Worldsheet theories of integrable strings

Ben Hoare

Humboldt-Universität zu Berlin

Supersymmetric Field Theories
15th August 2014
Worldsheet theories of integrable deformations of $\text{AdS}_3 \times S^3 \times M^4$ strings

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Integrability

Integrability methods for studying superstrings in $AdS_n \times M^{10-n}$ backgrounds have had remarkable success:

- $AdS_5 \times S^5$, $AdS_4 \times \mathbb{CP}^3$ – spectral problem “solved”
- $AdS_3 \times S^3 \times M^4$, $AdS_2 \times S^2 \times T^6$ – significant progress
- And more ...

All have largely the same structure controlled by Yangian symmetry

Preserve integrability while deforming the background/symmetry in a non-trivial way?
Theme ii

The symmetry algebra defining the S-matrix of the decompactified light-cone gauge $AdS_5 \times S^5$ superstring can be $q$-deformed

- Leads to a deformation of the S-matrix
  [Beisert, Koroteev '08; BH, Hollowood, Miramontes '11]

- Can consider TBA and finite size spectrum
  [Arutyunov, de Leeuw, van Tongeren '12]

- Connection with the Pohlmeyer reduction (relativistic symmetry)
  [Pohlmeyer '76, Grigoriev, Tseytlin '07, BH, Tseytlin '11]

- Superstring background described by $q(\eta)$-deformation
  [Delduc, Magro, Vicedo '13 '14, Arutyunov, Borsato, Frolov '13]
Theme iii

- $M_4 = T^4$ or $S^3 \times S^1$ – for pure RR flux GS action “partly” described by Metsaev-Tseytlin supercoset action

$$\frac{\hat{G} \times \hat{G}}{G_0} \quad \hat{G} = PSU(1,1|2) \text{ or } D(2,1;\alpha)$$

[Pesando '98, Rahmfeld, Rajaraman '98, Babichenko, Sfetanski, Zarembo '09, Zarembo '10]

- Interpolations to backgrounds supported by pure NSNS flux
  - simplification through NSR description

- black holes / condensed matter / higher spins

Ben Hoare

Humboldt-Universität zu Berlin

Worldsheet theories of integrable strings
Aim

- Some integrable deformations of these models already known
  - Addition of magnetic field (strength controlled by parameter $b$)
    
    $[\text{Cagnazzo, Zarembo '12}]$
  
  - Symmetric $q(\eta)$ deformation; $\mathcal{U}_q(\hat{G} \times \hat{G})$
    
    $[\text{Delduc, Magro, Vicedo '13, BH, Roiban, Tseytlin '14}]$

- Work in progress – can we find a three-parameter deformation?

  $\mathcal{U}_{q_L}(\hat{G}) \times \mathcal{U}_{q_R}(\hat{G}) + b\, H$

- Hope comes from fact that such a deformation of $S^3$ exists

  $[\text{Lukyanov '12, (Fateev '96, Klimcik '08, '14)}]$
Outline

- The undeformed model
- $q_L/q_R$-deformation
- Magnetic deformation
- S-matrices
- Comments
**Undeformed Lagrangian**

- Group-valued field
  
  \[ f \in \hat{G} \times \hat{G} \quad \rightarrow \quad \mathcal{J} = f^{-1}df \in \hat{g} \oplus \hat{g} \]

- \( \mathbb{Z}_4 \) decomposition of the algebra (\( P_k \) projectors)
  
  \[ \Omega(g_k) = i^k g_k \quad [g_k, g_l] \subset g_{k+l} \mod 4 \quad P_k \mathcal{J} \in g_k \quad k = 0, 1, 2, 3 \]

- Action constructed from the supertrace over products of \( P_{1,2,3}(\mathcal{J}) \)
  
  - \( G_0 \) gauge symmetry, bosonic diagonal subgroup of \( \hat{G} \times \hat{G} \)

