Exact results on the out-of-equilibrium Kondo model

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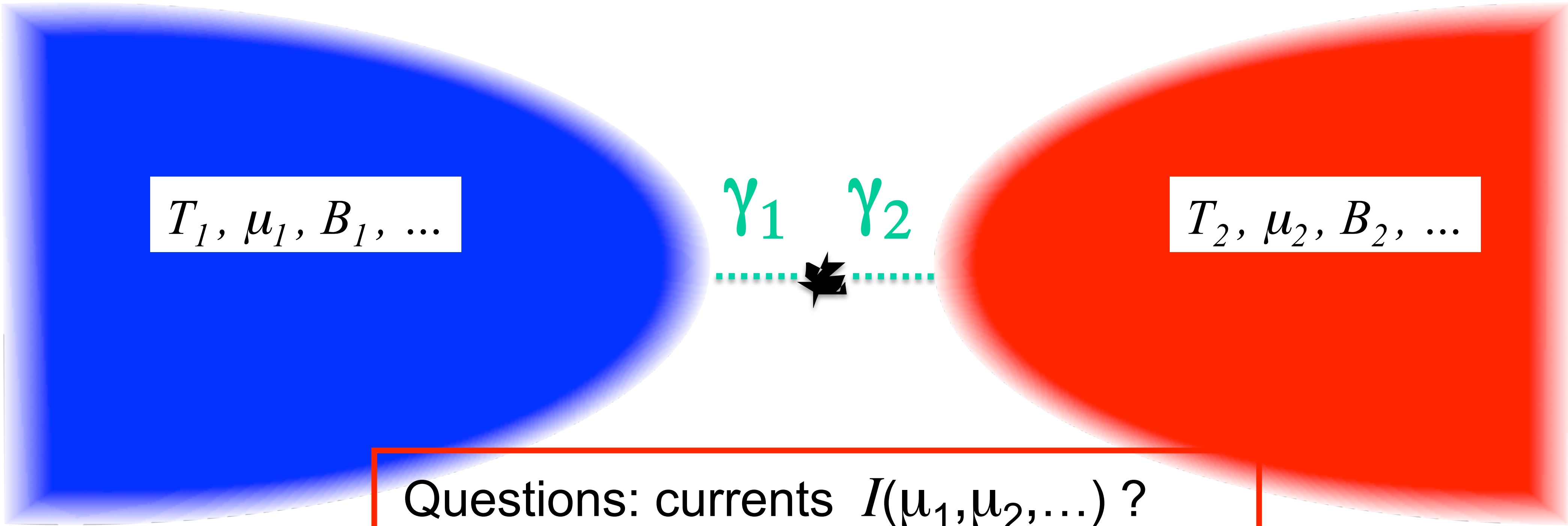
Impurity out-of-equilibrium

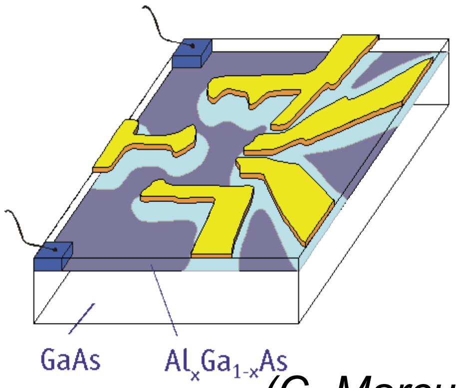
- Several baths (macroscopic, at equilibrium)
- Out-of-equilibrium forcing
- Flow (of charge, spin, energy, …) through impurity

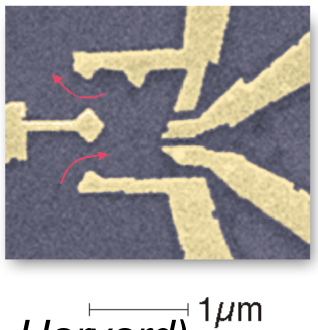


Questions: currents $I(\mu_1,\mu_2,\ldots)$?
fluctuations $\Delta I(\mu_1,\mu_2,\ldots)$?
state of the system ?

« Quantum impurities »

- Quantum dots:
  - ➔ 2D electron gas

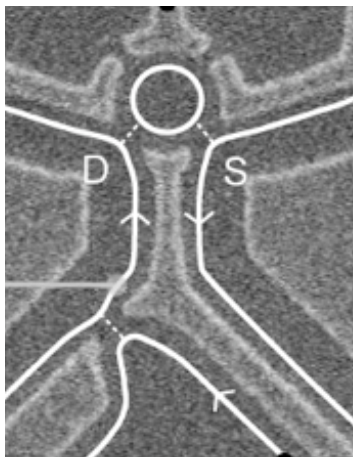




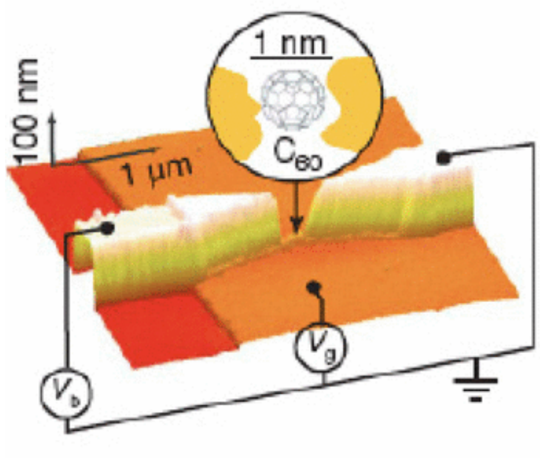
(C. Marcus, Harvard)

  - ➔ Quantum Hall edge states

(F. Pierre, LPN)

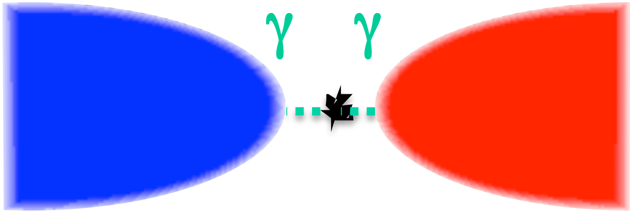


- Molecules: metallic electrodes
  - ➔ break junctions
  - ➔ electromigration



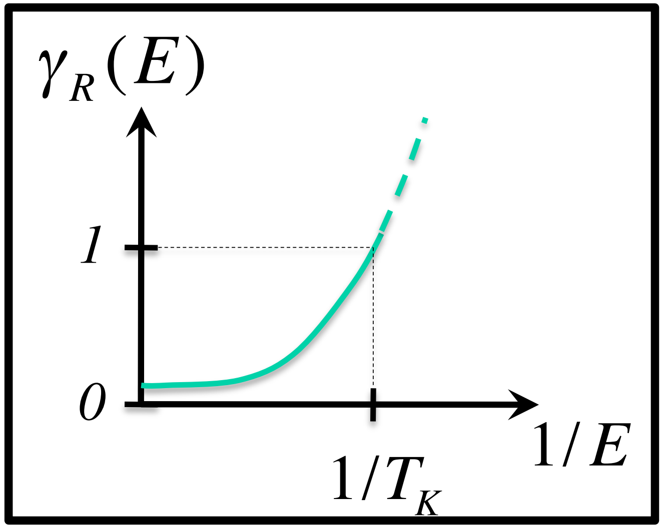
(W. Wernsdorfer, Institut Néel)

Weak - Strong Coupling

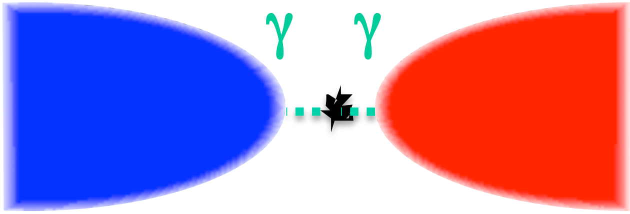


$$H = H_0 + \gamma H_K$$

baths impurity/bath coupling



Weak - Strong Coupling



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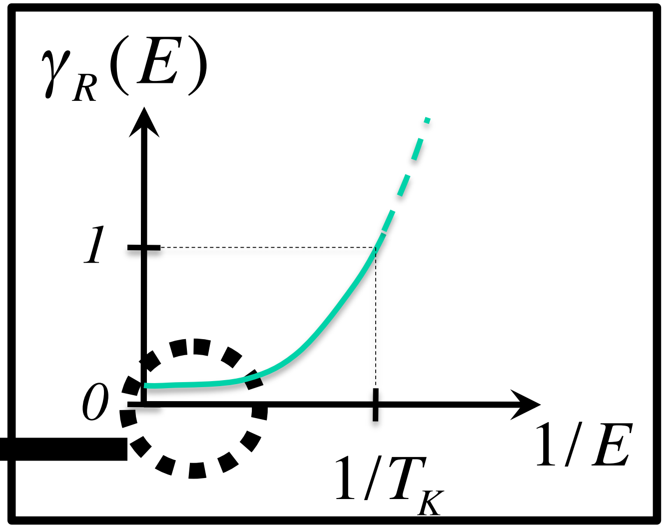
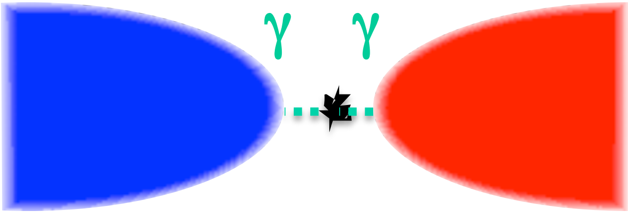


Weak coupling limit, $E >> T_K$

Weak - Strong Coupling



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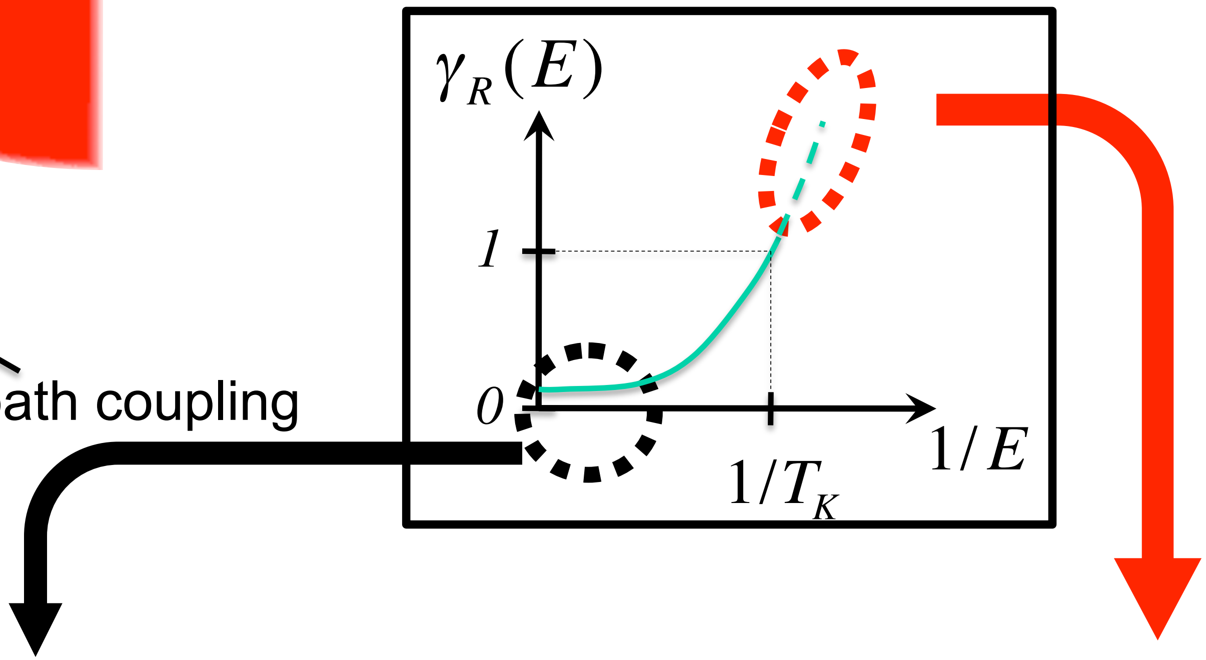
baths   impurity/bath coupling



