laboratoire pierre aigrain
électronique et photonique quantiques

Topological Kondo island Josephson-coupled to a superconductor

E. Eriksson, Christophe Mora,
A. Zazunov, R. Egger

ENS, Paris & Düsseldorf

Nordita, 2014

Outline of the talk

I. Majorana fermions in p-wave superconductors

II. Topological Kondo effect

III. Josephson coupling

Majorana fermions in p-wave superconductors

Majorana bound states

- Majorana fermions $\gamma = c + c^{+}$

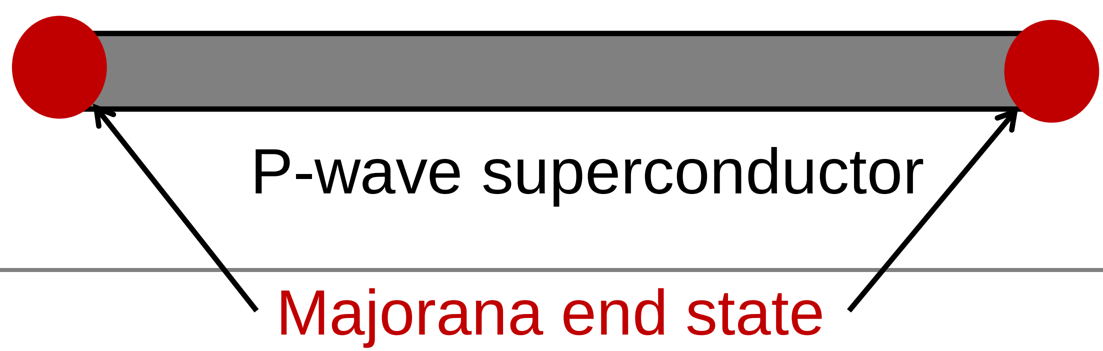
Beenakker, Ann. Rev. Con. Mat. Phys. 2013
Alicea, Rep. Prog. Phys. 2012
Leijnse & Flensberg, Semicond. Sci. Tech. 2012

$$\gamma^{+} = \gamma \quad \gamma^{2} = 1$$

- Natural solution of a Bogoliubov-de Gennes equation (superconductor) BUT requires zero energy and p-wave

→ { gap vanishes (local Majorana)
Pairs of Majorana

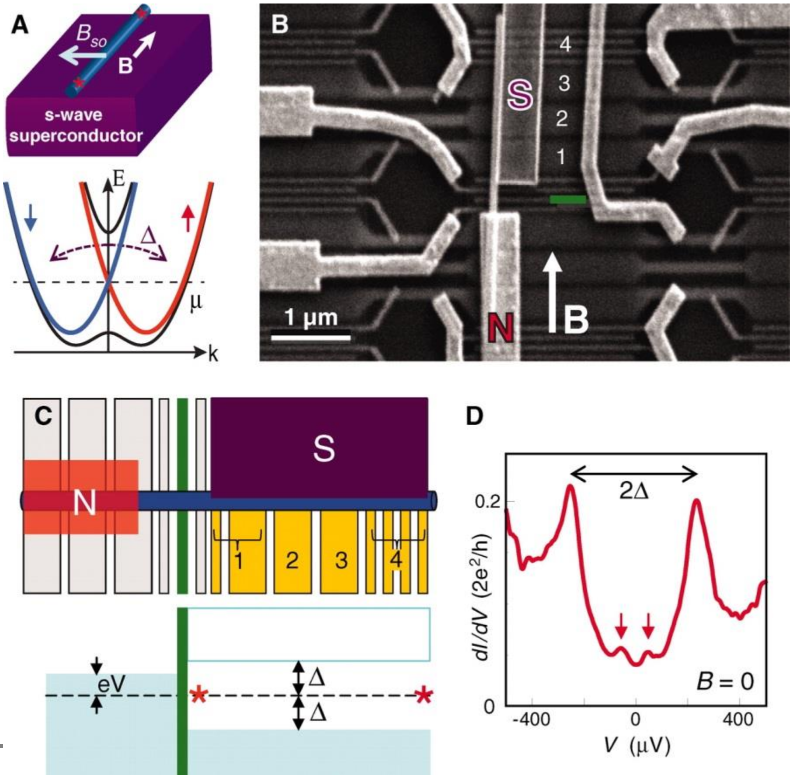
- Possible realization as end states of spinless 1D p-wave superconductor (Kitaev chain)



Experimental realization

- In practice, p-wave superconductors are inconvenient
- S-wave proximity effect on helical wire
- InSb/InAs nanowires are proposed as strong candidates for Majorana fermions
  - Strong spin-orbit coupling
  - Magnetic field
  - Proximity-induced pairing

Oreg, Refael & von Oppen, PRL 2010
Lutchyn, Sau & Das Sarma, PRL 2010



See also: *Rokhinson et al., Nat. Phys. 2012; Deng et al., Nano Lett. 2012; Das et al., Nat. Phys. 2012; Churchill et al., PRB 2013 Lee et al. Nat. Tech. 2014….*

