Holographic Accelerating Heavy Quark

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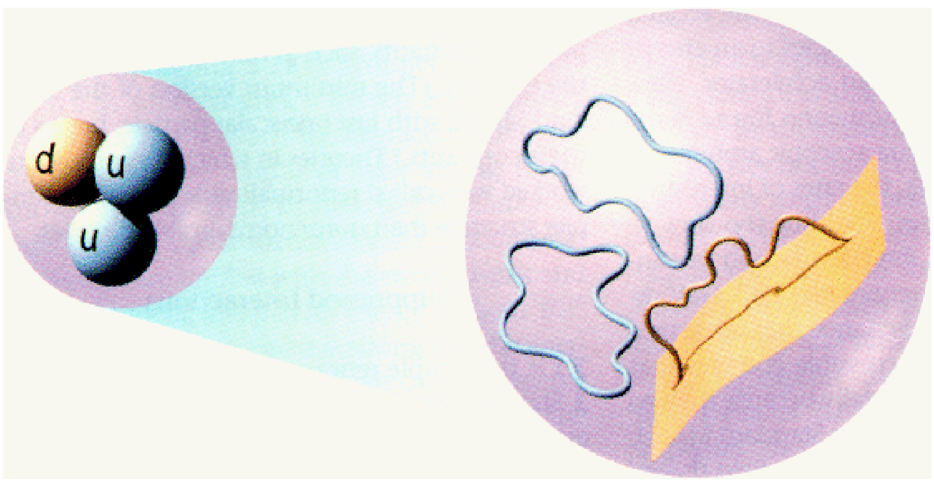
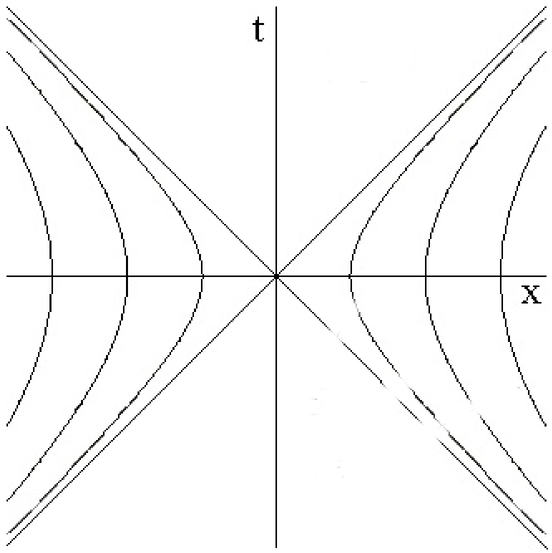


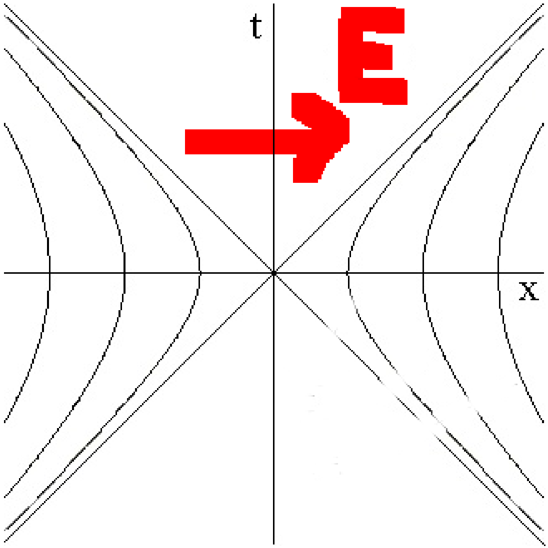
Supersymmetric Field Theories, NORDITA, August 14, 2014

Eternally accelerated heavy quark -anti-quark pair



- prepared in a maximally entangled incoming state
  $|\Psi>=\sum_1^N |a>|a>$, $S_{\text{entang.}} = \ln N$
- constant eternal proper acceleration
- The quark and anti-quark are never in causal contact.
- Do they influence each other?

Acceleration due to constant external electric field



- Proper acceleration $= \frac{E}{M}$
- Schwinger pair production $\mathcal{A} \sim \exp\left(-\frac{\pi M^2}{E}\right)$ suppressed when $\frac{E}{M} << M$, heavy quark, weak field limit

Quark as a W-Boson in N=4 SYM

- Consider $\mathcal{N}=4$ supersymmetric Yang-Mills theory with gauge group $U(N+1)$
- $U(N+1) \to U(N) \times U(1)$ by the Higgs mechanism
- $\exists$ Higgs and W-bosons.
- Quark is a scalar field in the W-boson supermultiplet.
- Electric field is in the unbroken $U(1)$.
- heavy quark limit, large N 't Hooft limit (suppresses W-boson loops and bremsstrahlung)

Semiclassical (heavy quark) limit

world line path integral

$$g(x_f, x_i) = \int_0^\infty \frac{dT}{T} \int dx^\mu e^{iS[x,T]}$$

$$S[x,T] = \int_0^1 d\tau \left[\frac{1}{4T} (\dot{x}^\mu)^2 - M^2 T + \frac{E}{2} x^- \dot{x}^+ \right]$$

