Double-copy structures of pure and matter-coupled supergravities

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Nordita workshop:
Supersymmetric
Field Theories

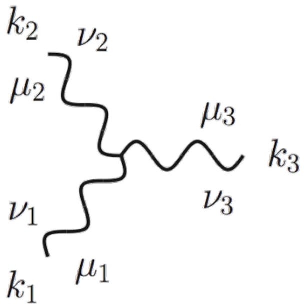
Based on: arXiv:1407.4772, HJ, A. Ochirov,
arXiv:1408.0764 M. Chiodaroli, M. Gunaydin, HJ, R. Roiban

Gravity double-copy structure

Graviton plane wave: $\varepsilon^{\mu}(p)\varepsilon^{\nu}(p)\, e^{ip\cdot x}$

Yang-Mills polarization

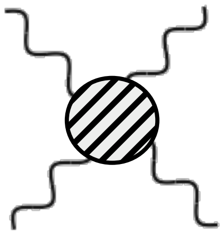
On-shell 3-graviton vertex:



$$= \quad i\kappa\Big(\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3}+\text{cyclic}\Big)\Big(\eta_{\nu_1\nu_2}(k_1-k_2)_{\nu_3}+\text{cyclic}\Big)$$

Yang-Mills vertex

Gravity scattering amplitude:



$$M_4^{\text{tree}}(1,2,3,4) = -i\frac{st}{u}A_4^{\text{tree}}(1,2,3,4)\tilde{A}_4^{\text{tree}}(1,2,3,4)$$

Yang-Mills amplitude

Kawai-Lewellen-Tye Relations ('86)

String theory tree-level identity:

closed string ~ (left open string) × (right open string)

$$A_n \sim \int \frac{dx_1 \cdots dx_n}{\mathcal{V}_{abc}} \prod_{1\le i<j\le n} |x_i - x_j|^{k_i\cdot k_j} \exp\Big[\sum_{i<j}\Big(\frac{\epsilon_i\cdot\epsilon_j}{(x_i-x_j)^2} + \frac{k_i\cdot\epsilon_j - k_j\cdot\epsilon_i}{(x_i-x_j)}\Big)\Big]\Big|_{\text{multi-linear}}$$

KLT relations emerge after nontrivial world-sheet integral identities

Field theory limit $\Rightarrow$ gravity theory ~ (gauge theory) × (gauge theory)

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12}A_4^{\text{tree}}(1,2,3,4)\,\widetilde{A}_4^{\text{tree}}(1,2,4,3)$$

$$M_5^{\text{tree}}(1,2,3,4,5) = is_{12}s_{34}A_5^{\text{tree}}(1,2,3,4,5)\,\widetilde{A}_5^{\text{tree}}(2,1,4,3,5)$$
$$+is_{13}s_{24}A_5^{\text{tree}}(1,3,2,4,5)\,\widetilde{A}_5^{\text{tree}}(3,1,4,2,5)$$

gravity states are products of gauge theory states:

$$|2\rangle = |1\rangle \otimes |1\rangle$$
$$|3/2\rangle = |1\rangle \otimes |1/2\rangle$$

etc...

Color-Kinematics Duality ('08)

Yang-Mills theories are controlled by a hidden kinematic Lie algebra

Bern, Carrasco, HJ

- **Amplitude represented by cubic graphs:**

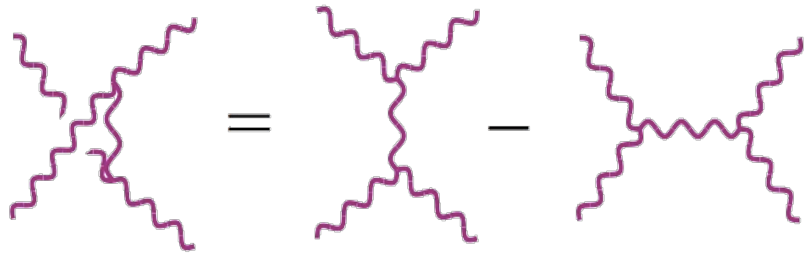
numerators

color factors

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

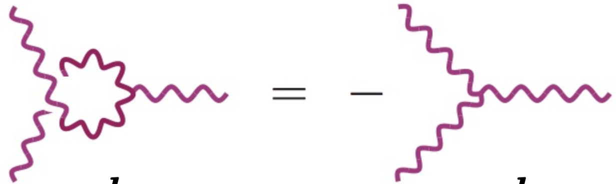
propagators

Color & kinematic numerators satisfy same relations:



Jacobi identity

$$f^{adc} f^{ceb} = f^{eac} f^{cbd} - f^{abc} f^{cde}$$



antisymmetry

$$f^{bac} \;=\; -\; f^{abc}$$

Algebra enforces (BCJ) relations on partial amplitudes → (n-3)! basis

(proven via string theory by Bjerrum-Bohr, Damgaard, Vanhove; Stieberger)

Gravity is a double copy of YM

- Gravity amplitudes obtained by replacing color with kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

BCJ

$$\mathcal{M}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

- The two numerators can belong to different theories:

n_i	$\tilde{n}_i$		
($\mathcal{N}$=4) ×	($\mathcal{N}$=4)	→	$\mathcal{N}$=8 sugra
($\mathcal{N}$=4) ×	($\mathcal{N}$=2)	→	$\mathcal{N}$=6 sugra
($\mathcal{N}$=4) ×	($\mathcal{N}$=0)	→	$\mathcal{N}$=4 sugra
($\mathcal{N}$=0) ×	($\mathcal{N}$=0)	→	Einstein gravity + axion+ dillaton

similar to Kawai-Lewellen-Tye but works at loop level

General or accident?

Is the double copy structure a generic feature of gravity ?
Or accident of maximal N=8 supergravity (and its truncations) ?

Some known limitations (even for tree-level KLT):

- N<4 supergravities contaminated by extra matter (dilaton, axion,…)
  - How are pure N<4 supergravities obtained ?
- Huge number of N=<4 supergravities that are not truncations of N=8 SG.
  - Can extra matter be introduced in N=<4 supergravities ?
    - Abelian
    - Non-abelian (gauged supergravities)
    - Tensor matter in D=6
- There are gauge-theory amplitudes that cannot be used in the KLT formula
  - e.g. fundamental matter amplitudes — do they have a purpose ?

This talk

Two generalizations of color-kinematics duality

1) Color-kinematics duality for fundamental rep. matter — HJ, Ochirov

→ "square" matter and vector states separately

e.g. gravitons ~ $(\text{vector})^2$

matter/vector ~ $(\text{matter})^2$

→ Pure N<4 supergravities

→ Generic abelian N=<4 matter (vectors, tensors, fermions…)

2) Color-kinematics duality for a certain bosonic YM + scalar theory

→ Supergravity coupled to abelian and non-abelian vectors

i.e. Maxwell-Einstein and Yang-Mills-Einstein supergravity

Chiodaroli, Gunaydin, HJ, Roiban

These address/resolve the mentioned problems!

Motivation II: (super)gravity UV behavior

Old results on UV properties:

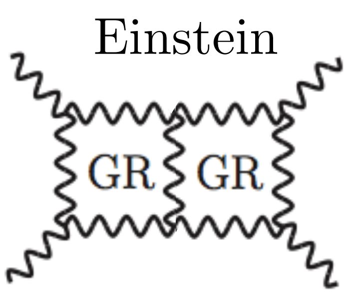
- susy forbids 1,2 loop div. $\cancel{R^2, R^3}$ Ferrara, Zumino, Deser, Kay, Stelle, Howe, Lindström, Green, Schwarz, Brink, Marcus, Sagnotti
- Pure gravity 1-loop finite, 2-loop divergent Goroff & Sagnotti, van de Ven
- With matter: 1-loop divergent 't Hooft & Veltman; (van Nieuwenhuizen; Fischler..)

