An order parameter for impurity systems at quantum criticality

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- **An order parameter for impurity systems at quantum criticality**
  A. Bayat, S. Bose, P. Sodano, H. Johannesson
  Nature Communications 5, 3784 (2014)

- **Entanglement probe of two-impurity Kondo physics in a spin chain**
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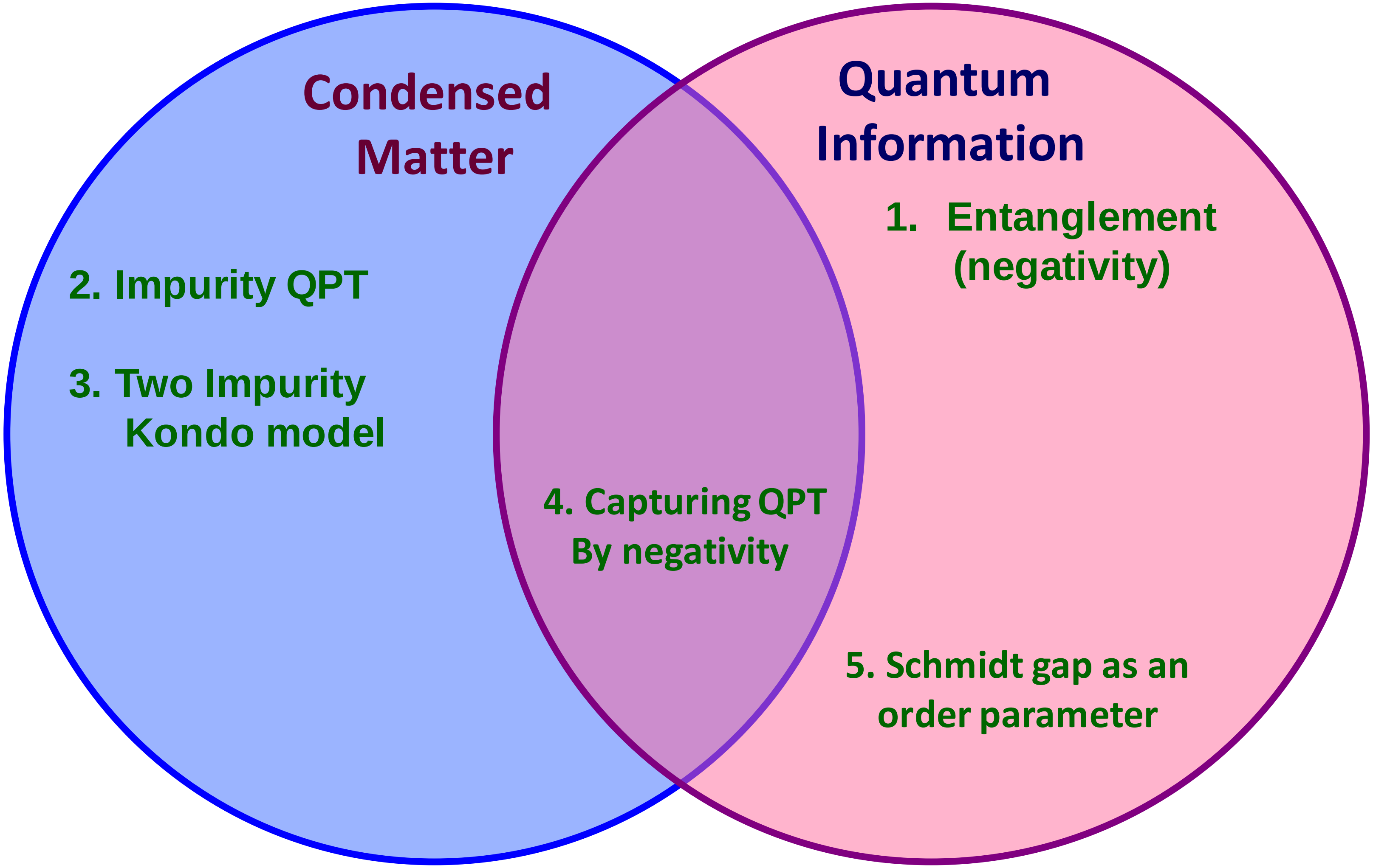
  Phys. Rev. Lett. 109, 066403 (2012)

Contents of the Talk



Properties of the Density Matrix

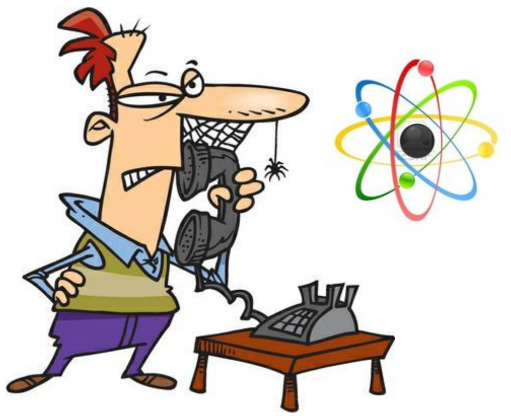
1. **Hermiticity:** $\rho = \rho^{+}$
2. **Trace one:** $Tr(\rho) = 1$
3. **Positivity:** $\rho \geq 0$

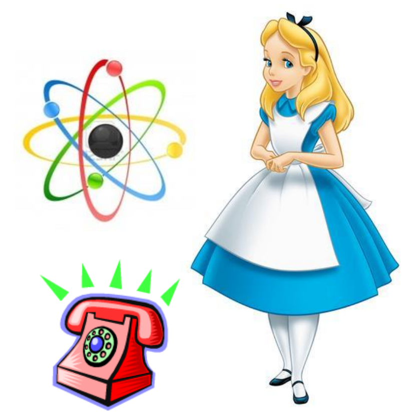
For any density matrix: ρ ⇒ ρ^{T} **is also a density matrix**

Separable Mixed States

Separable states: $$\rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

$$p_i \geq 0, \quad \sum_i p_i = 1$$





With local operations and classical communications Alice and Bob can produce these kind of states

Entanglement of Mixed States

Pure states:

$$\rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}|$$

$$E(\rho_{AB}) = S(\rho_A) = S(\rho_B)$$

Entangled states: $\rho_{AB} \neq \sum_i p_i \rho_i^A \otimes \rho_i^B$

How to quantify entanglement for a general mixed state?

There is not a unique entanglement measure

Negativity

Separable:

$$\rho = \sum_i p_i \, \rho_i^A \otimes \rho_i^B \;\Rightarrow\; \rho^{T_A} = \sum_i p_i \, (\rho_i^A)^t \otimes \rho_i^B \;\Rightarrow\; \rho^{T_A} \geq 0 \checkmark$$

Valid density matrix

Entangled:

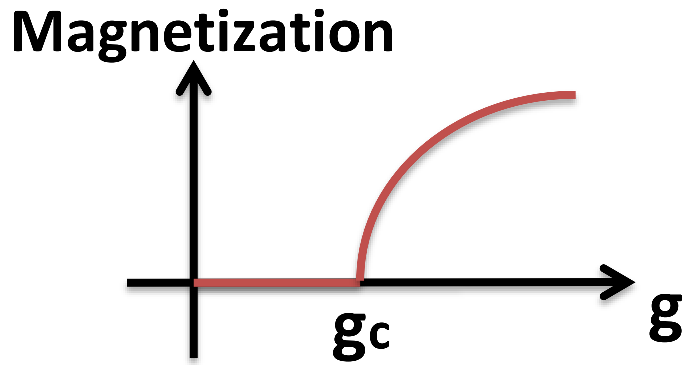
$$\rho^{T_A} |\lambda\rangle = \lambda |\lambda\rangle \quad (\lambda < 0)$$

