THE POWER OF MEASUREMENT IN QUANTUM CONTROL

18/8/14 NORDITA

DANIEL KLAUS BURGARTH

ABERYSTWYTH UNIVERSITY

WITH VITTORIO GIOVANNETTI, PAOLO FACCHI, SAVERIO PASCAZIO, KAZUYA YUASA, HIROMICHI NAKAZATO

ARXIV: 1403.5752

OUTLINE

PLATO'S CAVE

MEASUREMENT IN QM

COMPLEXITY OF DYNAMICS

QUANTUM ZENO

HAMILTONIAN PURIFICATION

ZENO MEETS PLATO

MANY-BODY IMPLEMENTATION

CONCLUSIONS

PLATO'S CAVE

PLATO'S CAVE

PLATO'S CAVE

PLATO'S CAVE

PLATO'S CAVE

PLATO'S CAVE

PROJECTIONS ARE POWERFUL :

PLATO'S CAVE

PROJECTIONS ARE POWERFUL :

PLATO'S CAVE

PROJECTIONS ARE POWERFUL :

IN THIS TALK : QUANTUM VERSION

QUANTUM PROJECTIONS

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: BACKACTION

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: <u>BACKACTION</u>

UNCERTAINTY PRINCIPLE

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: BACKACTION

UNCERTAINTY PRINCIPLE 

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: BACKACTION

UNCERTAINTY PRINCIPLE 

SUITABLE INITIAL STATE:

QUANTUM COMPUTING

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: BACKACTION

UNCERTAINTY PRINCIPLE 

SUITABLE INITIAL STATE:

QUANTUM COMPUTING 

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: BACKACTION

UNCERTAINTY PRINCIPLE 

SUITABLE INITIAL STATE:

QUANTUM COMPUTING 

$\boxed{\text{HOW DO MEASUREMENTS } \underline{\text{GENERALLY}} \text{ CHANGE A SYSTEM ?}}$

QUANTIFYING DYNAMICAL COMPLEXITY

QUANTIFYING DYNAMICAL COMPLEXITY

HARD TASK IN GENERAL;

QUANTIFYING DYNAMICAL COMPLEXITY

HARD TASK IN GENERAL;

CONTROLLED HAMILTONIAN DYNAMICS:

QUANTIFYING DYNAMICAL COMPLEXITY

HARD TASK IN GENERAL;

CONTROLLED HAMILTONIAN DYNAMICS:

$$f(t)\, H_1 + g(t)\, H_2$$

QUANTIFYING DYNAMICAL COMPLEXITY

HARD TASK IN GENERAL;

CONTROLLED HAMILTONIAN DYNAMICS:

$$f(t)\, H_1 + g(t)\, H_2$$

CONTROLS

QUANTIFYING DYNAMICAL COMPLEXITY

HARD TASK IN GENERAL;

CONTROLLED HAMILTONIAN DYNAMICS:

$$f(t)\,H_1 + g(t)\,H_2$$

CONTROLS

$L(iH_1, iH_2) \equiv$ SMALLEST REAL LIE ALGEBRA CONTAINING iH_1, iH_2

QUANTIFYING DYNAMICAL COMPLEXITY

HARD TASK IN GENERAL;

CONTROLLED HAMILTONIAN DYNAMICS:

$$f(t)\, H_1 + g(t)\, H_2$$

CONTROLS

$L(iH_1, iH_2) \equiv$ SMALLEST REAL LIE ALGEBRA CONTAINING iH_1, iH_2

$\dim L$

QUANTIFYING DYNAMICAL COMPLEXITY

HARD TASK IN GENERAL;

CONTROLLED HAMILTONIAN DYNAMICS:

$$f(t)\, H_1 + g(t)\, H_2$$

CONTROLS

$L(iH_1, iH_2) \equiv$ SMALLEST REAL LIE ALGEBRA CONTAINING iH_1, iH_2

$\dim L \sim$ COMPLEXITY

QUANTIFYING DYNAMICAL COMPLEXITY

HARD TASK IN GENERAL;

CONTROLLED HAMILTONIAN DYNAMICS:

$$\underbrace{f(t)}_{\text{CONTROLS}} H_1 + \underbrace{g(t)}_{\text{CONTROLS}} H_2$$

$L(iH_1, iH_2) \equiv$ SMALLEST REAL LIE ALGEBRA CONTAINING iH_1, iH_2

$\dim L \sim$ COMPLEXITY $\sim$ QUANTUM COMPUTATION

QUANTIFYING DYNAMICAL COMPLEXITY

HARD TASK IN GENERAL;

CONTROLLED HAMILTONIAN DYNAMICS:

$$f(t)\, H_1 + g(t)\, H_2$$

CONTROLS

$L(iH_1, iH_2) \equiv$ SMALLEST REAL LIE ALGEBRA CONTAINING iH_1, iH_2

$\dim L \sim$ COMPLEXITY $\sim$ QUANTUM COMPUTATION

$$\boxed{\text{WHAT HAPPENS IF WE CAN ALSO APPY } P^2 = P}$$

QUANTUM ZENO EFFECT

QUANTUM ZENO EFFECT

$$\left(1 + \frac{X}{n}\right)^n \longrightarrow e^X$$

QUANTUM ZENO EFFECT

$X_n \to X$: $\left(1 + \frac{X_n}{n}\right)^n \longrightarrow e^X$

QUANTUM ZENO EFFECT

$X_n \to X$:
$$\left(1 + \frac{X_n}{n}\right)^n \longrightarrow e^X$$
X_n, X MATRICES

QUANTUM ZENO EFFECT

$X_n \to X$: $\quad \left(1 + \frac{X_n}{n}\right)^n \longrightarrow e^X \quad X_n, X$ MATRICES

$$\left(P \exp\left(\frac{iHt}{n}\right) P\right)^n$$

QUANTUM ZENO EFFECT

$X_n \to X$: $\quad \left(1 + \frac{X_n}{n}\right)^n \longrightarrow e^X \qquad X_n, X$ MATRICES

$$\left(P \exp\left(\frac{iHt}{n}\right) P\right)^n = \left(P + \frac{iPHPt + \mathcal{O}(1/n)}{n}\right)^n$$

QUANTUM ZENO EFFECT

$X_n \to X$: $\quad \left(1 + \frac{X_n}{n}\right)^n \longrightarrow e^X \quad X_n, X$ MATRICES

$$\left(P \exp\left(\frac{iHt}{n}\right) P\right)^n = \left(P + \frac{iPHPt + O(1/n)}{n}\right)^n$$

$$= P\left(1 + \frac{iPHPt \quad + O(1/n)}{n}\right)^n P$$

QUANTUM ZENO EFFECT

$X_n \to X$: $\left(1 + \frac{X_n}{n}\right)^n \longrightarrow e^X$ X_n, X MATRICES

$$\left(P \exp\left(\frac{iHt}{n}\right) P\right)^n = \left(P + \frac{iPHPt + O(1/n)}{n}\right)^n$$

$$= P\left(1 + \frac{\overbrace{iPHPt}^{X_n} + O(1/n)}{n}\right)^n P$$

QUANTUM ZENO EFFECT

$X_n \to X$: $\quad \left(1 + \frac{X_n}{n}\right)^n \longrightarrow e^X \qquad X_n, X$ MATRICES

$$\left(P \exp\left(\frac{iHt}{n}\right) P\right)^n = \left(P + \frac{iPHPt + O(1/n)}{n}\right)^n$$

$$= P\left(1 + \frac{\overbrace{iPHPt + O(1/n)}^{X_n}}{n}\right)^n P \longrightarrow P \exp(iPHPt) P$$

QUANTUM ZENO EFFECT

$X_n \to X$:
$$\left(1 + \frac{X_n}{n}\right)^n \longrightarrow e^X \qquad X_n, X \text{ MATRICES}$$

$$\left(P \exp\left(\frac{iHt}{n}\right) P\right)^n = \left(P + \frac{iPHPt + O(1/n)}{n}\right)^n$$

$$= P\left(1 + \frac{\overbrace{iPHPt \quad + O(1/n)}^{X_n}}{n}\right)^n P \longrightarrow P\exp(iPHPt)P$$

$$\boxed{\text{FREQUENT MEASUREMENTS MODIFY THE HAMILTONIAN} \quad H \to PHP}$$

COMPLEXITY OF ZENO DYNAMICS

ZENO DYNAMICS TAKES PLACE IN SMALLER SPACE

PRP. IS IT <u>SIMPLER</u>?

