The superconformal bootstrap program

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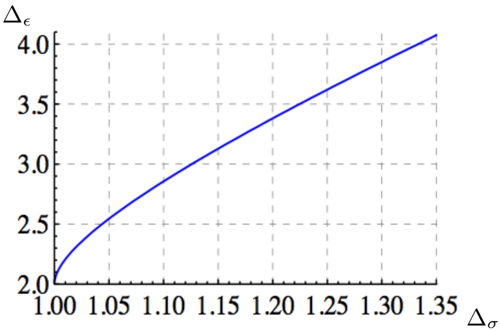
Based on work with C. Beem, M. Lemos, P. Liendo, W. Peelaers, L. Rastelli

Conformal field theory revisited

$$\langle \sigma(x_1)\sigma(x_2)\sigma(x_3)\sigma(x_4)\rangle$$

$$\sigma(x_1)\sigma(x_2) \sim \frac{1}{(x_1-x_2)^{2\Delta_\sigma}} + \frac{\epsilon(x_2)}{(x_1-x_2)^{2\Delta_\sigma-\Delta_\epsilon}} + \dots$$



[Rattazzi, Rychkov, Tonni, Vichi (2008)]

Back to the bootstrap

Conformal field theory revisited

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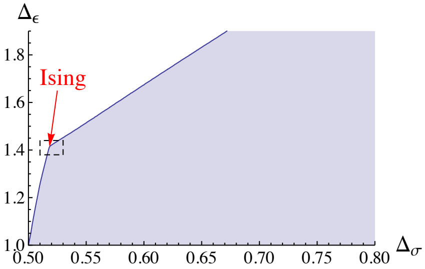


[El-Showk, Paulos, Poland, Simmons-Duffin, Rychkov, Vichi (2012)]

The superconformal bootstrap program

We are going to explore the consequences of crossing symmetry for superconformal field theories.

- Can we bootstrap specific superconformal theories?
- What can we learn about the space of all superconformal theories?

Spaces of superconformal field theories

The space of all $\mathcal{N}$-extended superconformal field theories in four dimensions

$\mathcal{N} = 4$

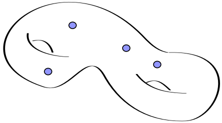
- Lagrangian theories are classified by

$$(G, \tau)/SL(2, \mathbb{Z})$$

  where G is a simple Lie group and $\tau \in H$
- No exotic theories?

$\mathcal{N} = 2$

- Quiver classification of Lagrangian theories [Bhardwaj, Tachikawa (2013)]
- Class ‘S’ theories obtained from six dimensions [Gaiotto (2008)]
- Many non-Lagrangian theories
- Do we have a complete classification?



Can the bootstrap program help us?

The superconformal bootstrap program

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The superconformal bootstrap

Is there a *protected, solvable* subsector of the crossing symmetry constraints for superconformal field theories?

Yes, for

$d = 4$ theories with $\mathcal{N} = 2$ susy

$d = 6$ theories with $(2, 0)$ susy

$d = 2$ theories with $(0, 4)$ susy

More precisely, we find that *twisted correlation functions* of certain *protected operators* become those of a two-dimensional *chiral algebra* and can be completely solved.

For example,

$$T(z)T(w) \sim \frac{c/2}{(z-w)^2} + \frac{2T}{(z-w)^2} + \frac{\partial T}{z-w}$$

completely determines all the correlation functions of $T(z)$.

[Beem, Lemos, Liendo, Peelaers, Rastelli, BvR (2013)]

The superconformal bootstrap program

Consequently, our program splits into two parts:

Minibootstrap		Maxibootstrap
• Protected	$\longrightarrow$	• Not protected
• Meromorphic		• Numerical
• Virasoro, Kac-Moody, W, ...		• Linear programming

Outline

1. Introduction
2. The minibootstrap in $d = 4$
   - Definition
   - Properties
   - Consequences for four-dimensional physics
3. The maxibootstrap
   - The $\mathcal{N} = 4$ maxibootstrap
   - The $\mathcal{N} = 2$ maxibootstrap
4. Conclusions

Outline

1 Introduction

2 The minibootstrap in $d = 4$
Definition
Properties
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The $\mathcal{N} = 4$ maxibootstrap
The $\mathcal{N} = 2$ maxibootstrap

4 Conclusions

Definition

Take an $\mathcal{N}=2$ superconformal field theory. Recall that the $\mathcal{N}=2$ superconformal algebra is $\mathfrak{su}(2,2|2)$ with maximal bosonic subgroup

$$\mathfrak{su}(2,2)\times\mathfrak{su}(2)_R\times\mathfrak{u}(1)_r$$

so irreps are labeled with Δ, (j_1,j_2) and (R,r).