Mourik et al., Science 2012

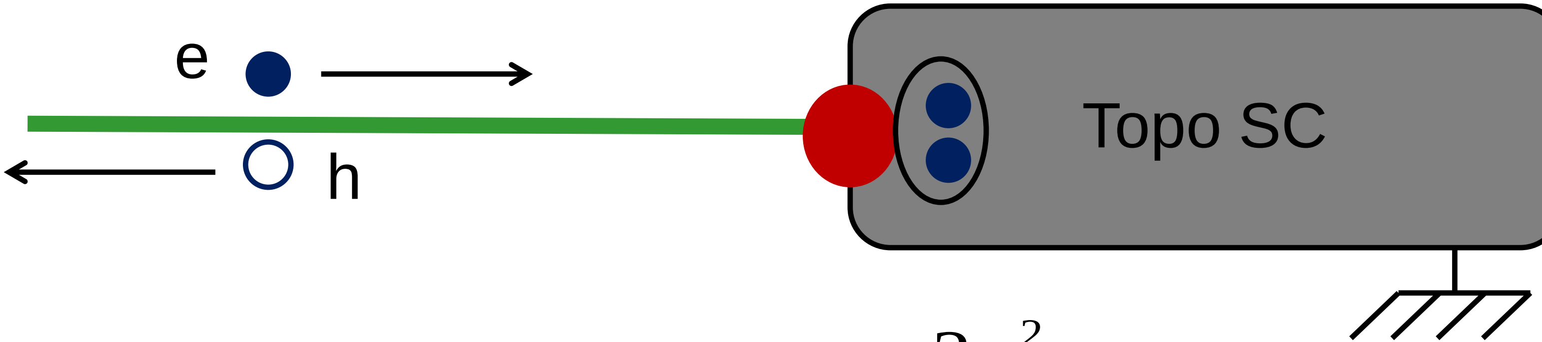
Majorana resonant level

- Coupling a Majorana end state to a metallic wire $\gamma \propto c + c^{+}$

ψ_0 is a chiral 1D field taken at x=0 $H_t = t\left(\psi_0 - \psi_0^{+}\right)\eta$

Same as Emery-Kivelson

- Andreev reflection at low energy



Cooper pair is transmitted $G = \frac{2e^2}{h}$

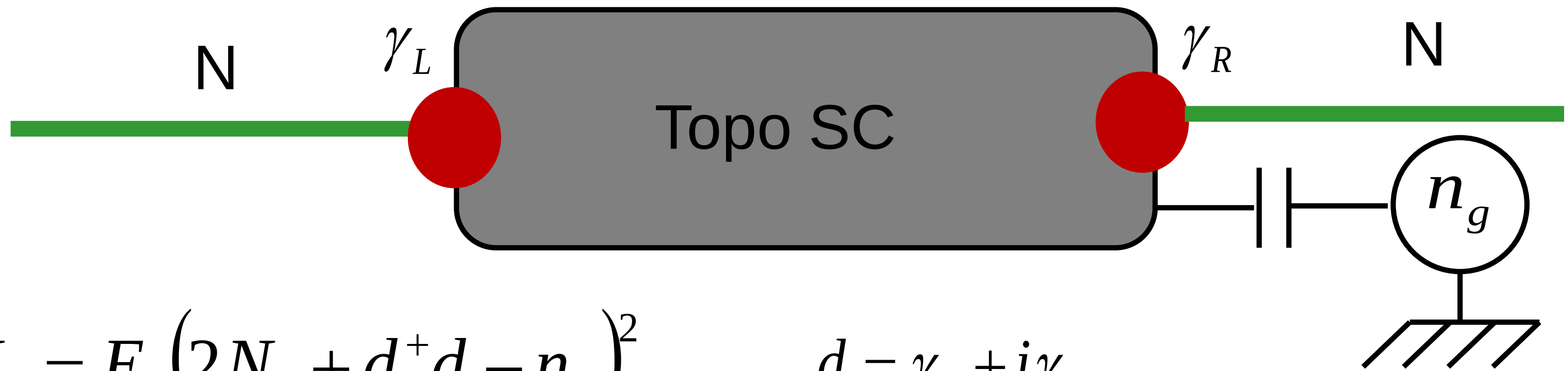
Grounded

Bolech & Demler, PRL 2007

Law, Lee & Ng, PRL 2009

Floating topological superconductor

- Andreev process is not favored when the SC is isolated



$$H_c = E_c\left(2N_c + d^{+}d - n_g\right)^2 \qquad d = \gamma_L + i\gamma_R$$

Accommodate an even number of electrons (no gap energy)