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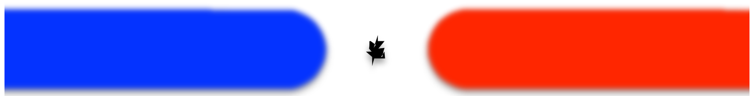
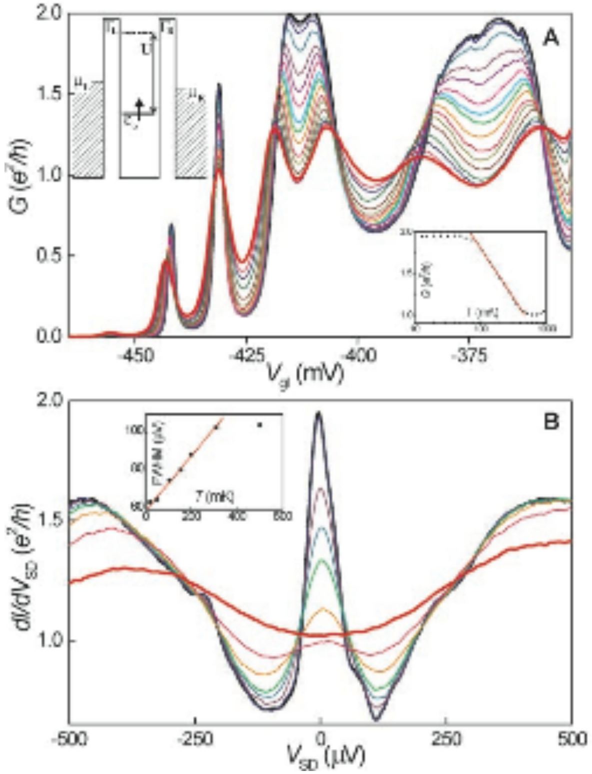


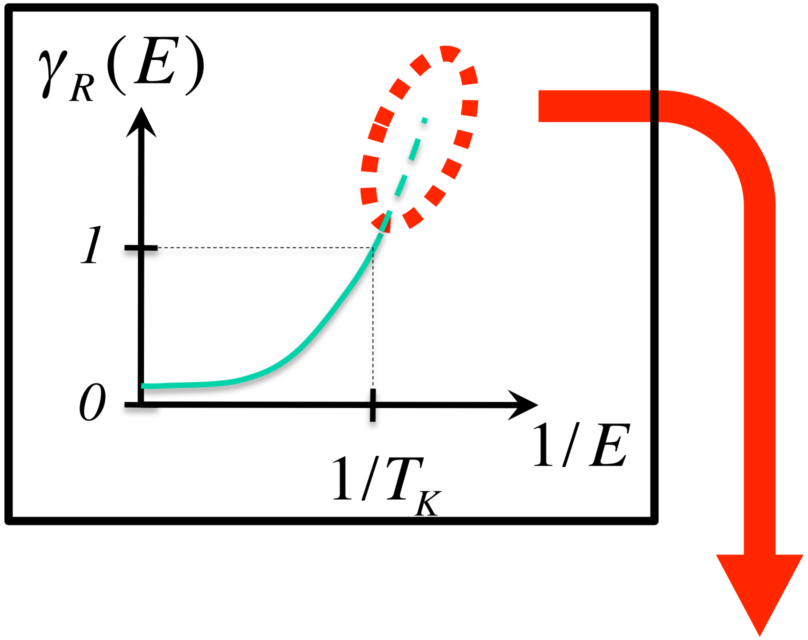
Strong coupling limit, $E << T_K$

At low energy :
strong coupling regime
“Physics is non perturbative”

Weak - Strong Coupling



« Kondo resonance is a strong coupling phenomenon »



Strong coupling limit, $E << T_K$

At low energy :
strong coupling regime
“Physics is non perturbative”

‘Standard’ perturbation theory

- Keldysh method:
  - allows for a formal expression of the out-of-equilibrium density matrix

$$\hat{\rho}(t) = \mathcal{U}(0,t)\ \hat{\rho}(0)\ \mathcal{U}(0,t)^{-1} \qquad\qquad \mathcal{U}(0,t) = \mathcal{P}\, e^{-i\gamma \int_0^t dt'\, H_B(t')}$$

  - but how to evaluate/resum the perturbative expansion?

    Fails in the strong coupling regime

- How to control approximate methods / other approaches?
  - truncated EOM, diagrammatic methods,
  - real-time RG, FRG,

- Integrability provides a non-perturbative approach

Non-perturbative approaches

a few available solutions !

- Dressed TBA
  - Quantum Hall edge states tunneling (P.Fendley, A.W.W.Ludwig, H.Saleur 1995)
  - Self-dual Interacting Resonant Level Model (E.B., P.Schmitteckert, H.Saleur 2008)
- Map to equilibrium problem
  - Boundary sine Gordon model (V.Bazhanov, S.Lukyanov, A.B.Zamolodchikov 1999)
- Effectively non-interacting systems (map to free fermions)
  - 1-ch Kondo (A. Schiller, U. Hershfield 1998) } Toulouse point
  - Luttinger Liquid (A. Komnik, O. Gogolin 2003) } Toulouse point
  - 2-ch Kondo (E. Sela, I. Affleck 2009) QCP & vicinity

- Out-of-equilibrium forcing generically destroys integrable quasiparticles!
- Dynamical forcing (AC…) ? Heat transport ?

The game is not over

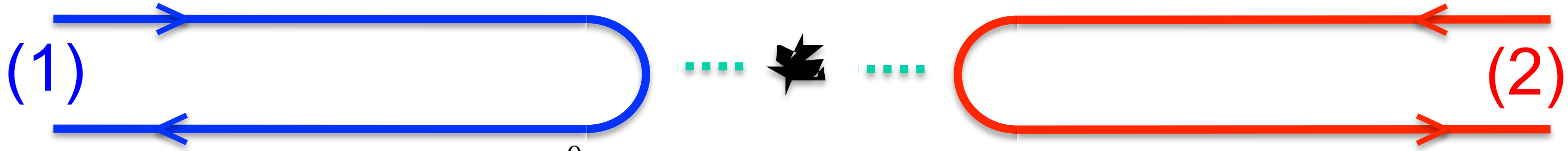
- Integrable theories have nevertheless a rich structure:
  - Infinite number of conserved quantities
  - Renormalization group flow is controlled non-perturbatively
- Can one use this rich structure to develop a controlled expansion out of equilibrium, in the strong coupling regime ?

→ **Yes (at least in some cases)**

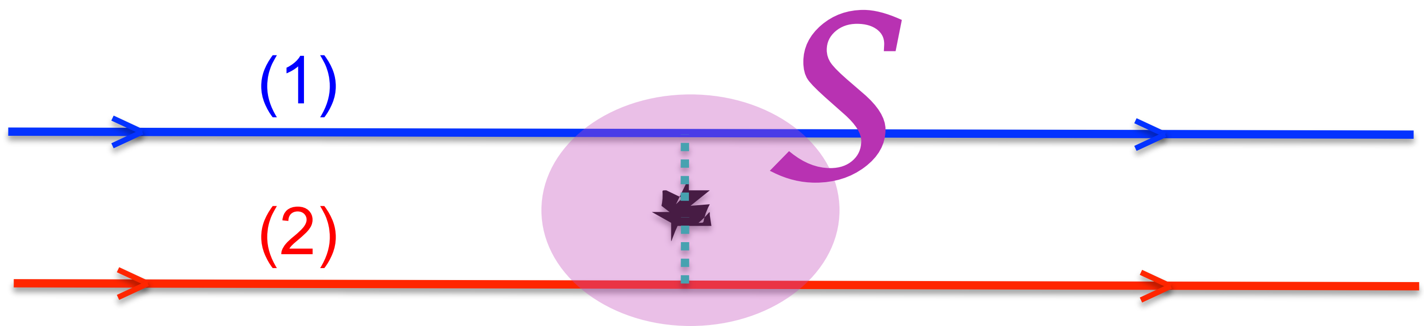
→ Integrable Strong Coupling Expansion $V, T, \omega, \ldots \leq T_K$

Modeling the baths

- Modes that couple to the impurity are 1D (conduction channel)
- Linearize the spectrum



$$H_0[\Psi] = \sum_{a=1,2} -iv_F \int_{-\infty}^{0} dx \left[\Psi_{aR}^{\dagger}(x)\partial_x \Psi_{aR}(x) - \Psi_{aL}^{\dagger}(x)\partial_x \Psi_{aL}(x) \right]$$



$$H_0[\Psi] = \sum_{a=1,2} -iv_F \int_{-\infty}^{\infty} dx \;\; \Psi_a^{\dagger}(x)\partial_x \Psi_a(x)$$

Chiral theory involving only right-moving fields: **scattering problem**

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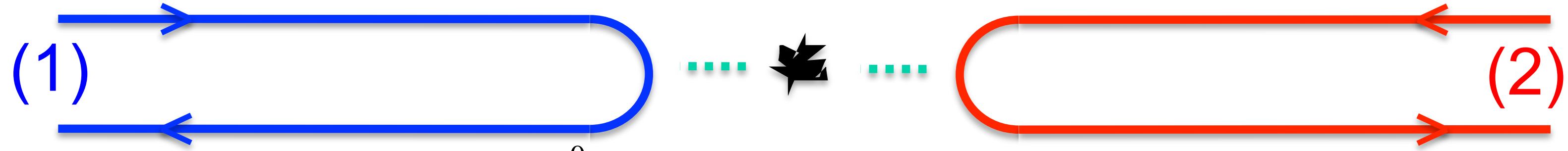

