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QUANTUM ENVIRONMENT
for
LONG-LASTING COHERENCE

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with

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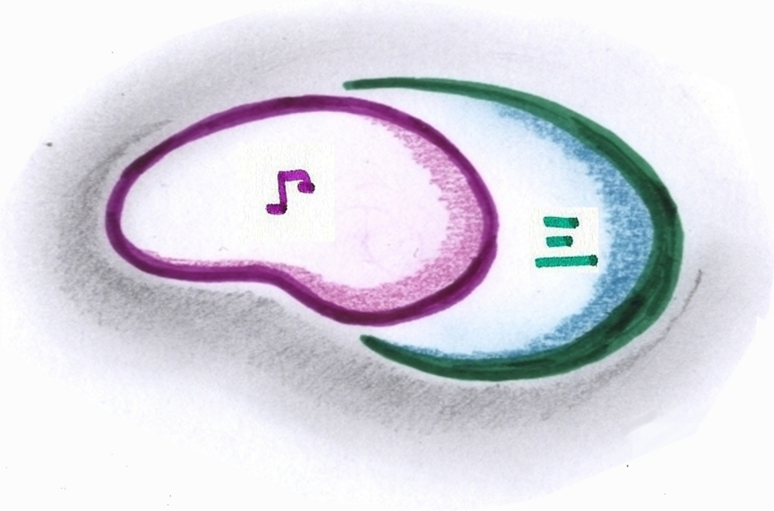
OPEN QUANTUM SYSTEMS

$\Psi = ♪ \cup \Xi$



$|\psi\rangle \in \mathcal{H}_{♪} \otimes \mathcal{H}_{\Xi}$

$\rho_{♪} = \mathrm{Tr}_{\Xi} |\psi\rangle\langle\psi|$

OPEN QUANTUM SYSTEMS

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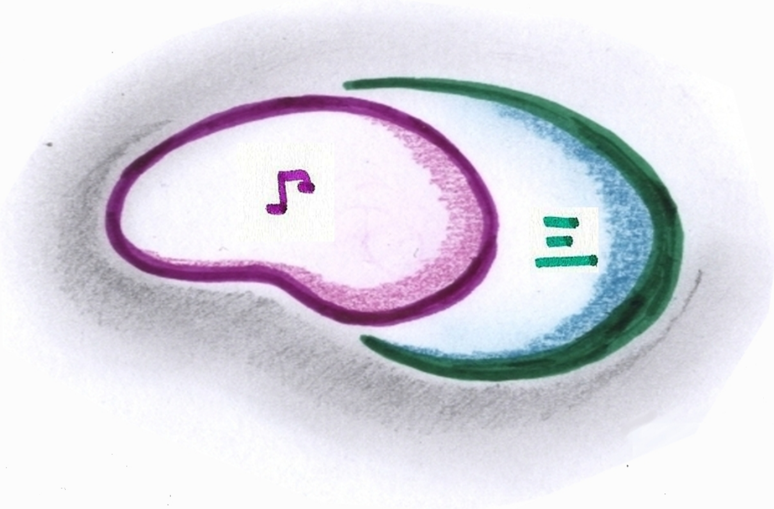
$H_{♪\Xi}$



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$|\psi(t)\rangle = \mathcal{U}(t)|\psi(0)\rangle$

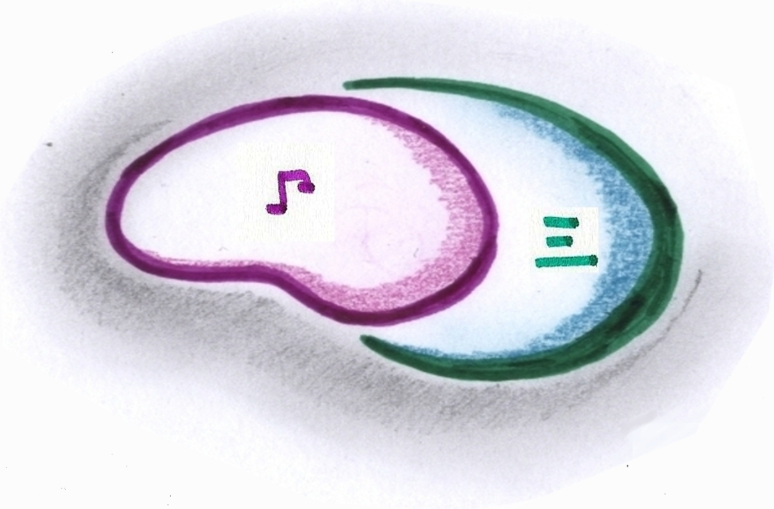
$$\rho_{♪} = \mathrm{Tr}_{\Xi} |\psi\rangle\langle\psi|$$

$$\rho_{♪}(t) = \mathrm{Tr}_{\Xi} \left\{ |\psi(t)\rangle\langle\psi(t)| \right\} = \mathrm{Tr}_{\Xi} \left\{ \mathcal{U}^{+}(t) |\psi(0)\rangle\langle\psi(0)| \mathcal{U}(t) \right\}$$

OPEN QUANTUM SYSTEMS

$\Psi = S \cup \Xi$

$H_{S\Xi}$



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?

$$\frac{\partial \rho_S}{\partial t} = -i \left[\mathcal{H}^{eff}_{S\Xi}, \rho_S \right] + \dots$$

or

$$\begin{cases} \rho_S(t) \sim |S(t)\rangle\langle S(t)| \\ |S(t)\rangle = T\left(e^{-i\int_0^t h_S(\tau,\xi)d\tau} |S(0)\rangle \right) \end{cases}$$

PARAMETRIC REPRESENTATION

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$$|\psi > = \sum_{\gamma\xi} c_{\gamma\xi} \, |\gamma > |\xi >$$

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suppose $\int_{\mathcal{M}} d\mu(\Omega) |\Omega\rangle\langle\Omega| = \mathbb{1}_{\mathcal{H}_\Xi}$ is provided, then

$$|\Psi\rangle = \int_{\mathcal{M}} d\mu(\Omega)\, \chi(\Omega) |\phi(\Omega)\rangle$$

with

$|\phi(\Omega)\rangle = \frac{1}{\chi(\Omega)} \sum_\gamma f^\gamma(\Omega) |\gamma\rangle$; $f^\gamma(\Omega) = \sum_\xi c_{\gamma\xi} \langle\Omega|\xi\rangle$; $\chi^2(\Omega) = \sum_\gamma |f^\gamma(\Omega)|^2$ so that