Floating topological superconductor

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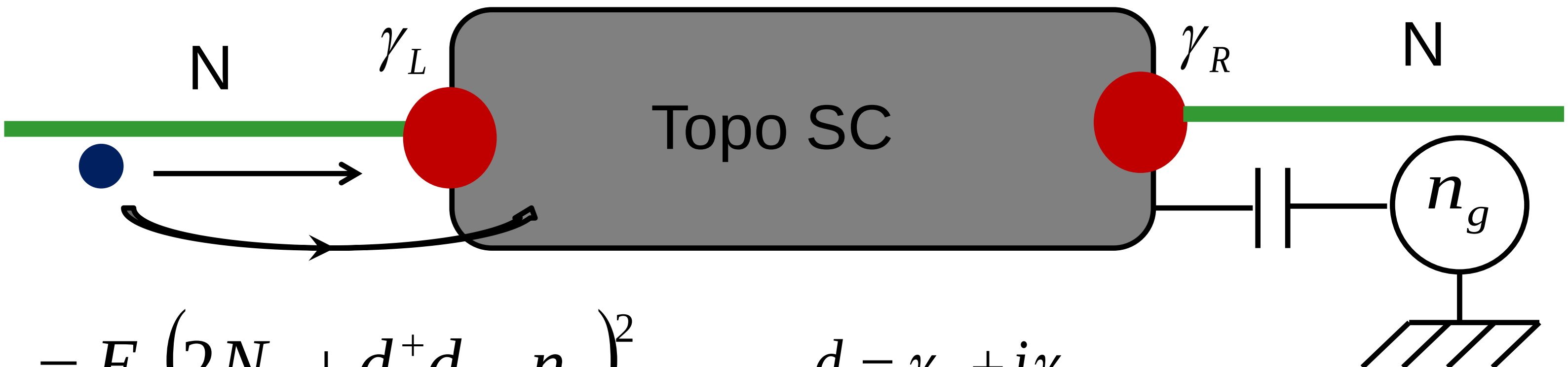

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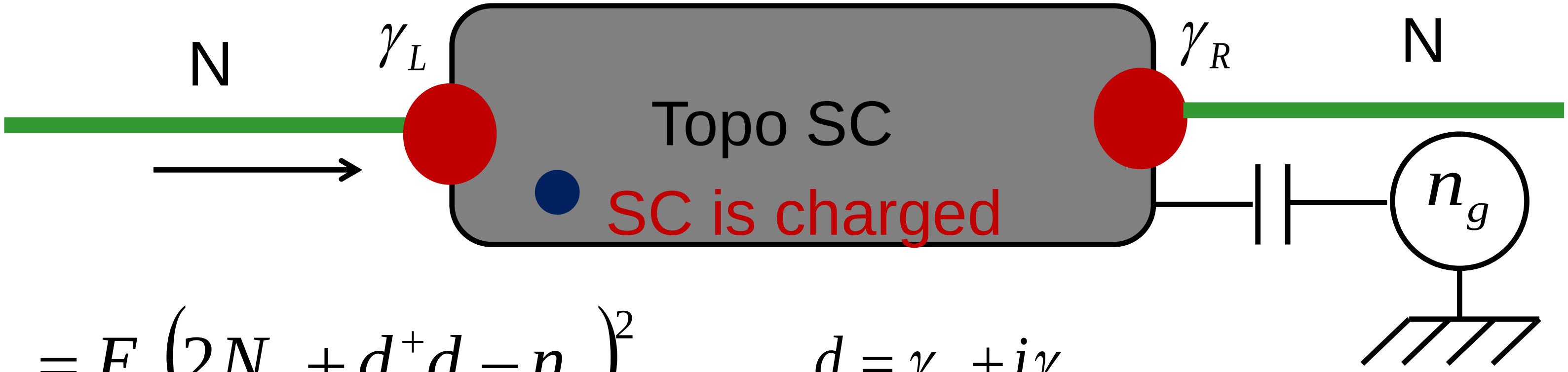

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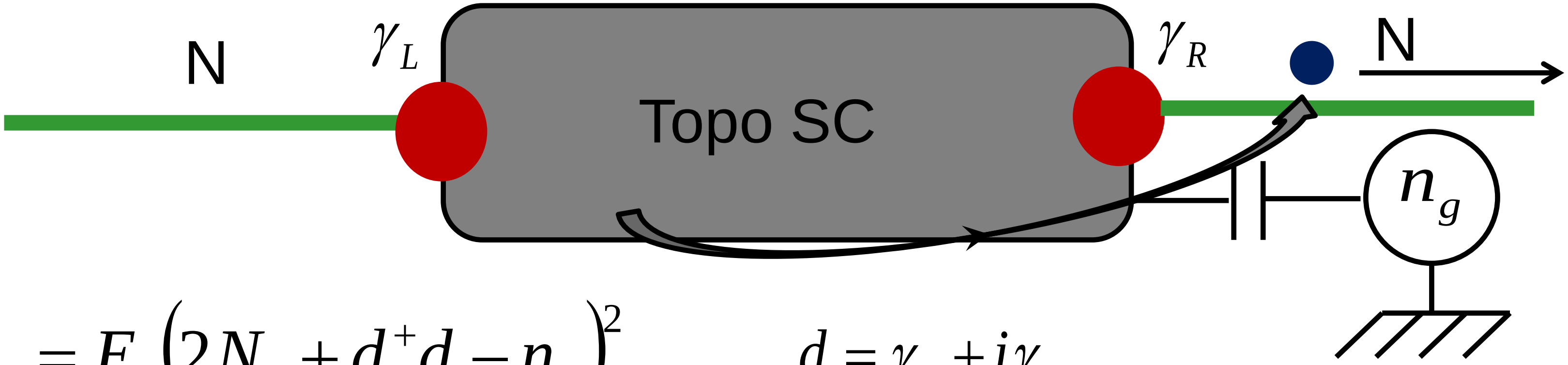


$$H_c = E_c\left(2N_c + d^+d - n_g\right)^2 \qquad d = \gamma_L + i\gamma_R$$

Accommodate an even number of electrons (no gap energy)