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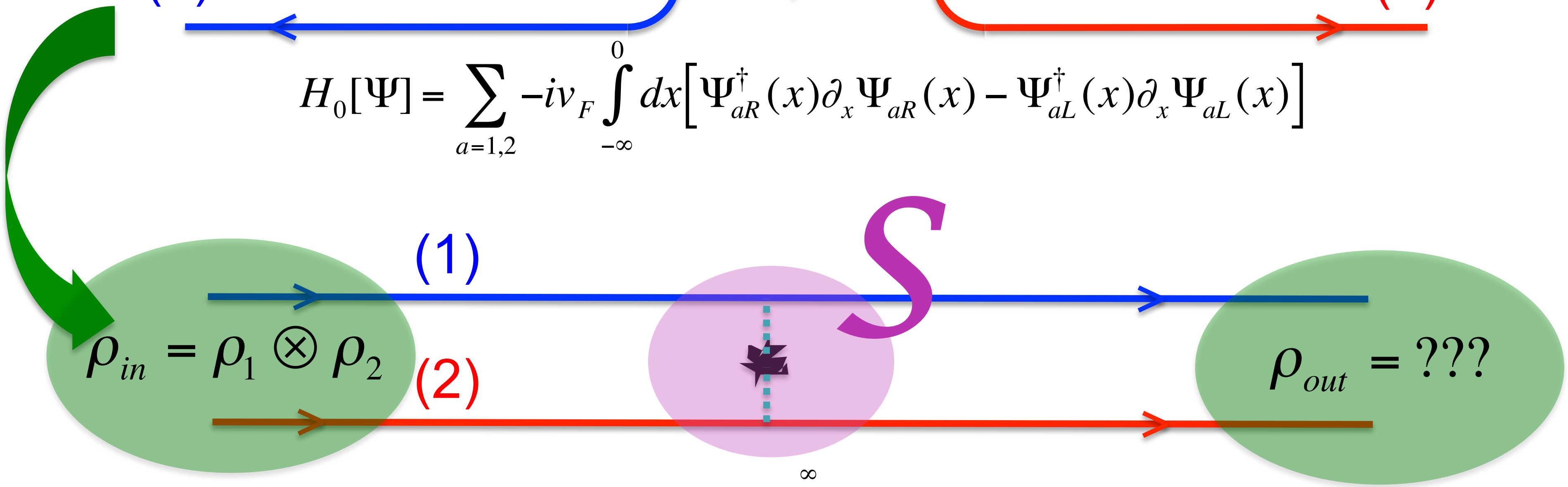

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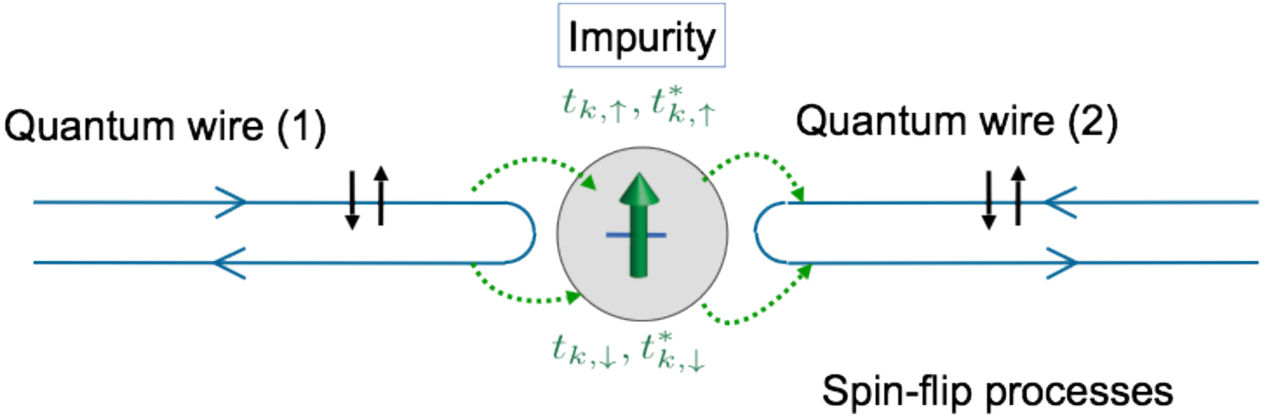
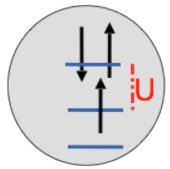
Chiral theory involving only right-moving fields: **scattering problem**

The s=½ Kondo model



Kondo stems from Anderson model



Parameters: - bare exchange coupling J
- anisotropy of couplings to the wires θ

Strategy

Want to describe the strong coupling regime $T, V, \omega \ldots \leq T_K$

1. Incoporate out-of-equilibrium forcing AT the fixed point
   → yields a deformed CFT

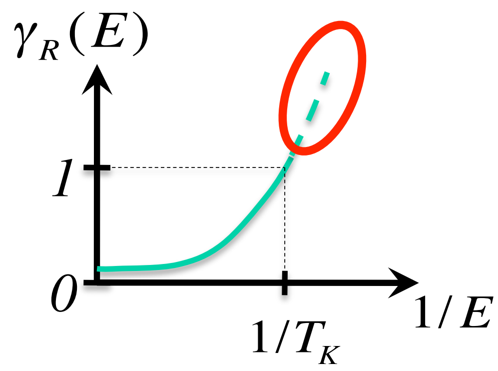
2. Use integrability to build the (many-body) S-matrix
   → incoporate (many-nody) back scattering

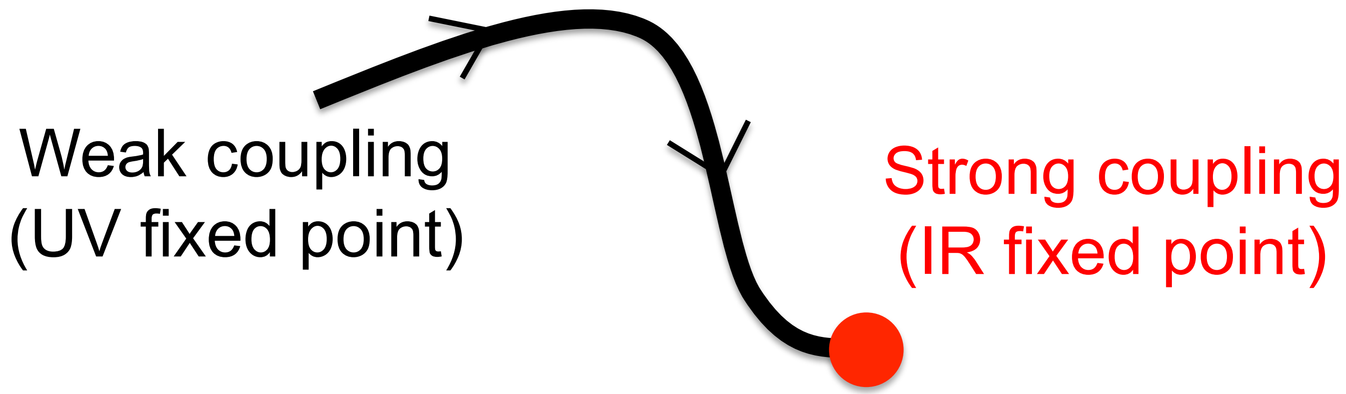
3. Expand in inverse powers of T_K

→ Net result: Taylor expansion of the universal scaling functions for local observables, at arbitrary order in principle

Strong coupling fixed point

- Perturbation is **relevant**
- Strong coupling fixed point described by BCFT





- Step 1: Out-of-equilibrium SC fixed point $(T_K = \infty)$

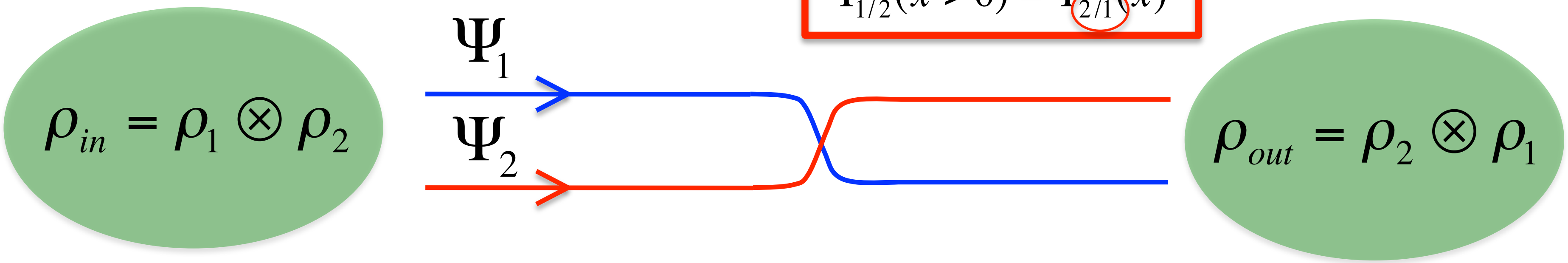
Strong coupling fixed point

- Boundary conditions: $\Phi(x=0^-) = \mathrm{B}\cdot\Phi(x=0^+)$
- **“Transparent fields”**: $\tilde{\Phi}\ (x<0) = \Phi\ (x)\quad ;\quad \tilde{\Phi}\ (x>0) = \mathrm{B}\cdot\Phi(x)$

  They don’t see the impurity!

  BC for fermions: π/2 phase shift

  $$\tilde{\Psi}_{1/2}(x<0) = \Psi_{1/2}(x)$$
  $$\tilde{\Psi}_{1/2}(x>0) = \Psi_{2/1}(x)$$



- Forcing out-of-equilibrium easily represented!
- Amounts to a gauge transformation $\mathcal{U}_{N.Equ}(z)$ for the transparent fields

$$\rho_{in} \propto e^{-\frac{H_0[\Psi_1]-\mu_1 Q_1}{T_1}} \otimes e^{-\frac{H_0[\Psi_2]-\mu_2 Q_2}{T_2}}$$

$$\langle I\rangle = \left(2e^2/h\right)\ \left(\mu_1(t)-\mu_2(t)\right)$$

Recover the linear regime for the charge current

"Doping" a CFT

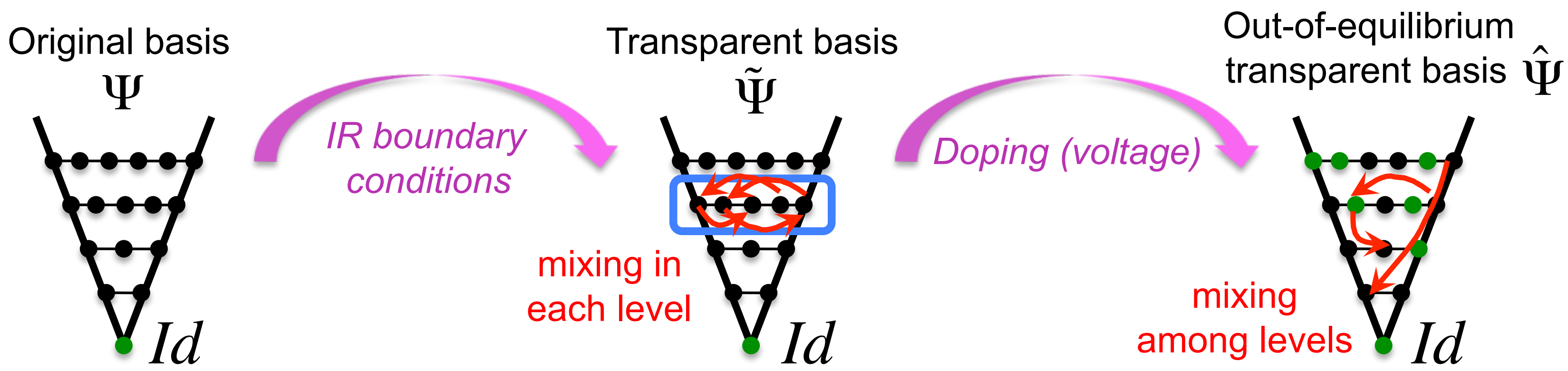
- The strong coupling fixed point has conformal symmetry ; transparent fields $\tilde{\Phi}$ are holomorphic (functions of $z = i(t\text{-}x)$)