$\langle\phi(\Omega)|\phi(\Omega)\rangle = 1 \quad \forall\, \Omega \in \mathcal{M}$ and $\int_{\mathcal{M}} d\mu(\Omega)\, \chi^2(\Omega) = 1$

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but

$$\rho_{\Gamma} = \iint_{\mathcal{M}} d\mu(\Omega) \, d\mu^{*}(\Omega') \quad \chi(\Omega)\chi(\Omega') |\phi(\Omega)\rangle\langle\phi(\Omega')|$$ ☹

PARAMETRIC REPRESENTATION
WITH
ENVIRONMENTAL COHERENT STATES

$$\Psi = \Gamma \cup \Xi \quad ; \quad \hat{H}_{\Gamma\Xi} = \sum_i g_i \, \hat{O}_i^{\Gamma} \otimes \hat{O}_i^{\Xi}$$

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ECS

in : G_Ξ s.t. $g_\Xi \supset \hat{O}_i^{\Xi} \; \forall i$; $\mathcal{H}_\Xi$; $|R\rangle \in \mathcal{H}_\Xi$

$$\text{out} : \begin{cases} F_\Xi \text{ s.t. } \hat{f}|R\rangle = e^{i\lambda_f}|R\rangle \;\; \forall \hat{f} \in F_\Xi \text{ (maximum stability subgroup)} \rightarrow G/F \\ |\Omega\rangle = \hat{\Omega}|R\rangle \text{ with } \hat{\Omega} \text{ s.t. } \hat{G} = \hat{\Omega}\hat{f} \;\; \forall \hat{G} \in G_\Xi \text{ and } \hat{f} \in F_\Xi \\ d\mu(\Omega) \text{ invariant measure on } G/F \text{ and } \int_{G/F} d\mu(\Omega)\,|\Omega\rangle\langle\Omega| = \mathbb{1}_{\mathcal{H}_\Xi} \end{cases}$$

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- $|\Omega\rangle = \hat{\Omega}\,|R\rangle$ with $\hat{\Omega}$ s.t. $\hat{G} = \hat{\Omega}\hat{f} \;\; \forall \hat{G} \in G_\Xi$ and $\hat{f} \in F_\Xi$
- $d\mu(\Omega)$ invariant measure on G/F and $\int_{G/F} d\mu(\Omega)\,|\Omega\rangle\langle\Omega| = \mathbb{1}_{\mathcal{H}_\Xi}$

PRECS

$$|\psi\rangle = \int_{G/F} d\mu(\Omega)\, \chi(\Omega)\, |\phi(\Omega)\rangle$$

PRECS : EXPLOIT COHERENT STATES PROPERTIES

$$|\Omega\rangle \quad \longleftrightarrow \quad \Omega \in G/F$$

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* —— *

overcomplete but such that

$$\rho_\Omega = \int_{G/F} d\mu(\Omega)\, \chi^2(\Omega) |\phi(\Omega)\rangle\langle\phi(\Omega)|$$

with $\chi^2(\Omega) = \langle\Omega|\rho_E|\Omega\rangle$: Husimi function for environmental density

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"ONCE A COHERENT STATE, ALWAYS A COHERENT STATE"

$$|\Omega(t)\rangle = \hat{\Omega}_t|\Omega(0)\rangle e^{i\varphi_t}\,, \quad \hat{\Omega}_t \text{ s.t. } \Omega_t \text{ solves } i\hbar\frac{\partial\Omega}{\partial t} = \frac{\partial H(\Omega)}{\partial\Omega^*}$$

with

$$\varphi_t = \int_0^t d\tau\, \langle\Omega|\hat{\Omega}_t^\dagger\left(i\frac{d}{d\Omega} - \hat{H}\right)\hat{\Omega}_t|\Omega\rangle\,, \quad H(\Omega) = \langle\Omega|\hat{H}|\Omega\rangle$$

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?

OFF - DIAGONAL DYNAMICS

$$\hat{H}_{\Omega\Xi} = \sum_i g_i \, \hat{O}_i^{\Omega} \otimes \hat{O}_i^{\Xi}$$

if $[\hat{O}_i^{\Omega}, \hat{O}_j^{\Omega}] = 0 \quad \forall i,j \qquad \exists \, \{|\gamma\rangle\}_{\mathcal{H}_\Omega} \;/\; \hat{O}_i^{\Omega}|\gamma\rangle = \omega_i^{\gamma}|\gamma\rangle \quad \forall i,\gamma$ and

$$|\psi(0)\rangle = |\Omega\rangle \otimes |R\rangle$$

$|\psi(t)\rangle = \sum_\gamma c_\gamma |\gamma\rangle |R_t^\gamma\rangle$ with $|R_t^\gamma\rangle = e^{-it\sum_i g_i \omega_i^\gamma \hat{O}_i^{\Xi}} |R\rangle$

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hence

$$\rho_\Omega(t) = \sum_\gamma |c_\gamma|^2 |\gamma><\gamma| + \sum_{\gamma \neq \gamma'} c_\gamma c_{\gamma'}^* \underline{\underline{\underline{<R_t^{\gamma'}|R_t^\gamma>}}} |\gamma><\gamma'|$$

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! DECOHERENCE & MEASUREMENT PROCESS !

