Worldsheet theories of integrable strings

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Supersymmetric Field Theories

15th August 2014

Worldsheet theories of integrable deformations of $AdS_3 \times S^3 \times M^4$ strings

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Theme i

Integrability

- Integrability methods for studying superstrings in $AdS_n \times M^{10-n}$ backgrounds have had remarkable success
  - $AdS_5 \times S^5$, $AdS_4 \times \mathbb{C}\mathbf{P}^3$ – spectral problem "solved"
  - $AdS_3 \times S^3 \times M^4$, $AdS_2 \times S^2 \times T^6$ – significant progress
  - And more ...
- All have largely the same structure controlled by Yangian symmetry
- Preserve integrability while deforming the background/symmetry in a non-trivial way?

Theme ii

q-deformation

- The symmetry algebra defining the S-matrix of the decompactified light-cone gauge $AdS_5 \times S^5$ superstring can be q-deformed [Beisert, Koroteev '08]
  - Leads to a deformation of the S-matrix [Beisert, Koroteev '08; BH, Hollowood, Miramontes '11]
  - Can consider TBA and finite size spectrum [Arutyunov, de Leeuw, van Tongeren '12]
  - Connection with the Pohlmeyer reduction (relativistic symmetry) [Pohlmeyer '76, Grigoriev, Tseytlin '07, BH, Tseytlin '11]
  - Superstring background described by $q(\eta)$-deformation [Delduc, Magro, Vicedo '13 '14, Arutyunov, Borsato, Frolov '13]

Theme iii

$$AdS_3 \times S^3 \times M^4$$

- $M_4 = T^4$ or $S^3 \times S^1$ – for pure RR flux GS action "partly" described by Metsaev-Tseytlin supercoset action

$$\frac{\widehat{G} \times \widehat{G}}{G_0} \qquad\qquad \widehat{G} = PSU(1,1|2) \text{ or } D(2,1;\alpha)$$

[Pesando '98, Rahmfeld, Rajaraman '98, Babichenko, Sfefanski, Zarembo '09, Zarembo '10]

- Interpolations to backgrounds supported by pure NSNS flux
  - simplification through NSR description

- black holes / condensed matter / higher spins

Aim

- Some integrable deformations of these models already known
  - Addition of magnetic field (strength controlled by parameter b)
    [Cagnazzo, Zarembo '12]
  - Symmetric $q(\eta)$ deformation; $\mathcal{U}_{q}(\widehat{G} \times \widehat{G})$
    [Delduc, Magro, Vicedo '13, BH, Roiban, Tseytlin '14]
- Work in progress – can we find a three-parameter deformation?

$$\mathcal{U}_{q_L}(\widehat{G}) \times \mathcal{U}_{q_R}(\widehat{G}) + \mathrm{b}\, H$$

  - Hope comes from fact that such a deformation of S^3 exists
    [Lukyanov '12, (Fateev '96, Klimcik '08, '14)]

Outline

- The undeformed model
- q_L/q_R-deformation
- Magnetic deformation
- S-matrices
- Comments

Undeformed Lagrangian

- Group-valued field

$$f \in \widehat{G} \times \widehat{G} \qquad \longrightarrow \qquad \mathcal{J} = f^{-1} df \in \hat{\mathfrak{g}} \oplus \hat{\mathfrak{g}}$$

- $\mathbb{Z}_4$ decomposition of the algebra (P_k projectors)

$$\Omega(\mathfrak{g}_k) = i^k \mathfrak{g}_k \qquad [\mathfrak{g}_k, \mathfrak{g}_l] \subset \mathfrak{g}_{k+l \text{ mod } 4} \qquad P_k \mathcal{J} \in \mathfrak{g}_k \qquad k = 0, 1, 2, 3$$

- Action constructed from the supertrace over products of $P_{1,2,3}(\mathcal{J})$
  - G_0 gauge symmetry, bosonic diagonal subgroup of $\widehat{G} \times \widehat{G}$