- The forcing out of equilibrium can be absorbed by a gauge transformation (« doping ») $\hat{\Psi}(z) = \mathcal{U}_{N.Equ}(z) \cdot \tilde{\Psi}(z)$

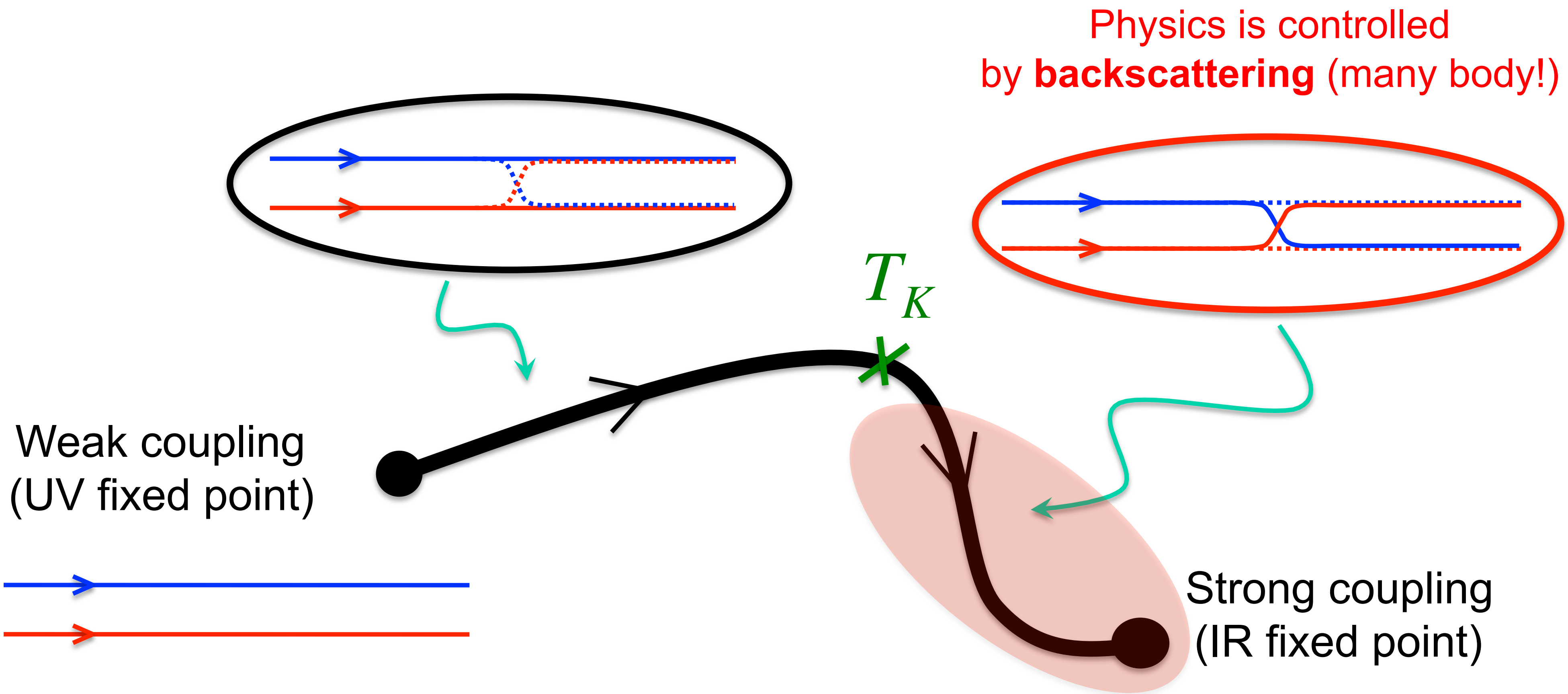
$$\mathcal{U}_{N.Equ}(z) = e^{i\oint_z d\omega\; \Xi_a(w)\,\tilde{Q}_a(w)} \quad ; \quad \Xi_a(z = i(t-x)) = \int_0^{t-x} dt'\,\mu_a(t')$$

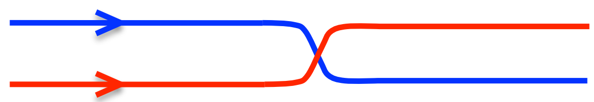
$$\left\langle A_1(x_1,t_1)\; A_2(x_2,t_2)\ldots\right\rangle_{N.Equ} = \left\langle \hat{A}_1(x_1,t_1)\; \hat{A}_2(x_2,t_2)\ldots\right\rangle_{Equ.} \quad ; \quad \hat{A} = \mathcal{U}_{N.Equ} \cdot A$$

- It's a deformation of the CFT (no geometrical interpretation unlike finite temperature CFT)



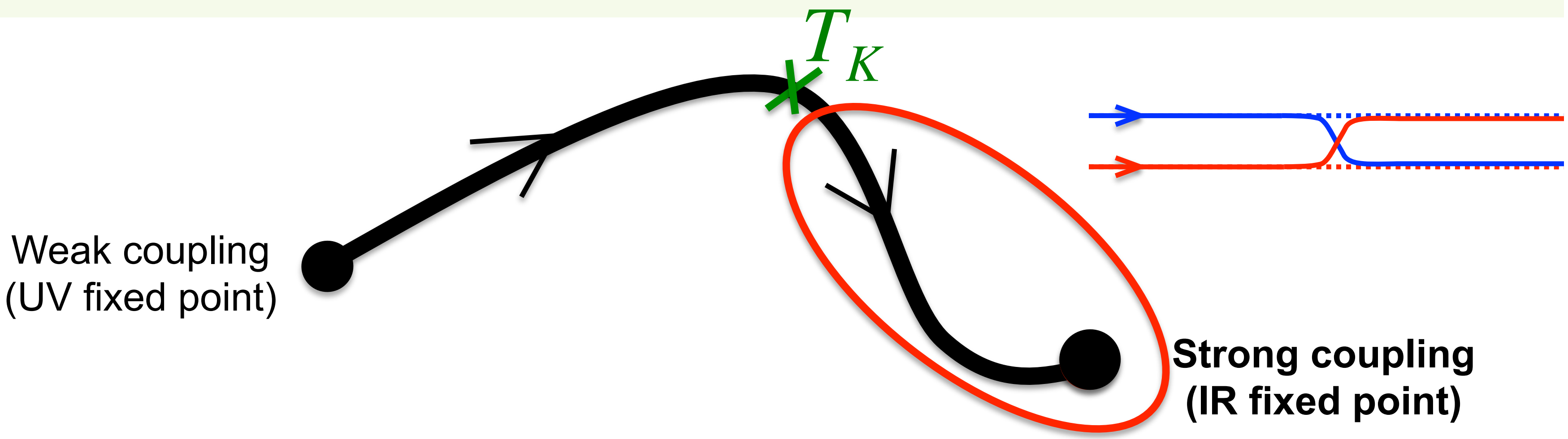
From weak to strong coupling





- Step 2: Incorporate backscattering

Backscattering: dual theory



Integrability completely fixes the RG flow

The full approach to the IR fixed point can be described exactly by a ***dual*** theory. (F. Lesage, H.Saleur 1999)

$$H = H_0^{SC} + H_B^{SC} \qquad\qquad H_B^{SC} = \sum_{n=1}^{\infty} \frac{g_{2n}}{(T_K)^{2n-1}} \hat{O}_{2n}(x=0)$$

Dual theory (2)

$$H = H_0^{SC} + H_B^{SC} \qquad H_B^{SC} = \sum_{n=1}^{\infty} \frac{g_{2n}}{(T_K)^{2n-1}} \hat{O}_{2n}(x=0)$$

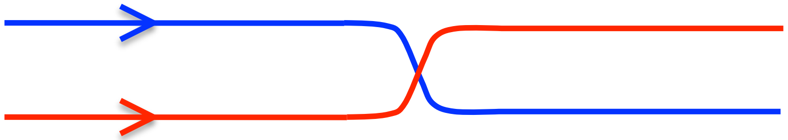
- The operators O_{2n} are the (infinitely many) conserved quantities stemming from integrability.
- The couplings g_n are pure numbers, **fixed** by integrability.
- Fermi liquid: the least irrelevant operator is $O_2{=}T$, an energy momentum tensor.
- Higher order processes have integer dimensions = 4,6,8,…

Backscattering transfers integer charges (electrons)

“SUPER FERMI LIQUID”

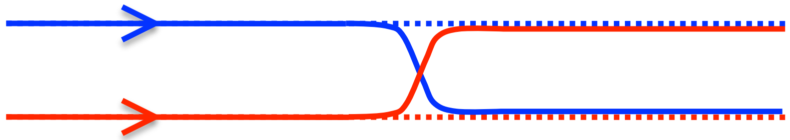
Keldysh expansion

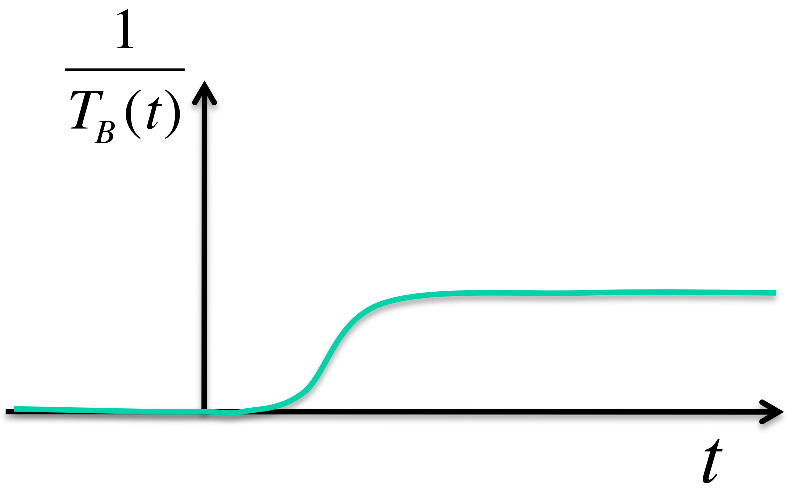
- Start at time $t = -\infty$ at the SC fixed point $(T_K = \infty)$



$$\rho(-\infty) = \rho_{SC} = e^{-H_0^{SC}/k_B T}$$

- Switch on backscattering at time t=0



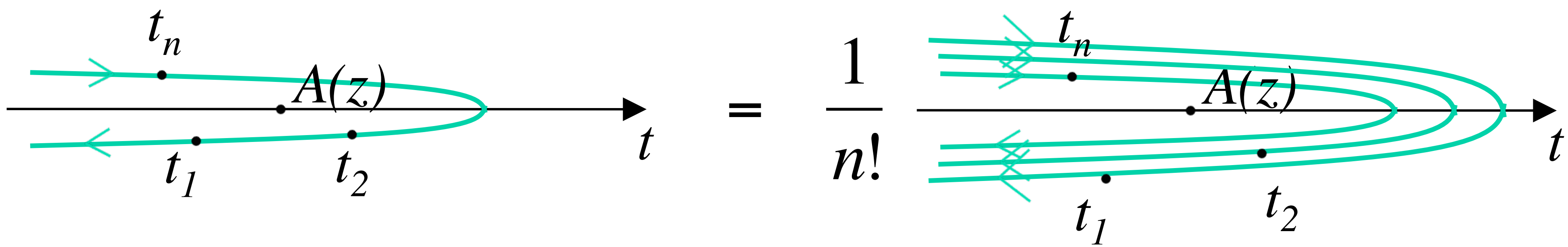


$$\rho(t) = U(t)\rho_{SC}U(t)^{\dagger}$$

$$U(t) = \mathcal{P}_K e^{-i\gamma\int_{-\infty}^{t} dt' H_B^{SC}(t')}$$

Effective operators

In a **super Fermi liquid**, the Keldysh expansion bears a simple form:

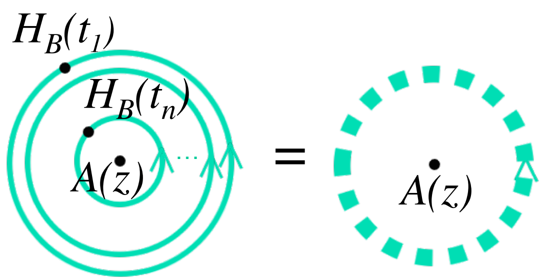


Each (local) operator can be replaced by an *effective* operator:

Complete many-body scattering

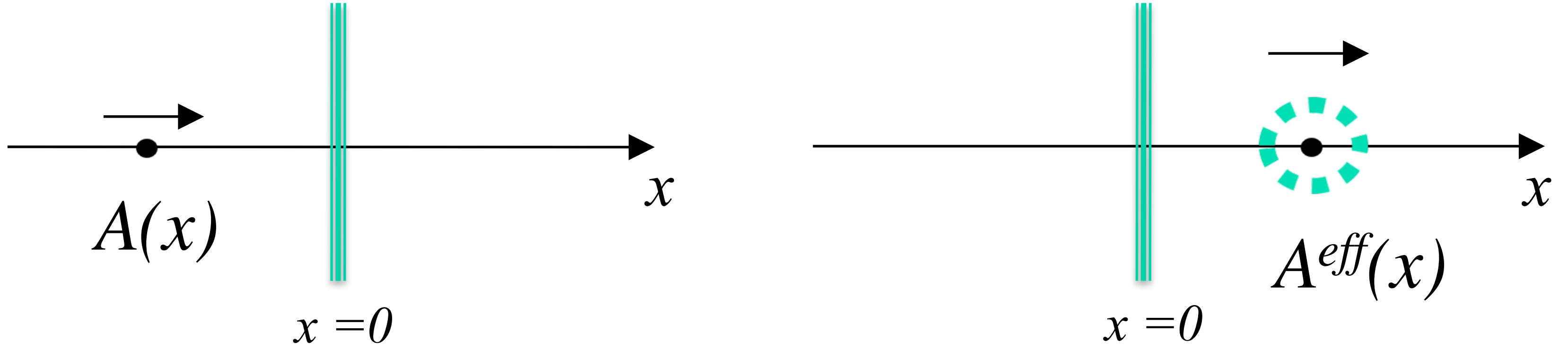
$$A^{eff}(z) = \mathcal{U}_{BS}(z) \bullet A(z)$$

$$= e^{-i\oint_z dt\, H_B(t)} \bullet A(z) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \oint_z dt_1 \ldots \oint_z dt_n$$

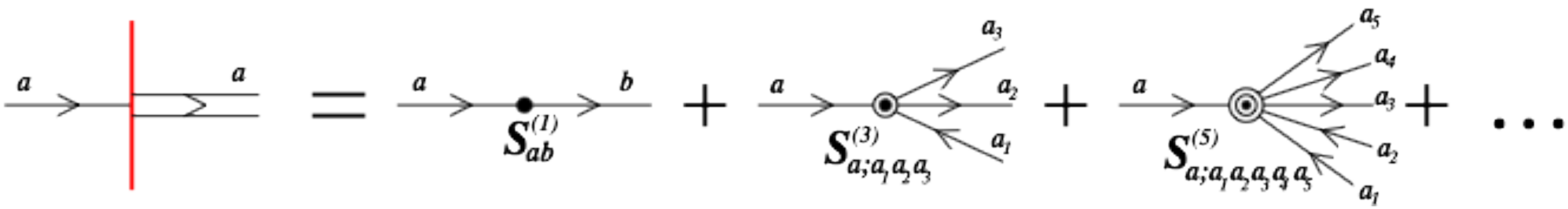


Effective operators

- Operators undergo a **dressing by scattering** when crossing the impurity:



$$\langle A(x,t)\rangle_{N.Equ} = \left\langle \mathcal{U}_{N.Equ}(z)\bullet \mathcal{U}_{BS}(z)\bullet A(z)\right\rangle_0$$



Summary

A simple formula

$$\langle A(t) \rangle_{\mathrm{n.eq.}} = \langle A_{eff} \rangle_0$$

can be expanded in powers of 1/ T_K : non-linear effects to arbitrary order !

Contours + CFT relations :

⇨ Algebraic (computer-friendly) reformulation

+

PT is *finite*: No UV divergence !

Exact formula ! **Directly gives universal results**

Versatile formulation $V, \mu, \omega, T_i, t, B, \epsilon_d \ldots$

−

Integrable models with integer conserved quantities only

Not always easy to find conserved quantities and couplings

The price to pay…

Electrical current operator : **initial fermions**

$$I_{BS} = 1/2((\psi^{\dagger}_{1\uparrow}\psi_{1\uparrow}) - (\psi^{\dagger}_{2\uparrow}\psi_{2\uparrow}) + (\psi^{\dagger}_{1\downarrow}\psi_{1\downarrow}) - (\psi^{\dagger}_{2\downarrow}\psi_{2\downarrow}))$$

The price to pay…

Electrical current operator : **Transparent fermions**

Zeroth order

$\overset{\times}{I}_{BS}$

$$
\begin{aligned}
&-\frac{1}{2}\sin(\theta)\cos(\theta)(\cos(\xi)-1)+\frac{1}{2}i\sin(\theta)\sin(\xi)((\Psi_{1\uparrow})^{\dagger}(\Psi_{2\uparrow}))\\
&+\frac{1}{2}\sin(\theta)\cos(\theta)(\cos(\xi)-1)+\frac{1}{2}i\sin(\theta)\sin(\xi)((\Psi_{1\uparrow})(\Psi_{2\uparrow})^{\dagger})\\
&+\frac{1}{2}\left(\sin^2(\theta)\cos(\xi)+\cos^2(\theta)\right)((\Psi_{1\uparrow})^{\dagger}(\Psi_{1\uparrow}))\\
&+\frac{1}{2}\left(-\sin^2(\theta)\cos(\xi)-\cos^2(\theta)\right)((\Psi_{2\uparrow})^{\dagger}(\Psi_{2\uparrow}))\\
&+-\frac{1}{2}\sin(\theta)\cos(\theta)(\cos(\xi)-1)-\frac{1}{2}i\sin(\theta)\sin(\xi)((\Psi_{1\downarrow})^{\dagger}(\Psi_{2\downarrow}))\\
&+\frac{1}{2}\sin(\theta)\cos(\theta)(\cos(\xi)-1)-\frac{1}{2}i\sin(\theta)\sin(\xi)((\Psi_{1\downarrow})(\Psi_{2\downarrow})^{\dagger})\\
&+\frac{1}{2}\left(\sin^2(\theta)\cos(\xi)+\cos^2(\theta)\right)((\Psi_{1\downarrow})^{\dagger}(\Psi_{1\downarrow}))\\
&+\frac{1}{2}\left(-\sin^2(\theta)\cos(\xi)-\cos^2(\theta)\right)((\Psi_{2\downarrow})^{\dagger}(\Psi_{2\downarrow}))
\end{aligned}
$$

The price to pay…

Electrical current operator : **Transparent fermions**

$-(\sin\theta * (2 * \cos[\theta/2]^2 * (\cos\xi - i * \sin\xi) + 2 * \cos[\theta/2]^2 * \cos\theta * (\cos\xi - i * \sin\xi) +$

$+(\sin\theta * (-2 * \cos[\theta/2]^2 * (\cos\xi + i * \sin\xi) + 2 * \cos[\theta/2]^2 * \cos\theta * (\cos\xi + i * \sin\xi) + \sin\theta^2 * (\cos$

$+-(\sin\theta * (-2 * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi) - 2 * \cos\theta * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi) +$

$+(\sin\theta * (2 * \sin[\theta/2]^2 * (\cos\xi + i * \sin\xi) - 2 * \cos\theta * \sin[\theta/2]^2 * (\cos\xi + i * \sin\xi) + \sin\theta^2 * (\cos$

$+-((\cos[\theta/2]^2 - \cos\theta - \sin[\theta/2]^2) * \sin\theta^2 * (\cos\xi -$

$-\sin[\theta/2]^2) * \sin\theta^2 * (\cos\xi +$

First order: 1/T_K

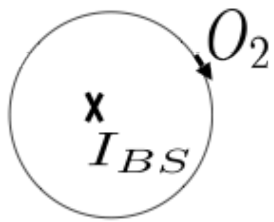
$+-(\sin\theta^2 * (-\cos\xi + 2 * \cos[\theta/2]^2 * (\cos\xi - i * \sin\xi) - \cos\theta * (\cos\xi - i * \sin\xi)$

$+2 * \cos[\theta/2]^2 * (\cos\xi + i * \sin\xi) - \cos\theta * (\cos\xi + i *$

$+(i/16) * \sin\theta^2 * (i * (\cos\xi - i * \sin\xi) + i * \cos\theta * (\cos\xi - i * \sin\xi) + (2 * i) * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi)) - (i/16) * \sin\theta^2 * ((-i) * (\cos\xi + i * \sin\xi) + i * \cos\theta * (\cos\xi + i * \sin\xi) + (2 * i) * \sin[\theta/2]$

$+(\sin\theta * (2 * \cos[\theta/2]^2 * (\cos\xi - i * \sin\xi) + 2 * \cos[\theta/2]^2 * \cos\theta * (\cos\xi - i * \sin\xi)))/16 - (\sin\theta * ($

$+2 * \cos[\theta/2]^2 * \cos\theta * (\cos\xi + i * \sin\xi)))/16 + (\sin\theta^3 * (\cos\xi - i * \sin\xi))/16 - (\sin\theta^3 * (\cos\xi + i$



$+-(\sin\theta * (-2 * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi) - 2 * \cos\theta * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi)))/1$

$+i * \sin\xi) - 2 * \cos\theta * \sin[\theta/2]^2 * (\cos\xi + i * \sin\xi)))/16 - (\sin\theta^3 * (\cos\xi$

$+i$

$+(\sin\theta^2 * (\cos\xi - \cos\theta * (\cos\xi - i * \sin\xi) - i * \sin\xi))/16 + (\sin\theta^2 * (\cos\xi + \cos\theta *$

$-(\sin\theta^2 * (-\cos\xi - \cos\theta * (\cos\xi + i * \sin\xi) - i * \sin\xi))/16 - (\sin\theta^2 * (-\cos\xi + \cos\theta * (\cos\xi + i * \sin\xi) - i$

$+(\sin\theta^2 * (\cos\xi + \cos\theta * (\cos\xi - i * \sin\xi) - i * \sin\xi))/16 - (\sin\theta^2 * (-\cos\xi + \cos\theta *$

$+(\sin\theta^2 * (-\cos\xi - \cos\theta * (\cos\xi - i * \sin\xi) + i * \sin\xi))/16 - (\sin\theta^2 *$

