Non-abelian physics between one and two dimensions

Ady Stern (Weizmann)

Outline:

1. The context
2. Fractionalized Majorana modes and the associated ground state degeneracy

   with N. Lindner, E. Berg, G. Refael

   See also: Barkeshli and Qi

   Clarke, Alicea & Shtengel , Meng , Vaezi
3. Shrinking two dimensions into one, trying to keep non-abelian physics alive

   (with Y. Oreg, E. Sagi, E. Sela, BI Halperin)

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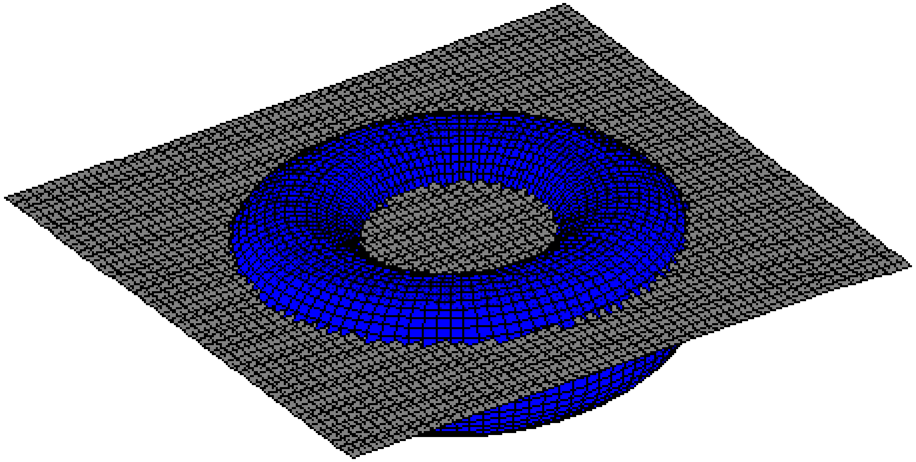
The context

- Non-abelian physics is the (unavoidable) consequence of ground state degeneracy that is topologically protected.
- Following ground state degeneracy – topological unitary transformations associated with braiding, topological quantum computation etc.

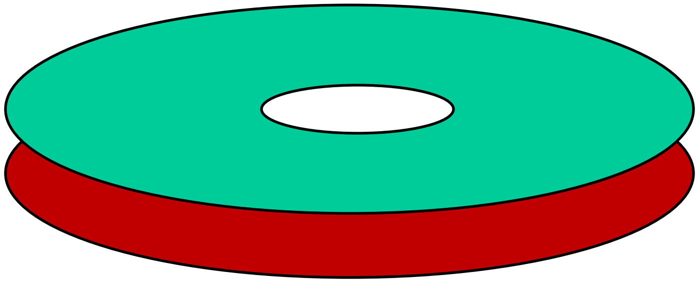
The origin of ground state degeneracy – topologically non-trivial geometries

Fractionalized states imply ground state degeneracy on a torus.

A torus may be obtained by stitching together two annuli.



A annulus-shaped bi-layer electron-hole system



Four gapless modes, two on the exterior, two on the interior. Stitching by mutually gapping each pair (exterior & interior)

Two distinct mechanisms for gapping – backscattering (+spinflip, through coupling to a ferro-magnet) and coupling to a super-conductor

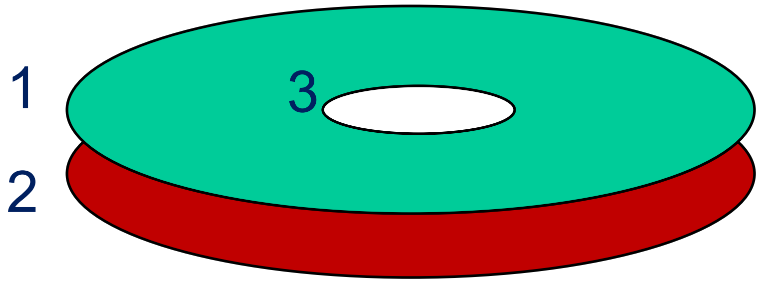
F & F – three-fold degeneracy, distinguished by the fractional dipole moment on each edge

F & S – no degeneracy

S & S – three-fold degeneracy, distinguished by the fractional total charge on each edge

The nature of the degenerate states (for example, $\nu=1/3$) :

Four edges, four charges – q_1, q_2, q_3, q_4



$$q_1 + q_3 = integer$$

$$q_2 + q_4 = integer$$

Super-conducting coupling makes $q_1 - q_2 = 0$ and $q_1 + q_2$ defined only modulo 2.