Negativity:

$$N(\rho) = 2 \sum_{\lambda < 0} |\lambda|, \qquad \rho^{T_A} |\lambda\rangle = \lambda |\lambda\rangle$$

Quantum vs. Classical Phase Transitions

Quantum: $H(g), |GS(g)\rangle$ → External parameter g is the control parameter



Phase transition is captured by a local order parameter: $m = \sum_i \sigma_z^i$

Order Parameter

Order parameter is:

1- Observable
2- Is zero in one phase and non-zero in the other
3- Scales at criticality

Landau-Ginzburg paradigm:

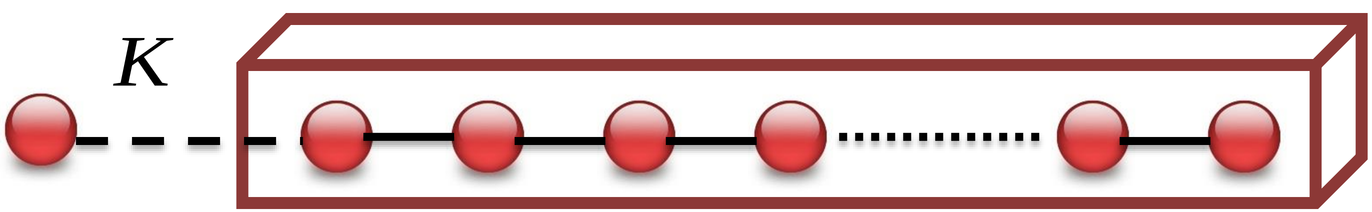
4- Order parameter is local
5- Order parameter is associated with a spontaneous symmetry breaking

Bulk vs. Boundary QPT

Bulk phase transition: a global parameter induces the QPT

$$H_{I\sin g} = \sum_i \sigma_z^i \sigma_z^{i+1} + B \sum_i \sigma_x^i$$

Boundary phase transition: a local parameter induces the QPT



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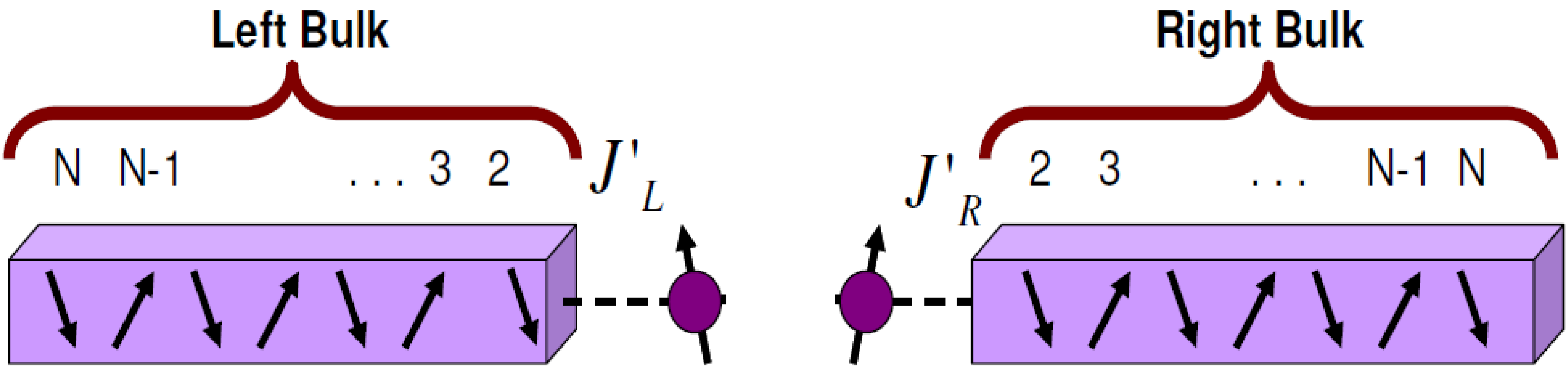
Impurity Phase Transitions

Impurity phase transitions are an example of boundary QPT:

- **There is no order parameter (either local or non-local)**
- **There is no spontaneous symmetry breaking**

Two Impurity Kondo Model



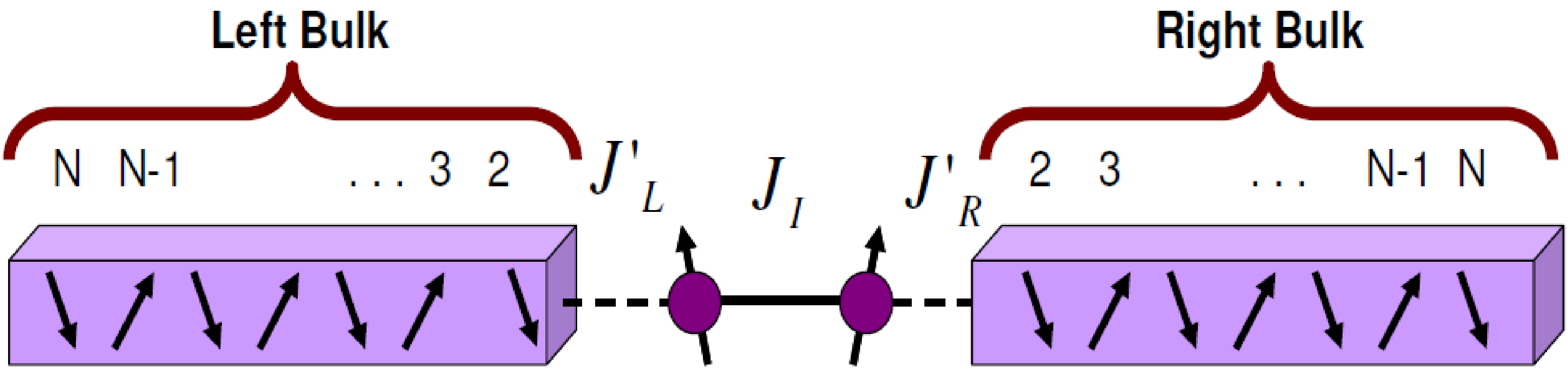
$$H_L = J'_L J_1 \sigma_1^L . \sigma_2^L + \sum_{i=2}^{N_L - 1} J_1 \sigma_i^L . \sigma_{i+1}^L$$

$$H_R = J'_R J_1 \sigma_1^R . \sigma_2^R + \sum_{i=2}^{N_R - 1} J_1 \sigma_i^R . \sigma_{i+1}^R$$

