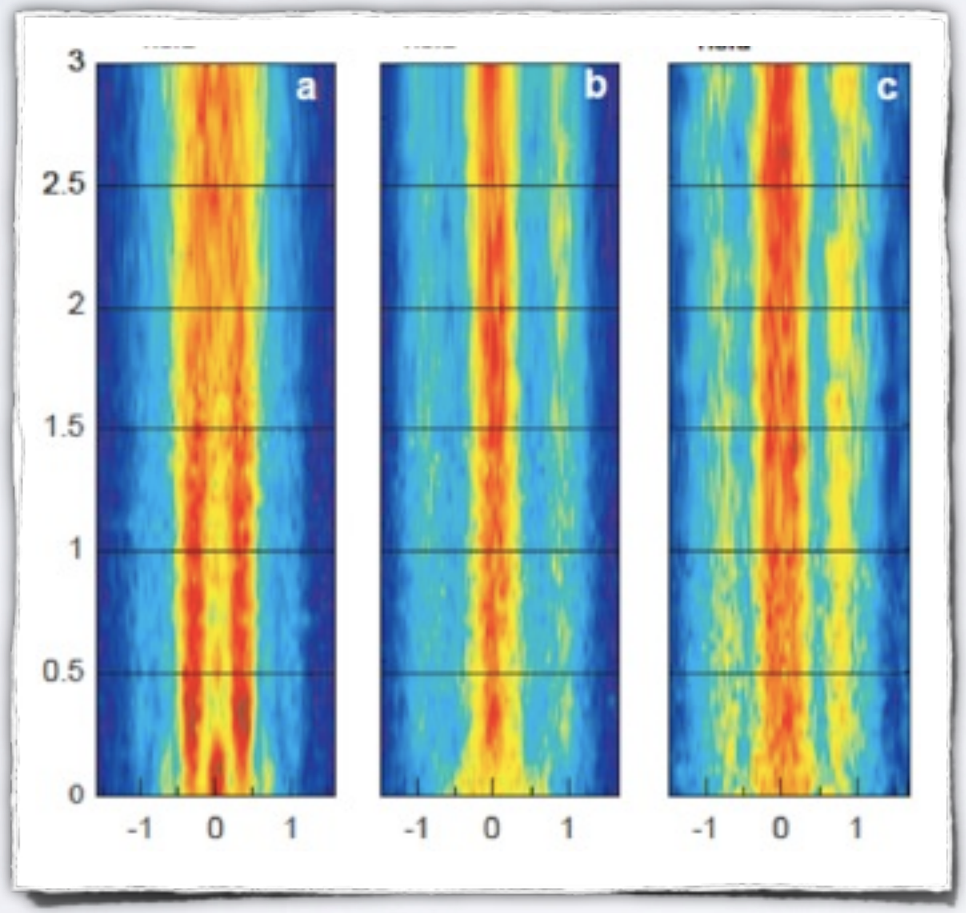
Dynamical analog quantum simulators

Perspectives on engineering quantum states and devices



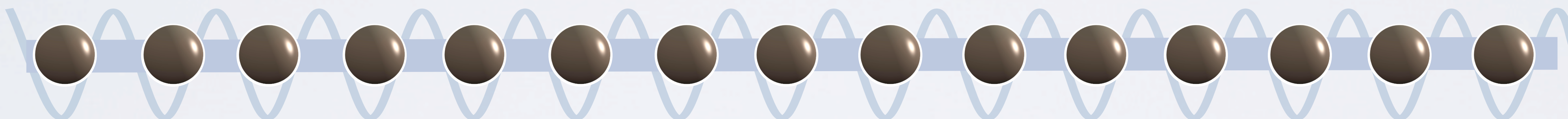
Jens Eisert

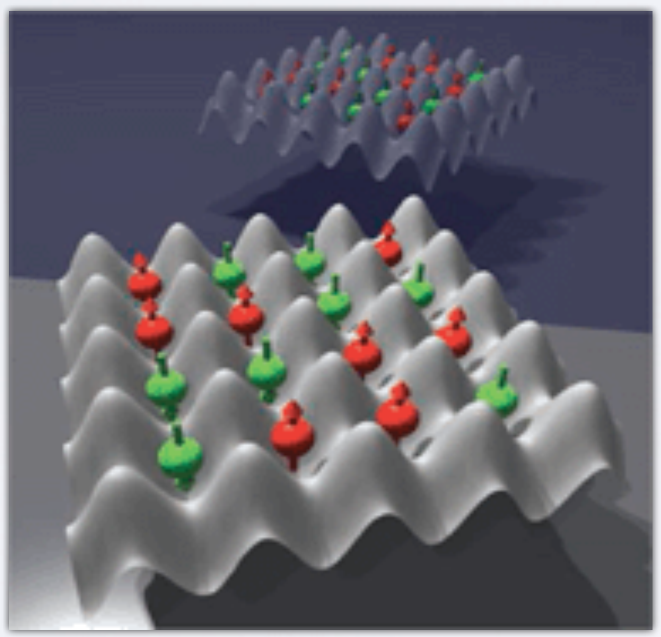
Freie Universität Berlin

Conference on quantum engineering of states and devices, Nordita, Stockholm, 2014

Mentions joint work with M. Friesdorf, C. Gogolin, M. Kliesch, A. Riera, M. del Rey, S. Braun, S. Trotzky, I. McCulloch, A. Flesch, Y.-U. Chen, U. Schollwoeck, U. Schneider, and I. Bloch

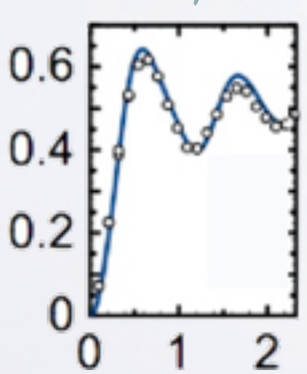
Analog(ue) quantum simulators as engineered devices





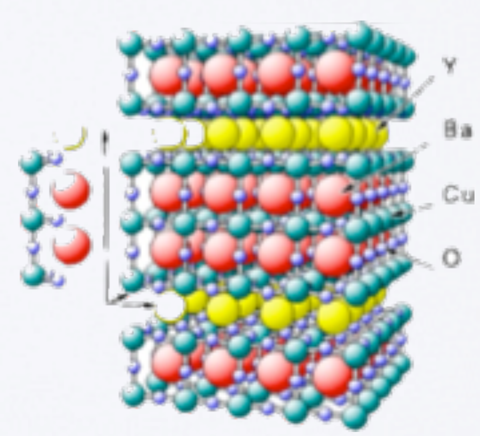
Dynamical property of time-evolving quantum system

Efficient



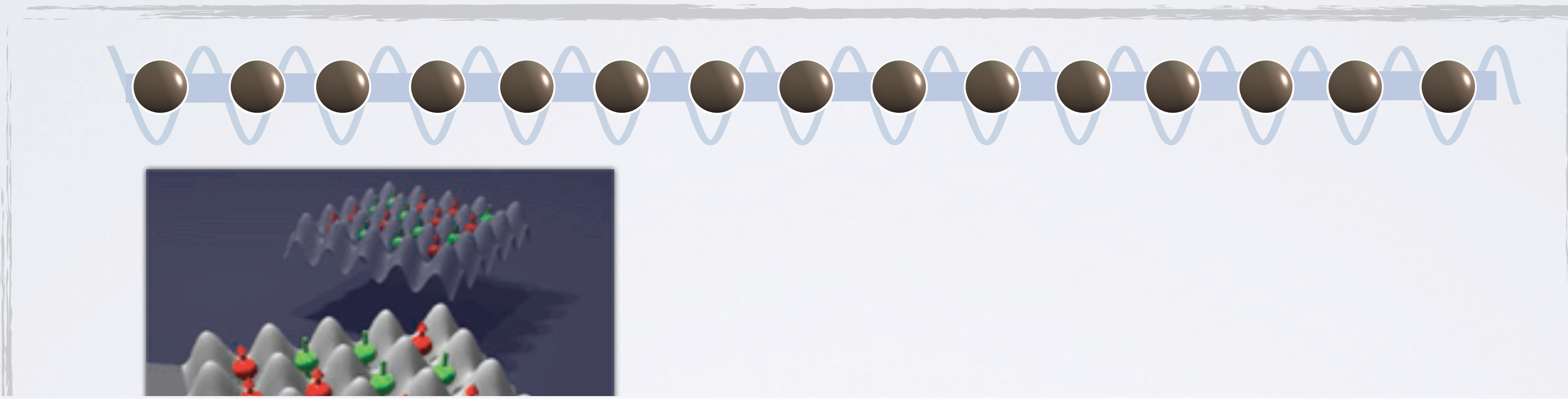
$\langle\psi|A|\psi\rangle$

Expectation value



Dynamical features

Computational power of analogue simulators



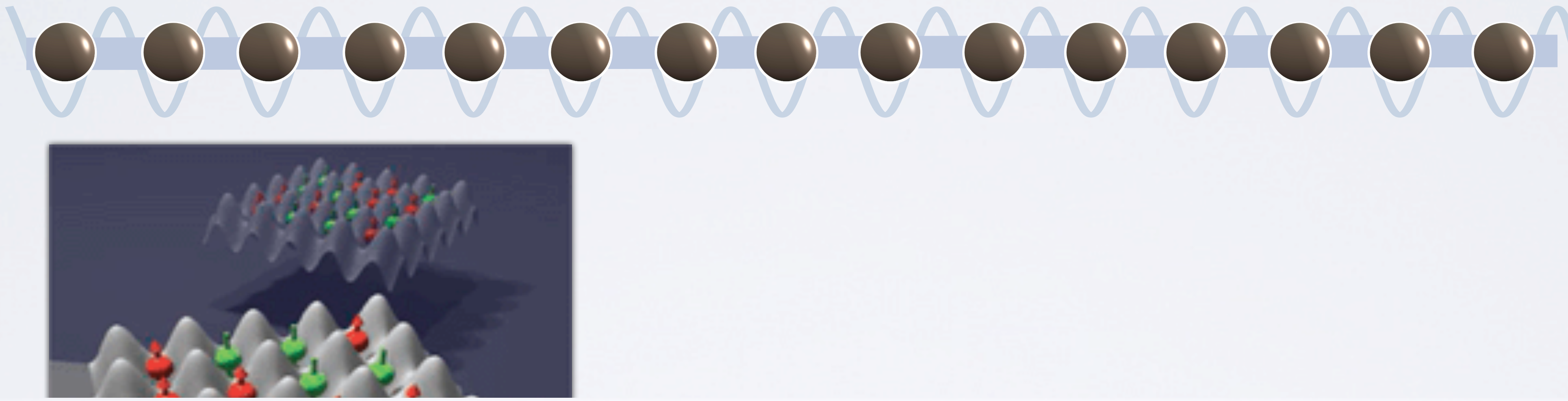
- **"Hardness problem"**

  Quantum simulator should solve problem that is "classically inaccessible"

  (or good reasons to believe so)

- **Error correction** out of scope - simulate "robust" features?
- Devices without error correction presumably not **BQP complete**

Computational power of analogue simulators



- **"Certification problem"**

  How can one find out whether one has done the right thing?