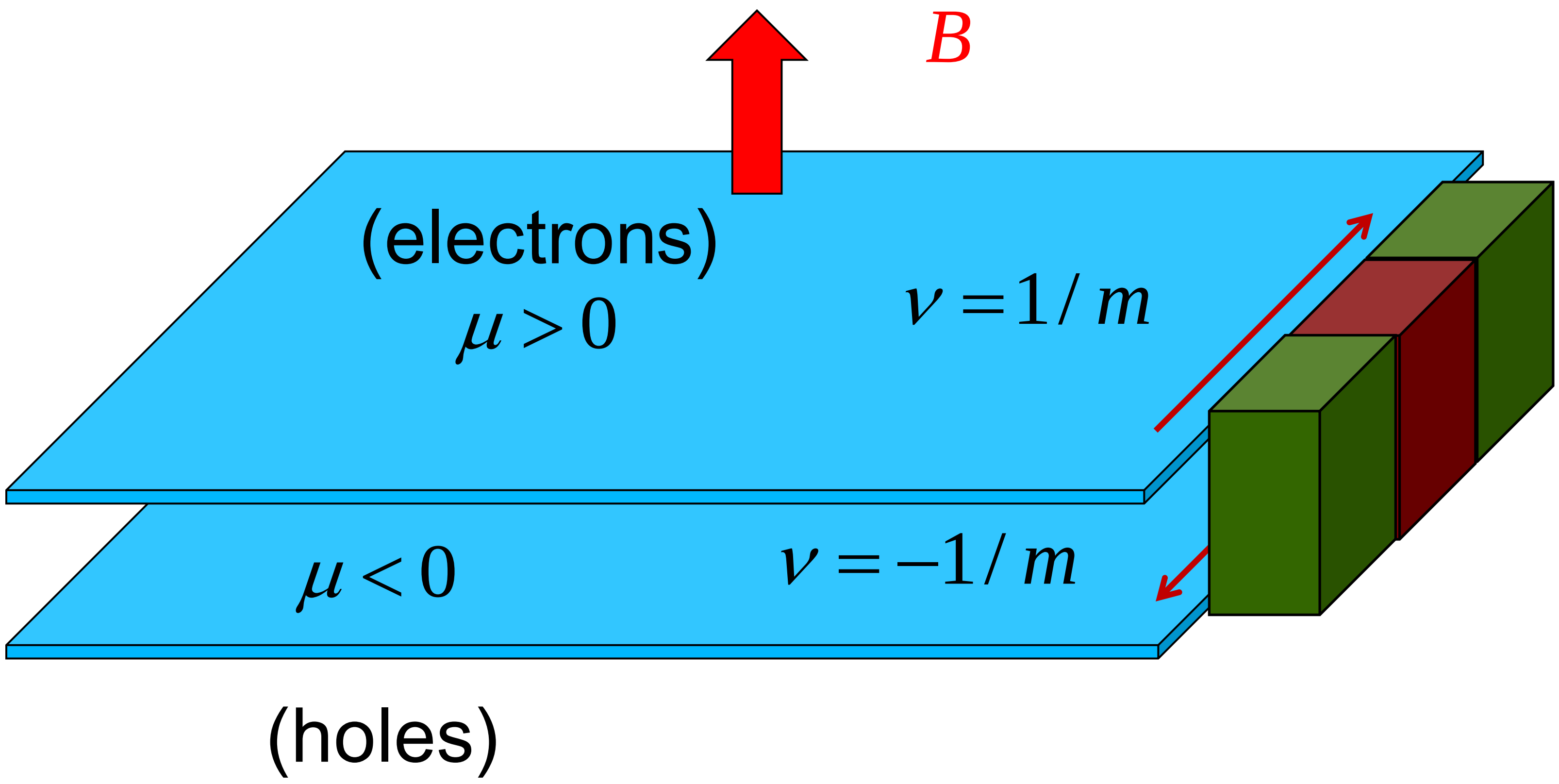
➡ The three degenerate states are characterized by the fractional charge of $q_1 + q_2$. Possible values are $0, \frac{2}{3}, \frac{4}{3}$.

Removal of degeneracy – hopping of 2/3 charge between edges

What if we mix mechanisms on the same edge?