Consider now an n-point correlation function

$$\langle\mathcal{O}^{I_1}(x_1)\ldots\mathcal{O}^{I_n}(x_n)\rangle$$

and restrict it in the following way:

1. Take all operators to be 'Schur' operators satisfying $\Delta=2R+j_1+j_2$.
2. Take all n points to lie in a two-plane $\mathbf{R}^2\subset\mathbf{R}^4$.
3. Contract the $\mathfrak{su}(2)_R$ indices with *position-dependent* vectors $v_I(\bar{z})$. For example, for the fundamental representation $v(\bar{z})=(1,\bar{z})$.

Claim: the resulting correlation function is *meromorphic* in all the positions.

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$$v_{I_1}(\bar{z}_1) \dots v_{I_n}(\bar{z}_n) \langle \mathcal{O}^{I_1}(z_1, \bar{z}_1) \dots \mathcal{O}^{I_n}(z_n, \bar{z}_n) \rangle$$

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$$\frac{\partial}{\partial \bar{z}_k} \left(v_{I_1}(\bar{z}_1) \dots v_{I_n}(\bar{z}_n) \langle \mathcal{O}^{I_1}(z_1, \bar{z}_1) \dots \mathcal{O}^{I_n}(z_n, \bar{z}_n) \rangle \right) = 0$$

and restrict it in the following way:

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Claim: the resulting correlation function is *meromorphic* in all the positions.

Example: free hypermultiplet

In a free hypermultiplet the scalars $Q^I = (Q, \tilde{Q}^*)$ and $\tilde{Q}^J = (\tilde{Q}, -Q^*)$ form two $\mathfrak{su}(2)_R$ doublets and satisfy $\Delta = 2R + j_1 + j_2$. Their OPE is

$$Q^I(z,\bar{z})\tilde{Q}^J(0) \sim \frac{-\epsilon^{IJ}}{z\bar{z}}$$

so

$$v_I(\bar{z})Q^I(z,\bar{z})v_J(0)\tilde{Q}^J(0) \sim \frac{-v_I(\bar{z})v_J(0)\epsilon^{IJ}}{z\bar{z}} = \frac{1}{z}$$

Defining $q(z) = v_I Q^I$ and $\tilde{q}(z) = v_I \tilde{Q}^I$ we find the two-dimensional OPE

$$q(z)\tilde{q}(0) \sim \frac{1}{z}$$

corresponding to a (non-unitary) pair of symplectic bosons of dimension 1/2.

Definition

Claim:

$$\frac{\partial}{\partial \bar{z}_k} \langle v_{I_1}(\bar{z}_1)\mathcal{O}^{I_1}(z_1,\bar{z}_1)\ldots\ldots v_{I_n}(\bar{z}_n)\mathcal{O}^{I_n}(z_n,\bar{z}_n)\rangle = 0$$

Proof:

- There exists a particular nilpotent supercharge $\mathbb{Q}$ such that

  $$\{\mathbb{Q},\mathbb{Q}^\dagger\} = \mathcal{H} - 2R - \mathcal{M}_+{}^+ - \mathcal{M}_{\dot{+}}{}^{\dot{+}}$$

  so necessarily $\Delta - 2R - j_1 - j_2 \geq 0$ and a Schur operator satisfies

  $$[\mathbb{Q}, \mathcal{O}^{1\ldots1}_{+\ldots+\dot{+}\ldots\dot{+}}(0)\} = 0\,.$$

  We can pick $\mathbb{Q} = \mathcal{Q}^1_- - \tilde{\mathcal{S}}^{\dot{2}-}$.
- Holomorphic translations are $\mathbb{Q}$ closed

  $$[\mathbb{Q}, P_z] = 0$$

- In the antiholomorphic direction we find that

  $$\partial_{\bar{z}}\left(v_I(\bar{z})\mathcal{O}^I(z,\bar{z})\right) = v_I(\bar{z})[P_{\bar{z}} + R^-, \mathcal{O}^I(\bar{z})]$$

  and such *twisted* antiholomorphic translations are $\mathbb{Q}$ exact

  $$P_{\bar{z}} + R^- = \{\mathbb{Q},\ldots\}$$

Meromorphicity then follows from the usual cohomological argument.

