LIV studies with HESS II

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J. Bolmont
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Outline

LIV in Fundamental Theories

LIV tests with Cherenkov Telescopes

HESS II performance: Monte Carlo Studies

Summary and Conclusions
LIV in Fundamental Theories

- Lorentz Invariance/Symmetry: Einstein's Relativity & Standard Model

- At Planck scale ~ $10^{-35}$ m ($10^{19}$ GeV), nature of space-time needs to account for microscopic effects
  - Quantum Gravity (QG)

- Some models of QG lead to Lorentz Invariance Violation
  - D branes String model (foamy structure of space-time)
  - Non-commutative geometry
  - Spontaneous symmetry breaking (SME)
  - LQG

- LIV can be tested in different ways:
  - Photon decay, Vacuum Cherenkov Radiation
  - Modified GZK cutoff, and TeV γ-ray spectra of extra-galactic sources.
  - Vacuum birefringence
  - Dispersion of light in vacuum
Modification of dispersion relations in vacuum

- LIV modifies dispersion relation for the photon:

\[ c^2 p^2 = E^2 \left( 1 + \xi \left( \frac{E}{E_{planck}} \right) + \zeta \left( \frac{E}{E_{planck}} \right)^2 + \ldots \right) \]

- Leading order corrections to the speed of light (c) in vacuum:

\[ \nu = \frac{\delta E}{\delta p} = c \left( 1 - \xi \left( \frac{E}{E_{planck}} \right) - \zeta \left( \frac{E}{E_{planck}} \right)^2 \right) \]

- Figure of merit of LIV:

\[ \xi \approx \frac{c E_p}{d} \frac{\Delta t}{\Delta E} \]

Best sensitivity for:
- Fast variability sources
- Distant sources
- Energetic sources
### Vacuum dispersion

- 2 photons of energies $E_1$ and $E_2 (> E_1)$ emitted at time $t$
  - observed with a relative $\Delta t_{\text{LIV}} = t_2 - t_1$ ($>0$ for subluminal, $<0$ for superluminal)

![Diagram showing vacuum dispersion](image)

- Possible source intrinsic delays

![Diagram showing source intrinsic delays](image)

- Source effect is major caveat: only redshift dependence study can distinguish
LIV tests with Cherenkov Telescopes

- Energy dispersion of time of arrivals in observed gamma rays.

**PKS2155-304 – Big flare (HESS I)**

\[ E_{QG,l} > 2.1 \times 10^{18} \text{ GeV} \]

- Maximum Likelihood method

**Crab-pulsed emission (VERITAS)**

\[ E_{QG,l} > 1.9 \times 10^{17} \text{ GeV} \]

- DisCan method

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Present QG limits on linear term

- Apart from large redshift GRBs & PKS 2155-304, Crab pulsar good competitor.
## Astrophysical probes of LIV with HESS II

<table>
<thead>
<tr>
<th></th>
<th>Pulsar</th>
<th>Active Galactic Nuclei</th>
<th>Gamma Ray Burst</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ad.</strong></td>
<td>• Permanent pulsations</td>
<td>• Extragalactic</td>
<td>• Extragalactic</td>
</tr>
<tr>
<td></td>
<td>• Distinguish between LIV/source effects</td>
<td>• Up to TeV</td>
<td>• Up to TeV ?</td>
</tr>
<tr>
<td><strong>Disad.</strong></td>
<td>• Galactic</td>
<td>• Source effects</td>
<td>• Source effects</td>
</tr>
<tr>
<td></td>
<td>• Up to 400 GeV (Crab) to be confirmed with H.E.S.S.2</td>
<td>• Random transient evts</td>
<td>• Obs. based on luck</td>
</tr>
</tbody>
</table>
| **HESS II running mode** | • Mono  
   ➔ Access lower energies (crucial with pulsars) | • Hybrid  
   ➔ Access higher energies (crucial with AGNs) | • Hybrid                             |
The method of time-lag measurement