DECOHERENCE BY THE PRECS

reference state for ECS

take $|\psi(0)\rangle = |\Omega\rangle \otimes |R\rangle = \sum_{\gamma} c_{\gamma} |\gamma\rangle \otimes |R\rangle$

eigenstates of all $\hat{O}_i^{\Omega}$

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eigenstates of all $\hat{O}_i^\Omega$

get $$\rho_S(t) = \int_{G/F} d\mu(\Omega)\, \underbrace{\chi_t^2(\Omega)}_{\underbrace{\sum_\gamma |c_\gamma|^2 |\langle\Omega|R_t^\gamma\rangle|^2 \equiv \sum_\gamma |c_\gamma|^2 h_t^\gamma(\Omega)}_{R_t^\gamma \text{ solutions of } i m \frac{d\Omega}{dt} = \frac{\partial}{\partial\Omega^*} \langle\Omega|\hat{H}^\gamma|\Omega\rangle}}\, |\phi(\Omega,t)\rangle\langle\phi(\Omega,t)|$$

$H^\gamma = \langle\gamma|\hat{H}_{S\Sigma}|\gamma\rangle$ and $R_t^\gamma = 0 \ \forall \gamma$

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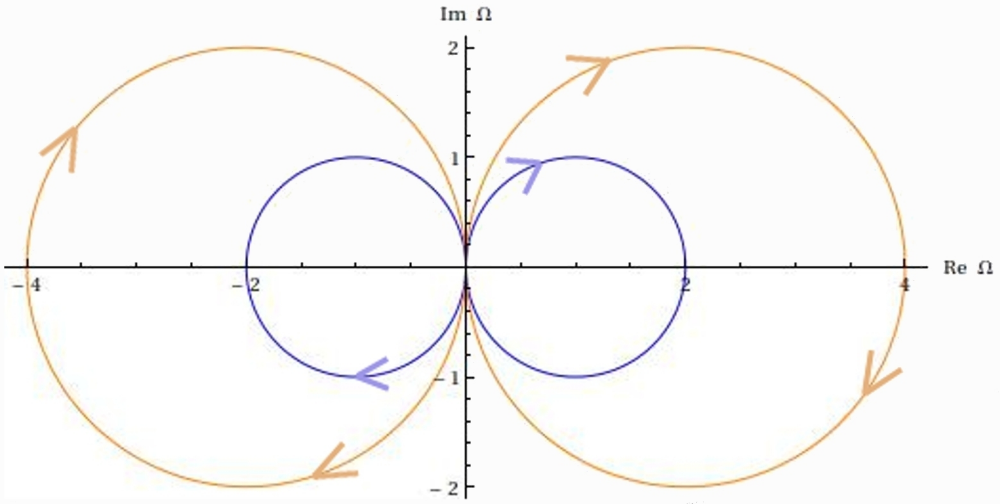
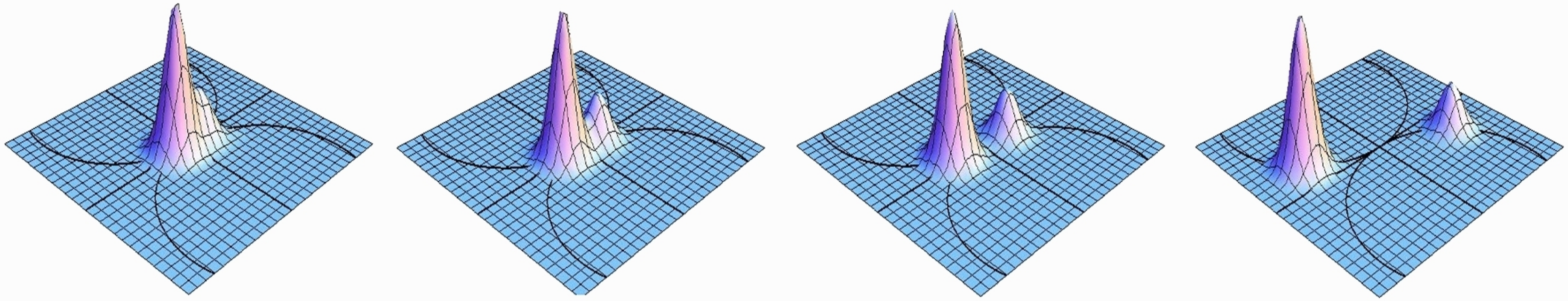
$H^\gamma = \langle\gamma|\hat{H}_{SE}|\gamma\rangle$ and $R_t^\gamma = 0 \quad \forall\gamma$



$$\hat{H}_{SE} = \omega \hat{b}^\dagger \hat{b} + q\hat{\sigma}^z(\hat{b} + \hat{b}^\dagger)$$

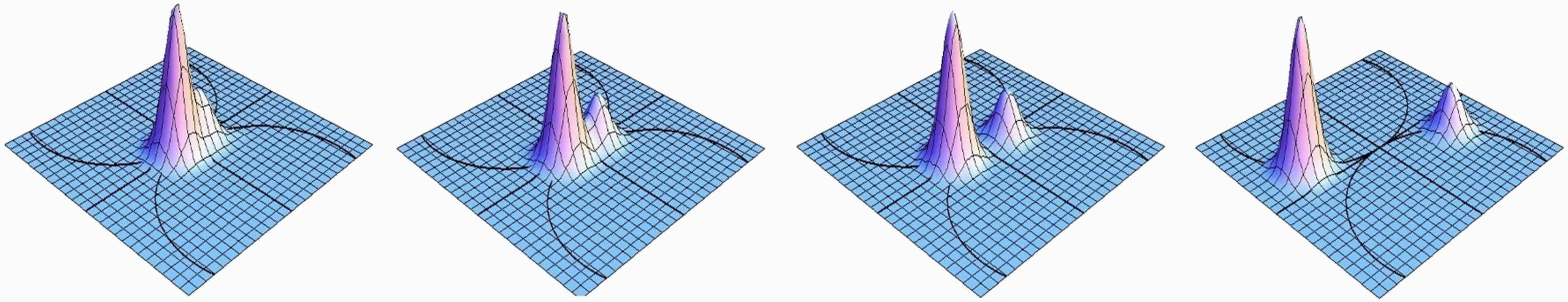
$$[\hat{b}, \hat{b}^\dagger] = 1 \quad ; \quad [\hat{\sigma}^\alpha, \hat{\sigma}^\beta] = i\varepsilon^{\alpha\beta\gamma}\hat{\sigma}^\gamma$$