COMPLEXITY OF ZENO DYNAMICS

ZENO DYNAMICS TAKES PLACE IN SMALLER SPACE

$P\mathcal{H}P$. IS IT SIMPLER?

DEPENDS ON RANK OF P:

COMPLEXITY OF ZENO DYNAMICS

ZENO DYNAMICS TAKES PLACE IN SMALLER SPACE $P\mathcal{H}P$. IS IT <u>SIMPLER</u>?

DEPENDS ON RANK OF P:

$P = |\phi\rangle\langle\phi|$ $\rightarrow$ $H_Z = PHP$ TRIVIAL ON $P\mathcal{H}P$

COMPLEXITY OF ZENO DYNAMICS

ZENO DYNAMICS TAKES PLACE IN SMALLER SPACE $P\mathcal{H}P$. IS IT SIMPLER?

DEPENDS ON RANK OF P:

$P = |\phi\rangle\langle\phi|$ $\rightarrow$ $H_Z = PHP$ TRIVIAL ON $P\mathcal{H}P$

"ZENO'S PARADOX"

COMPLEXITY OF ZENO DYNAMICS

ZENO DYNAMICS TAKES PLACE IN SMALLER SPACE $P\mathcal{H}P$. IS IT SIMPLER?

DEPENDS ON RANK OF P:

$P = |\phi\rangle\langle\phi| \rightarrow H_Z = PHP$ TRIVIAL ON $P\mathcal{H}P$

"ZENO'S PARADOX"

EXAMPLE $P = |\phi\rangle\langle\phi|_1 \otimes \mathbb{1}_2$, RANK 2:

COMPLEXITY OF ZENO DYNAMICS

ZENO DYNAMICS TAKES PLACE IN SMALLER SPACE $P\mathcal{H}P$. IS IT SIMPLER?

DEPENDS ON RANK OF P:

$P = |\phi\rangle\langle\phi|$ $\quad\rightarrow\quad$ $H_Z = PHP$ TRIVIAL ON $P\mathcal{H}P$

"ZENO'S PARADOX"

EXAMPLE $\quad P = |\phi\rangle\langle\phi|_1 \otimes \mathbb{1}_2$, $\quad$ RANK 2:

$|\phi\rangle$ qubit: $\quad \langle\phi|X|\phi\rangle = \langle\phi|Z|\phi\rangle = 1/\sqrt{2}$