$$H_I = J_I \sigma_1^L . \sigma_1^R$$

RKKY interaction

Impurities



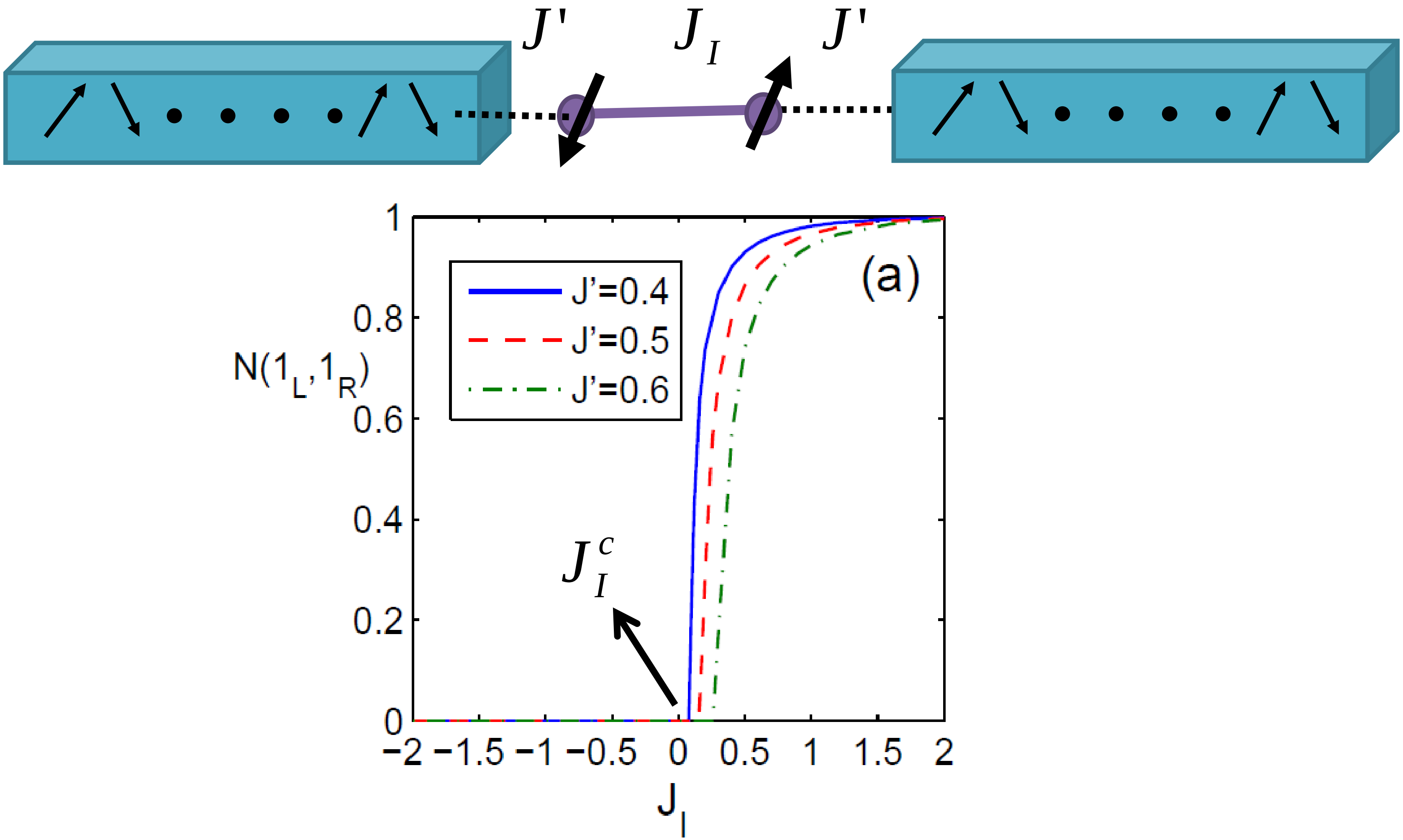
$$|GS\rangle \Rightarrow \rho_{1_L 1_R} = p|\psi^-\rangle\langle\psi^-| + \frac{1-p}{3}\sum_{k=0,\pm}|T^k\rangle\langle T^k|$$

Entanglement

$$p \le \frac{1}{2} \Rightarrow N(\rho_{1_L 1_R}) = 0$$

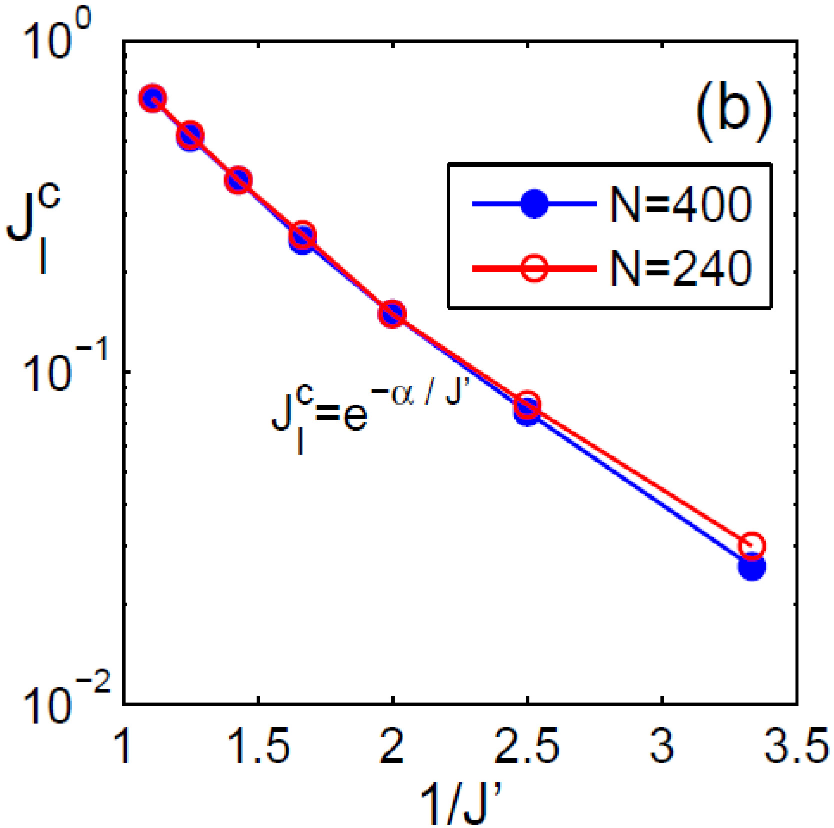
$$p > \frac{1}{2} \Rightarrow N(\rho_{1_L 1_R}) > 0$$

Entanglement of Impurities



Entanglement can be used for distinguishing differentiating phases

Scaling at the Phase Transition



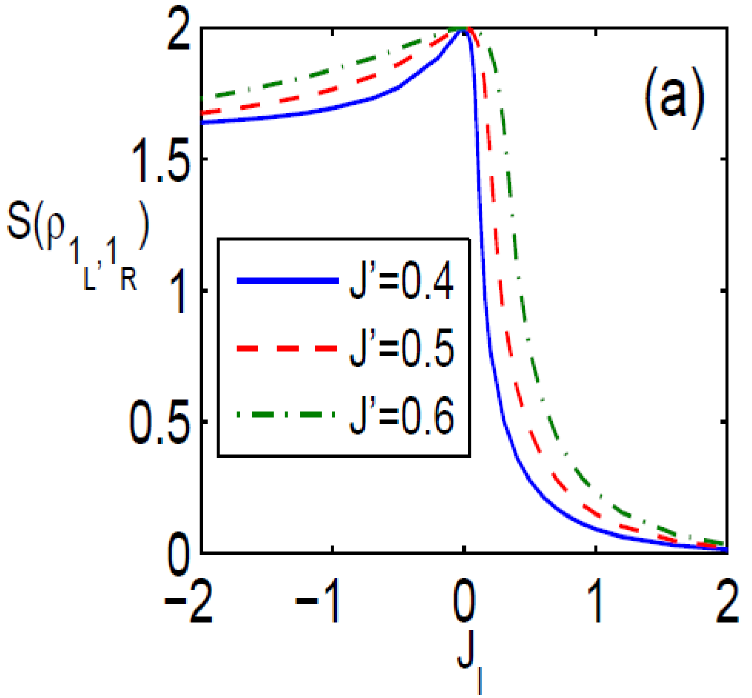
$$J_I^c \propto T_K \propto \frac{1}{\xi_K} \propto e^{-\alpha / J'}$$