Definition

In fact, by restricting ourselves to $\mathbf{R}^2 \subset \mathbf{R}^4$ we preserve

$$\mathfrak{sl}(2)_L \times \mathfrak{sl}(2|2)_R \subset \mathfrak{su}(2,2|2)$$

The entire $\mathfrak{sl}(2)_L$ is closed

$$[\mathbb{Q}, L_{-1}] = 0 \qquad [\mathbb{Q}, L_0] = 0 \qquad [\mathbb{Q}, L_1] = 0$$

and the entire *twisted* $\mathfrak{sl}(2)_R$ is exact

$$\bar{L}_{-1} + R^- = \{\mathbb{Q}, \dots\} \qquad \bar{L}_0 - R = \{\mathbb{Q}, \dots\} \qquad \bar{L}_1 - R^+ = \{\mathbb{Q}, \dots\}$$

→ We have a *superconformal* twist. Notice that

$$L_0 = \frac{1}{2}\left(\mathcal{H} + \mathcal{M}_+{}^+ + \mathcal{M}_{\dot{+}}{}^{\dot{+}}\right) \qquad \rightarrow \qquad h = \frac{1}{2}(\Delta + j_1 + j_2)$$

$$\bar{L}_0 - R = \frac{1}{2}\left(\mathcal{H} - 2R - \mathcal{M}_+{}^+ - \mathcal{M}_{\dot{+}}{}^{\dot{+}}\right) = \frac{1}{2}\{\mathbb{Q}, \mathbb{Q}^\dagger\} = 0$$

→ The twist works for any superconformal algebra with an $\mathfrak{sl}(2|2)$ subalgebra, so chiral algebras also exist for $(2,0)$ SUSY in $d = 6$ and $(0,4)$ SUSY in $d = 2$.

Dictionary

Hypermultiplet	$\rightarrow$	Symplectic bosons
Vector multiplet	$\rightarrow$	(b, c) ghost system of type $(1, 0)$
Flavor symmetry G	$\rightarrow$	affine Kac-Moody symmetry G $j^A(z)j^B(0) \sim \frac{k_{2d}}{z^2} + \frac{f^{AB}{}_C}{z} j^C(0)$ $k_{2d} = -k_{4d}/2$
Stress tensor	$\rightarrow$	Virasoro stress tensor $T(z)T(0) \sim \frac{c_{2d}}{z^4} + \frac{2T(0)}{z^2} + \frac{\partial T(0)}{z}$ $c_{2d} = -12c_{4d}$
Higgs branch chiral ring	$\rightarrow$	Virasoro primaries
Chiral ring relations	$\rightarrow$	Null states
Coulomb branch chiral ring	$\not\rightarrow$	

...

Consequences for four-dimensional physics

To summarize, $\mathcal{N} = 2$ SCFTs in $d = 4$ always have infinite chiral symmetry in a protected sector. In particular we have Virasoro symmetry, but there is often much more.

Practical consequences?

- Representation theory: Schur operators form Virasoro (or AKM) irreps
- OPE coefficients: infinite classes are fixed by meromorphicity
- New unitarity bounds:
  for example, for $SU(N)$ flavor symmetry with $N \geq 3$ we have $k_{4d} \geq N$

Outlook

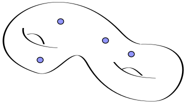
What else do we know?

- Chiral algebras in six dimensions: $W_{\mathfrak{g}}$ for $A_{\mathfrak{g}}$ type theories
- Chiral algebras on defects: AKM at a critical level $k = -h^{\vee}$
- Nonmaximal punctures in class S: QDS reduction
- Class S structure: a generalized TQFT with values in chiral algebras
- ...

[Beem, Rastelli, BvR (2014)]
[Beem, Peelaers, Rastelli, BvR (to appear)]

What don't we know yet?

- Proofs for chiral algebras for specific theories
- Classifications of chiral algebras?
- Connection to AGT?
- Connection to geometric Langlands?