$H_1 = X_1 X_2$

$$H_1 = X_1 X_2$$

$$H_2 = Z_1 Z_2$$

$H_1 = X_1 X_2$

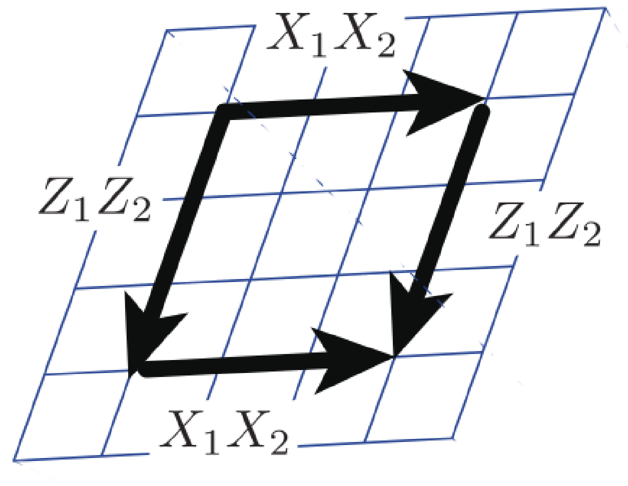
$H_2 = Z_1 Z_2$

$\Rightarrow \quad \dim L(i X_1 X_2, \; i Z_1 Z_2) = 2$

$H_1 = X_1 X_2$

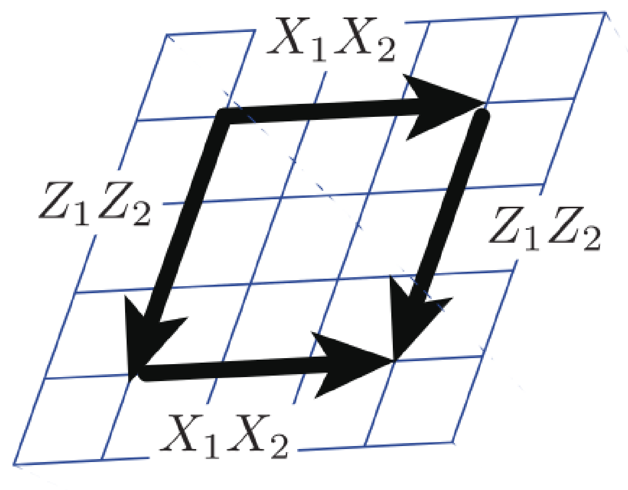
$H_2 = Z_1 Z_2$

$$\Rightarrow \quad \dim L(iX_1X_2,\ iZ_1Z_2) = 2$$



$$H_1 = X_1 X_2 \qquad \Rightarrow \quad \dim L(i X_1 X_2,\ i Z_1 Z_2) = 2$$

$$H_2 = Z_1 Z_2$$

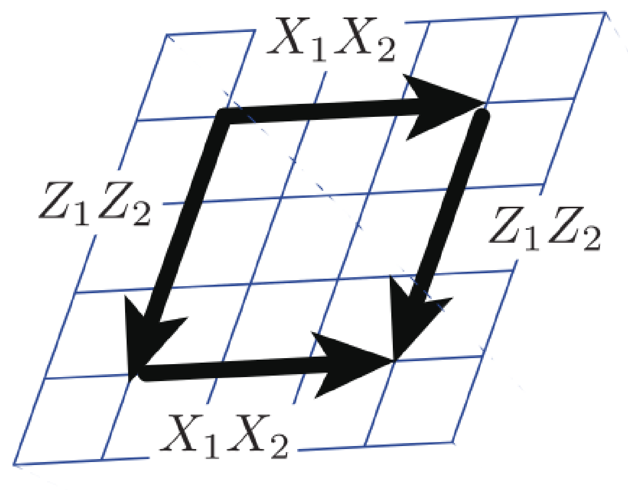


$$P H_1 P \sim P X_2 \equiv H_{z1}$$

$$H_1 = X_1 X_2$$

$$H_2 = Z_1 Z_2$$

$$\Rightarrow \quad \dim L(iX_1X_2,\ iZ_1Z_2) = 2$$



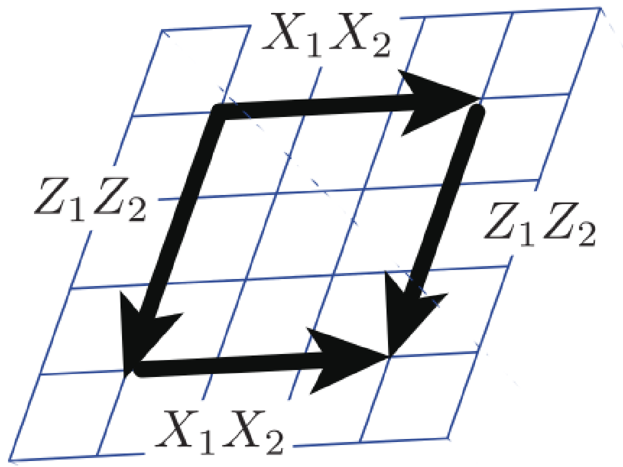
$$PH_1P \sim PX_2 \equiv H_{z1}$$

$$PH_2P \sim PZ_2 \equiv H_{z2}$$

$H_1 = X_1 X_2$

$H_2 = Z_1 Z_2$

$\Rightarrow \dim L(iX_1X_2,\ iZ_1Z_2) = 2$



$PH_1P \sim PX_2 \equiv H_{z1}$

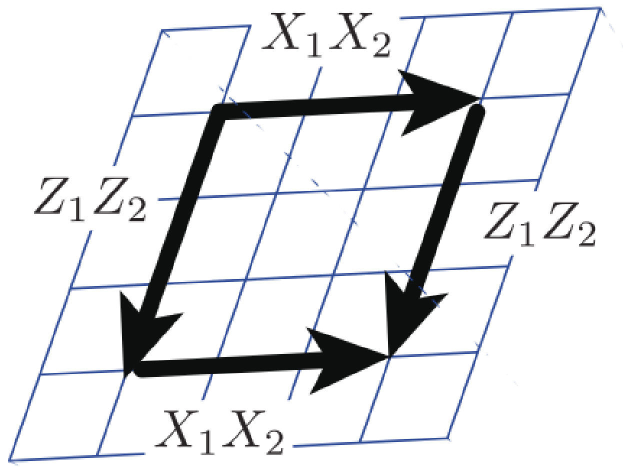
$PH_2P \sim PZ_2 \equiv H_{z2}$

FREEZING QUBIT 1 IN $|0\rangle$

$H_1 = X_1 X_2$

$H_2 = Z_1 Z_2$

$\Rightarrow \dim L(i X_1 X_2, i Z_1 Z_2) = 2$



$P H_1 P \sim P X_2 \equiv H_{Z1}$

$P H_2 P \sim P Z_2 \equiv H_{Z2}$

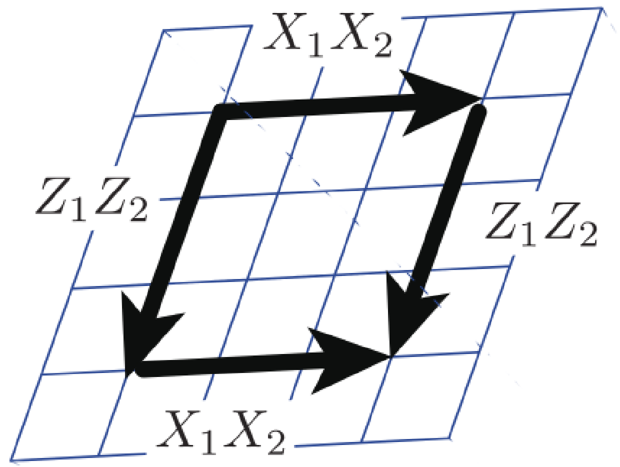
FREEZING QUBIT 1 IN $|\phi\rangle$

$[H_{Z1}, H_{Z2}] \sim P Y_2$

$H_1 = X_1 X_2$

$H_2 = Z_1 Z_2$

$\Rightarrow \dim L(i X_1 X_2, i Z_1 Z_2) = 2$



$P H_1 P \sim P X_2 \equiv H_{z1}$

$P H_2 P \sim P Z_2 \equiv H_{z2}$

FREEZING QUBIT
1 IN $|0\rangle$

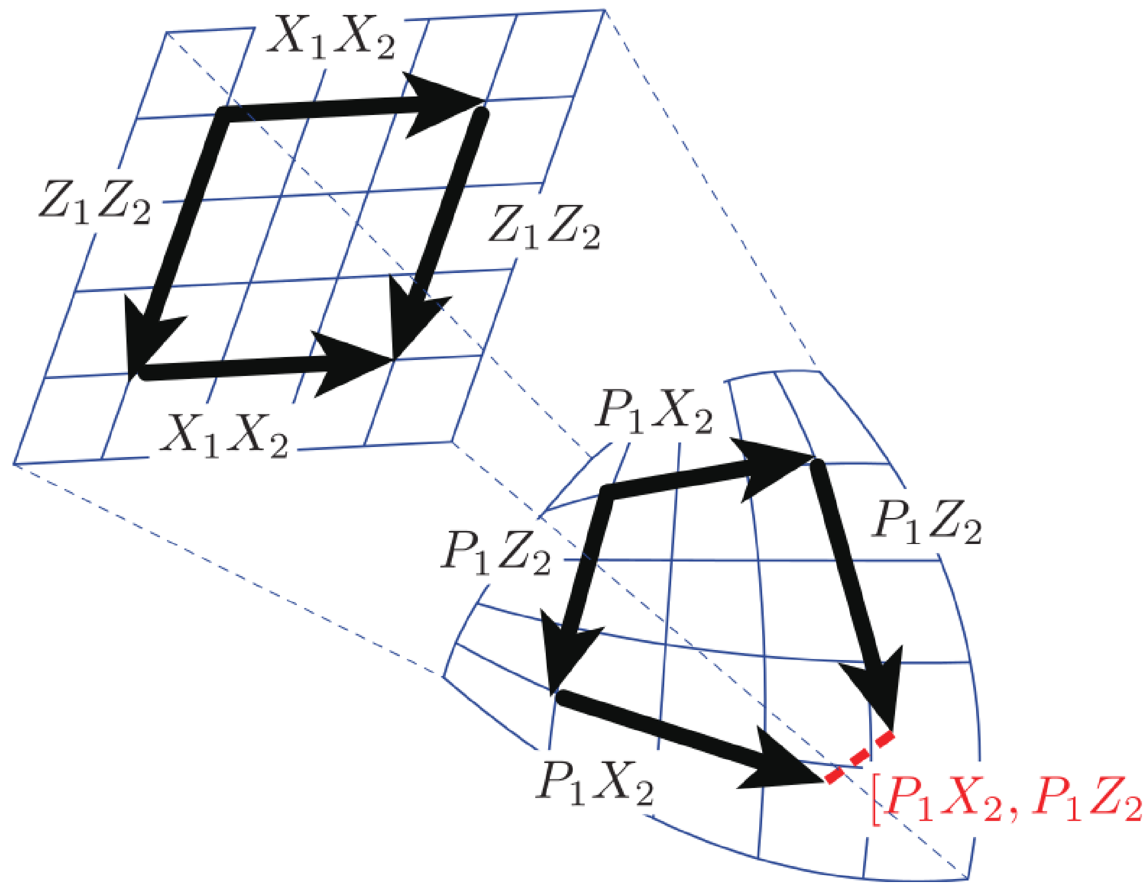
$[H_{z1}, H_{z2}] \sim P Y_2$

$\rightarrow \dim L_z = 3$, MORE COMPLEX DYNAMICS IN SMALLER SPACE

$H_1 = X_1 X_2$

$H_2 = Z_1 Z_2$

$\Rightarrow \dim L(iX_1X_2, iZ_1Z_2) = 2$



$PH_1P \sim PX_2 \equiv H_{z1}$

$PH_2P \sim PZ_2 \equiv H_{z2}$

FREEZING QUBIT 1 IN $|0\rangle$

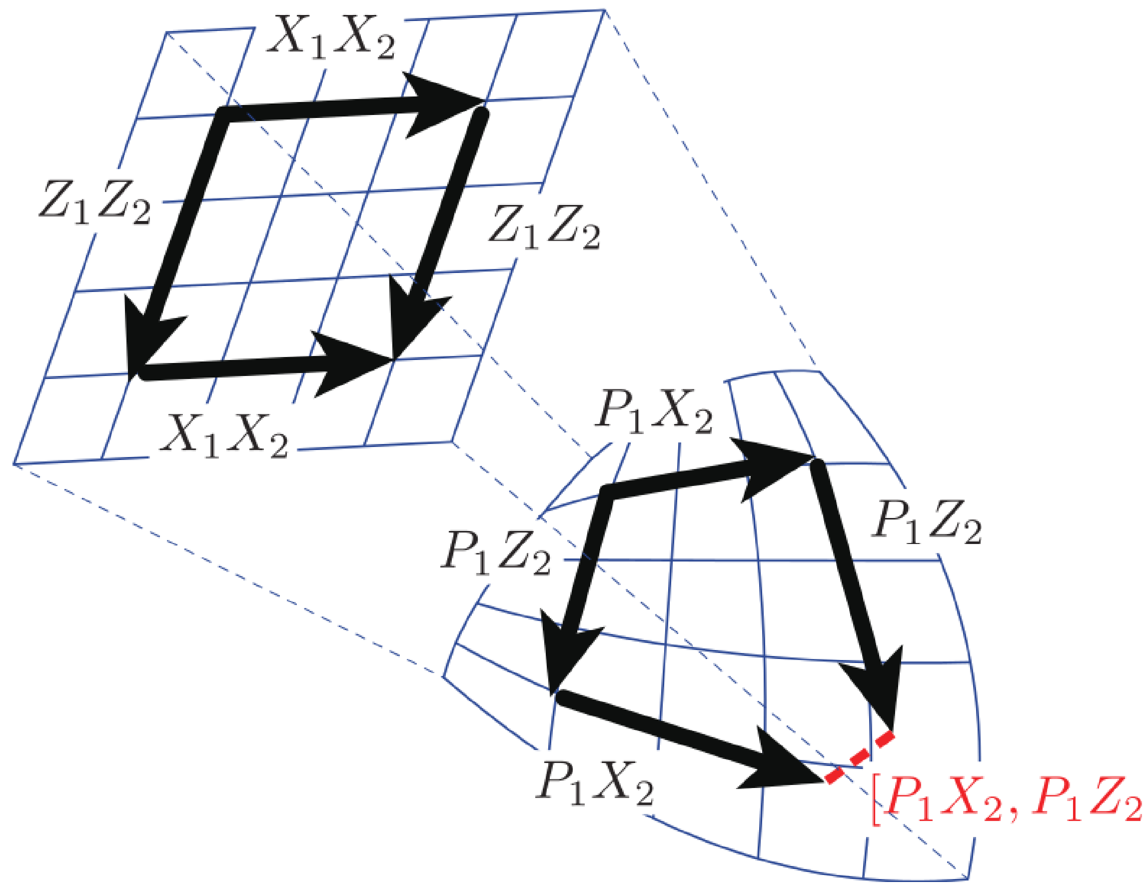
$[H_{z1}, H_{z2}] \sim PY_2$

$\rightarrow \dim L_z = 3$, MORE COMPLEX DYNAMICS IN SMALLER SPACE

$H_1 = X_1 X_2$

$H_2 = Z_1 Z_2$

$\Rightarrow \dim L(i X_1 X_2, i Z_1 Z_2) = 2$



$P H_1 P \sim P X_2 \equiv H_{z1}$

$P H_2 P \sim P Z_2 \equiv H_{z2}$

FREEZING QUBIT 1 IN $|\phi\rangle$

$[H_{z1}, H_{z2}] \sim P Y_2$

$\rightarrow \dim L_z = 3$, MORE COMPLEX DYNAMICS IN SMALLER SPACE

$\rightarrow$ PROJECTION INTRODUCES CURVATURE

HOW GENERAL IS THIS

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1 , h_2 ON $d \times d$ GENERIC

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1, h_2 ON $d\times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1, h_2 ON $d \times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

$$H_2 \equiv \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_1 + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_2$$

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1 , h_2 ON $d\times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

$$H_2 \equiv \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_1 + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_2$$

$$\Rightarrow [H_1 , H_2] =$$

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1, h_2 ON $d \times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

$$H_2 \equiv \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_1 + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_2$$

$$\Rightarrow [H_1, H_2] = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes [h_1, h_2] + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}^2 \otimes [h_2, h_1]$$

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1 , h_2 ON $d \times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

$$H_2 \equiv \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_1 + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_2$$

$$\Rightarrow [H_1, H_2] = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes [h_1, h_2] + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}^2 \otimes [h_2, h_1] = 0$$

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1 , h_2 ON $d \times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

$$H_2 \equiv \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_1 + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_2$$