New results on $N \geq 4$ UV properties:

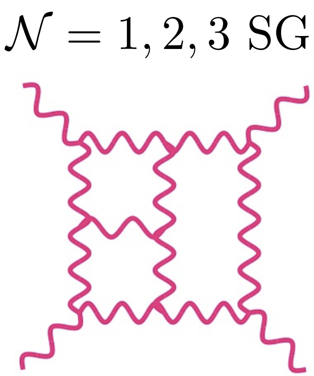
- $\mathcal{N}$=8 SG and $\mathcal{N}$=4 SG 3-loop finite! Bern, Carrasco, Dixon, HJ, Kosower, Roiban; Bern, Davies, Dennen, Huang
- $\mathcal{N}$=8 SG: no divergence before 7 loops Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Björnsson, Green, Bossard, Howe, Stelle, Vanhove Kallosh, Ramond, Lindström, Berkovits, Grisaru, Siegel, Russo, and more....
- First $\mathcal{N}$=4 SG divergence at 4 loops Bern, Davies, Dennen, Smirnov, Smirnov
  (unclear interpretation, U(1) anomaly?) Carrasco, Kallosh, Roiban, Tseytlin

However, no new results in pure $N < 4$ SG

Need double-copy construction for pure $\mathcal{N} < 4$ supergravity



Dissect Goroff & Sagnotti; van de Ven



Outline

- Motivation
- Color-kinematics duality for fundamental rep.
  - Gravity-matter amplitudes
  - Application to pure gravities in *D*=4
- Color-kinematics duality for bosonic YM+scalar theory
  - Gauged supergravity (generic Jordan family of SGs)
- Explicit loop-level checks
- Conclusion

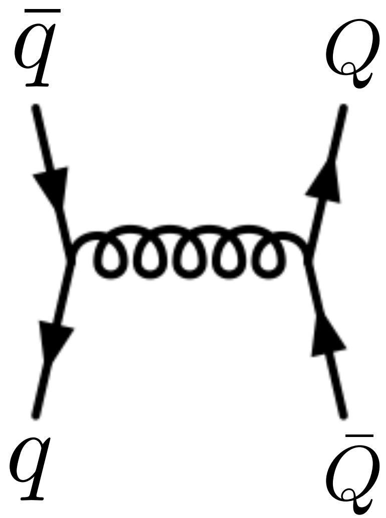
Color-kinematics duality for fundamental rep.

4pt matter ampl’s

Since 2008, large body of evidence for color-kinematics duality

However, 4pt tree-level YM with matter seems broken?

$$A(\bar{q}, q, \bar{Q}, Q) =$$



$$= \frac{n_s c_s}{s}$$

Naïve application of color-kinematics duality

$$n_t = n_u = 0$$

$$n_s + n_t + n_u = 0$$

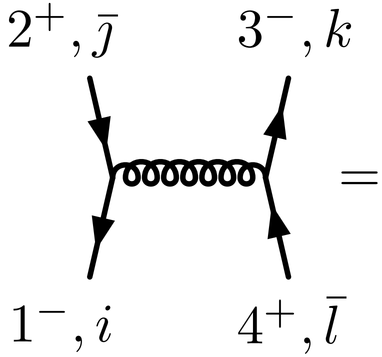
→ YM amplitude forced to vanish.

→ Color-kinematics duality not generic ?

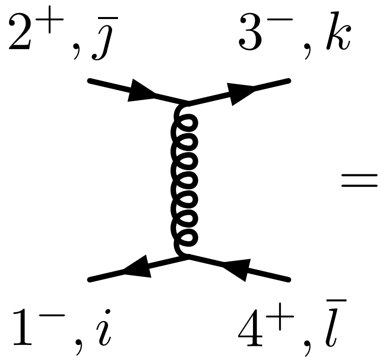
or are we thinking about it incorrectly ?

