Anyonics: Designing exotic circuitry with non-Abelian anyons

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Support:
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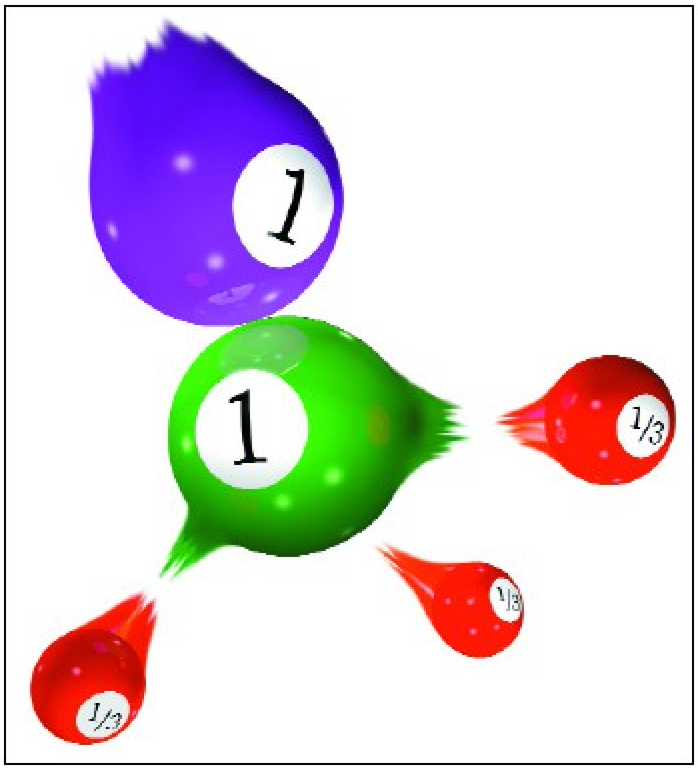
Quantum Engineering of States and Devices

From electronics to anyonics

Particles called anyons that do not fit into the usual categories of fermions and bosons may lead to high-performance quantum computers, explains **Frank Wilczek**

Oddballs
In 2D systems, electrons can behave as if they carry fractional electric charge.



Textbooks on quantum mechanics traditionally divide elementary particles into two types: fermions and bosons. Fermions such as electrons have antisymmetric wavefunctions, which means that a minus sign (i.e. a phase of π) is introduced into a system when two fer-

These developments spawned a vast literature, featuring beautiful and elaborate mathematics.

Until very recently, however, the subject of anyons had still been almost entirely theoretical. Suddenly, over the last few months, that has changed with the appearance of serious – though not entirely uncontroversial – claims that anyons have been observed directly. Meanwhile, several groups have proposed a new generation of experiments that will be more decisive in proving that anyons exist.

Fractional fluids

Strange things happen in semiconductors that are very pure, very cold and subject to strong magnetic fields. In particular, a phase of matter called a fractional quantum Hall effect fluid appears. In this state, electrons as we know them decompose so that electric charge is no longer transported in discreet lumps of charge e, but in fractions of that unit (see *Physics World* March 2000 pp37–43).

The fact that these fractional electrons are anyons could lead to a new kind of semiconductor technology that goes beyond electronics. An "anyonic" circuit would operate in ways that are impossible for conventional electronic circuits, and it is precisely this behaviour that physicists are ultimately hoping to exploit. Indeed, conceptual designs for anyonic quantum computers are

Frank Wilczek, Physics World (2006)

Majorana zero modes

Most likely experimental candidates for non-Ableian statistics:
Majorana zero modes

Ettore Majorana,1937:

Majorana fermion – a fermion which is its own antiparticle

$$\gamma = \gamma^\dagger, \quad \gamma^2 = 1$$

$$\gamma = c + c^\dagger \qquad c = (\gamma_1 + i\gamma_2)/\sqrt{2}$$

A superposition of an electron and a hole - need a superconductor!

Majorana wires

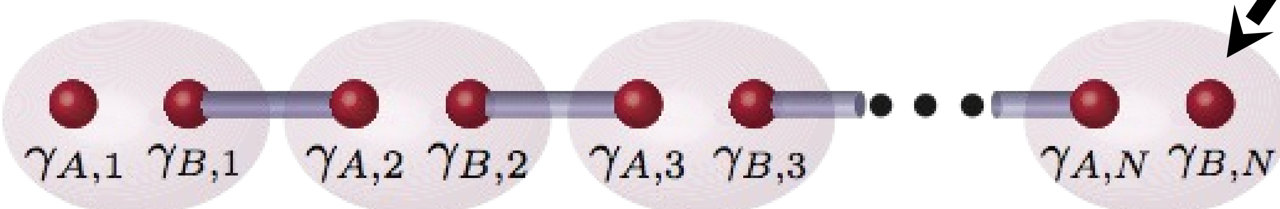
1D spinless p-wave superconductor(Kitaev 2001):

$$H = \mu \sum_{x=1}^{N} c_x^\dagger c_x - \sum_{x=1}^{N-1} \left(t c_x^\dagger c_{x+1} + |\Delta| e^{i\phi} c_x c_{x+1} + h.c. \right)$$

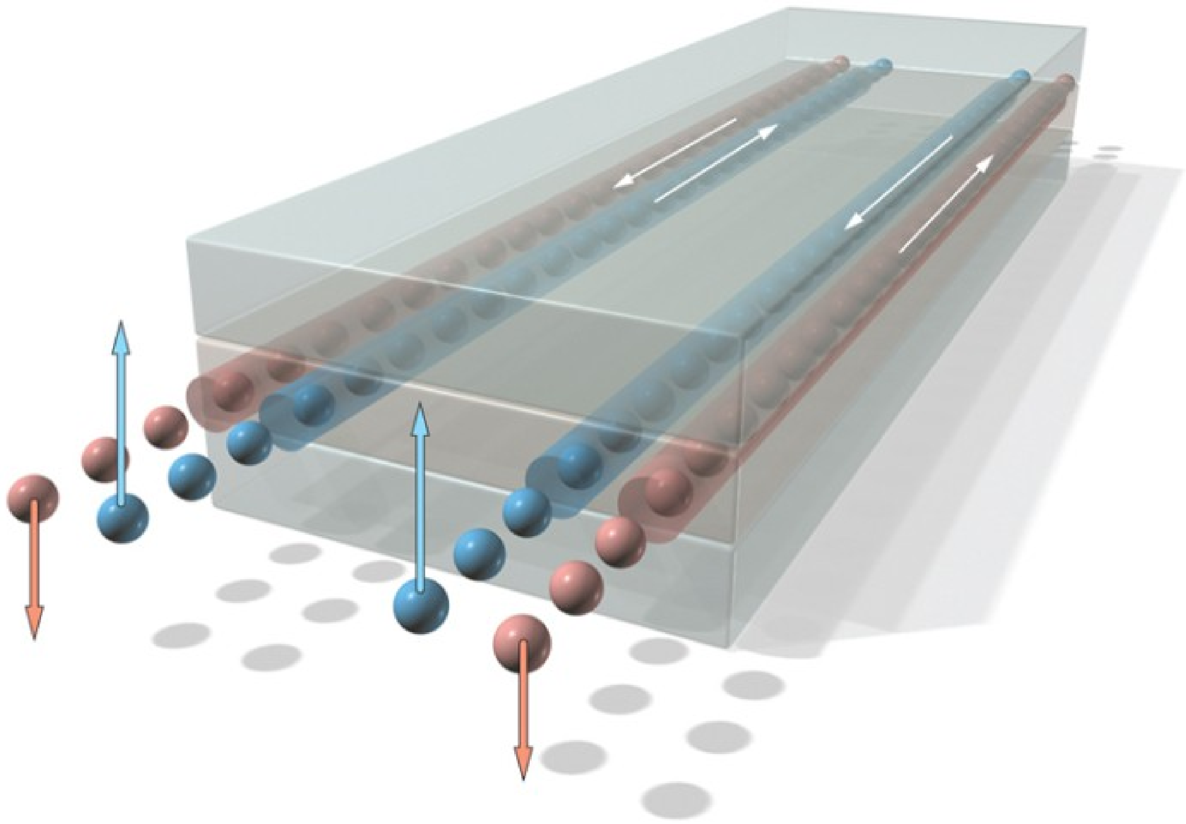
$$\boxed{\begin{array}{ll} \mu = 0 & \\ t = |\Delta| & \qquad c_x = \frac{1}{2} e^{-i\frac{\phi}{2}} (\gamma_{B,x} + i\gamma_{A,x}) \end{array}}$$

$$\Rightarrow \quad H = -it \sum_{x=1}^{N-1} \gamma_{B,x} \gamma_{A,x+1}$$

Unpaired Majorana fermions at the ends!

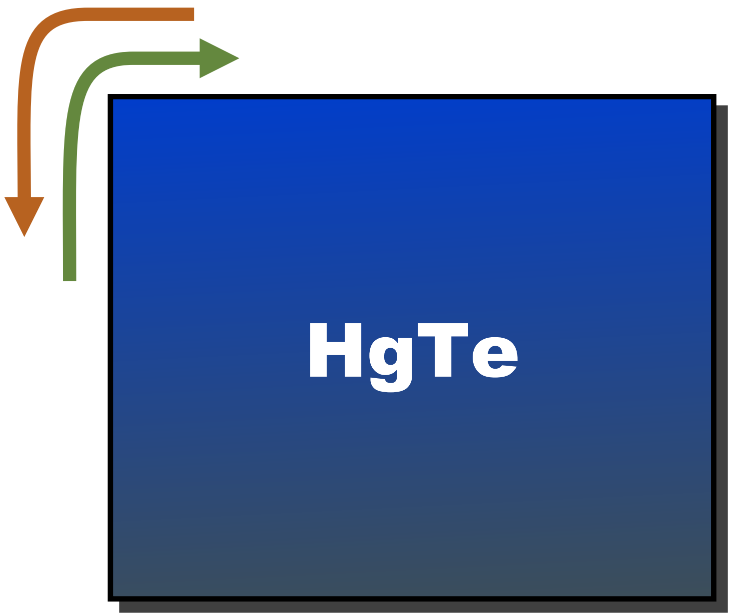


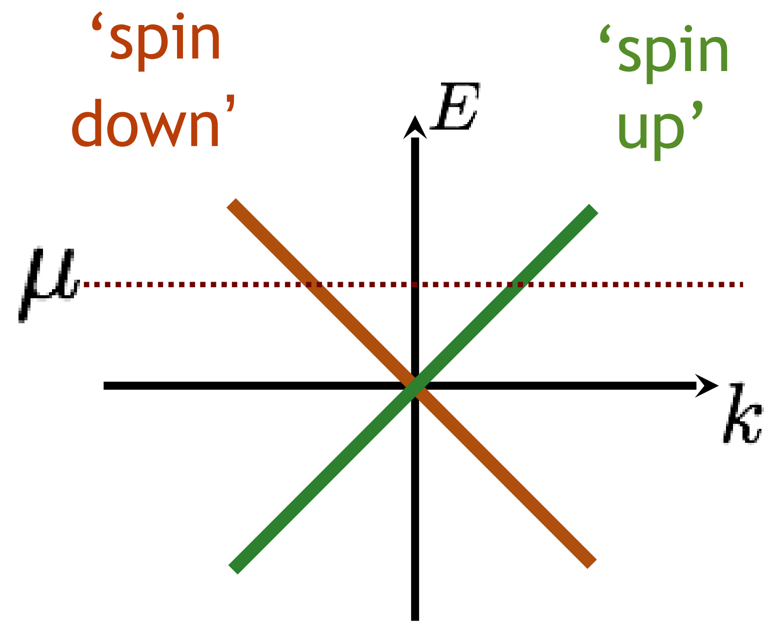
Realization in topological insulator edges