$+i$

$+-(\sin\theta * (-2 * \cos[\theta/2]^2 * (\cos\xi - i * \sin\xi) + 2 * \cos[\theta/2]^2 * \cos\theta * (\cos\xi - i * \sin\xi) + \sin\theta^2 * (\cos\xi - i * \sin\xi)))/16 + (\sin\theta * (2 * \cos[\theta/2]^2 * (\cos\xi + i * \sin\xi) + 2 * \cos[\theta/2]^2 * \cos\theta * (\cos\xi + i * \sin\xi) + \sin\theta^2 * ($

$+-(\sin\theta * (2 * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi) - 2 * \cos\theta * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi) + \sin\theta^2 * (\cos\xi - i * \sin\xi)))/16 + (\sin\theta * (-2 * \sin[\theta/2]^2 * (\cos\xi + i * \sin\xi) - 2 * \cos\theta * \sin[\theta/2]^2 * (\cos\xi + i * \sin\xi) + \sin\theta^2 *$

$+(\sin\theta^2 * (\cos\xi + 2 * \cos[\theta/2]^2 * (\cos\xi - i * \sin\xi) - \cos\theta * (\cos\xi - i * \sin\xi) - i * \sin\xi))/16 - (\sin\theta^2 * (-\cos\xi + 2 * \cos[\theta/2]^2 * (\cos\xi + i * \sin\xi) - \cos\theta * (\cos\xi + i$

$+(\sin\theta^2 * (-\cos\xi + \cos\theta * (\cos\xi - i * \sin\xi) + 2 * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi) + i * \sin\xi))/16 - (\sin\theta^2 * (\cos\xi + \cos\theta * (\cos\xi + i * \sin\xi) + 2 * \sin[\theta/2]^2 * (\cos\xi + i$

$+-(\sin\theta * (-2 * \cos[\theta/2]^2 * (\cos\xi - i * \sin\xi) + 2 * \cos[\theta/2]^2 * \cos\theta * (\cos\xi - i * \sin\xi)))/16 + (\sin\theta * (2 * \cos[\theta/2]^2 * (\cos\xi + i * \sin\xi) + 2 * \cos[\theta/2]^2 * \cos\theta * (\cos\xi + i * \sin\xi)))/16 - (\sin\theta^3 * (\cos\xi - i * \sin\xi))/16 + (\sin\theta^3 * (\cos\xi +$

$+(\sin\theta * (2 * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi) - 2 * \cos\theta * \sin[\theta/2]^2 * (\cos\xi - i * \sin\xi)))/16 - (\sin\theta * (-2 * \sin[\theta/2]^2 * (\cos\xi + i * \sin\xi) - 2 * \cos\theta * \sin[\theta/2]^2 * (\cos\xi + i * \sin\xi)))/16 + (\sin\theta^3 * (\cos\xi - i * \sin\xi))/16 - (\sin\theta^3 * (\cos\xi +$

The price to pay…

Electrical current operator : **Transparent fermions**

$$\ast\sin\xi)-i\ast\sin\xi))/16+(\sin\theta^2\ast(-\cos\xi+\cos\theta\ast(\cos\xi-i\ast\sin\xi)+i\ast\sin\xi))/16-(\sin\theta^2\ast(\cos\xi+\cos\theta\ast(\cos\xi+i\ast\sin\xi)+i\ast\sin\xi))/16\ast(\psi_{1\uparrow}(\psi^\dagger_{2\uparrow}(\psi^\dagger_{1\downarrow}\psi_{2\downarrow})))$$

$$i\ast\sin\xi)+i\ast\sin\xi))/16-(\sin\theta^2\ast(\cos\xi-\cos\theta\ast(\cos\xi+i\ast\sin\xi)+i\ast\sin\xi))/16-(\sin\theta^2\ast(\cos\xi+\cos\theta\ast(\cos\xi+i\ast\sin\xi)+i\ast\sin\xi))/16\ast(\psi_{1\uparrow}(\psi^\dagger_{2\uparrow}(\psi_{1\downarrow}\psi^\dagger_{2\downarrow})))$$

$$-i\ast\sin\xi))/8+(\cos[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi-i\ast\sin\xi))/8+(\cos[\theta/2]^2\ast\sin\theta^2\ast(-\cos\xi+i\ast\sin\xi))/8-(\cos[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi+i\ast\sin\xi))/8\ast(\psi^\dagger_{1\uparrow}(\psi_{1\uparrow}(\psi^\dagger_{1\downarrow}\psi_{1\downarrow})))$$

$$\xi-i\ast\sin\xi))/8-(\sin[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi-i\ast\sin\xi))/8+(\cos[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi+i\ast\sin\xi))/8+(\sin[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi+i\ast\sin\xi))/8\ast(\psi^\dagger_{1\uparrow}(\psi_{1\uparrow}(\psi^\dagger_{2\downarrow}\psi_{2\downarrow})))$$

First order: $1/T_K$

$$2\ast\cos[\theta/2]^2 \ldots [\theta/2]^2\ast\cos\theta\ast(\cos\xi+i\ast\sin\xi)))/16-(\sin\theta^3\ast(\cos\xi-i\ast\sin\xi))/16+(\sin\theta^3\ast(\cos\xi+i\ast\sin\xi))/16\ast(\psi^\dagger_{1\uparrow}(\psi_{1\uparrow}(\psi^\dagger_{1\downarrow}\psi_{2\downarrow})))$$

$$2\ast\cos[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)+2\ast\cos[\theta/2]^2\ast\cos\theta\ast(\cos\xi+i\ast\sin\xi)))/16+(\sin\theta^3\ast(\cos\xi-i\ast\sin\xi))/16-(\sin\theta^3\ast(\cos\xi+i\ast\sin\xi))/16\ast(\psi^\dagger_{1\uparrow}(\psi_{1\uparrow}(\psi_{1\downarrow}\psi^\dagger_{2\downarrow})))$$

$$\xi-i\ast\sin\xi))/8+(\sin[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi-i\ast\sin\xi))/8-(\cos[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi+i\ast\sin\xi))/8-(\sin[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi+i\ast\sin\xi))/8\ast(\psi^\dagger_{2\uparrow}(\psi_{2\uparrow}(\psi^\dagger_{1\downarrow}\psi_{1\downarrow})))$$

$$2]^2\ast\sin\theta^2\ast(-\cos\xi-i\ast\sin\xi))/8-(\sin[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi-i\ast\sin\xi))/8-(\sin[\theta/2]^2\ast\sin\theta^2\ast(-\cos\xi+i\ast\sin\xi))/8+(\sin[\theta/2]^2\ast\sin\theta^2\ast(\cos\xi+i\ast\sin\xi))/8\ast(\psi^\dagger_{2\uparrow}(\psi_{2\uparrow}(\psi^\dagger_{2\downarrow}\psi_{2\downarrow})))$$

$$n\xi)))/16-(\sin\theta\ast(-2\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)-2\ast\cos\theta\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)))/16+(\sin\theta^3\ast(\cos\xi-i\ast\sin\xi))/16-(\sin\theta^3\ast(\cos\xi+i\ast\sin\xi))/16\ast(\psi^\dagger_{2\uparrow}(\psi_{2\uparrow}(\psi^\dagger_{1\downarrow}\psi_{2\downarrow})))$$

$$\sin\xi)))/16+(\sin\theta\ast(2\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)-2\ast\cos\theta\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)))/16-(\sin\theta^3\ast(\cos\xi-i\ast\sin\xi))/16+(\sin\theta^3\ast(\cos\xi+i\ast\sin\xi))/16\ast(\psi^\dagger_{2\uparrow}(\psi_{2\uparrow}(\psi_{1\downarrow}\psi^\dagger_{2\downarrow})))$$

$$\ast\cos\theta\ast(\cos\xi-i\ast\sin\xi)+\sin\theta^2\ast(\cos\xi-i\ast\sin\xi)))/16-(\sin\theta\ast(2\ast\cos[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)+2\ast\cos[\theta/2]^2\ast\cos\theta\ast(\cos\xi+i\ast\sin\xi)+\sin\theta^2\ast(\cos\xi+i\ast\sin\xi)))/16\ast(d\psi^\dagger_{1\downarrow}\psi_{2\downarrow})$$

$$/2]^2\ast(\cos\xi-i\ast\sin\xi)+\sin\theta^2\ast(\cos\xi-i\ast\sin\xi)))/16-(\sin\theta\ast(-2\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)-2\ast\cos\theta\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)+\sin\theta^2\ast(\cos\xi+i\ast\sin\xi)))/16\ast(\psi^\dagger_{1\downarrow}d\psi_{2\downarrow})$$

$$+((\cos[\theta/2]^2-\cos\theta-\sin[\theta/2]^2)\ast\sin\theta^2\ast(\cos\xi-i\ast\sin\xi))/8-((\cos[\theta/2]^2-\cos\theta-\sin[\theta/2]^2)\ast\sin\theta^2\ast(\cos\xi+i\ast\sin\xi))/8\ast(\psi^\dagger_{1\downarrow}(\psi_{1\downarrow}(\psi^\dagger_{2\downarrow}\psi_{2\downarrow})))$$

$$s\xi+2\ast\cos[\theta/2]^2\ast(\cos\xi-i\ast\sin\xi)-\cos\theta\ast(\cos\xi-i\ast\sin\xi)-i\ast\sin\xi))/16-(\sin\theta^2\ast(-\cos\xi+2\ast\cos[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)-\cos\theta\ast(\cos\xi+i\ast\sin\xi)-i\ast\sin\xi))/16\ast(\psi^\dagger_{1\downarrow}d\psi_{1\downarrow})$$

$$\cos\xi+\cos\theta\ast(\cos\xi-i\ast\sin\xi)+2\ast\sin[\theta/2]^2\ast(\cos\xi-i\ast\sin\xi)+i\ast\sin\xi))/16-(\sin\theta^2\ast(\cos\xi+\cos\theta\ast(\cos\xi+i\ast\sin\xi)+2\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)+i\ast\sin\xi))/16\ast(d\psi^\dagger_{2\downarrow}\psi_{2\downarrow})$$

$$\cos\theta\ast(\cos\xi-i\ast\sin\xi)+\sin\theta^2\ast(\cos\xi-i\ast\sin\xi)))/16-(\sin\theta\ast(-2\ast\cos[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)+2\ast\cos[\theta/2]^2\ast\cos\theta\ast(\cos\xi+i\ast\sin\xi)+\sin\theta^2\ast(\cos\xi+i\ast\sin\xi)))/16\ast(d\psi_{1\downarrow}\psi^\dagger_{2\downarrow})$$