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The $\mathcal{N} = 4$ maxibootstrap

In theories with $\mathcal{N} = 4$ superconformal symmetry, the primary $\mathcal{O}^i_{\mathbf{20'}}$ is a *universal* operator. So let's bootstrap its four-point function,

$$\langle\, \mathcal{O}^{i_1}_{\mathbf{20'}}(x_1)\mathcal{O}^{i_2}_{\mathbf{20'}}(x_2)\mathcal{O}^{i_3}_{\mathbf{20'}}(x_3)\mathcal{O}^{i_4}_{\mathbf{20'}}(x_4)\,\rangle = \frac{A^{i_1 i_2 i_3 i_4}(u,v)}{x_{12}^4 x_{34}^4}$$

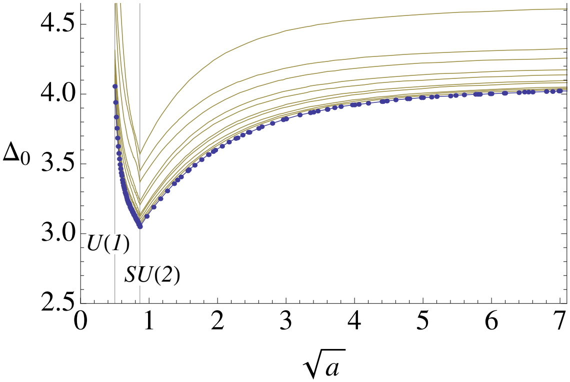
A priori there are 6 different functions but they are fixed in terms of a *single* unconstrained function $A(u,v)$ and two meromorphic functions $f_1(z)$ and $f_2(z)$.

- The $f_i(z)$ are fixed by the chiral algebra in terms of $a = \dim G/4$ so they are *input* for the numerical bootstrap.
- The unconstrained function $A(u,v)$ contains information on the *unprotected* operators only and is analyzed numerically.

Note: the only long multiplets that can appear have $\mathbf{R} = 0$ and even spin. Examples would be Konishi $\mathrm{Tr}\,(\Phi^I \Phi_I)$ or the double-trace operator $\mathrm{Tr}\,(\Phi^{\{I}\Phi^{J\}})\mathrm{Tr}\,(\Phi_{\{I}\Phi_{J\}})$.

The $\mathcal{N} = 4$ maxibootstrap

Results for the first unprotected scalar with $\mathbf{R} = 0$



[Beem, Rastelli, BvR (2013)]

$$\Delta_{\text{kon}} = 2 + \frac{3Ng}{\pi} - \frac{3N^2g^2}{\pi^2} + \frac{21N^3g^3}{4\pi^3} + \left(-39 + 9\,\zeta(3) - 45\,\zeta(5)\left(\frac{1}{2} + \frac{6}{N^2}\right)\right)\frac{N^4g^4}{4\pi^4} + \cdots\,,$$

[Velizhanin, . . .]

The $\mathcal{N} = 2$ maxibootstrap

In theories with $\mathcal{N} = 2$ superconformal symmetry, a flavor symmetry multiplet contains a dimension two scalar $\mu^{A,IJ}$ in the triplet of $\mathfrak{su}(2)_R$ known as the moment map.

Its four-point function is decomposed into a set of meromorphic functions $f^{ABCD}(z)$ and unconstrained functions $\mathcal{G}^{ABCD}(u,v)$. The meromorphic functions are fixed from

$$\langle j^A(0) j^B(z) j^C(1) j^D(\infty) \rangle = f^{ABCD}(z)$$

and we analyze the two-variable functions numerically.

Input parameters:

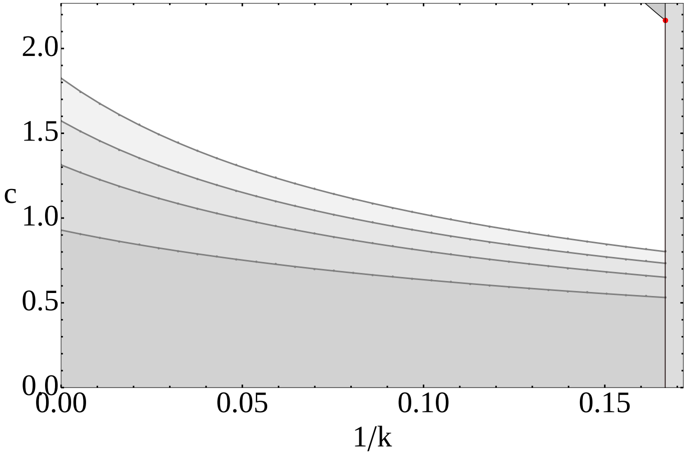
- global symmetry algebra G
- flavor central charge k
- central charge c

Output:

- Can the theory exist?
- Bounds on e.g. scalar operators
- ...

