Mirror dual (string) sigma models

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Supersymmetric Field Theories, Nordita
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1403.6104 with Gleb Arutyunov and Marius de Leeuw
1406.2304 and 14xx.xxxx with Gleb Arutyunov

Supersymmetric field theories

$$
\begin{aligned}
ds^2 = & -(1+\rho^2)dt^2 + \frac{d\rho^2}{1+\rho^2} + \rho^2 d\Omega_3 \\
& + (1-r^2)d\phi^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega_3,
\end{aligned}
$$

Supersymmetric? field theories

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\begin{aligned}
ds^2 = {} & \frac{1}{1-r^2}(-dt^2 + dr^2) + r^2 d\Omega_3 \\
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\end{aligned}
$$

Outline

- The $\mathrm{AdS}_5 \times \mathrm{S}^5$ string and the mirror model
- $\left(\mathrm{AdS}_5 \times \mathrm{S}^5\right)_\eta$
- Mirror duality
- Mirror $\mathrm{AdS}_5 \times \mathrm{S}^5$

A closed superstring on $\mathrm{AdS}_5 \times S^5$

- Sigma model on $\frac{\mathrm{PSU}(2,2|4)}{\mathrm{SO}(4,1)\times\mathrm{SO}(5)}$

  Metsaev, Tseytlin '98

- Classically integrable field theory

  Bena, Polchinski, Roiban '03

- Light-cone gauge: massive 2d QFT (8b+8f)

- 8b+8f form fundamental (short) rep of $\mathfrak{psu}(2|2)^{\oplus 2}_{c.e.}$

- $\mathfrak{psu}(2|2)^{\oplus 2}_{c.e.}$: exact two-body S-matrix and dispersion

  Beisert '05 ('08); Arutyunov, Frolov, Plefka, Zamaklar '07; Arutyunov, Frolov, Zamaklar '07

- Compatible with quantum integrability

Parametrization

- Dispersion relation

$$E(p) = \sqrt{1 + 4g^2 \sin^2 \frac{p}{2}}$$

- Convenient parametrization $x^\pm$

$$E = ig(x^- - x^+) - 1$$
$$p = -i \log x^+/x^-$$

- Nice variable $u = x + 1/x$, $x^\pm = x(u \pm i/g)$

$$\Rightarrow x(u) \text{ branch points } \pm 2$$

- $\to E(u_i), p(u_i), S(u_i, u_j)$

Integrable quantum field theory

- Factorized scattering on large circle ($L \gg 1$) gives BY eqs

$$e^{ip_m L} \prod_{n \neq m} S_{mn} = 1$$

- Finite L? Mirror trick (double Wick rotation)

$$E \to i\tilde{p}\,, \;\; p \to i\tilde{E}$$

- $E_0(L) = \tilde{f}|_{\tilde{\beta}=L}$ (mirror volume is infinite!)
- Mirror free energy from thermodynamic Bethe ansatz
- Entire spectrum by ‘analytic continuation’

The mirror model

Double Wick rotation of light-cone world sheet theory

- Spectral problem

  Ambjorn, Janik, Kristjansen '06, Arutyunov, Frolov '07
- Polygonal WL/scattering amplitude problem

  Basso, Sever, Vieira '13

mirror model $=$ model of ??

A closed superstring on $\left(\mathrm{Ad}S_5 \times S^5\right)_\eta$

- Deformation of coset sigma model — Delduc, Magro, Vicedo '13
- Classically integrable field theory
- $\mathrm{PSU}_q(2,2|4)$ symmetry
- $\mathfrak{psu}_q(2|2)^{\oplus 2}_{c.e.}$: S-matrix and dispersion — Arutyunov, Borsato, Frolov '13 (Beisert, Koroteev '08, Hoare, Hollowood, Miramontes '11)
- Compatible with quantum integrability