- Bosonic piece is just usual coset model for
  
  \[
  \frac{G \times G}{G_0} \quad (SU(2) \cong S^3 \quad SU(1, 1) \cong AdS_3)
  \]

\[ S = \frac{T}{2} \int d^2x \left( \sqrt{-g} g^{ab} - \epsilon^{ab} \right) \text{STr} [\mathcal{J}_a P \mathcal{J}_b] \quad P = P_1 + 2P_2 - P_3 \]
\[ \mathbb{Z}_4 \text{ decomposition} \]

Take an element of the superalgebra

\[ \mathcal{A} = \begin{pmatrix} \mathcal{A} & 0 \\ 0 & \tilde{\mathcal{A}} \end{pmatrix} \in \mathfrak{g} \oplus \mathfrak{g} \]

and decompose

\[ P_0 \mathcal{A} = \begin{pmatrix} \mathcal{A}_0 & 0 \\ 0 & \mathcal{A}_0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} P_e(\mathcal{A} + \tilde{\mathcal{A}}) & 0 \\ 0 & P_e(\tilde{\mathcal{A}} + \mathcal{A}) \end{pmatrix} \]

\[ P_1 \mathcal{A} = \begin{pmatrix} \mathcal{A}_1 & 0 \\ 0 & -i\mathcal{A}_1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} P_o(\mathcal{A} + i\tilde{\mathcal{A}}) & 0 \\ 0 & P_o(\tilde{\mathcal{A}} - i\mathcal{A}) \end{pmatrix} \]

\[ P_2 \mathcal{A} = \begin{pmatrix} \mathcal{A}_2 & 0 \\ 0 & -\mathcal{A}_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} P_e(\mathcal{A} - \tilde{\mathcal{A}}) & 0 \\ 0 & P_e(\tilde{\mathcal{A}} - \mathcal{A}) \end{pmatrix} \]

\[ P_3 \mathcal{A} = \begin{pmatrix} \mathcal{A}_3 & 0 \\ 0 & i\mathcal{A}_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} P_o(\mathcal{A} - i\tilde{\mathcal{A}}) & 0 \\ 0 & P_o(\tilde{\mathcal{A}} + i\mathcal{A}) \end{pmatrix} \]

\( P_e \) and \( P_o \) are projections onto Grassmann-even and Grassmann-odd subspaces.

[Babichenko, Stefanski, Zarembo ’09]
**Lax connection**

- Compute equations of motion and flatness equation for $J$
- Decompose under $\mathbb{Z}_4$ decomposition: $J \rightarrow J_{0,1,2,3}$
- Resulting equations follow from the flatness of the following Lax connection

\[ \mathcal{L}_\pm = J_{0\pm} + z^{-1}J_{1\pm} + zJ_{3\pm} + z^{\pm 2}J_{2\pm} \]
Symmetric q-deformation

\[ S = \frac{T}{2} \int d^2x (\sqrt{-g} g^{ab} - \epsilon^{ab}) \text{STr} \left[ \partial_a (P_\eta \frac{1}{1 - \eta^2} R_f P_\eta \partial_b) \right] \]

\[ P_\eta = (1 - \eta^2)(P_1 - P_3) + 2P_2 \]

- \( R_f = \text{Ad}_f^{-1} R \text{Ad}_f \), where R is an antisymmetric constant solution of the (non-split) modified classical Yang-Baxter equation

\[ [RM, RN] - R([RM, N] + [M, RN]) = [M, N], \quad M, N \in \mathfrak{g} \oplus \mathfrak{g}, \]

- “Take \( R \) to be the restriction to [the real form] of the operator acting on the complexified algebra by \(-i\) on generators associated with positive roots, \(+i\) on generators associated with negative roots, and 0 on Cartan generators.”