The $\mathcal{N} = 2$ maxibootstrap

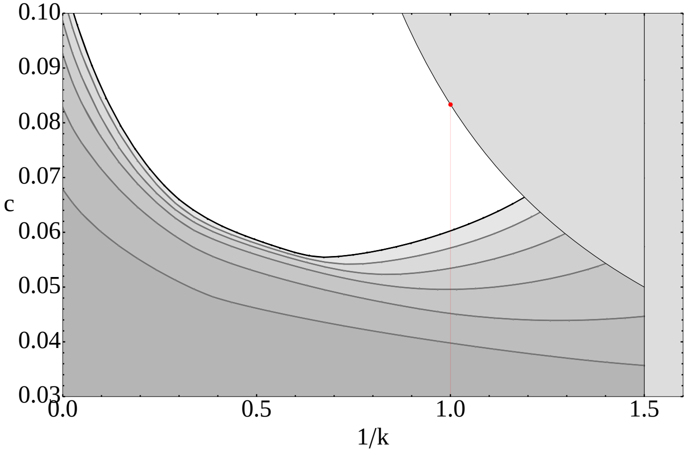
Global symmetry group: E_6



[Beem, Lemos, Liendo, Rastelli, BvR (2013)]

The $\mathcal{N} = 2$ maxibootstrap

Global symmetry group: $\mathfrak{su}(2)$



[Beem, Lemos, Liendo, Rastelli, BvR (2013)]

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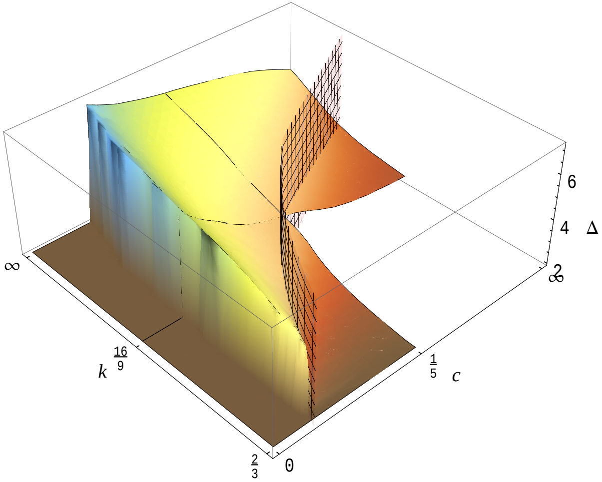


[Beem, Lemos, Liendo, Rastelli, BvR (2013)]

Other results

Other work on the superconformal maxibootstrap:

- $\mathcal{N} = 1$ in $d = 4$
  [Poland, Simmons-Duffin, Vichi (2010-2011);
  Berkooz, Yacoby, Zait (2014)]
- $\mathcal{N} = 4$ in $d = 4$ [Alday, Bissi (2013-2014)]
- $\mathcal{N} = 8$ in $d = 3$ [Chester, Lee, Pufu, Yacoby (2014)]
- $(2, 0)$ in $d = 6$ [Beem, Lemos, Rastelli, BvR (to appear)]
- ...

Lots of analytic work, for example on the computation of superconformal blocks, the lightcone limit, etc.
[Fitzpatrick, Kaplan, Khandker, Komargodski, Li, Poland, Simmons-Duffin, Zhiboedov, ... (2010-2014)]

Conclusions

We are only beginning to understand the consequences of crossing symmetry for superconformal theories. The results so far have been very promising.

Grand questions:

- Spaces of SCFTs?
- Microscopic derivation of AGT?
- From bounds to solutions?