- One finds Coulomb blockade-like physics

$$G = \frac{e^2}{h}$$

Hützen et al., PRL 2010

« Electron teleportation »

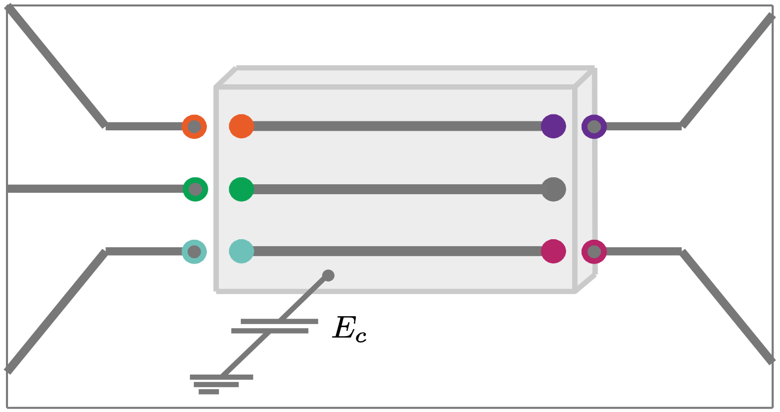
Fu, PRL 2010

Topological Kondo effect

Beri & Cooper, PRL 2012; Beri, PRL 2013
Zazunov, Altland & Egger, NJP 2014
Altland & Egger, PRL 2013
Altland, Beri, Egger & Tsvelik, PRL and J. Phys. A 2014

Many topological wires

- Array of Majoranas coupled individually to metallic leads



N wires

2N Majorana

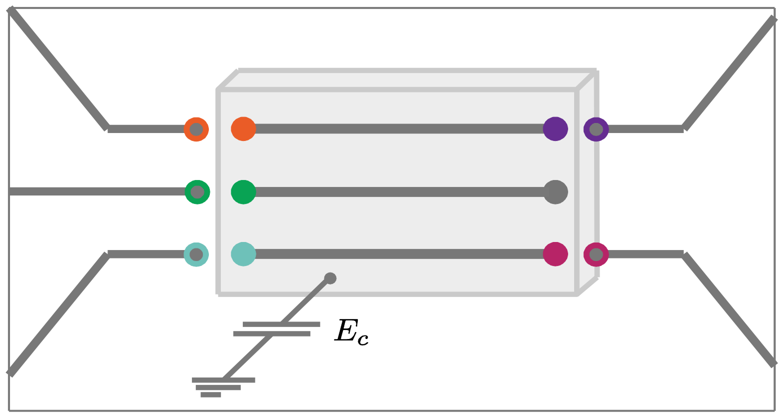
M leads

$$H_c = E_c\left(2N_c + \hat{n} - n_g\right)^2$$

$$\hat{n} = \sum_{i=1,N} d_i^+ d_i \qquad d_i = \gamma_{2i-1} + i\,\gamma_{2i}$$

Many topological wires

- Array of Majoranas coupled individually to metallic leads



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Non-local spin representation

$$SU(2)\ \text{spin} \sim \gamma_{1,2,3}$$

$$S_a = \frac{-i}{2}\varepsilon_{abc}\gamma_b\gamma_c$$

Schrieffer-Wolff projection

- Tunneling term

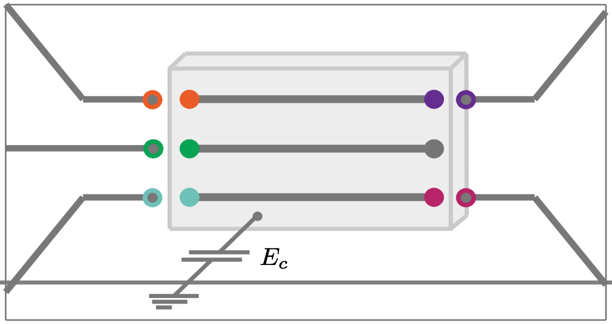
Change number of electrons

$$H_t = \sum_{j=1,M} t_j \gamma_j \left(e^{-i\varphi/2} \psi_j^+(0) - e^{i\varphi/2} \psi_j(0) \right)$$

- Project out the charge excited states, gives Kondo model

$$H_K = \sum_{j \neq k} \lambda_{jk} \psi_j^+(0) \psi_k(0) \gamma_j \gamma_k \overset{M=3}{\sim} \lambda \vec{J}(0).\vec{S} \qquad \lambda_{jk} \sim t_j t_k / E_c$$

- Non-local conductivity



$$G_{jk} = e \frac{\partial I_j}{\partial \mu_k} = \frac{2e^2}{h} \left[\delta_{jk} - \frac{1}{M} \right]$$

