Quantum Spectral Curve in N=4 SYM at Small Spin

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based on
1402.0871 (N.Gromov, F.L.-M., G.Sizov, S.Valatka)
1305.1944 (N.Gromov, F.L.-M., G.Sizov)

"Supersymmetric Field Theories" workshop, Nordita, 13th-16th August 2014

Introduction

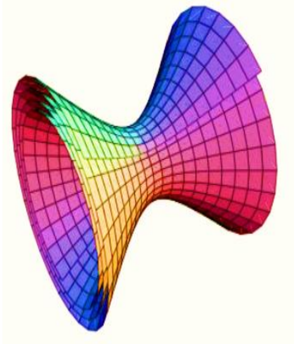
$\mathcal{N}=4$ Yang-Mills theory
in four dimensions
(planar limit)

$N_c \to \infty$, $\lambda = g_{YM}^2 \, N_c$

$\Longleftrightarrow$

Type II B superstring theory in

$AdS_5 \times S^5$



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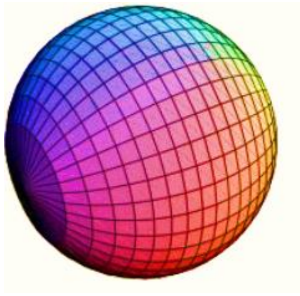


Operator conformal dimensions $\Delta_i(\lambda)$ $\Longleftrightarrow$ spectrum of string energies $E_i(\lambda)$

Integrability $\Longrightarrow$ exact solution for the spectrum!

We study twist operators

$$\mathcal{O} = \mathrm{Tr}(D^S Z^L) + permutations$$

in the near-BPS limit when $S \to 0$

Some motivation:

- can get results exact in λ, see interpolation from gauge to string theory
- predictions for states with fixed spin, e.g. Konishi (S=2)
- strong test of proposed Quantum Spectral Curve equations

Talk outline:

- Origins of the Quantum Spectral Curve approach
- Application: exact conformal dimensions in N=4 SYM at small spin
- Other near-BPS exact results: $q\bar{q}$ potential, ABJM

1. The Quantum Spectral Curve

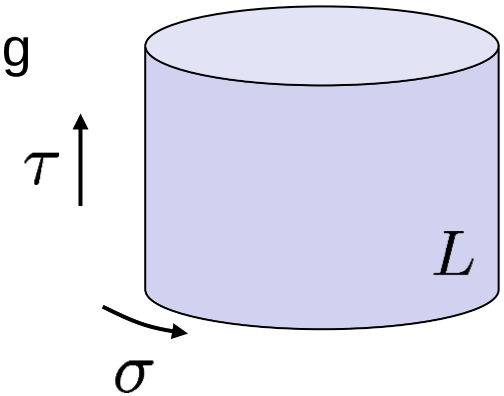
Quantum Integrability

weak coupling: single trace operators in N=4 SYM $\mathrm{Tr}(XYZ...)$

eigenstates of integrable spin chains

Minahan,Zarembo; Beisert,Kristjanssen,Staudacher;…

For exact finite-length spectrum use integrability of string sigma model in a finite volume L



Exact S-matrix

Beisert,Eden,Staudacher;…

Asymptotic Bethe ansatz for $L \to \infty$, any λ

Beisert,Staudacher;…

Thermodynamic Bethe ansatz (TBA) – infinite set of nonlinear integral equations.

From TBA to Quantum Spectral Curve

$$\begin{aligned}
\log Y_{\otimes} &= +K_{m-1} * \log \frac{1+1/Y_{\circ_m}}{1+Y_{\triangle_m}} + \mathcal{R}^{(01)}_{1m} * \log(1+Y_{\bullet_m}) + \left[\log \frac{R^{(+)}}{R^{(-)}}\right] + \log(-1) && (40)\\
\log Y_{\oplus} &= -K_{m-1} * \log \frac{1+1/Y_{\circ_m}}{1+Y_{\triangle_m}} - \mathcal{B}^{(01)}_{1m} * \log(1+Y_{\bullet_m}) - \left[\log \frac{B^{(+)}}{B^{(-)}}\right] - \log(-1) && (41)\\
\log Y_{\triangle_n} &= -K_{n-1,m-1} * \log(1+Y_{\triangle_m}) - K_{n-1} * \log \frac{1+Y_{\otimes}}{1+1/Y_{\oplus}} + \left(\mathcal{R}^{(01)}_{nm} + \mathcal{B}^{(01)}_{n-2,m}\right) * \log(1+Y_{\bullet_m}) \\
&+ \left[\sum_{k=-\frac{n-1}{2}}^{\frac{n-1}{2}} \log \frac{R^{(+)}(u+ik)}{R^{(-)}(u+ik)} + \sum_{k=-\frac{n-3}{2}}^{\frac{n-3}{2}} \log \frac{B^{(+)}(u+ik)}{B^{(-)}(u+ik)}\right] && (42)\\
\log Y_{\circ_n} &= K_{n-1,m-1} * \log(1+1/Y_{\circ_m}) + K_{n-1} * \log \frac{1+Y_{\otimes}}{1+1/Y_{\oplus}} && (43)\\
\log Y_{\bullet_n} &= L \log \frac{x^{[-n]}}{x^{[+n]}} + \left(2\mathcal{S}_{nm} - \mathcal{R}^{(11)}_{nm} + \mathcal{B}^{(11)}_{nm}\right) * \log(1+Y_{\bullet_m}) + \left[\sum_{k=-\frac{n-1}{2}}^{\frac{n-1}{2}} i\Phi(u+ik)\right] && (44)\\
&+ 2\left(\mathcal{R}^{(10)}_{n1} * \log(1+Y_{\otimes}) - \mathcal{B}^{(10)}_{n1} * \log(1+1/Y_{\oplus}) + \left(\mathcal{R}^{(10)}_{nm} + \mathcal{B}^{(10)}_{n,m-2}\right) * \log(1+Y_{\triangle_m})\right)
\end{aligned}$$