Extra slides

Conformal multiplets

The conformal group in four dimensions is $SU(2,2) \sim SO(4,2)$ with generators

$$P^\mu \qquad M_{\mu\nu} \qquad D \qquad K_\mu$$

Consider the set of local operators in a CFT

$$\{\mathcal{O}_i^{\Delta,j_1,j_2}(x)\}$$

where $[D,\mathcal{O}] = \Delta\mathcal{O}$ and (j_1,j_2) are the Lorentz quantum numbers.

They can be organized in conformal multiplets consisting of

primary:	$[K_\mu, \mathcal{O}_i^{\Delta,j_1,j_2}(0)] = 0$
descendants:	$\partial_{\mu_1}\ldots\partial_{\mu_n}\mathcal{O}_i^{\Delta,j_1,j_2}(0)$

Sometimes representations are *short*, e.g.

$$\partial_\mu J^\mu = 0 \qquad\qquad \Box\phi = 0$$

and then the dimensions are fixed

$$[D, J^\mu] = 3 \qquad\qquad [D,\phi] = 1$$

Superconformal multiplets

The $\mathcal{N} = 4$ superconformal group in four dimensions is $PSU(2,2|4)$ with generators

$$P^\mu \qquad M_{\mu\nu} \qquad D \qquad K_\mu \qquad Q^I_\alpha \qquad \tilde{Q}_{\dot\alpha I} \qquad S^\alpha_I \qquad \tilde{S}^{\dot\alpha I} \qquad R^J_I$$

The local operators

$$\{\mathcal{O}_i^{\Delta, j_1, j_2, \mathbf{R}}(x)\}$$

can be organized in superconformal multiplets consisting of

superconformal primary: $[S^\alpha_I, \mathcal{O}_i^{\Delta,j_1,j_2,\mathbf{R}}(0)] = 0 \quad [\tilde{S}^{\dot\alpha I}, \mathcal{O}_i^{\Delta,j_1,j_2,\mathbf{R}}(0)] = 0$

superconformal descendants: $Q \ldots Q \tilde{Q} \ldots \tilde{Q}\, \mathcal{O}_i^{\Delta,j_1,j_2,\mathbf{R}}$

Generic superconformal multiplets contain 2^8 conformal multiplets.

Sometimes representations are *short* or *semishort*, e.g.

$$Q^3_\alpha \mathcal{O} = Q^4_\alpha \mathcal{O} = 0 \qquad\qquad \tilde{Q}_{\dot\alpha 3}\mathcal{O} = \tilde{Q}_{\dot\alpha 4}\mathcal{O} = 0$$

(but $Q^1_\alpha \mathcal{O}$, $Q^2_\alpha \mathcal{O}$, $\tilde{Q}_{\dot\alpha 1}\mathcal{O}$, $\tilde{Q}_{\dot\alpha 2}\mathcal{O} \neq 0$).

Then there are relations between the quantum numbers. For this case:

$$j_1 = j_2 = 0 \qquad R = [0, p, 0] \qquad \Delta = p$$

(These are the chiral primaries, with $\mathcal{O} = \mathrm{Tr}\,(\Phi^{\{I_1} \ldots \Phi^{I_p\}})$ in $\mathcal{N} = 4$ SYM.)

Gauging prescription

In a free vectormultiplet the Schur operators are the gauginos λ_{α}^{A} and $\tilde{\lambda}_{\dot{\alpha}}^{A}$. Defining

$$b^{A}(z) \sim v(\bar{z}) \cdot \lambda_{+}^{A}(z, \bar{z}) \qquad \partial c^{A}(z) \sim v(\bar{z}) \cdot \tilde{\lambda}_{\dot{+}}^{A}(z, \bar{z})$$

we find a (small) (b, c) ghost system with dimensions $(1, 0)$ and OPE

$$b^{A}(z) c^{B}(0) \sim \frac{\delta^{AB}}{z}$$

Gauging of a flavor symmetry now corresponds to a restriction to the cohomology of

$$Q_{\text{BRST}} = \frac{1}{2\pi i} \oint dz \left(c_A J^A - \frac{1}{2} f^{AB}{}_{C} c_A c_B b^C \right)$$

in the chiral algebra.

This operator is nilpotent precisely when the four-dimensional beta function vanishes!