Kane & Mele, 2005; Bernevig, Hughes, Zhang, 2006; Fu & Kane, 2008

Realization in topological insulator edges



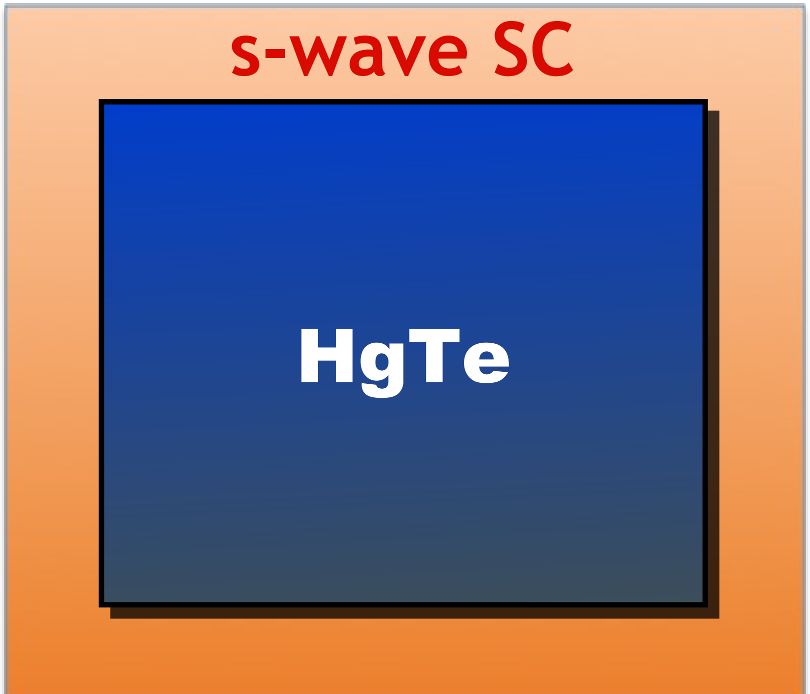


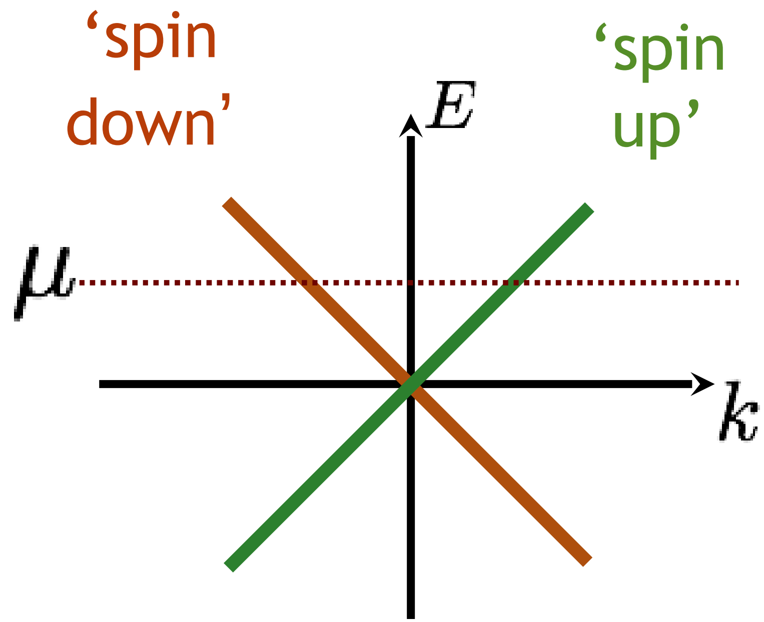
$$H_{\rm edge} = \int dx[-\mu(\psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L) - i\hbar v(\psi_R^\dagger\partial_x\psi_R - \psi_L^\dagger\partial_x\psi_L)]$$

1D and effectively 'spinless'! Just need superconductivity...

Fu & Kane, 2008

Realization in topological insulator edges





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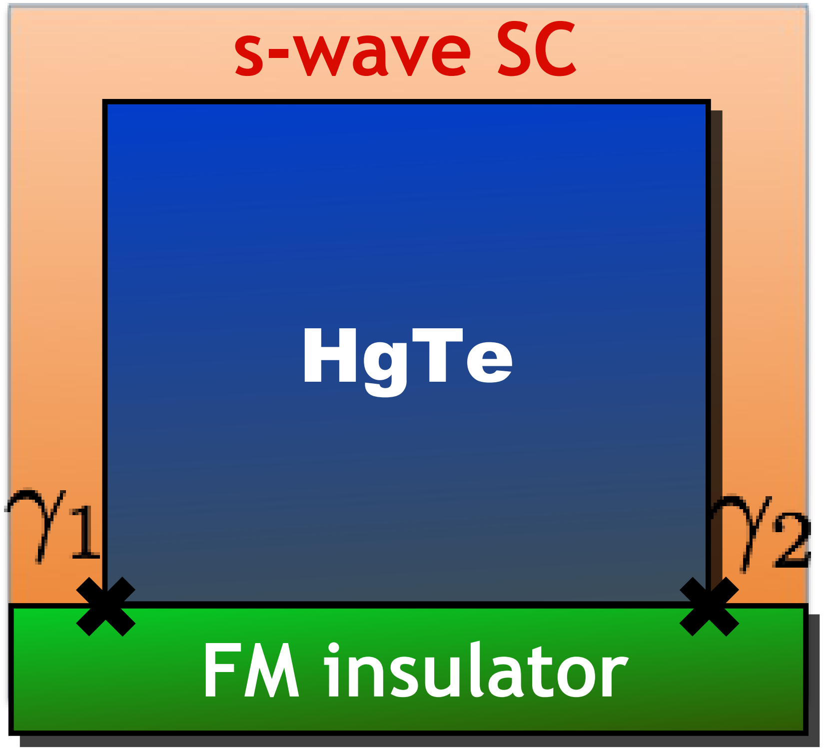
$$+[\Delta\psi_R\psi_L + h.c.]$$

Fu & Kane, 2008

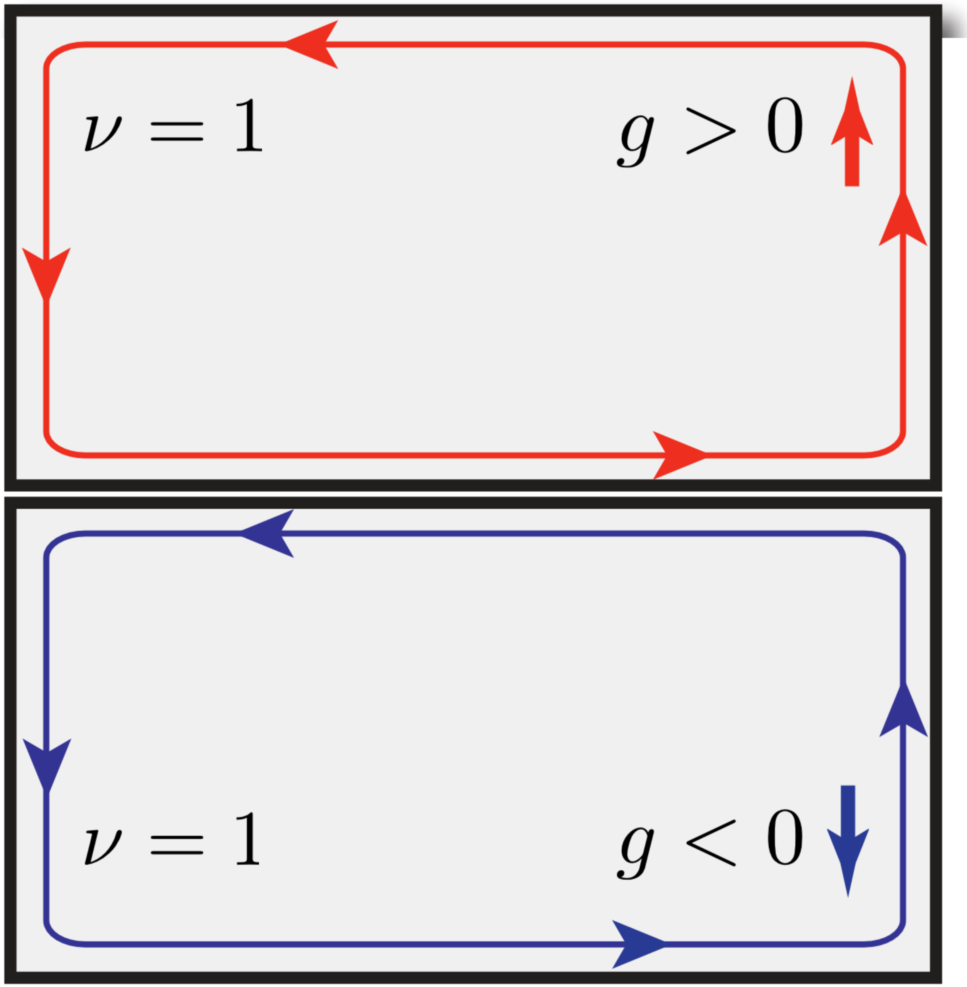
Realization in topological insulator edges



"Terminating" the SC wire by a magnetic gap: Majorana zero modes localised at the ends

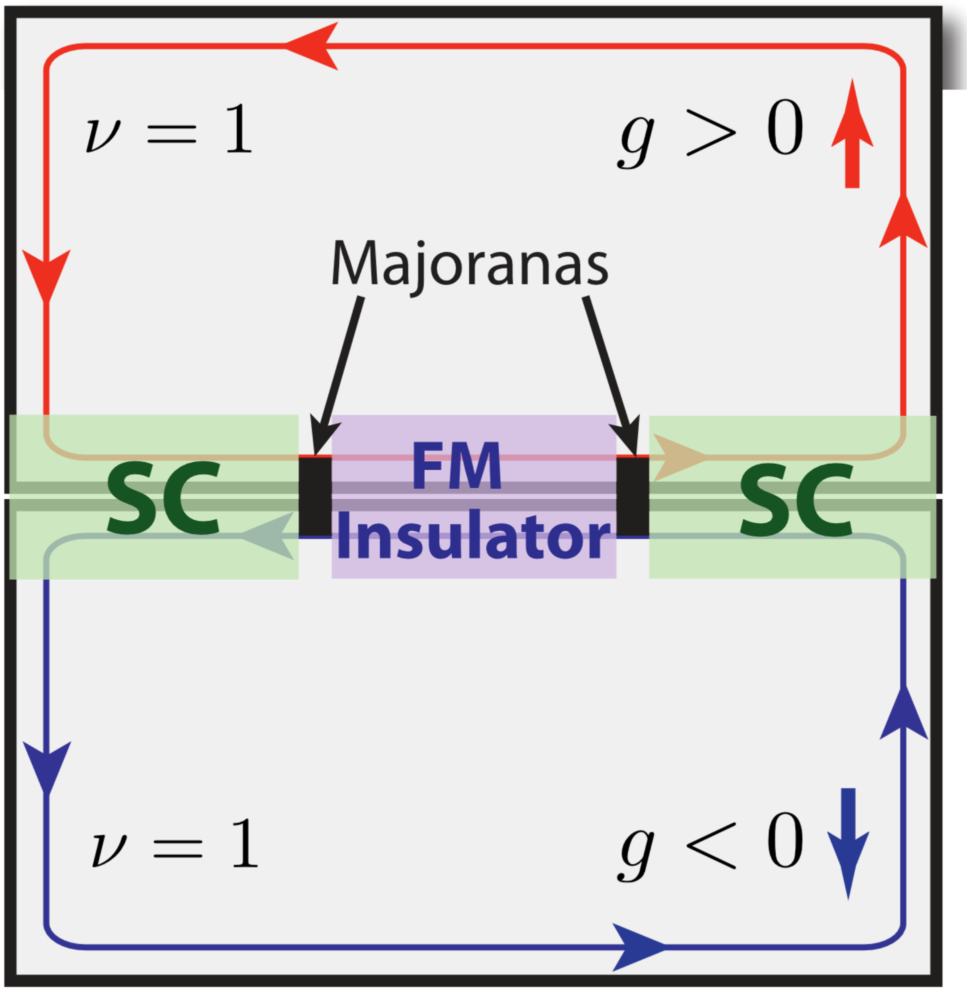
Fu & Kane, 2008

Realization in quantum Hall edges



Counter-propagating edge modes at the boundary between $g > 0$ and $g < 0$. The sign of g can be changed by stress.