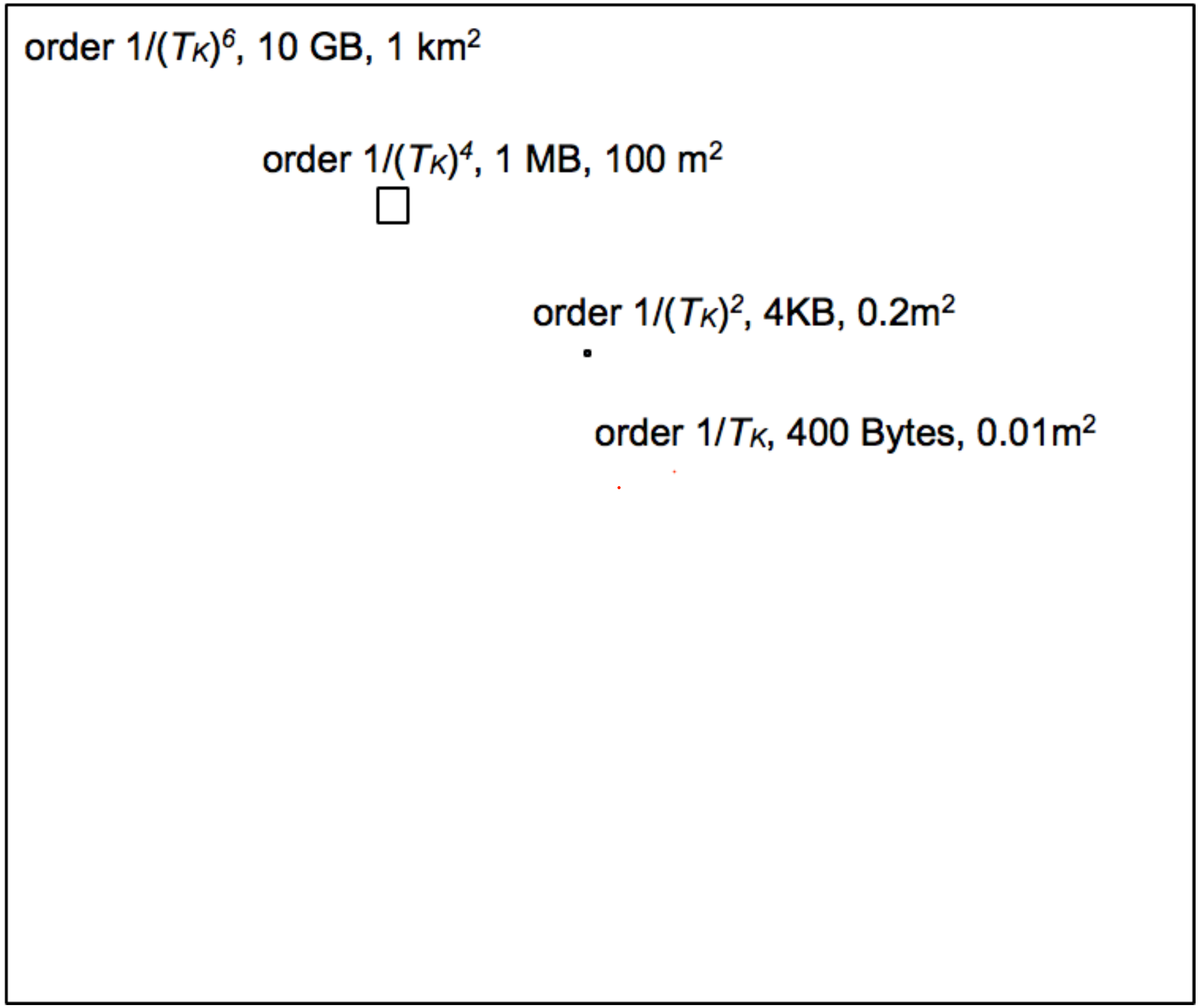
$$[\theta/2]^2\ast(\cos\xi-i\ast\sin\xi)+\sin\theta^2\ast(\cos\xi-i\ast\sin\xi)))/16-(\sin\theta\ast(2\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)-2\ast\cos\theta\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)+\sin\theta^2\ast(\cos\xi+i\ast\sin\xi)))/16\ast(\psi_{1\downarrow}d\psi^\dagger_{2\downarrow})$$

$$\cos\xi+2\ast\cos[\theta/2]^2\ast(\cos\xi-i\ast\sin\xi)-\cos\theta\ast(\cos\xi-i\ast\sin\xi)+i\ast\sin\xi))/16+(\sin\theta^2\ast(\cos\xi+2\ast\cos[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi)-\cos\theta\ast(\cos\xi+i\ast\sin\xi)+i\ast\sin\xi))/16\ast(d\psi^\dagger_{1\downarrow}\psi_{1\downarrow})$$

$$\ast\cos\theta\ast(\cos\xi-i\ast\sin\xi)-(2\ast i)\ast\sin[\theta/2]^2\ast(\cos\xi-i\ast\sin\xi))+(i/16)\ast\sin\theta^2\ast(i\ast(\cos\xi+i\ast\sin\xi)-i\ast\cos\theta\ast(\cos\xi+i\ast\sin\xi)-(2\ast i)\ast\sin[\theta/2]^2\ast(\cos\xi+i\ast\sin\xi))\ast(\psi^\dagger_{2\downarrow}d\psi_{2\downarrow})$$

The price to pay…

The price to pay…



Results

DC bias applied to the two baths

Net analytical result : current

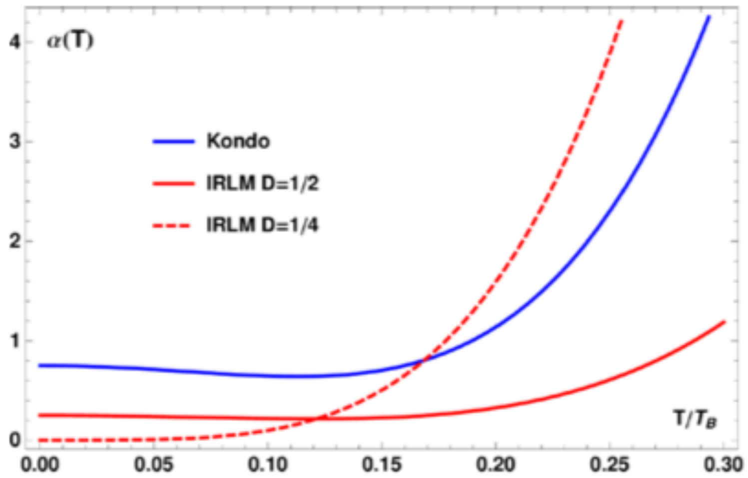
$$\mathrm{I(V)}{=}2\ \sin^2\theta V - \frac{\sin^2\theta V^3}{4T_B^2} + \frac{\left(252\sqrt{3}+65\pi+5\pi\cos 2\theta\right)\sin[\theta]^2 V^5}{1920\pi T_B^4} + O[V^7/T_B^7]$$

linear response

Fermi Liquid result

Temperature dependence of non linearity captured !

$$I(V)/I_0 V = (1 + \alpha(T) V^2/T_B^2 + \ldots)$$



Results

DC bias: Universal differential conductance G=dI/dV

Rescaled quantities

$$\overline{X} = X / T_K$$

Fixed point — Fermi liquid corrections

$$\frac{G(\overline{V},\overline{T},\overline{h})}{G_0} = 1 - c_T\overline{T}^2 - \alpha_V c_T \overline{V}^2 - \alpha_h c_T \overline{h}^2$$

NEW !

$$+\overline{V}^2 c_T^2(\gamma_V\overline{V}^2+\gamma_T\overline{T}^2+\gamma_h\overline{h}^2)+\rho_{Th}c_T^2\overline{T}^2\overline{h}^2+\rho_h c_T^2\overline{h}^4+\rho_T c_T^2\overline{T}^4$$
$$-\overline{V}^4 c_T^3(\kappa_V\overline{V}^2+\kappa_T\overline{T}^2+\kappa_h\overline{h}^2)-\overline{T}^4 c_T^3(\beta_V\overline{V}^2+\beta_T\overline{T}^2+\beta_h\overline{h}^2)$$
$$-\overline{h}^4 c_T^3(\chi_V\overline{V}^2+\chi_T\overline{T}^2+\chi_h\overline{h}^2)-\gamma_{Th}c_T^3\overline{V}^2\overline{T}^2\overline{h}^2+\mathcal{O}(T_K^{-7})$$

$\alpha_V = 3/2\pi^2 \approx 0.15$ $\quad c_T = \pi^2/4 \approx 2.46$ $\quad \alpha_h = 1/\pi^2 \approx 0.10$

$\gamma_V = \frac{252\sqrt{3}+65\pi+5\pi\cos 2\theta}{48\pi^5} \approx 0.04$ $\quad \gamma_T = \frac{72\sqrt{3}+17\pi+\pi\cos 2\theta}{4\pi^3} \approx 1.44^{\pm 0.03}$ $\quad \gamma_h = 3(5\sqrt{3}+\pi)/\pi^5 \approx 0.12$

$\rho_{Th} = 2(5\sqrt{3}+\pi)/\pi^3 \approx 1.52$ $\quad \rho_T = \frac{5}{3} + \frac{36\sqrt{3}}{5\pi} \approx 5.64$ $\quad \rho_h = (6\sqrt{3}+\pi)/3\pi^5 \approx 0.02$

Results

DC bias: Universal differential conductance G=dI/dV

Fixed point — Fermi liquid corrections

$$\frac{G(\bar{V},\bar{T},\bar{h})}{G_0} = 1 - c_T\bar{T}^2 - \alpha_V c_T\bar{V}^2 - \alpha_h c_T\bar{h}^2$$

NEW !

$$+\bar{V}^2 c_T^2(\gamma_V\bar{V}^2 + \gamma_T\bar{T}^2 + \gamma_h\bar{h}^2) + \rho_{Th}c_T^2\bar{T}^2\bar{h}^2 + \rho_h c_T^2\bar{h}^4 + \rho_T c_T^2\bar{T}^4$$
$$-\bar{V}^4 c_T^3(\kappa_V\bar{V}^2 + \kappa_T\bar{T}^2 + \kappa_h\bar{h}^2) - \bar{T}^4 c_T^3(\beta_V\bar{V}^2 + \beta_T\bar{T}^2 + \beta_h\bar{h}^2)$$
$$-\bar{h}^4 c_T^3(\chi_V\bar{V}^2 + \chi_T\bar{T}^2 + \chi_h\bar{h}^2) - \gamma_{Th}c_T^3\bar{V}^2\bar{T}^2\bar{h}^2 + \mathcal{O}(T_K^{-7})$$

$\kappa_V = \frac{12960+19025\sqrt{5}+7308\sqrt{3}\pi+420\pi^2}{720\pi^8} \approx 0.01$

$\kappa_T = \frac{5265+7375\sqrt{5}+2826\sqrt{3}\pi+150\pi^2}{36\pi^6} \approx 1.11$

$\kappa_h = \frac{1215+1850\sqrt{5}+756\sqrt{3}\pi+35\pi^2}{120\pi^8} \approx 0.08$

$\beta_V = \frac{4455+6300\sqrt{5}+2376\sqrt{3}\pi+121\pi^2}{15\pi^4} \approx 22.36$

$\beta_T = \frac{2(14580+21250\sqrt{5}+8316\sqrt{3}\pi+427\pi^2)}{315\pi^2} \approx 71.77$

$\beta_h = \frac{2(3240+4600\sqrt{5}+1674\sqrt{3}\pi+75\pi^2)}{45\pi^4} \approx 10.66$