The big picture

For specific theories we have precise claims for the chiral algebra:

- $\mathfrak{su}(2)$ with four fundamental flavors:

  $\mathfrak{so}(8)_{-2}$ AKM algebra

- Minahan-Nemeschansky E_6 theory:

  $(\mathfrak{e}_6)_{-3}$ AKM algebra

- $\mathfrak{su}(N_c)$ with $N_f = 2N_c$ fundamental flavors:

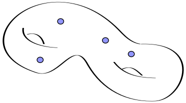
  $\mathfrak{u}(1) \times \mathfrak{su}(N_f)_{-N_c}$ AKM algebra + baryons

- $\mathcal{N} = 4$ SYM theories:

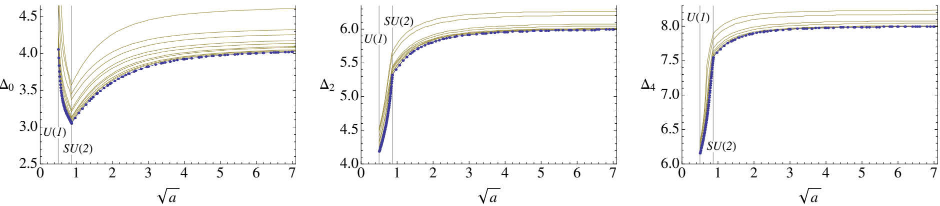
  small $\mathcal{N} = 4$ algebra + primaries from half BPS chiral ring

Furthermore, for class S theories:

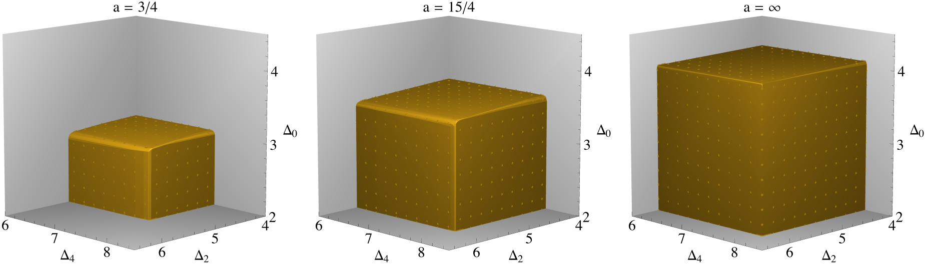
- Chiral algebra of T_N?
- Gauging: as before
- Maximal puncture: AKM algebra at $k = -h^\vee$
- Closing puncture: quantum Drinfeld-Sokolov reduction



Results for the first three spins

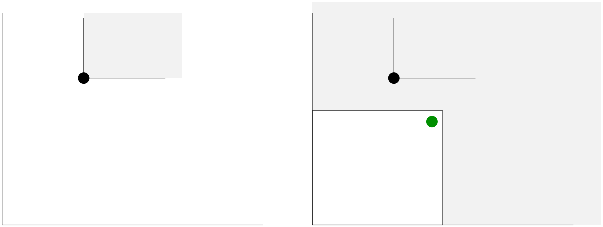


Combining spins



Why the cubes?

We are looking at *bounds*



$\rightarrow$ there is a special solution to crossing symmetry at the corner

We conjecture that it corresponds to a self-dual point of $\mathcal{N} = 4$ SYM.

This leads e.g to

$$\Delta \lesssim 2.90$$

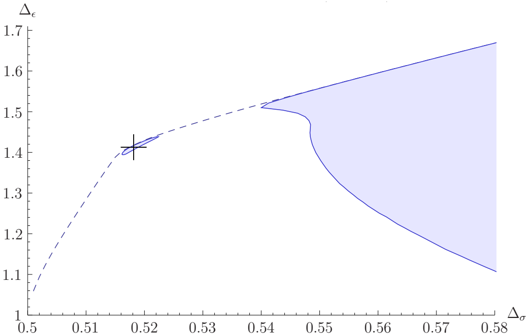
for the Konishi operator $\mathrm{Tr}\,(\Phi^I \Phi_I)$ in $SU(2)$ $\mathcal{N} = 4$ SYM at $\tau = i$ or at $\tau = \exp(i\pi/3)$.

Not in disagreement with resumming the four-loop result...
[Beem, Rastelli, Sen, BvR (2013), Alday, Bissi (2013)]

Can we find the rest of the conformal manifold?