Realization in quantum Hall edges



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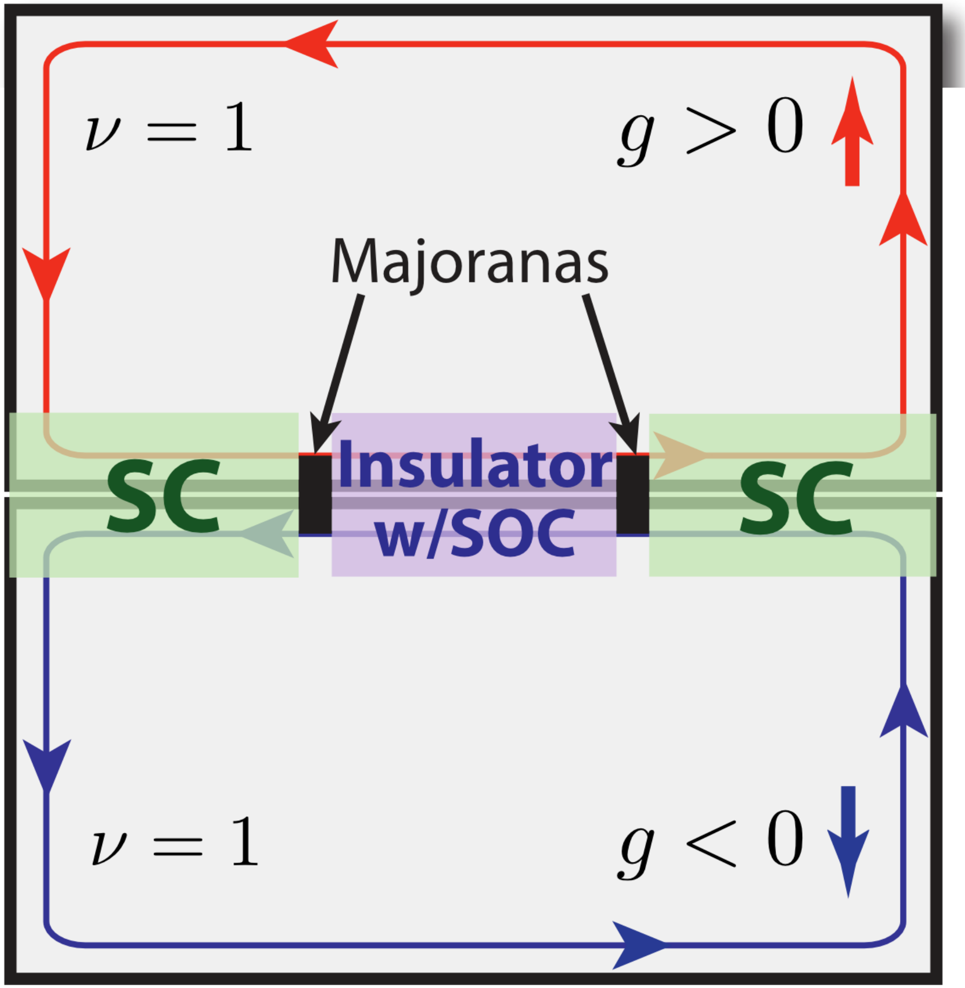
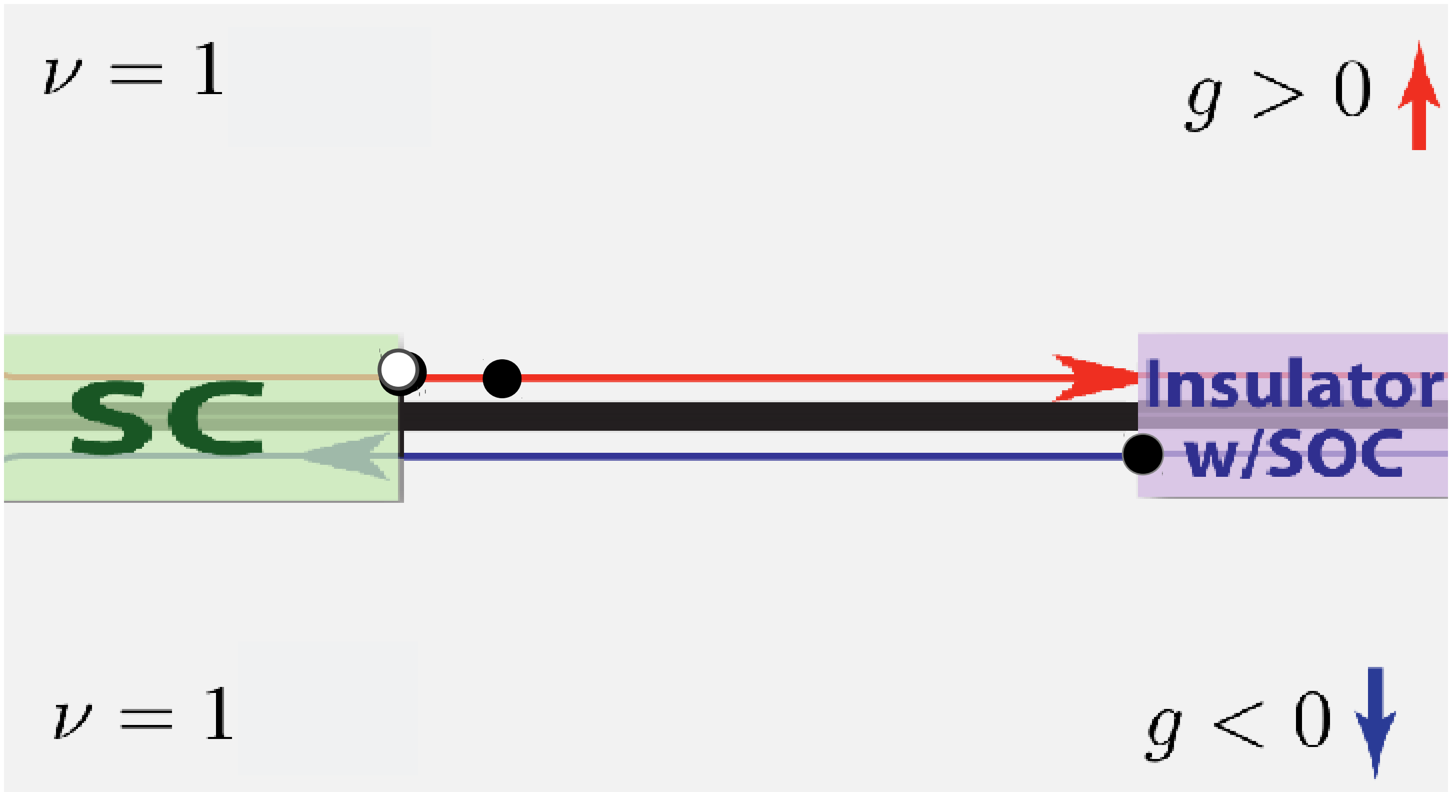


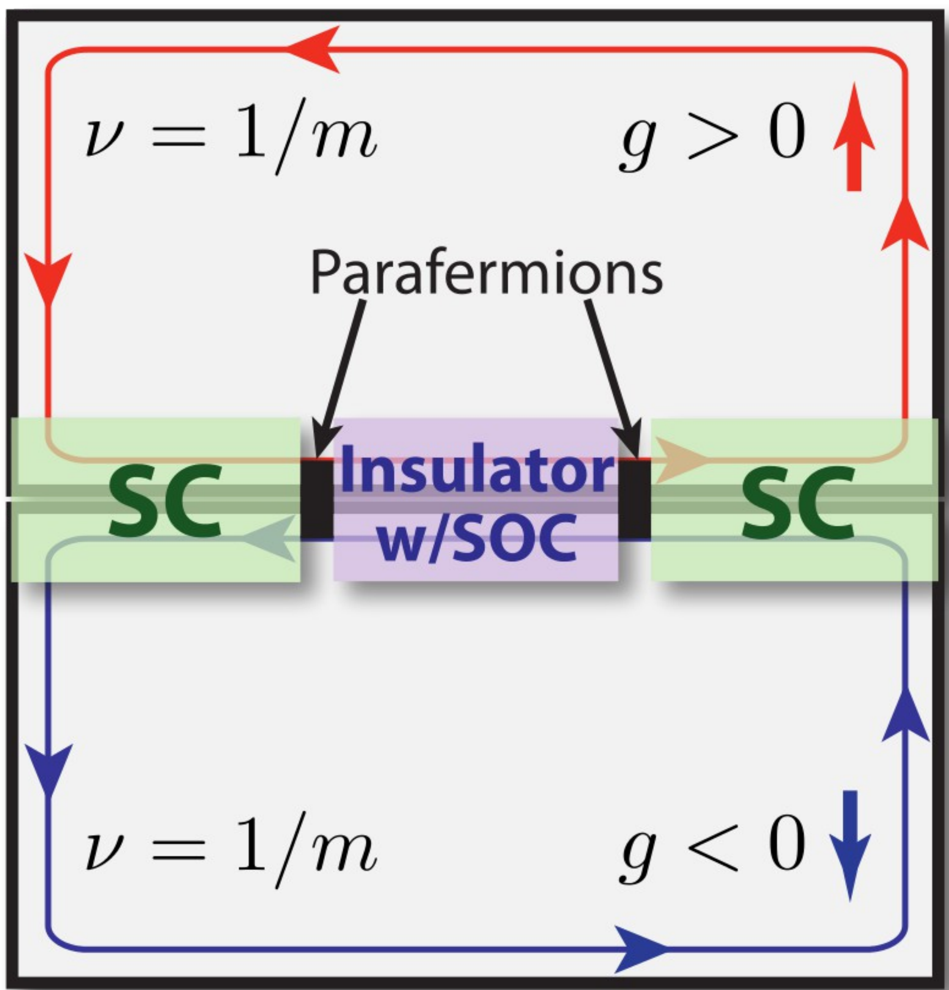
Counter-propagating edge modes at the boundary between $g > 0$ and $g < 0$. The sign of g can be changed by stress.