Strategy adapted from *Martinez & Errando (Astropart.Phys. 31 (2009) 226)*

\[
P(E,t) = N \int_{0}^{\infty} A(E_s) \Gamma(E_s) G(E - E_s, \sigma(E_s)) F_s(t - \tau_n E_s^n) dE_s
\]

\(A(E_s):\) Acceptance of telescope
\(\Gamma(E_s):\) Spectrum at source
\(G(E-E_s):\) Energy smearing function

The time-lag parameter:
(s/TeV for \(n=1\))
(s/TeV^2 for \(n=2\))

\[
\tau_n = \frac{\Delta t}{\Delta E} \approx \frac{(n+1) \xi}{2 E_p^2 H_o} \int_{0}^{z} \frac{(1+z)}{\sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}} \, dz
\]

1) Parametrize Template Light curve \(F_s(t_s)\) at low energy and spectrum \(A(E_s)\)
2) Use **Maximum Likelihood** at high energy to estimate the time lag parameter.

The likelihood is the product of the p.d.f's over all the photons in the fit:

\[
L = \prod_{i} P(E,t)
\]
HESS II performances with pulsars

The method

- Time delay due to LIV:
  - phase delay between photons of ≠ energies in the reconstructed phasogram.

- Define a phase-lag parameter:
  - (TeV\(^{-1}\) for n=1)
  - (TeV\(^{-2}\) for n=2)

\[ \varphi_n = \frac{\tau_n}{P(t)} \]

\[ P(t) \approx P + \dot{P}t \quad \text{For short time scale: } P(t) \approx P \]

- Parametrize Template Phasogram \( F_s \) at low energy and spectrum \( A(E_s) \)

\[ F_s(t_s - \tau_n E^n_s) \rightarrow F_s(\Phi_s - \varphi_n E^n_s) \]

\[ P(E, t) \rightarrow P(E, \Phi) \]

- Maximum Likelihood at high energy gives estimate on the phase-lag parameter.

\[ L = \prod_i P(E, \Phi) \]
1 single pulse in phasogram
\[ \sigma_{\text{pulse}} = 2 \times 10^{-2} \text{ (rotational phases)} \]
\[ \text{Power law spectrum } E^{-\Gamma} \Gamma = 3.3 \]

Acceptance & energy resolution
\[ \text{H.E.S.S.2 mono} \]
\[ \Delta E/E \sim 35\% \]

2 studies:
\[ \text{B1 model: } S/B = \infty \text{ (>30 GeV)} \]
\[ \text{B2 model: } S/B = 1 \text{ (>30 GeV)} \]

Model is optimistic:
\[ \text{Pulse shape not Gaussian} \]
\[ \text{S/B could be } >1 \text{ due to hadron bckg suppression problems} \]
HESS II performances with pulsars

Template phasogram and spectrum

B1: S/B=∞:
- Fit phasogram using Gaussian pulse.
- Fit spectrum with power law (>55GeV)

B2: S/B=1:
- Fit phasogram using 
  \((1-\beta) \times \text{Gaussian} (\Phi) + \beta \times \text{Uniform} (\Phi)\)
Estimate on phase-lag parameter given by minimum of $-2\Delta\ln(L)$.

- Red: B1 model (no background)
- Blue: B2 model (S/B=1)
- Wider “parabola” due to background contamination.
HESS II performances with pulsars
Calibration of the method

Nb: 0.05 TeV⁻¹ ~ 2 ms/TeV⁻¹ for a “Crab like” pulsar.