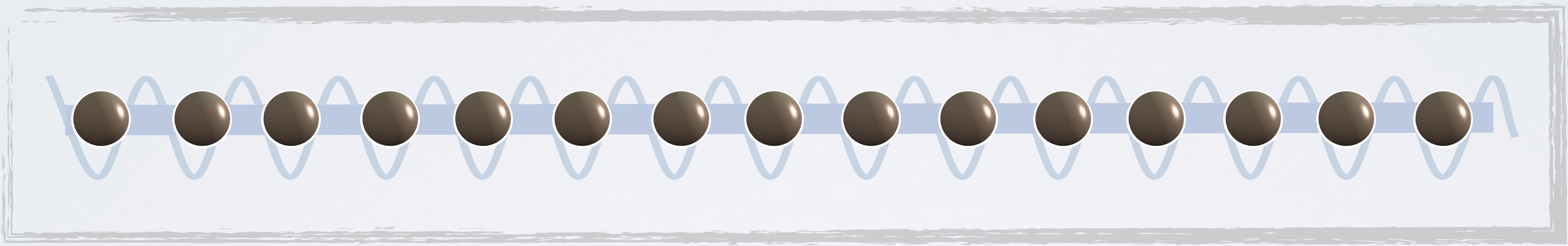
  (or good reasons to believe so)

- **Error correction** out of scope - simulate "robust" features?
- Devices without error correction presumably not **BQP complete**

1. Probing questions of non-equilibrium

Kliesch, Kastoryano, Gogolin, Riera, Eisert, arXiv:1309:0816
Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)
Cramer, Eisert, *New J Phys* **12**, 055020 (2010)
Cramer, Dawson, Eisert, Osborne, *Phys Rev Lett* **100**, 030602 (2008)
Riera, Gogolin, Eisert, *Phys Rev Lett* **108**, 080402 (2012)

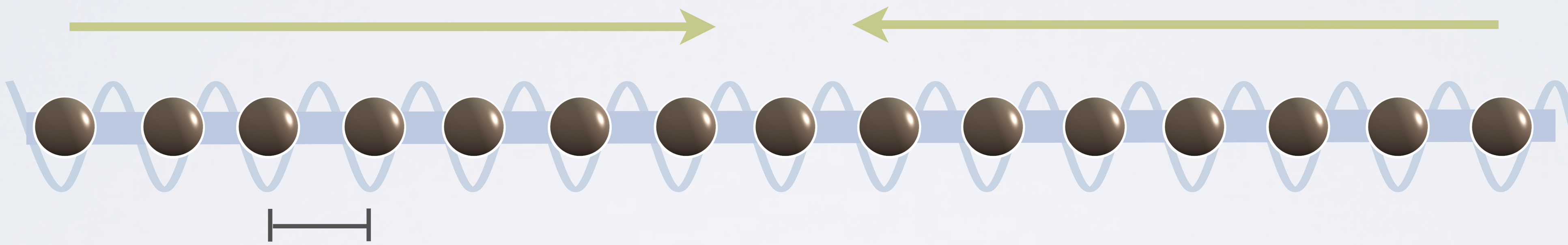
Old physics questions and new answers



- How do closed quantum many-body system come to **equilibrium?**
- How does **temperature** dynamically emerge?
- How can systems go from one **phase** to another?

- New mathematical methods/numerics/quantum simulations

Cold atoms in optical lattices

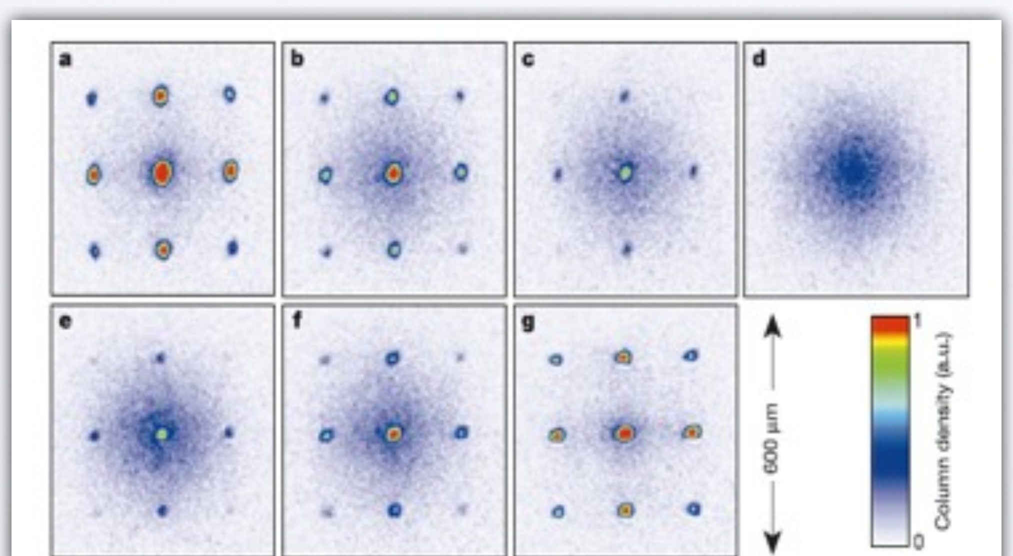


$a = k/2$ $\lambda = 2\pi/k$

- **Condensed-matter systems** under controlled conditions in the **laboratory**

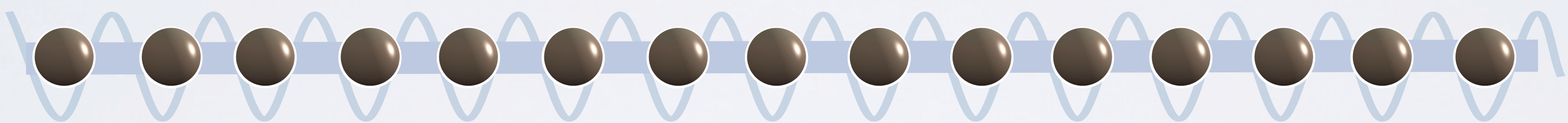
$$H = -J \sum_{\langle j,k \rangle} b_j^\dagger b_k + \frac{U}{2} \sum_k b_k^\dagger b_k (b_k^\dagger b_k - 1) - \mu \sum_k b_k^\dagger b_k$$

- *Lattice depth, superlattices, Feshbach resonances:* "Build" Hamiltonian in simulation



Greiner, Mandel, Haensch, Bloch, *Nature* **419** (2002)
Trotzky, Cheinet, Foelling, ... , Lukin, Bloch, *Science* **319** (2008)

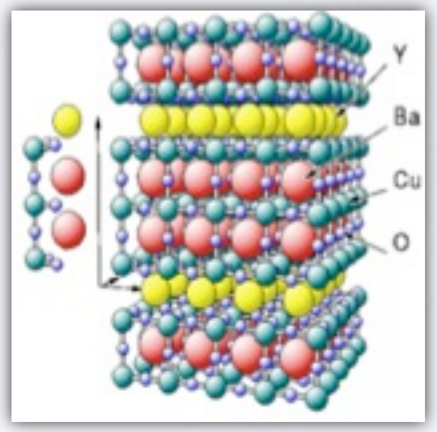
Sudden quenches



- Start with **some initial state** with clustering correlations
- Non-equilibrium (closed system) dynamics after a **sudden quench**

$$\rho(t) = e^{-iHt}\rho(0)e^{iHt}, \quad H = \sum_i h_i$$

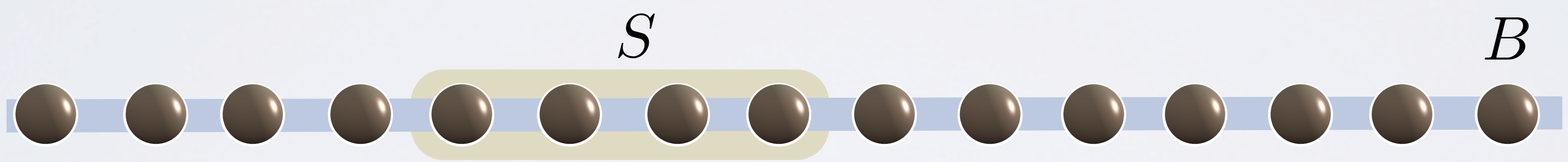
- Say, Mott to superfluid quench in Bose-Hubbard model



Local Hamiltonians, models for strongly correlated matter

- **What happens?** Does it equilibrate?

Local relaxation in quantum many-body systems



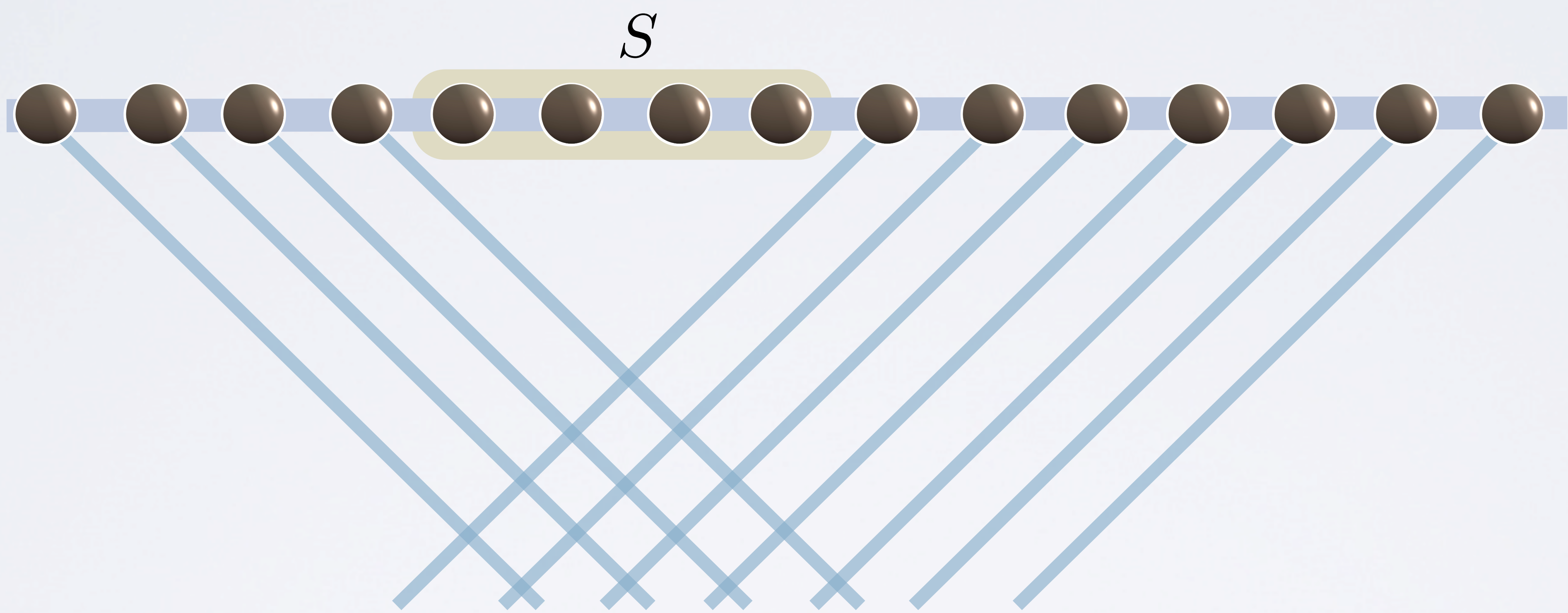
$$H = H_S + H_B + H_I$$

- **Local relaxation:** Local expectation values of subsystems relax and "perfectly look like their time average"

$$\omega = \lim_{T\to\infty} \frac{1}{T} \int_0^T dt\, e^{-iHt} \rho(0) e^{iHt}$$