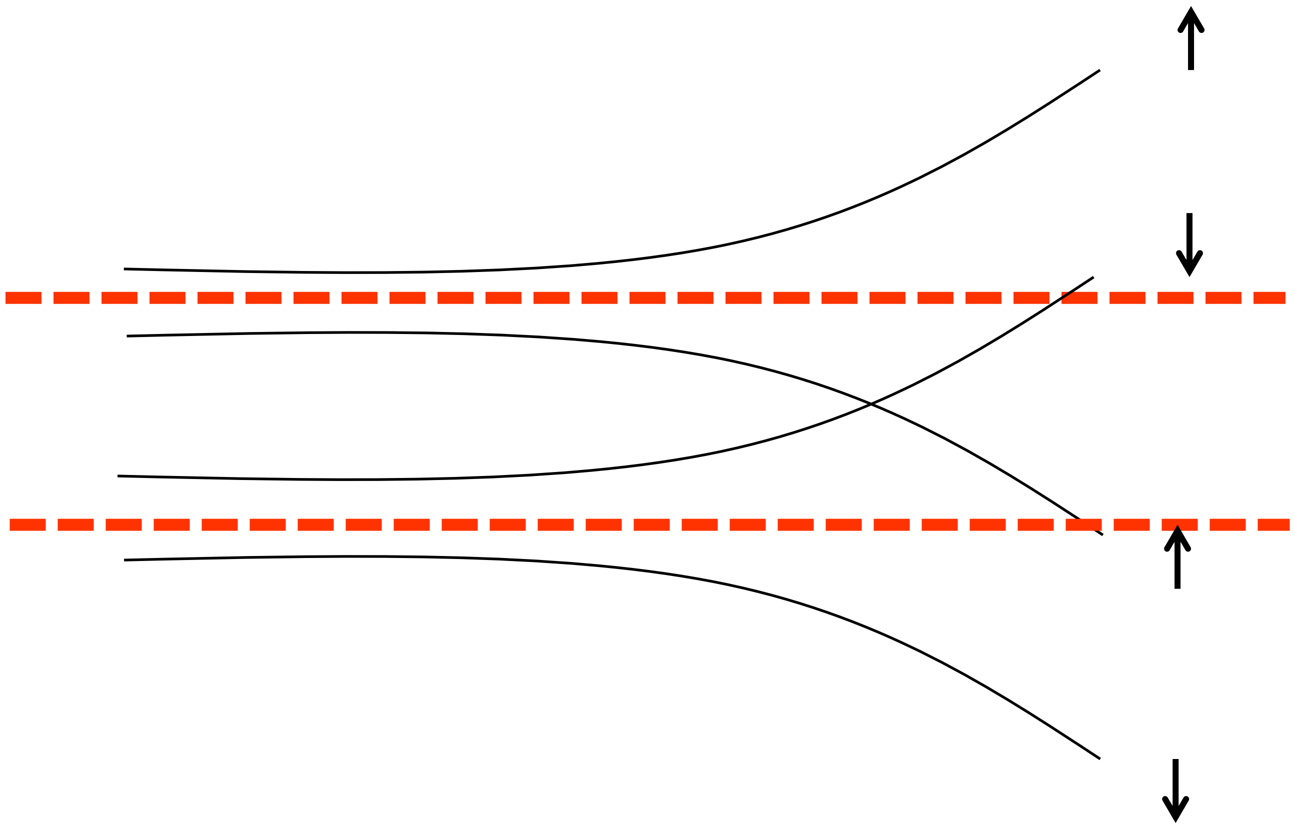
Topological defects on FQHE edge modes

Bi-layer graphene electron-hole system



(Superconductivity not unavoidable)

Electron-Hole symmetric bi-layer – a pair of counter-propagating edge channels of opposite spin direction



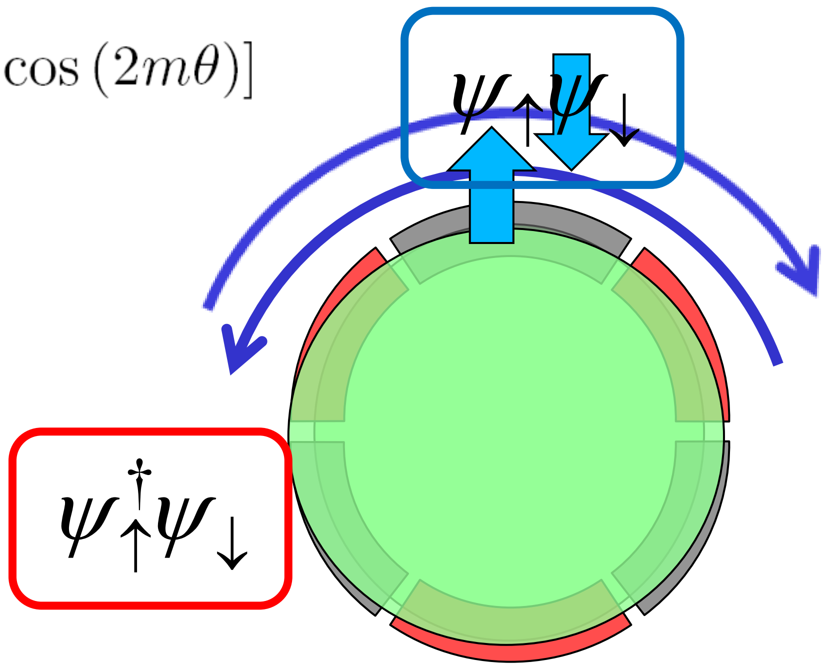
Fractional Majorana modes and the associated ground state degeneracy

Effective Edge Model

$$H = \frac{u}{2\pi\nu} \int dx \left[K(x) \left(\partial_x \phi\right)^2 + \frac{1}{K(x)} \left(\partial_x \theta\right)^2 \right]$$

$$- \int dx \left[g_S(x) \cos(2m\phi) + g_F(x) \cos(2m\theta) \right]$$

$$\left[\phi(x), \theta(x')\right] = \frac{i\pi}{m} \Theta(x' - x)$$



Density: $\rho(x) = \frac{1}{\pi} \partial_x \theta(x)$

Spin density: $\sigma(x) = \frac{1}{\pi} \partial_x \phi(x)$

Electron: $\psi_\pm \propto e^{im(\phi \pm \theta)}$ (charge 1, spin 1)

Laughlin q.p.: $\chi_\pm \propto e^{i(\phi \pm \theta)}$ (charge 1/m, spin 1/m)

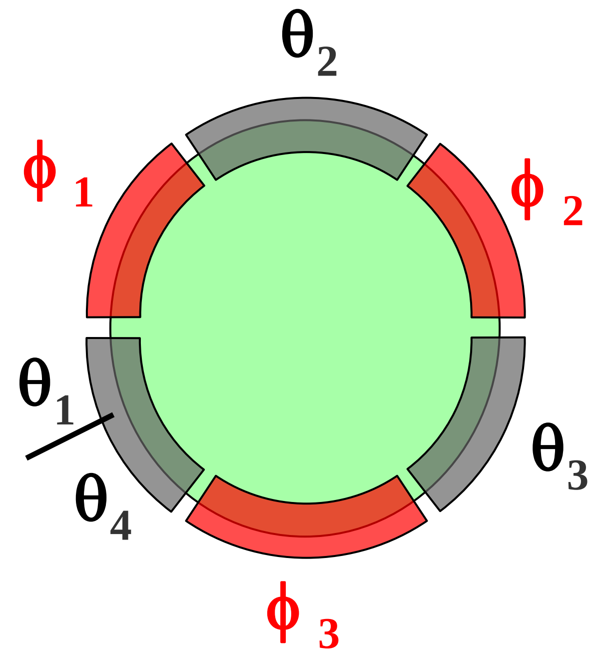
Effective Edge Model

Large cosine terms (strong coupling to SC/FM)

