Quantum Valley Hall Effect and Emergent Phenomena Induced by the Electronic EM Interactions in Graphene

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Acknowledgements

Brazilian Funding Agencies

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

FAPERJ

CAPES

CIÊNCIA SEM FRONTEIRAS

Dutch Funding Agency

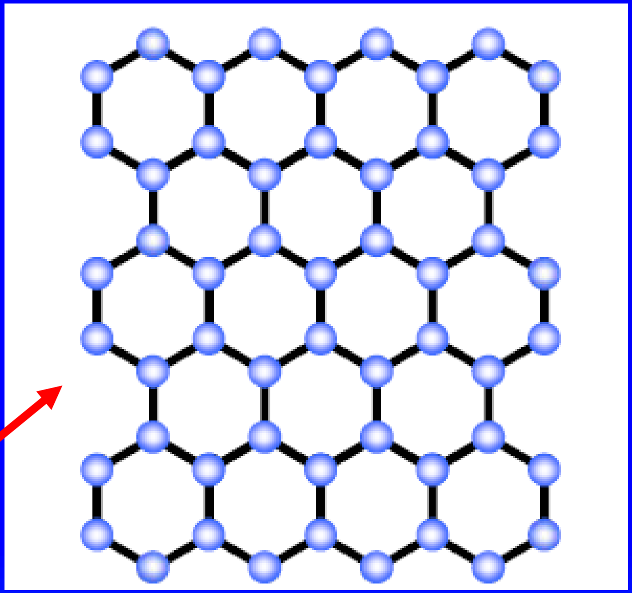
NWO

Summary

1) Introduction
2) Non-Interacting Electrons in Graphene: Dirac Fermions
3) Electromagnetic Interactions in Graphene: Pseudo Quantum Electrodynamics
4) The Minimal DC Conductivity
5) The Dynamical Gap Generation
6) The Quantum Valley Hall Effect
7) Outlook

1) Introduction

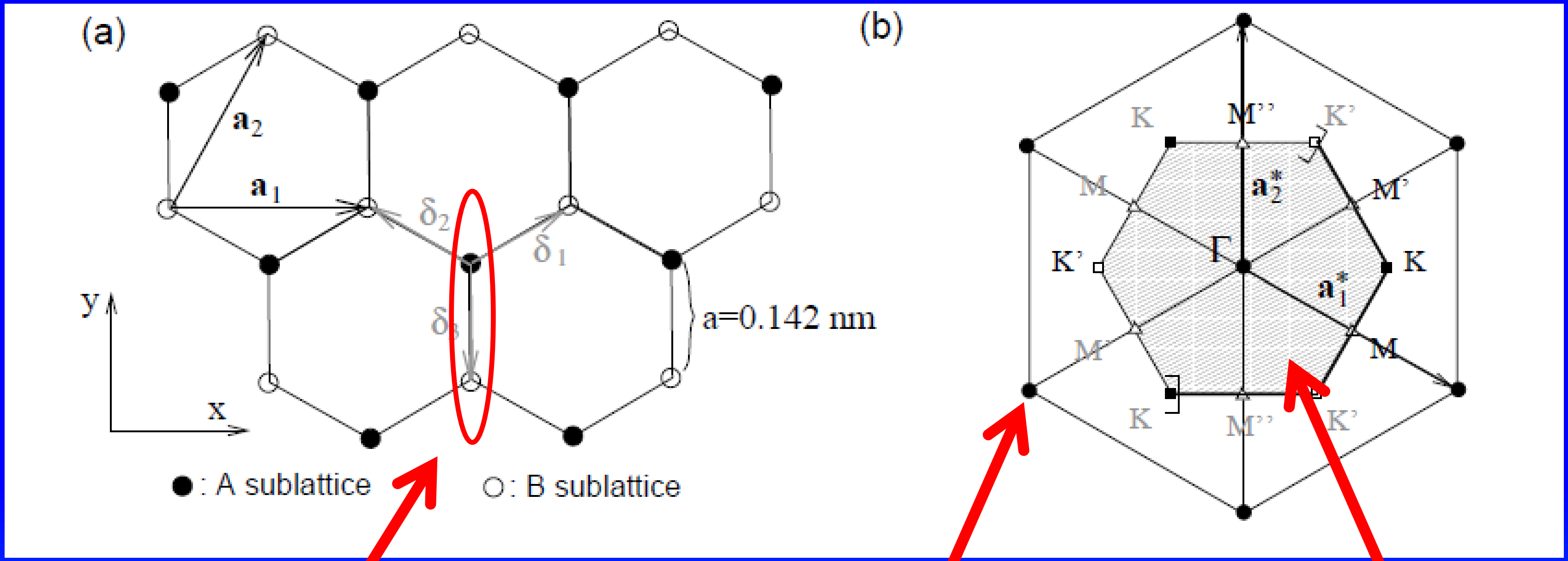
Graphene is a one-atom wide C sheet!!!