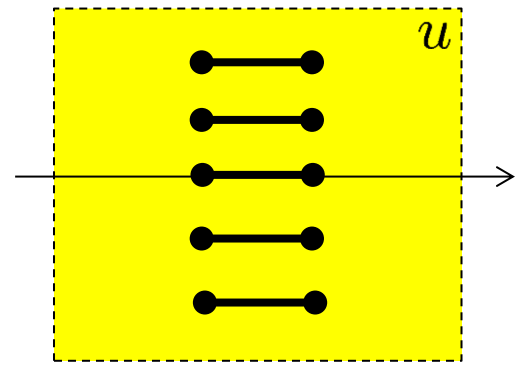
TBA is exact for all L and λ

Complicated system for infinitely many Y-functions $Y_{a,s}(u)$

Gromov,Kazakov,Vieira 09 Arutyunov,Frolov 09
Gromov,Kazakov,Kozak,Vieira 09 Bombardelli,Fioravanti,Tateo 09

Rich underlying algebraic structure (Y-system, T-system/Hirota)

+ ensure correct analytical properties in u



drastic simplification!

Reformulated as Quantum Spectral Curve equations

Gromov, Kazakov,Leurent,Volin 2013

Quantum Spectral Curve/Pμ system

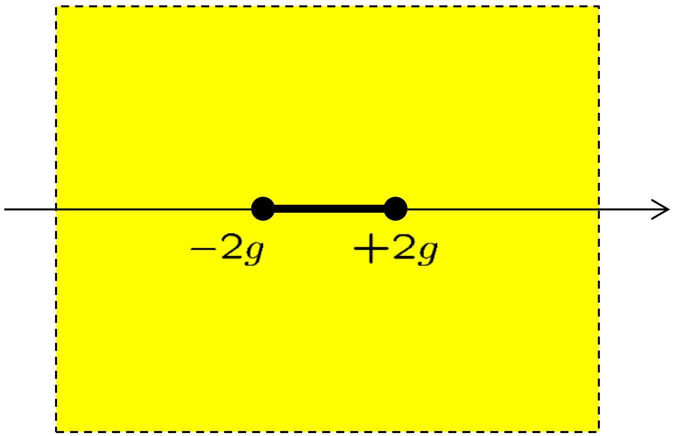
Gromov, Kazakov,Leurent,Volin 2013

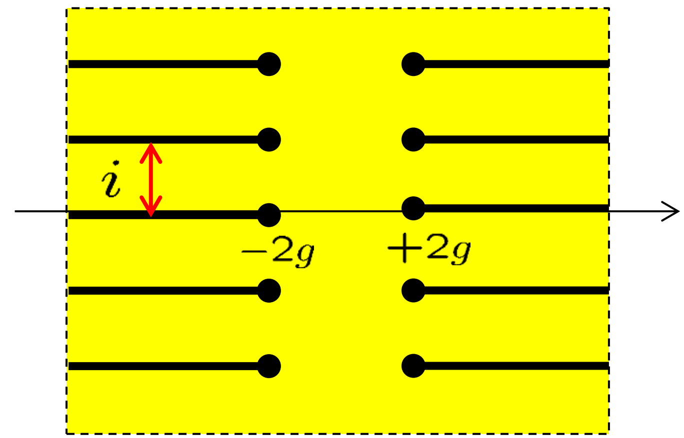
TBA equations reduced to only 4+6 functions:

$$a, b = 1, \ldots, 4$$

$$\mathbf{P}_a(u)$$

$$\mu_{ab}(u) = -\mu_{ba}(u)$$





$$g = \frac{\sqrt{\lambda}}{4\pi}$$

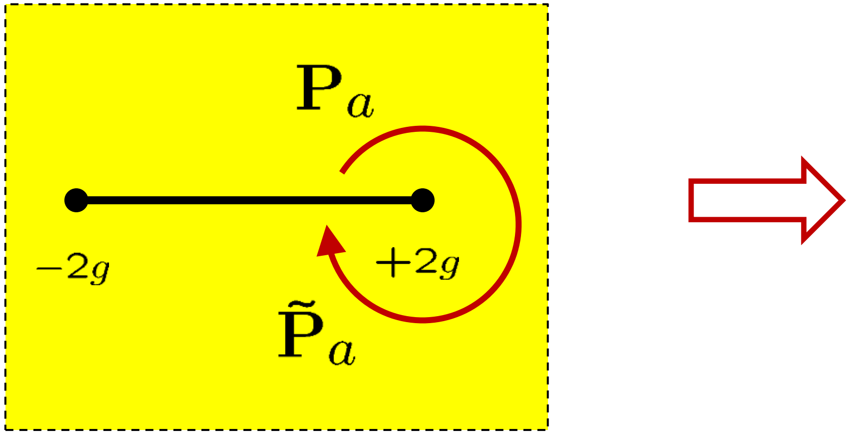
$$\mu_{ab}(u) = \mu_{ab}(u+i)$$

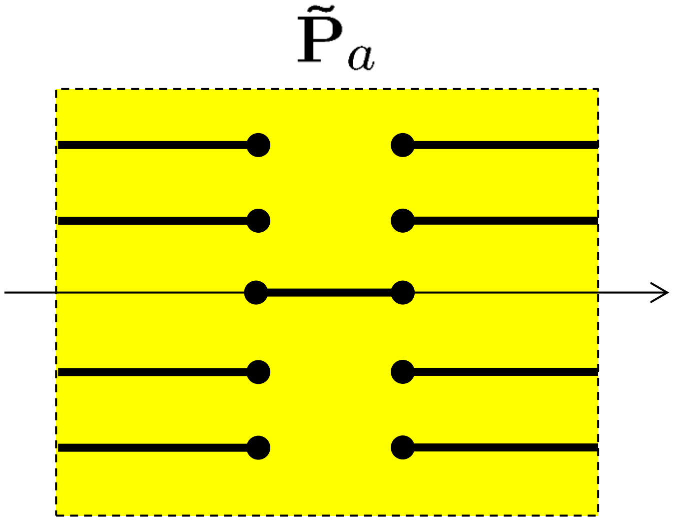
$$\mu_{12}\mu_{34} - \mu_{13}\mu_{24} + \mu_{14}^2 = 1$$

$$\mu_{14} = \mu_{23}$$

Pμ system equations

Analytic continuation around branchpoint:





$$\tilde{\mathbf{P}}_a = -\mu_{ab}\chi^{bc}\mathbf{P}_c$$

$$\tilde{\mu}_{a,b} = \mu_{a,b} + \mathbf{P}_a\tilde{\mathbf{P}}_b - \mathbf{P}_b\tilde{\mathbf{P}}_a$$

closed system of equations
(Riemann-Hilbert problem)

$\chi^{14} = -\chi^{23} = \chi^{32} = -\chi^{41} = 1$

All branchpoints are quadratic

Exact energy is found from asymptotics:

$\mathbf{P}_a \sim (\text{conformal charges})u^{\mathrm{R-charges}}$
for $u \to \infty$

The energy from Pμ-system

E.g. for twist operators $\text{Tr}(D^S Z^L) + permutations$

(where $Z = \Phi_1 + i\Phi_2$)

$$\mathbf{P}_1 \simeq A_1 u^{-L/2}$$
$$\mathbf{P}_2 \simeq A_2 u^{-L/2-1}$$
$$\mathbf{P}_3 \simeq A_3 u^{+L/2}$$
$$\mathbf{P}_4 \simeq A_4 u^{+L/2-1}$$

$$(\mu_{12}, \mu_{13}, \mu_{14}, \mu_{24}, \mu_{34}) \sim (u^{\Delta-L}, u^{\Delta+1}, u^{\Delta}, u^{\Delta-1}, u^{\Delta+L})$$