- **Maximum entropy state** given **all** constants of motion

Strong and weak equilibration



- **Strong equilibration for free, integrable bosonic models**

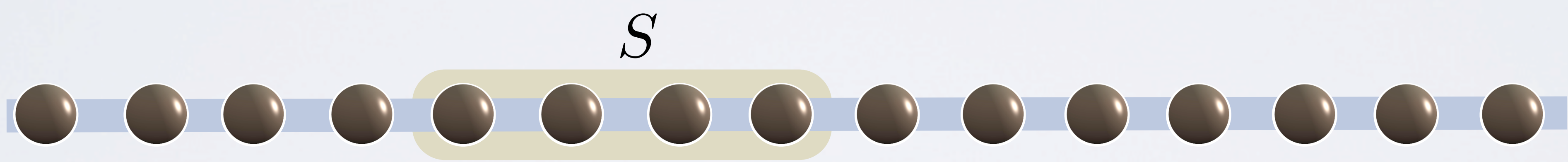
For all initial states with second and fourth order correlations decaying $O(\mathrm{dist}^{-2})$, for all $\varepsilon > 0$ find interval $[t_1, t_2]$

$$\|\rho_S(t) - \omega_S\|_1 < \varepsilon, \quad \forall t \in [t_1, t_2]$$

Methods: Lieb-Robinson bounds, non-commutative central limit theorems

Cramer, Eisert, *New J Phys* **12**, 055020 (2010)
Cramer, Dawson, Eisert, Osborne, *Phys Rev Lett* **100**, 030602 (2008)

Strong and weak equilibration



- **Equilibration in expectation:** For Hamiltonians with non-degenerate energy gaps)

$$\mathbb{E}(\|\rho_S(t) - \omega_S\|_1) \leq \frac{1}{2}\sqrt{\frac{d_S^2}{d^{\text{eff}}}}\,, \qquad d^{\text{eff}} = \frac{1}{\sum_k |\langle E_k|\psi_0\rangle|^4}$$

Linden, Popescu, Short, Winter, *Phys Rev E* **79**, 061103 (2009)
Short, Farrelly, arXiv:1110.5759
Reimann, *Phys Rev Lett* **101**, 190403 (2008)
Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)

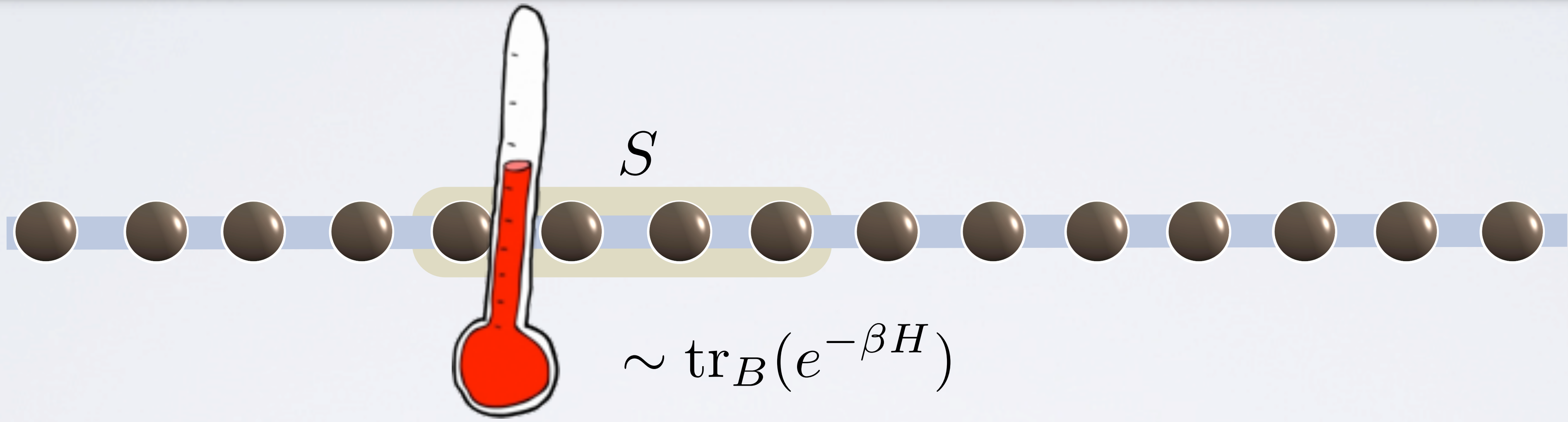
Lessons

- **Lesson**: Systems generically "appear relaxed", although dynamics is unitary

- Proven in *strong sense* for general states in *integrable limit* of Bose-Hubbard model
- True in slightly weaker sense generically

- **Time scales for non-integrable models?**

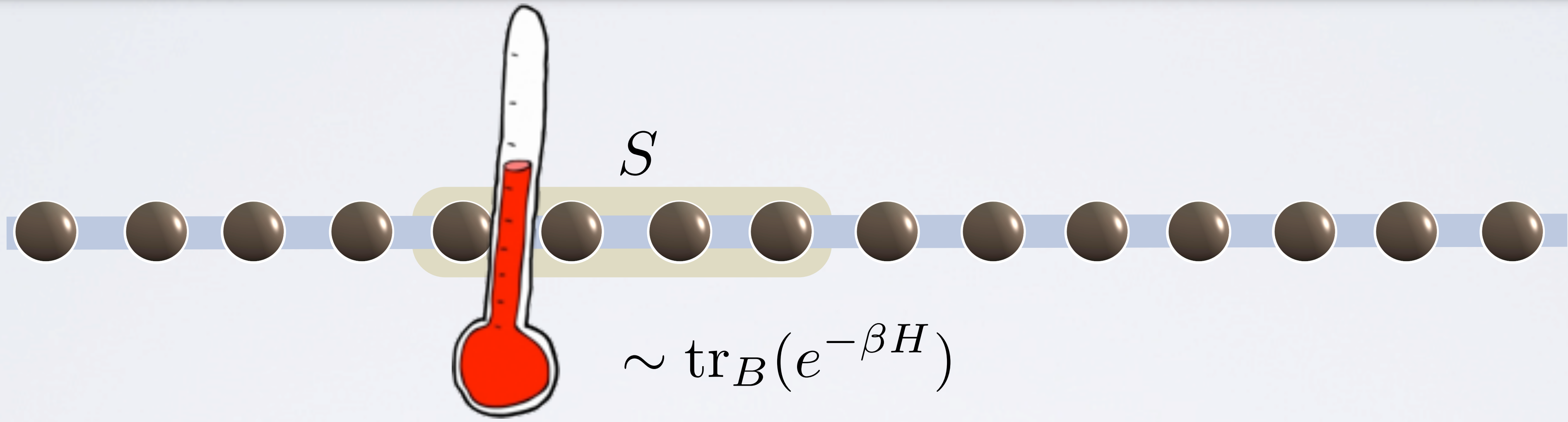
Thermalization



- When do systems **thermalize?**
- In addition to equilibration...
- ...local state indistinguishable from global (non-generalised) **Gibbs state?**

Riera, Gogolin, Eisert, *Phys Rev Lett* **108**, 080402 (2012)
Tasaki, *Phys Rev Lett* **80**, 1373 (1998)
Popescu, Short, Winter, *Nature Phys* **2**, 754 (2006)
Goldstein, Lebowitz, Tumulka, Zanghi, *Phys Rev Lett* **96**, 050403 (2006)
Reimann, *Phys Rev Lett* **101**, 190403 (2008)

Thermalization



- **Progress on thermalisation question**

- Consider **1D Hamiltonians** H of n sites
- Initial states with a flat energy distribution in $[nE, n(E+\Delta)]$ locally equilibrate towards a Gibbs state, even if they are **initially far from equilibrium:** can bound

$$\|\mathrm{tr}_B(e^{-\beta H})/Z - \omega_S\|_1 \leq \delta$$

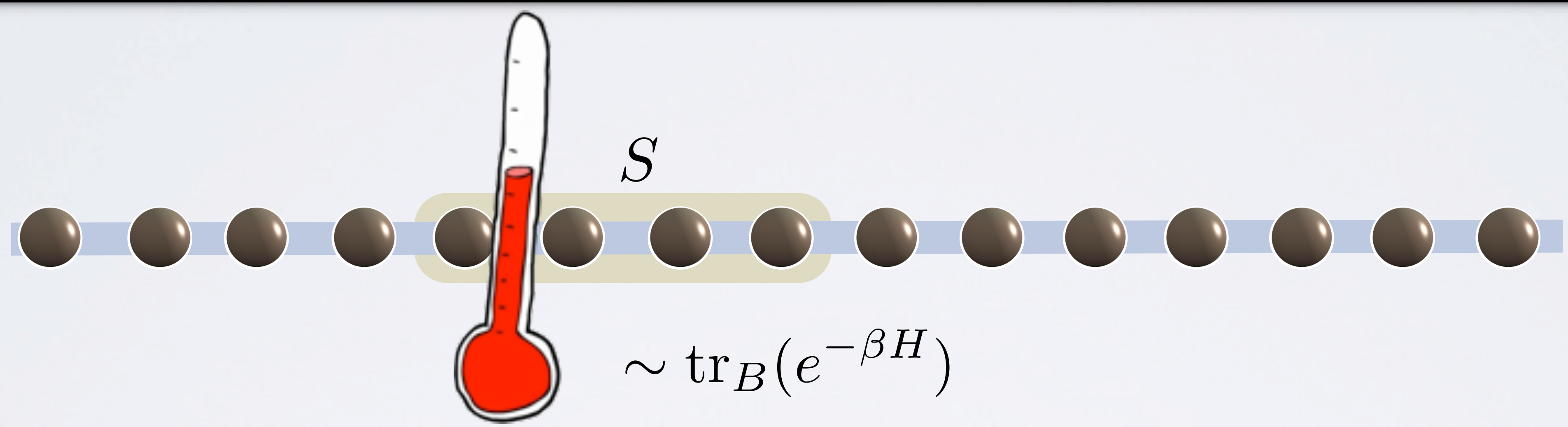
with δ that depends on Δ , E, and basic spectral properties of $H = \sum_j h_j$

Methods: Spectral perturbation theory, concentration of measure and typicality tools

Riera, Gogolin, Eisert, *Phys Rev Lett* **108**, 080402 (2012)
Riera, Gogolin, Eisert, in preparation (2014)
Mueller, Adlam, Masanes, Wiebe, arXiv:1312.7420