Josephson coupling

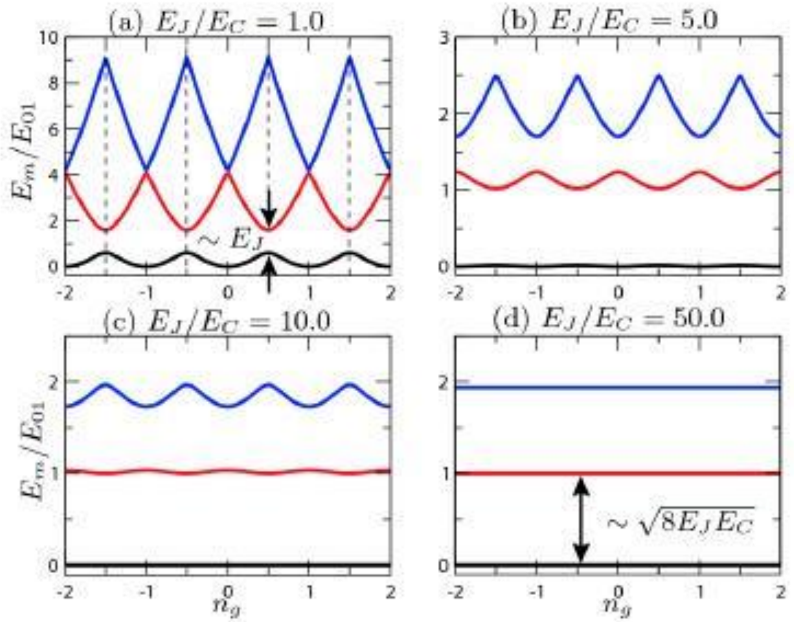
Eriksson, Mora, Zazunov, Egger, PRL 2014

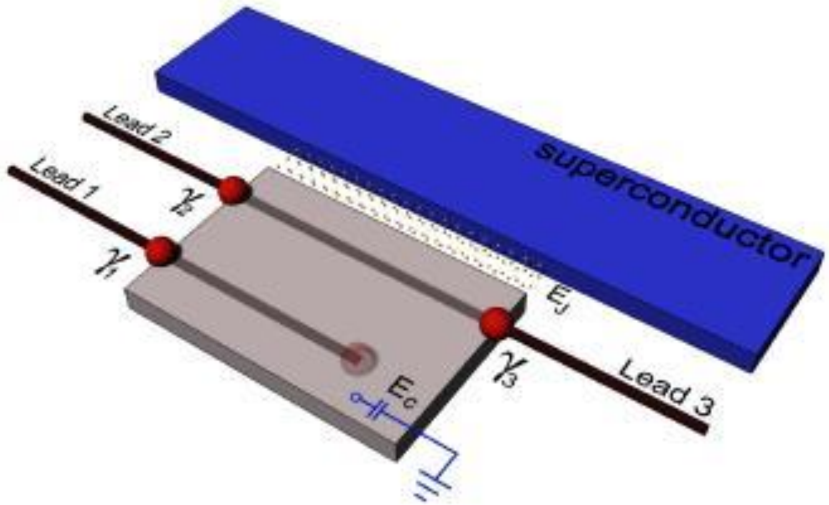
Cooper pair box

- Topo SC Josephson-coupled to a bulk superconductor

$$H_c = E_c\left(2N_c + \hat{n} - n_g\right)^2 - E_J \cos\varphi$$

Topological Cooper pair box





Koch et al, PRA 2007

Cooper pair box

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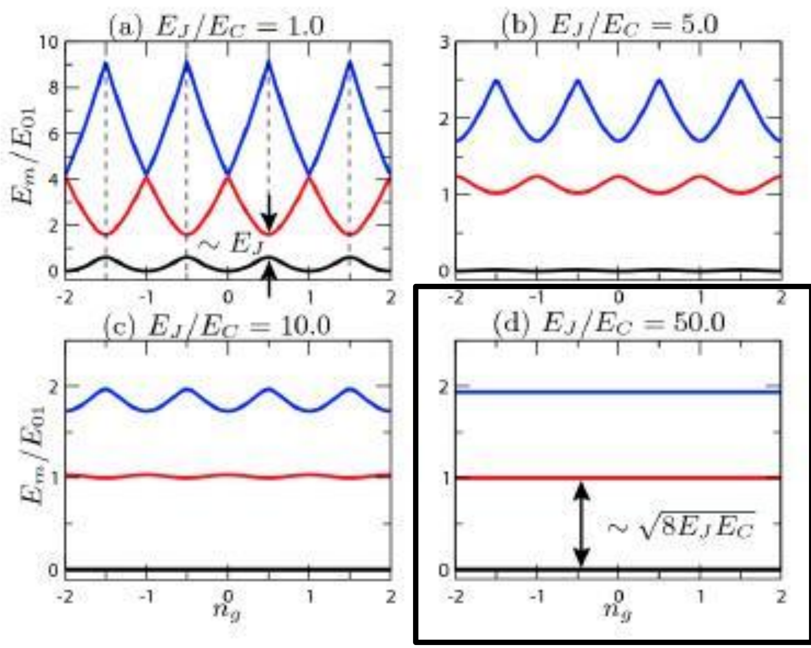