The critical RKKY coupling scales just as Kondo temperature does

Entropy of Impurities

$$\rho_{1_L 1_R} = p|\psi^-\rangle\langle\psi^-| + \frac{1-p}{3}\sum_{k=0,\pm}|T^k\rangle\langle T^k|$$

$$S(\rho_{1_L,1_R}) = -p\log(p) - (1-p)\log(1-p)$$



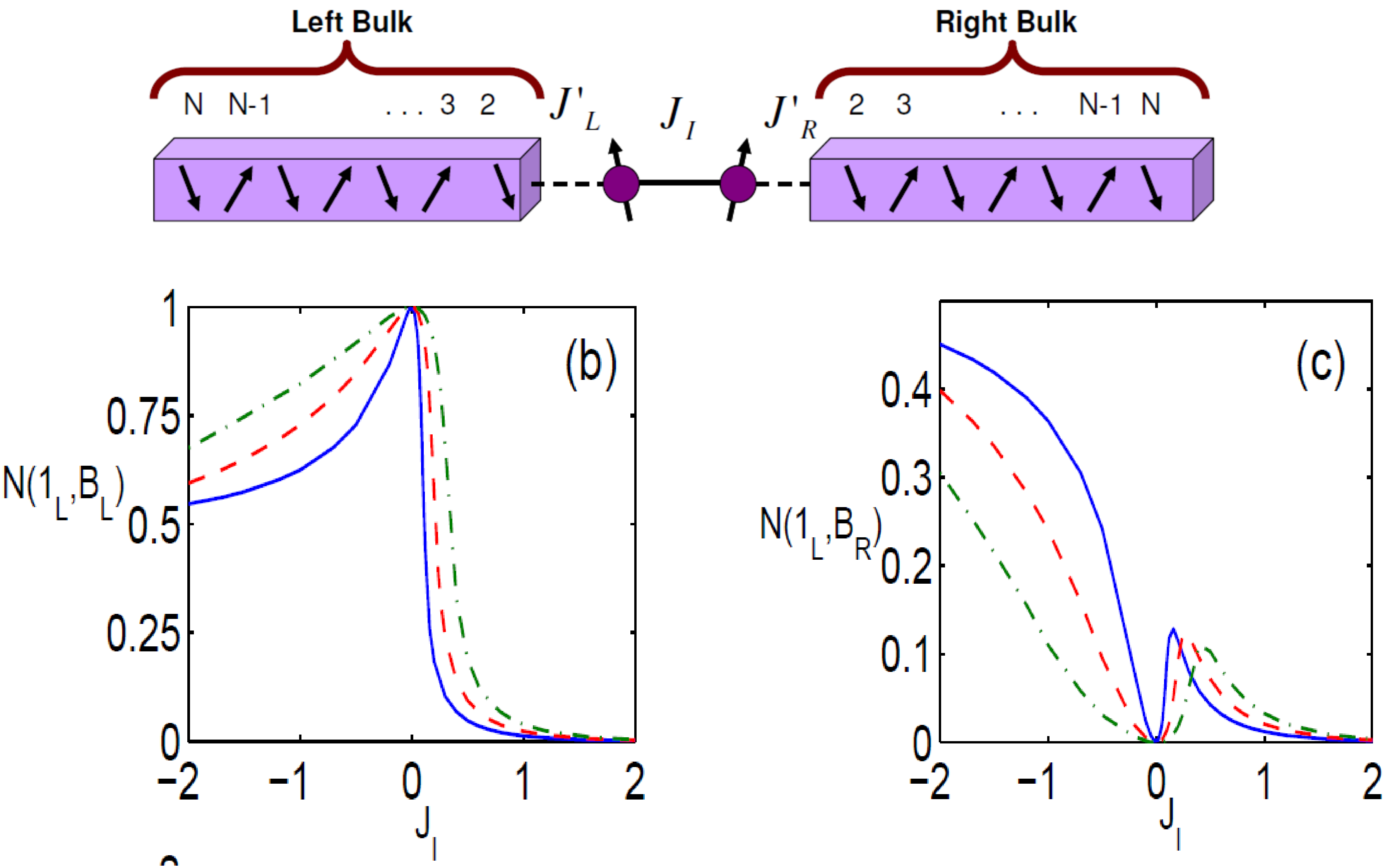
$$\begin{cases} J_I << 0: & S(\rho_{1_L,1_R}) = \log(3) \\ J_I = 0: & S(\rho_{1_L,1_R}) = 2 \\ J_I >> 0: & S(\rho_{1_L,1_R}) = 0 \end{cases} \Rightarrow \begin{cases} J_I << 0: & p = 0 \\ J_I = 0: & p = 1/4 \\ J_I >> 0: & p = 1 \end{cases}$$