$$\Rightarrow [H_1 , H_2] = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes [h_1, h_2] + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}^2 \otimes [h_2, h_1] = 0$$

$$P \equiv \begin{pmatrix} 1 & \\ & \end{pmatrix} \otimes \mathbb{1}$$

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1 , h_2 ON $d \times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

$$H_2 \equiv \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_1 + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_2$$

$$\Rightarrow [H_1 , H_2] = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes [h_1, h_2] + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}^2 \otimes [h_2, h_1] = 0$$

$$P \equiv \begin{pmatrix} 1 & \\ & \end{pmatrix} \otimes \mathbb{1} \qquad \Rightarrow \qquad P H_i P = P \otimes h_i$$

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1, h_2 ON $d \times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

$$H_2 \equiv \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_1 + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_2$$

$$\Rightarrow [H_1, H_2] = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes [h_1, h_2] + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}^2 \otimes [h_2, h_1] = 0$$

$$P \equiv \begin{pmatrix} 1 & \\ & \end{pmatrix} \otimes \mathbb{1} \quad \Rightarrow \quad P H_i P = P \otimes h_i$$

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1, h_2 ON $d\times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

$$H_2 \equiv \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_1 + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_2$$

$$\Rightarrow [H_1, H_2] = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes [h_1, h_2] + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}^2 \otimes [h_2, h_1] = 0$$

$$P \equiv \begin{pmatrix} 1 & \\ & \end{pmatrix} \otimes \mathbb{1} \quad \Rightarrow \quad P H_i P = P \otimes h_i$$

"HAMILTONIAN PURIFICATION"

NON-COMMUTATIVE BECOMES SIMPLE ON LARGER SPACE

HOW GENERAL IS THIS

PURIFICATION LEMMA: h_1, h_2 ON $d \times d$ GENERIC

$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

$$H_2 \equiv \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_1 + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_2$$

$$\Rightarrow [H_1, H_2] = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes [h_1, h_2] + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}^2 \otimes [h_2, h_1] = 0$$

$$P \equiv \begin{pmatrix} 1 & \\ & \end{pmatrix} \otimes \mathbb{1} \quad \Rightarrow \quad P H_i P = P \otimes h_i$$

"HAMILTONIAN PURIFICATION"

NON-COMMUTATIVE BECOMES SIMPLE ON LARGER SPACE

FREQUENT MEASUREMENTS BRING US BACK

$$\boxed{\text{KURANISHI ('52)}: \exists\, h_1, h_2 : L = u(d) \text{ MAXIMAL}}$$

$\boxed{\text{KURANISHI ('52)}: \exists\, h_1, h_2 : L = u(d) \text{ MAXIMAL}}$

LEMMA $\Rightarrow \exists\, H_1, H_2, P$ ON $\mathcal{H} = \mathbb{C}^{2d}$:

$\boxed{\text{KURANISHI ('52)} : \exists\, h_1, h_2 : L = \mathfrak{u}(d) \text{ MAXIMAL}}$

LEMMA $\Rightarrow \exists\, H_1, H_2, P$ ON $\mathcal{H} = \mathbb{C}^{2d}$:

$[H_1, H_2] = 0$ BUT PH_1P & PH_2P GENERATE $P\,\mathfrak{u}(d)$

$\boxed{\text{KURANISHI ('52)}: \exists\, h_1, h_2 : L = u(d) \text{ MAXIMAL}}$

LEMMA $\Rightarrow \exists\, H_1, H_2, P$ ON $\mathcal{H} = \mathbb{C}^{2d}$:

$[H_1, H_2] = 0$ BUT PH_1P & PH_2P GENERATE $P\,u(d)$

DETERMINANT IS A POLYNOMIAL $\Rightarrow$

$\boxed{\text{KURANISHI ('52): } \exists\, h_1, h_2 : L = u(d) \text{ MAXIMAL}}$

LEMMA $\Rightarrow \exists\, H_1, H_2, P$ ON $\mathcal{H} = \mathbb{C}^{2d}$:

$[H_1, H_2] = 0$ BUT PH_1P & PH_2P GENERATE $P\,u(d)$

DETERMINANT IS A POLYNOMIAL $\Rightarrow$

$\boxed{\text{KURANISHI ('52): } \exists\, h_1, h_2 : L = u(d) \text{ MAXIMAL}}$

LEMMA $\Rightarrow \exists\, H_1, H_2, P$ ON $\mathcal{H} = \mathbb{C}^{2d}$:

$[H_1, H_2] = 0$ BUT PH_1P & PH_2P GENERATE $P\,u(d)$

DETERMINANT IS A POLYNOMIAL $\Rightarrow$

FREQUENT MEASUREMENTS ON SINGLE QUBIT ALMOST ALWAYS TURN A COMMUTATIVE SYSTEM INTO A QUANTUM COMPUTER.