Carbon

sp2
hybridization

Crystal Lattice of Graphene



Base
+
Triangular
Lattice

Reciprocal Lattice

1st Brillouin Zone

2) Non-Interacting Electrons in Graphene: Dirac Fermions

The tight-binding approximation:
Non-interacting electrons on a lattice

$$H_{TB} = \epsilon_n \sum_{\mathbf{R}_i} |n;\mathbf{R}_i\rangle\langle n;\mathbf{R}_i| - t \sum_{\langle \mathbf{R}_i \mathbf{R}_j \rangle} |n;\mathbf{R}_i\rangle\langle n;\mathbf{R}_j|$$

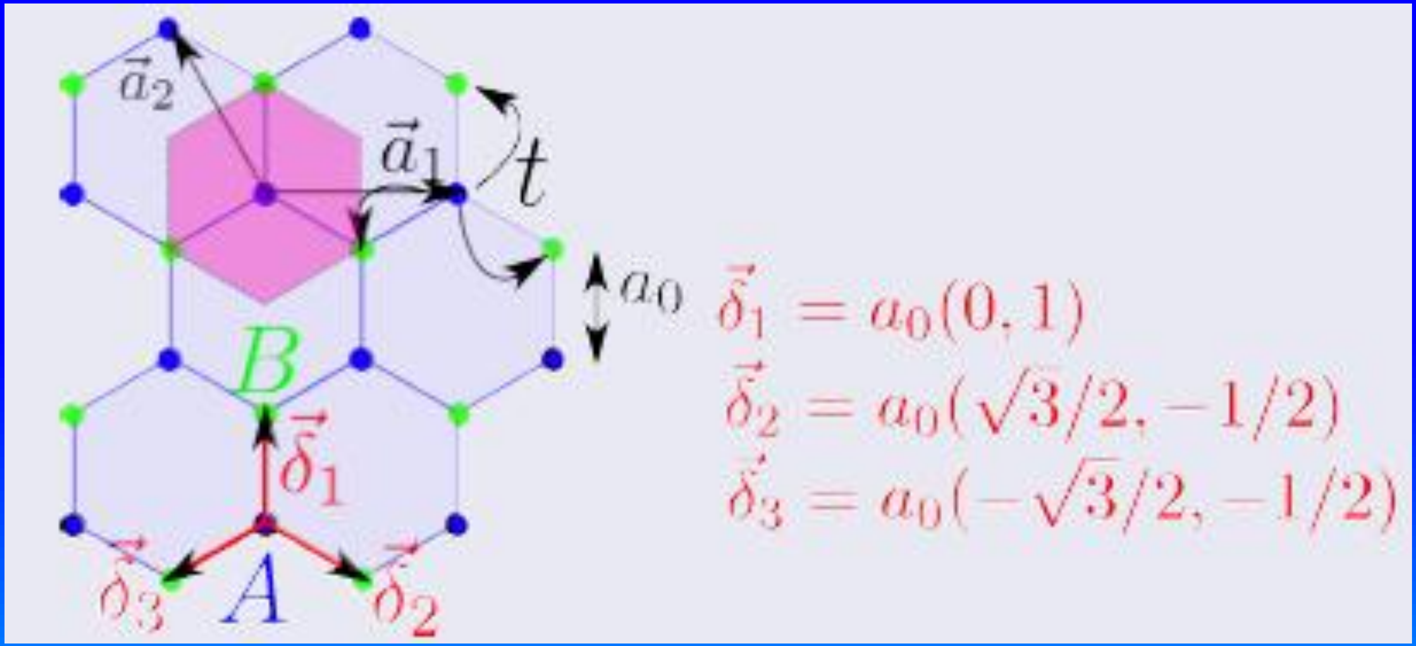
Lattice Translation Invariance

$$[H_{TB}, T(\mathbf{R})] = 0 \quad ; \quad T(\mathbf{R}) = \exp\left\{\frac{i}{\hbar}\mathbf{P}\cdot\mathbf{R}\right\}$$

H_{TB} and T(R) have common eigenvectors!!!

The Tight-Binding Hamiltonian in Graphene

$$H = t\sum_{\vec{R}}\left\{|A,\vec{R}\rangle\left[\langle B,\vec{R}+\vec{\delta}_1| + \langle B,\vec{R}+\vec{\delta}_2| + \langle B,\vec{R}+\vec{\delta}_3|\right] + \text{H.c.}\right\}$$