Triplet

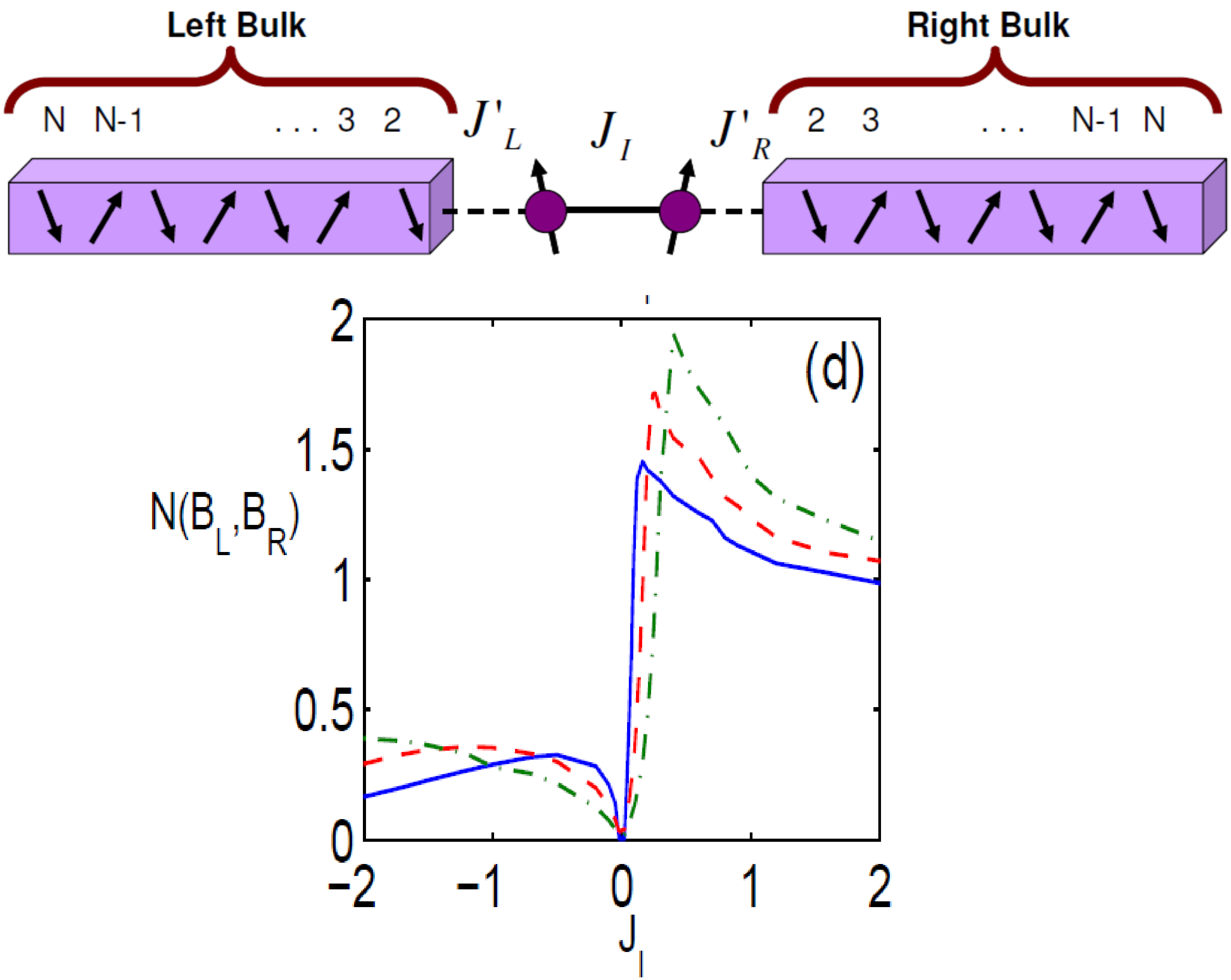
Identity

Singlet

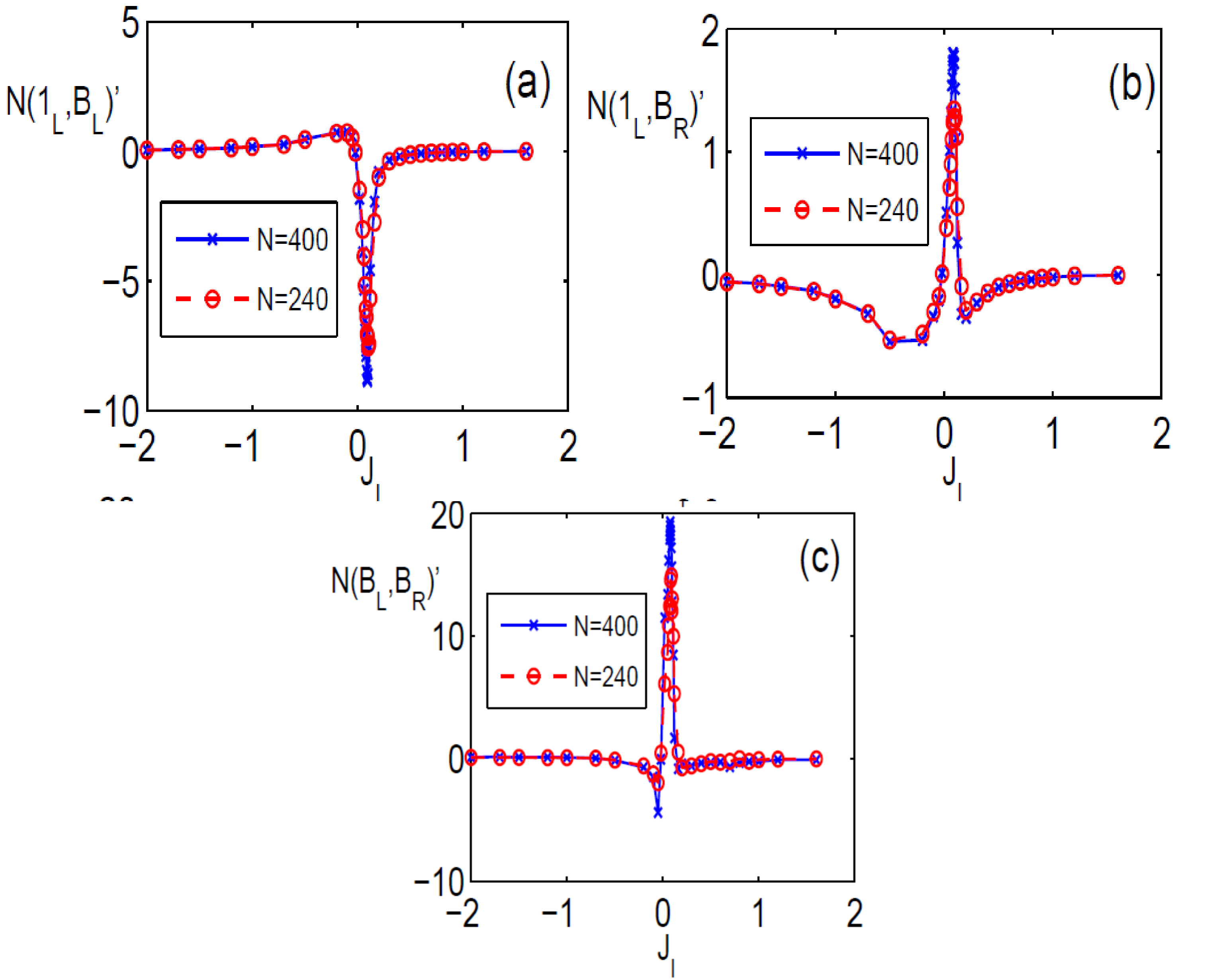
Impurity-Block Entanglement



Block-Block Entanglement

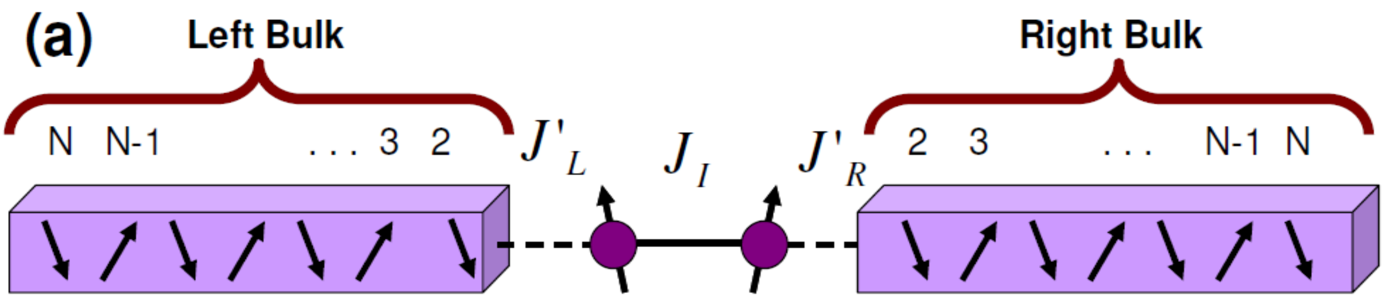


2^{nd} Order Phase Transition



Order Parameter for Two Impurity Kondo Model