DECOHERENCE



$$x_t^2(\Omega) = \sum_\gamma |c_\gamma|^2 \, h_t^\gamma(\Omega)$$

DECOHERENCE



$$\chi_t^2(\Omega) = \sum_\gamma |c_\gamma|^2 \, h_t^\gamma(\Omega)$$

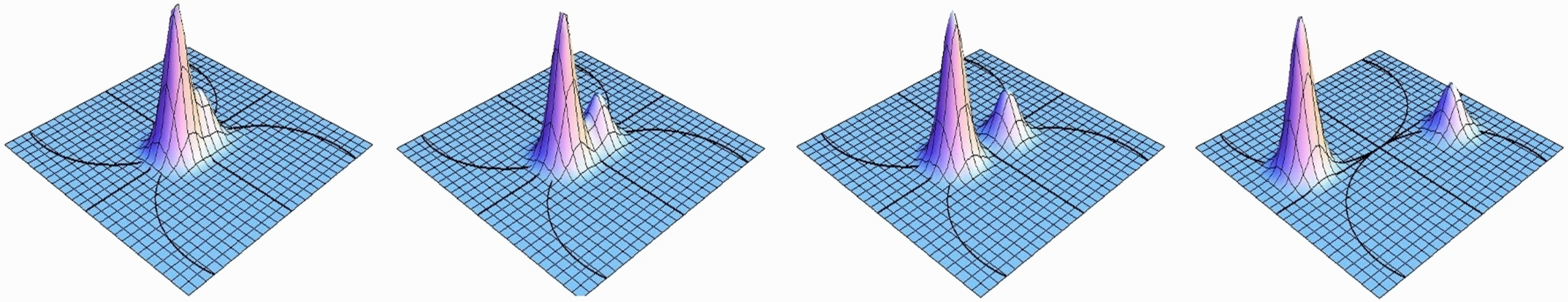
when $h_t^\gamma(\Omega)$ do not significantly overlap

it is

$$\rho_\Omega(t) = \sum_\gamma |c_\gamma|^2 \int_{S_\gamma} d\mu(\Omega)\, h_t^\gamma(\Omega) |\phi(\Omega,t)\rangle\langle\phi(\Omega,t)| = \sum_\gamma |c_\gamma|^2 |\gamma\rangle\langle\gamma|$$

DECOHERENCE has OCCURRED

DECOHERENCE



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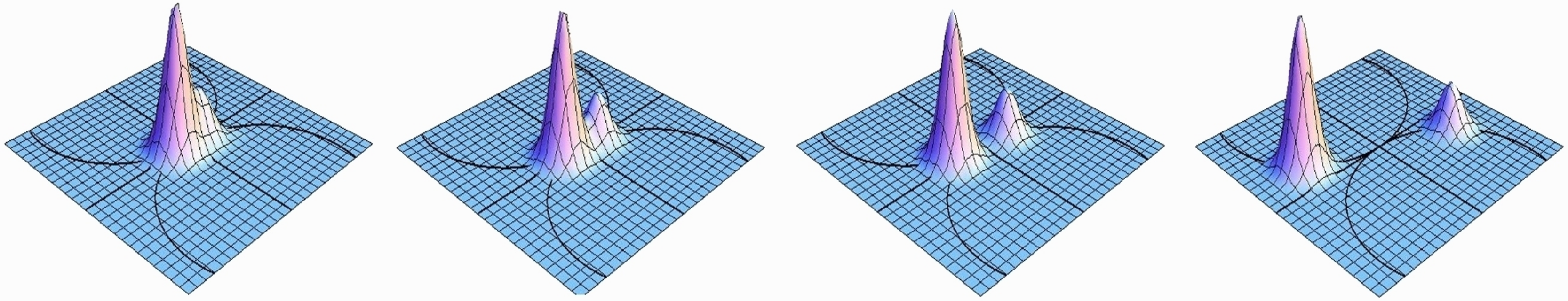
DECOHERENCE has OCCURRED

$$S_\gamma \subset G/F \leftrightarrow \gamma$$

DECOHERENCE TIME



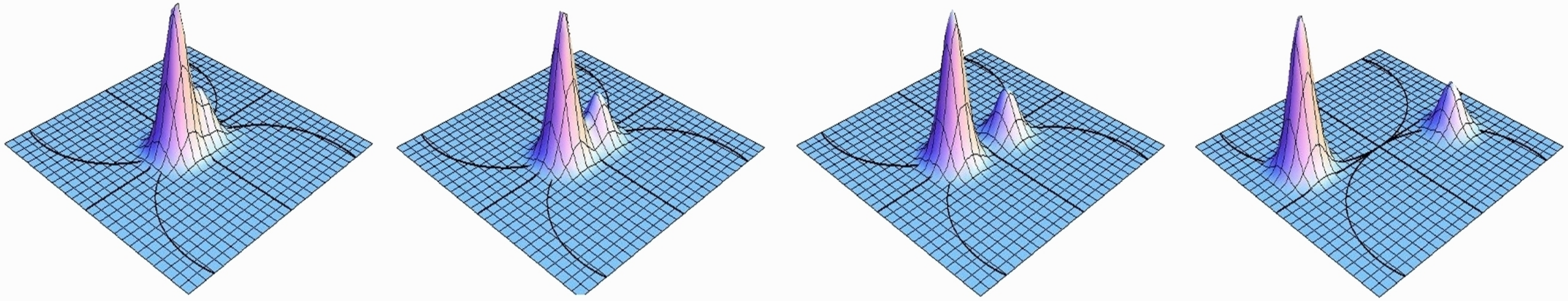
$$\chi_t^2(\Omega) = \sum_\gamma |c_\gamma|^2 \, h_t^\gamma(\Omega)$$

define $\tilde{\tau}_D$ by requiring

DISTANCE > 2 WIDTH

$$d_{\gamma\gamma'}(t) > 2\sigma_\gamma(t)$$

DECOHERENCE TIME



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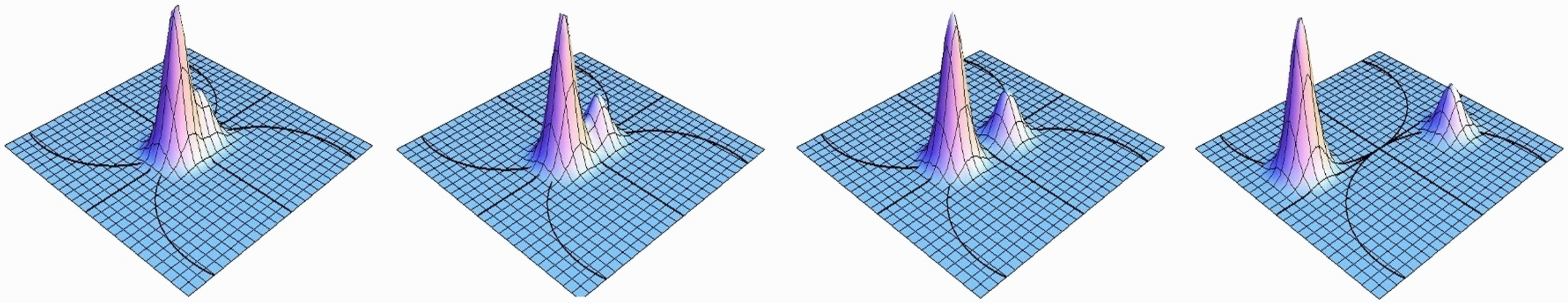