Majorana zero mode



$$\gamma = c + c^\dagger$$

What about fractional quantum Hall edges?



Counter-propagating ***fractionalized*** edge modes at the boundary between $g > 0$ and $g < 0$.

(In GaAs the sign of g can be changed by stress.)

- D. Clarke, J. Alicea & KS, arXiv:1204.5479, Nature Commun. 2013
- N. Lindner, E. Berg, G. Refael & A. Stern, arXiv:1204.5733, PRX 2012
- M. Cheng, arXiv:1204.6084, PRB 2012

Parafermions vs Majoranas

__Upshot:__

Majorana Fermions: $$\gamma^2 = 1$$

$$\gamma_y \gamma_x = -\gamma_x \gamma_y$$

Parafermions: $$\alpha^N = 1$$

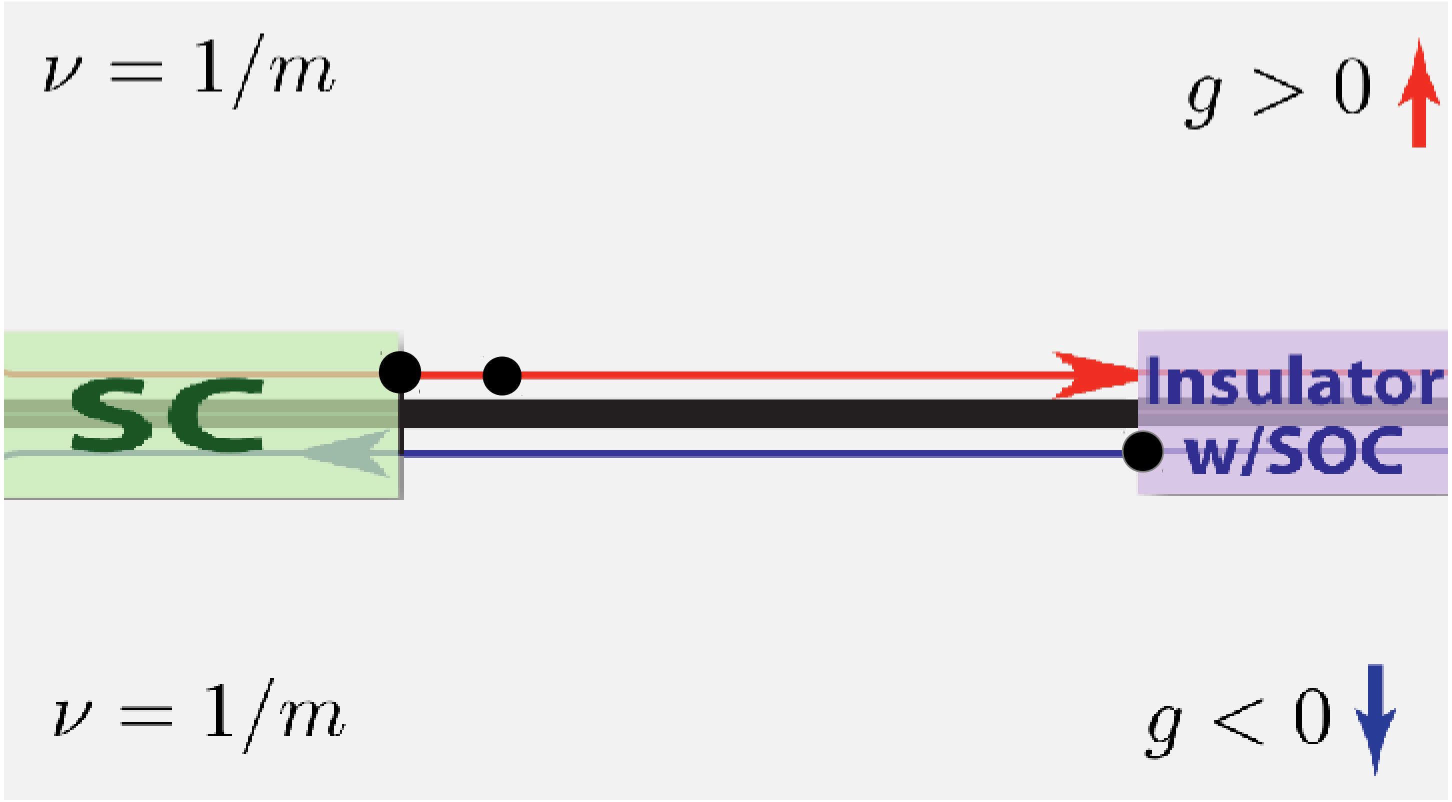
$$\alpha_y \alpha_x = \alpha_x \alpha_y e^{\frac{2\pi i}{N} \mathrm{sgn}(x-y)}$$

Majoranas ⟷ 1D quantum Ising model

Parafermions ⟷ 1D quantum Clock/Potts model

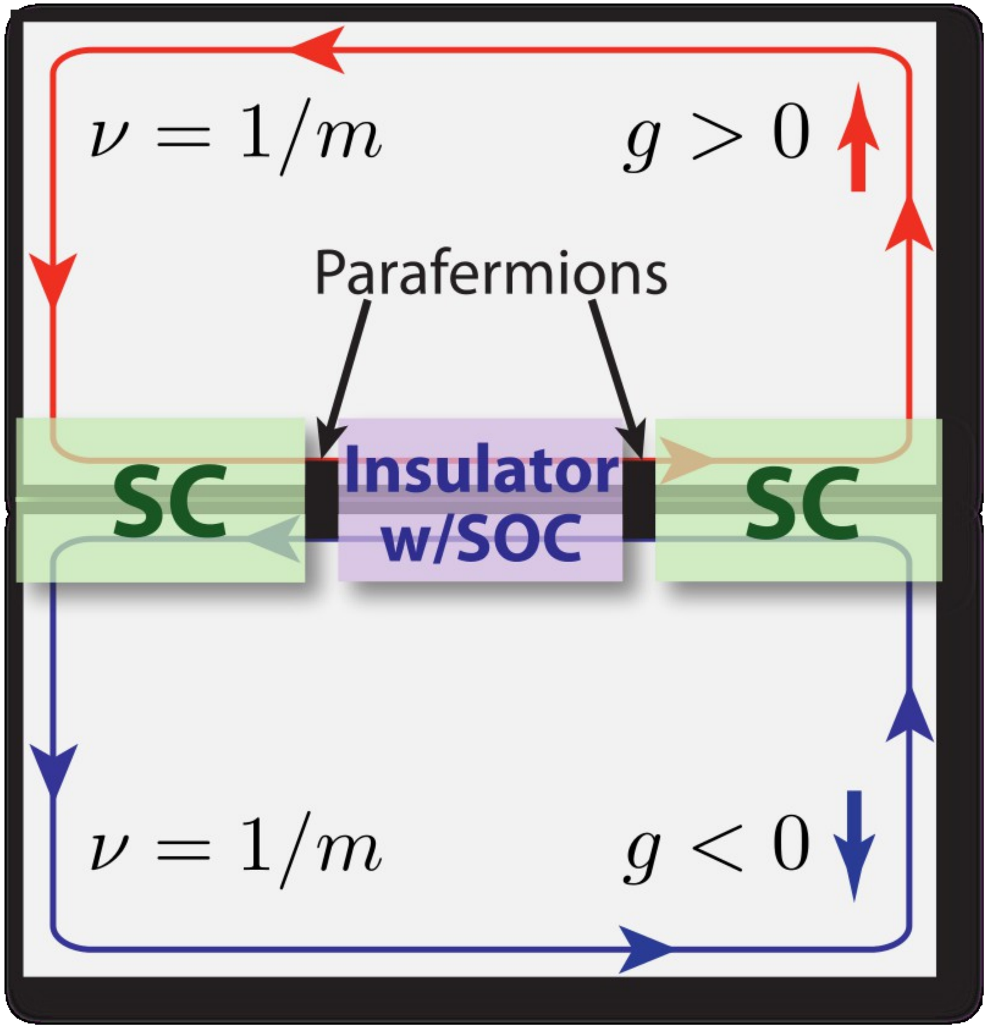
Paul Fendley (2013)

Parafermionic zero mode



Experimental realizations?

Is this feasible?

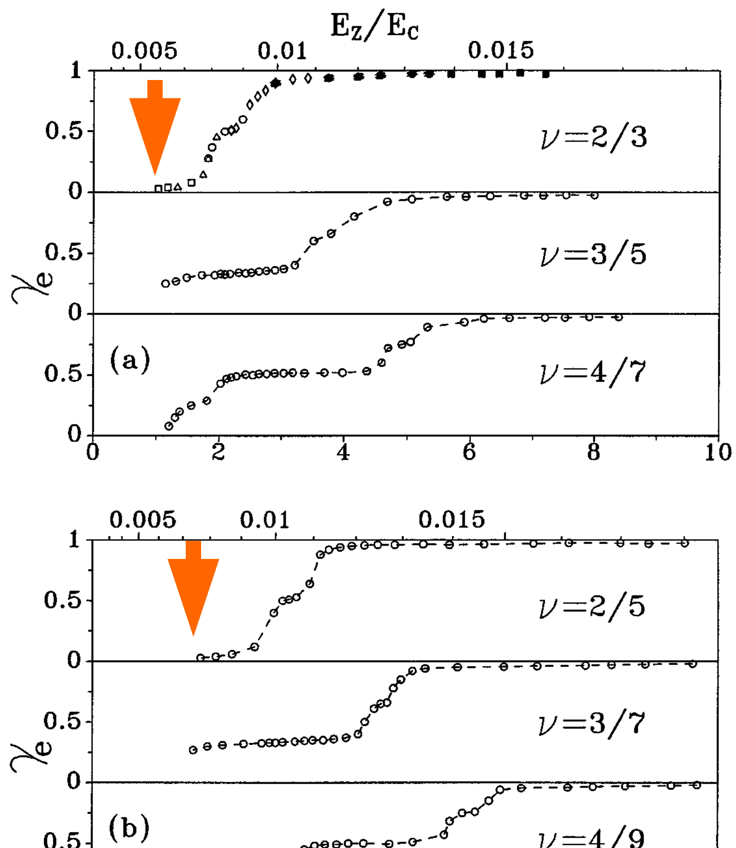


Which parts are really necessary?