Entanglement Spectrum

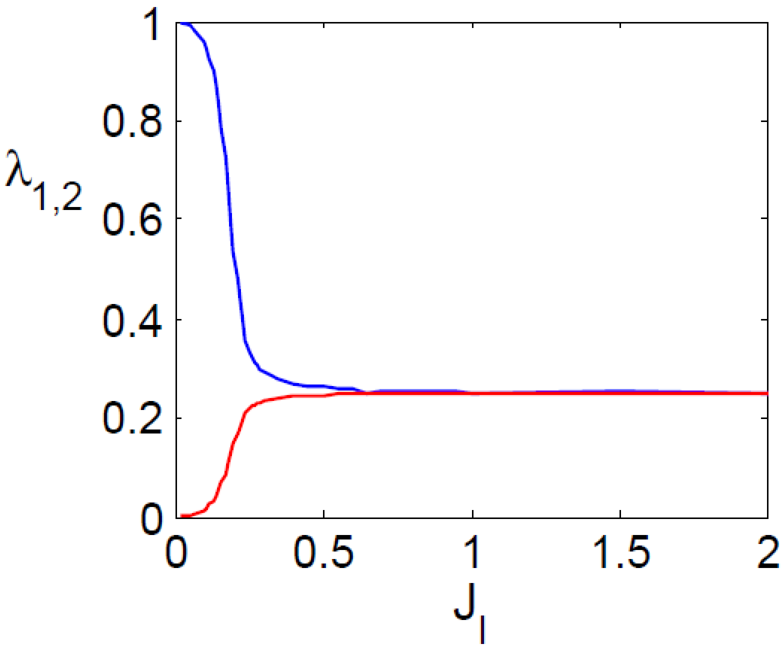


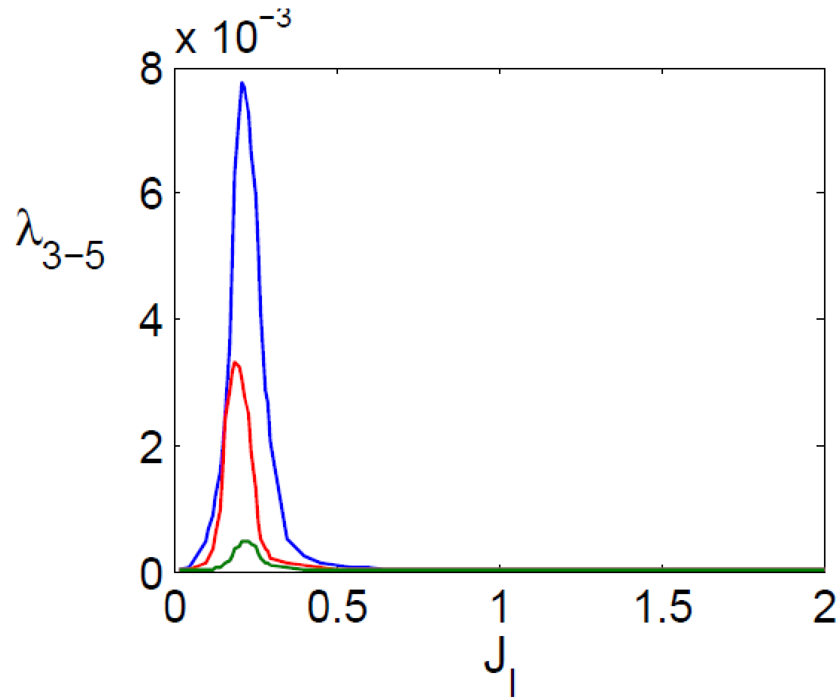
$$|GS\rangle = \sum_k \sqrt{\lambda_k} |A_k\rangle \otimes |B_k\rangle, \quad \lambda_k \geq 0$$

$$\rho_\alpha = \sum_k \lambda_k |\alpha_k\rangle\langle\alpha_k|, \quad \alpha = A, B.$$