$x(1) = x_f$, $x(0) = x_i$

Classical Solution:

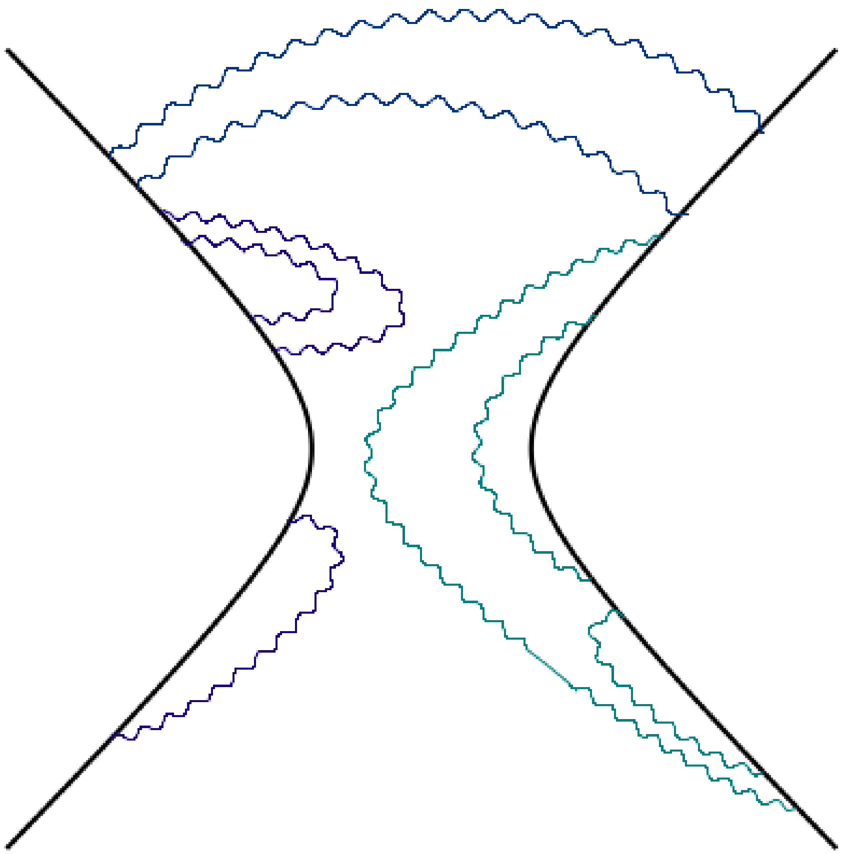
$$x_0^\mu = (t,x,y,z) = \tfrac{M}{E}\left(\sinh \tfrac{E}{M}\tau_P\tau, \cosh \tfrac{E}{M}\tau_P\tau, 0, 0\right) \quad , \quad T = \frac{\tau_P}{2M}$$

Semiclassical limit:

$$g(x_f, x_i) \sim \exp\left(-i \left[M - \frac{M}{2} \right] \tau_P \right)$$

corrections suppressed when $\frac{E}{M^2} << 1$

four-point function



four-point function

$$g(x_f,\tilde{x}_f;x_i,\tilde{x}_i) = \int_0^\infty \frac{dT}{T}\int_0^\infty \frac{d\tilde{T}}{\tilde{T}}\int dx^\mu \int d\tilde{x}^\mu e^{iS[x,T]+i\tilde{S}[\tilde{x},\tilde{T}]}W[x,\tilde{x}]$$

$$S[x,T] = \int_0^1 d\tau \left[\frac{1}{4T}(\dot{x}^\mu)^2 - M^2T + \frac{E}{2}x^-\dot{x}^+\right]$$

$$\tilde{S}[\tilde{x},\tilde{T}] = \int_0^1 d\tau \left[\frac{1}{4\tilde{T}}(\dot{\tilde{x}}^\mu)^2 - M^2\tilde{T} - \frac{E}{2}\tilde{x}^-\dot{\tilde{x}}^+\right]$$

$$W[x,\tilde{x}] = \langle 0|\,\mathrm{Tr}\; U[x]\tilde{U}[\tilde{x}]\,|0\rangle$$

$$U[x] = \mathcal{T}e^{i\int_{-\infty}^{\infty} d\tau\left[A_\mu(x(\tau))\dot{x}^\mu(\tau)+\sqrt{-\dot{x}(\tau)^2}\phi^1(x(\tau))\right]}$$

$$\tilde{U}[\tilde{x}] = \left(\mathcal{T}e^{i\int_{-\infty}^{\infty} d\tau\left[A_\mu(\tilde{x}(\tau))\dot{\tilde{x}}^\mu(\tau)-\sqrt{-\dot{\tilde{x}}(\tau)^2}\phi^1(\tilde{x}(\tau))\right]}\right)^\dagger$$

four-point function

$$g(x_f,\tilde{x}_f;x_i,\tilde{x}_i)=\int_0^\infty \frac{dT}{T}\int_0^\infty \frac{d\tilde{T}}{\tilde{T}}\int dx^\mu\int d\tilde{x}^\mu e^{iS[x,T]+i\tilde{S}[\tilde{x},\tilde{T}]}W[x,\tilde{x}]$$

$$S[x,T]=\int_0^1 d\tau\left[\frac{1}{4T}(\dot{x}^\mu)^2-M^2T+\frac{E}{2}x^-\dot{x}^+\right]$$