- For reality \( \eta \in [0, 1) \) (Pohlmeyer limit given by \( \eta \to i \))
Some comments

- Originates from existence of two compatible Poisson brackets, therefore can take linear combination with arbitrary parameter

[Delduc, Magro, Vicedo '13, '14, Hollowood, Miramontes '14]

- Integrability is preserved by construction

- $G_0$ gauge symmetry $\rightarrow$ same number of bosonic degrees of freedom
- $\kappa$-symmetry $\rightarrow$ same number of fermionic degrees of freedom

Can we add in appropriate number of flat directions to give a string in a Type IIB supergravity background?

- Global symmetry algebra q-deformed, $q = \exp(-\frac{\kappa}{T})$  

$\mathcal{U}_q(\hat{G} \times \hat{G})$

- Natural to write bosonic part of the model in terms of $\kappa = \frac{2\eta}{1-\eta^2}$
Metric for deformed $AdS_3$ and deformed $S^3$

- Choose matrix representative of coset element
- Positive roots to correspond to upper triangular entries, negative to lower
- Invert the operator and compute metric and B-field:

\[
\begin{align*}
\text{ds}^2_{\text{AdS}} &= -\frac{(1 + \rho^2)}{1 - \kappa^2 \rho^2} dt^2 + \frac{1}{(1 + \rho^2)(1 - \kappa^2 \rho^2)} d\rho^2 + \rho^2 d\psi^2 \\
\text{ds}^2_{\text{S}} &= \frac{(1 - r^2)}{1 + \kappa^2 r^2} d\phi^2 + \frac{1}{(1 - r^2)(1 + \kappa^2 r^2)} dr^2 + r^2 d\phi^2
\end{align*}
\]

[Arutyunov, Borsato, Frolov '13, BH, Roiban, Tseytlin '14]

- No magnetic field (total derivative)
- Manifest $U(1)^2$ symmetry in each case
  - Cartans of $SU(2)^2$ and $SU(1,1)^2$, consistent with q-deformation
- Singularity at $\rho = \frac{1}{\kappa}$

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Worldsheet theories of integrable strings
Fermions

- Either need to invert operator including fermionic fields or solve Type IIB supergravity equations of motion

- Both computationally difficult problems
  - Inverting large matrices with Grassmann odd entries
  - Questions of uniqueness (already for bosonic non-compact sector)

- Solution of dilaton equation \( R + 4\nabla^2 \Phi - 4(\nabla_m \Phi)^2 - \frac{1}{12} H_{mnk}^2 = 0 \) is not simple

- Consider certain limits and truncations ...
Maximal deformation limit

- **Method 1**: outside the singularity
  - $\bar{\rho} \equiv \rho^{-1}$, $\bar{\psi} \equiv \kappa \psi$, $\bar{r} \equiv r^{-1}$, $\bar{\phi} \equiv \kappa \phi$
  - $\kappa \rightarrow \infty$
  - T-duality in $\bar{\psi}$ and $\bar{\phi}$ gives $dS_3$ and $H^3$

- **Method 2**: inside the singularity
  - First rescale $t$, $\rho$, $\varphi$ and $r$ by $\kappa^{-1}$
  - $\kappa \rightarrow \infty$ gives the mirror model of the light-cone gauge superstring
  - T-duality in $t$ and $\varphi$ gives $dS_3$ and $H^3$

- $dS_3 \times H^3 \times T^4$ solution of Type IIB* supergravity

[BH, Roiban, Tseytlin '14]

[Arutyunov, de Leeuw, van Tongeren '14]

[Hull '98]
Imaginary deformation for $\text{AdS}_3 \times S^3$

$$\eta \rightarrow i, \kappa \rightarrow i$$

$$\kappa^2 = -1 + \epsilon^2, \quad t = \epsilon^{-1} x^+ - \epsilon x^-, \quad \varphi = \epsilon^{-1} x^+ + \epsilon x^- \quad \epsilon \rightarrow 0$$