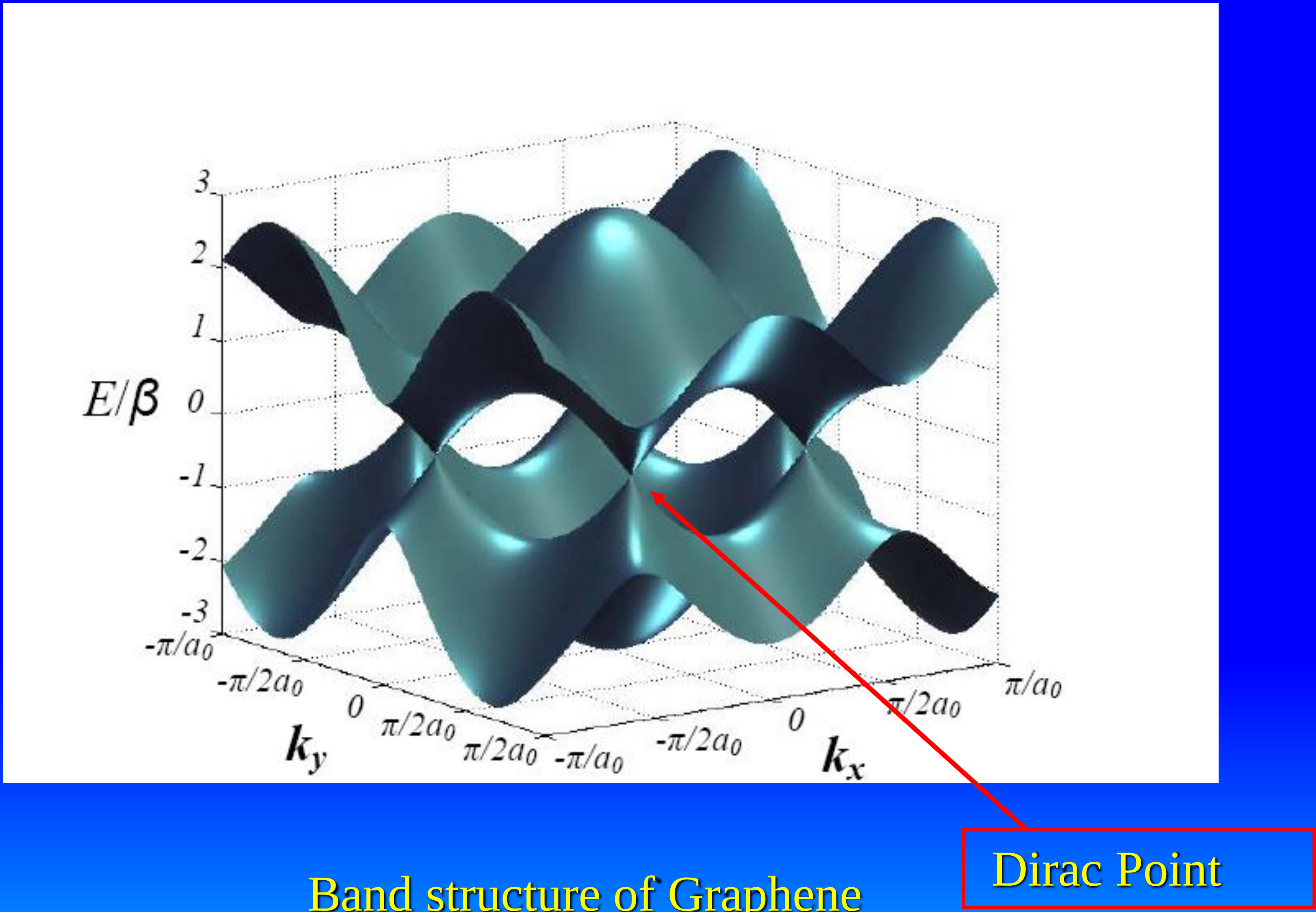


Energy eigenvalues

$$E = \pm t|\phi(\vec{k})|$$

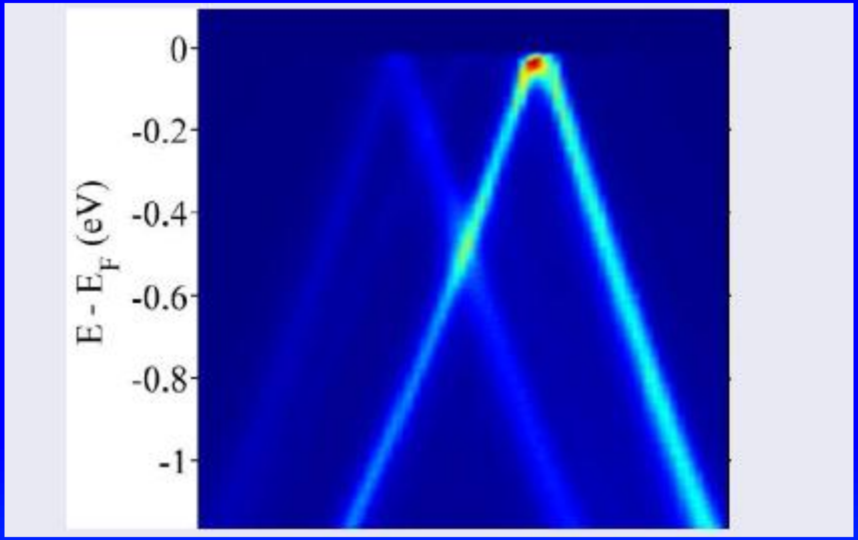
$$\phi(\vec{k}) = e^{i\vec{k}\cdot\vec{\delta}_1} + e^{i\vec{k}\cdot\vec{\delta}_2} + e^{i\vec{k}\cdot\vec{\delta}_3}$$

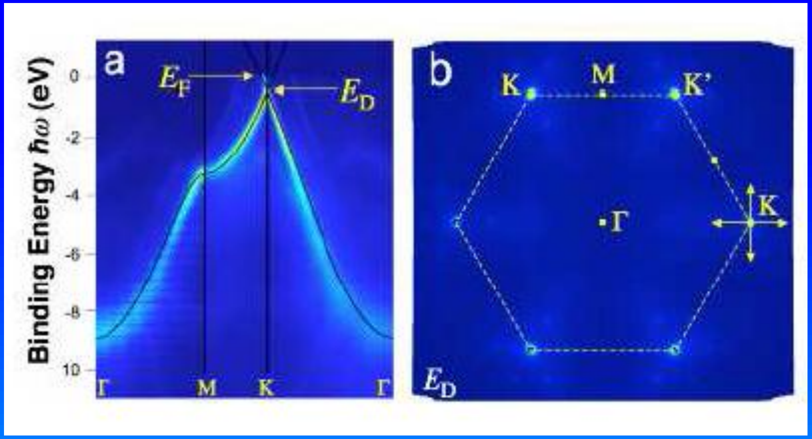
$$|\phi(\vec{k})| = \frac{1}{2}\sqrt{1 + 4\left(\cos\frac{\sqrt{3}\,k_x}{2}\right)^2 + 4\cos\frac{3k_y}{2}\cos\frac{\sqrt{3}\,k_z}{2}}$$