Wilson loop $W=\left\langle \mathrm{Tr}\mathcal{P}e^{i\oint d\tau(A_\mu(x(\tau))\dot{x}^\mu(\tau)+\Phi^1(x(\tau))|\dot{x}(\tau)|}\cdot\tilde{U}[\tilde{x}]\right\rangle$

$$\frac{\delta}{\delta x^\mu}\ln W[x,\tilde{x}]\Big|_{x=x_0,\tilde{x}=\tilde{x}_0}=0\quad,\quad\frac{\delta}{\delta\tilde{x}^\mu}\ln W[x,\tilde{x}]\Big|_{x=x_0,\tilde{x}=\tilde{x}_0}=0$$

classical solution is not modified by Wilson loop

$$x_0^\mu=\frac{M}{E}\left(\sinh\tfrac{E}{M}\tau_P\tau,\cosh\tfrac{E}{M}\tau_P\tau,0,0\right)\quad,\quad T=\frac{\tau_P}{2M}$$

$$\tilde{x}_0^\mu=\frac{M}{E}\left(\sinh\tfrac{E}{M}\tau_P\tau,-\cosh\tfrac{E}{M}\tau_P\tau,0,0\right)\quad,\quad T=\frac{\tau_P}{2M}$$

four-point function in heavy quark limit

$$g(x_f,\tilde{x}_f;x_i,\tilde{x}_i) \approx \exp\left(-2i\cdot\left[M-\frac{M}{2}\right]\tau_P\right)\cdot W[x_0,\tilde{x}_0]$$

we are left with the problem of computing the Wilson loop for Rindler trajectories

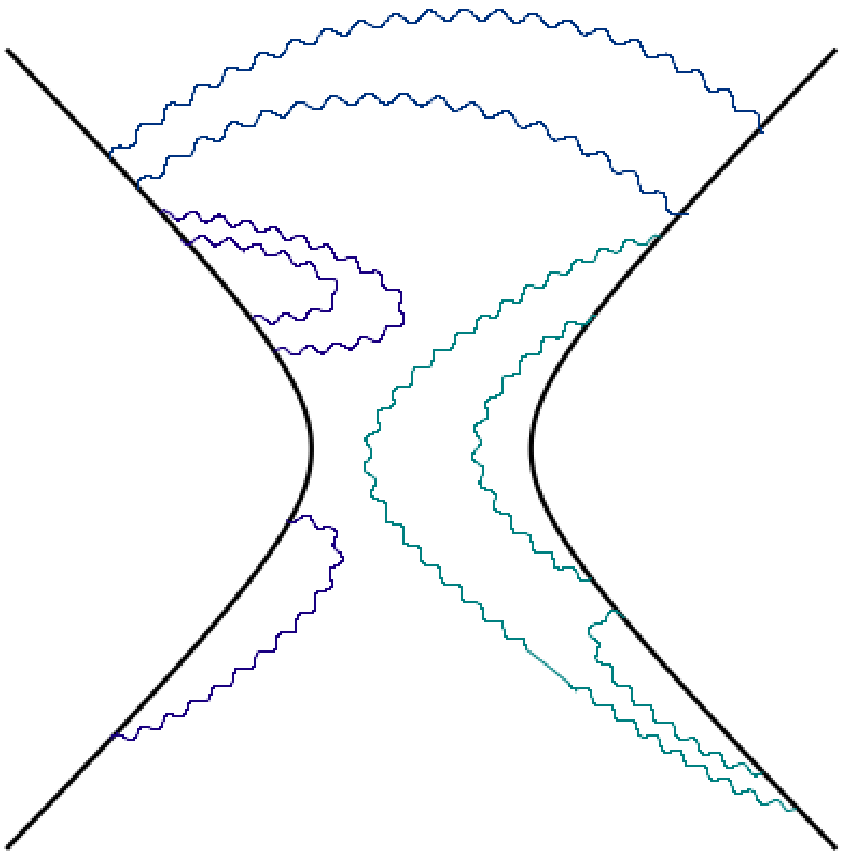
$$x_0^\mu = \frac{M}{E}\left(\sinh\tfrac{E}{M}\tau_P\tau, \cosh\tfrac{E}{M}\tau_P\tau, 0, 0\right) \quad , \quad T=\frac{\tau_P}{2M}$$

$$\tilde{x}_0^\mu = \frac{M}{E}\left(\sinh\tfrac{E}{M}\tau_P\tau, -\cosh\tfrac{E}{M}\tau_P\tau, 0, 0\right) \quad , \quad T=\frac{\tau_P}{2M}$$

Heavy quark limit

$$\frac{E}{M^2} << \sqrt{\lambda}\frac{E}{M^2} << 1$$

computation of Wilson loop



computation of the Wilson loop

$$W[x,\tilde{x}] = \langle 0|\,\mathrm{Tr}\; U[x]\tilde{U}[\tilde{x}]\,|0\rangle$$

$$U[x] = \mathcal{T}e^{i\int_{-\infty}^{\infty} d\tau\left[A_\mu(x(\tau))\dot{x}^\mu(\tau)+\sqrt{-\dot{x}(\tau)^2}\phi^1(x(\tau))\right]}$$