Entanglement spectrum: $\lambda_1 \geq \lambda_2 \geq \ldots$

Entanglement Spectrum





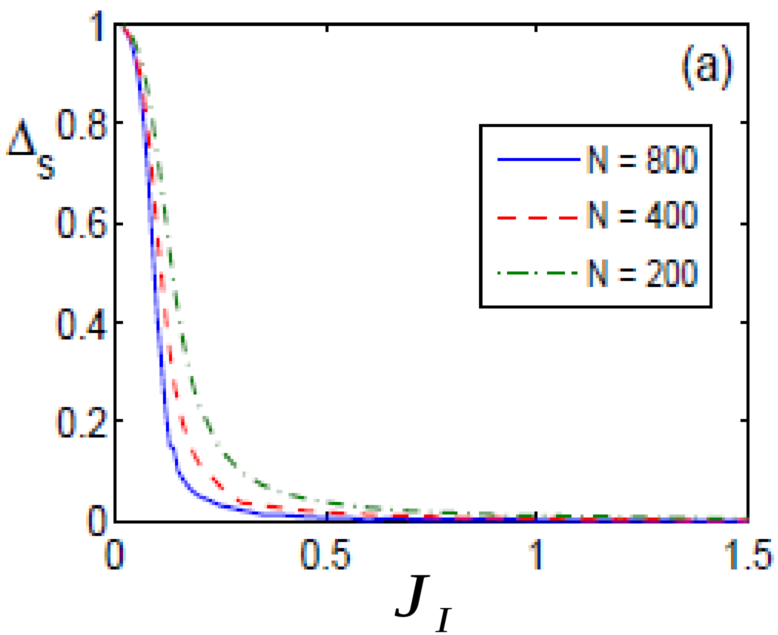
N_A=N_B=400

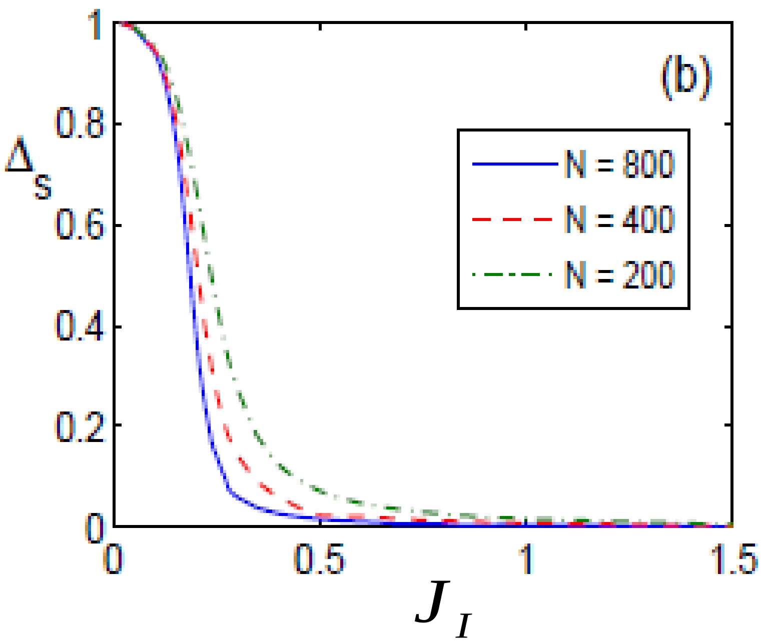
J'=0.5

Thermodynamic Behaviour

Schmidt gap: $\Delta_S = \lambda_1 - \lambda_2$

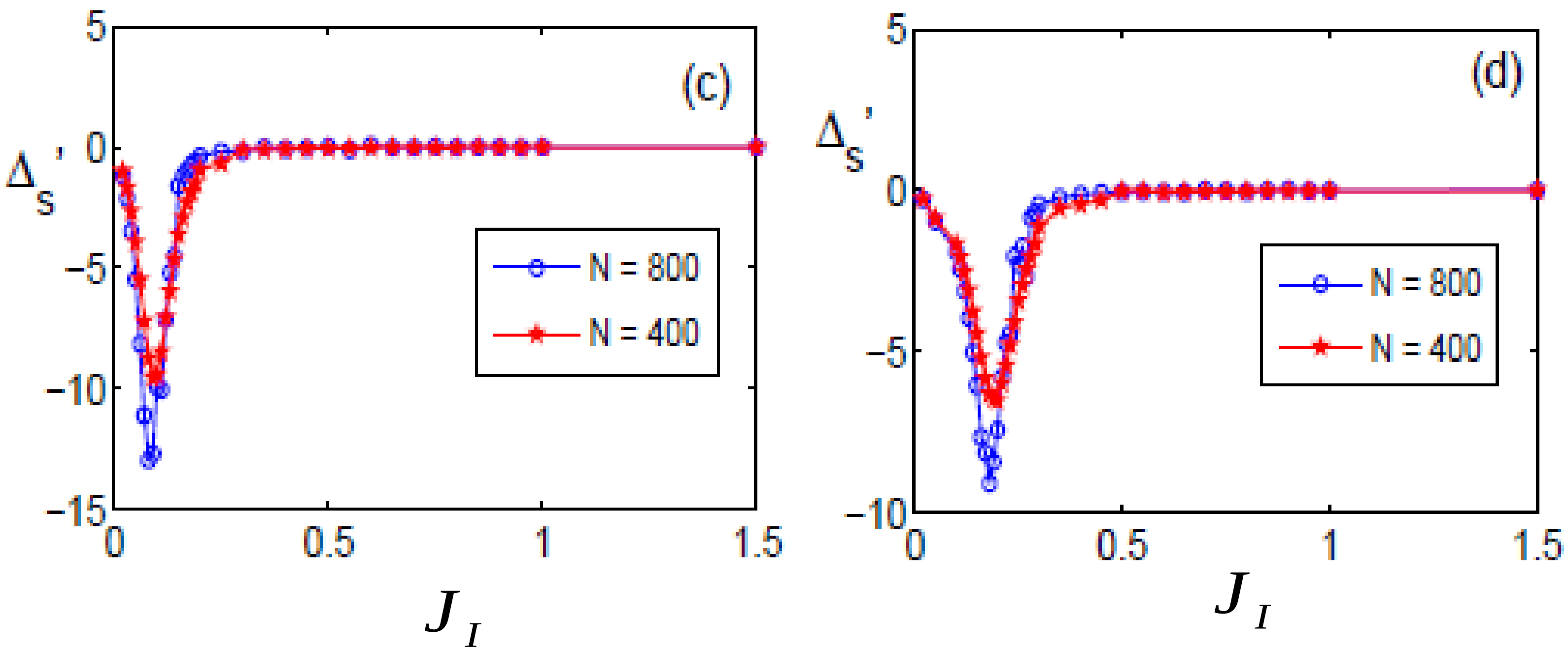
J'=0.4 | **J'=0.5**





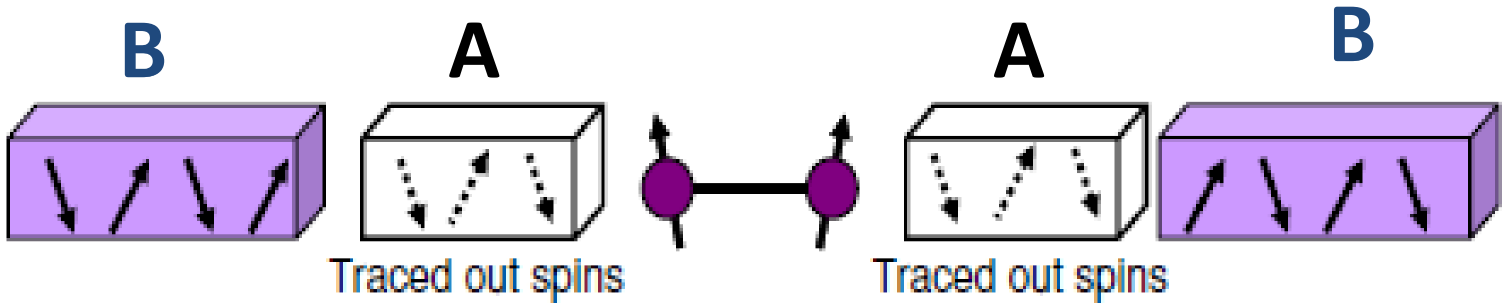
In the thermodynamic limit Schmidt gap vanishes in the RKKY regime