- Bosonic piece is just usual coset model for

$$\frac{G \times G}{G_0} \qquad (SU(2) \cong S^3 \quad SU(1,1) \cong AdS_3)$$

$$\mathcal{S} = \frac{T}{2} \int d^2x (\sqrt{-g} g^{ab} - \epsilon^{ab}) \, \mathrm{STr} \, [\mathcal{J}_a P \mathcal{J}_b] \qquad P = P_1 + 2P_2 - P_3$$

$\mathbb{Z}_4$ decomposition

Take an element of the superalgebra

$$\mathcal{A} = \begin{pmatrix} \mathcal{A} & 0 \\ 0 & \tilde{\mathcal{A}} \end{pmatrix} \in \hat{\mathfrak{g}} \oplus \hat{\mathfrak{g}}$$

and decompose

$$P_0\mathcal{A} = \begin{pmatrix} \mathcal{A}_0 & 0 \\ 0 & \mathcal{A}_0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} P_e(\mathcal{A}+\tilde{\mathcal{A}}) & 0 \\ 0 & P_e(\tilde{\mathcal{A}}+\mathcal{A}) \end{pmatrix}$$

$$P_1\mathcal{A} = \begin{pmatrix} \mathcal{A}_1 & 0 \\ 0 & -i\mathcal{A}_1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} P_o(\mathcal{A}+i\tilde{\mathcal{A}}) & 0 \\ 0 & P_o(\tilde{\mathcal{A}}-i\mathcal{A}) \end{pmatrix}$$

$$P_2\mathcal{A} = \begin{pmatrix} \mathcal{A}_2 & 0 \\ 0 & -\mathcal{A}_2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} P_e(\mathcal{A}-\tilde{\mathcal{A}}) & 0 \\ 0 & P_e(\tilde{\mathcal{A}}-\mathcal{A}) \end{pmatrix}$$

$$P_3\mathcal{A} = \begin{pmatrix} \mathcal{A}_3 & 0 \\ 0 & i\mathcal{A}_3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} P_o(\mathcal{A}-i\tilde{\mathcal{A}}) & 0 \\ 0 & P_o(\tilde{\mathcal{A}}+i\mathcal{A}) \end{pmatrix}$$

P_e and P_o are projections onto Grassmann-even and Grassmann-odd subspaces.

[Babichenko, Stefanski, Zarembo '09]

Lax connection

- Compute equations of motion and flatness equation for $\mathcal{J}$
- Decompose under $\mathbb{Z}_4$ decomposition: $\mathcal{J} \to \mathcal{J}_{0,1,2,3}$
- Resulting equations follow from the flatness of the following Lax connection

$$\mathcal{L}_\pm = \mathcal{J}_{0\pm} + z^{-1}\mathcal{J}_{1\pm} + z\mathcal{J}_{3\pm} + z^{\pm 2}\mathcal{J}_{2\pm}$$

Symmetric q-deformation

$$\mathcal{S} = \frac{T}{2}\int d^2x(\sqrt{-g}g^{ab} - \epsilon^{ab})\,\mathrm{STr}\,[\mathcal{J}_a(P_\eta \frac{1}{1-\frac{\eta}{1-\eta^2}R_f P_\eta}\mathcal{J}_b)]$$

$$P_\eta = (1-\eta^2)(P_1 - P_3) + 2P_2$$

[Delduc, Magro, Vicedo '13, '14]

- $R_f = \mathrm{Ad}_f^{-1}\, R\, \mathrm{Ad}_f$, where R is an antisymmetric constant solution of the (non-split) modified classical Yang-Baxter equation

$$[RM, RN] - R([RM, N] + [M, RN]) = [M, N]\ , \qquad M,\ N \in \hat{\mathfrak{g}} \oplus \hat{\mathfrak{g}}\ ,$$

- "Take R to be the restriction to [the real form] of the operator acting on the complexified algebra by $-i$ on generators associated with positive roots, $+i$ on generators associated with negative roots, and 0 on Cartan generators."