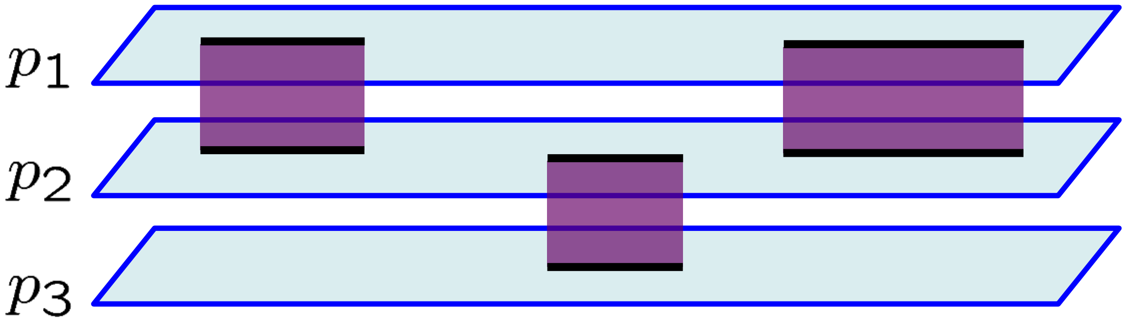
And anomalous dimension Δ is found from

$$A_2 A_3 = \frac{[(L-S+2)^2 - \Delta^2][(L+S)^2 - \Delta^2)]}{16iL(L+1)}$$
$$A_4 A_1 = \frac{[(L+S-2)^2 - \Delta^2][(L-S)^2 - \Delta^2]}{16iL(L-1)}.$$

Relation to classical spectral curve

Classical string solution $\longleftrightarrow$ algebraic curve

Beisert,Kazakov,Sakai,Zarembo



In the classical limit $\mathbf{P}_a(u) \simeq e^{\int^u p_a(v)dv}$

S^5 quasimomenta

$\mathbf{P}\mu$ system may be viewed as a quantum version of the curve

Expect that $\mathbf{P}_a$ should be the exact Baxter Q-functions
= wavefunctions in separated variables

Gromov, Kazakov,Leurent,Volin 2013, 2014

2. Application: Small Spin Limit

Twist operators at small spin

$$\mathcal{O} = \text{Tr}(D^S Z^L) + permutations$$

For $S = 0$ this operator is protected,
we study the near-BPS limit when $S \to 0$

$$\Delta = L + S\Delta^{(1)}(\lambda) + S^2\Delta^{(2)}(\lambda) + \dots$$

$$A_2 A_3 = \frac{[(L-S+2)^2 - \Delta^2][(L+S)^2 - \Delta^2)]}{16iL(L+1)}$$
$$A_4 A_1 = \frac{[(L+S-2)^2 - \Delta^2][(L-S)^2 - \Delta^2]}{16iL(L-1)}.$$

$\Rightarrow$ Key point: All $\mathbf{P}_a$ are small

$$\tilde{\mu}_{ab} = \mu_{ab} + \mathbf{P}_a \tilde{\mathbf{P}}_b - \mathbf{P}_b \tilde{\mathbf{P}}_a$$
$$\mu_{ab} = \mu_{ab}(u+i)$$

$\Rightarrow$ μ_{ab} are trivial at leading order

Gromov, F.L.-M., Sizov, Valatka 2014

$$\mu_{12} = 1,\ \mu_{13} = 0,\ \mu_{14} = -1,\ \mu_{24} = \sinh(2\pi u),\ \mu_{34} = 0$$

Solution at leading order

Gromov, F.L.-M.
Sizov, Valatka '14

$$\begin{cases} \tilde{\mathbf{P}}_1 = -\mu_{1,2}\mathbf{P}_3 + \mu_{1,3}\mathbf{P}_2 - \mu_{1,4}\mathbf{P}_1, \\ \tilde{\mathbf{P}}_2 = -\mu_{1,2}\mathbf{P}_4 + \mu_{2,3}\mathbf{P}_2 - \mu_{2,4}\mathbf{P}_1, \\ \tilde{\mathbf{P}}_3 = -\mu_{1,3}\mathbf{P}_4 + \mu_{2,3}\mathbf{P}_3 - \mu_{3,4}\mathbf{P}_1, \\ \tilde{\mathbf{P}}_4 = -\mu_{1,4}\mathbf{P}_4 + \mu_{2,4}\mathbf{P}_3 - \mu_{3,4}\mathbf{P}_2. \end{cases} \Rightarrow \begin{cases} \tilde{\mathbf{P}}_1 = -\mathbf{P}_3 + \mathbf{P}_1, \\ \tilde{\mathbf{P}}_2 = -\mathbf{P}_4 - \mathbf{P}_2 - \mathbf{P}_1 \sinh(2\pi u), \\ \tilde{\mathbf{P}}_3 = \qquad -\mathbf{P}_3, \\ \tilde{\mathbf{P}}_4 = \qquad +\mathbf{P}_4 + \mathbf{P}_3 \sinh(2\pi u). \end{cases}$$

Easy to solve using Zhukovsky variable $x + \frac{1}{x} = \frac{u}{g}, \ \tilde{x} = 1/x$