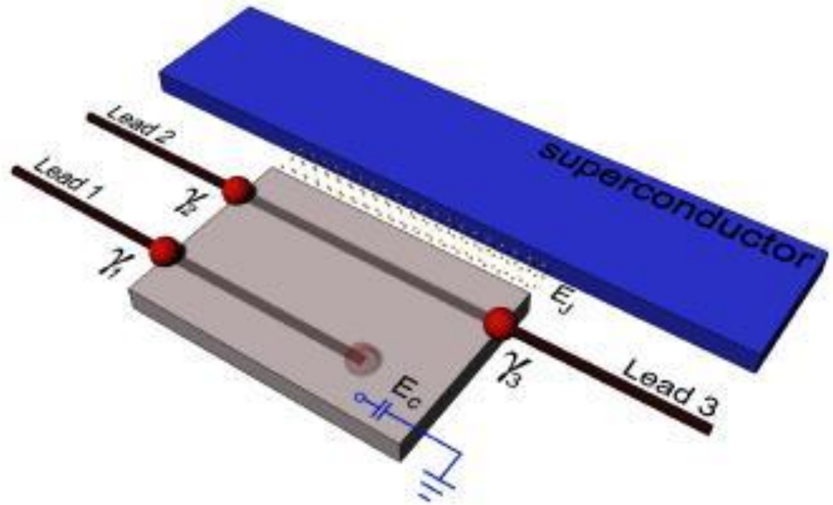

Harmonic oscillator

$$H_t = \sum_{j=1,M} t_j \gamma_j \begin{pmatrix} e^{-i\varphi/2}\psi_j^+(0) \\ -e^{i\varphi/2}\psi_j(0) \end{pmatrix}$$

Koch et al, PRA 2007

$$\Gamma_j << \sqrt{8E_J E_c}$$

Low energy projection

- Perturbation theory to zeroth and first order

$$H_A = \sum_{j=1,M} t_j \gamma_j \left(\psi_j^+(0) - \psi_j(0) \right)$$ Andreev term

$$H_K = \sum_{j \neq k} \lambda_{jk} \left(\psi_j^+(0) + \psi_j(0) \right) \left(\psi_k^+(0) + \psi_k(0) \right) \gamma_j \gamma_k$$ Kondo term

$$\left(\text{topo Kondo } H_K = \sum_{j \neq k} \lambda_{jk} \psi_j^+(0) \psi_k(0) \gamma_j \gamma_k \right)$$

- Two different fixed points (FP)
  - Majorana fixed point (resonant Andreev reflection) $\Delta_{LIO} = 2$
  - Kondo fixed point $\Delta_{LIO} = 3/2 \, (M = 3)$ $\Delta_{LIO} = 2 \, (M > 3)$

Γ / T_K controls the competition between FPs

Stability of fixed points

- At the Majorana FP: local Majoranas are screened and the Kondo term is irrelevant

- At the Kondo FP: the Andreev term is marginal, dimension of leading irrelevant operator is changed (orthogonality catastrophy)

Bosonisation of leads (fusion of Majoranas)

$$H_K + H_A \propto -\sum_j \sqrt{\Gamma_j} \sin\Theta_j - \sqrt{T_K} \sum_{j \neq k} \cos\Theta_j \cos\Theta_k$$

- Quantum brownian motion (QBM) picture: bosonic fields are pinned to the minima of this potential
  - Scaling dimensions of operators read off from distances between minima

Yi and Kane, PRB 1998, Yi, PRB 2002

Potential minima

- What are the potential minima ?

$$M = 3$$

$$H_K + H_A \propto -\sum_j \sqrt{\Gamma_j}\sin\Theta_j - \sqrt{T_K}\sum_{j \neq k}\cos\Theta_j \cos\Theta_k$$

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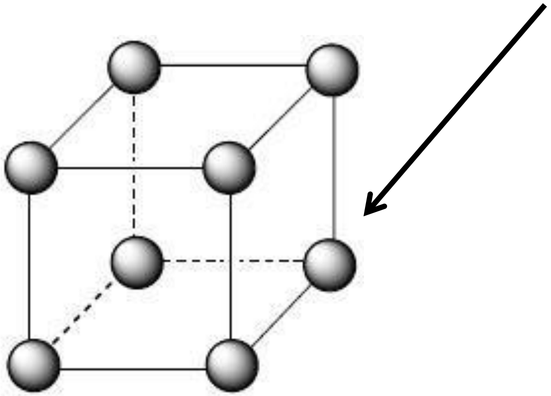

Cubic lattice (Majorana FP)

$$\Delta = \frac{d^2}{2\pi^2} = 2$$

Potential minima

- What are the potential minima ?

$$M = 3$$

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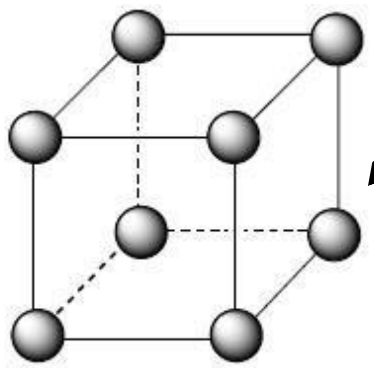
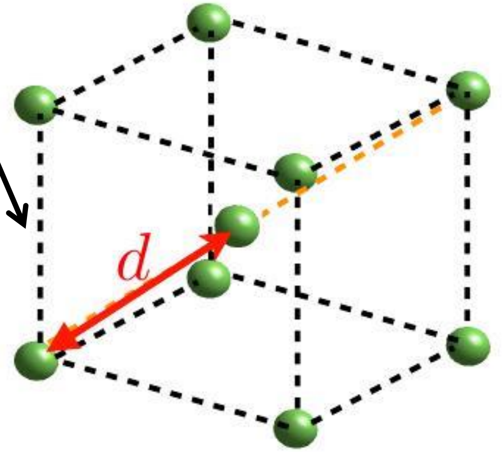