- For reality $\eta \in [0, 1)$ (Pohlmeyer limit given by $\eta \to i$)

Some comments

- Originates from existence of two compatible Poisson brackets, therefore can take linear combination with arbitrary parameter

[Delduc, Magro, Vicedo '13, '14, Hollowood, Miramontes '14]

- Integrability is preserved by construction

- G_0 gauge symmetry $\longrightarrow$ same number of bosonic degrees of freedom
- κ-symmetry $\longrightarrow$ same number of fermionic degrees of freedom

Can we add in appropriate number of flat directions to give a string in a Type IIB supergravity background?

- Global symmetry algebra q-deformed, $q = \exp(-\frac{\varkappa}{T})$ [Delduc, Magro, Vicedo '13, '14]

$$\mathcal{U}_q(\widehat{G} \times \widehat{G})$$

- Natural to write bosonic part of the model in terms of $\varkappa = \frac{2\eta}{1-\eta^2}$

Metric for deformed AdS_3 and deformed S^3

- Choose matrix representative of coset element
- Positive roots to correspond to upper triangular entries, negative to lower
- Invert the operator and compute metric and B-field:

$$ds^2_{AdS} = -\frac{(1+\rho^2)}{1-\varkappa^2\rho^2}dt^2 + \frac{1}{(1+\rho^2)(1-\varkappa^2\rho^2)}d\rho^2 + \rho^2 d\psi^2$$
$$ds^2_{S} = \frac{(1-r^2)}{1+\varkappa^2 r^2}d\varphi^2 + \frac{1}{(1-r^2)(1+\varkappa^2 r^2)}dr^2 + r^2 d\phi^2$$

[Arutyunov, Borsato, Frolov '13, BH, Roiban, Tseytlin '14]

- No magnetic field (total derivative)
- Manifest $U(1)^2$ symmetry in each case
  - Cartans of $SU(2)^2$ and $SU(1,1)^2$, consistent with q-deformation
- Singularity at $\rho = \frac{1}{\varkappa}$

Fermions

- Either need to invert operator including fermionic fields
  or solve Type IIB supergravity equations of motion
- Both computationally difficult problems
  - Inverting large matrices with Grassmann odd entries
  - Questions of uniqueness (already for bosonic non-compact sector)
    [Delduc, Magro, Vicedo '14]
  - Solution of dilaton equation $R + 4\nabla^2\Phi - 4(\nabla_m\Phi)^2 - \frac{1}{12}H_{mnk}^2 = 0$ is not simple
    [Lunin, Roiban, Tseytlin (unpublished)]
- Consider certain limits and truncations ...

Maximal deformation limit

$$\eta \to 1, \ \varkappa \to \infty$$

- **Method 1:** *outside the singularity* [BH, Roiban, Tseytlin '14]
  - $\bar{\rho} \equiv \rho^{-1}$, $\bar{\psi} \equiv \varkappa\psi$, $\bar{r} \equiv r^{-1}$, $\bar{\phi} \equiv \varkappa\phi$
  - $\varkappa \to \infty$
  - T-duality in $\bar{\psi}$ and $\bar{\phi}$ gives dS_3 and H^3
- **Method 2:** *inside the singularity* [Arutyunov, de Leeuw, van Tongeren '14]
  - First rescale t, ρ, φ and r by $\varkappa^{-1}$
  - $\varkappa \to \infty$ gives the mirror model of the light-cone gauge superstring
  - T-duality in t and φ gives dS_3 and H^3
- $dS_3 \times H^3 \times T^4$ solution of Type IIB* supergravity [Hull '98]

Imaginary deformation for $AdS_3 \times S^3$

$$\eta \to i, \; \varkappa \to i$$

$$\varkappa^2 = -1 + \epsilon^2 \qquad t = \epsilon^{-1} x^+ - \epsilon x^- \qquad \varphi = \epsilon^{-1} x^+ + \epsilon x^- \qquad \epsilon \to 0$$

- Gives pp-wave type background ($\rho \equiv \tan\alpha$, $r \equiv \tanh\beta$)

$$ds^2 = 4dx^+dx^- - \left[\sin^2\alpha + \sinh^2\beta\right] dx^{+2} + ds^2_{A\perp} + ds^2_{S\perp}$$
$$ds^2_{A\perp} = d\alpha^2 + \tan^2\alpha \, d\psi^2 \qquad ds^2_{S\perp} = d\beta^2 + \tanh^2\beta \, d\phi^2$$