E.g. $\tilde{\mathbf{P}}_3 = -\mathbf{P}_3, \quad \mathbf{P}_3 \sim u \Rightarrow \mathbf{P}_3 = C(x - 1/x)$

Using $\sinh(2\pi u) = \sum_{n=-\infty}^{\infty} I_{2n+1}(\sqrt{\lambda}) x^{2n+1}$ we find all P's

$$A_2 A_3 = \frac{[(L-S+2)^2 - \Delta^2][(L+S)^2 - \Delta^2)]}{16iL(L+1)}$$
$$A_4 A_1 = \frac{[(L+S-2)^2 - \Delta^2][(L-S)^2 - \Delta^2]}{16iL(L-1)}.$$

$$\Rightarrow \quad \Delta = L + S\sqrt{\lambda}\frac{I_{L+1}(\sqrt{\lambda})}{L I_L(\sqrt{\lambda})}$$

Basso's slope function!

Basso 2011

For leading order asymptotic Bethe ansatz is enough

Basso 2012
Gromov 2012

The S^2 term is much more involved – sensitive to dressing phase and finite-size effects (wrapping)

Exact result at next order

$$\mu_{ab} = \mu_{ab}^{(0)} + S^2\mu_{ab}^{(1)} + \ldots, \qquad \mathbf{P}_a = S\mathbf{P}_a^{(0)} + S^3\mathbf{P}_a^{(1)} + \ldots$$

$$\tilde{\mu}_{ab} = \mu_{ab} + \mathbf{P}_a\tilde{\mathbf{P}}_b - \mathbf{P}_b\tilde{\mathbf{P}}_a \qquad \tilde{\mathbf{P}}_a = -\mu_{ab}\chi^{bc}\mathbf{P}_c$$

Leading order $\mathbf{P}_a$ ⇨ correction to μ_{ab} ⇨ correction to $\mathbf{P}_a$

$$\Delta = L + S\Delta^{(1)}(g) + S^2\Delta^{(2)}(g) + \ldots$$

We found the S^2 term at all loops for L=2,3,4

Gromov, F.L.-M. Sizov, Valatka '14

$$\Delta^{(2)}_{L=2} = \oint \frac{du_x}{2\pi i}\oint \frac{du_y}{2\pi i}\left[\frac{8\pi^3 I_1(\sqrt{\lambda})^2\left(x^3-\left(x^2+1\right)y\right)\left(2\pi g I_1(\sqrt{\lambda})-I_2(\sqrt{\lambda})\right)}{I_2(\sqrt{\lambda})^3\left(x^3-x\right)y^2}\right.$$
$$\left.+\ldots-\frac{4\pi^3(\mathrm{sh}_-^x)^2\left(x^2+1\right)y^2}{I_2(\sqrt{\lambda})^2\left(x^2-1\right)}\right]\frac{1}{4\pi i}\partial_u\log\frac{\Gamma(iu_x-iu_y+1)}{\Gamma(1-iu_x+iu_y)}$$

Similar to dressing phase!

Tests

At weak coupling we find

$$\Delta^{(2)}_{J=2} = -8g^2\zeta(3) + g^4\left(140\zeta(5) - \frac{32\pi^2\zeta(3)}{3}\right) + g^6\left(200\pi^2\zeta(5) - 2016\zeta(7)\right)$$
$$+ \; g^8\left(-\frac{16\pi^6\zeta(3)}{45} - \frac{88\pi^4\zeta(5)}{9} - \frac{9296\pi^2\zeta(7)}{3} + 27720\zeta(9)\right)$$
$$+ \; g^{10}\left(\frac{208\pi^8\zeta(3)}{405} + \frac{160\pi^6\zeta(5)}{27} + 144\pi^4\zeta(7) + 45440\pi^2\zeta(9) - 377520\zeta(11)\right) + \dots$$

Matches known ABA + wrapping (checked to 4 loops)

[Kotikov, Lipatov, Onishenko, Velizhanin]
[Moch, Vermaseren, Vogt] [Staudacher]
[Kotikov, Lipatov, Rej, Staudacher, Velizhanin]
[Bajnok, Janik, Lukowski]
[Lukowski, Rej, Velizhanin] […]

$$\Delta - S - L = 8g^2\, S_1$$
$$-16g^4(S_3 + S_{-3} - 2\,S_{-2,1} + 2\,S_1\,(S_2 + S_{-2}))$$
$$-64g^6\left(2\,S_{-3}\,S_2 - S_5 - 2\,S_{-2}\,S_3 - 3\,S_{-5} + 24\,S_{-2,1,1,1} + 6\left(S_{-4,1} + S_{-3,2} + S_{-2,3}\right) - \dots\right)$$
$$+128g^8\left(4S_{-7} + 6S_7 + 2S_{-3,1,3} + \ldots\right)$$
$$+256g^{10}\left(20480S_1^4S_{-5} - 8192S_1^4S_{-3}S_{-2} + \ldots\right)$$

