + U' \sum_i c_i^\dagger c_i c_{i+1}^\dagger c_{i+1}$$

mapping to Majorana fermions

$$H' = \mathrm{i}t \sum_i \gamma_{i,b}\gamma_{i+1,a} - \frac{\mathrm{i}}{2}(\mu' - U') \sum_i \gamma_{i,a}\gamma_{i,b} - \frac{U'}{4}\sum_i \gamma_{i,a}\gamma_{i,b}\gamma_{i+1,a}\gamma_{i+1,b} + \mathrm{const}$$

$$\mu' \approx 2t + \left(\frac{8}{3\pi} + 1\right) U'$$



relation of parameters

$$\mu = \mu' - U'$$

$$U = U'/4$$

phase diagram invariant under

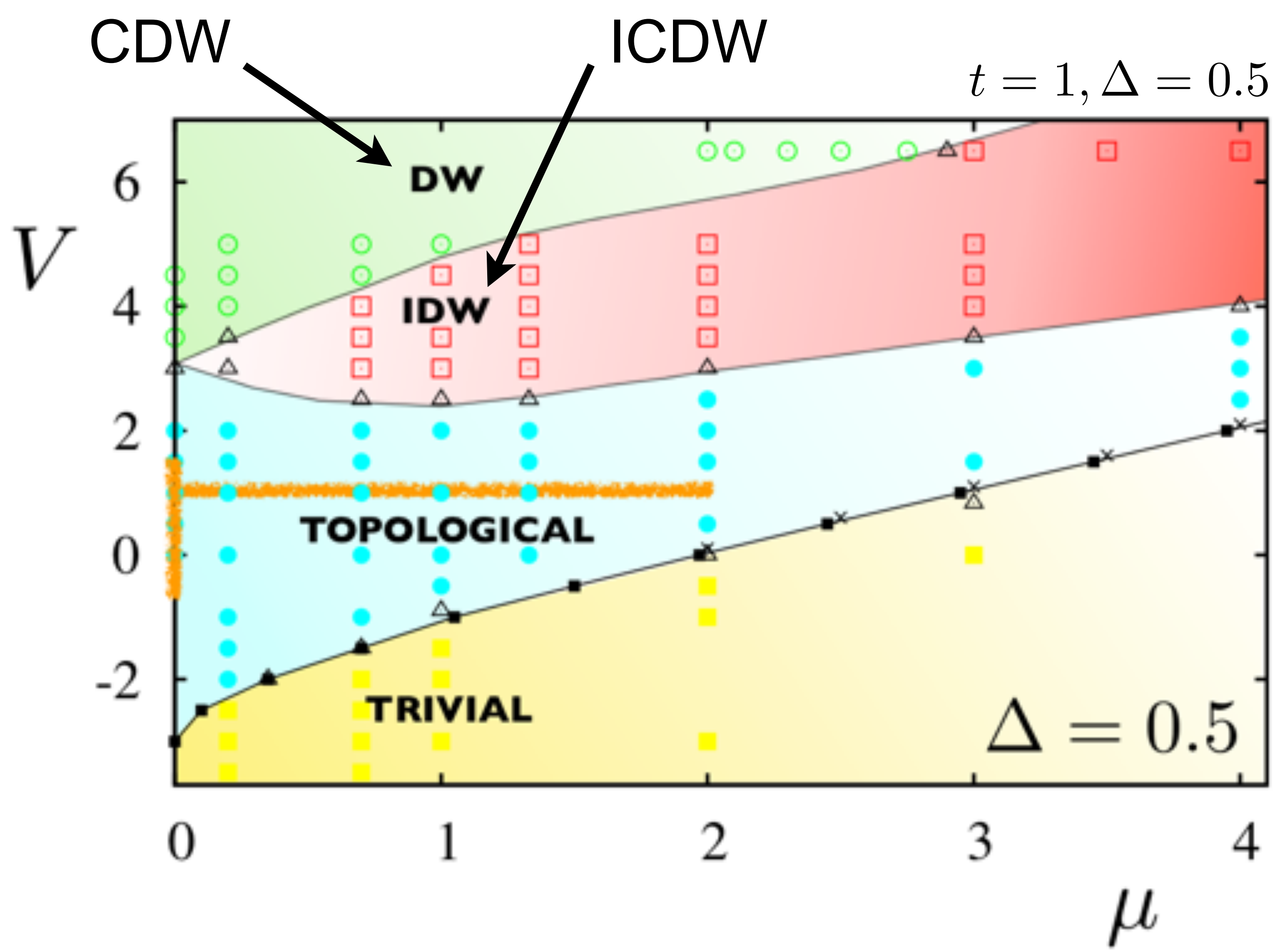
$$(U', \mu') \to (U', -\mu' + 2U)$$

remark: *H'* is not invariant under particle-hole transformations $c_i \to (-1)^i c_i^\dagger$