KURANISHI ('52): $\exists\ h_1, h_2$: $L = u(d)$ MAXIMAL

LEMMA $\Rightarrow \exists\ H_1, H_2, P$ ON $\mathcal{H} = \mathbb{C}^{2d}$:

$[H_1, H_2] = 0$ BUT PH_1P & PH_2P GENERATE $P\,u(d)$

DETERMINANT IS A POLYNOMIAL $\Rightarrow$

FREQUENT MEASUREMENTS ON SINGLE QUBIT ALMOST ALWAYS TURN A COMMUTATIVE SYSTEM INTO A QUANTUM COMPUTER.

CONVERSELY ANY COMPLEX DYNAMICS CAN BE THOUGHT OF ARRISING THIS WAY.

$$\boxed{\text{KURANISHI ('52)}: \exists\, h_1, h_2 : \ L = u(d) \text{ MAXIMAL}}$$

LEMMA $\Rightarrow \exists\, H_1, H_2, P$ ON $\mathcal{H} = \mathbb{C}^{2d}$:

$[H_1, H_2] = 0$ BUT PH_1P & PH_2P GENERATE $P\,u(d)$

DETERMINANT IS A POLYNOMIAL $\Rightarrow$

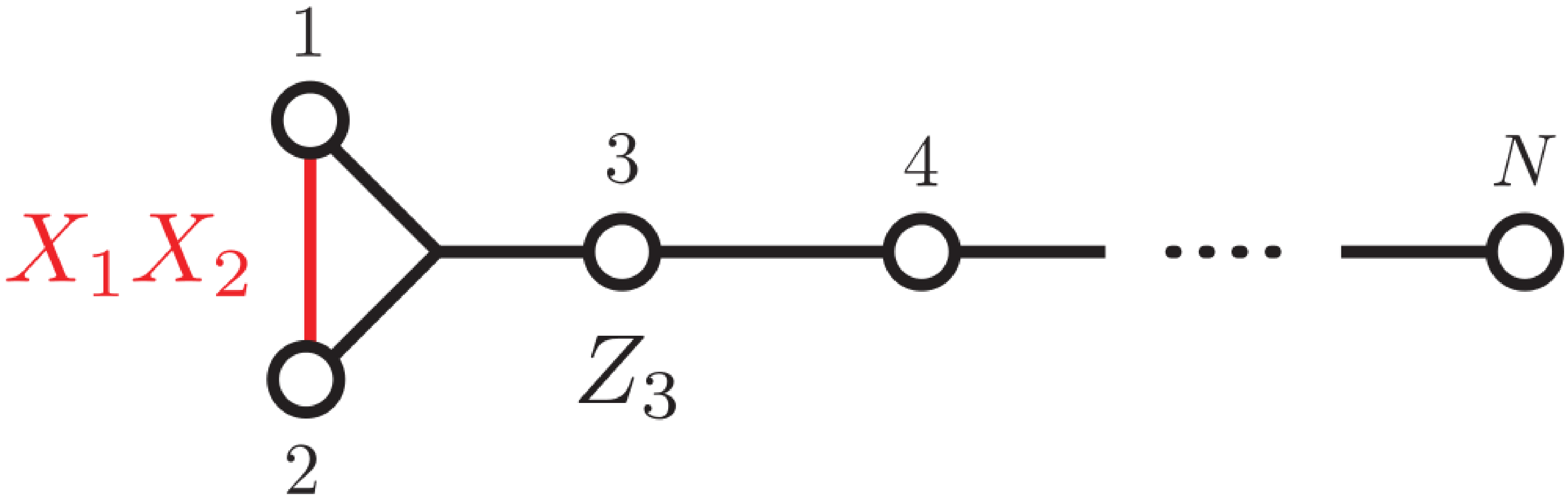
FREQUENT MEASUREMENTS ON SINGLE QUBIT ALMOST ALWAYS TURN A COMMUTATIVE SYSTEM INTO A QUANTUM COMPUTER.

CONVERSELY ANY COMPLEX DYNAMICS CAN BE THOUGHT OF ARRISING THIS WAY.