Tasaki, *Phys Rev Lett* **80**, 1373 (1998)
Popescu, Short, Winter, *Nature Phys* **2**, 754 (2006)
Goldstein, Lebowitz, Tumulka, Zanghi, *Phys Rev Lett* **96**, 050403 (2006)

Thermalization



- **Lesson**: Very weakly coupled systems with narrow initial energies thermalise

- Many-body localized models not thermalising

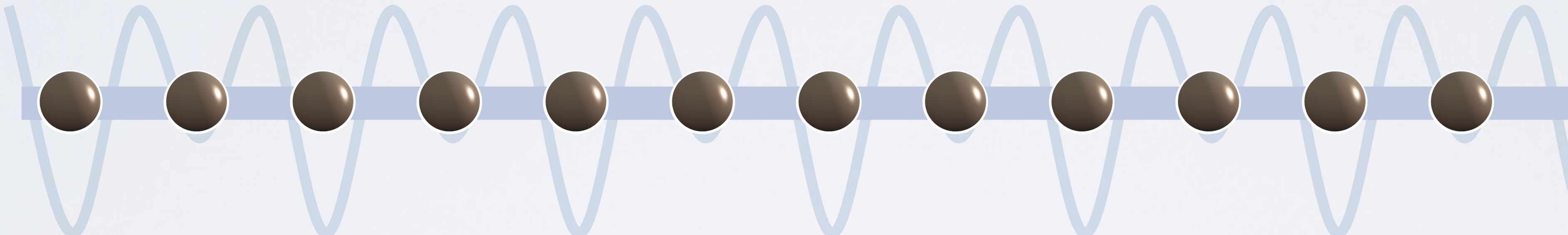
  Nandkishore, Huse, arXiv:1404.0686
  Gogolin, Mueller, Eisert, Phys Rev Lett **106**, 040401 (2011)

- Do non-integrable models thermalize?
- **Many questions wide open**

2. Simulations with sudden quenches

Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)
Cramer, Flesch, McCulloch, Schollwoeck, Eisert, *Phys Rev Lett* **101**, 063001 (2008)

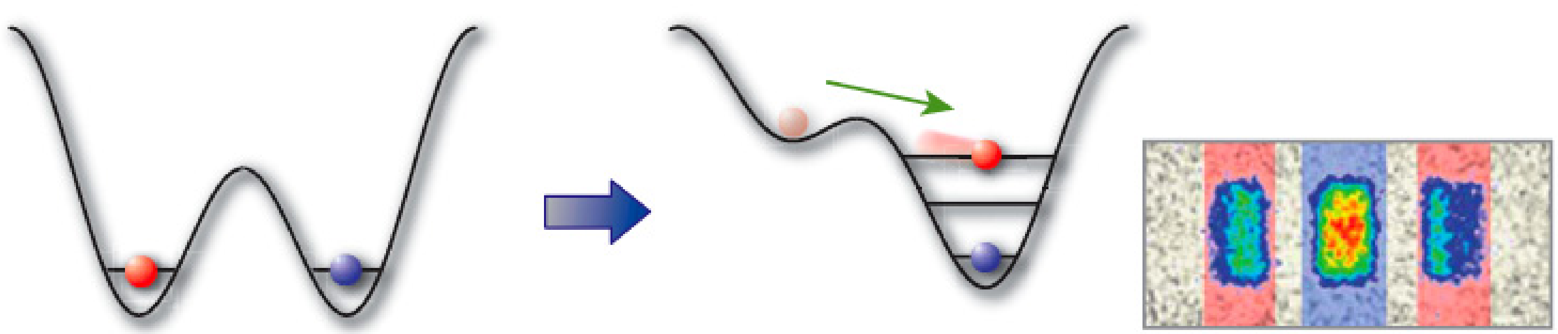
An experiment probing dynamics of a strongly correlated model



- Quench to full **strongly-correlated Bose-Hubbard Hamiltonian**...

$$|\psi(t)\rangle = e^{-iHt}|1,0,1,0,\ldots,1,0\rangle$$

read out and preparation with period 2: Densities, correlators, currents...

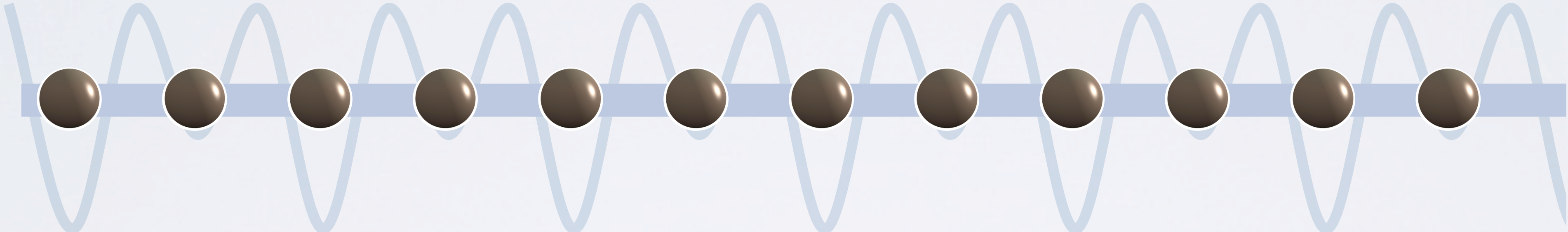


- Bias superlattice
- Unload to higher band
- Time-of-flight measurement: mapping to different Brillouin zones

Foelling et al, *Nature Phys* **448**, 1029 (2007)

Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

An experiment probing dynamics of a strongly correlated model



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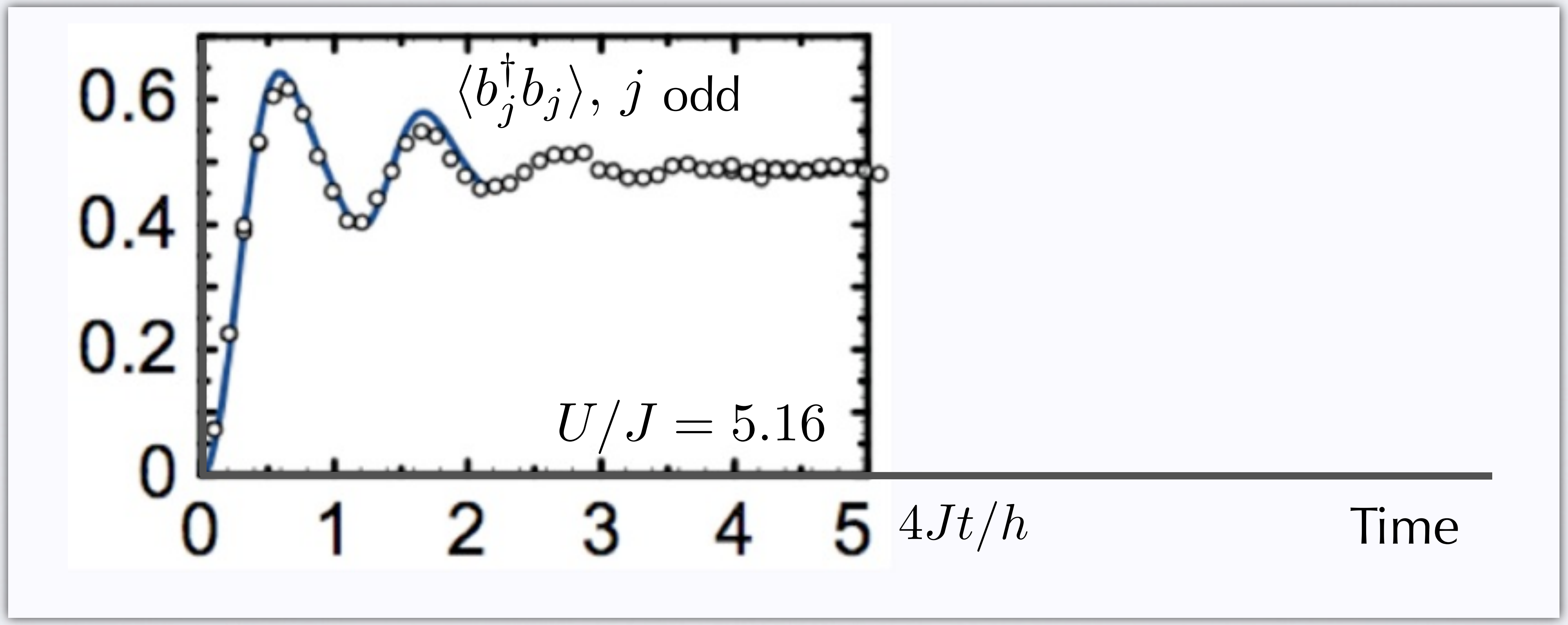
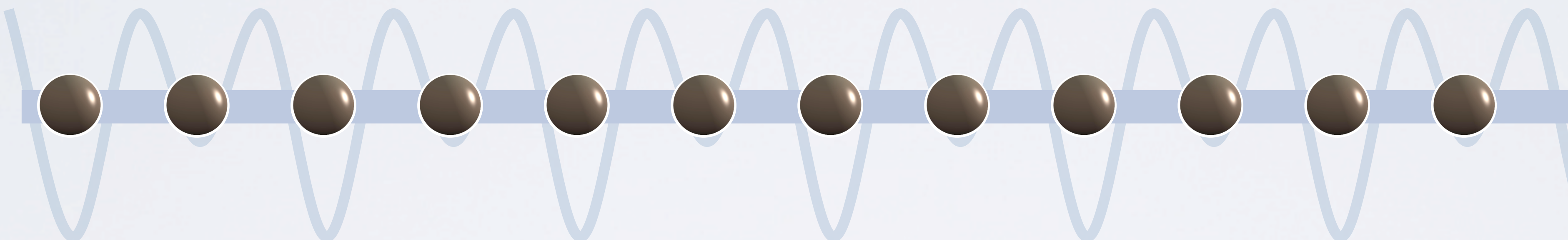
$$|\psi(t)\rangle = e^{-iHt}|1,0,1,0,\ldots,1,0\rangle$$