$$-\int dx \left[g_S(x) \cos(2m\phi) + g_F(x) \cos(2m\theta) \right]$$

θ pinned : $e^{i\pi Q_j} = e^{i\left(\theta_{j+1} - \theta_j\right)}$

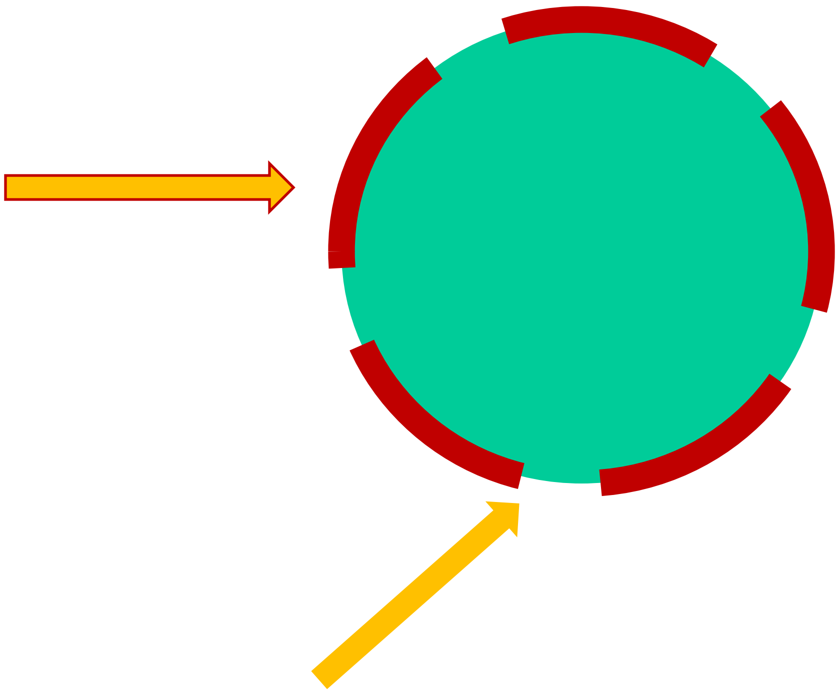
Ground states: $\left\langle e^{i\pi Q_j} \right\rangle = e^{i\pi q_j / m}$



Degeneracy between states of different values of this operator

Physical picture:

Super-conducting segments, where charge is quantized in units of *e/m,* spin is zero, and the ground state energy is insensitive to the charge. Six possible value of the charge. Degeneracy 6^{N-1}.



Incompressible insulators, where charge is zero, spin is quantized in units of 1/m, and the ground state energy is insensitive to the spin.

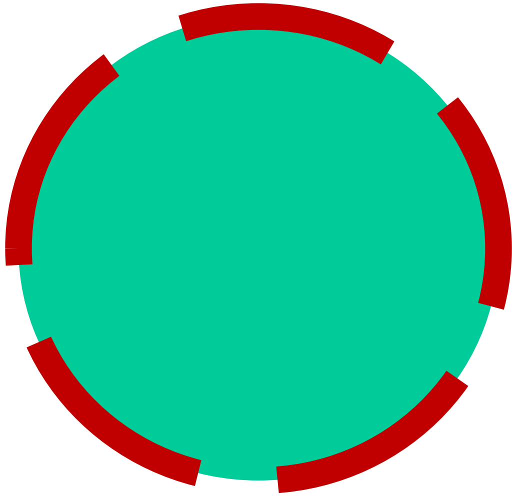
Charges and spins cannot be simultaneously defined

- The degeneracy is topological, i.e., it cannot be lifted by a local perturbation. Measuring operators charge on a super-conducting quantum dot is a non-local operation.

- Coupling different ground states requires tunneling of fractional quasi-particles between different super-conducting segments.

- The unitary transformation that corresponds to interchanging two nearest-neighbor interfaces