- Gives pp-wave type background ($\rho \equiv \tan \alpha, \ r \equiv \tanh \beta$)

$$ds^2 = 4dx^+ dx^- - \left[ \sin^2 \alpha + \sinh^2 \beta \right] dx^{+2} + ds^2_{A \perp} + ds^2_{S \perp}$$

$$ds^2_{A \perp} = d\alpha^2 + \tan^2 \alpha \ d\psi^2 \quad ds^2_{S \perp} = d\beta^2 + \tanh^2 \beta \ d\phi^2$$

- After light-cone gauge-fixing we find the following relativistic model

$$\partial_+ \alpha \partial_- \alpha + \tan^2 \alpha \ \partial_+ \psi \partial_- \psi + \mu^2 \sin^2 \alpha + \partial_+ \beta \partial_- \beta + \tanh^2 \beta \ \partial_+ \phi \partial_- \phi + \mu^2 \sinh^2 \beta$$

- complex sine-Gordon + complex sinh-Gordon, Pohlmeyer reduction of $\text{AdS}_3 \times S^3$
Imaginary deformation – supergravity solution

- Metric on previous slide can be extended to solution of 6-d supergravity supported by RR 3-form flux

\[ u = \sin \alpha \, e^{i\psi} \quad w = \sinh \beta \, e^{i\phi} \]

\[ \Phi = \Phi_0 - \frac{1}{2} \log(1 - |u|^2) - \frac{1}{2} \log(1 + |w|^2) \]

\[ C_2 = \frac{i}{2} e^{-\Phi_0} \left[ (1 + |w|^2)(ud\bar{u} - \bar{u}du) + (1 - |u|^2)(wd\bar{w} - \bar{w}dw) \right] \wedge dx^+ \]

- Solution + $T^4$ can be extended to 10-dimensional Type IIB supergravity
- Taking GS action and fixing light-cone gauge, we recover Pohlmeyer reduction of $AdS_3 \times S^3$ superstring
- Expansion of coset still needs to be checked
Asymmetric q-deformation

\[
S = \frac{T}{2} \int d^2 x (\sqrt{-g} g^{ab} - \epsilon^{ab}) \text{STr} \left[ J_a \left( P_{\eta_{L,R}} \frac{1}{1 - I_{\eta_{L,R}} R_f P_{\eta_{L,R}}} \right) \right]
\]

\[
P_{\eta_{L,R}} = \sqrt{1 - \eta_L^2} \sqrt{1 - \eta_R^2} (P_1 - P_3) + 2P_2
\]

\[
I_{\eta_{L,R}} = \frac{1}{\sqrt{1 - \eta_L^2} \sqrt{1 - \eta_R^2}} \begin{pmatrix} \eta_L & 1 & 0 \\ 0 & 1 & \eta_R \\ \end{pmatrix}
\]

- Model is classically integrable
- \(G_0\) gauge symmetry \(\rightarrow\) same number of bosonic degrees of freedom
- \(\kappa\)-symmetry \(\rightarrow\) same number of fermionic degrees of freedom

**Can we add in appropriate number of flat directions to give a string in a Type IIB supergravity background?**

- Conjecture that the global symmetry algebra q-deformed, \(q_{L,R} = \exp(-\frac{\kappa_{L,R}}{T})\)

\[
\mathcal{U}_{q_L}(\hat{G}) \times \mathcal{U}_{q_R}(\hat{G})
\]
Metric for asymmetrically deformed $S^3$

- Bosonic model naturally written in terms of

$$\kappa_{L,R} = \frac{\eta_{L,R}}{\sqrt{1 - \eta_L^2/\sqrt{1 - \eta_R^2}}}$$

- Deformation of the three-sphere:

$$ds^2 = \frac{4}{4 + (\kappa_L - \kappa_R)^2 + 4\kappa_L \kappa_R r^2} \left[ \frac{dr^2}{1 - r^2} + \frac{1}{2} r^2 (1 - r^2)(\kappa_L^2 - \kappa_R^2) d\varphi d\phi 
  + (1 - r^2)(1 + \frac{1}{4} (1 - r^2)(\kappa_L - \kappa_R)^2) d\varphi^2 + r^2 (1 + \frac{1}{4} r^2 (\kappa_L + \kappa_R)^2) d\phi^2 \right]$$