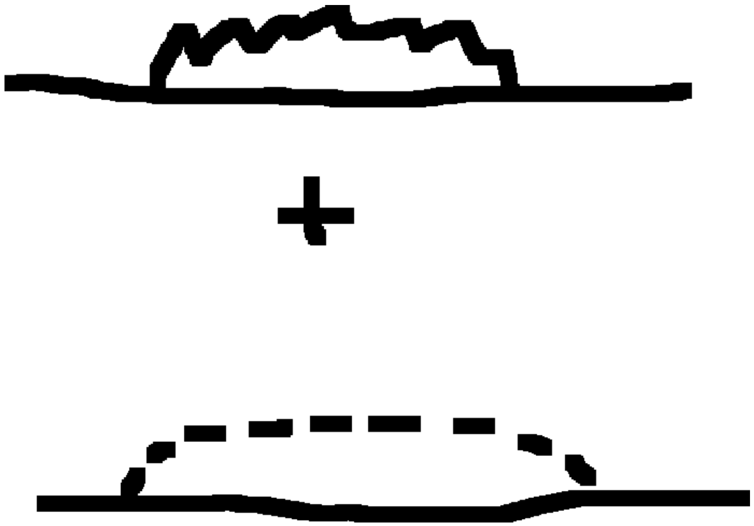
$$\tilde{U}[\tilde{x}] = \left(\mathcal{T}e^{i\int_{-\infty}^{\infty} d\tau\left[A_\mu(\tilde{x}(\tau))\dot{\tilde{x}}^\mu(\tau)-\sqrt{-\dot{\tilde{x}}(\tau)^2}\phi^1(\tilde{x}(\tau))\right]}\right)^\dagger$$

Propagators:

$$\left\langle \mathcal{T}\phi^{ab}(x)\phi^{cd}(y)\right\rangle_0 = \frac{g^2\delta^{ad}\delta^{bc}}{8\pi^2\left[x-y\right]^2+i\epsilon}$$

$$\left\langle \mathcal{T}A_\mu^{ab}(x)A_\nu^{cd}(y)\right\rangle_0 = \frac{g^2\delta^{ad}\delta^{bc}g_{\mu\nu}}{8\pi^2\left[x-y\right]^2+i\epsilon}$$

Exchange of one "gluon"



$$-\frac{g^2N^2}{2}\int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{x}(\tau_2)^2}+\dot{x}_\mu(\tau_1)\dot{x}^\mu(\tau_2)}{8\pi^2\left[x(\tau_1)-x(\tau_2)\right]^2+i\epsilon}$$

Exchange of one “gluon”

Perturbation theory

$$\delta W = -\frac{g^2N^2}{2}\int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{x}(\tau_2)^2}+\dot{x}_\mu(\tau_1)\dot{x}^\mu(\tau_2)}{8\pi^2\left[x(\tau_1)-x(\tau_2)\right]^2+i\epsilon}$$

$$-\frac{g^2N^2}{2}\int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{\tilde{x}}(\tau_1)^2}\sqrt{-\dot{\tilde{x}}(\tau_2)^2}+\dot{\tilde{x}}_\mu(\tau_1)\dot{\tilde{x}}^\mu(\tau_2)}{8\pi^2\left[\tilde{x}(\tau_1)-\tilde{x}(\tau_2)\right]^2+i\epsilon}$$

$$-g^2N^2\int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{\tilde{x}}(\tau_2)^2}-\dot{x}_\mu(\tau_1)\dot{\tilde{x}}^\mu(\tau_2)}{8\pi^2\left[x(\tau_1)-\tilde{x}(\tau_2)\right]^2+i\epsilon}$$

Propagators = constants: $\frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{x}(\tau_2)^2}+\dot{x}_\mu(\tau_1)\dot{x}^\mu(\tau_2)}{8\pi^2[x(\tau_1)-x(\tau_2)]^2+i\epsilon} = \frac{E^2}{16\pi^2M^2}$

$$W[x_0,\tilde{x}_0] = N\left[1-\lambda\left(\frac{E}{2\pi M}\tau_P\right)^2 + \ldots\right]$$

Perturbation theory

$$\delta W = -\frac{g^2 N^2}{2} \int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{x}(\tau_2)^2} + \dot{x}_\mu(\tau_1)\dot{x}^\mu(\tau_2)}{8\pi^2 \left[x(\tau_1) - x(\tau_2)\right]^2 + i\epsilon}$$

$$-\frac{g^2 N^2}{2} \int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{\tilde{x}}(\tau_1)^2}\sqrt{-\dot{\tilde{x}}(\tau_2)^2} + \dot{\tilde{x}}_\mu(\tau_1)\dot{\tilde{x}}^\mu(\tau_2)}{8\pi^2 \left[\tilde{x}(\tau_1) - \tilde{x}(\tau_2)\right]^2 + i\epsilon}$$

$$-g^2 N^2 \int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{\tilde{x}}(\tau_2)^2} - \dot{x}_\mu(\tau_1)\dot{\tilde{x}}^\mu(\tau_2)}{8\pi^2 \left[x(\tau_1) - \tilde{x}(\tau_2)\right]^2 + i\epsilon}$$