- After light-cone gauge-fixing we find the following relativistic model

$$\partial_+\alpha\partial_-\alpha + \tan^2\alpha \, \partial_+\psi\partial_-\psi + \mu^2 \sin^2\alpha + \partial_+\beta\partial_-\beta + \tanh^2\beta \, \partial_+\phi\partial_-\phi + \mu^2 \sinh^2\beta$$

- complex sine-Gordon + complex sinh-Gordon, Pohlmeyer reduction of $AdS_3 \times S^3$

Imaginary deformation – supergravity solution

- Metric on previous slide can be extended to solution of 6-d supergravity supported by RR 3-form flux [BH, Roiban, Tseytlin '14]

$$u = \sin\alpha\, e^{i\psi} \qquad w = \sinh\beta\, e^{i\phi}$$

$$\Phi = \Phi_0 - \frac{1}{2}\log(1-|u|^2) - \frac{1}{2}\log(1+|w|^2)$$

$$C_2 = \frac{i}{2}e^{-\Phi_0}\left[(1+|w|^2)(ud\bar{u}-\bar{u}du) + (1-|u|^2)(wd\bar{w}-\bar{w}dw)\right]\wedge dx^+$$

- Solution + T^4 can be extended to 10-dimensional Type IIB supergravity
- Taking GS action and fixing light-cone gauge, we recover Pohlmeyer reduction of $AdS_3\times S^3$ superstring [Grigoriev, Tseytlin '08]
- Expansion of coset still needs to be checked

Asymmetric q-deformation

$$\mathcal{S} = \frac{T}{2}\int d^2x(\sqrt{-g}g^{ab} - \epsilon^{ab})\,\mathrm{STr}\,[\mathcal{J}_a(P_{\eta_{L,R}}\frac{1}{1 - I_{\eta_{L,R}} R_f P_{\eta_{L,R}}}\mathcal{J}_b)]$$

$$P_{\eta_{L,R}} = \sqrt{1-\eta_L^2}\sqrt{1-\eta_R^2}(P_1 - P_3) + 2P_2$$

$$I_{\eta_{L,R}} = \frac{1}{\sqrt{1-\eta_L^2}\sqrt{1-\eta_R^2}}\begin{pmatrix}\eta_L \mathbf{1} & 0 \\ 0 & \eta_R \mathbf{1}\end{pmatrix}$$

[BH (unpublished)]

- Model is classically integrable
- G_0 gauge symmetry $\longrightarrow$ same number of bosonic degrees of freedom
- κ-symmetry $\longrightarrow$ same number of fermionic degrees of freedom

Can we add in appropriate number of flat directions to give a string in a Type IIB supergravity background?

- Conjecture that the global symmetry algebra q-deformed, $q_{L,R} = \exp(-\frac{\varkappa_{L,R}}{T})$

$$\mathcal{U}_{q_L}(\widehat{G}) \times \mathcal{U}_{q_R}(\widehat{G})$$

Metric for asymmetrically deformed S^3

- Bosonic model naturally written in terms of

$$\varkappa_{L,R} = \frac{\eta_{L,R}}{\sqrt{1-\eta_L^2}\sqrt{1-\eta_R^2}}$$

- Deformation of the three-sphere:

$$ds^2 = \frac{4}{4+(\varkappa_L-\varkappa_R)^2+4\varkappa_L\varkappa_R r^2}\Big[\frac{dr^2}{1-r^2}+\frac{1}{2}r^2(1-r^2)(\varkappa_L^2-\varkappa_R^2)d\varphi d\phi$$
$$+(1-r^2)(1+\frac{1}{4}(1-r^2)(\varkappa_L-\varkappa_R)^2)d\varphi^2+r^2(1+\frac{1}{4}r^2(\varkappa_L+\varkappa_R)^2)d\phi^2\Big]$$

- No magnetic field (total derivative)
- Manifest $U(1)^2$ symmetry
  - Cartan subalgebra of $SU(2)^2$, consistent with q-deformation