<table>
<thead>
<tr>
<th>Model</th>
<th>S/B</th>
<th>N_{evt}</th>
<th>Calibrated error $\sigma_{\phi}$ ($\phi_{\text{inj}} = 0$)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$\infty$</td>
<td>2449</td>
<td>0.0047 ± 0.0001 TeV⁻¹</td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>1</td>
<td>4195</td>
<td>0.0072 ± 0.0002 TeV⁻¹</td>
<td>~35% deterioration</td>
</tr>
<tr>
<td>Quadratic</td>
<td>$\infty$</td>
<td>2216</td>
<td>0.0193 ± 0.0006 TeV⁻²</td>
<td></td>
</tr>
<tr>
<td>Quadratic</td>
<td>1</td>
<td>4038</td>
<td>0.0285 ± 0.0006 TeV⁻²</td>
<td>~32% deterioration</td>
</tr>
</tbody>
</table>
95% CL upper/lower limits on phase lag parameter are derived from $-2\Delta\ln(L)$. Improper coverage, mainly due to:
- Template phasogram uncertainties
- Spectrum parametrization

Refine threshold on $-2\Delta\ln(L)$ to get proper coverage.
- Derive mean upper/lower limits on linear and quadratic phase lag parameter
- Lower limits on quantum gravity scale $E_{QG}$
HESS II performances with pulsars
Sensitivity (linear correction, subluminal)

B1: $S/B=\infty$

$E_{95\% LL}^{EG}$, GeV

Period (ms)

Distance (pc)

Crab

Vela

J1809-2332

J1826-1856

J1709-4429

M.Chrétien, 2nd DIAS workshop
# HESS II performances with pulsars

<table>
<thead>
<tr>
<th>$E_{\text{QG}}^{95% \text{ LL}}$ (GeV) for H.E.S.S.2 pulsar candidates</th>
<th>Linear</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S/B=\infty$</td>
<td>$S/B=1$</td>
</tr>
<tr>
<td>Crab</td>
<td>$1.04 \times 10^{18}$</td>
<td>$5.47 \times 10^{17}$</td>
</tr>
<tr>
<td>PSR J1826-1256*</td>
<td>$&lt; 3.18 \times 10^{18}$</td>
<td>$&lt; 1.83 \times 10^{18}$</td>
</tr>
<tr>
<td>PSR J1709-4429</td>
<td>$3.19 \times 10^{17}$</td>
<td>$1.84 \times 10^{17}$</td>
</tr>
<tr>
<td>PSR J1809-2332</td>
<td>$1.64 \times 10^{17}$</td>
<td>$9.5 \times 10^{16}$</td>
</tr>
<tr>
<td>Vela</td>
<td>$4.69 \times 10^{16}$</td>
<td>$2.71 \times 10^{16}$</td>
</tr>
</tbody>
</table>

* from published upper limit on distance (Fermi 2\textsuperscript{nd} year catalog), distance to the Galaxy’s edge
Modest flare from PKS 2155-304:

- 1 gaussian pulse in 25 min
  - 1000 events > 0.3 TeV
  - $\sigma_{\text{flare}} = 250$ s
  - Power law spectrum $E^{-\Gamma}$, $\Gamma = 3.2$

Acceptance and resolution: HESS II hybrid/mono

- Estimation of no of events in the low energy range for a Template LC
- HESS I/ HESS II sensitivity ratio in 0.15 – 1.0 TeV range ~ 2
- Safe range for likelihood fit (> 0.15 TeV) with respect to: Efficient background suppression
  Assuming a power law spectrum reconstruction
HESS II performances with AGNs

Error calibration

- Statistical precision measurement: calibrated error p.d.f.s

**HESS 1** $E > 0.3$ TeV

**HESS 2** mono $E > 0.15$ TeV

\[ \sigma_T = 8.7 \pm 0.2 \]

\[ \sigma_T = 14.2 \pm 0.9 \text{ s/TeV} \]
HESS II performances with AGNs
Error calibration