Propagators = constants: $\frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{x}(\tau_2)^2}+\dot{x}_\mu(\tau_1)\dot{x}^\mu(\tau_2)}{8\pi^2[x(\tau_1)-x(\tau_2)]^2+i\epsilon} = \frac{E^2}{16\pi^2 M^2}$

$$W[x_0, \tilde{x}_0] = N\left[1 - \lambda\left(\frac{E}{2\pi M}\tau_P\right)^2 + \ldots\right]$$

N=1:

$$W[x_0, \tilde{x}_0] = \ e^{-\lambda\left(\frac{E}{2\pi M}\tau_P\right)^2} \ \approx \ 0$$

Perturbation theory

$$\delta W = -\frac{g^2N^2}{2}\int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{x}(\tau_2)^2}+\dot{x}_\mu(\tau_1)\dot{x}^\mu(\tau_2)}{8\pi^2\left[x(\tau_1)-x(\tau_2)\right]^2+i\epsilon}$$

$$-\frac{g^2N^2}{2}\int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{\tilde{x}}(\tau_1)^2}\sqrt{-\dot{\tilde{x}}(\tau_2)^2}+\dot{\tilde{x}}_\mu(\tau_1)\dot{\tilde{x}}^\mu(\tau_2)}{8\pi^2\left[\tilde{x}(\tau_1)-\tilde{x}(\tau_2)\right]^2+i\epsilon}$$

$$-g^2N^2\int d\tau_1 d\tau_2 \frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{\tilde{x}}(\tau_2)^2}-\dot{x}_\mu(\tau_1)\dot{\tilde{x}}^\mu(\tau_2)}{8\pi^2\left[x(\tau_1)-\tilde{x}(\tau_2)\right]^2+i\epsilon}$$

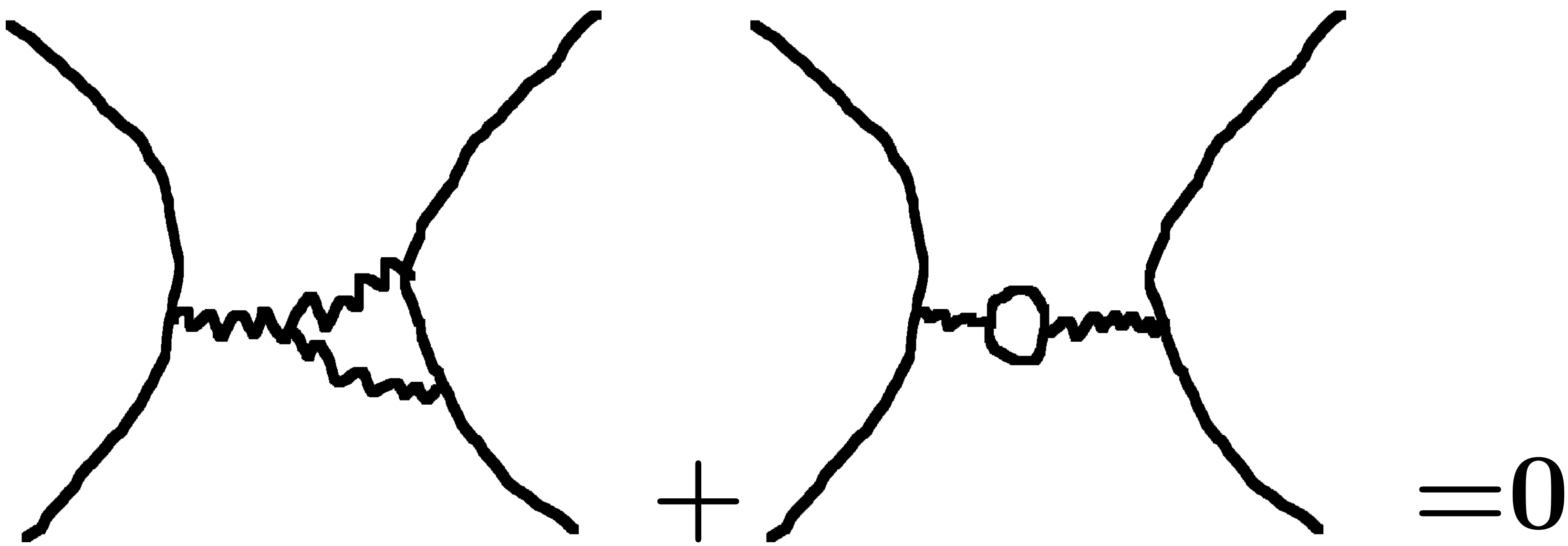
Propagators = constants: $\frac{\sqrt{-\dot{x}(\tau_1)^2}\sqrt{-\dot{x}(\tau_2)^2}+\dot{x}_\mu(\tau_1)\dot{x}^\mu(\tau_2)}{8\pi^2[x(\tau_1)-x(\tau_2)]^2+i\epsilon} = \frac{E^2}{16\pi^2M^2}$

$$W[x_0,\tilde{x}_0] = N\left[1-\lambda\left(\frac{E}{2\pi M}\tau_P\right)^2 + \ldots\right]$$

sum all diagrams with no internal vertices (matrix model)

$$W[x_0,\tilde{x}_0] = NL^1_{N-1}\left[\frac{\lambda}{N}\left(\frac{E}{2\pi M}\tau_P\right)^2\right] e^{-\frac{\lambda}{N}\left(\frac{E}{2\pi M}\tau_P\right)^2}$$