Some comments

- Corresponding bosonic sigma model is known:
  - integrable deformation of the $O(4)$ sigma model preserving $U(1)^2$ symmetry, preserved under RG flow [Fateev '96]
  - $SU(2)$ case of the bi-Yang-Baxter sigma model (Poisson-Lie deformation of $SU(2)$ principal chiral model) [Klimcik '02, '08, '14]
  - vanishing magnetic field case of Lukyanov's three parameter deformation of the $O(4)$ sigma model. [Lukyanov '14]

- Limits:
  - $\varkappa_L = 0$ or $\varkappa_R = 0$ gives squashed S^3 [Cherednik '81]
  - $\varkappa_L = \pm\varkappa_R$ gives symmetric q-deformation

- $\varkappa_L = \varkappa_R = 0, \infty$ are IR and UV asymptotics, $\varkappa_L = \varkappa_R = i$ is UV fixed point

Lax connection

- Compute equations of motion and flatness equation for $\mathcal{J}$
- Change of variables

$$\mathcal{K}_- = \frac{1}{1 - I_{\eta_{L,R}} R_f P_{\eta_{L,R}}} \mathcal{J}_- \ , \qquad \mathcal{K}_+ = \frac{1}{1 + I_{\eta_{L,R}} R_f \tilde{P}_{\eta_{L,R}}} \mathcal{J}_+$$

- Decompose under $\mathbb{Z}_4$ decomposition: $\mathcal{K} \to \mathcal{K}_{0,1,2,3}$
- Resulting equations follow from the flatness of the following Lax connection

$$\begin{aligned}\mathcal{L}_\pm = \mathcal{K}_{0\pm} + \bar{\eta}\mathcal{K}_{2\pm} &+ \widehat{\eta} z^{\pm 2}\mathcal{K}_{2\pm} \\ &+ \widehat{\eta}^{\frac{1}{2}} \left[z^{-1}(\eta_+\mathcal{K}_{1\pm} - \eta_-\mathcal{K}_{3\pm}) + z(\eta_+\mathcal{K}_{3\pm} - \eta_-\mathcal{K}_{1\pm}) \right]\end{aligned}$$

$$\bar{\eta} = \frac{\eta_L^2 - \eta_R^2}{(1-\eta_L^2)(1-\eta_R^2)} \qquad \widehat{\eta} = \frac{1 - \eta_L^2\eta_R^2}{(1-\eta_L^2)(1-\eta_R^2)} \qquad \eta_\pm = \frac{\sqrt{1-\eta_L^2} \pm \sqrt{1-\eta_R^2}}{2}$$

- Same form as undeformed model [BH (unpublished)]
- Key point is that we had supercoset based on product supergroup, $\widehat{G} \times \widehat{G}$

Magnetic deformation

$$\mathcal{S} = \frac{T}{2}\Big[\int d^2x(\sqrt{-g}g^{ab} - \epsilon^{ab})\,\mathrm{STr}\,[\mathcal{J}_a P_{\mathrm{b}} \mathcal{J}_b]$$
$$- 2\mathrm{b}\int d^3x\,\epsilon^{abc}\,\widetilde{\mathrm{STr}}[\frac{2}{3}(P_2\mathcal{J}_a)(P_2\mathcal{J}_b)(P_2\mathcal{J}_c) + [(P_1\mathcal{J}_a),(P_3\mathcal{J}_b)](P_2\mathcal{J}_c)]\Big]$$
$$P_{\mathrm{b}} = \sqrt{1-\mathrm{b}^2}(P_1 - P_3) + 2P_2$$

- parameter b controlling strength of magnetic field [Cagnazzo, Zarembo '12]
- $\mathrm{b} \in [0,1]$, $\mathrm{b} = 0$ pure RR flux, $\mathrm{b} = 1$ pure NSNS flux
- $\widetilde{\mathrm{STr}}[\mathcal{A}] = \mathrm{STr}[W\mathcal{A}] \qquad W = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1}\end{pmatrix}$
- Integrable, G_0 gauge symmetry, κ-symmetry, SUGRA background known (S-duality)
- Global $\widehat{G} \times \widehat{G}$ symmetry is undeformed