For $L = 4$ our prediction was confirmed from ABA very recently

$$\Delta^{(2)} = g^2\left(-\frac{14\zeta_3}{5} + \frac{48\zeta_5}{\pi^2} - \frac{252\zeta_7}{\pi^4}\right) + \dots$$

Beccaria,Macorini 2014

Tests at strong coupling

Basso's conjecture links the strong coupling and small spin regimes:

$$\Delta^2 = L+S\left(A_1\sqrt{\lambda}+A_2+\ldots\right)+S^2\left(B_1+\frac{B_2}{\sqrt{\lambda}}+\ldots\right)+S^3\left(\frac{C_1}{\lambda^{1/2}}+\frac{C_2}{\lambda^{3/2}}+\ldots\right)+O(S^4)$$

Basso 2011

Our result gives:

$$\Delta^{(2)}_{L=2} = \overbrace{-\pi^2 g^2+\frac{\pi g}{4}+\frac{1}{8}-\frac{1}{\pi g}\left(\frac{3\zeta_3}{16}+\frac{3}{512}\right)}^{\text{matches known data! (slope + 1 loop folded string)}}$$
$$- \frac{1}{\pi^2 g^2}\underbrace{\left(\frac{9\zeta_3}{128}+\frac{21}{512}\right)+\frac{1}{\pi^3 g^3}\left(\frac{3\zeta_3}{2048}+\frac{15\zeta_5}{512}-\frac{3957}{131072}\right)}_{\text{new prediction}}$$

Konishi dimension

Re-expansion of small S result ⇨ predictions for operators with finite S

Simplest unprotected operator, L=2, S=2

$$\mathcal{O}_{Konishi} = \mathrm{Tr}(ZD^2Z) + perm.$$

Our prediction for string theory

$$\Delta_{Konishi} = 2\lambda^{1/4} + \frac{2}{\lambda^{1/4}} + \frac{-3\zeta(3)+\frac{1}{2}}{\lambda^{3/4}} + \frac{\frac{15\,\zeta(5)}{2}+6\,\zeta(3)-\frac{1}{2}}{\lambda^{5/4}}$$

Gubser,Klebanov, Polyakov 98

Gromov,Serban, Shenderovich,Volin '11 Roiban,Tseytlin '11 Mazzuchato,Vallilo '11

Gromov, Valatka'11 (via Basso'11)

Gromov, F.L.-M. Sizov, Valatka '14

Similarly we find for any L and S a new prediction for the 3-loop coefficient in

$$\Delta = C_0\lambda^{1/4} + C_1\lambda^{-1/4} + C_2\lambda^{-3/4} + C_3\lambda^{-5/4} + \ldots$$

E.g. for S=2

$$C_3 = \frac{1}{512}\left(L^6 - 20L^4 + 48L^2(4\zeta_3 - 1) + 192(12\zeta_3 + 20\zeta_5 + 1)\right)$$

BFKL pomeron intercept

$$j(\Delta) \equiv 2 + S(\Delta)$$

With our results we can compute the intercept $j(0)$ at strong coupling:

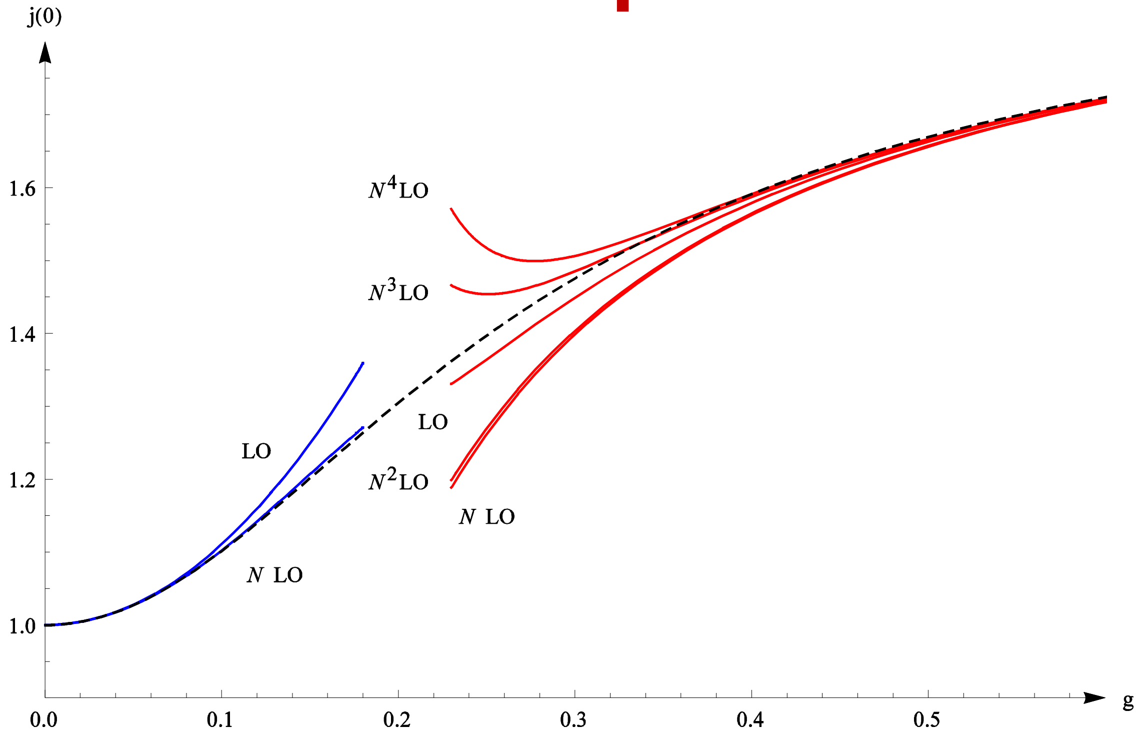
Costa,Goncalves,Penedones 2012

Kotikov,Lipatov 2013

$$j(0) = 2 + S(0) = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\,\lambda^{3/2}} + (6\zeta_3 + 2)\,\frac{1}{\lambda^2}$$
$$+ \left(18\,\zeta_3 + \frac{361}{64}\right)\frac{1}{\lambda^{5/2}} + \left(39\,\zeta_3 + \frac{447}{32}\right)\frac{1}{\lambda^3} + \mathcal{O}\left(\frac{1}{\lambda^{7/2}}\right)$$

New result
[Gromov,F.L.-M.,Sizov,Valatka 2014]

BFKL pomeron intercept



Extensive studies of BFKL limit are in progress

$$S \to -1, \ \frac{g^2}{S+1} \simeq 1$$

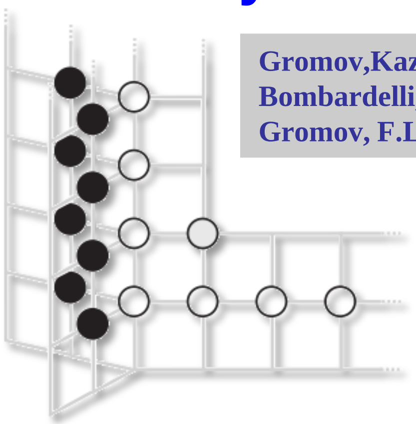
Alfimov,Gromov,Kazakov 2014
Gromov,Sizov

Our results already provide some guidance for analytic continuation to non-integer S

3. Other near-BPS exact solutions

ABJM theory

Y-system / TBA



Gromov,Kazakov,Vieira 2009
Bombardelli,Fioravanti,Tateo 2009
Gromov, F.L.-M. 2009

$\Rightarrow$

Quantum Spectral Curve

Cavaglia, Fioravanti, Gromov, Tateo 2014

Exact slope function (i.e. leading order in S) was recently computed from the QSC

Gromov,Sizov 2014

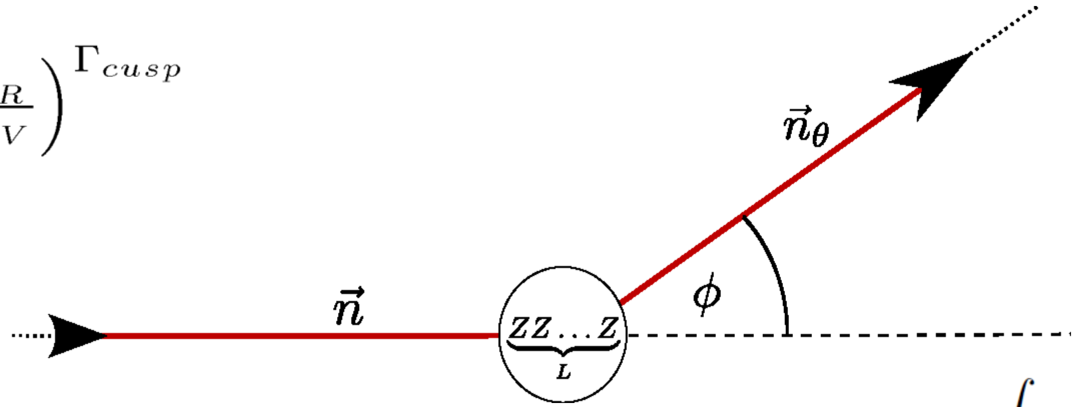
Comparison with localization result for 1/6 BPS Wilson loop gives conjecture for exact interpolating function $h(\lambda)$