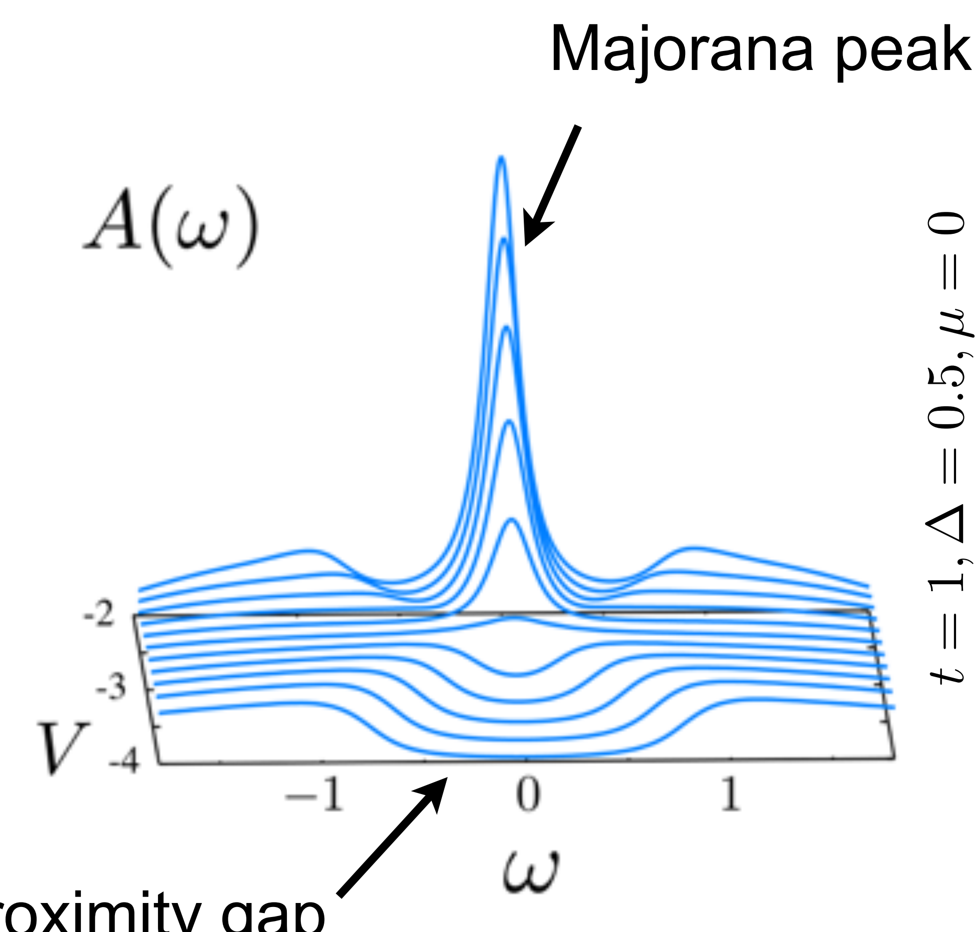
Appendix: DMRG study

Thomale et al., PRB 2013

$$H = \sum_i \left[-t\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i\right) + \Delta\left(c_i c_{i+1} + c_{i+1}^\dagger c_i^\dagger\right)\right] - \mu \sum_i c_i^\dagger c_i + V \sum_i c_i^\dagger c_i c_{i+1}^\dagger c_{i+1}$$