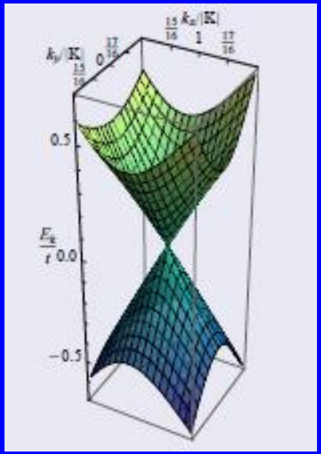
Band structure of Graphene

ARPES : Photoemission Measurements





Dirac Electrons



$$H^{2D} = c\boldsymbol{\sigma} \cdot \mathrm{p} + mc^2\sigma^z$$ Hamiltonian

$$E^{\pm} = \pm\sqrt{m^2c^4 + \hbar^2c^2\mathbf{k}^2},$$ Eigenvalues

$m = 0$

$$E^{\pm} = \pm\hbar\, \mathrm{c}\, |\, \vec{k}|$$

3) Electromagnetic Interactions in Graphene: Pseudo Quantum Electrodynamics

EM is the natural interaction among electrons!!

But……………!!

Electrons

2D

Electromagnetic Field

3D

3.1) Pseudo Quantum Electrodynamics

Start from 3+1D QED

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e\, j^{\mu}_{3+1} A_{\mu} + \mathcal{L}_m ,$$

Particles Constrained to a Plane

$$j^{\mu}_{3+1}(\xi) = \begin{cases} j^{\mu}(x,y,\tau)\delta(z) & \mu = 0,1,2 \\ 0 & \mu = 3 \end{cases}$$

$\xi = (x, y, z, \tau).$

Current Partition Functional

$$Z_{QED}[j^{\mu}_{3+1}] = Z_0^{-1} \int DA_{\mu} \exp\left\{ - \int d^4\xi \mathcal{L}_{QED} \right\},$$

$$Z_{QED}[j^{\mu}_{3+1}] = \exp\left\{ -\frac{e^2}{2} \int d^4\xi d^4\xi' \; j^{\mu}_{3+1}(\xi) \times G^{\mu\nu}_{QED}(\xi - \xi') j^{\nu}_{3+1}(\xi') \right\},$$

QED – EM field propagator

$$G^{\mu\nu}_{QED}(\xi - \xi') = \delta^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(\xi-\xi')}}{k^2}$$

$$\xi = (x, y, z, \tau).$$

Inserting

$$j^{\mu}_{3+1}(\xi) = \begin{cases} j^{\mu}(x,y,\tau)\delta(z) & \mu = 0,1,2 \\ 0 & \mu = 3 \end{cases}$$

in

$$Z_{QED}[j^{\mu}_{3+1}] = \exp\left\{-\frac{e^2}{2}\int d^4\xi d^4\xi' \; j^{\mu}_{3+1}(\xi) \times G^{\mu\nu}_{QED}(\xi-\xi')j^{\nu}_{3+1}(\xi')\right\},$$

$$\xi = (x,y,z,\tau).$$

… integrating on z, z'

$$G^{\mu\nu}_{QED}(\eta-\eta'; z=z'=0) = \frac{\delta^{\mu\nu}}{8\pi^2|\eta-\eta'|^2} + \text{gt}$$

$$\eta = (x,y,\tau)$$

Full 2+1D Description

$$\eta = (x, y, \tau)$$

$$\frac{1}{8\pi^2|\eta - \eta'|^2} = \int \frac{d^3k_{3D}}{(2\pi)^3} \frac{e^{ik_{3D}\cdot(\eta-\eta')}}{4\sqrt{k_{3D}^2}}$$

$$Z_{QED}[j^\mu] = Z_0^{-1} \int DA_\mu \exp\left\{-\int d^3\eta \mathcal{L}_{PQED}\right\}$$

This Functional is fully generated by PQED

$$\mathcal{L}_{PQED} = -\frac{1}{4}F_{\mu\nu}\left[\frac{4}{(-\Box)^{1/2}}\right]F^{\mu\nu} - e\,j^\mu A_\mu + \mathcal{L}_m$$

PQED

- Respects causality: support on the light-cone
- Respects unitarity: Optical Theorem
- Satisfies Huygens Principle
- Static Potential: V(r) = 1/r
- Has no dimensionful parameters
- Vacuum Polarization Tensor: $\Pi^{ij} \rightarrow \omega\, \delta^{ij}$
  (Kubo Formula in dc-limit!)

$$\mathcal{L}_{PQED} = -\frac{1}{4} F_{\mu\nu} \left[\frac{4}{(-\Box)^{1/2}} \right] F^{\mu\nu} - e\, j^{\mu} A_{\mu} + \mathcal{L}_m$$

A brief history of PQED

1990

PHYSICAL REVIEW B VOLUME 42, NUMBER 7 1 SEPTEMBER 1990

Kosterlitz-Thouless mechanism of two-dimensional superconductivity

A. Kovner and B. Rosenstein
Physics Department, University of British Columbia, Vancouver, British Columbia, Canada V6T 2A6
(Received 7 May 1990)

The possibility of a nonzero T_c superconducting phase transition in a purely two-dimensional system is discussed. We present a parity-invariant model that exhibits perfect diamagnetism and superconductivity due to the fact that the electromagnetic $U_E(1)$ group is realized in the Kosterlitz-Thouless (KT) mode in the vacuum. The superconducting phase transition is of the KT type, and the transition temperature is calculated through the parameters of the model. The connection with the anyon superconductivity mechanism is discussed.