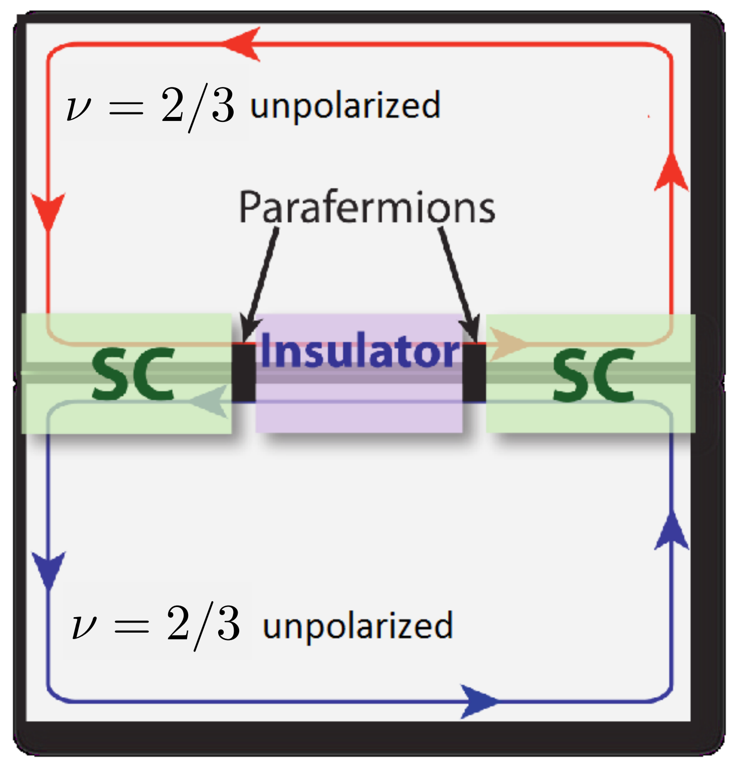
Do we need opposite g-factors?

No! Use spin-unpolarized states, like 2/3 or 2/5

- R. Mong et al, PRX 4, 011036 (2014);
- D. Clarke, J. Alicea, KS, arXiv:1312.6123, to appear in Nature Physics



Spin polarisation vs. Zeeman energy
Kukushkin, von Klitzing, Eberl (1999)



Which parts are really necessary?

So do we really need the insulator at all?

No! Use spin-unpolarized states, like 2/3 or 2/5

- R. Mong et al, PRX 4, 011036 (2014);
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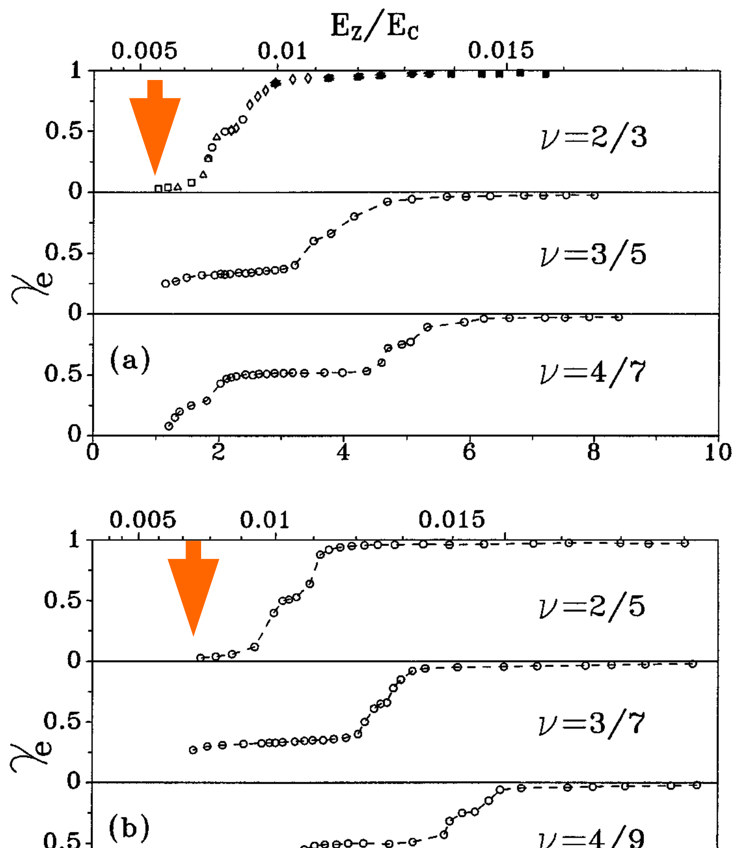
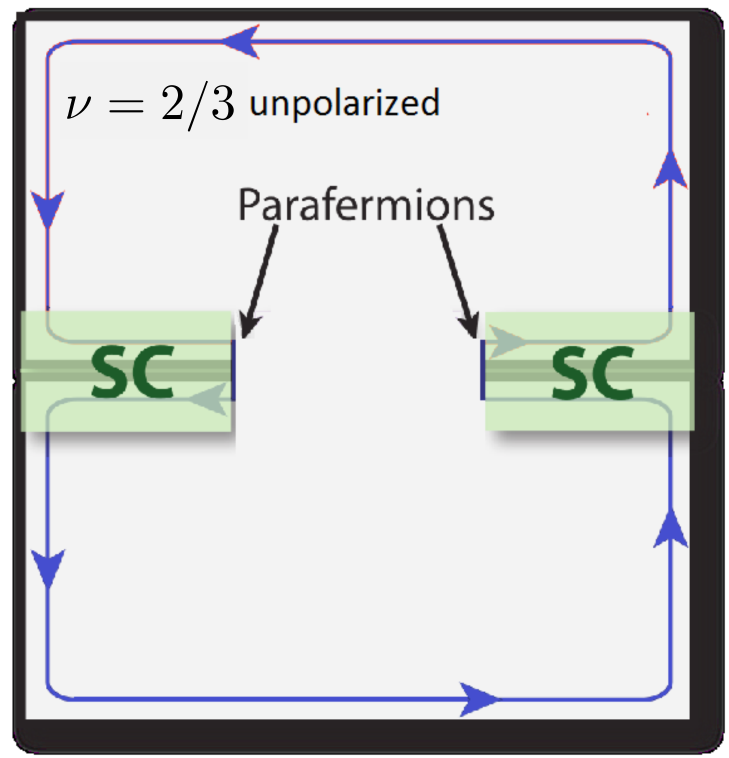
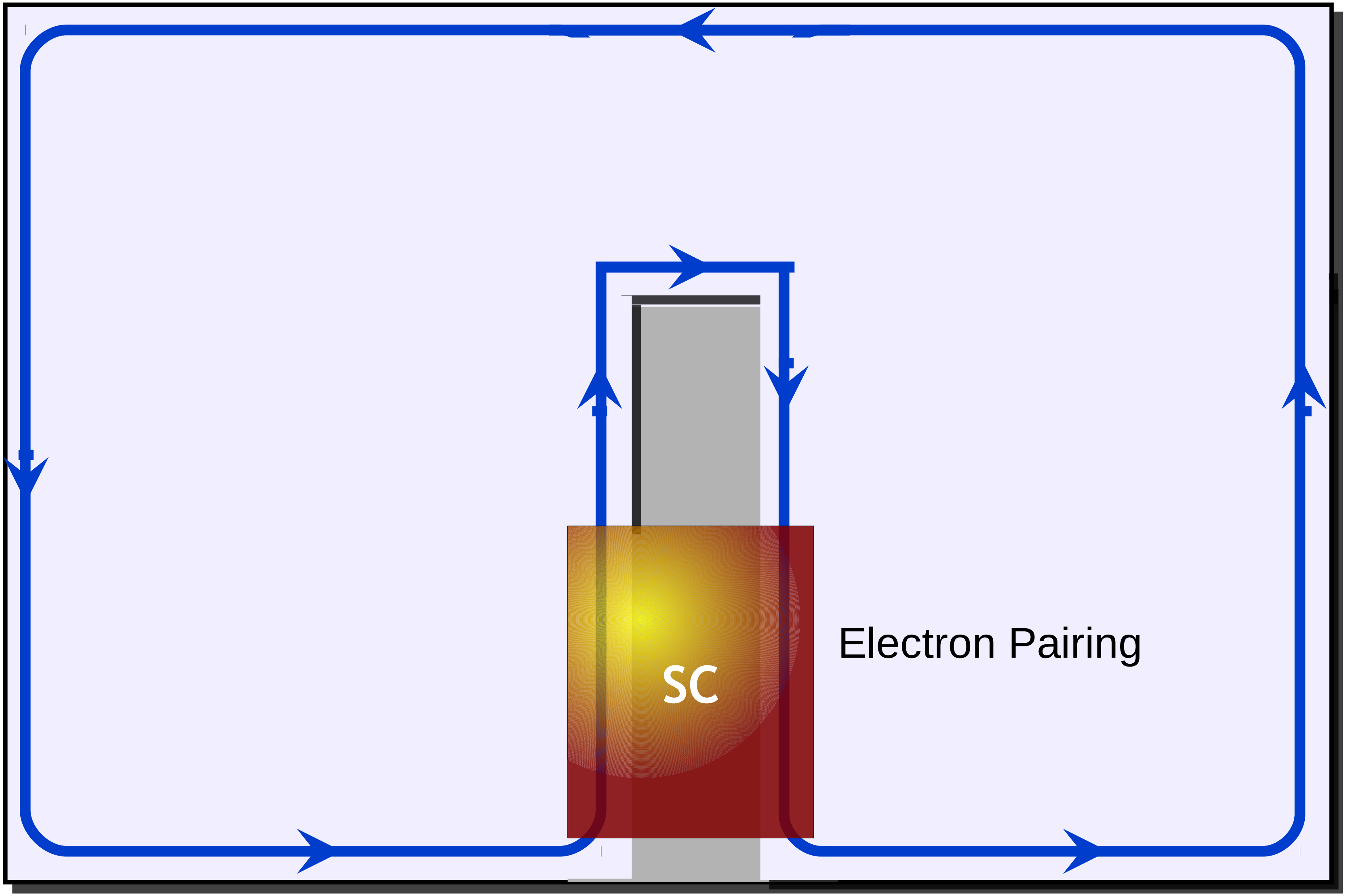
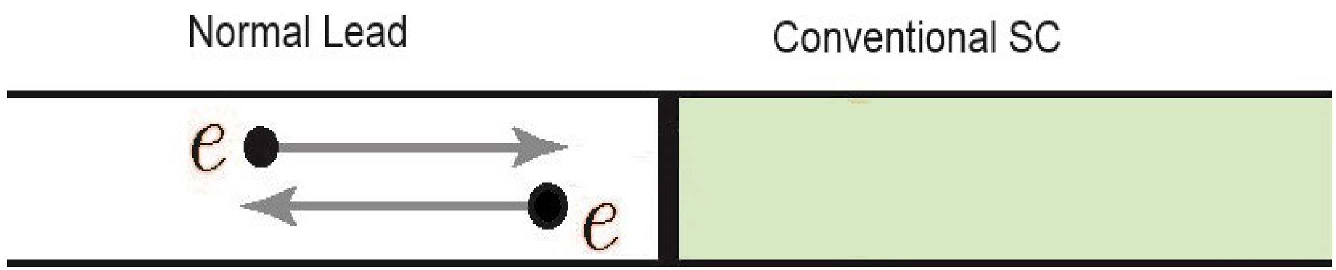