Gromov,Sizov 2014

$$\lambda = \frac{\sinh(2\pi h)}{2\pi}\ {}_3F_2\left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}; 1, \tfrac{3}{2}; -\sinh^2(2\pi h)\right)$$

Cusp anomalous dimension in N=4 SYM

$$\langle W \rangle \sim \left(\frac{\Lambda_{IR}}{\Lambda_{UV}} \right)^{\Gamma_{cusp}}$$



$$W = \mathrm{Tr}\,\mathcal{P} \exp \int dt \left[iA \cdot \dot{x}_q + \vec{\Phi} \cdot \vec{n}\, |\dot{x}_q| \right]$$

- Cusp angle ϕ
- Angle θ between the couplings to scalars on two rays
- R-charge L of a local operator inserted at the cusp
- 't Hooft coupling λ

For $\phi = \theta$ this observable is BPS

TBA equations were proposed in **Correa,Maldacena,Sever 12** **Drukker 12**

In the near-BPS limit $\phi \approx \theta$ they can be solved analytically

All loop near-BPS solution

[$\phi \to 0, \theta = 0$ Gromov,Sever 2012] [$\phi \approx \theta$ Gromov,F.L-.M.,Sizov 2013]

$$\Gamma_{cusp} = \frac{\phi-\theta}{4} \partial_\theta \log \frac{\det \mathcal{M}_{2L+1}}{\det \mathcal{M}_{2L-1}}$$

$$\mathcal{M}_N = \begin{pmatrix} I_1^\theta & I_0^\theta & \cdots & I_{2-N}^\theta & I_{1-N}^\theta \\ I_2^\theta & I_1^\theta & \cdots & I_{3-N}^\theta & I_{2-N}^\theta \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ I_N^\theta & I_{N-1}^\theta & \cdots & I_1^\theta & I_0^\theta \\ I_{N+1}^\theta & I_N^\theta & \cdots & I_2^\theta & I_1^\theta \end{pmatrix}$$

$$I_n^\theta = \tfrac{1}{2} I_n\left(\sqrt{\lambda}\sqrt{1-\tfrac{\theta^2}{\pi^2}}\right)\left[\left(\sqrt{\tfrac{\pi+\theta}{\pi-\theta}}\right)^n - (-1)^n\left(\sqrt{\tfrac{\pi-\theta}{\pi+\theta}}\right)^n\right].$$

Matches localization results at L=0

Correa,Henn,Maldacena,Sever 12
Pestun 07; Drukker,Zarembo,…

Interpolates between gauge and string theory

$$\Gamma_L = (\phi-\theta) g^{2L+2} \frac{(-1)^L (4\pi)^{1+2L}}{(1+2L)!} B_{1+2L}\left(\frac{\pi-\theta}{2\pi}\right)$$

Correa,Henn,Maldacena,Sever 12

$$\begin{aligned} \frac{\Gamma_L}{2(\phi-\theta)\theta} = & \left(-\frac{g}{\pi} + \frac{3L}{4\pi^2} - \frac{9L^2}{64 g\pi^3} - \frac{5L^3}{256 g^2\pi^4} + \frac{45L^4}{16384 g^3\pi^5}\right) \\ & + \theta^2\left(-\frac{g}{2\pi^3} + \frac{3L}{4\pi^4} - \frac{21L^2}{128 g\pi^5} - \frac{L^3}{16 g^2\pi^6} - \frac{105L^4}{32768 g^3\pi^7}\right) \\ & + \theta^4\left(-\frac{3g}{8\pi^5} + \frac{3L}{4\pi^6} - \frac{99L^2}{512 g\pi^7} - \frac{3L^3}{32 g^2\pi^8} - \frac{2085L^4}{131072 g^3\pi^9}\right) \end{aligned}$$

Classical curve found from matrix model **Sizov,Valatka 2013**

Also corresponds to a solution of $\mathbf{P}\mu$ system
$\mathbf{P}\mu$ asymptotics for any angles are yet to be understood

Gromov,Kazakov,Leurent,Volin 2013
Gromov,F.L.-M.,Sizov 2013

Conclusions

- Computed the S^2 term in twist operator dimension at any coupling from $\mathbf{P}\mu$ system
- New strong coupling predictions: Konishi operator, BFKL intercept
- Other applications: generalized cusp anomalous dimension, ABJM

Future:

- Iteratively generate corrections in S; strong coupling expansion
- P's as exact wavefunctions -> 3 pt correlators?
- $\mathbf{P}\mu$ for cusp anomalous dimension; access Regge trajectories? (see talk of J. Henn)