$$S=\int d^2x\,dt\left[\mathcal{L}_0+eA_\mu J_\mu-F_{\mu\nu}\frac{1}{\sqrt{\partial^2}}F^{\mu\nu}\right]. \qquad (12)$$

tegration over the z coordinate is performed. One can easily see that this action leads to a $1/r$ potential in the static case. The derivation of Eq. (12) is given elsewhere.[2] Now performing the steps that led from Eq. (8)

1991

Volume 263, number 1 PHYSICS LETTERS B 4 July 1991

Complete bosonization of the Dirac fermion field in 2+1 dimensions ☆

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Received 23 March 1991

Observe that with our choice of a nonlocal $\tilde{J}^{\mu\nu}$, we ensure that the real constants a and b are both dimensionless, provided the dimension of W_μ is one. W_μ would have this dimension for example, for $\mathcal{L}[W_\mu] = -\frac{1}{4} W^2_{\mu\nu}(-\Box)^{-1/2}$ which corresponds to the nonlocal Maxwell equation above. As we will see this nonlocality will be fundamental in the process of bosonization. In the previous expressions, by

1992

Nuclear Physics B 386 (1992) 614–680
North-Holland

NUCLEAR
PHYSICS B

QED$_3$ and two-dimensional superconductivity without parity violation

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Received 28 October 1991
Accepted for publication 28 July 1992

$$\frac{1}{\sqrt{\nabla^2}}\nabla\times\boldsymbol{B}=\frac{1}{c}\boldsymbol{J},\qquad \frac{1}{\sqrt{\nabla^2}}\nabla\cdot\boldsymbol{E}=\rho, \tag{4.15}$$

where we have suppressed the label 2D). These field equations follow, in the static case, from the covariant (euclidean) action,

$$S_{\mathrm{EM}}=-\int \mathrm{d}^3x\frac{1}{4\sqrt{\partial^2}}F_{\mu\nu}F^{\mu\nu}, \tag{4.16}$$

where $\partial^2=\nabla^2+(1/c^2)\partial^2/\partial t^2$. The arguments leading to this result have a straightforward generalisation to the nonstatic case, although for the present purposes all we shall require are the two-dimensional equations of motion (4.15).

1993

Nuclear Physics B408 (1993) [FS] 551–564
North-Holland

NUCLEAR
PHYSICS B [FS]

Quantum electrodynamics of particles on a plane and the Chern–Simons theory

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Received 6 January 1993
Accepted for publication 7 July 1993

We study the electrodynamics of generic charged particles (bosons, fermions, relativistic or not) constrained to move on an infinite plane. An effective gauge theory in (2 + 1)-dimensional space-time which describes the real electromagnetic interaction of these particles is obtained. explored. It is shown that the QED lagrangian per se produces the Chern–Simons constraint relating the current to the effective gauge field in 2 + 1 dimensions. It is also shown that the geometry of the system unavoidably induces a contribution from the topological θ-term that generates an explicit Chern–Simons term for the effective (2 + 1)-dimensional gauge field as well as a minimal coupling of the matter to it. The possible relation of the effective three-dimensional theory with the bosonization of the Dirac fermion field in 2 + 1 dimensions is briefly discussed as well as the potential applications in condensed matter systems.

3.2) A Model for Graphene

Graphene: PQED of Dirac electrons

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} \left[\frac{4}{\sqrt{-\Box}} \right] F^{\mu\nu} + \bar{\psi}_a \left(i\partial\!\!\!/ + e\,\gamma^\mu A_\mu \right) \psi_a .$$

4 flavors: a = ↑, ↓, K, K' (2 spins; 2 valleys)

2 components: A and B sublattices

$$\psi_a = \begin{pmatrix} \psi_{a,A} \\ \psi_{a,B} \end{pmatrix}$$

4) The Minimal DC Conductivity

Kubo's Formula

$$\sigma^{ik} = \lim_{\omega\to 0,\mathbf{p}\to 0} \frac{i\langle j^i j^k\rangle}{\omega}.$$

Vacuum Polarization

$$\langle j_\mu j_\nu\rangle_{1PI} = \Pi_{\mu\nu}.$$

Schwinger-Dyson Equation

$$G^{-1}_{\mu\nu} - G^{-1}_{0,\mu\nu} = -e^2\Pi_{\mu\nu},$$

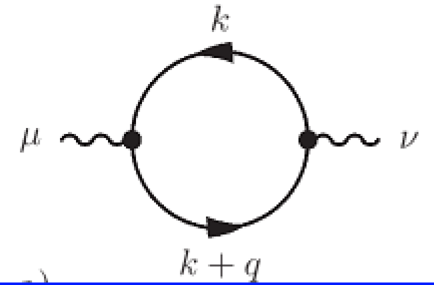
One-Loop

$$\Pi^{(1)}_{\mu\nu}(p) = A(p)\, P_{\mu\nu} + B\, \epsilon_{\mu\nu\alpha} p^{\alpha} \,,$$