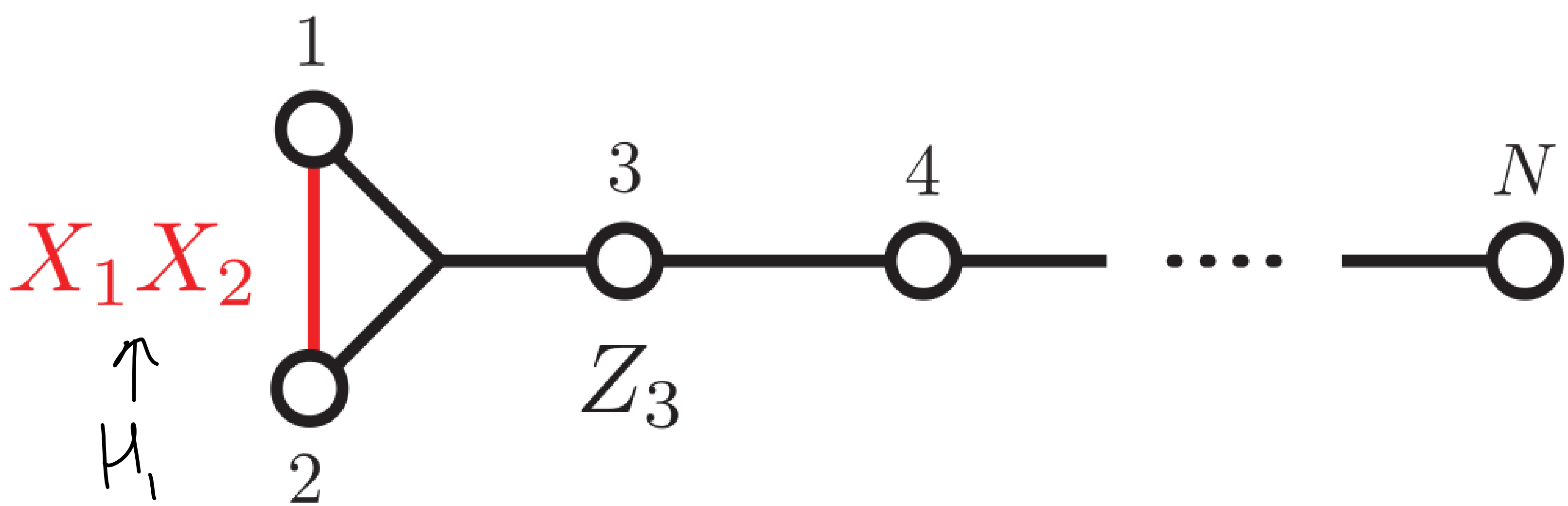
$\Rightarrow$ QUANTUM PLATO CAVE

HOW MIGHT THESE LOOK LIKE

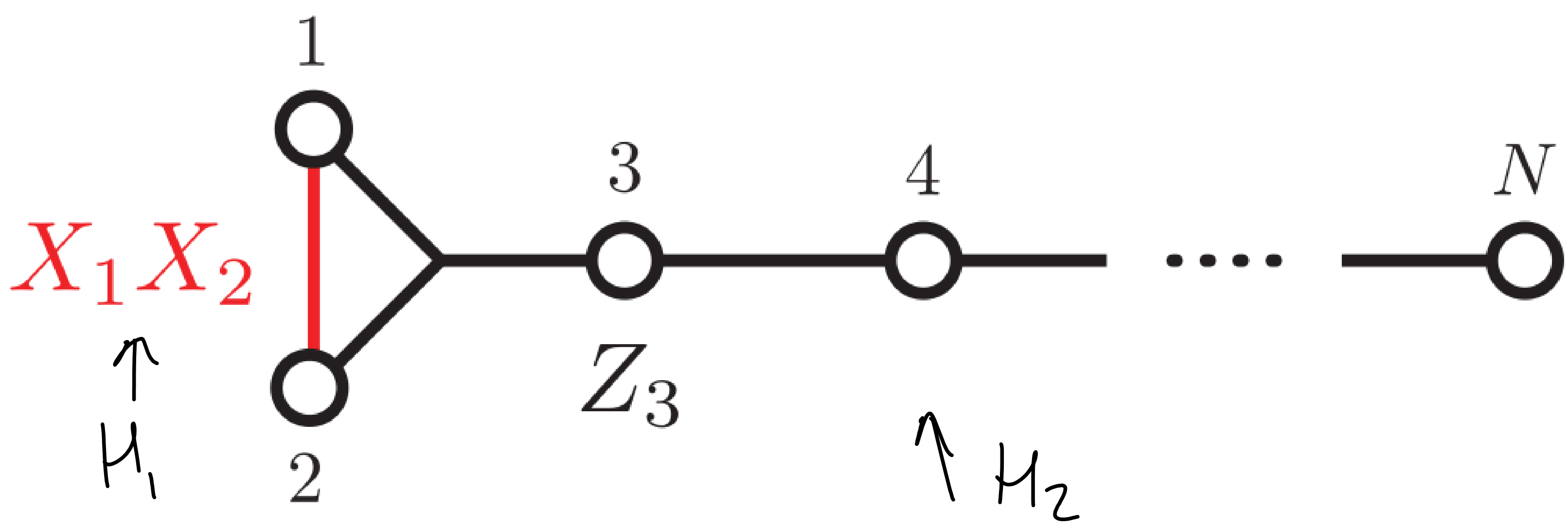
HOW MIGHT THESE LOOK LIKE



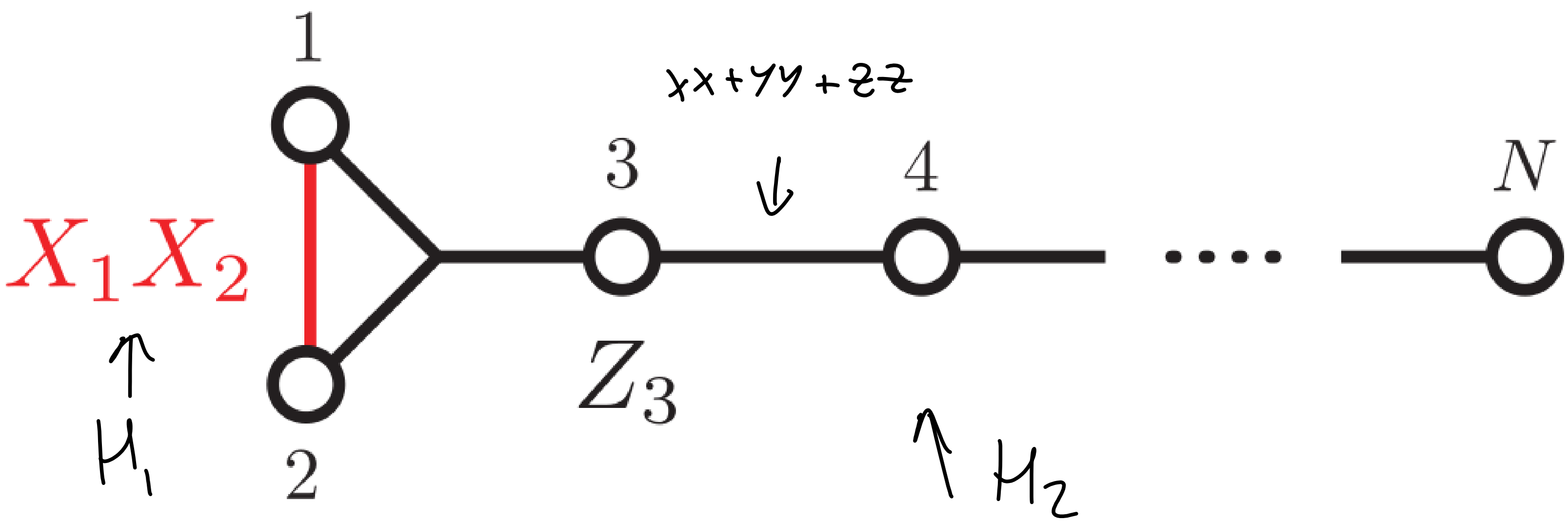
HOW MIGHT THESE LOOK LIKE



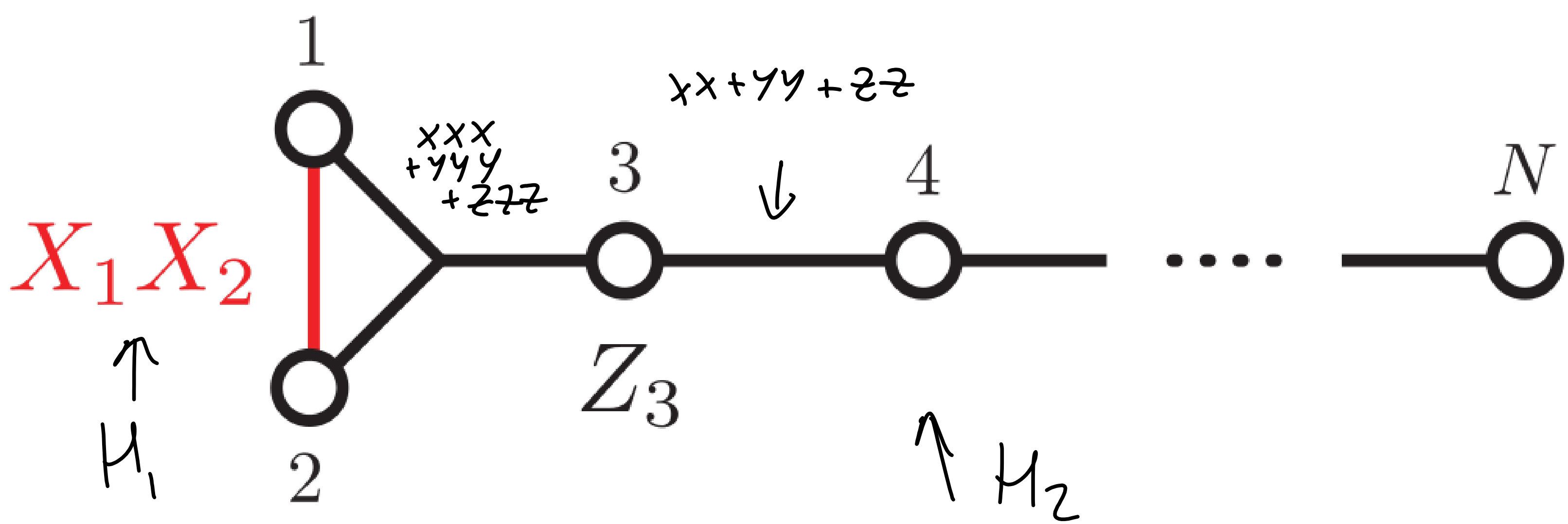
HOW MIGHT THESE LOOK LIKE