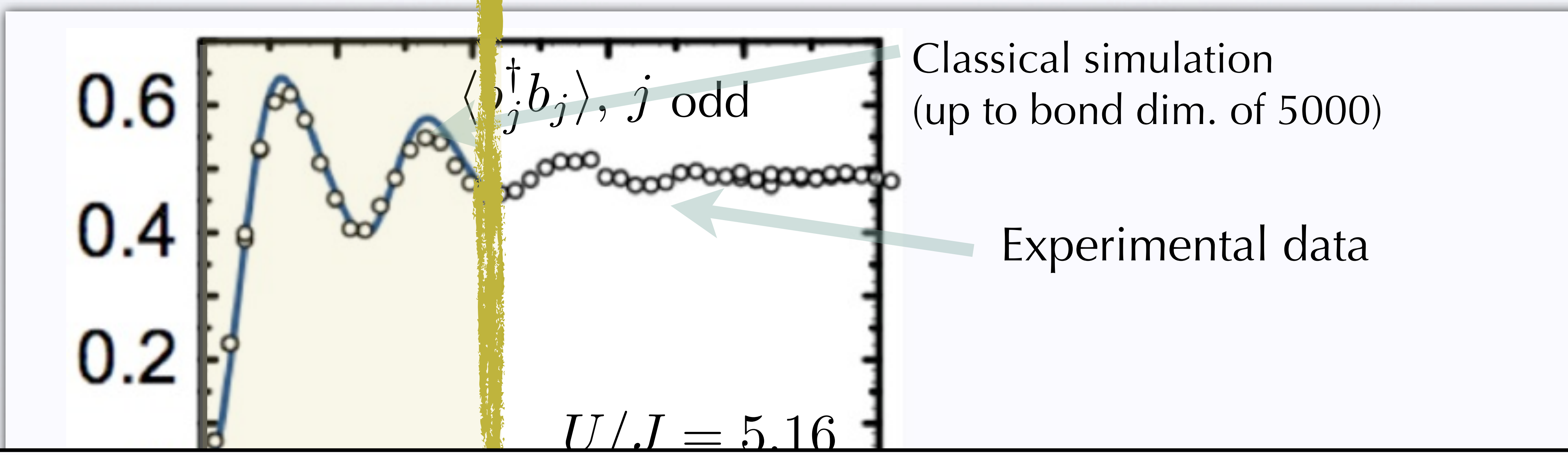
Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

MPS classical simulation



- Quench to full **strongly-correlated Bose-Hubbard Hamiltonian**...

$$|\psi(t)\rangle = e^{-iHt}|1,0,1,0,\ldots,1,0\rangle$$

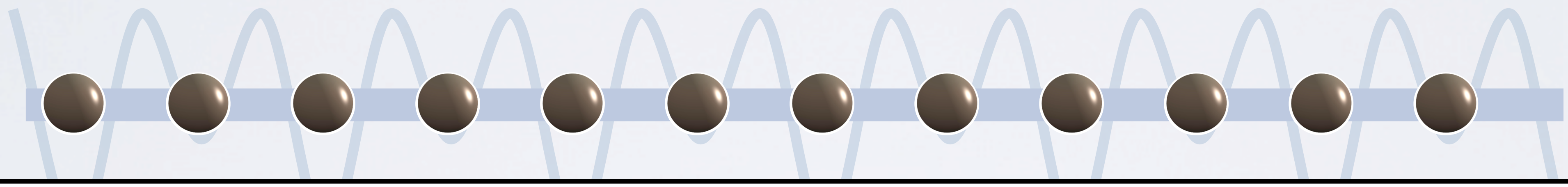


- **Short times matrix-product state (MPS) simulation**

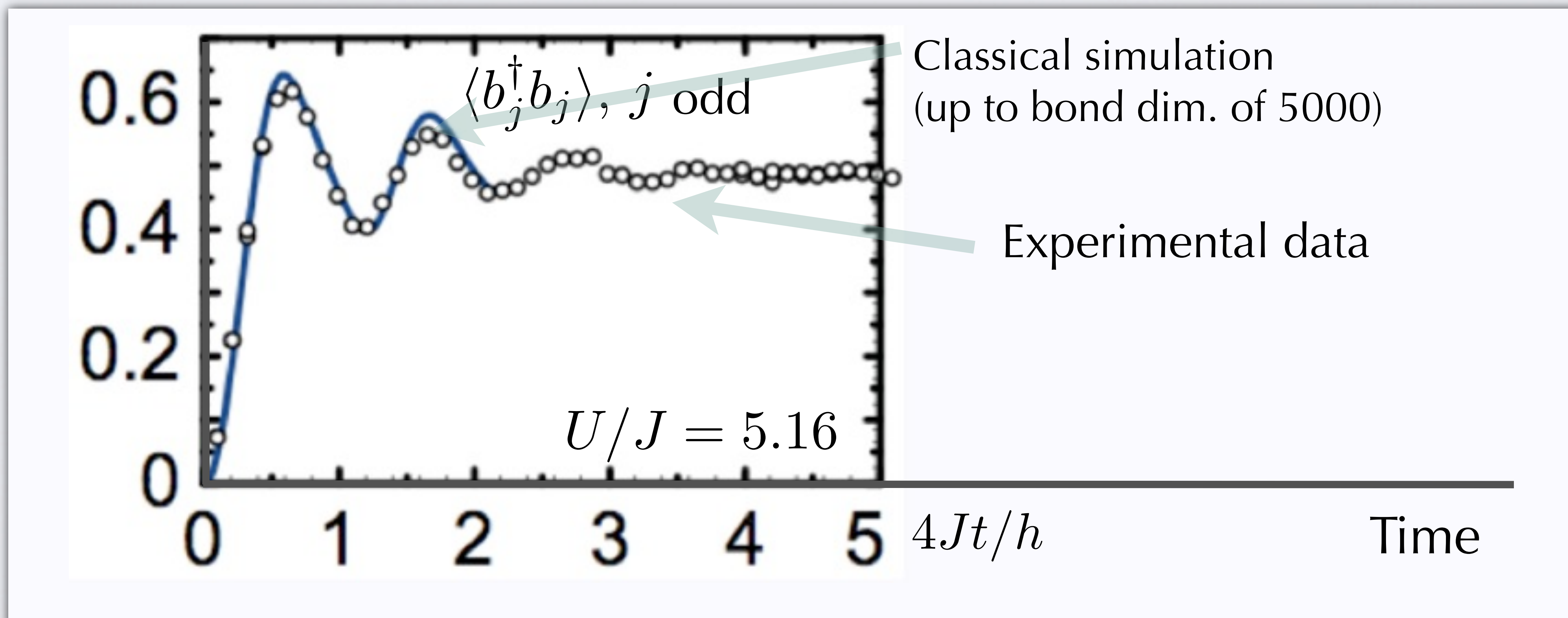
- **Short times:** Machine precision t-DMRG, rigorous error bounds, "building trust"
- **Long times:** Bond dimensions for faithful MPS grow **exponentially** in time

2012)

MPS classical simulation

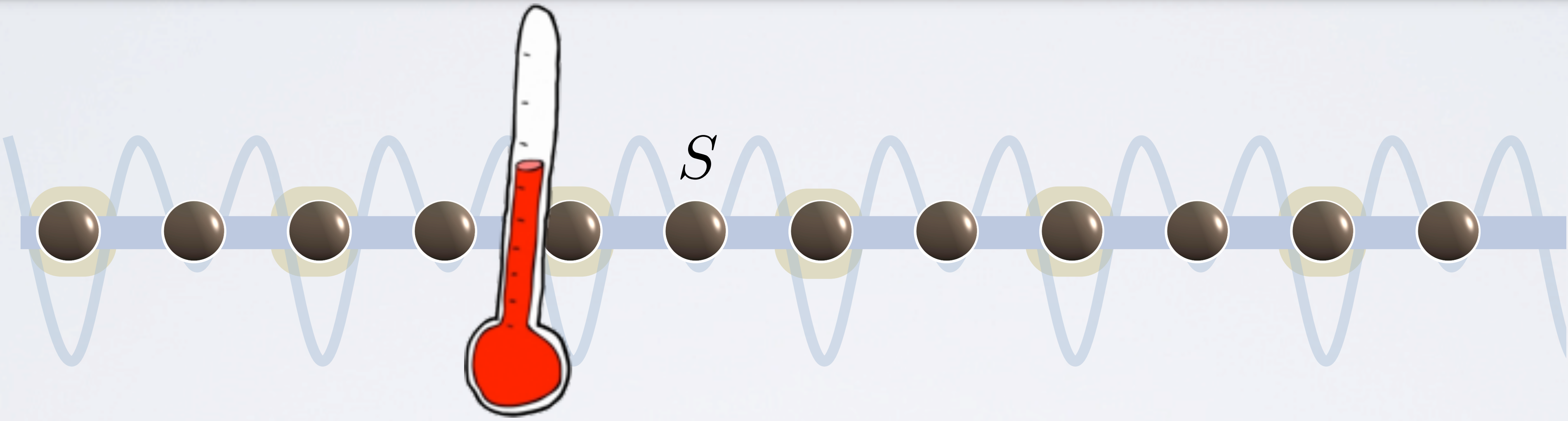


- **Long time dynamics of many-body dynamics in experiment**
- Can accurately probe dynamics for long times (exp vs poly decay, ...)
- "Outperforms" **best available classical simulation** on supercomputers

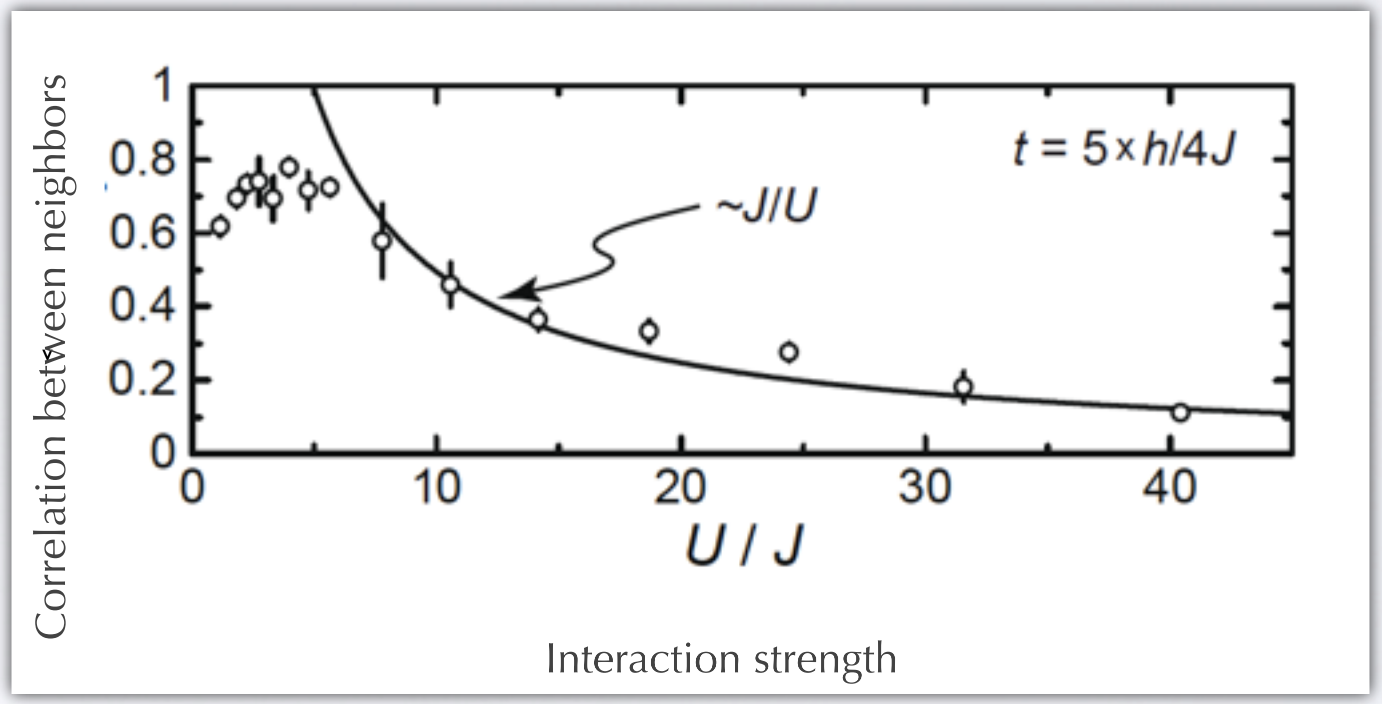


Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

Thermalisation?