$$A(p) = \sqrt{p^2}/16,$$

$$B = (1/2\pi)\,(n + 1/2),$$

$$P_{\mu\nu} = \delta_{\mu\nu} - p_\mu\, p_\nu / p^2 .$$



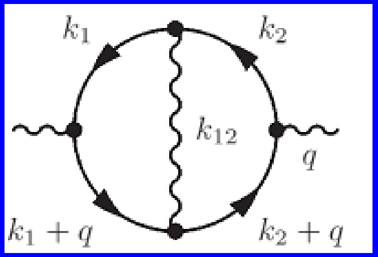
$$p^2 = \omega^2 + |\vec{p}|^2$$

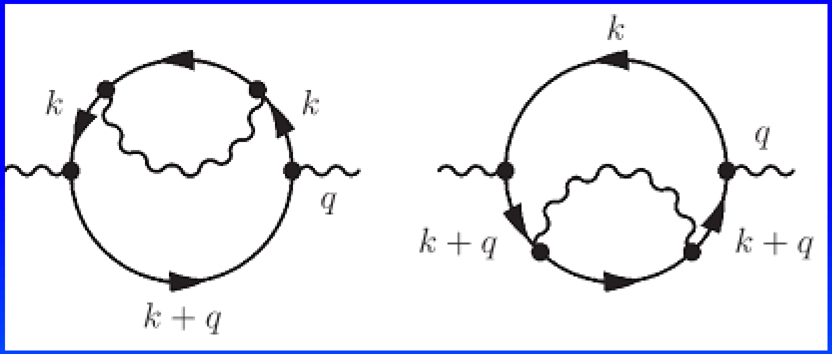
Coste, Luscher, NPB 323, 631 (1989)

Coleman-Hill Theorem: B is exact!!!!!

Two Loops

$$\Pi^{(2)}_{\mu\nu} = A(p)\, C_\alpha \alpha_g P_{\mu\nu}\,,$$





$$C_\alpha = \left(\frac{92 - 9\pi^2}{18\pi}\right) = 0.056$$

$$\alpha_g \sim 300/137 = 2.189.$$

$$P_{\mu\nu} = \delta_{\mu\nu} - p_\mu\, p_\nu / p^2\,.$$

S. Teber, PRD 86, 025005 (2012)

The Minimal DC Conductivity

Contributions from Each Valley

$$\sigma^{ik} = \lim_{\omega\to 0, \mathbf{p}\to 0} \left\{ \frac{i\langle j^i j^k \rangle}{\omega} + \frac{i\langle j^i j^k \rangle^T}{\omega} \right\},$$

$$\langle j^i j^j \rangle \xrightarrow[|\vec{p}|\to 0]{} \omega\left[a\delta^{ij} + b\epsilon^{ij}\right]$$

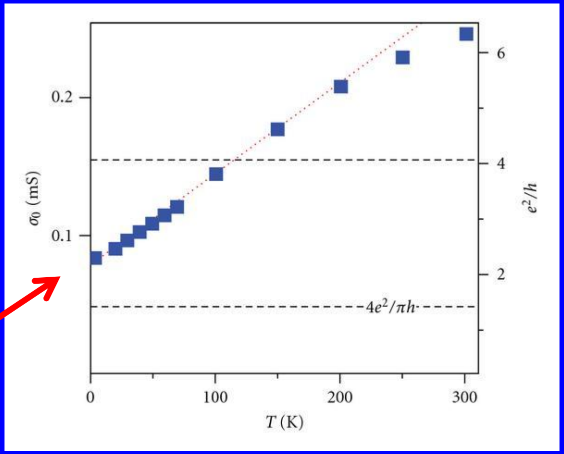
T-symmetry $a^T = a$; $b^T = -b$

$$\sigma^{xx} = \left(\frac{\pi}{2}\frac{e^2}{h}\right)\left[1 + \left(\frac{92 - 9\pi^2}{18\pi}\right)\alpha_g + \mathcal{O}(e^4)\right]$$

$$\sigma^{xy} = 0.$$

The Minimal T= 0 DC Conductivity: Theory x Experiment

A) Experiment



x = 2.16

$$\sigma_0 = x\left(\frac{e^2}{h}\right)$$

B) Existing Theoretical Values

$$x = \left(\frac{4}{\pi}\right) = 1.27$$

$$x = \left(\frac{\pi}{2}\right) = 1.57$$

X. Du, I. Skachko, A. Barker,
E. Y Andrei, Nature Nanotech. 3, 491 (2008)

C) Our Result

$$\sigma_0 = x\left(\frac{e^2}{h}\right)$$

$$x = \left(\frac{\pi}{2}\right)\left[1 + 0.056\alpha_g\right] \cong 1.76$$

$x_{exp} = 2.16$

B) Existing Theoretical Values

$$\begin{cases} x = \left(\frac{4}{\pi}\right) = 1.27 \\ x = \left(\frac{\pi}{2}\right) = 1.57 \end{cases}$$

5) The Dynamical Generation of an Electron Gap

Schwinger-
Dyson Equation

$$S_F^{-1}(p) = S_{0F}^{-1}(p) - \Sigma(p),$$

Expanding the electron self-energy…

$$\Sigma(p) = \Sigma(p=\epsilon) + (\gamma^\mu p_\mu - \epsilon)\frac{\partial \Sigma(p)}{\partial p}|_{p=\epsilon} + \ldots$$

… and imposing

$$\Sigma(p=\epsilon) = \epsilon,$$

SD – Equation

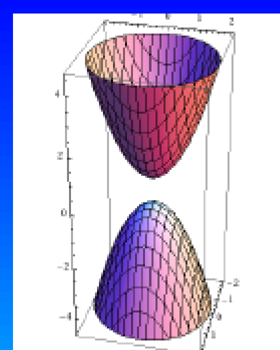
+

$$\Sigma(p=\epsilon)=\epsilon,$$

$$\begin{aligned} S_F(p) &= \frac{1}{\gamma^\mu p_\mu - \Sigma(p)} \\ &= \frac{1}{(\gamma^\mu p_\mu - \epsilon)(1-\frac{\partial \Sigma(p)}{\partial p}|_{p=\epsilon}+\ldots)} \\ &= \frac{\gamma^\mu p_\mu + \epsilon}{(p^2-\epsilon^2)(1-\frac{\partial \Sigma(p)}{\partial p}|_{p=\epsilon}+\ldots)}. \end{aligned}$$

The pole at $\omega = \pm|p|$ is shifted to $\omega = \pm[p^2+\epsilon^2]^{1/2}$

Gapped energy spectrum!!!