HOW MIGHT THESE LOOK LIKE

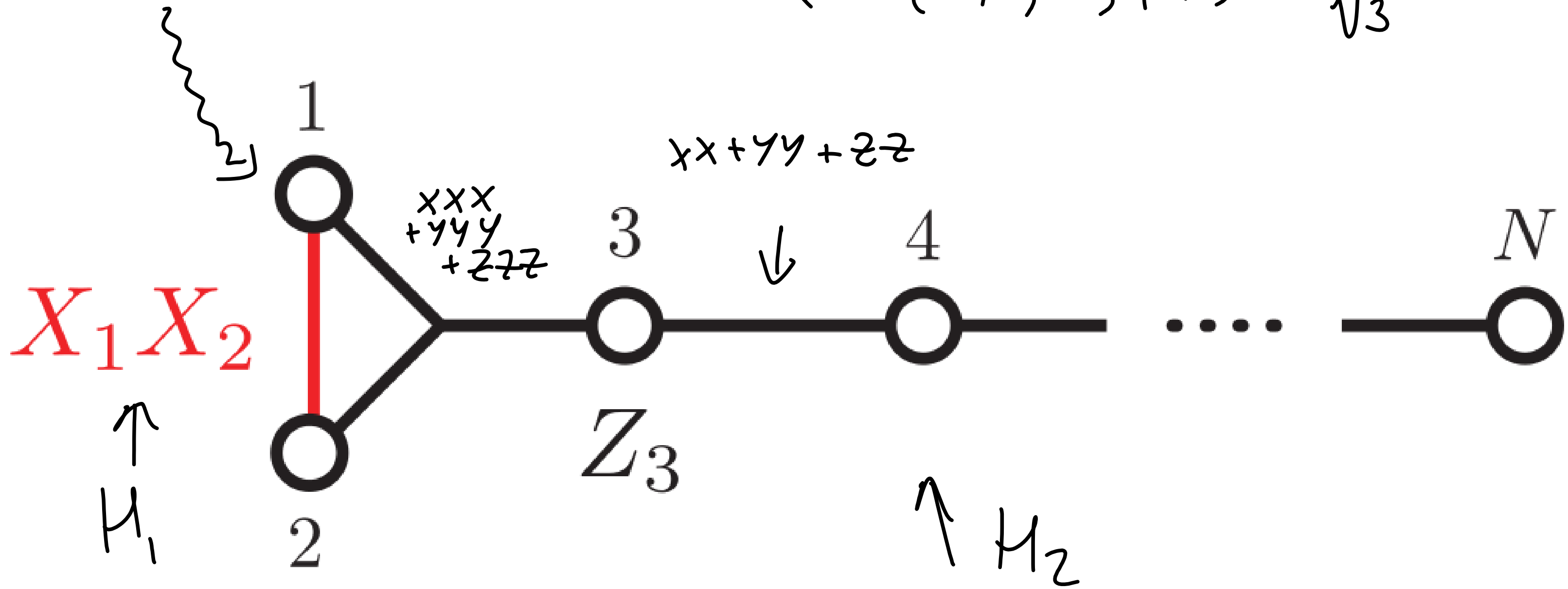


HOW MIGHT THESE LOOK LIKE

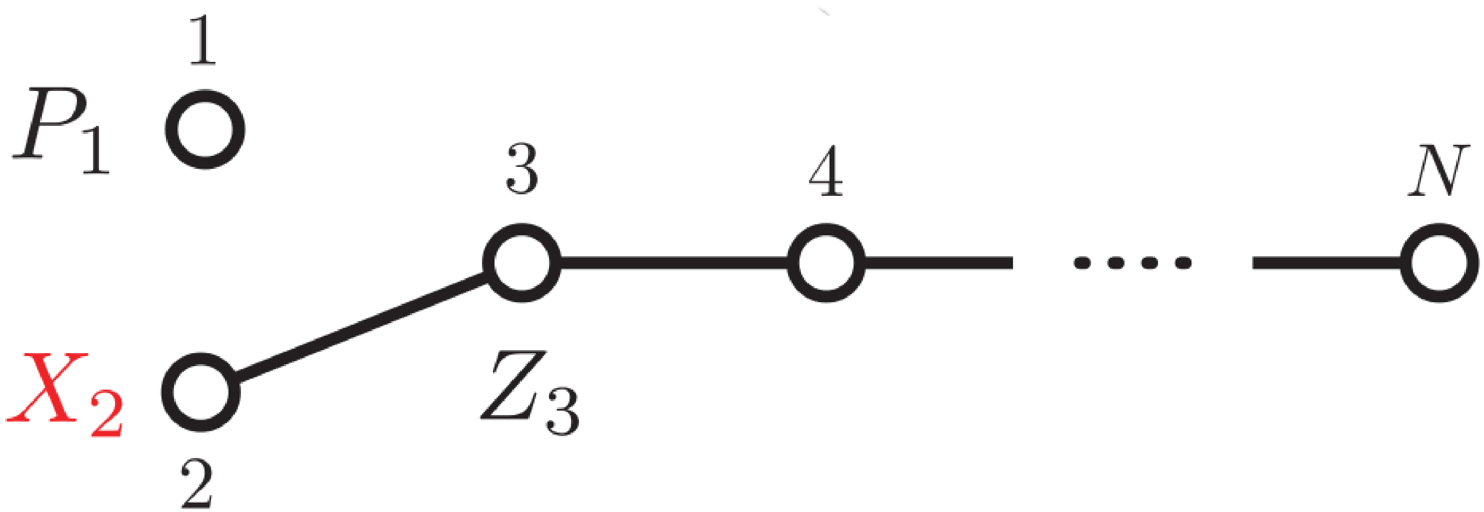


HOW MIGHT THESE LOOK LIKE

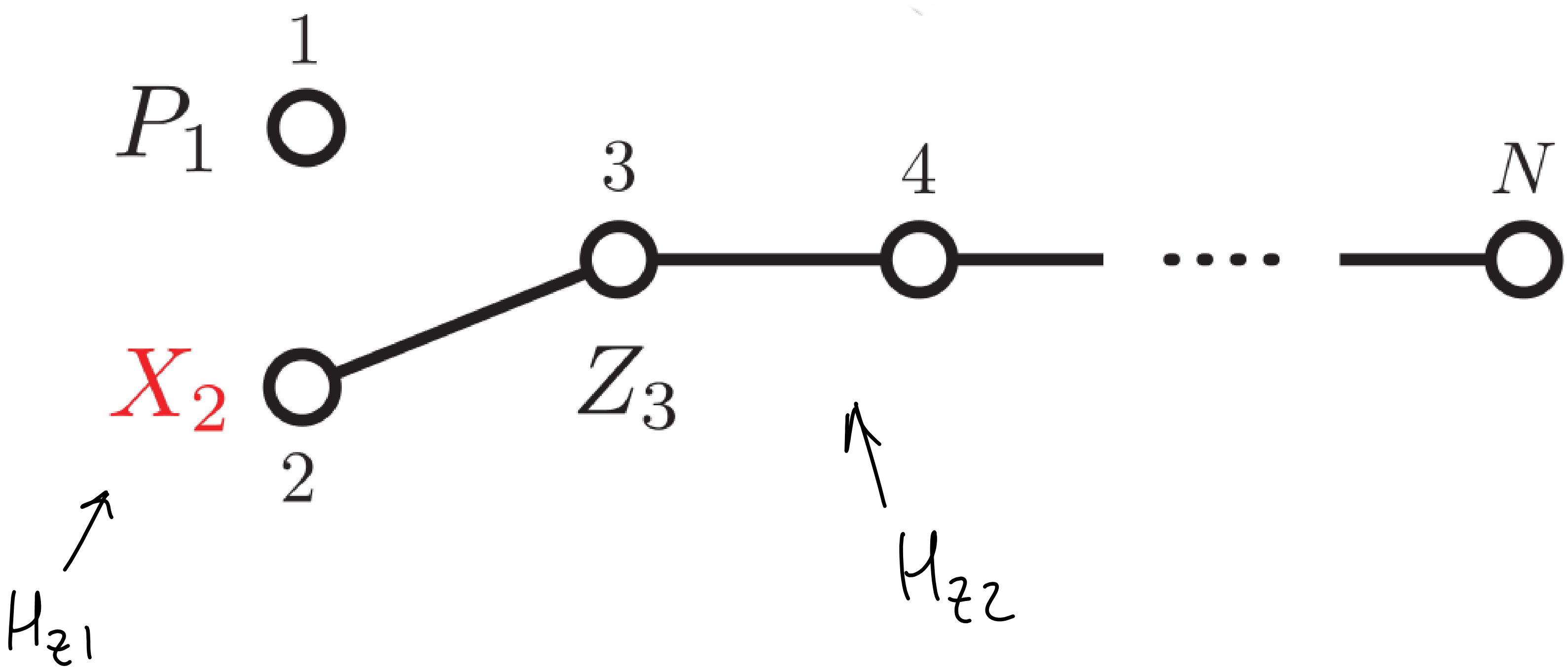
MEASURE 1 QUBIT IN $\langle \phi | \{X, Y, Z\} | \phi \rangle = \frac{1}{\sqrt{3}}$