**HESS 1 + HESS 2 mono**

**HESS 2 hybrid E > 0.2 TeV**

\[ \sigma_T = 8.0 \pm 0.2 \]

\[ \sigma_T = 7.2 \pm 0.2 \]
**HESS II performances with AGNs**

**Summary**

A. Jacholkowska

<table>
<thead>
<tr>
<th>Mode</th>
<th>N_{evts}</th>
<th>\text{Calibrated error} (\sigma_T) (s/TeV)</th>
<th>Template LC range (TeV)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>HESS 1 (E &gt; 0.3\text{ TeV})</td>
<td>1000</td>
<td>(8.7 \pm 0.2)</td>
<td>0.15 – 0.25</td>
<td>Low intensity flare BF calibration: 5.5 s/TeV</td>
</tr>
<tr>
<td>HESS 2 (\text{mono}) (E &gt; 0.15\text{ TeV})</td>
<td>1100</td>
<td>(14.2 \pm 0.9)</td>
<td>0.05 – 0.12</td>
<td>Not competitive alone</td>
</tr>
<tr>
<td>HESS 1 (\text{+ HESS 2 mono}) (E &gt; 0.3\text{ TeV}) (E &gt; 0.15\text{ TeV})</td>
<td>2100</td>
<td>(8.0 \pm 0.2)</td>
<td>0.15 – 0.25</td>
<td>Suitable for tests: 2 template LCs</td>
</tr>
<tr>
<td>HESS 2 (\text{hybrid}) (E &gt; 0.2\text{ TeV})</td>
<td>3600</td>
<td>(7.2 \pm 0.2)</td>
<td>0.05 – 0.12</td>
<td>Error: 25% improvement</td>
</tr>
</tbody>
</table>

With systematics \(\sigma_{syst} \approx \sigma_{stat}\) \(E_{QG} > 3.50 \times 10^{18}\text{ GeV}\) (95\% CL)
Summary and Conclusions

- **Overall:**
  - Increase in sensitivity <0.2 TeV, better Template at low energy
  - Larger statistics

- If **pulsars** confirmed with HESS II (**Mono running mode**)
  - Permanent pulsations, low systematics
  - “Crab like” competitors to AGNs
  - With millisecond pulsars, could reach the Planck scale.

- With **AGNs** (PKS 2155-304)
  - Hybrid running mode
  - 25% improvement on statistical error compared to HESS I

- With **GRBs**
  - Work is ongoing, preliminary result (z=0.5): $E_{QG} > 1.02 \times 10^{20}$ GeV

- White paper before end of the year
Thanks
Tack!

“That’s a violation of the law of Lorentz invariance, baby”
Futurama, “Law and Oracle” (2011)
Backup slides

HESS II telescope running modes

- Hybrid
  - Higher energy threshold
  - Access higher energies

- Mono
  - Lower energy threshold (smaller effective area)
  - Access lower energies (~<100 GeV)
Backup slides
Calibration curves

![Graph showing calibration curves for B1 with a quadratic fit.](image-url)
Backup slides
Calibration curves

![Graph showing calibration curves with a shaded area and a line indicating a quadratic relationship between reconstructed and injected phase lag.](image)
Backup slides

Distribution of reconstructed phase lag (no LIV)

\[ \chi^2 / \text{ndf} = 6.741 / 12 \]

Constant: \( 82.61 \pm 4.83 \)
Mean: \( 0.0006132 \pm 0.0003319 \)
Sigma: \( 0.00719 \pm 0.00028 \)

B2
LINEAR

\( \phi_i (\text{TeV}^{-1}) \)
Backup slides
Distribution of reconstructed phase lag (no LIV)

B2
QUADRATIC
Backup slides

Effect of LIV on phasogram

Template phasogram

high energy band
phase lag=-1 TeV$^{-1}$

high energy band
phase lag=1 TeV$^{-1}$

high energy band
phase lag=10 TeV$^{-1}$
Generate 500 realizations.
- Inject phase-lag parameter from -0.05 to 0.05 TeV\(^{-1}\). 
- 2 energy intervals: low energy (30 – 55 GeV) and high energy (55 GeV – 1 TeV)
# Backup slides