Diverging Derivative

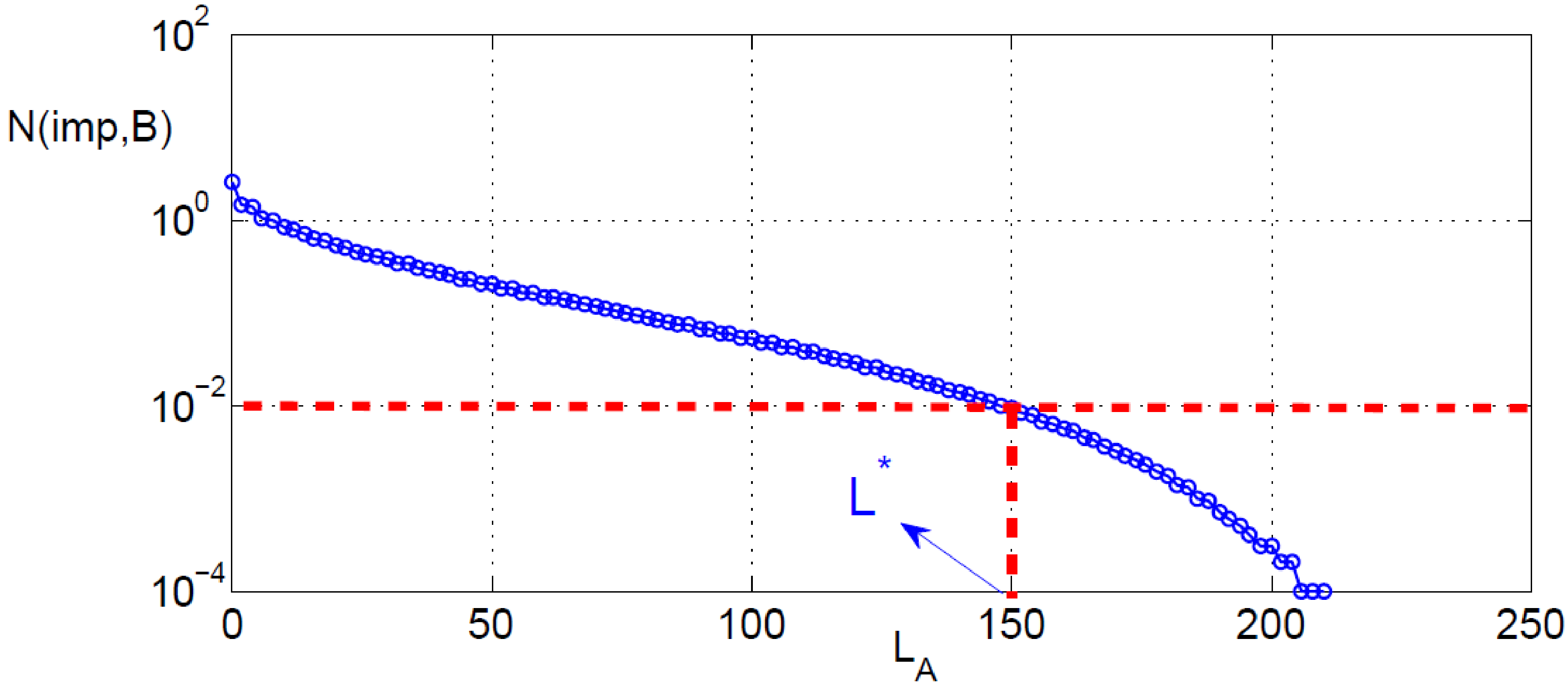


In the thermodynamic limit the first derivative of Schmidt gap diverges

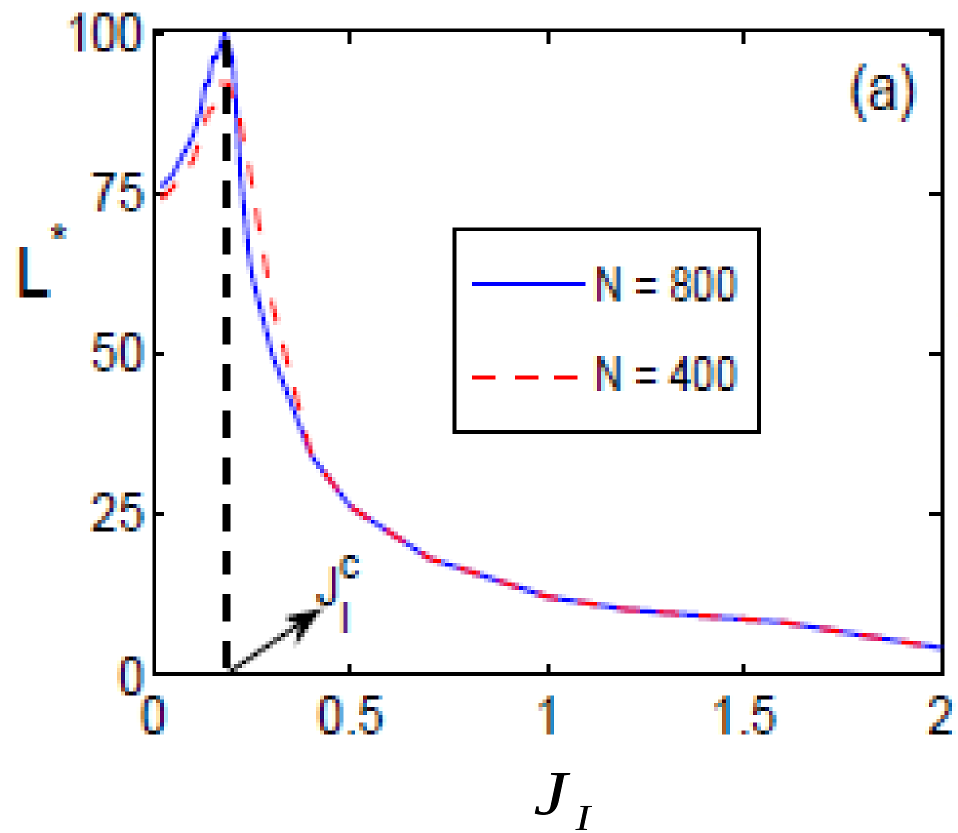
Kondo Length



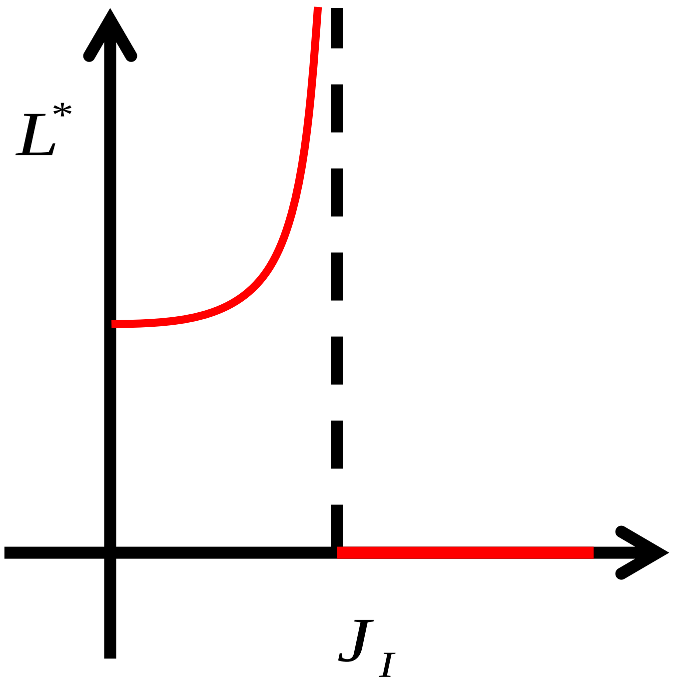
Negativity between the two impurities and the block B



Diverging Kondo Length



Finite size

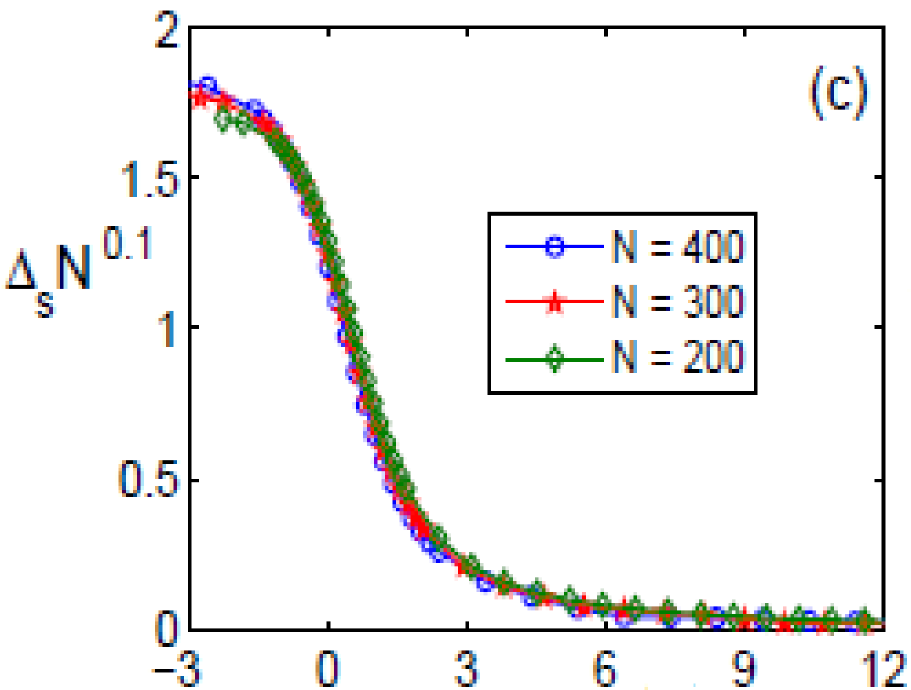


Thermodynamic limit

Finite Size Scaling

$$\Delta_S = N^{-\beta/\nu} f(|J_I - J_I^c| N^{1/\nu}) \Rightarrow$$

$$\Delta_S N^{\beta/\nu} = f(|J_I - J_I^c| N^{1/\nu})$$



$$(J_I - J_I^c) N^{0.5}$$

$$\Delta_S = |J_I - J_I^c|^{\beta}$$

$$\xi = |J_I - J_I^c|^{-\nu}$$

$$\beta = 0.2$$

$$\nu = 2$$

Schmidt Gap as an Observable

$$|GS\rangle = \sum_k \sqrt{\lambda_k}|A_k\rangle \otimes |B_k\rangle, \quad \lambda_k \geq 0$$

$$\mathcal{O} \equiv |A_1\rangle\langle A_1| - |A_2\rangle\langle A_2|$$

$$\langle GS|\mathcal{O}|GS\rangle \;=\; \lambda_1 - \lambda_2$$

Conventional Quantum Phase Transitions

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Entanglement Spectrum, Critical Exponents, and Order Parameters in Quantum Spin Chains

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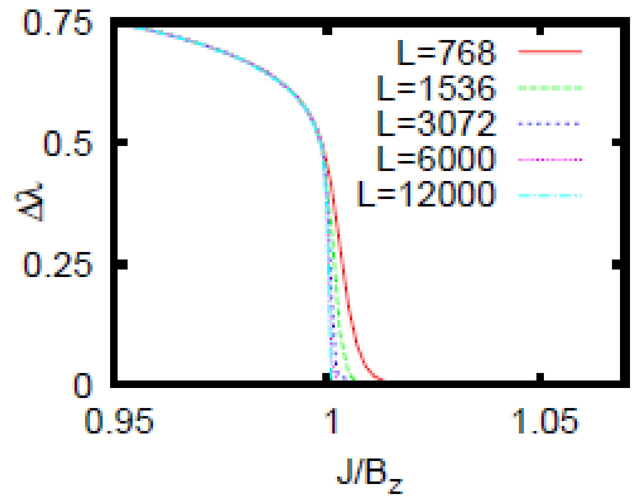
$$H_{Ising} = -J\sum_i \sigma_x^i \sigma_x^{i+1} - B_z \sum_i \sigma_z^i$$

$$H = J\sum_{i=1}^{L-1}\left[\cos(\theta) S_i \cdot S_{i+1} + \sin(\theta)(S_i \cdot S_{i+1})^2\right] + D\sum_{i=1}^{L} S_{zi}^2$$



Summary

- Impurity systems show exotic quantum phase transitions which do not fit in the Landau-Ginzburg paradigm.
- Entanglement captures the quantum phase transition in two impurity Kondo model though capturing the scaling is tricky!!
- Schmidt gap, as an observable, shows scaling with the right exponents at the critical point.

References:

- **An order parameter for impurity systems at quantum criticality**
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