Eigenenergies:

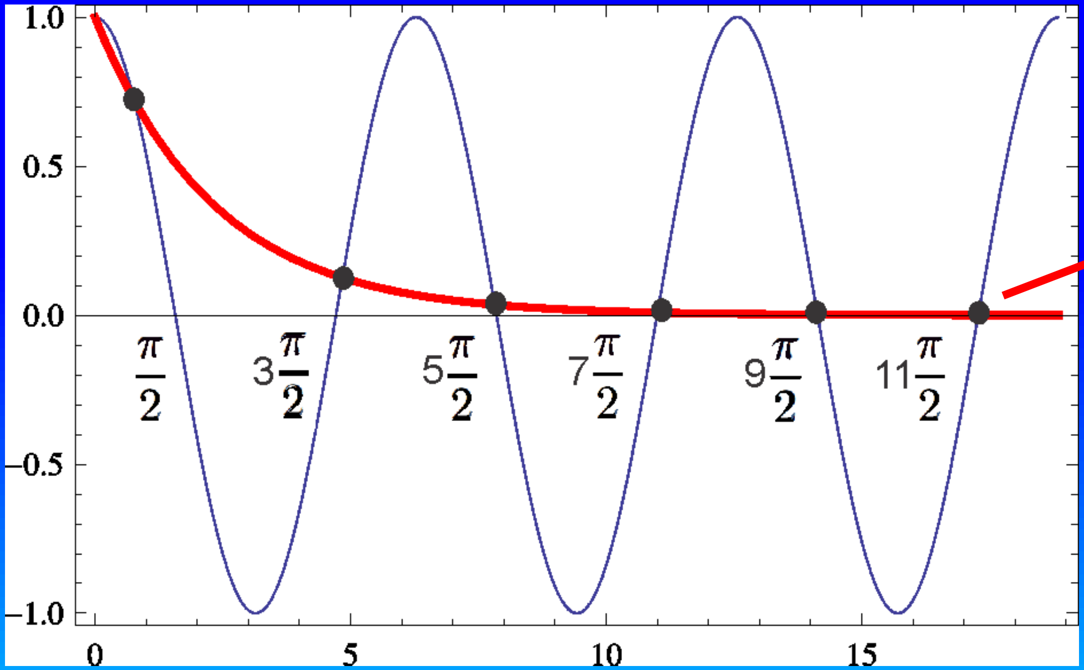
$$\Sigma(p=\epsilon)=\epsilon,$$

$$|\epsilon| = \Lambda\ \exp\left(-\frac{z}{\gamma}\right)$$

Z's:

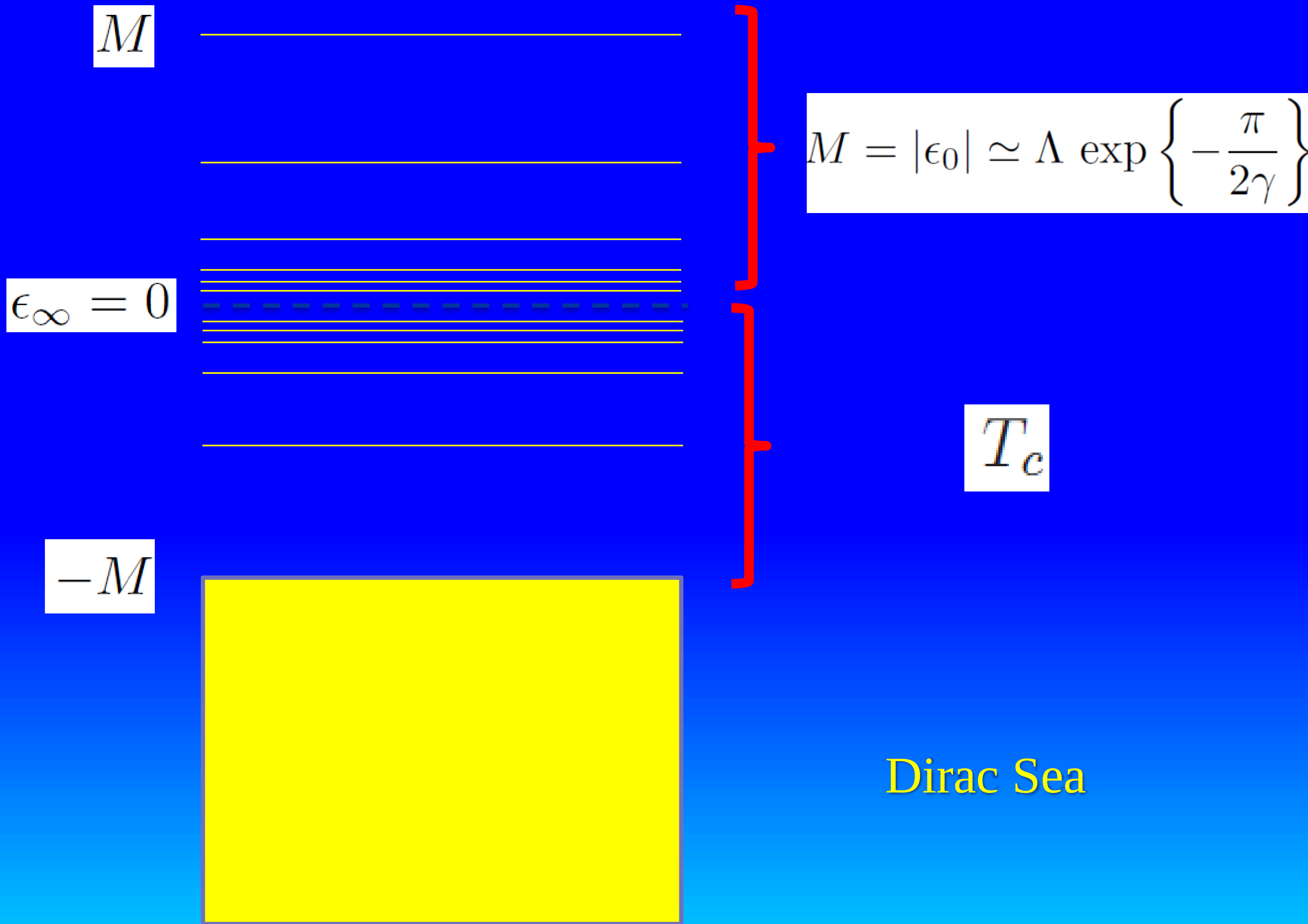
$$\exp\left(-\frac{3z}{2\gamma}\right) = \cos z,$$