- No magnetic field (total derivative)

- Manifest $U(1)^2$ symmetry
  - Cartan subalgebra of $SU(2)^2$, consistent with $q$-deformation
Some comments

- Corresponding bosonic sigma model is known:
  - integrable deformation of the \(O(4)\) sigma model preserving \(U(1)^2\) symmetry, preserved under RG flow \[\text{[Fateev '96]}\]
  - \(SU(2)\) case of the bi-Yang-Baxter sigma model
    (Poisson-Lie deformation of \(SU(2)\) principal chiral model) \[\text{[Klimcik '02, '08, '14]}\]
  - vanishing magnetic field case of Lukyanov’s three parameter deformation of the \(O(4)\) sigma model. \[\text{[Lukyanov '14]}\]

- Limits:
  - \(\kappa_L = 0\) or \(\kappa_R = 0\) gives squashed \(S^3\) \[\text{[Cherednik '81]}\]
  - \(\kappa_L = \pm \kappa_R\) gives symmetric q-deformation

- \(\kappa_L = \kappa_R = 0, \infty\) are IR and UV asymptotics, \(\kappa_L = \kappa_R = i\) is UV fixed point
Lax connection

- Compute equations of motion and flatness equation for $J$
- Change of variables

\[ \mathcal{K}_- = \frac{1}{1 - I_{\eta_{L,R}} R_f P \eta L R} J_- , \quad \mathcal{K}_+ = \frac{1}{1 + I_{\eta_{L,R}} R_f \tilde{P} \eta_{L,R}} J_+ \]

- Decompose under $\mathbb{Z}_4$ decomposition: $\mathcal{K} \rightarrow \mathcal{K}_{0,1,2,3}$
- Resulting equations follow from the flatness of the following Lax connection

\[ \mathcal{L}_\pm = \mathcal{K}_{0\pm} + \bar{\eta} \mathcal{K}_{2\pm} + \bar{\eta} z^{\pm 2} \mathcal{K}_{2\pm} \]
\[ + \bar{\eta}^{\frac{1}{2}} [ z^{-1} (\eta_+ \mathcal{K}_{1\pm} - \eta_- \mathcal{K}_{3\pm}) + z (\eta_+ \mathcal{K}_{3\pm} - \eta_- \mathcal{K}_{1\pm}) ] \]

\[ \bar{\eta} = \frac{\eta^2 L - \eta^2 R}{(1 - \eta^2 L)(1 - \eta^2 R)} \quad \hat{\eta} = \frac{1 - \eta^2 L \eta^2 R}{(1 - \eta^2 L)(1 - \eta^2 R)} \quad \eta_\pm = \frac{\sqrt{1 - \eta^2 L} \pm \sqrt{1 - \eta^2 R}}{2} \]

- Same form as undeformed model
  
  \[ \text{[BH (unpublished)]} \]

- Key point is that we had supercoset based on product supergroup, $\hat{G} \times \hat{G}$
Magnetic deformation

\[ S = \frac{T}{2} \left[ \int d^2 x (\sqrt{-g} g^{ab} - \epsilon^{ab}) S\text{Tr} [J_a P_b J_b] \right. \\
\left. - 2b \int d^3 x \epsilon^{abc} \widetilde{S}\text{Tr} \left[ \frac{2}{3} (P_2 J_a)(P_2 J_b)(P_2 J_c) + [(P_1 J_a), (P_3 J_b)](P_2 J_c) \right] \right] \]

\[ P_b = \sqrt{1 - b^2} (P_1 - P_3) + 2P_2 \]