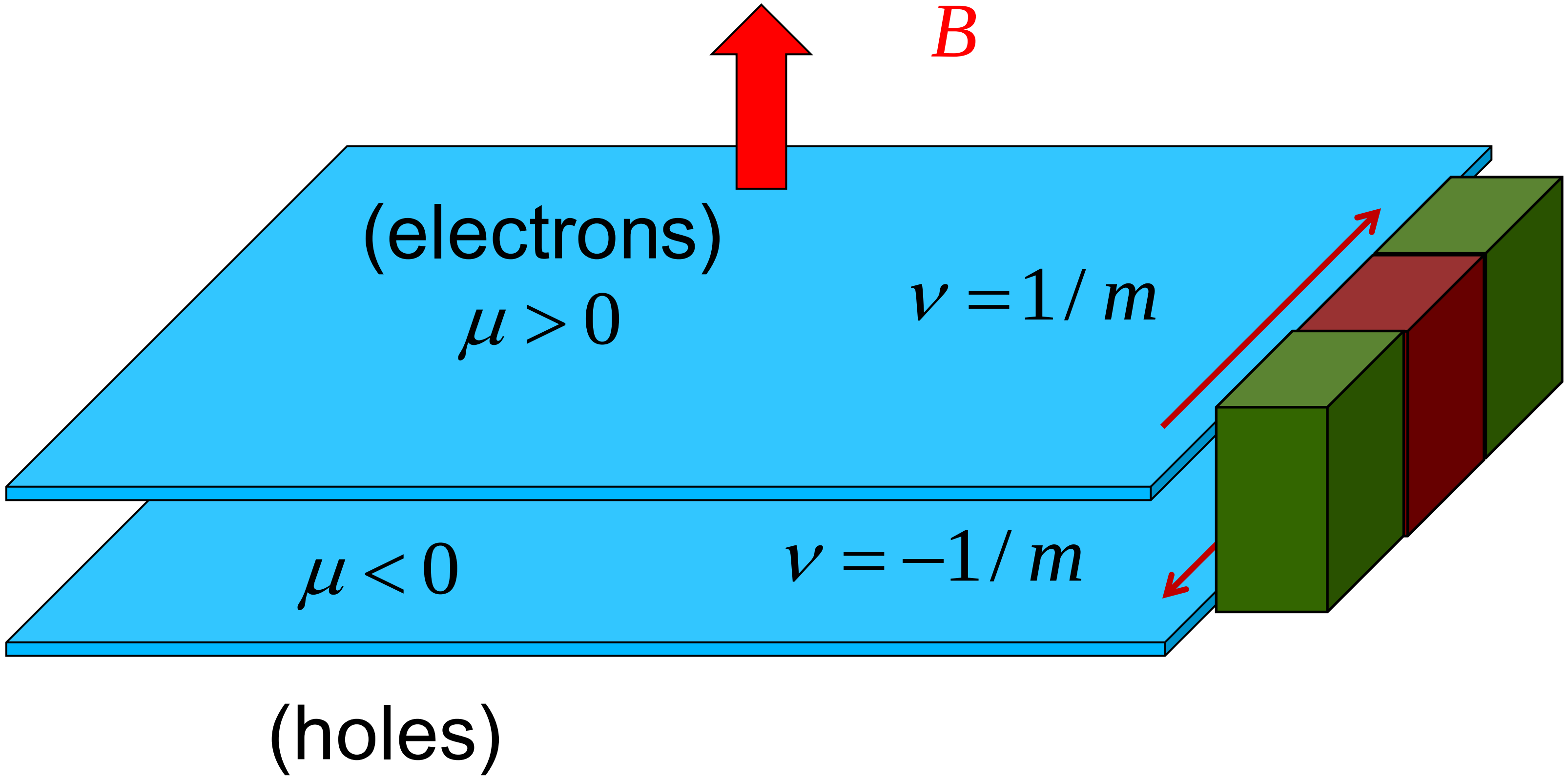
$$U = exp\left(i\pi \frac{m}{2} Q^2\right)$$



such that U^{2m} is the parity, and U^{4m} is unity

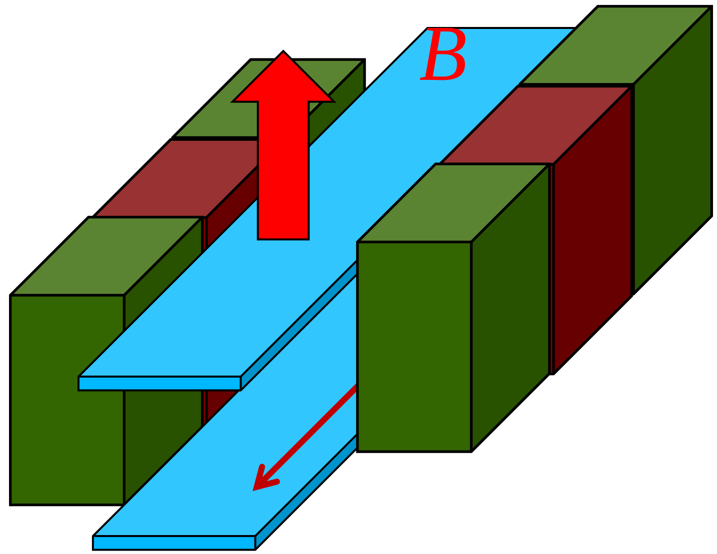
Richer than Majorana fermions

Going down in dimensionality

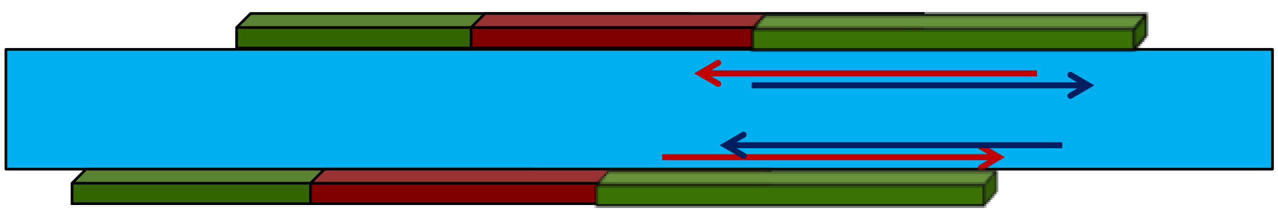


$$\nu = 1/m$$

$$\nu = -1/m$$

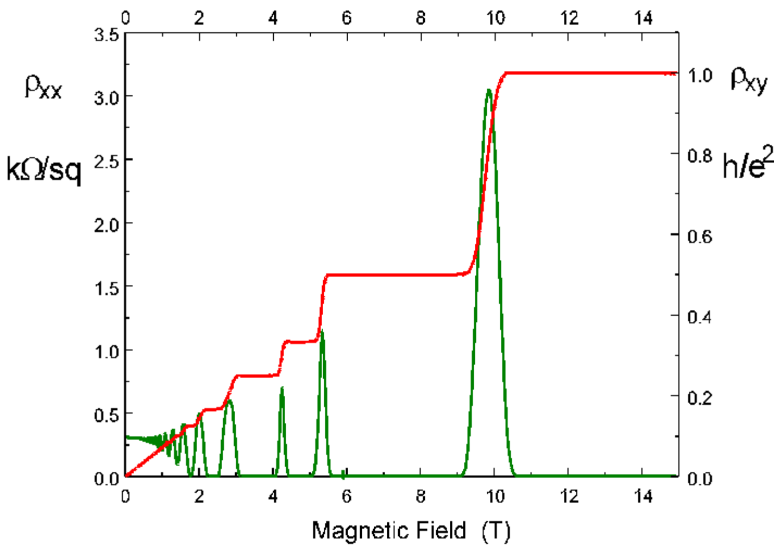


Top view:

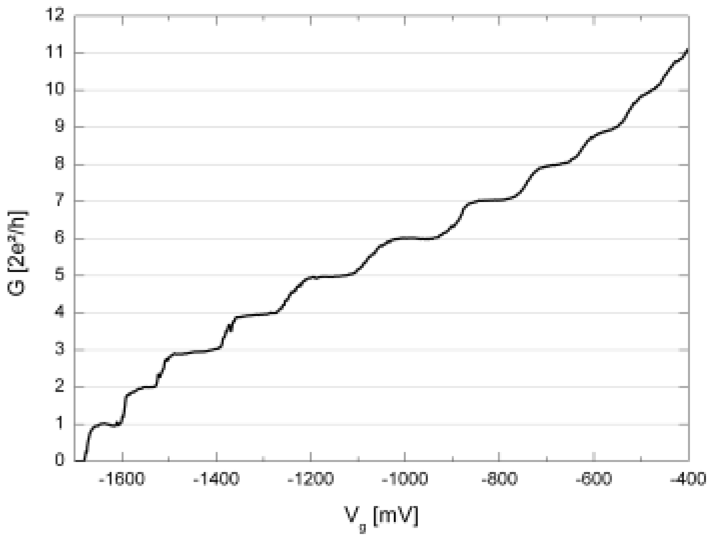


Quantization of the conductance in…

The quantum Hall effect

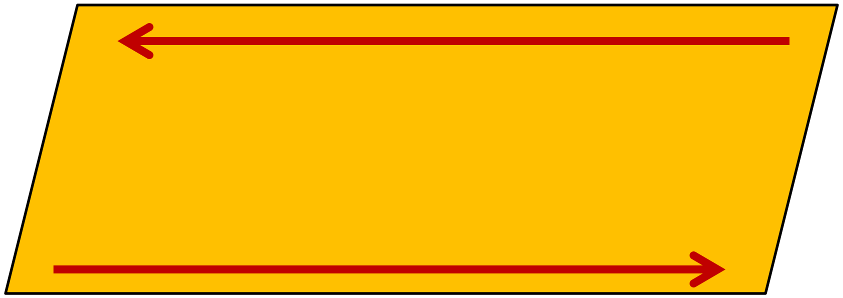


Short 1D constrictions



(Rossler et al)

Protected by distance



Protected by cleanliness and adiabaticity

One dimensional limit

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Fractional Quantum Hall Effect in an Array of Quantum Wires

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+ Teo&Kane (2012)

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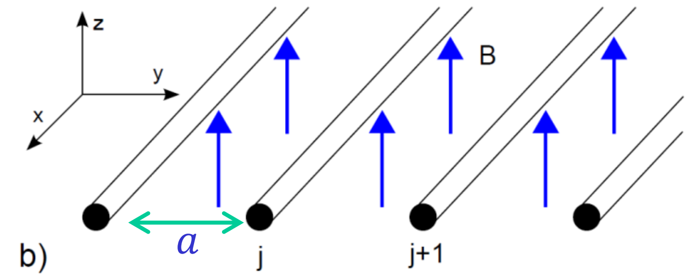
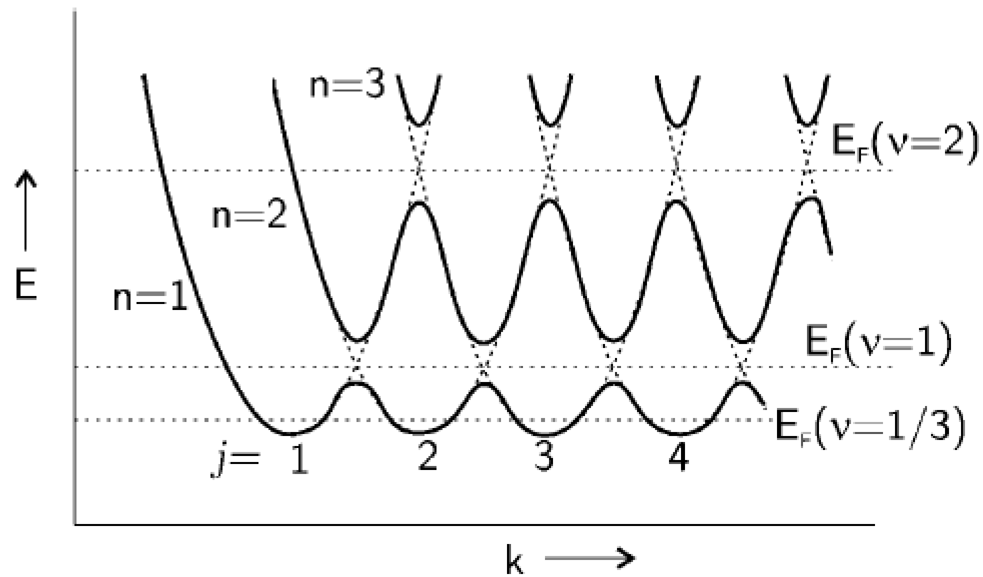
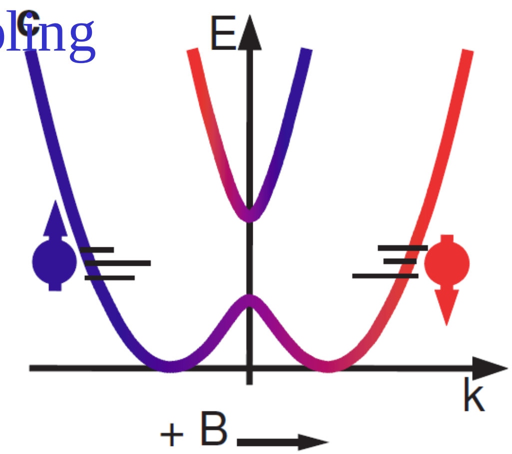
We demonstrate the emergence of the quantum Hall (QH) hierarchy in a 2D model of coupled quantum wires in a perpendicular magnetic field. At commensurate values of the magnetic field, the system can develop instabilities to appropriate interwire electron hopping processes that drive the system into a variety of QH states. Some of the QH states are not included in the Haldane-Halperin hierarchy. In