Cubic lattice (Majorana FP)

$$\Delta = \frac{d^2}{2\pi^2} = 2$$

Bcc lattice (Kondo FP)

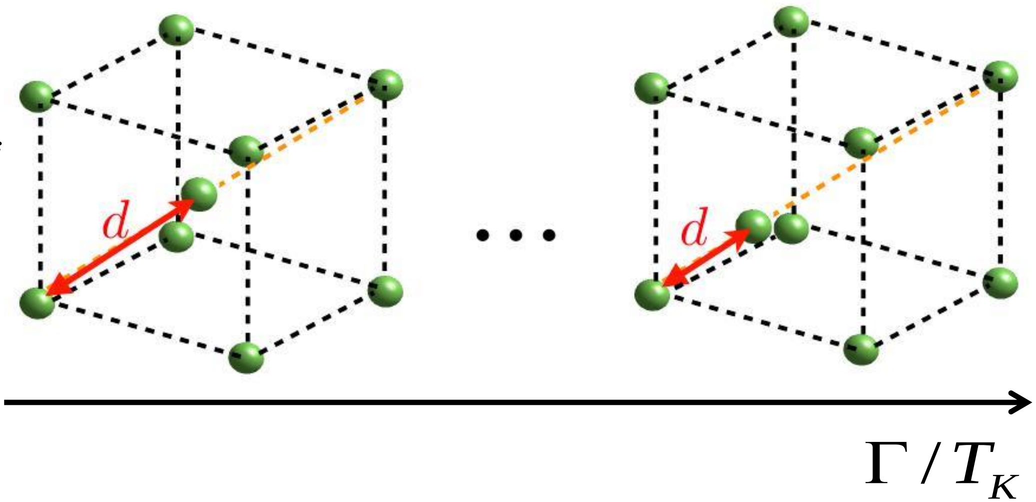
$$\Delta = \frac{d^2}{2\pi^2} = \frac{3}{2}$$

NFL manifold

$$M = 3 \qquad H_K + H_A \propto -\sum_j \sqrt{\Gamma_j} \sin\Theta_j - \sqrt{T_K} \sum_{j \neq k} \cos\Theta_j \cos\Theta_k$$

- Central point is moved by tunning lead tunnel coupling Γ_j
- Scaling dimensions of the irrelevant operators change

$$\Delta\left(\frac{\Gamma_j}{T_K}\right) = \frac{d^2}{2\pi^2}$$



- NFL becomes unstable when Δ reaches 1

NFL manifold

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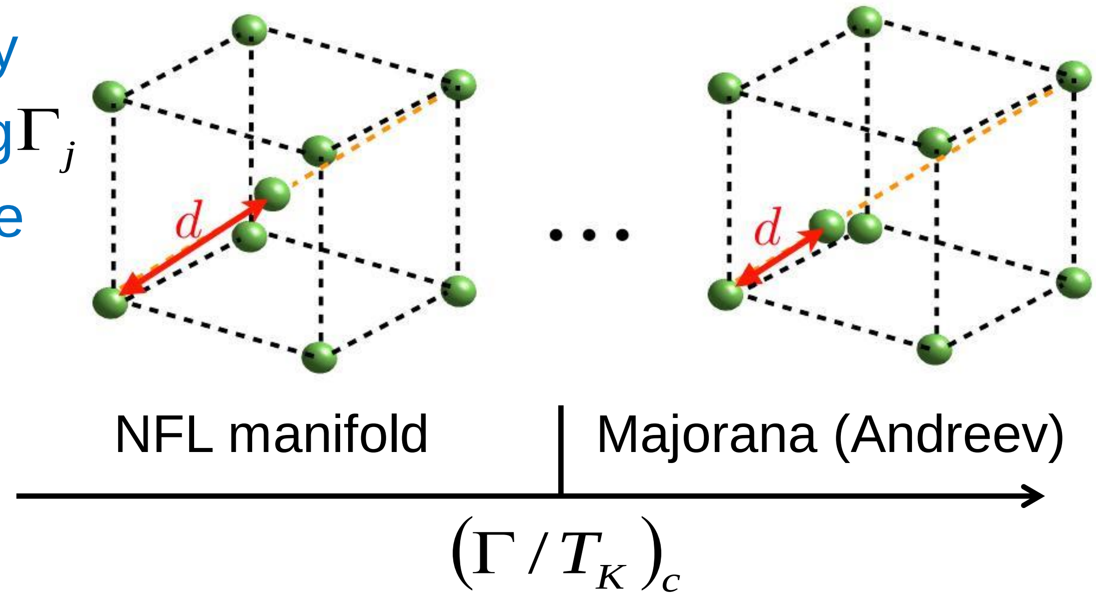

$$\Delta\left(\frac{\Gamma_j}{T_K}\right) = \frac{d^2}{2\pi^2}$$

- NFL becomes unstable when Δ reaches 1
- Non-local conductance

$$G_{jk} = \frac{2e^2}{h}\left[\delta_{jk} - A\left(\frac{T}{T_K}\right)^{2(\Delta-1)}\right]$$