## Pulsar candidates for HESS II

<table>
<thead>
<tr>
<th>Name (PSR)</th>
<th>( \text{zenith}_{\text{culm}} ) (^{\circ} )</th>
<th>( \log_{10}(F_{10-100 GeV}) ) ( \text{(cm}^{-2}\text{s}^{-1}) )</th>
<th>( \log_{10}(F_{1-100 GeV}) ) ( \text{(cm}^{-2}\text{s}^{-1}) )</th>
<th>([2])</th>
<th>( \Delta \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>J0835-4510*</td>
<td>22</td>
<td>-8.01</td>
<td>-5.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J1709-4429</td>
<td>21</td>
<td>-8.63</td>
<td>-6.72</td>
<td>3.20-3.70</td>
<td></td>
</tr>
<tr>
<td>J1809-2332</td>
<td>0</td>
<td>-9.28</td>
<td>-7.16</td>
<td>3.50-3.70</td>
<td></td>
</tr>
<tr>
<td>J1907+0602</td>
<td>29</td>
<td>-9.47</td>
<td>-7.42</td>
<td>2.90-3.50</td>
<td></td>
</tr>
<tr>
<td>J1826-1256</td>
<td>10</td>
<td>-9.51</td>
<td>-7.27</td>
<td>3.00-3.60</td>
<td></td>
</tr>
<tr>
<td>J1732-3131</td>
<td>8</td>
<td>-9.57</td>
<td>-7.43</td>
<td>3.10-3.50</td>
<td></td>
</tr>
<tr>
<td>J1833-1034</td>
<td>13</td>
<td>-9.63</td>
<td>-7.99</td>
<td>2.30-2.70</td>
<td></td>
</tr>
<tr>
<td>J0633+0632</td>
<td>30</td>
<td>-9.72</td>
<td>-7.81</td>
<td>3.00-3.10</td>
<td></td>
</tr>
<tr>
<td>J1614-2230</td>
<td>1</td>
<td>-10.11</td>
<td>-8.34</td>
<td>2.60-2.70</td>
<td></td>
</tr>
<tr>
<td>J2124-3358</td>
<td>11</td>
<td>-10.16</td>
<td>-8.13</td>
<td>2.10-2.30</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Top-ten list of the best candidates to reach \( 5\sigma \) in 100 hours for observation zenith angles \(<30^{\circ}\). The columns represent (by left-right order): the source culmination zenith angle, the logarithm of the energy flux between 1 and 100 GeV (\( F_{1-100 GeV} \) for Crab pulsar is \( 1.8\times10^{-7} \text{ cm}^{-2}\text{s}^{-1} \)) and the range of values in \( \beta \) allowed in our simulation for each pulsar. The asterisk (*) following Vela pulsar’s name indicates that it is the top-ten list besides of having a strong indication of an exponential cut-off at high energies, since it is the most energetic one in the Southern hemisphere.
### Backup slides