$$\chi^2_t(\Omega) = \sum_\gamma |c_\gamma|^2 \, h^\gamma_t(\Omega)$$

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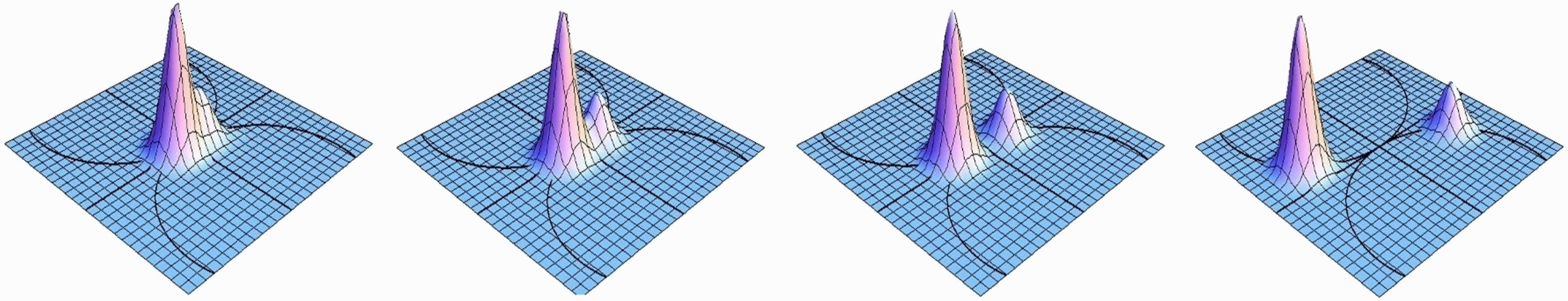
$$d_{G/F}(R^\gamma_t, R^{\gamma'}_t) = d_{\gamma\gamma'}(t) > 2\sigma_\gamma(t) = \left[\int_{G/F} d\mu(\Omega)\, d^2_{G/F}(\Omega, R^\gamma_t)\, h^\gamma_t(\Omega)\right]^{1/2}$$

$$t > \tau_D$$

τ_D is A DECOHERENCE TIME

DECOHERENCE TIME



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$$t > \tau_D$$

τ_D is A DECOHERENCE TIME

$$\frac{1}{\nu} \arccos\left(1 - \frac{\nu^2}{q^2}\right)$$

QUBIT AND SPIN - J

- a magnetic environment -

$$\hat{H}_{SE} = \omega \hat{J}_0 + g\hat{\sigma}^z \otimes (\hat{J}_+ + \hat{J}_-)$$

$$[\hat{J}_0, \hat{J}_\pm] = \pm \hat{J}_\pm , \quad [\hat{J}_+, \hat{J}_-] = 2\hat{J}_0 \quad ; \quad |\Omega\rangle = e^{\Omega \hat{J}_- - \Omega^* \hat{J}_+} |R\rangle , \quad \Omega = \frac{\theta}{2} e^{i\varphi} \rightarrow \vec{n}(\Omega)$$

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take

$|\psi(0)\rangle = (c_+|+\rangle + c_-|-\rangle) \otimes |J\rangle$, $|J\rangle = |R\rangle$ / $\hat{J}_0 |J\rangle = J|J\rangle$

get

$\theta_\pm(t)$, $\varphi_\pm(t) \longrightarrow \vec{n}(\theta_\pm(t), \varphi_\pm(t)) \equiv \vec{n}(\Omega_t^\pm)$

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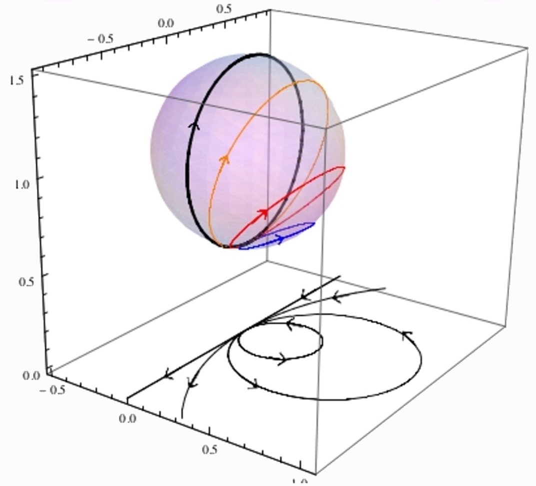

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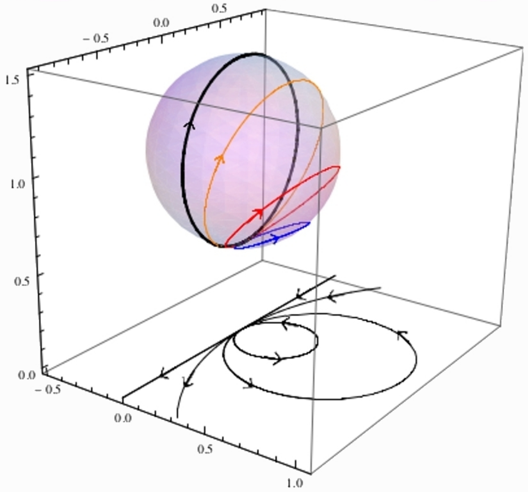
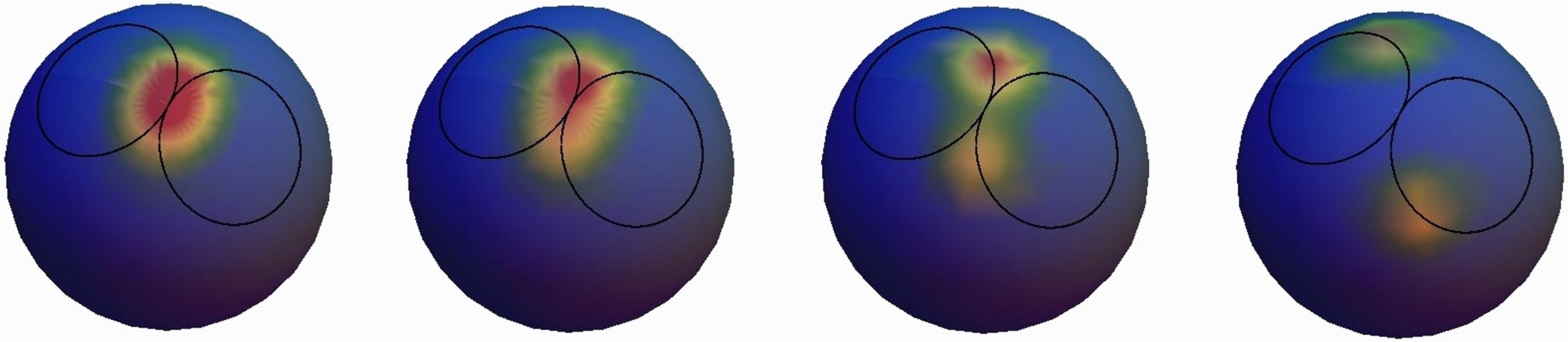