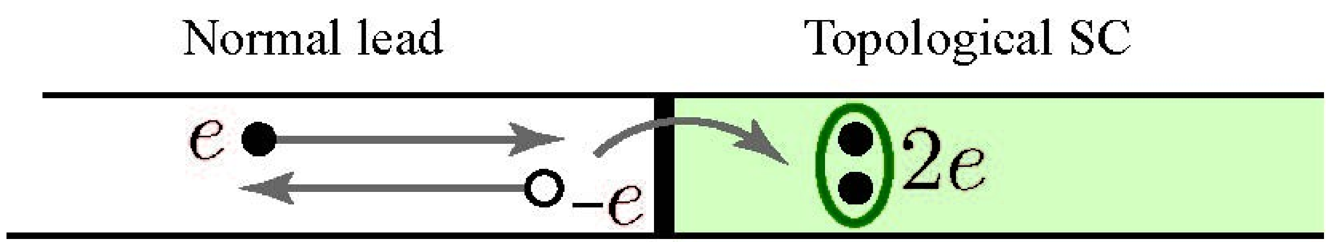


Realization in quantum Hall edges



Normal vs. Andreev reflection

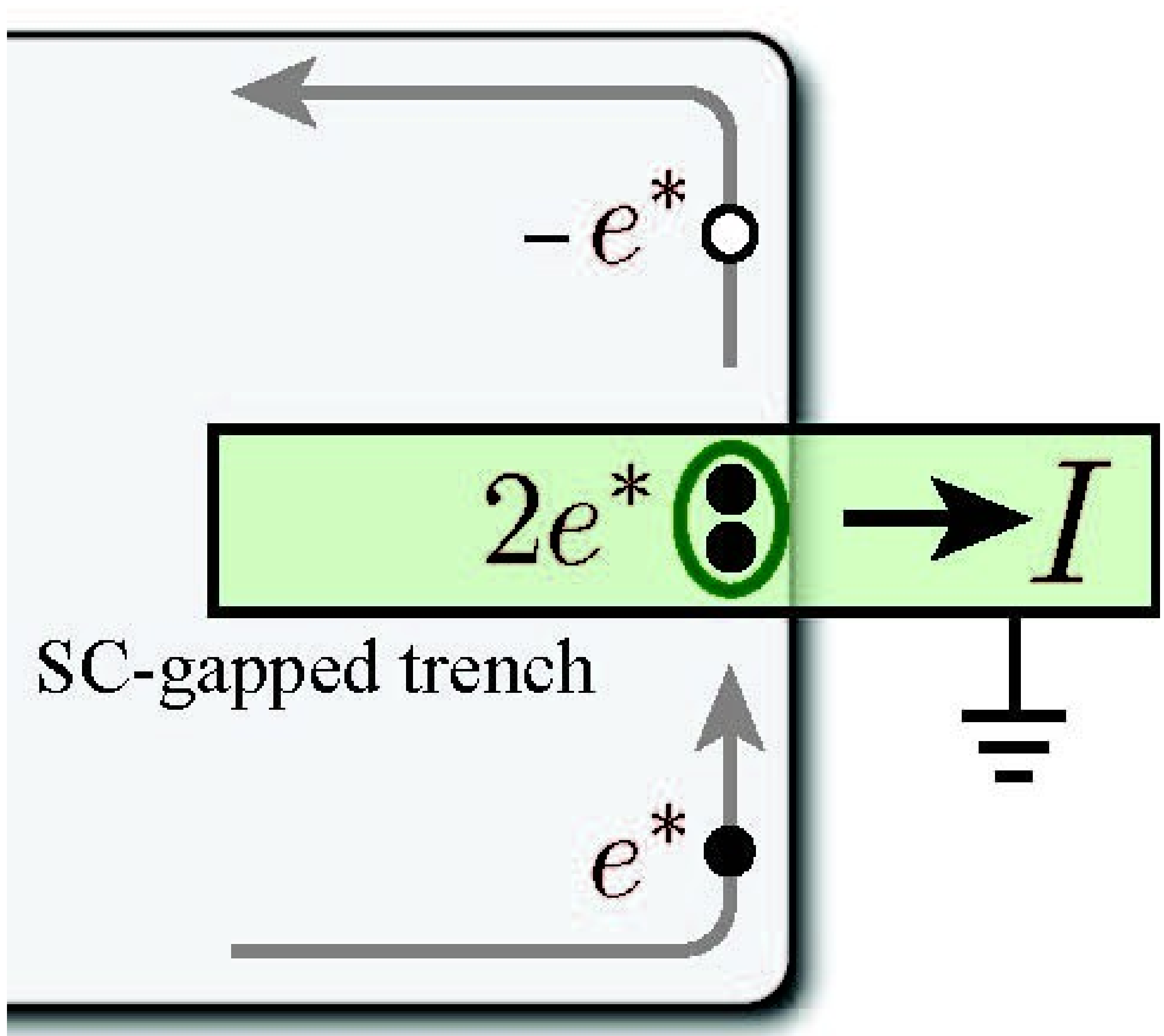




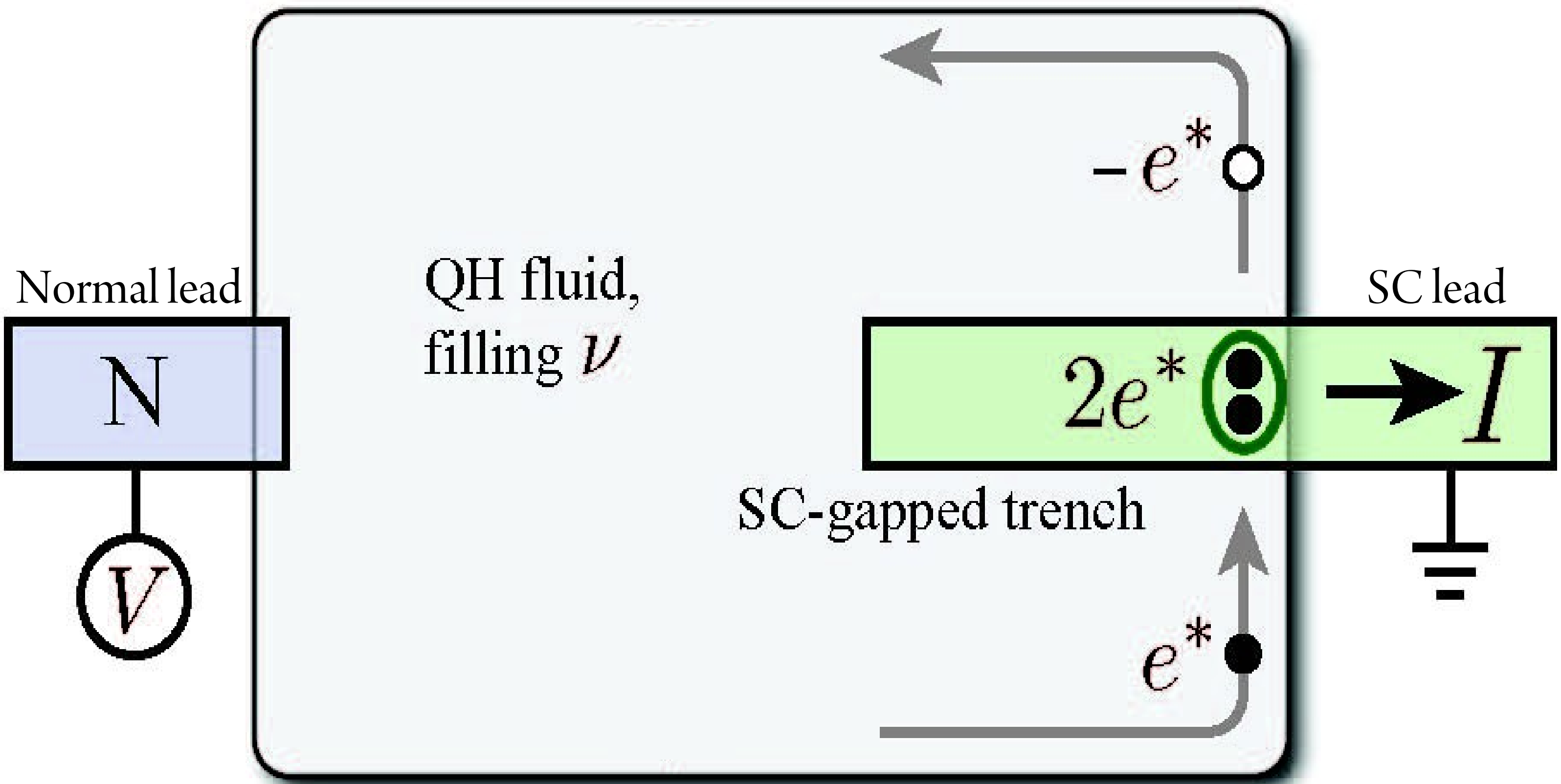
Conductance is *doubled* relative to a single-channel wire

Andreev Conversion on QH edge

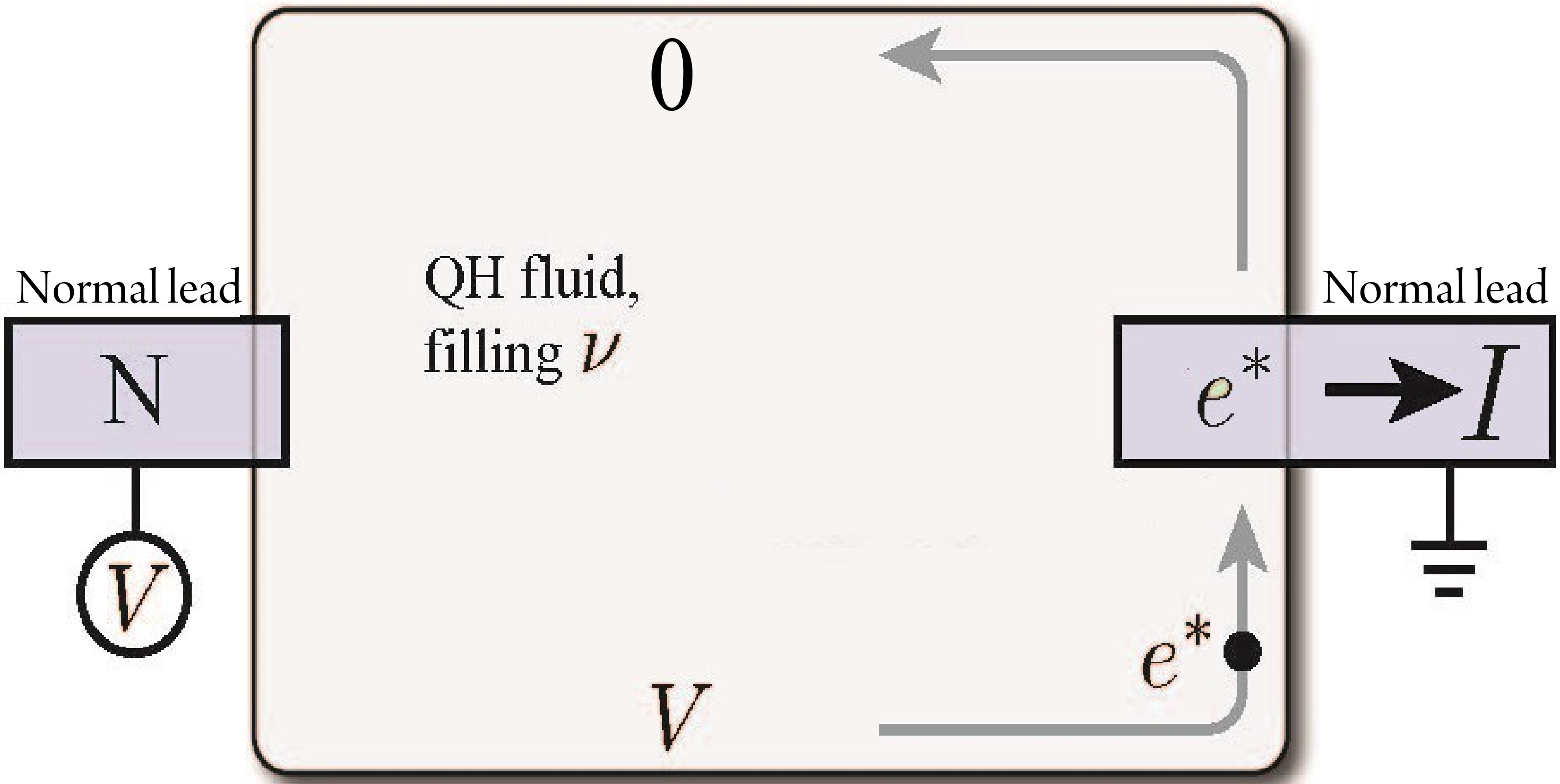
For fully chiral edge:



Two-terminal Hall Conductance

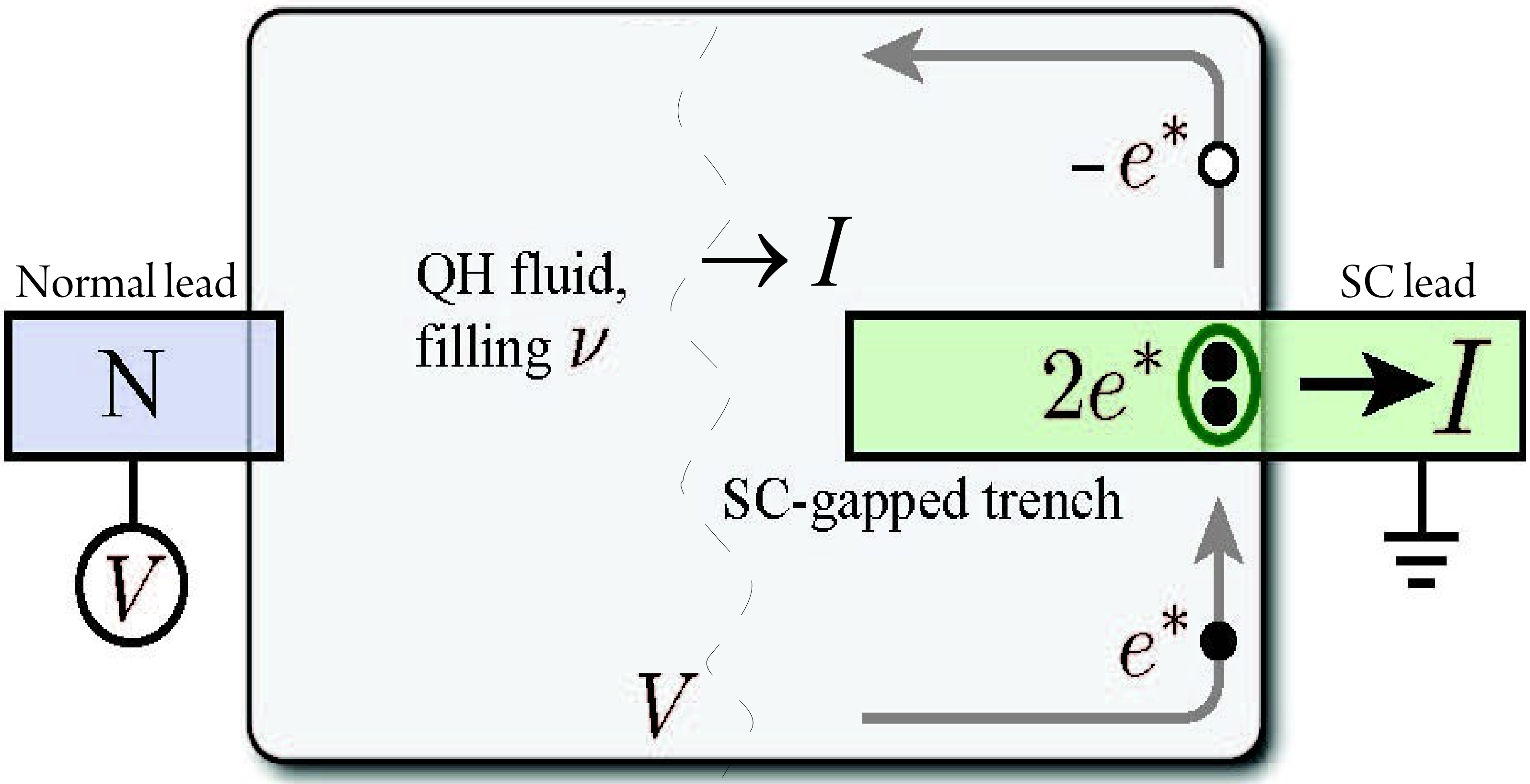


Two-terminal Hall Conductance



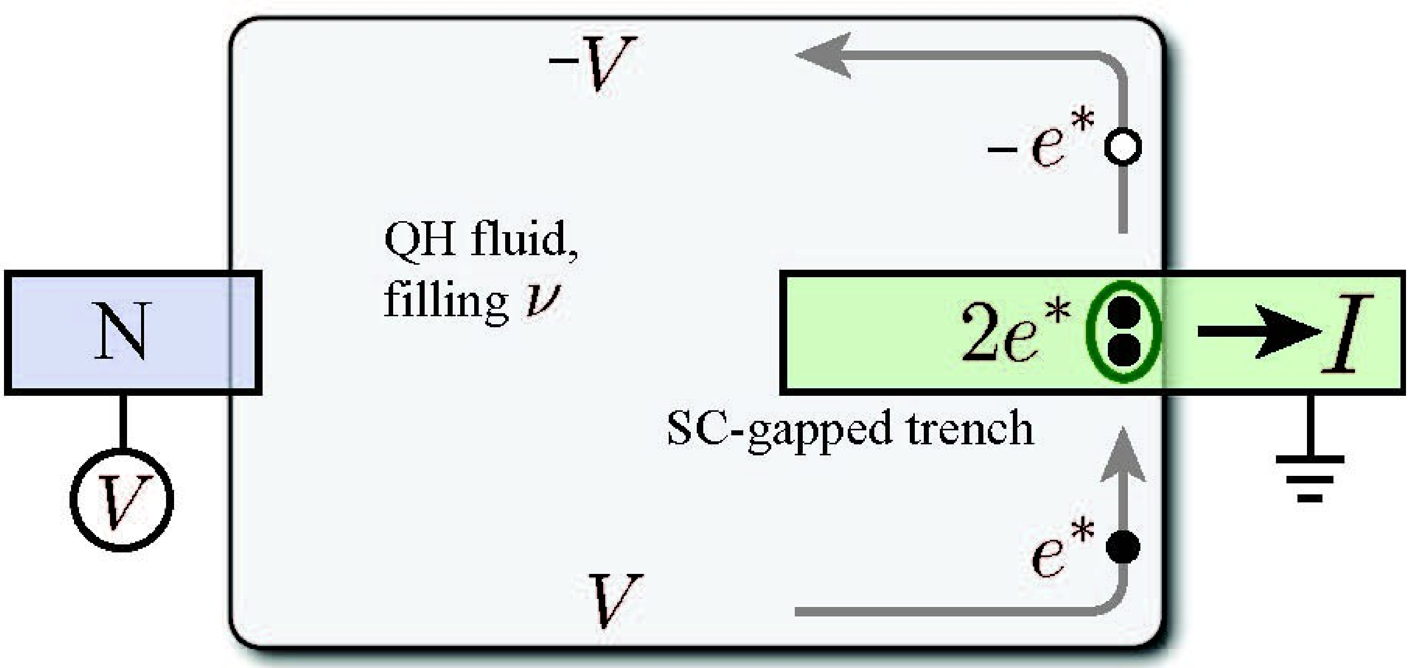
$$I = \sigma_{\mathrm{H}} V \qquad \sigma_{\mathrm{H}} = \nu e^2/h$$

Doubled Hall Voltage



$$I = 2\sigma_H V = \sigma_H (V_{Bottom} - V_{Top})$$

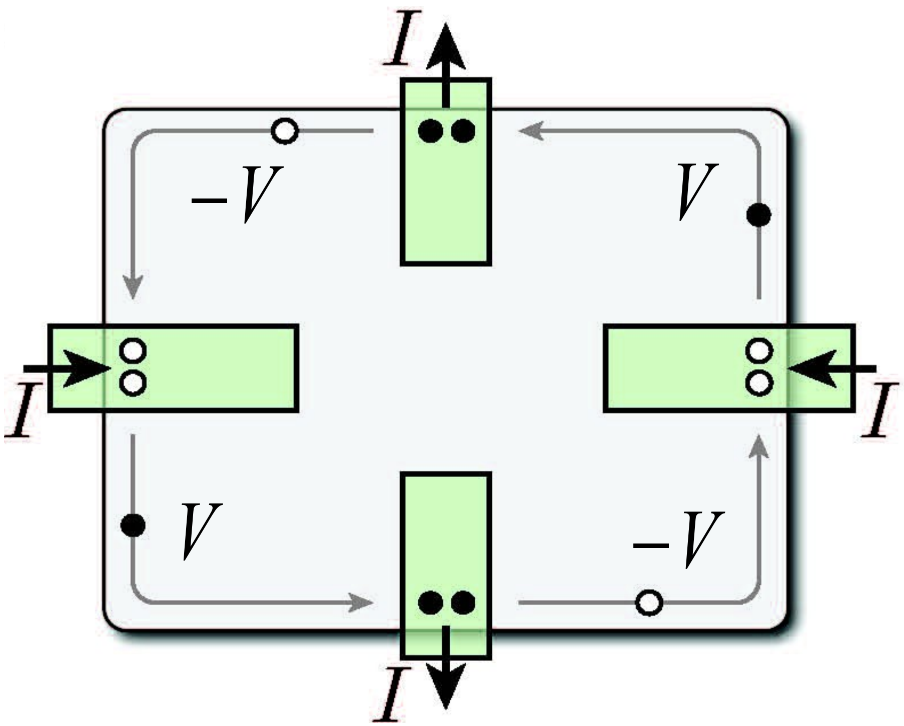
Doubled Hall Voltage



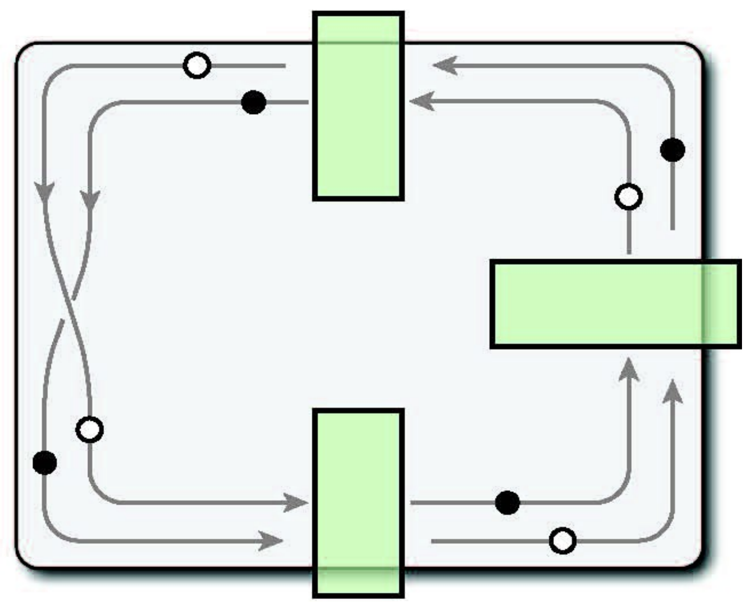
Equilibration and energy dissipation occur only at the normal lead! (in the ideal case)