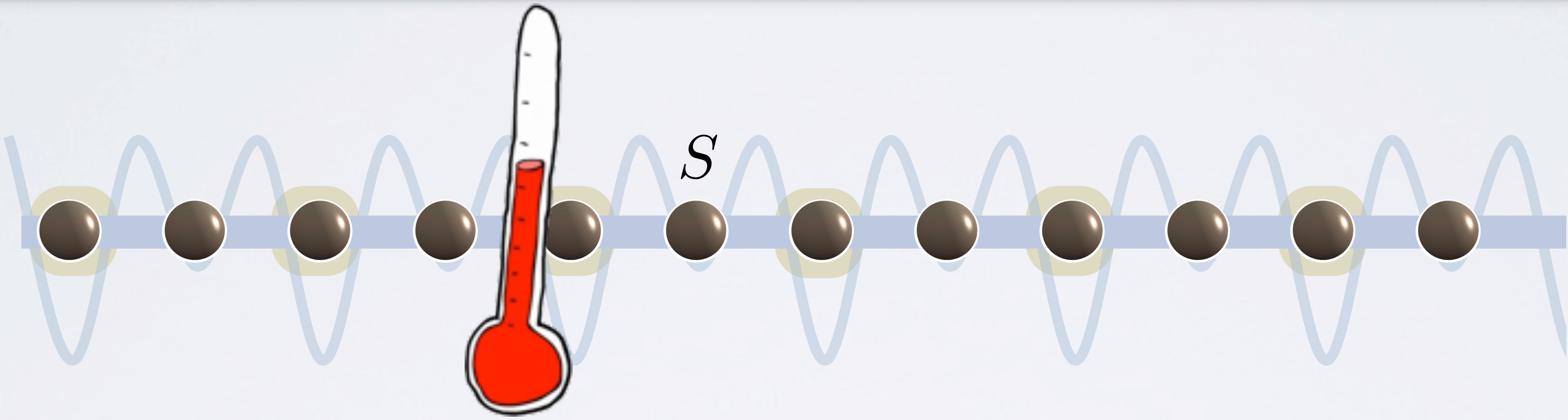


- Nearest-neighbor correlators for long times compatible with thermalisation



Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

Quantum simulation of fast quenches



- **Lesson**: Cold atoms analog[ue] simulators with 10^5 particles allow to probe non-equilibrium features that "outperform classical computers"

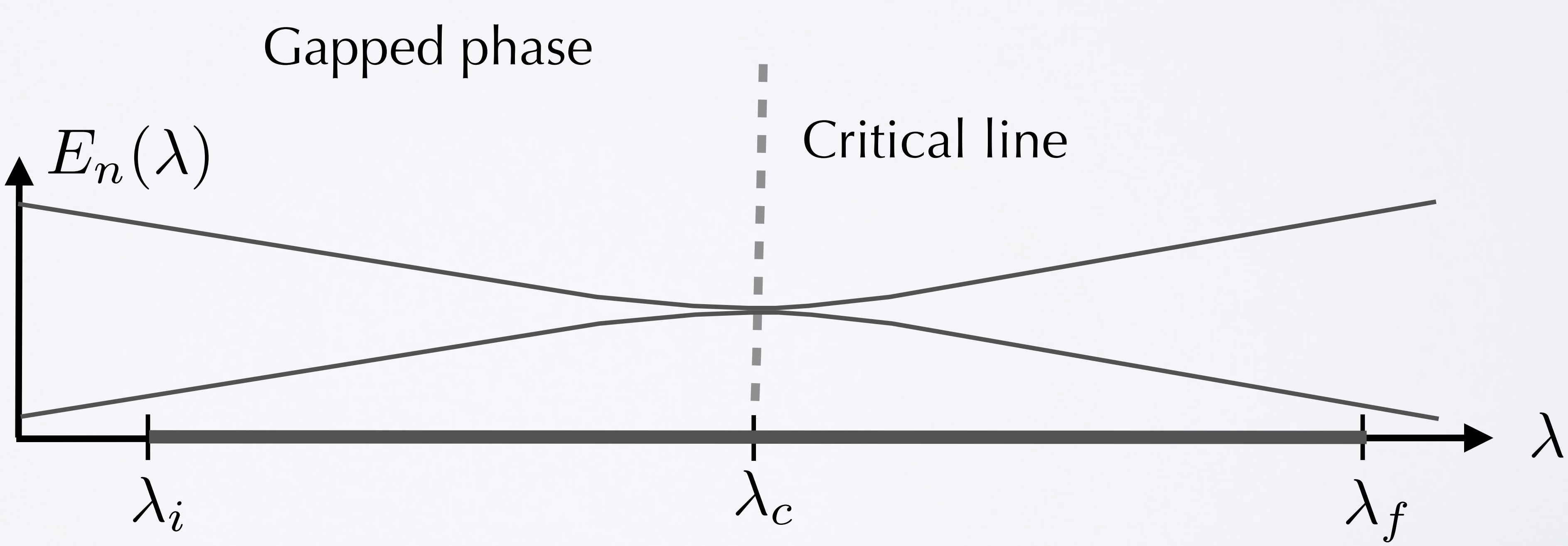
3. The puzzle of slow quenches

Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, Schneider, arxiv:1304.7199

Crossing critical lines

- Slow quenches, follow **parametrized curve** $t \mapsto H(t)$

- How do **phase "transitions"** happen?

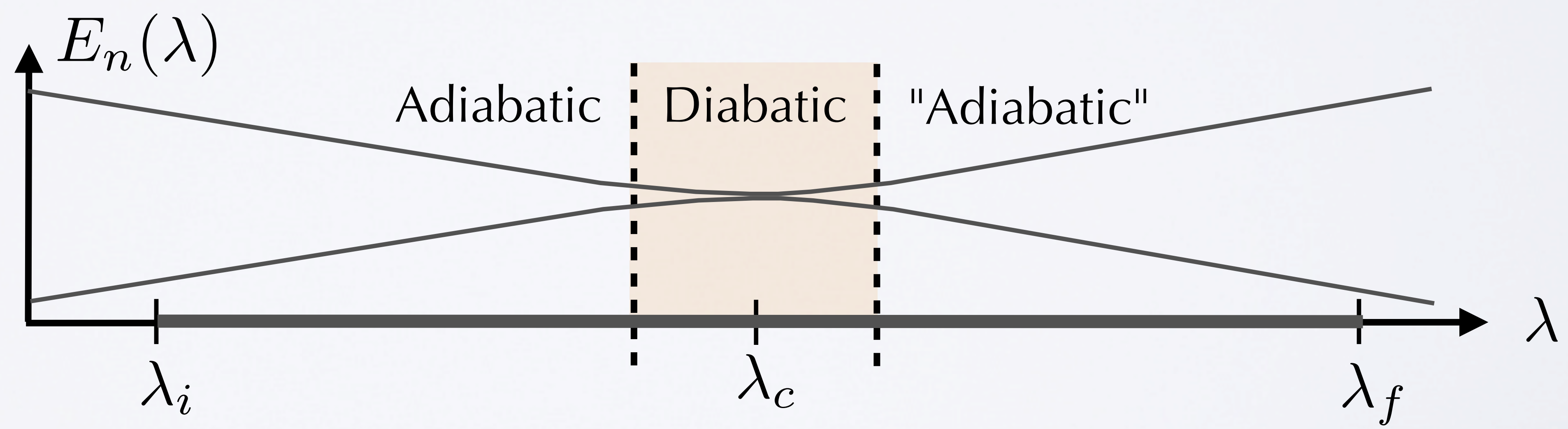


Kibble Zurek narrative

- **Crude intuitive picture:** Dynamics are simplified to
  - **Adiabatic regime** $\frac{d\Delta}{dt} \ll \Delta$
  - **Diabatic regime**
- Universality close to critical point $\xi \sim |\lambda - \lambda_c|^{-\nu}$

$$\Delta \sim |\lambda - \lambda_c|^{z\nu}$$

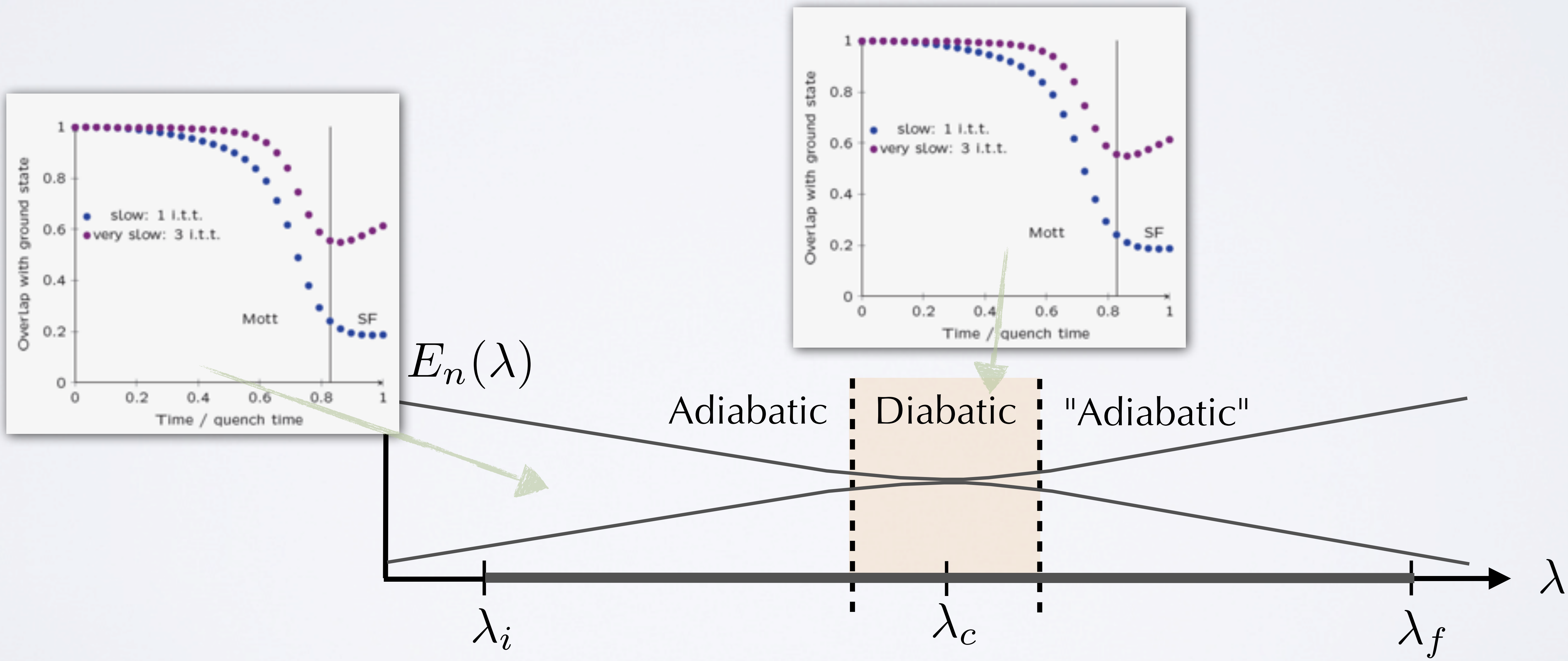
- Predicts **power laws** for correlation lengths in quench time