Corrections vanish



Exact Wilson loop

$$W[x_0, \tilde{x}_0] = N L^1_{N-1} \left[\frac{\lambda}{N} \left(\frac{E}{2\pi M} \tau_P \right)^2 \right] e^{-\frac{\lambda}{N}\left(\frac{E}{2\pi M}\tau_P\right)^2}$$

We are supposed to take τ_P large and extract the asymptotic behaviour

$$W \sim e^{-i\mathcal{E}\tau_P}$$

Here, $W \to 0$

bremsstrahlung
The amplitude for Q-$\bar{Q}$ without additional photons and gluons in final state is zero
bremsstrahlung suppressed at large N

Take the large N limit

$$W[x_0,\tilde{x}_0] = N L^1_{N-1}\left[\frac{\lambda}{N}\left(\frac{E}{2\pi M}\tau_P\right)^2\right] e^{-\frac{\lambda}{N}\left(\frac{E}{2\pi M}\tau_P\right)^2}$$

$$W[x_0,\tilde{x}_0] = \frac{N}{\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}\tau_P} J_1\left(2\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}\tau_P\right) + \mathcal{O}(1/N)$$

Take the large τ_P asymptotic:

$$W[x_0,\tilde{x}_0] \approx N\frac{-i}{\sqrt{4\pi}\left(\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}\tau_P\right)^{\frac{3}{2}}}\; e^{i\; 2\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}\tau_P}$$

four-point function in heavy quark limit

$$g(x_f,\tilde{x}_f;x_i,\tilde{x}_i) \approx \exp\left(-2i\cdot\left[M-\frac{M}{2}\right]\tau_P\right)\cdot W[x_0,\tilde{x}_0] \sim e^{-i\mathcal{E}\tau_P}$$

Wilson loop for Rindler trajectories

$W[x_0,\tilde{x}_0] \approx \exp\left(2i\cdot\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M^2}\ M\tau_P\right)$

Is significant (larger than $\frac{E}{M^2}$ corrections when $\frac{\sqrt{\lambda}}{2\pi} >> 1$)