$$\epsilon_n = \pm\Lambda\ \exp\left\{-\frac{Z_n}{\gamma}\right\}$$



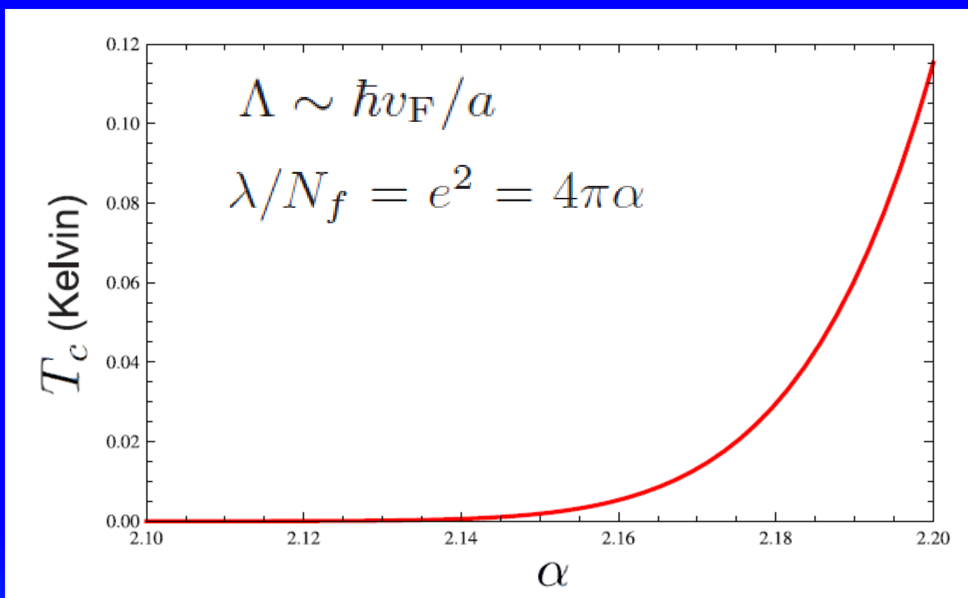
$$\gamma = \frac{1}{2}\sqrt{\frac{N_c}{N_f}-1}$$

Midgap States



The Critical Temperature

Critical Temperature



$$N = N_0 \exp\left[-\frac{\Delta_\infty}{k_B T}\right]$$

$$T_c = \frac{\Delta_\infty}{k_B \ln(N_0/N)} \simeq \frac{\Delta_\infty}{k_B x}$$

$$T_c \simeq \frac{\Lambda}{k_B x} \exp\left[-\frac{\pi}{2\gamma}\right].$$

$Tc \cong 0.05\ K$

6) The Quantum Valley Hall Effect

Net Valley Current

$$\langle J_V^i \rangle = \langle 0 | j_K^i | 0 \rangle - \langle 0 | j_{K'}^i | 0 \rangle$$

$$\langle j^i j^j \rangle \xrightarrow[|\vec{p}| \to 0]{} \omega [a \delta^{ij} + b \epsilon^{ij}]$$

T-symmetry $\quad a^T = a \;\; ; \quad b^T = - b$

Valley Hall Conductivity (Exact!)

$$\sigma_V^{xy} = 4 \left(n + \frac{1}{2} \right) \frac{e^2}{h},$$

$$\sigma_V^{xx} = 0.$$

7) Outlook

Electronic Interactions play a crucial role in the physics of Graphene

The natural interaction among electrons is the full EM interaction. Using only static Coulomb leads to problems with the Kubo formula

This is correctly and completely described in 2D by Pseudo Quantum Electrodynamics

New effects (exclusive of PQED!!!):
1)Quantum Valley Hall Effect: emergent!
2)Dynamical generation of an electron gap
3)Corrections to DC minimal conductivity: best result ever!!!