Zurek, Dorner, Zoller, *Phys Rev Lett* **95**, 105701 (2005)
Dziarmaga, Rams, *New J Phys* **12**, 055007 (2010)
Del Campo, De Chiara, Morigi, Plenio, Retzker, *Phys Rev Lett* **105**, 075701 (2010)

Kibble Zurek narrative

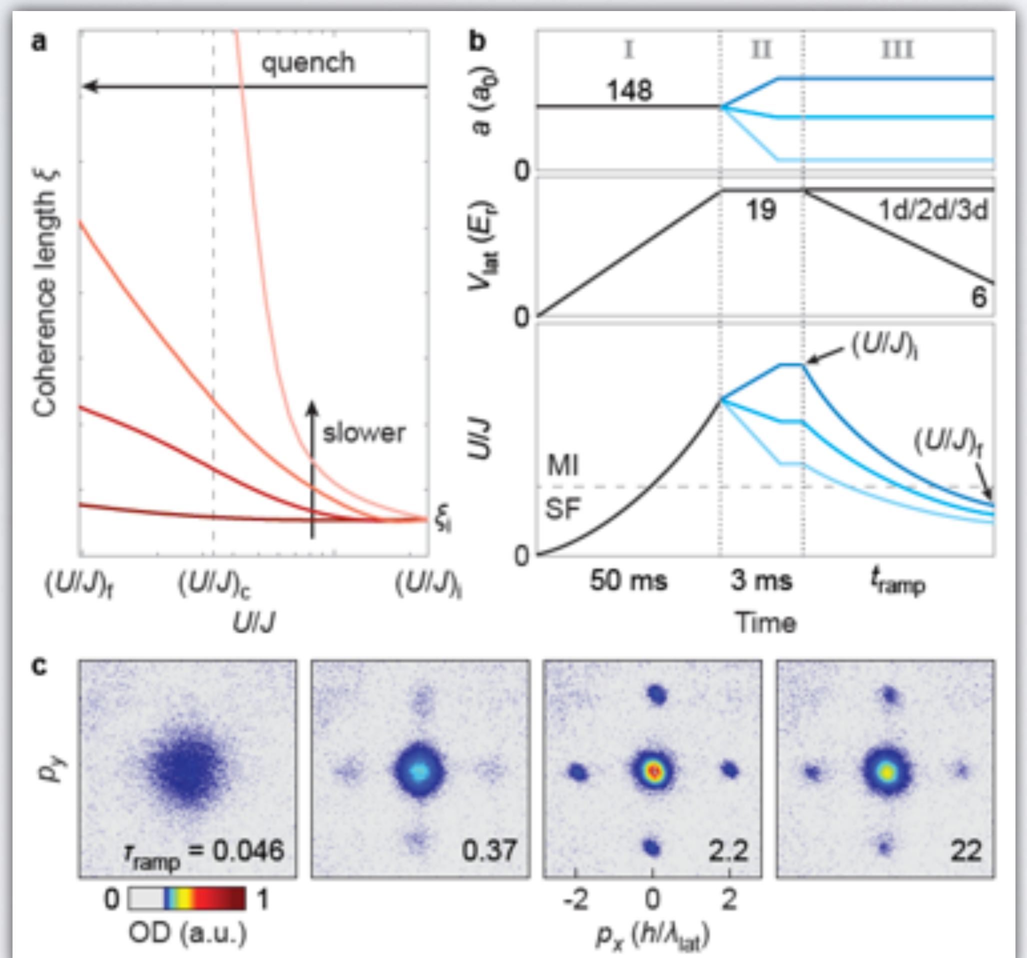
- **Crude picture** is often too crude
- Huge literature, no complete understanding
- Inequivalent approaches: Free models, adibatic perturbation theory, scaling collapse

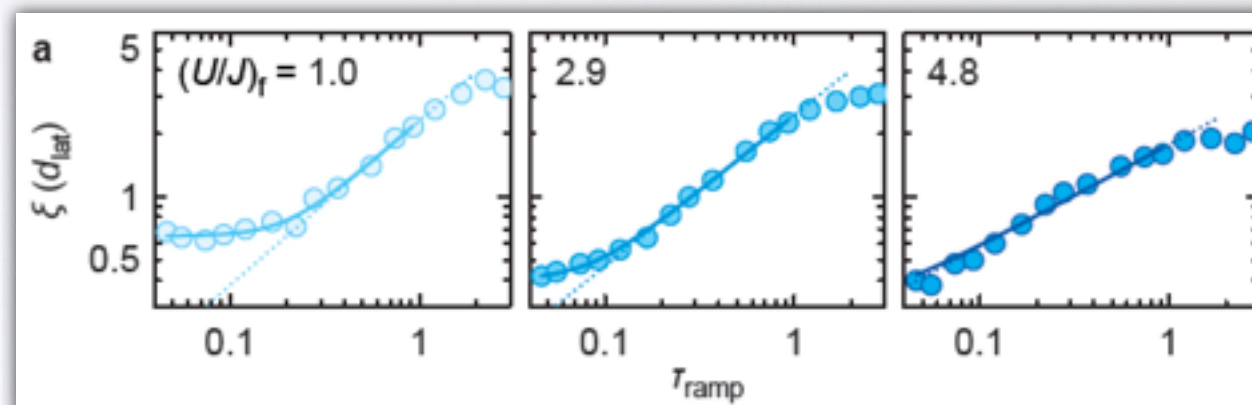


Zurek, Dorner, Zoller, *Phys Rev Lett* **95**, 105701 (2005)
Dziarmaga, Rams, *New J Phys* **12**, 055007 (2010)
Del Campo, De Chiara, Morigi, Plenio, Retzker, *Phys Rev Lett* **105**, 075701 (2010)

An experiment

- **Experiment in Immanuel Bloch/Uli Schneider's lab:** Mott to superfluid quench





- Heavy numerics: DMRG and **exact diagonalisation** in 1D

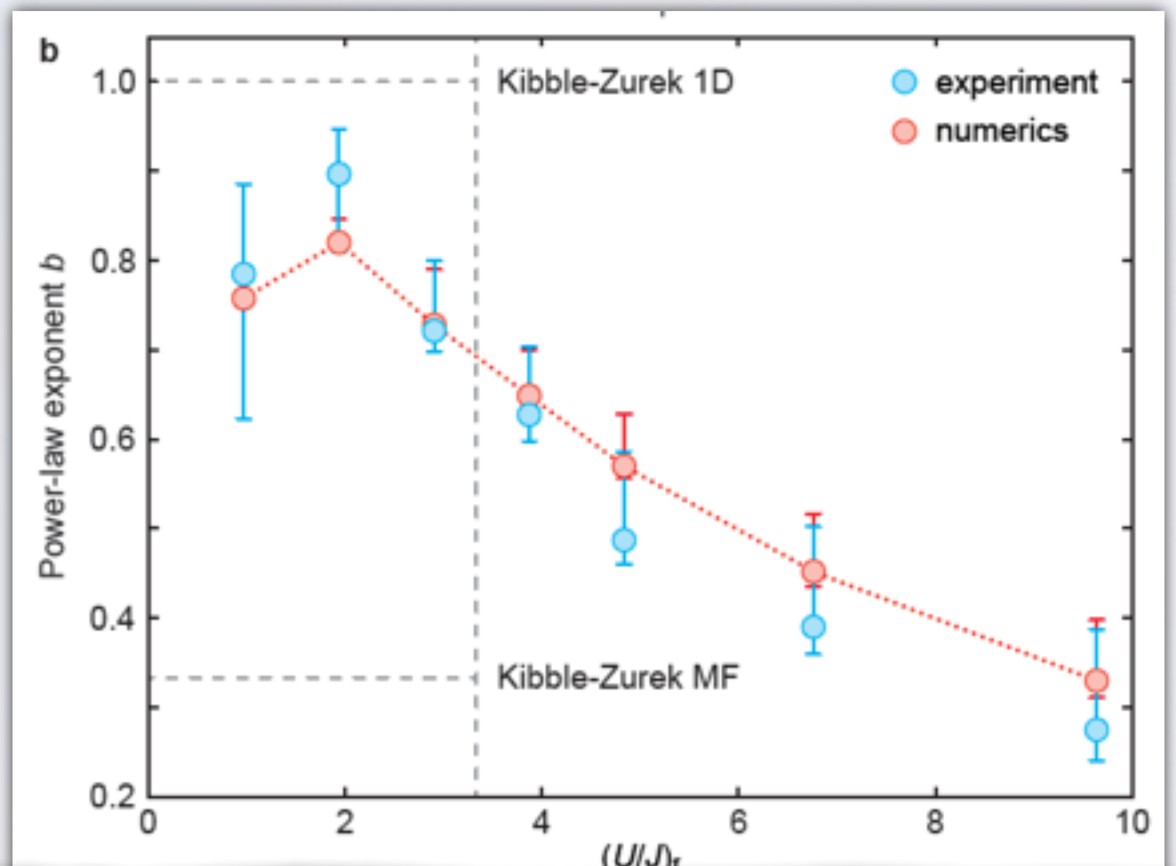
(15 sites, truncation at 9 bosons, 2.581.186 basis states, 5.5 GB size of involved matrices)

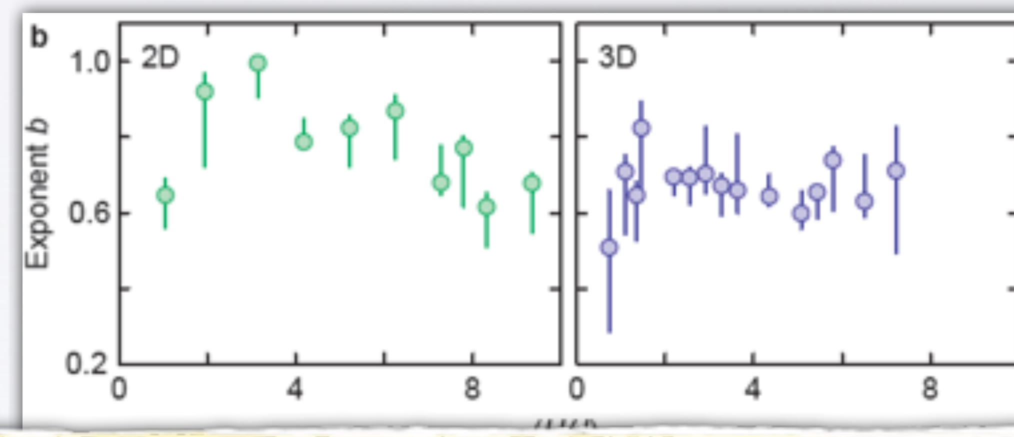
- Careful study of effects of trap ..., excellent **agreement** with data

Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, Schneider, arxiv:1304.7199

Dynamical analogue quantum simulator

- Finds **accurate power laws** over wide range of parameters
- But **very different** (!) from naive reading of Kibble-Zurek





- **Lesson**: Finds power laws, but different from naive reading of Kibble-Zurek picture
- **"Quantum simulation":** Build trust in correctness of simulation in 1D, experiment allows for assessment of 2D, 3D, alternative schedules etc

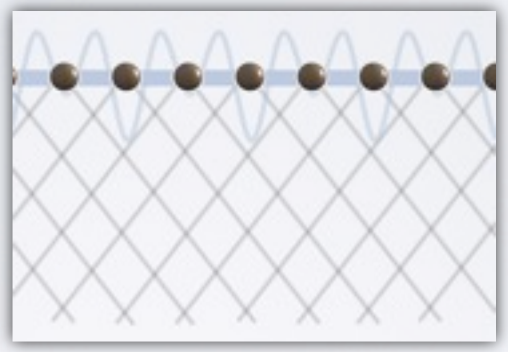
Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, Schneider, arxiv:1304.7199

4. Summary and some musings

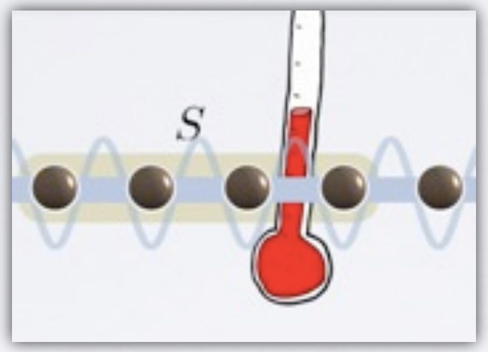
Gogolin, Kliesch, Aolita, Eisert, arXiv:1306.3995
Gogolin, Kliesch, Aolita, Eisert, in preparation (2014)

Summary

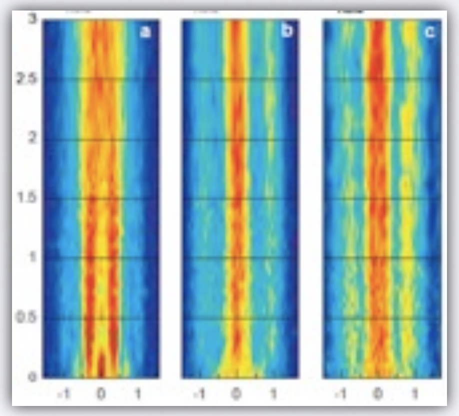
- **This talk:** Quantum simulation on physically motivated questions



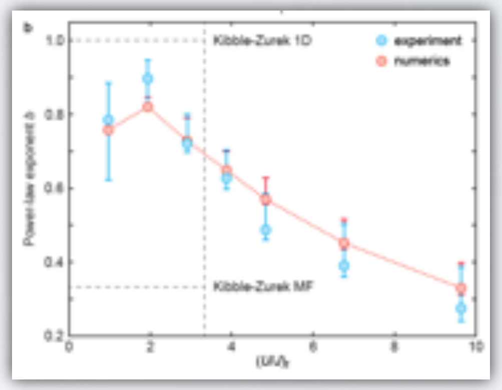
- Equilibration



- Thermalization



- Experimental fast quenches



- "Crime story" of slow quenches

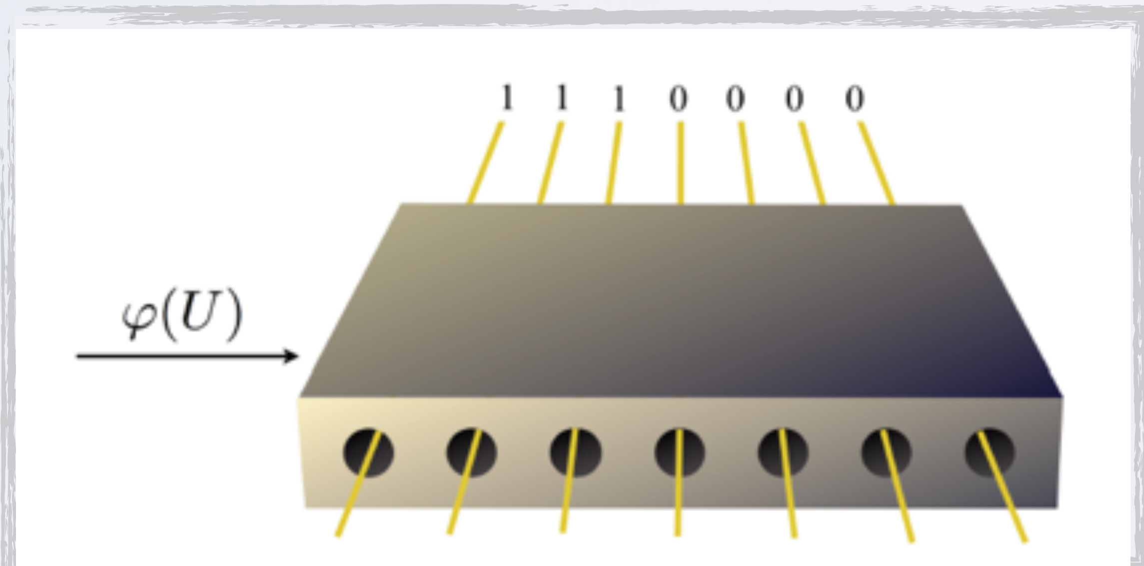
- Evidence of **quantum devices** "outperforming" **classical supercomputers**

  ("quantum supremacy", in John Preskill's words)

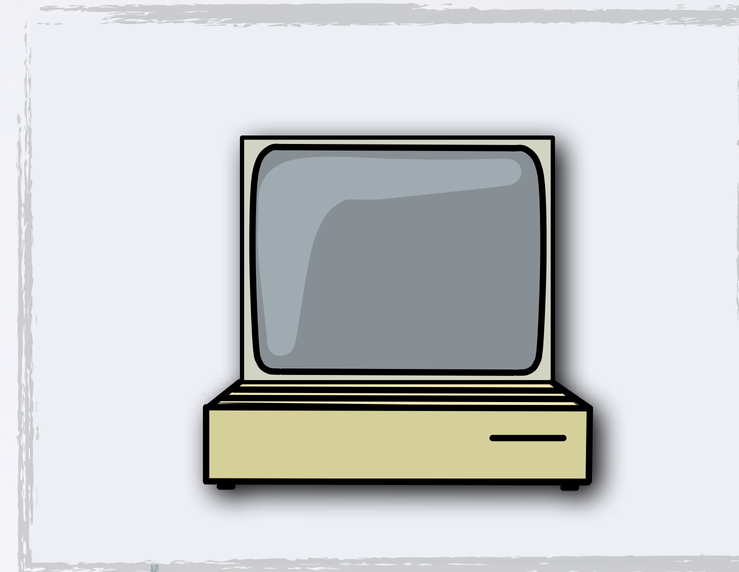
- **Quantum many-body dynamics** appears great arena for this
- Complexity classes?

Power of quantum devices terms of complexity classes?

- **Experiments on boson sampling:** Cannot be efficiently classically sampled (to constant error in 1-norm), unless collapse of polynomial hierarchy



Efficient sampling

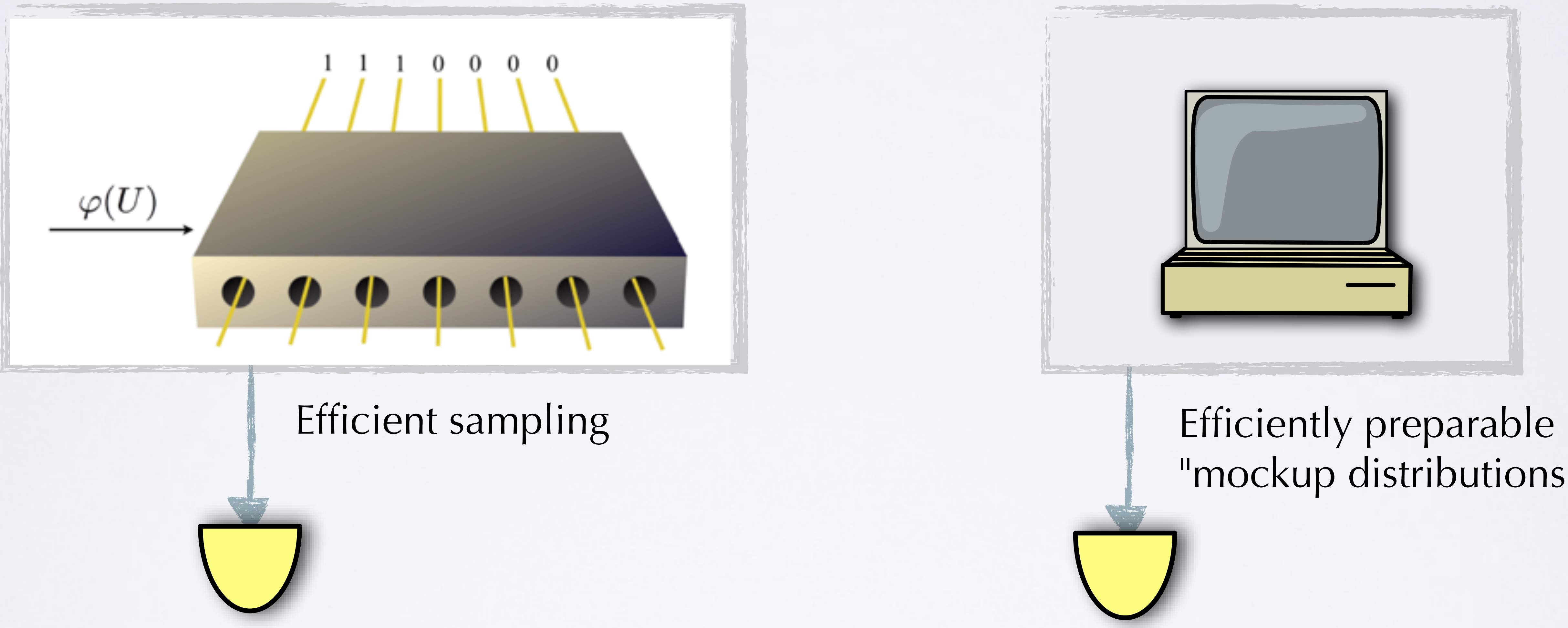


Presumably inefficient sampling

Aaronson, Arkhipov, Proc of ACM Symp Th Comp, STOC (2011)

Classical certification?

- **Experiments on boson sampling:** Cannot be efficiently classically sampled (to constant error in 1-norm), unless collapse of polynomial hierarchy



- **Classical efficient certification unlikely:** Efficiently classically create distributions that cannot be distinguished from true distribution with circuits of any finite size

Gogolin, Kliesch, Aolita, Eisert, arXiv:1306.3995
Gogolin, Kliesch, Aolita, Eisert, in preparation (2014)
Aaronson, Arkhipov, arXiv:1309.7460
Brandao, private communication (2013)

What is a quantum simulation?

- This talk

Thanks for your attention!