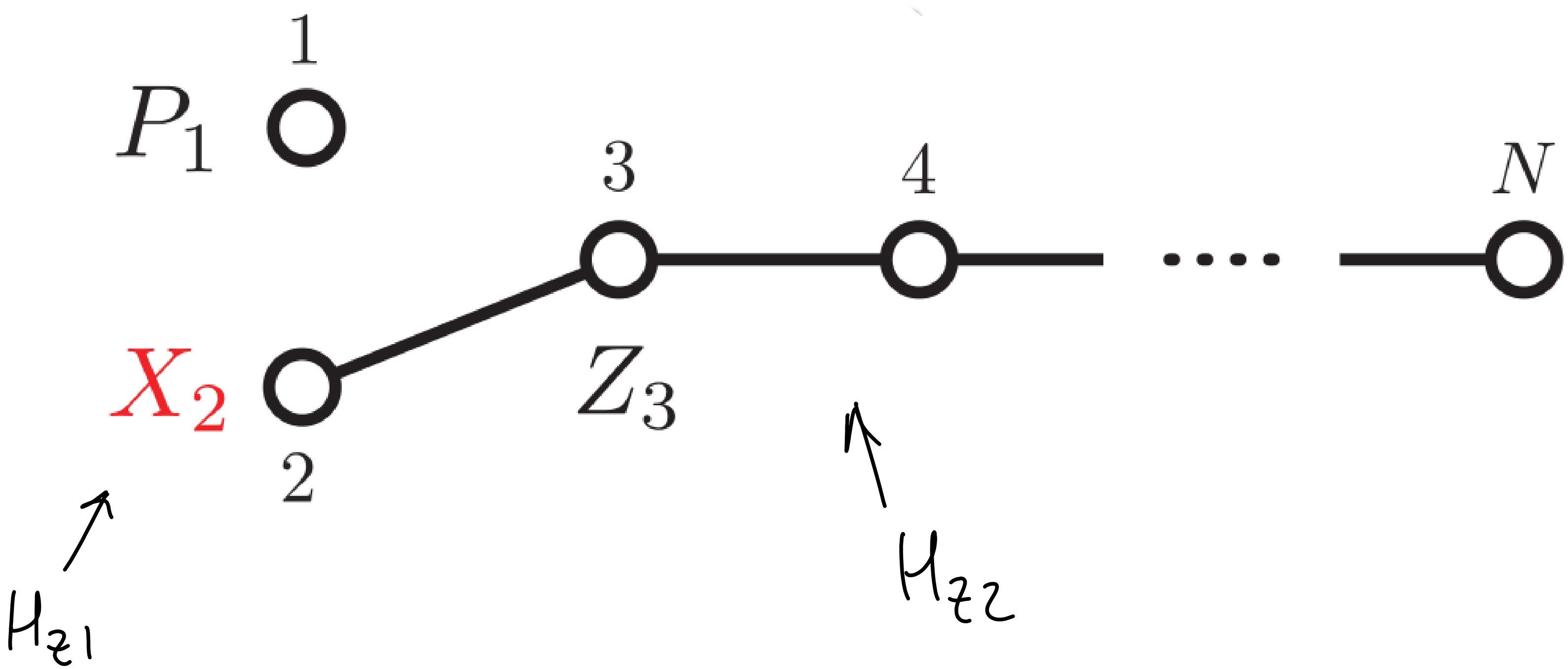
HOW MIGHT THESE LOOK LIKE



HOW MIGHT THESE LOOK LIKE

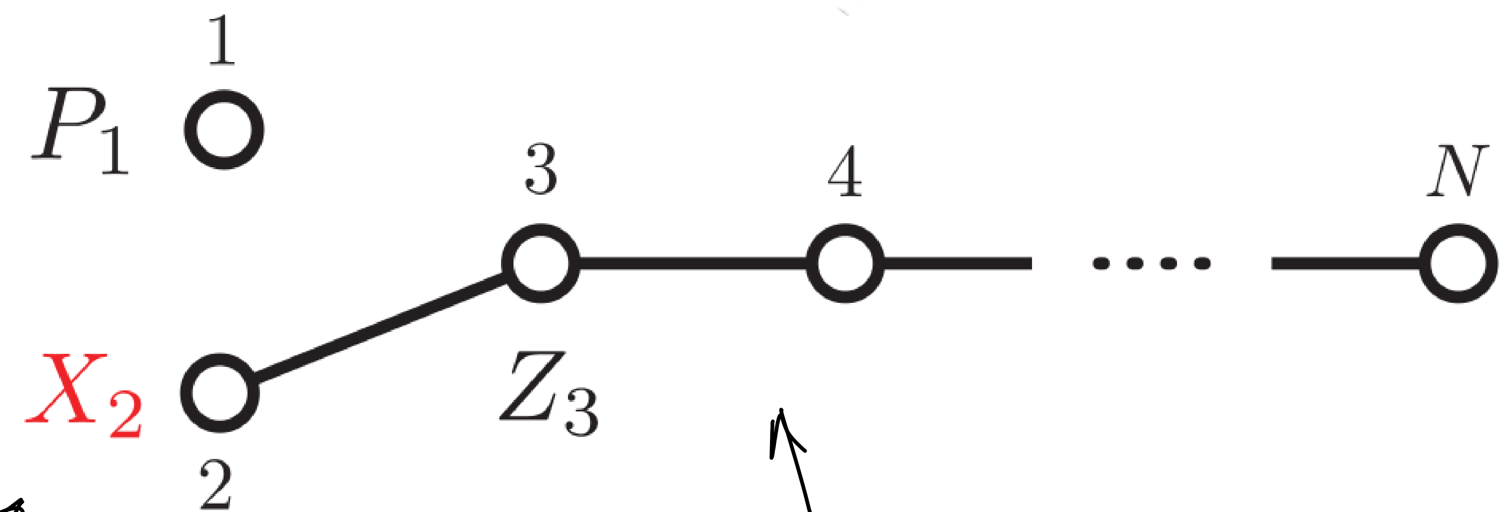


HOW MIGHT THESE LOOK LIKE



... THESE GENERATE $su(2^{N-1})$ ON $2, \ldots, N$

HOW MIGHT THESE LOOK LIKE



H_{z1}

H_{z2}

... THESE GENERATE $su(2^{N-1})$ ON $2, \ldots, N$

(ALSO HAVE 2-BODY ONLY EXAMPLES)

CONCLUSIONS

ARXIV : 1403.5752

CONCLUSIONS ARXIV: 1403.5752

CONFIRMS POWER OF MEASUREMENT

CONCLUSIONS   ARXIV: 1403.5752

CONFIRMS POWER OF MEASUREMENT

$$\mathcal{R}(H_1, H_2, P) \supset \mathcal{R}(PH_1P, PH_2P) \sim \text{LARGE}$$

CONCLUSIONS ARXIV: 1403.5752

CONFIRMS POWER OF MEASUREMENT

$$\mathcal{R}(H_1, H_2, P) \supset \mathcal{R}(PH_1P, PH_2P) \sim \text{LARGE}$$

GENERIC EFFECT

CONCLUSIONS ARXIV: 1403.5752

CONFIRMS POWER OF MEASUREMENT

$$\mathcal{R}(H_1, H_2, P) \supset \mathcal{R}(PH_1P, PH_2P) \sim \text{LARGE}$$

GENERIC EFFECT

CONCLUSIONS

ARXIV: 1403.5752

CONFIRMS POWER OF MEASUREMENT

$$\mathcal{R}(H_1, H_2, P) \supset \mathcal{R}(PH_1P, PH_2P) \sim \text{LARGE}$$

GENERIC EFFECT

PURIFICATION OF NON-COMMUTATIVITY

CONCLUSIONS

ARXIV: 1403.5752

CONFIRMS POWER OF MEASUREMENT

$\mathcal{R}(H_1, H_2, P) \supset \mathcal{R}(PH_1P, PH_2P) \sim$ LARGE

GENERIC EFFECT

PURIFICATION OF NON-COMMUTATIVITY

SPIN CHAIN IMPLEMENTATIONS

CONCLUSIONS ARXIV: 1403.5752

CONFIRMS POWER OF MEASUREMENT

$\mathcal{R}(H_1, H_2, P) \supset \mathcal{R}(PH_1P, PH_2P) \sim$ LARGE

GENERIC EFFECT

PURIFICATION OF NON-COMMUTATIVITY

SPIN CHAIN IMPLEMENTATIONS

THANKS!