Deformed parametrization

- Parameters: tension (T) and $q \to \xi(T,q), q$
- Semi-classically

Arutyunov, Borsato, Frolov '13

$$q = e^{-\varkappa/T}\,, \quad \xi = i\varkappa$$

- Beyond semi-classics? unitarity $\Rightarrow \xi$ imaginary $\equiv i \tan \frac{\vartheta}{2}$
- Treat generic $\vartheta \in (0,\pi)$ and (positive) $a = -\log q$

Dispersion relation

- Deformed dispersion relation:

$$\cos^2\frac{\vartheta}{2}\sinh^2\frac{aE}{2}-\sin^2\frac{\vartheta}{2}\sin^2\frac{p}{2}=\sinh^2\frac{a}{2}$$

- Matches giant magnon

$$E=2T\left(\varkappa+\frac{1}{\varkappa}\right)\operatorname{arcsinh}\varkappa\left|\sin\frac{p}{2}\right|$$

- Mirror dispersion relation?

Mirror duality

- $\vartheta \leftrightarrow \pi - \vartheta \quad \Leftrightarrow \quad$ string $\leftrightarrow$ mirror (dispersion)
- Parametrization: $x(u)$ branch points $\pm\vartheta$, $x^{\pm} = x(u \pm ia)$
- *(Under a suitable mapping of charges) the mirror theory at ϑ_0 is nothing but the 'string' theory at $\pi - \vartheta_0$!*

Scattering duality

- S-matrix respects mirror duality
  - In particular $\sigma|_{\vartheta=\vartheta_0} = \tilde{\Sigma}^{-1}|_{\vartheta=\pi-\vartheta_0}$

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$$\Phi(x_1,x_2) = i \oint_{\mathcal{C}_\vartheta} \frac{dz}{2\pi i} \oint_{\mathcal{C}_\vartheta} \frac{dw}{2\pi i} \frac{1}{z-x_1} \frac{1}{w-x_2} \log \frac{\Gamma_{q^2}(1+\frac{i}{2a}(u_z-u_w))}{\Gamma_{q^2}(1-\frac{i}{2a}(u_z-u_w))}$$

$$\Psi(x_1,x_2) = i \oint_{\mathcal{C}_\vartheta} \frac{dz}{2\pi i} \frac{1}{z-x_2} \log \frac{\Gamma_{q^2}(1+\frac{i}{2a}(u_1-u_z))}{\Gamma_{q^2}(1-\frac{i}{2a}(u_1-u_z))}$$

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$$\Phi(x_1^+,x_2^+)-\Phi(x_1^-,x_2^+)-\Phi(x_1^+,x_2^-)+\Phi(x_1^-,x_2^-)\Big|_{x=x_s,\vartheta=\vartheta_0}$$

$$=$$

$$-\Big(\Phi(x_1^+,x_2^+)-\Phi(x_1^-,x_2^+)-\Phi(x_1^+,x_2^-)+\Phi(x_1^-,x_2^-)$$
$$+\Psi(x_2^+,x_1^+)-\Psi(x_1^+,x_2^+)+\Psi(x_1^+,x_2^-)-\Psi(x_2^+,x_1^-)$$
$$+i\log\frac{\Gamma_{q^2}(1+\frac{i}{2a}(u_1-u_2))}{\Gamma_{q^2}(1-\frac{i}{2a}(u_1-u_2))}-i\log\frac{1-\frac{1}{x_1^+x_2^-}}{1-\frac{1}{x_1^-x_2^+}}\Big)\Bigg|_{x=x_m,\vartheta=\pi-\vartheta_0}$$

Deformed TBA (exact spectrum)

- Deformation:
  - plane $\rightarrow$ cylinder
  - $2 \rightarrow \vartheta$
  - $\pm i/g \rightarrow \pm ia$
- Deformed integration kernels, e.g.

$$s(u) \equiv \frac{1}{4\cosh\frac{\pi u}{2}} \rightarrow \sum_n \frac{1}{4a\cosh\frac{\pi(u+2\pi n)}{2a}}$$

- Mirror duality: spectrum $\vartheta_0 \sim$ thermodynamics at $\pi - \vartheta_0$