$$\chi^2_\pm(\Omega) = \frac{2J+1}{4\pi} |c_+|^2 \left(\frac{1 + \vec{n}(\Omega) \cdot \vec{n}(\Omega^*_t)}{2} \right)^{2J} + [+ \to -]$$

DECOHERENCE : MAGNETIC ENVIRONMENT

$$\hat{H}_{SE} = \omega \hat{J}_0 + g \hat{\sigma}^z \otimes (\hat{J}_+ + \hat{J}_-)$$



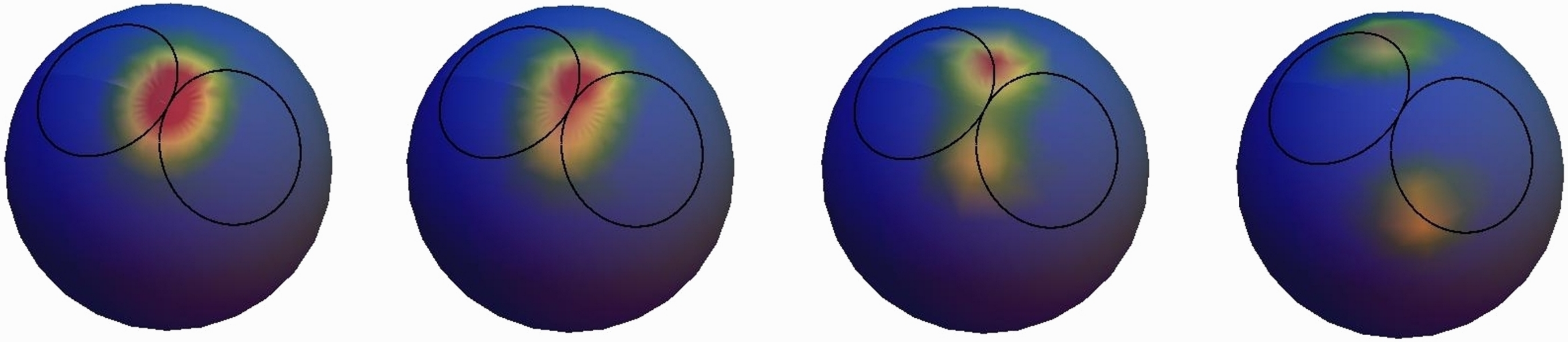
$$\chi^2_t(\Omega) = \frac{2J+1}{4\pi} |c_+|^2 \left(\frac{1 + \vec{n}(\Omega) \cdot \vec{n}(\Omega^*_t)}{2} \right)^{2J} + [+ \to -]$$

$g = 1 \qquad \omega = 4 \qquad J = 25$

$|\psi(0)\rangle = (c_+|+\rangle + c_-|-\rangle) \otimes |J\rangle \qquad |c_+|^2 = 2/5 \qquad |c_-|^2 = 3/5$

DECOHERENCE : MAGNETIC ENVIRONMENT

$$\hat{H}_{SE} = \omega \hat{J}_0 + g \hat{\sigma}^z \otimes (\hat{J}_+ + \hat{J}_-)$$



$$\chi_t^2(\Omega) = \frac{2J+1}{4\pi} |c_+|^2 \left(\frac{1 + \vec{n}(\Omega) \cdot \vec{n}(\Omega^*_t)}{2} \right)^{2J} + [+ \rightarrow -]$$

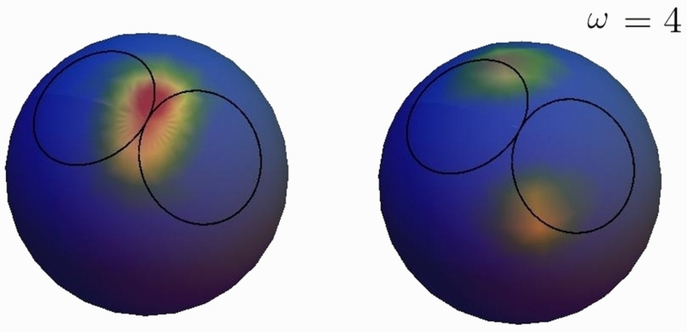
$$\min \left\{ 1, \frac{d_{\gamma\gamma'}(t)}{2\sigma} \right\}$$

$g = 1$ $\omega = 4$ $J = 25$

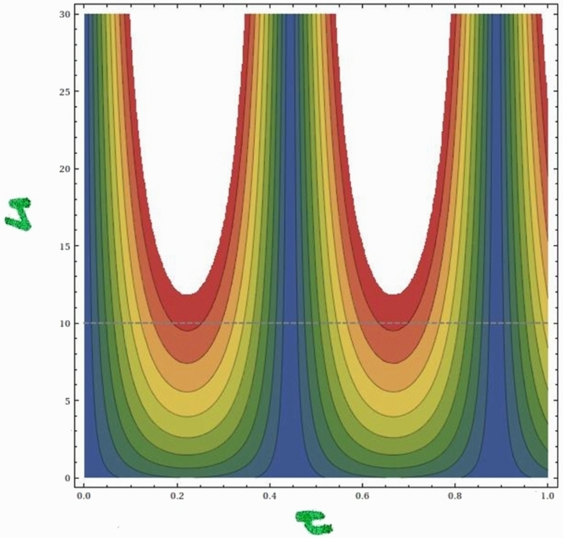
$|\psi(0)\rangle = (c_+|+\rangle + c_-|-\rangle) \otimes |\Xi\rangle$ $|c_+|^2 = 2/5$ $|c_-|^2 = 3/5$