$$V_{\text{out}} - V_{\text{SC}} = V_{\text{SC}} - V_{\text{in}}$$

Even/Odd effect for Grounded Superconductors



$$I = 2\sigma_H V$$

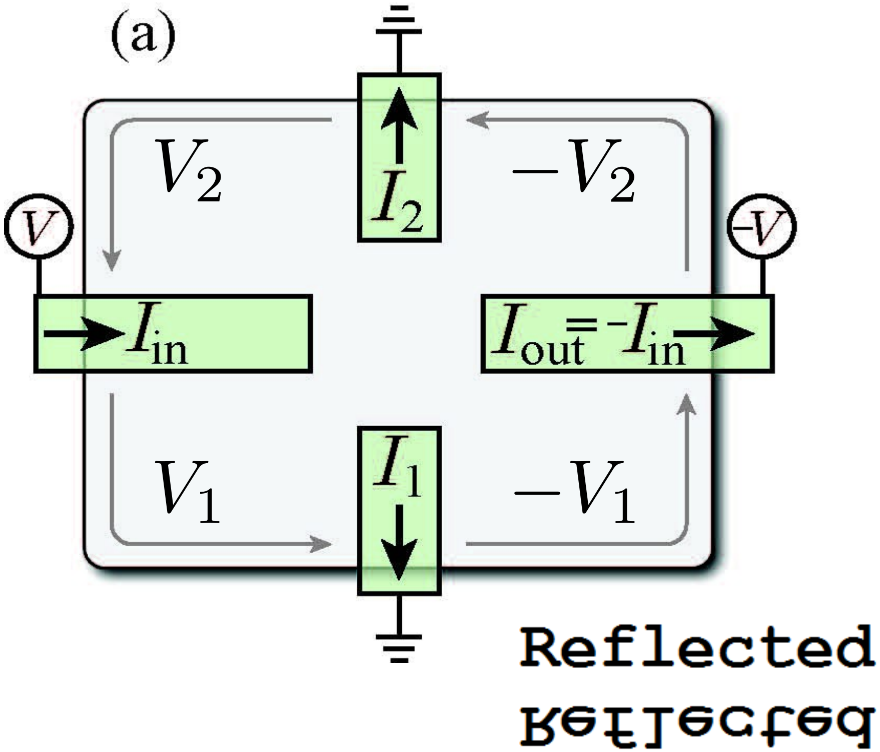


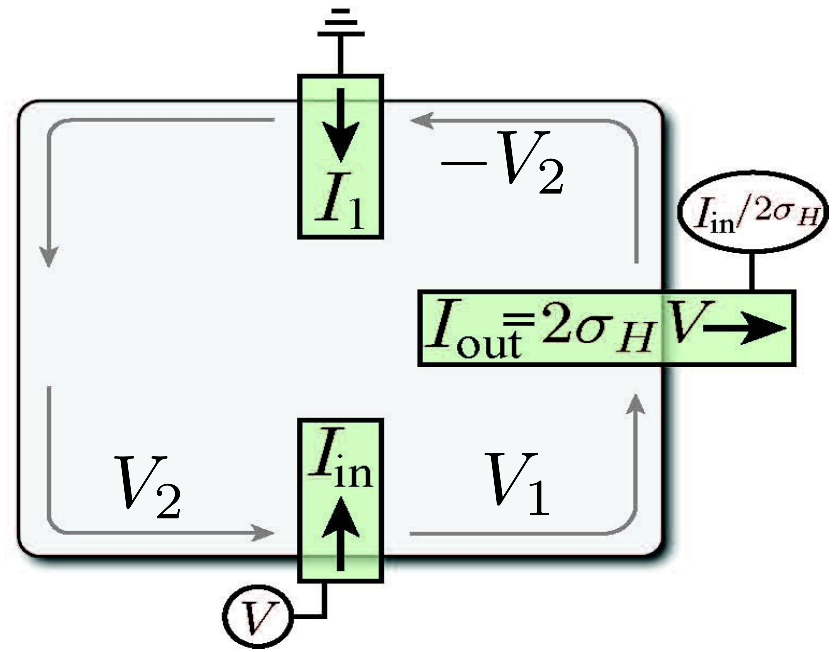
$$I = 0$$

(Grounded ideal superconductors)

$$V_{\mathrm{out}} - V_{\mathrm{SC}} = V_{\mathrm{SC}} - V_{\mathrm{in}}$$

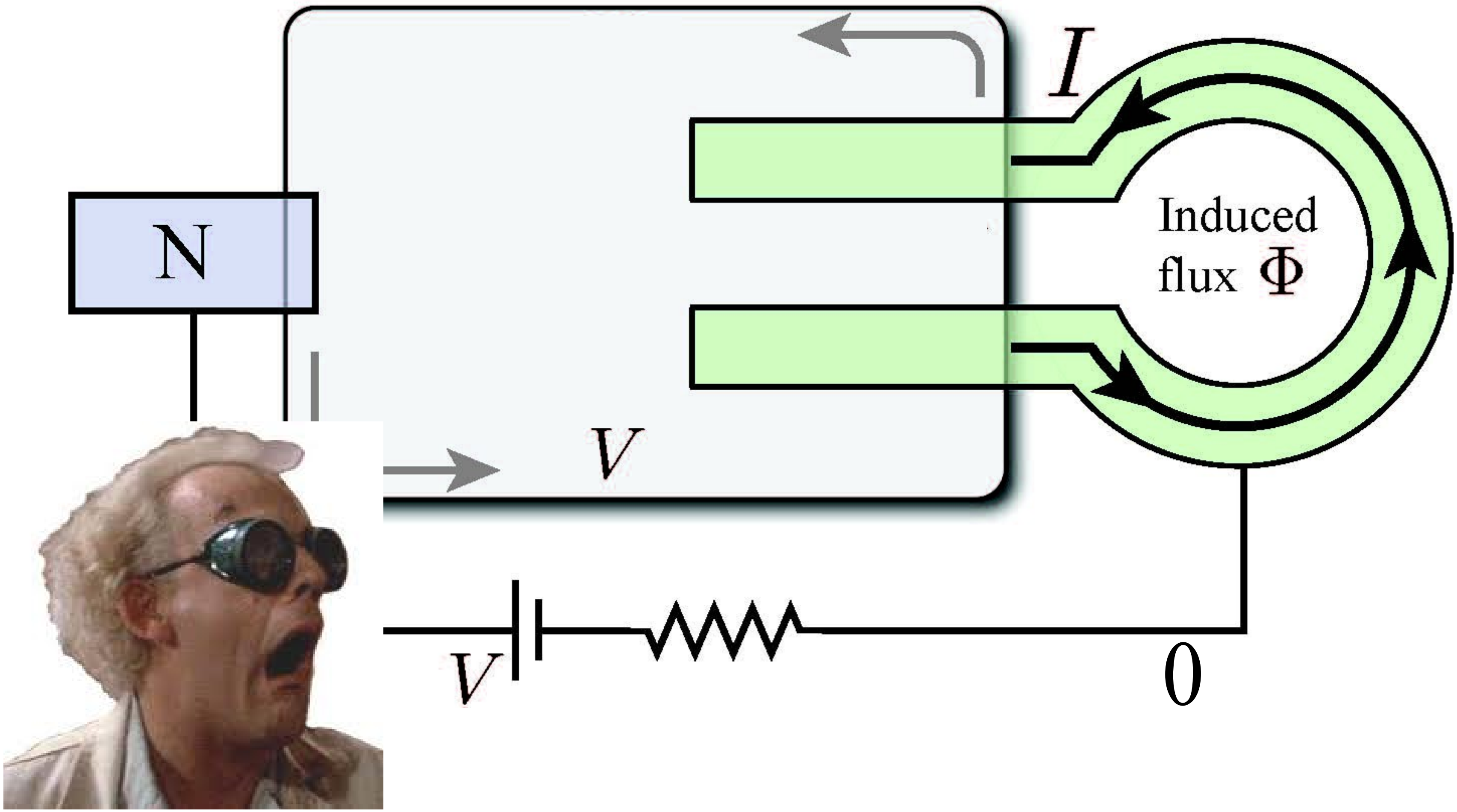
Current and Voltage Manipulation





Unusual circuit elements follow from Kirchhoff's rules!

Capacitive Flux Storage



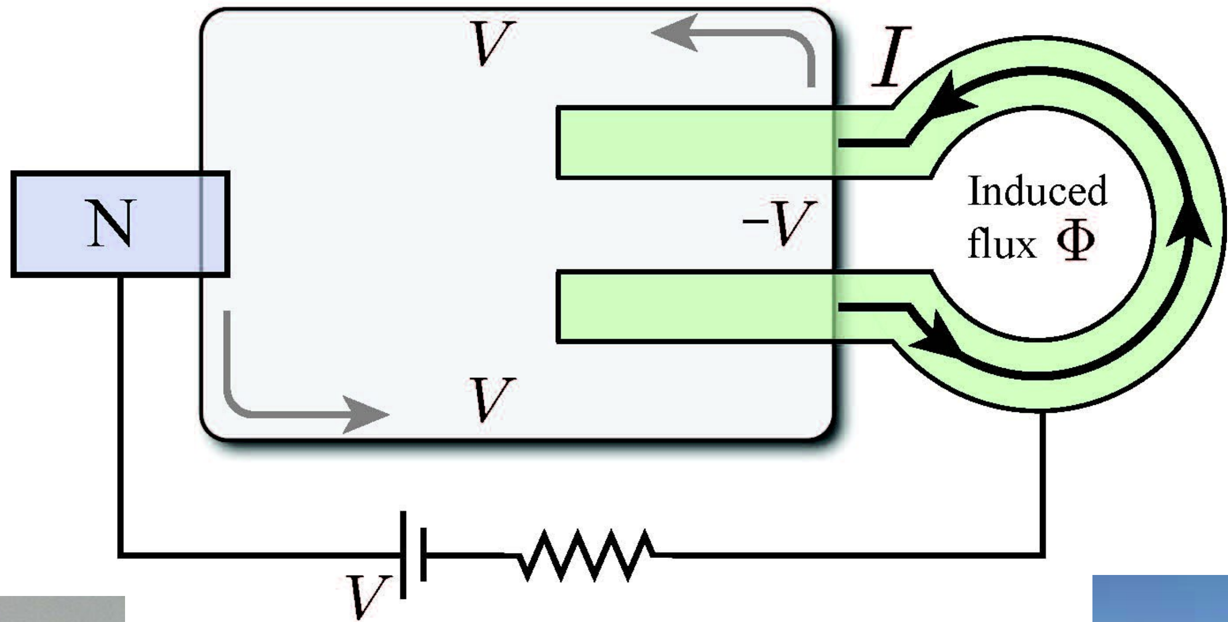
Flux Capacitor!

E.L. Brown, unpublished

(Possible) Applications of new circuit elements

- *Low Temperature and Low Power Amplifiers, Transistors, Logic Gates, etc.*
- *"On Chip" control elements for anyons*
- *Detection and utilisation of local magnetic fields*
- *New transport signatures of non-Abelian zero modes in hybrid FQH/SC systems*
- *A new use for non-Abelian anyons!*

Short of a quantum computer (for now)



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