$$E_j(k) = \frac{\hbar^2}{2m}(k - bj)^2 \qquad b = eaB/\hbar c$$

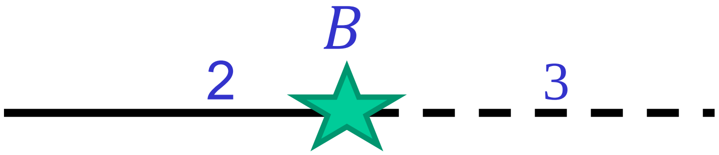




ν=1 QHE in a wire with SO coupling

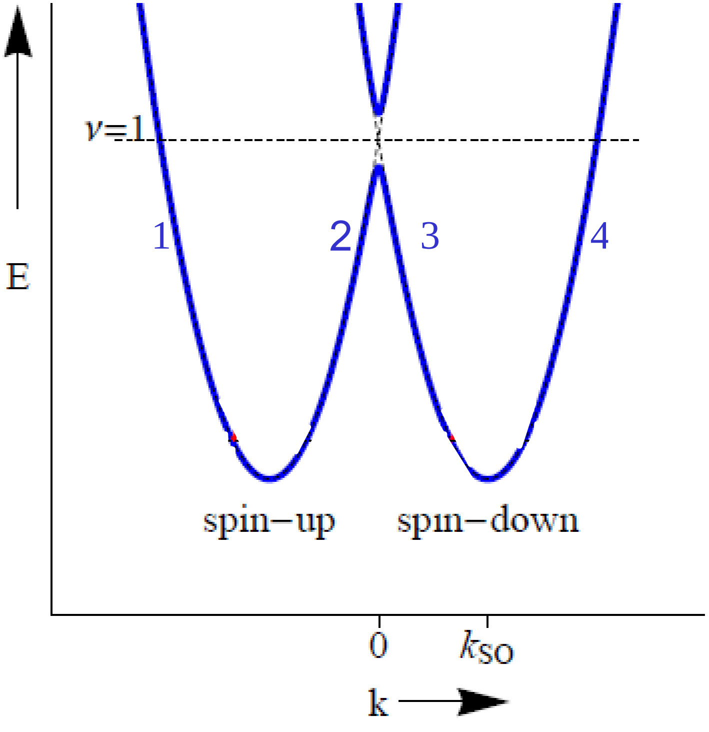


Interactions



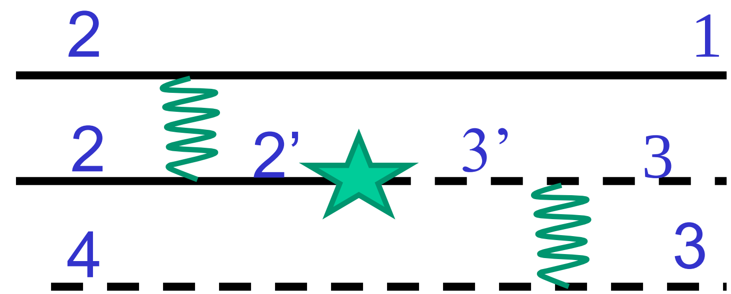
Magnetic field:

$$\psi_2 \psi_3^\dagger$$



Interactions

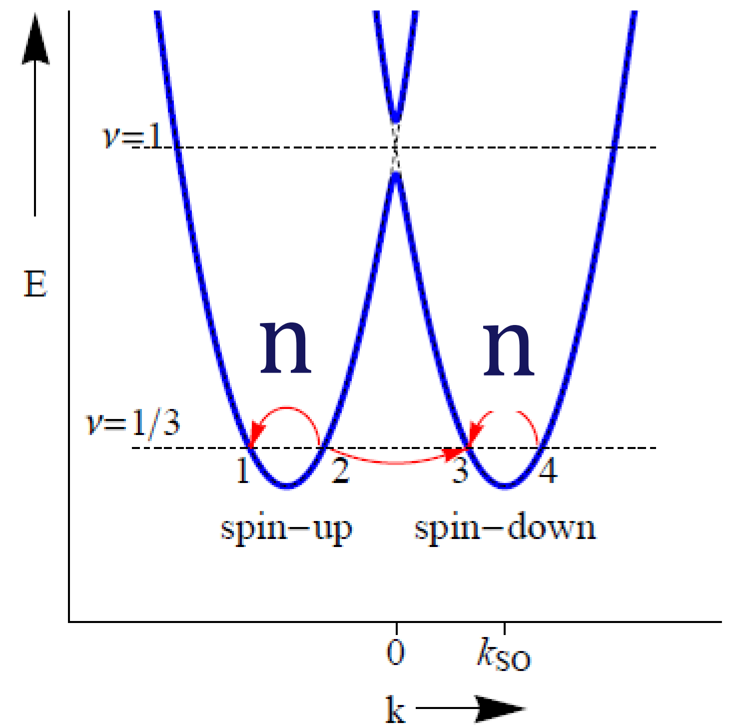
$$g_B \propto B U_{2k_F}^2$$



$$\mathcal{O}^B = g_B \left(\psi_1^\dagger \psi_2\right) \;\; \psi_2 \psi_3^\dagger \left(\psi_3^\dagger \psi_4\right)$$

conserves momentum at

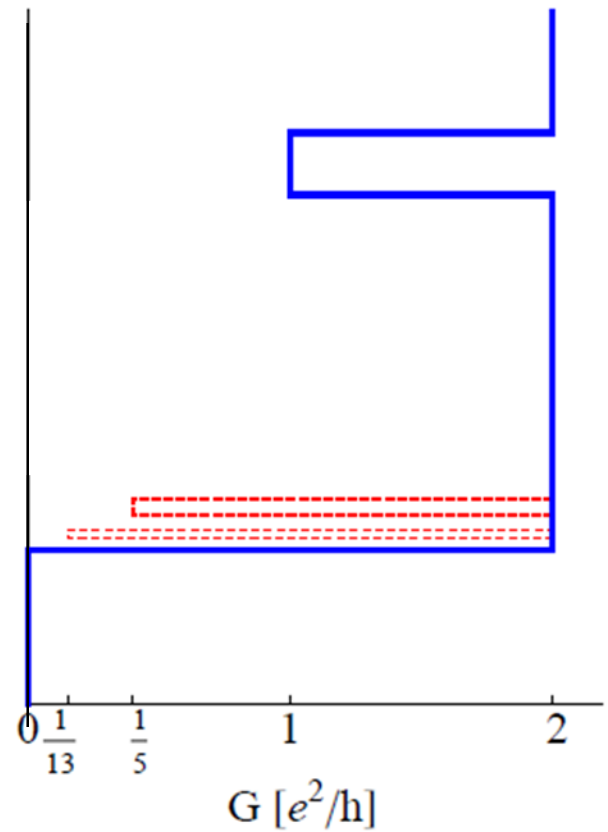
$$\nu = \frac{k_F}{k_{SO}} = \frac{1}{2n+1}$$



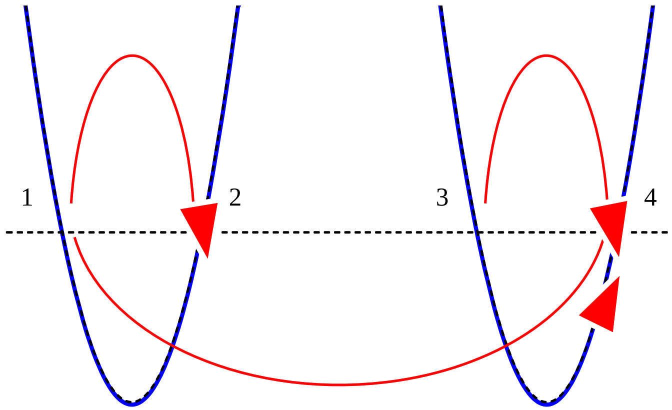
what is the strong coupling phase of g_B?

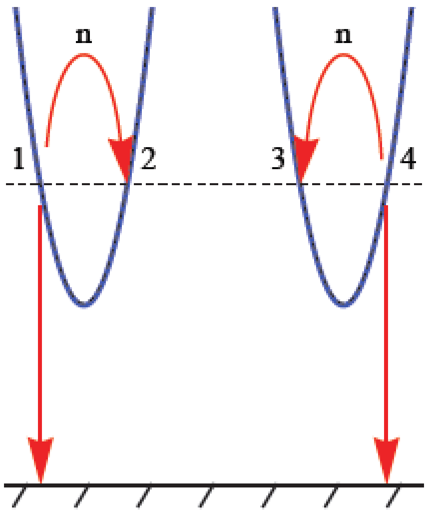
bosonize: $g_B \cos\{\phi_3 - \phi_2 + n[(\phi_3 - \phi_4) - (\phi_2 - \phi_1)] + kx\}$

- One cosine term gaps two of the four modes, so the system is still gapless
- The 2-terminal conductance is $e^2/5h$, converging to the expected $e^2/3h$ when the number of wires is around 5.
- This value is disorder sensitive
- Next – gapping the remaining gapless modes. We need two gapping mechanisms, by a super-conductor and by a ferromagnet, that commute with the g_B term.



Gapping the remaining modes – fractional Majorana modes





Proximity coupling to super-conductor

staggered field $B_{stag}(x) \propto Cos(4\, k_{SO}\, x)$

Translational symmetry is broken. The six degenerate ground states per super-conducting island are two for parity (non-local) and three that differ by a translation (locally distinguishable)

Summary:

1. Non-abelian anyons on the edges of abelian quantum (spin) Hall states.
2. Ground state degeneracy richer than that of Majorana fermions.
3. Similar states in one dimension, without the topological protection from disorder.
4. Topological manipulations that mimic exchanges