Lax connection

- Compute equations of motion and flatness equation for $\mathcal{J}$
- Decompose under $\mathbb{Z}_4$ decomposition: $\mathcal{J} \to \mathcal{J}_{0,1,2,3}$
- Resulting equations follow from the flatness of the following Lax connection

$$\mathcal{L}_\pm = \mathcal{J}_{0\pm} \pm \mathrm{b}\mathcal{J}_{2\pm} + \widehat{\mathrm{b}} z^{\pm 2}\mathcal{J}_{2\pm} + \widehat{\mathrm{b}}^{\frac{1}{2}}\left[z^{-1}(\mathrm{b}_+\mathcal{J}_{1\pm} + \mathrm{b}_-\mathcal{J}_{3\pm}) + z(\mathrm{b}_+\mathcal{J}_{3\pm} - \mathrm{b}_-\mathcal{J}_{1\pm})\right]$$

$$\widehat{\mathrm{b}} = \sqrt{1-\mathrm{b}^2} \qquad \mathrm{b}_\pm = \sqrt{\frac{1\pm\sqrt{1-\mathrm{b}^2}}{2}}$$

[Cagnazzo, Zarembo '12]

- Same form as undeformed model
- Again key point is that we had supercoset with product supergroup, $\widehat{G} \times \widehat{G}$
- Shifts in different directions, compared to asymmetric q-deformation, which had shifts in the same direction

Lax connections

Asymmetric q-deformation

$$\mathcal{L}_\pm = \mathcal{K}_{0\pm} + \bar{\eta}\mathcal{K}_{2\pm} + \widehat{\eta} z^{\pm 2}\mathcal{K}_{2\pm} + \widehat{\eta}^{\frac{1}{2}}\left[z^{-1}(\eta_+\mathcal{K}_{1\pm} - \eta_-\mathcal{K}_{3\pm}) + z(\eta_+\mathcal{K}_{3\pm} - \eta_-\mathcal{K}_{1\pm})\right]$$

Magnetic deformation

$$\mathcal{L}_\pm = \mathcal{J}_{0\pm} \pm \mathrm{b}\mathcal{J}_{2\pm} + \widehat{\mathrm{b}} z^{\pm 2}\mathcal{J}_{2\pm} + \widehat{\mathrm{b}}^{\frac{1}{2}}\left[z^{-1}(\mathrm{b}_+\mathcal{J}_{1\pm} + \mathrm{b}_-\mathcal{J}_{3\pm}) + z(\mathrm{b}_+\mathcal{J}_{3\pm} - \mathrm{b}_-\mathcal{J}_{1\pm})\right]$$

Three-parameter deformation of Lax connection, based on Lukyanov's model?

$$\mathcal{L}_\pm = \mathcal{K}_{0\pm} + \delta_\pm\mathcal{K}_{2\pm} + \widehat{\delta} z^{\pm 2}\mathcal{K}_{2\pm} + \widehat{\delta}^{\frac{1}{2}}\left[z^{-1}(\delta'\mathcal{K}_{1\pm} + \delta'_1\mathcal{K}_{3\pm}) + (\delta'\mathcal{K}_{3\pm} + \delta'_3\mathcal{K}_{1\pm})\right]$$

Generalities i

Starting point: Superstring world-sheet action for $AdS_3 \times S^3 \times M^4$

$$\frac{T}{2}\int d^2x\sqrt{-g}g^{\alpha\beta}G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu + \text{ fermions}$$

RR flux $\longrightarrow$ use Green-Schwarz action to include fermions

Expansion around background: GS action means we need to expand around bosonic background to consider perturbation theory (fermion kinetic terms)

Gauge-fixing: Conformal gauge leads to non-unitary world-sheet S-matrix
$\longrightarrow$ implementation of Virasoro conditions
Alternative is to consider physical "light-cone" gauge
$\longrightarrow$ introduces mass scale ($x^+ = p^+\tau$) $\longrightarrow$ massive modes