$\chi_V = \frac{8(117+175\sqrt{5}+56\sqrt{3}\pi+2\pi^2)}{\pi^8} \approx 0.70$

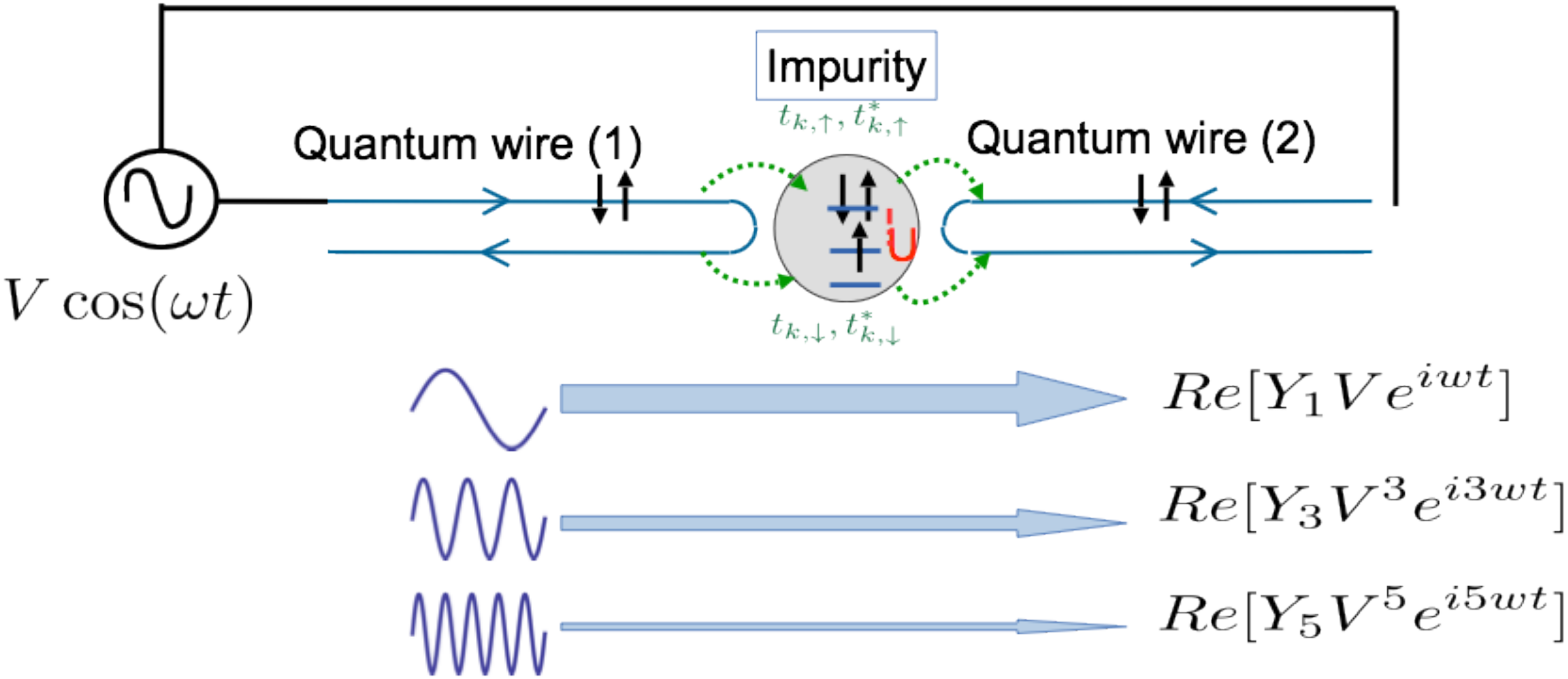
$\chi_T = \frac{16(117+175\sqrt{5}+56\sqrt{3}\pi+2\pi^2)}{3\pi^6} \approx 4.62$

$\chi_h = \frac{(45(3+5\sqrt{5})+60\sqrt{3}\pi+2\pi^2)}{45\pi^8} \approx 0.002$

$\gamma_{Th} = \frac{4428+6050\sqrt{5}+2160\sqrt{3}\pi+96\pi^2}{12\pi^6} \approx 2.66$

Results

AC bias applied to the two baths

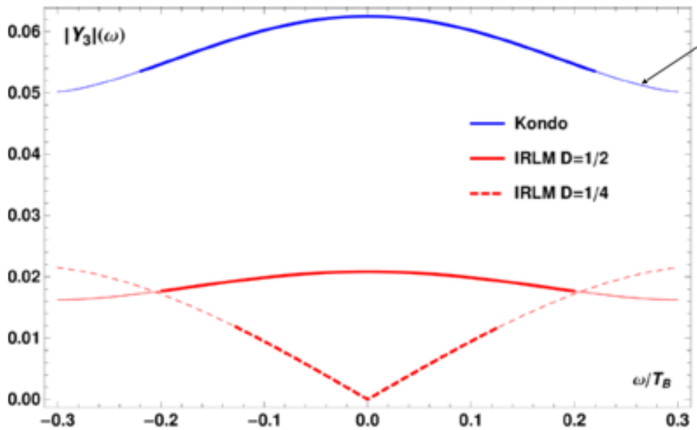


Higher harmonics can be captured ! $Y_a(V, \omega, T)$

Results

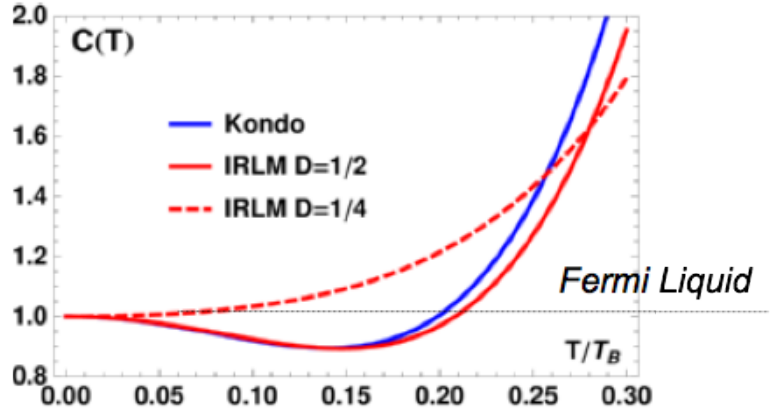
AC bias applied to the two baths

3rd harmonic :



5 % discrepancy with p.o. order

Capacitance C(T) , imaginary part of $Y_1(T)$



Results

Exact results for the noise !

Symmetric noise :

$$\mathrm{S}(\tau) = (I(t+\tau)I(t) - I^2) + (I(t)I(t+\tau) - I^2)$$

Noise spectrum :

$$S(\omega) = \int e^{i\omega\tau} S(\tau) d\tau$$

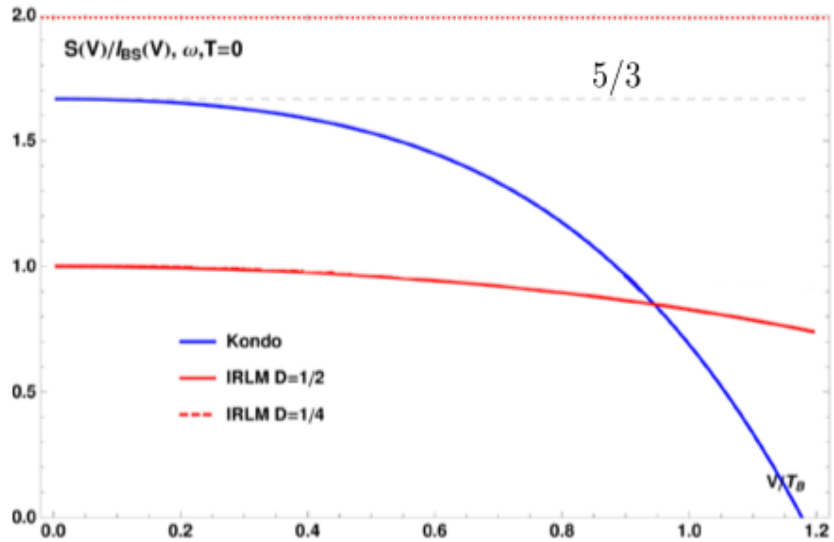
Results

Fano factor

$$F = \frac{1}{2e}\frac{S}{I_{BS}}$$

At V=0: $$F = \frac{5}{3}$$

Sela, Oreg, von Oppen, Koch, + Gogolin, Komnik: PRL 2006



Corrections : $C = \cos\theta$

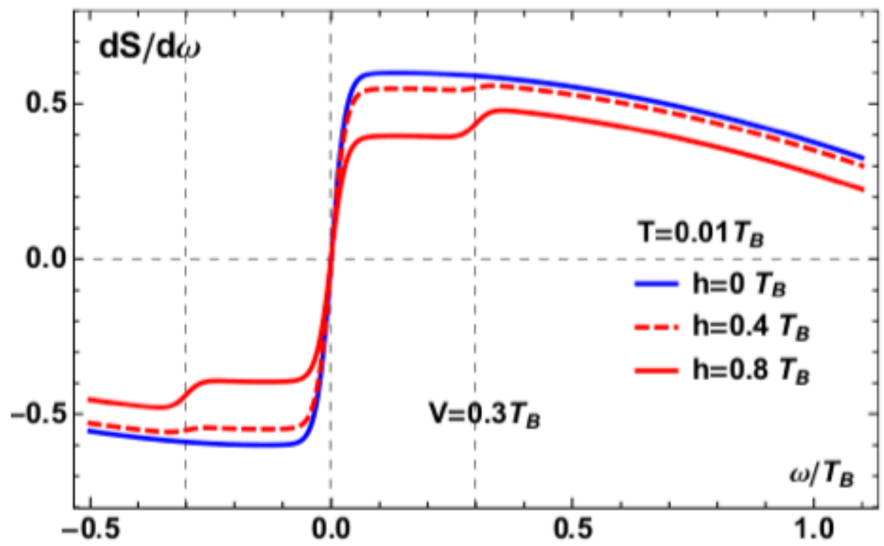
$$F = \frac{1}{3}\left(5 - 8C^2\right) + \frac{\left(C^2-1\right)\left(\pi\left(25C^2+153\right) - 18\sqrt{3}\right)V^2}{360\pi T_B^2}$$
$$+ \frac{\left(\left(513297 + 782875\sqrt{5} + 376866\sqrt{3}\pi + 37275\pi^2\right)C^2 - 42630\pi^2 - 313866\sqrt{3}\pi - 475625\sqrt{5} - 276372\right)V^4}{604800\pi^2 T_B^4}$$
$$+ \frac{\left(875\pi^2 C^6 - 70\left(972 + 500\sqrt{5} - 882\sqrt{3}\pi - 163\pi^2\right)C^4\right)V^4}{604800\pi^2 T_B^4}$$

Results

At T=0 : non-analycities

$$S^{(0)}(\omega,V,0)=\frac{4\sin^2(\theta)}{\pi\left(W^2+4\right)^2}\left((|V-\omega|+|V+\omega|)\left(4\cos^2(\theta)+W^2\right)+8|\omega|\sin^2(\theta)\right)$$
$$-\frac{W\sin^2(\theta)\cos(\theta)}{\pi\left(W^2+4\right)^2T_B}\Big(|V-\omega|\left(8V\cos(2\theta)+W^2(V-\omega)-4(V+\omega)\right)$$
$$+|V+\omega|\left(8V\cos(2\theta)+W^2(V+\omega)-4(V-\omega)\right)+32V|\omega|\sin^2(\theta)\Big)$$

Derivative of the noise at low temperature :
Non analycities at T=0 are rounded by temperature



Conclusions

✓ A generic **– but perturbative** – method for some integrable systems: super Fermi liquids

✓ Gives exact (formal) expression for the out-of-equilibrium density matrix

✓Yields exact results in variety of conditions:

- ✓Voltage (AC/DC)
- ✓Finite temperature(s)
- ✓Particle-hole asymmetry
- ✓Magnetic field

✓Perspectives:

- ✓(Slow) quenches
- ✓Non Fermi liquid fixed points?