We still need $\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M^2} << 1$

$$\frac{E}{M^2} << \frac{\sqrt{\lambda}}{2\pi}\frac{E}{M^2} << 1$$

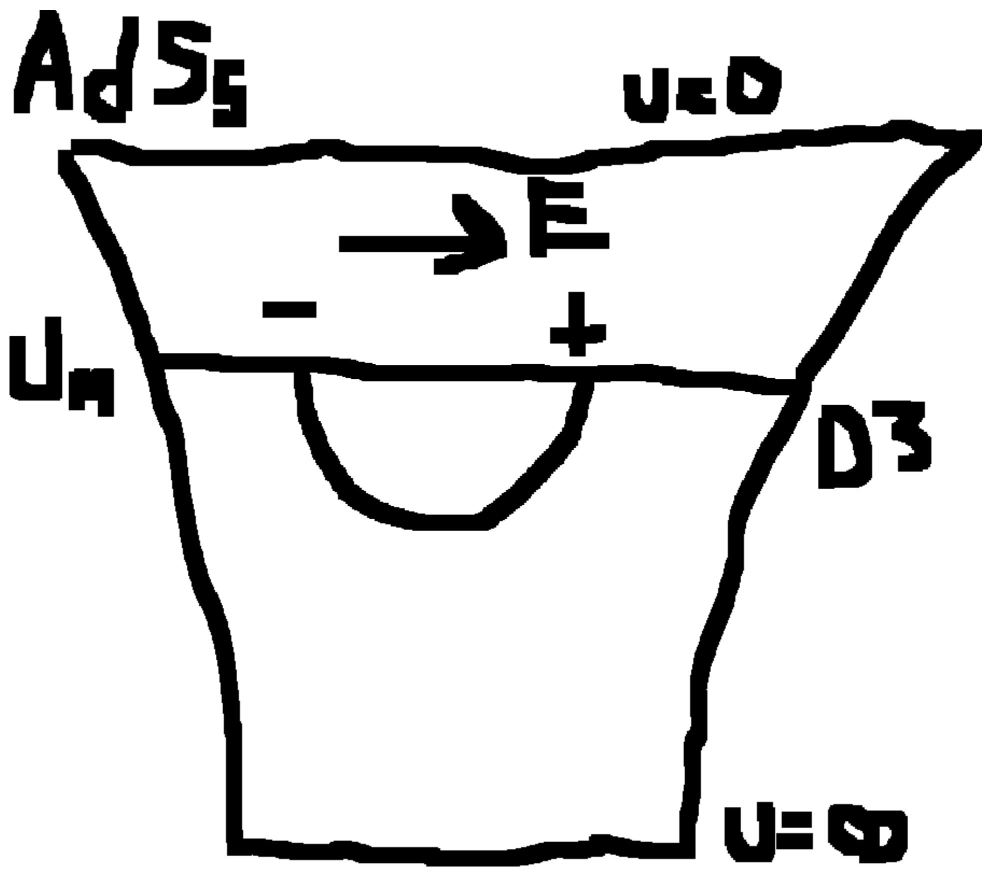
Energy of Q-$\bar{Q}$ pair is

$$\mathcal{E} = 2\cdot\left[M-\frac{M}{2}-\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M^2}M\right]+\ldots$$

String theory dual

- IIB string theory on $AdS_5 \times S^5$ background
- metric of AdS: $ds^2 = \sqrt{\lambda}\alpha' \left[\frac{du^2 + d\vec{x}^2 - dt^2}{u^2} \right]$
- Probe D3 brane suspended at constant AdS radius u_M
- W-boson = open string suspended between probe brane and Poincare horizon $u_M = \frac{\sqrt{\lambda}}{2\pi M}$
- turn off string loop diagrams ('t Hooft limit in Yang Mills)
- IIB string sigma model is semi-classical when $\sqrt{\lambda}$ is large
- Compute the disc amplitude where the boundary of the disc is located on the probe D3 brane and is subject to a probe brane world volume electric field E,
- E does not modify brane embedding $\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M^2} << 1$
- Disc amplitude $\sim \exp\left(-i\mathcal{E}\tau_P\right)$. What is $\mathcal{E}$?

String theory dual



String theory dual

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{-\det g} + \oint \frac{E}{2} x^- \dot{x}^+$$

Solution is AdS_2:

$$u^2 + x^2 - t^2 = \frac{M^2}{E^2} \quad ,$$

E enters through the boundary condition.

This solution is mostly fixed by symmetry

At the boundary: $u = u_M = \sqrt{\lambda}/2\pi M$

$$x^2 - t^2 = \frac{M^2}{E^2}\left(1 - \left(\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M^2}\right)^2\right)$$

$$(t,x) = \frac{M}{E}\sqrt{1 - \left(\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M^2}\right)^2}\left(\sinh\frac{\frac{E}{M}\tau}{\sqrt{1-\left(\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M^2}\right)^2}}, \pm\cosh\frac{\frac{E}{M}\tau}{\sqrt{1-\left(\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M^2}\right)^2}}\right)$$

World sheet metric

near quark trajectory

$$ds^2 = \sqrt{\lambda}\alpha' \left[-\frac{2}{u^2} \frac{du d\tau}{\sqrt{1 - \left(\frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2} \right)^2}} - \left(\frac{1}{u^2} - \frac{E^2}{M^2} \right) \frac{d\tau^2}{1 - \left(\frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2} \right)^2} \right]$$

world sheet event horizons at $u = \frac{M}{E}$

proper area **exterior** to event horizons gives the energy

$$\mathcal{E} = 2M \sqrt{\frac{1 - \frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2}}{1 + \frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2}}} - M \sqrt{1 - \left(\frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2} \right)^2}$$

$$\boxed{\mathcal{E} = 2 \left[M - \frac{M}{2} - \frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2} \, M \right] + \ldots}$$

This matches the Yang-Mills Calculation!

Questions: I)

Why just the area outside of the event horizon?

$$ds^2 = \sqrt{\lambda}\alpha' \left[-\frac{2}{u^2} du d\tau - \left(\frac{1}{u^2} - \frac{E^2}{M^2} \right) d\tau^2 \right]$$

$$\mathcal{E}_{\rm entire} = \mathcal{E} + \mathcal{E}_{YM} \quad = \quad 2\left[M - \tfrac{M}{2}\right] \quad , \quad \mathcal{E}_{\rm YM} = 2M \tfrac{\sqrt{\lambda}}{2\pi} \tfrac{E}{M^2}$$

$$\boxed{\mathcal{E} \approx \left[2M - M - 2\frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2} \ M \right] + \ldots} \quad \boxed{\mathcal{E}_{\rm YM} \approx 2M \left(\frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2} \right) + \ldots}$$

$\mathcal{E}_{\rm YM}$ is the energy stored in classical gauge and scalar fields

$S = \int {\rm Tr}(-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - D_\mu \bar{\Phi} D^\mu \Phi)$

Questions: II)

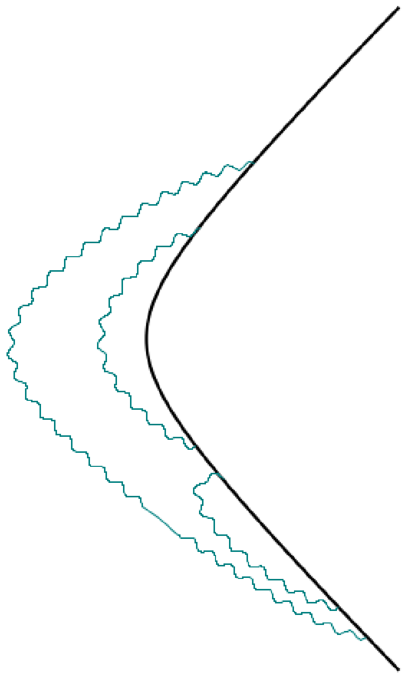
Do Q and $\bar{Q}$ talk to each other?