Mirror duality again

- (Simple) proof without assumed integrable description?
- $\mathrm{AdS}_5 \times \mathrm{S}^5$ ($\varkappa \to 0$) limit?
- Note: semi-classically $(\varkappa, T) \to (1/\varkappa, T/\varkappa^2)$

Bosonic string in a light-cone gauge

- Consider space with
  - isometries in t and ϕ
  - $g_{t\phi} = g_{t\mu} = g_{\phi\mu} = 0$
- Action

$$S = -\frac{T}{2} \int \mathrm{d}\tau \mathrm{d}\sigma \, g_{MN} \, dx^M dx^N \, , \qquad M, N = t, \phi, \mu \, , \;\; (+, -, \mu)$$

- Fix gauge $x^+ = \tau$, $p_- = 1$

$$S = T \int \mathrm{d}\tau \mathrm{d}\sigma \left(1 - \sqrt{(\dot{x}_\mu x'^\mu)^2 - (\dot{x}_\mu \dot{x}^\mu - g_{tt})(x'_\nu x'^\nu + 1/g_{\phi\phi})} \right)$$

- Shows: double Wick rotation $\Leftrightarrow$ $g_{tt} \leftrightarrow 1/g_{\phi\phi}$

Mirror $\mathrm{AdS}_5 \times \mathrm{S}^5$

- Metric

$$ds^2 = \frac{1}{1-r^2}(-dt^2 + dr^2) + r^2 d\Omega_3 + \frac{1}{1+\rho^2}\left(d\phi^2 + d\rho^2\right) + \rho^2 d\Omega_3,$$

- Integrable sigma model with $\mathfrak{psu}(2|2)^{\oplus 2}_{c.e.}$ symmetry
- Naked singularity at $r = 1$

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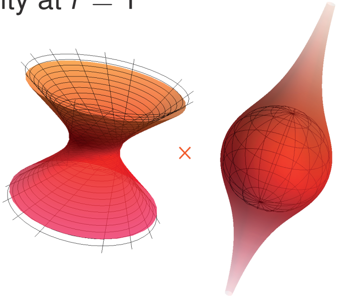

Mirror $\mathrm{AdS}_5 \times S^5$ in type IIB

- Metric

$$ds^2 = \frac{1}{1-r^2}(-dt^2 + dr^2) + r^2 d\Omega_3 + \frac{1}{1+\rho^2}\left(d\phi^2 + d\rho^2\right) + \rho^2 d\Omega_3,$$

- Solution of type IIB sugra
  - $\Phi = \Phi_0 - \frac{1}{2}\log(1-r^2)(1+\rho^2)$
  - $F = 4e^{-\Phi}(\omega_\phi - \omega_t)$
  - No Killing spinors
- GS fermions match Wick rotated $\mathrm{AdS}_5 \times S^5$ ones!

The $\mathrm{AdS}_5 \times \mathrm{S}^5$ mirror model as a string

Integrable deformation of $\mathrm{AdS}_5 \times \mathrm{S}^5$ string sigma model

- (Equations for the exact spectrum)
- Mirror duality of the integrable model
- Mirror transformation in terms of geometry
- Mirror $\mathrm{AdS}_5 \times \mathrm{S}^5$ (IIB background)

Outlook

- Quantum $a(T,\varkappa)$ and $\vartheta(T,\varkappa)$? (cannot be semi-classics!)
  - mirror duality $\sim a(T,\varkappa) = a(T/\varkappa)$, $\vartheta(a,\varkappa) = \pi - \vartheta(a, 1/\varkappa)$
  - maybe $\xi = i\varkappa$, $a = \text{arcsinh}\,\varkappa/T$?
- $(P\mu)_q$ system? (mirror duality: $P\mu \leftrightarrow Q\omega$?)

  Gromov, Kazakov, Leurent, Volin '13
- Other backgrounds?
- $(\text{AdS/CFT})_q$? Mirror planar $\mathcal{N} = 4$ SYM?