Decompactification limit: to define world-sheet S-matrix

Generalities ii

Pertubation theory: BMN light-cone gauge-fix, take decompactification limit and compute [Rughoonauth, Sundin, Wulff '12, '13, '14]

- 8+8 modes – some massive, some massless, everything coupled and not Lorentz invariant
- Loops using unitarity methods [Engelund, McKeown, Roiban '13, Bianchi, BH '14]
- or perturbation theory (finiteness) [Roiban, Sundin, Tseytlin, Wulff '14]

Integrability: Part of the GS superstring action for $AdS_3 \times S^3 \times M^4$ can be written as a classically integrable supercoset sigma model based on

$$\frac{\widehat{G} \times \widehat{G}}{G_0}$$

- Again consider BMN light-cone gauge
- Identifying residual symmetries and their action allows the S-matrix to be fixed up to phases

Undeformed model

- Light-cone gauge S-matrix for massive modes is built from R-matrices invariant under

$$\mathfrak{u}(1) \Subset \mathfrak{psu}(1|1)^2 \ltimes \mathfrak{u}(1) \ltimes \mathbb{R}^3$$

[Borsato, Ohlsson-Sax, Sfondrini '12, Borsato, Ohlsson-Sax, Sfondrini, Stefanski, Torrielli '13]

- Phases found from solving crossing equation for $AdS_3 \times S^3 \times T^4$

[Borsato, Ohlsson-Sax, Sfondrini, Stefanski, Torrielli '13]

- Phases for $AdS_3 \times S^3 \times S^3 \times S^1$ should basically be the same
- Consistent with semiclassical, perturbation theory and unitarity computations

[Beccaria, Levkovich-Maslyuk, Macorini, Tseytlin '12, Abbott '13]

[Rughoonauth, Sundin, Wulff '12, '13, '14, Roiban, Sundin, Wulff, Tseytlin '14]

[Engelund, Roiban, McKeown '13, Bianchi, BH '14]

Model with magnetic field

- Symmetry algebra not deformed
  - R-matrix is same, deformation parameter appears in the representation

[BH, Tseytlin, Stepanchuk '13]

- Related to extra $\mathfrak{u}(1)$ central element
  - does not constrain the S-matrix

- Phase more complicated, fixed to one-loop by classical integrability and unitarity methods [Babichenko, Dekel, Ohlsson-Sax '14, Bianchi, BH '14]

Symmetric q-deformed model

- In the symmetric case symmetry algebra can be q-deformed and corresponding S-matrix constructed [BH (unpublished)]
- Still have freedom of central $\mathfrak{u}(1)$
  - could be used to include magnetic deformation parameter
- Not clear how to deform the phases, $\Gamma \to \Gamma_q$ not really an option any more

State of play for $AdS_3 \times S^3(\times S^3)$ integrable deformations

Theory	Lax	supercoset	SUGRA	S-matrix	Phases
Undeformed	✓	✓	✓	✓	✓
Magnetic (b)	✓	✓	✓	✓	1-loop
Symmetric q ($q(\eta)$)	✓	✓	(✓)	(✓)	?
Sym. q + mag. ($q(\eta)$, b)	✓	?	?	(✓/?)	?
Asymmetric q (q_L, q_R)	✓	(✓)	?	?	?
Asym. q + mag. (q_L, q_R, b)	(✓/?)	?	?	?	?

Broader open questions

- Related deformations – R can also be solution of classical YBE
  - TsT transformations, duals of non-commutative gauge theories
  - non-integrable backgrounds [Crichigno,Kawaguchi,Matsumoto,Yoshida '14]
  - Unified construction?

- Poisson-Lie dualities – generalization of non-abelian T-duality to Poisson-Lie deformed (q-deformed) symmetry [Klimcik,...]

- Alternative deformation starting from first-order form of the Lagrangian [Sfetsos '13]
  - Appropriate setting for imaginary deformation (Pohlmeyer) [Hollowood, Miramontes, Schmidtt '14]
  - What is relation to q-deformation?

- Uniqueness?
- Supergravity background?
- Dual gauge theory?

Thank you!