CFT+bosonization approach

$M = 3$

- Real Majorana basis for leads $\psi_j(x) = \eta_j(x) + i\rho_j(x)$
- Majorana representation for spin $\vec{S} = (-i/2)\,\vec{\gamma}\times\vec{\gamma} \qquad \hat{b} = -2i\gamma_1\gamma_2\gamma_3$

$$\underbrace{H = -\frac{i}{2}\int dx\left[\vec{\eta}\partial_x\vec{\eta} + \vec{\rho}\partial_x\vec{\rho}\right] + J\,\vec{S}.\left[\frac{-i}{2}\vec{\gamma}\times\vec{\gamma}\right]} + i\,t\,\hat{b}\,\vec{S}.\vec{\rho}(0)$$

Two-channel Kondo model (compactified version)

Coleman, Ioffe, Tsvelik, PRB 1995

- At strong coupling: $\vec{S} = \hat{a}\,\vec{\eta}(0)$

$$H = H_0 + \frac{1}{\sqrt{T_K}}\hat{a}\,\eta_1\eta_2\eta_3 + i\,t\,\hat{b}\hat{a}\,\vec{\eta}(0).\vec{\rho}(0)$$

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Two-channel Kondo model (compactified version)

Coleman, Ioffe, Tsvelik, PRB 1995

- At strong coupling: $\vec{S} = \hat{a}\,\vec{\eta}(0)$

$$d = \hat{a} - i\hat{b} \qquad \psi \sim e^{i\phi}$$

$$H = H_0 + \frac{1}{\sqrt{T_K}}\hat{a}\,\eta_1\eta_2\eta_3 + i\,t\,\hat{b}\hat{a}\,\vec{\eta}(0).\vec{\rho}(0) \qquad \left(d^+d - \frac{1}{2}\right)\frac{\partial\phi}{\partial x}$$

X-ray edge sing.

- Formal similarity with two-impurity, two-channel Kondo model

Georges, Sengupta, PRL 1995, NPB 1997

Comparison

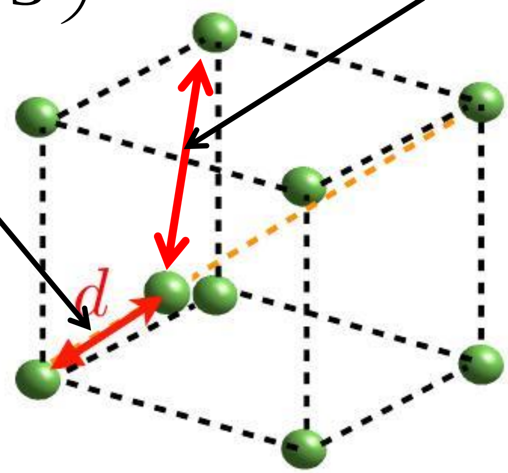
- Unitary transformation to remove the marginal term (boundary condition changing operator) *Affleck, Ludwig 1994, Affleck, 1996*

$$U\,\hat{a}\,\eta_1\eta_2\eta_3\,U^{-1}$$

$$\Delta = \frac{3}{2}\left(1 \pm \frac{\bar{\delta}}{3}\right)^2$$

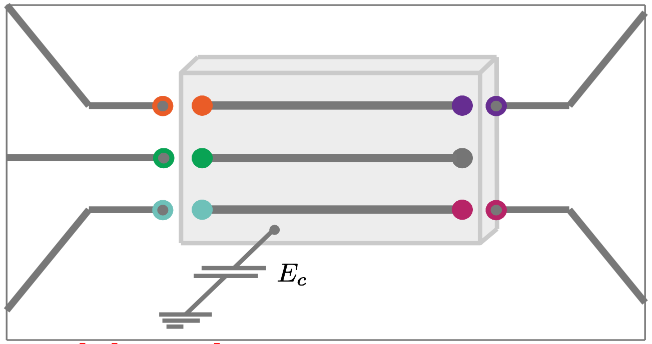
$$\Delta = \frac{4}{3} + \frac{\left(1 \pm \bar{\delta}\right)^2}{6}$$

$$\Delta = \frac{d^2}{2\pi^2}$$

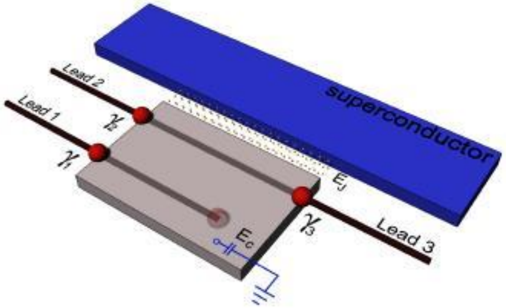


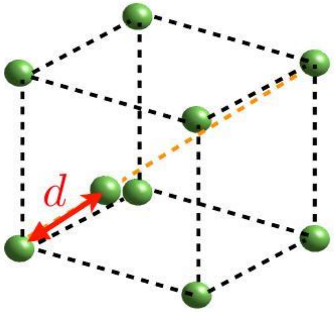
Conclusions

- Floating superconducting island with Majoranas gives rise to a topological Kondo effect



- Topological Cooper pair box: competition between Andreev and Kondo leads to a NFL manifold





- Study of LDOS (with A. Nava)

Eriksson, Mora, Zazunov, Egger, PRL 2014

Thank you for your attention

and

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