- parameter \( b \) controlling strength of magnetic field
- \( b \in [0, 1] \), \( b = 0 \) pure RR flux, \( b = 1 \) pure NSNS flux
- \( \widetilde{S}\text{Tr}[A] = S\text{Tr}[W A] \)
- \( W = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \)
- Integrable, \( G_0 \) gauge symmetry, \( \kappa \)-symmetry, SUGRA background known (S-duality)
- Global \( \hat{G} \times \hat{G} \) symmetry is undeformed

[Cagnazzo, Zarembo '12]
Lax connection

- Compute equations of motion and flatness equation for $\mathcal{J}$
- Decompose under $\mathbb{Z}_4$ decomposition: $\mathcal{J} \rightarrow \mathcal{J}_{0,1,2,3}$
- Resulting equations follow from the flatness of the following Lax connection

$$
\mathcal{L}_\pm = J_{0\pm} \pm b J_{2\pm} + \hat{b} z^{\pm 2} J_{2\pm} + \hat{b}^{1/2} \left[ z^{-1} (b_+ J_{1\pm} + b_- J_{3\pm}) + z (b_+ J_{3\pm} - b_- J_{1\pm}) \right]
$$

$$
\hat{b} = \sqrt{1 - b^2} \quad b_\pm = \sqrt{\frac{1 \pm \sqrt{1 - b^2}}{2}}
$$

- Same form as undeformed model
- Again key point is that we had supercoset with product supergroup, $\hat{G} \times \hat{G}$
- Shifts in different directions, compared to asymmetric q-deformation, which had shifts in the same direction

[Cagnazzo, Zarembo '12]
Lax connections

Asymmetric q-deformation

\[ L_\pm = \mathcal{K}_0 \pm + \hat{\eta} \mathcal{K}_2 \pm + \hat{\eta} z^{\pm 2} \mathcal{K}_2 \pm + \hat{\eta}^{\frac{1}{2}} \left[ z^{-1} (\eta_+ \mathcal{K}_1 \pm - \eta_- \mathcal{K}_3 \pm) + z (\eta_+ \mathcal{K}_3 \pm - \eta_- \mathcal{K}_1 \pm) \right] \]

Magnetic deformation

\[ L_\pm = \mathcal{J}_0 \pm \pm b \mathcal{J}_2 \pm + \hat{b} z^{\pm 2} \mathcal{J}_2 \pm + b^{\frac{1}{2}} \left[ z^{-1} (b_+ \mathcal{J}_1 \pm + b_- \mathcal{J}_3 \pm) + z (b_+ \mathcal{J}_3 \pm - b_- \mathcal{J}_1 \pm) \right] \]

Three-parameter deformation of Lax connection, based on Lukyanov’s model?

\[ L_\pm = \mathcal{K}_0 \pm + \delta_\pm \mathcal{K}_2 \pm + \hat{\delta} z^{\pm 2} \mathcal{K}_2 \pm + \hat{\delta}^{\frac{1}{2}} \left[ z^{-1} (\delta'_+ \mathcal{K}_1 \pm + \delta'_- \mathcal{K}_3 \pm) + (\delta'_+ \mathcal{K}_3 \pm + \delta'_- \mathcal{K}_1 \pm) \right] \]
Generalities i

**Starting point:** Superstring world-sheet action for $AdS_3 \times S^3 \times M^4$

$$\frac{T}{2} \int d^2x \sqrt{-g} g^{\alpha\beta} G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + \text{fermions}$$

RR flux $\rightarrow$ use Green-Schwarz action to include fermions

**Expansion around background:** GS action means we need to expand around bosonic background to consider perturbation theory (fermion kinetic terms)