Conformal field theory revisited

$$\sigma(x_1)\sigma(x_2) \sim \frac{1}{(x_1 - x_2)^{2\Delta_\sigma}} + \frac{\epsilon(x_2)}{(x_1 - x_2)^{2\Delta_\sigma - \Delta_\epsilon}} + \ldots$$



[Kos, Poland, Simmons-Duffin (2014)]

The bootstrap in two minutes

The conformal bootstrap

We are interested in correlation functions of local operators

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$$

These are heavily constrained by conformal symmetry, for example

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(y) \rangle = \frac{\delta_{ij}}{(x-y)^{2\Delta}}$$

Conformal invariance further guarantees the existence of a convergent *operator product expansion* (or OPE) of the form

$$\mathcal{O}_i(x)\mathcal{O}_j(y) \sim \sum_k \lambda_{ij}^{\ k}\, C[x-y, \partial_y]\mathcal{O}_k(y)$$

We can use the OPE to decompose correlation functions as

$$\begin{aligned}&\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle \\ &\qquad = \sum_k \lambda_{12}^{\ k}\lambda_{34}^{\ k} C[x_1-x_2, \partial_2] C[x_3-x_4, \partial_4] \langle \mathcal{O}_k(x_2)\mathcal{O}_k(x_4)\rangle\end{aligned}$$

The conformal bootstrap

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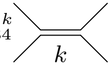

The conformal bootstrap

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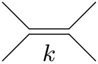
$$= \sum_k \lambda_{12}^{\ \ k}\lambda_{34}^{\ \ k}$$ 

The conformal bootstrap

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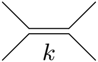
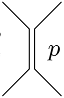
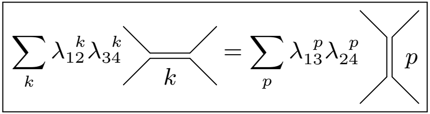


$$= \sum_p \lambda_{13}^{\ \ p}\lambda_{24}^{\ \ p}$$





Crossing symmetry: an infinite set of constraints for Δ_k and $\lambda_{ij}^{\;k}$

Can we solve them? Could we determine the theory using

- global symmetries
- unitarity
- crossing symmetry

and nothing else? In other words, can we *bootstrap* the theory?

[Ferrara, Gatto, Grillo, Parisi (1972); Polyakov (1974)]

Success stories

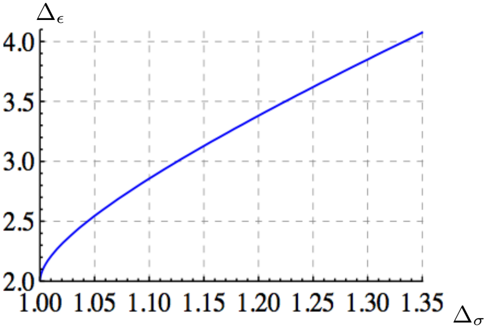
- Minimal models in two dimensions
  [Belavin, Polyakov, Zamolodchikov (1984)]
- Rational CFTs in two dimensions [Moore, Seiberg (1989), . . .]

Success stories

Conformal field theory revisited

$$\langle \sigma(x_1)\sigma(x_2)\sigma(x_3)\sigma(x_4) \rangle$$

$$\sigma(x_1)\sigma(x_2) \sim \frac{1}{(x_1 - x_2)^{2\Delta_\sigma}} + \frac{\epsilon(x_2)}{(x_1 - x_2)^{2\Delta_\sigma - \Delta_\epsilon}} + \dots$$



[Rattazzi, Rychkov, Tonni, Vichi (2008)]

Bootstrapping the 3d Ising model

Conformal field theory revisited

$$\langle \sigma(x_1)\sigma(x_2)\sigma(x_3)\sigma(x_4) \rangle$$

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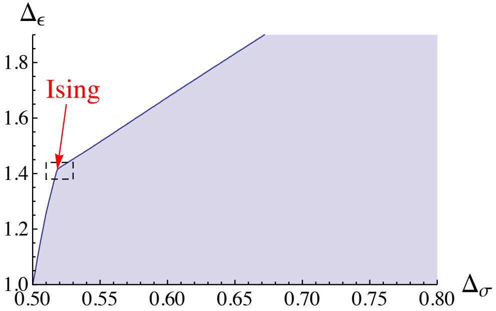

[El-Showk, Paulos, Poland, Simmons-Duffin, Rychkov, Vichi (2012)]