DECOHERENCE TIME : DIFFERENT FIELDS

$$\hat{H}_{SE} = \omega \hat{J}_0 + g\hat{\sigma}^z \otimes (\hat{J}_+ + \hat{J}_-)$$



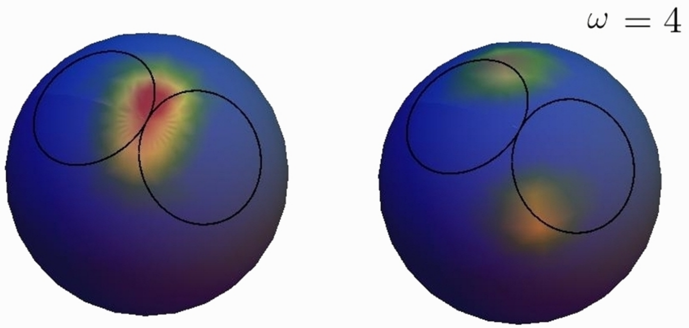
$\chi^2_t(\Omega)$

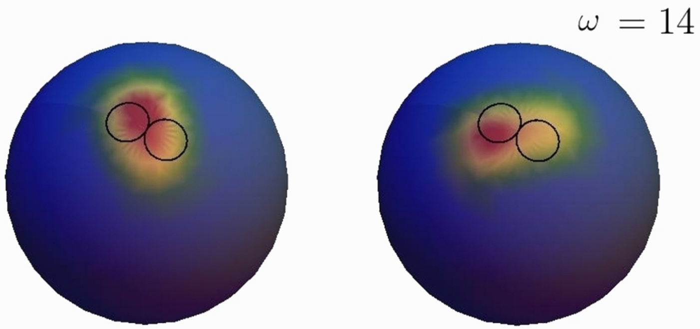


$$\min\left\{1, \frac{d_{\gamma\gamma'}(\tau)}{2\sigma}\right\}$$

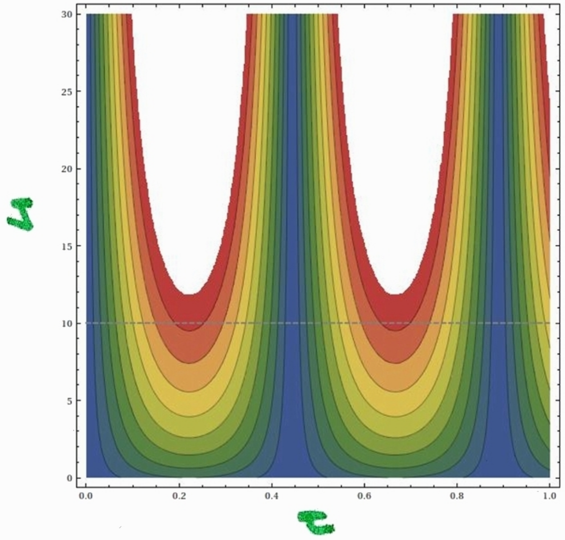
DECOHERENCE TIME : DIFFERENT FIELDS

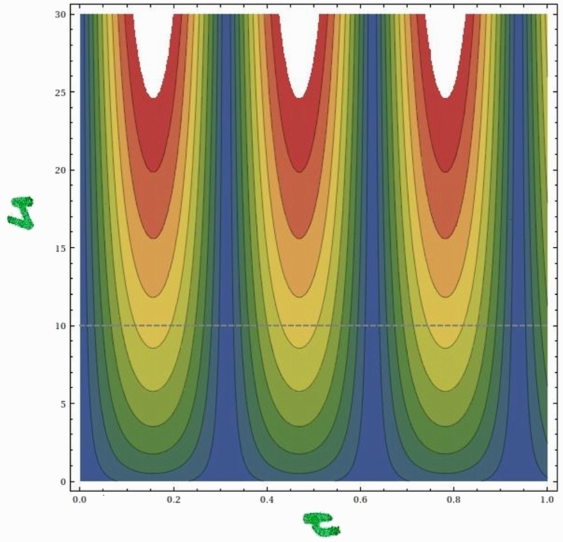
$$\hat{H}_{SE} = \omega \hat{J}_0 + g\hat{\sigma}^z \otimes (\hat{J}_+ + \hat{J}_-)$$





$\chi^2_t(\Omega)$

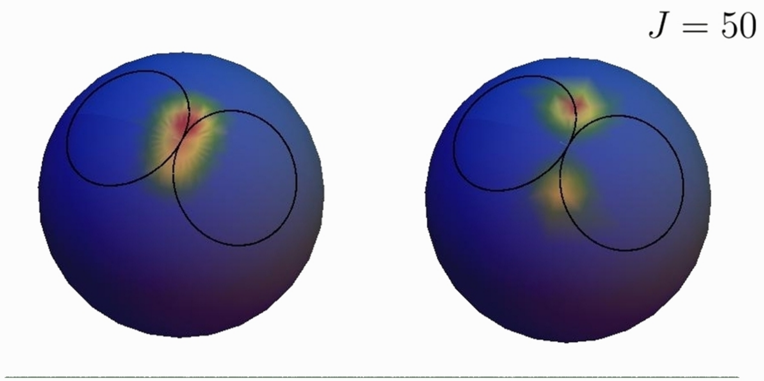




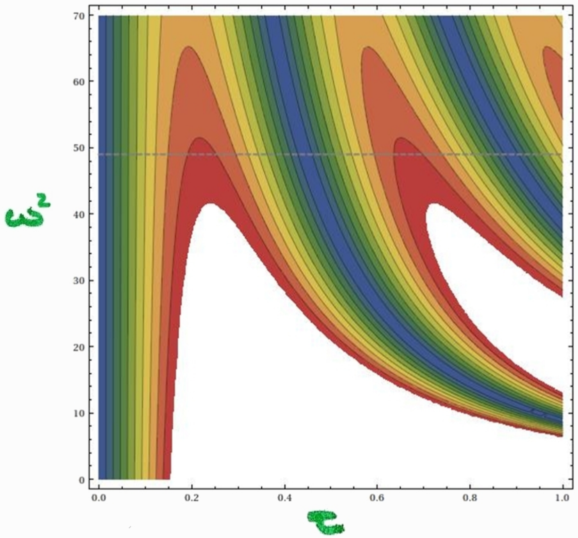
$$\min\left\{1, \frac{d_{\gamma\gamma'}(\tau)}{2\sigma}\right\}$$