#### 6 Fermi millisecond pulsars

<table>
<thead>
<tr>
<th>Pulsar name</th>
<th>$l$, $b$</th>
<th>$P$ (ms)</th>
<th>$d$ (pc)</th>
<th>$\log \dot{E}$ (ergs s$^{-1}$)</th>
<th>$\delta$</th>
<th>$\Delta$</th>
<th>Photon flux $&gt;0.1$ GeV (10$^{-8}$ photons cm$^{-2}$ s$^{-1}$)</th>
<th>Energy flux $&gt;0.1$ GeV (10$^{-11}$ ergs cm$^{-2}$ s$^{-1}$)</th>
<th>Spectral index</th>
<th>Exponential cutoff energy (GeV)</th>
<th>$\eta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J0030+0451</td>
<td>113.1°, −57.6°</td>
<td>4.865</td>
<td>300 ± 90</td>
<td>33.54</td>
<td>0.16</td>
<td>0.45</td>
<td>$5.5 \pm 0.7$</td>
<td>$4.9 \pm 0.3$</td>
<td>$1.3 \pm 0.2$</td>
<td>$1.9 \pm 0.4$</td>
<td>15 ± 9</td>
</tr>
<tr>
<td>J0218+4232</td>
<td>139.5°, −17.5°</td>
<td>2.323</td>
<td>2700 ± 600*</td>
<td>35.39</td>
<td>0.50</td>
<td>—</td>
<td>$5.6 \pm 1.3$</td>
<td>$3.5 \pm 0.5$</td>
<td>$2.0 \pm 0.2$</td>
<td>$7 \pm 4$</td>
<td>13 ± 6</td>
</tr>
<tr>
<td>J0437−4715</td>
<td>253.4°, −42.0°</td>
<td>5.757</td>
<td>156 ± 2</td>
<td>33.46</td>
<td>0.45</td>
<td>—</td>
<td>$4.4 \pm 1.0$</td>
<td>$1.9 \pm 0.3$</td>
<td>$2.1 \pm 0.3$</td>
<td>$2.1 \pm 1.1$</td>
<td>1.9 ± 0.3</td>
</tr>
<tr>
<td>J0613−0200</td>
<td>210.4°, −9.3°</td>
<td>3.061</td>
<td>480 ± 140</td>
<td>34.10</td>
<td>0.42</td>
<td>—</td>
<td>$3.1 \pm 0.7$</td>
<td>$3.1 \pm 0.3$</td>
<td>$1.4 \pm 0.2$</td>
<td>$2.9 \pm 0.7$</td>
<td>7 ± 4</td>
</tr>
<tr>
<td>J0751+1807</td>
<td>202.7°, 21.1°</td>
<td>3.479</td>
<td>620 ± 310</td>
<td>33.85</td>
<td>0.42</td>
<td>—</td>
<td>$2.0 \pm 0.7$</td>
<td>$1.7 \pm 0.2$</td>
<td>$1.6 \pm 0.2$</td>
<td>$3.4 \pm 1.2$</td>
<td>11 ± 11</td>
</tr>
<tr>
<td>J1614−2230</td>
<td>352.5°, 20.3°</td>
<td>3.151</td>
<td>1300 ± 250*</td>
<td>33.7</td>
<td>0.20</td>
<td>0.48</td>
<td>$2.3 \pm 2.1$</td>
<td>$2.5 \pm 0.8$</td>
<td>$1.0 \pm 0.3$</td>
<td>$1.2 \pm 0.5$</td>
<td>100 ± 80</td>
</tr>
<tr>
<td>J1744−1134</td>
<td>14.8°, 9.2°</td>
<td>4.075</td>
<td>470 ± 90</td>
<td>33.60</td>
<td>0.85</td>
<td>—</td>
<td>$7.1 \pm 1.4$</td>
<td>$4.0 \pm 1.0$</td>
<td>$1.5 \pm 0.2$</td>
<td>$1.1 \pm 0.2$</td>
<td>27 ± 12</td>
</tr>
<tr>
<td>J2124−3358</td>
<td>10.9°, −45.4°</td>
<td>4.931</td>
<td>250 ± 125</td>
<td>33.6</td>
<td>0.85</td>
<td>—</td>
<td>$2.9 \pm 0.5$</td>
<td>$3.4 \pm 0.3$</td>
<td>$1.3 \pm 0.2$</td>
<td>$2.9 \pm 0.9$</td>
<td>6 ± 6</td>
</tr>
</tbody>
</table>

* A. A. Abdo et al., Science 325, 848 (2009);
Backup slides

Sensitivity (linear correction)

M.Chrétien, 2nd DIAS workshop
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Quadratic term

\[ E^{95\% LL}_{QG,q} \]

\[ B1: \ S/B=\infty \]

\[ B2: \ S/B=1 \]
Distinguish between LIV and source intrinsic delays

- **LIV delay:**
  - \( P(t) = P + \frac{dP}{dt} t \) and \( \Delta \Phi(t) = \frac{\Delta t}{P(t)} \) in pulsar frame
  - \( \Delta \Phi \) decreases with time for LIV delays.

- **Source Intrinsic delay:**
  - \( \Delta \Phi = \) Constant in pulsar frame (if not correlated with period increase)
  - No change with time

![Graphs showing LIV and intrinsic delays over time](image)

- **T1 epoch**
- **T2 > T1 epoch**
- **T3 > T2 epoch**