**Gauge-fixing:** Conformal gauge leads to non-unitary world-sheet S-matrix

$\rightarrow$ implementation of Virasoro conditions

Alternative is to consider physical “light-cone” gauge

$\rightarrow$ introduces mass scale $(x^+ = p^+ \tau) \rightarrow$ massive modes

**Decompactification limit:** to define world-sheet S-matrix
Generalities ii

**Perturbation theory:** BMN light-cone gauge-fix, take decompactification limit and compute

- 8+8 modes – some massive, some massless, everything coupled and not Lorentz invariant
- Loops using unitarity methods
- or perturbation theory (finiteness)

**Integrability:** Part of the GS superstring action for $AdS_3 \times S^3 \times M^4$ can be written as a classically integrable supercoset sigma model based on

$$\frac{\hat{G} \times \hat{G}}{G_0}$$

- Again consider BMN light-cone gauge
- Identifying residual symmetries and their action allows the S-matrix to be fixed up to phases
Motivation

Undeformed model

- Light-cone gauge S-matrix for massive modes is built from R-matrices invariant under
  \[ u(1) \in \text{psu}(1|1)^2 \ltimes u(1) \ltimes \mathbb{R}^3 \]

  [Borsato, Ohlsson-Sax, Sfondrini '12, Borsato, Ohlsson-Sax, Sfondrini, Stefanski, Torrielli '13]

- Phases found from solving crossing equation for \( \text{AdS}_3 \times S^3 \times T^4 \)

  [Borsato, Ohlsson-Sax, Sfondrini, Stefanski, Torrielli '13]

- Phases for \( \text{AdS}_3 \times S^3 \times S^3 \times S^1 \) should basically be the same

- Consistent with semiclassical, perturbation theory and unitarity computations

  [Beccaria, Levkovich-Maslyuk, Macorini, Tseytlin '12, Abbott '13]
  [Rughoonauth, Sundin, Wulff '12, '13, '14, Roiban, Sundin, Wulff, Tseytlin '14]
  [Engelund, Roiban, McKeown '13, Bianchi, BH '14]
Model with magnetic field

- Symmetry algebra not deformed
  - $R$-matrix is same, deformation parameter appears in the representation
    \[ \text{[BH, Tseytlin, Stepanchuk '13]} \]
- Related to extra $u(1)$ central element
  - does not constrain the $S$-matrix
- Phase more complicated, fixed to one-loop by classical integrability and unitarity methods
  \[ \text{[Babichenko, Dekel, Ohlsson-Sax '14, Bianchi, BH '14]} \]

Symmetric q-deformed model

- In the symmetric case symmetry algebra can be q-deformed and corresponding $S$-matrix constructed
  \[ \text{[BH (unpublished)]} \]
- Still have freedom of central $u(1)$
  - could be used to include magnetic deformation parameter
- Not clear how to deform the phases, $\Gamma \rightarrow \Gamma_q$ not really an option any more
State of play for $AdS_3 \times S^3(\times S^3)$ integrable deformations

<table>
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<th>Theory</th>
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<th>supercoset</th>
<th>SUGRA</th>
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Ben Hoare
Humboldt-Universität zu Berlin

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Motivation    Undeformed    q-deformation    Magnetic    S-matrices    Comments

[Image 0x-0 to 363x272]
**Broader open questions**

- Related deformations – $R$ can also be solution of classical YBE
  - TsT transformations, duals of non-commutative gauge theories
  - non-integrable backgrounds
- Unified construction? [Crichigno, Kawaguchi, Matsumoto, Yoshida '14]

- Poisson-Lie dualities – generalization of non-abelian T-duality to Poisson-Lie deformed (q-deformed) symmetry [Klimcik, ...]

- Alternative deformation starting from first-order form of the Lagrangian [Sfetsos '13]
  - Appropriate setting for imaginary deformation (Pohlmeyer)
  - What is relation to q-deformation? [Hollowood, Miramontes, Schmidtt '14]

- Uniqueness?
- Supergravity background?
- Dual gauge theory?
Thank you!

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