Worldsheet is a wormhole. Q and $\bar{Q}$, which are the endpoints of the string are separated from the remainder of the world sheet (and from each other) by event horizons. Q and $\bar{Q}$ cannot talk to each other by exchanging worldsheet excitations.

Questions: II)

Do Q and $\bar{Q}$ talk to each other?

Compare Q-$\bar{Q}$ with Q



Do Q and $\bar{Q}$ talk to each other?

Sum ladder diagrams:

$$W[x_0,\tilde{x}_0] = N L^1_{N-1}\left[\frac{\lambda}{4N}\left(\frac{E}{2\pi M}\tau_P\right)^2\right] e^{-\frac{\lambda}{4N}\left(\frac{E}{2\pi M}\tau_P\right)^2}$$

$$W[x_0,\tilde{x}_0] = \frac{N}{\frac{\sqrt{\lambda}}{4\pi}\frac{E}{M}\tau_P} J_1\left(\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}\tau_P\right) + \mathcal{O}(1/N)$$

$$W[x_0,\tilde{x}_0] \approx N \frac{-i}{\sqrt{4\pi}\left(\frac{\sqrt{\lambda}}{4\pi}\frac{E}{M}\tau_P\right)^{\frac{3}{2}}}\ e^{i\ \frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}\tau_P}$$

For Q-$\bar{Q}$ $\mathcal{E} = 2[M - M/2 - \frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}]$

For Q, $\mathcal{E} = [M - M/2 - \frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}]$

Do Q and $\bar{Q}$ talk to each other?

Sum ladder diagrams:

$$W[x_0,\tilde{x}_0] = NL^1_{N-1}\left[\frac{\lambda}{4N}\left(\frac{E}{2\pi M}\tau_P\right)^2\right] e^{-\frac{\lambda}{4N}\left(\frac{E}{2\pi M}\tau_P\right)^2}$$

$$W[x_0,\tilde{x}_0] = \frac{N}{\frac{\sqrt{\lambda}}{4\pi}\frac{E}{M}\tau_P} J_1\left(\frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}\tau_P\right) + \mathcal{O}(1/N)$$

$$W[x_0,\tilde{x}_0] \approx N\frac{-i}{\sqrt{4\pi}\left(\frac{\sqrt{\lambda}}{4\pi}\frac{E}{M}\tau_P\right)^{\frac{3}{2}}}\; e^{i\ \frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}\tau_P}$$

For Q-$\bar{Q}$ $\mathcal{E} = 2[M - M/2 - \frac{\sqrt{\lambda}}{2\pi}\frac{E}{M}]$

For Q, $\mathcal{E} = [M - M/2 - \frac{\sqrt{\lambda}}{2\pi}\frac{E}{M} + ?]$

But, the contribution of Feynman diagrams with interactions do not cancel.

Do Q and $\bar{Q}$ talk to each other?

Sum ladder diagrams:

$$W[x_0, \tilde{x}_0] = N L^1_{N-1} \left[\frac{\lambda}{4N} \left(\frac{E}{2\pi M} \tau_P \right)^2 \right] \, e^{-\frac{\lambda}{4N} \left(\frac{E}{2\pi M} \tau_P \right)^2}$$

$$W[x_0, \tilde{x}_0] = \frac{N}{\frac{\sqrt{\lambda}}{4\pi} \frac{E}{M} \tau_P} J_1 \left(\frac{\sqrt{\lambda}}{2\pi} \frac{E}{M} \tau_P \right) + \mathcal{O}(1/N)$$

$$W[x_0, \tilde{x}_0] \approx N \frac{-i}{\sqrt{4\pi} \left(\frac{\sqrt{\lambda}}{4\pi} \frac{E}{M} \tau_P \right)^{\frac{3}{2}}} \; e^{i \, \frac{\sqrt{\lambda}}{2\pi} \frac{E}{M} \tau_P}$$

For Q-$\bar{Q}$ $\mathcal{E} = 2[M - M/2 - \frac{\sqrt{\lambda}}{2\pi} \frac{E}{M}]$

For Q, $\mathcal{E} = [M - M/2 - \frac{\sqrt{\lambda}}{2\pi} \frac{E}{M} + ?]$

But, the contribution of Feynman diagrams with interactions do cancel.

The question is still open...

Questions: II)

What about the Unruh temperature?

$T_U = \frac{\text{acceleration}}{2\pi} = \frac{E}{2\pi M}$

Energy of the quark:

$$\boxed{\mathcal{E} = \left[M - M/2 - \sqrt{\lambda} T_U + \ldots \right]}$$

Free energy:

$$H = E - TS$$

Entropy:

$$S = \sqrt{\lambda}$$

Interpret as entanglement entropy?

Thank You