DECOHERENCE TIME : DIFFERENT J

$$\hat{H}_{SE} = \omega \hat{J}_0 + g\hat{\sigma}^z \otimes (\hat{J}_+ + \hat{J}_-)$$



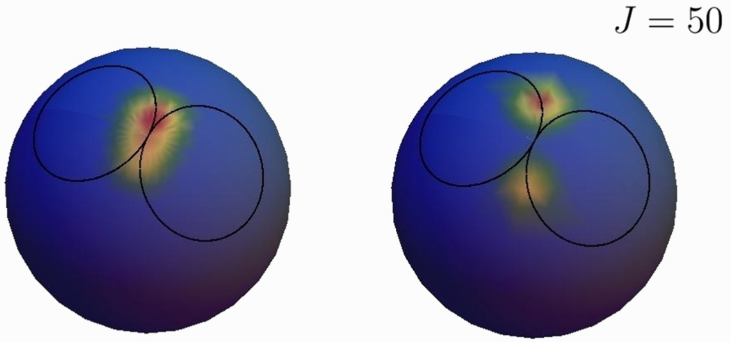
$\chi^2_t(\Omega)$

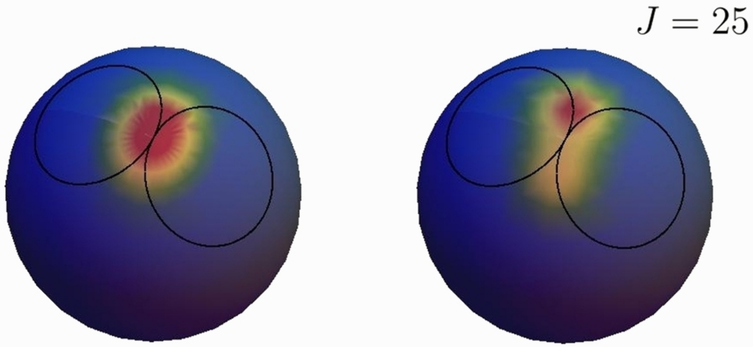


$$\min\left\{1, \frac{d_{\gamma\gamma'}(\tau)}{2\sigma}\right\}$$

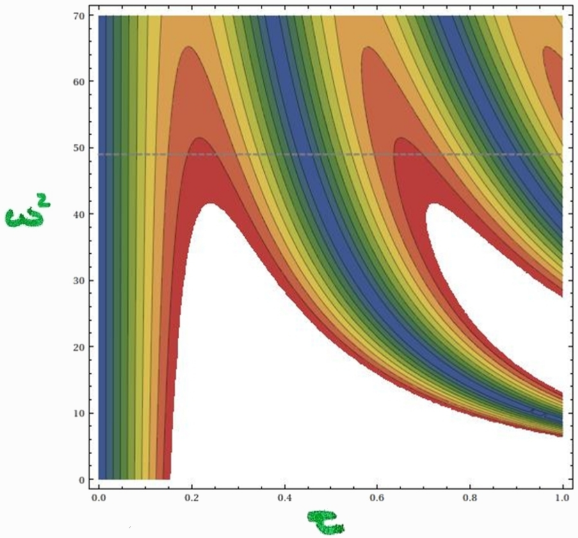
DECOHERENCE TIME : DIFFERENT J

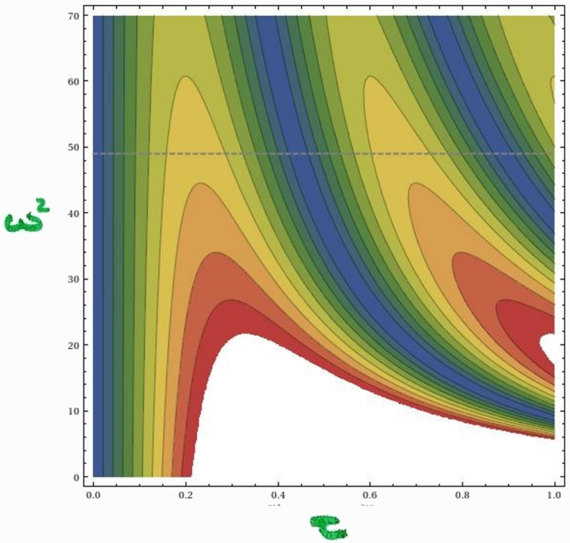
$$\hat{H}_{SE} = \omega \hat{J}_0 + g\hat{\sigma}^z \otimes (\hat{J}_+ + \hat{J}_-)$$





$\chi^2_t(\Omega)$





$$\min\left\{1, \frac{d_{\gamma\gamma'}(\tau)}{2\sigma}\right\}$$

CONCLUSIONS

- decoherence of the principal system ↔ dynamics of its environment
- decoherence time ↔ control the environment to get a long-lasting coherence
    - ★ high fields NATURE 476, 76 (2011)
    - ★ low J PRL 101, 47601 (2008)
    - ...

FURTHER DEVELOPMENTS

- specific : realistic environments with non-trivial spectral densities
- foundational : measurement process and Zeno effect

...

THANK YOU

VERY ESSENTIAL BIBLIOGRAPHY

M. Schlosshauer : DECOHERENCE AND THE QUANTUM TO CLASSICAL TRANSITION

Zhang, Feng, Gilmore : REV. MOD. PHYS. 62 , 867 (1990)

Zurek : REV. MOD. PHYS. 75, 715 (2003)

SOME REFERENCES

more coming soon on the arXiv

PRECS : PNAS 110 , 6748 (2013)

DYNAMICS with PRECS : OSID 20, 03 (2013)

THIS WORK : P. Liuzzo-Scorpo MASTER THESIS (2014), on the web: ask for link