phase diagram very similar to our finding for $t=\Delta$



DMRG study for up to 96 sites investigated also $t \neq \Delta$

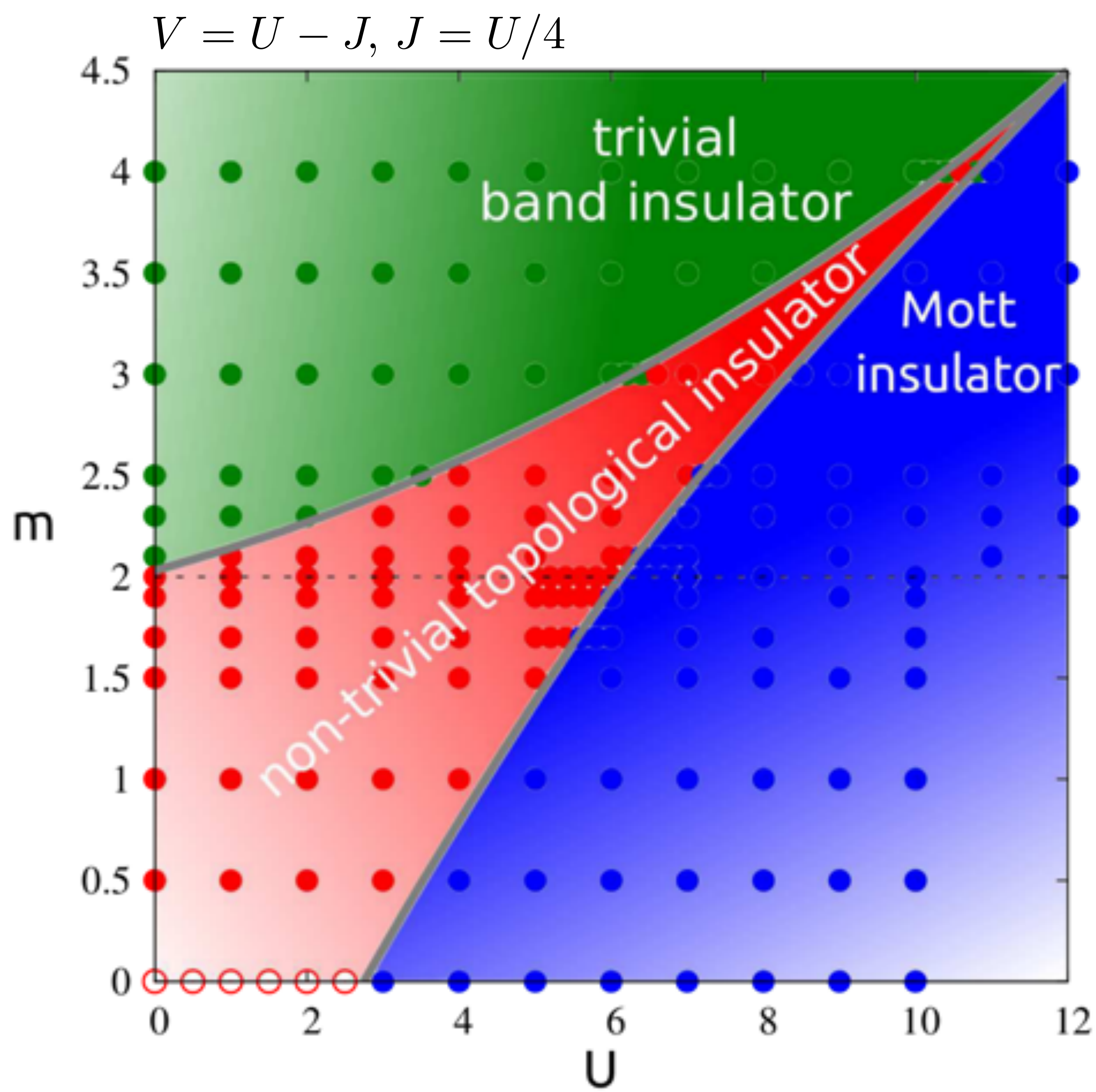
mean-field treatment: Manolescu et al., J Phys: Condens Matter 2014

Appendix: Hund insulator

Budich et al., PRB 2013

$$H = H_{\mathrm{BHZ}} + H_U + H_V + H_J$$

- H_{BHZ}: electron and hole orbital
- H_U: intra-band interaction
- H_V: inter-band interaction
- H_J: Hund coupling



model for HgTe quantum wells

DMFT study of self energy

system can be driven into topological phase by interactions

relation to interacting Majorana chain unclear