Switch to fundamental representation

The four-quark tree amplitude:



$$= -i\, T^a_{i\bar\jmath} T^a_{k\bar l} \frac{\langle 13\rangle\, [24]}{s} = \frac{c_1 n_1}{D_1}$$

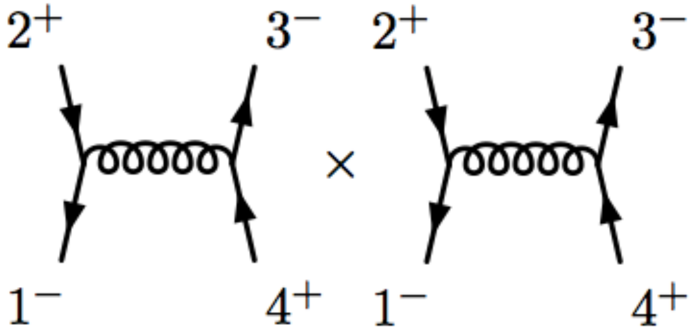


$$= -i\, T^a_{i\bar l} T^a_{k\bar\jmath} \frac{\langle 13\rangle\, [24]}{t} = \frac{c_2 n_2}{D_2}$$

Note: 1) diagrams are gauge invariant – each is a partial amplitude
2) not meaningful to impose constraints on numerators
3) any numerator relation have to be manifest
4) this amplitude must have manifest color-kinematics duality

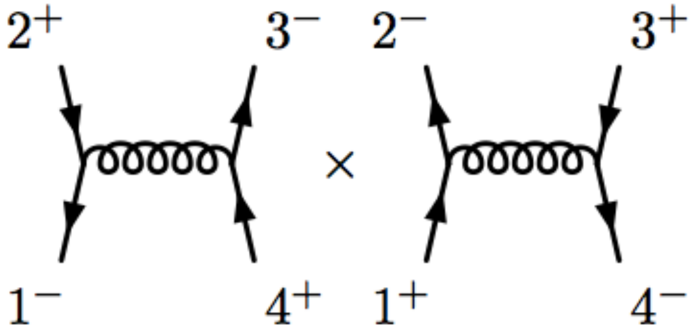
Gravity amplitudes from double copy

Four-photon amplitude in GR (distinguishable matter):



$$A(1^-_\gamma, 2^+_\gamma, 3^-_{\gamma'}, 4^+_{\gamma'}) = \frac{n_s^2}{s} = \frac{\langle 13\rangle^2 [24]^2}{s}$$

Four-scalar amplitude in GR (distinguishable matter):



$$A(1^{-+}_\phi, 2^{+-}_\phi, 3^{-+}_{\phi'}, 4^{+-}_{\phi'}) = \frac{n_s \bar{n}_s}{s} = \frac{u^2}{s}$$

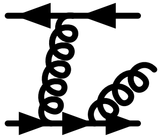
indistinguishable matter:

$$A(1^{-+}_\phi, 2^{+-}_\phi, 3^{-+}_\phi, 4^{+-}_\phi) = \frac{n_s \bar{n}_s}{s} + \frac{n_s \bar{n}_s}{t} = \frac{u^2}{s} + \frac{u^2}{t}$$

More complicated 5pt example

Look at 3 Feynman diagrams out of 10 in total:

$3^-, k$ $4^+, \bar{l}$



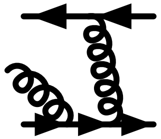
$5, a$

$2^+, \bar{\jmath}$ $1^-, i$

$$= \frac{i}{\sqrt{2}} \frac{1}{s_{15} s_{34}} T^a_{i\bar{m}} T^b_{m\bar{\jmath}} T^b_{k\bar{l}} \, \langle 1|\varepsilon_5|1{+}5|3\rangle \, [24] = \frac{c_1 n_1}{D_1}$$

$3^-, k$ $4^+, \bar{l}$

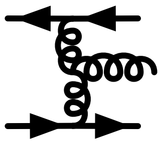
$5, a$



$2^+, \bar{\jmath}$ $1^-, i$

$$= -\frac{i}{\sqrt{2}} \frac{1}{s_{25} s_{34}} T^b_{i\bar{m}} T^a_{m\bar{\jmath}} T^b_{k\bar{l}} \, \langle 13\rangle \, [2|\varepsilon_5|2{+}5|4] = \frac{c_2 n_2}{D_2}$$

$3^-, k$ $4^+, \bar{l}$



$5, a$

$2^+, \bar{\jmath}$ $1^-, i$

$$= \frac{i}{\sqrt{2}} \frac{1}{s_{12} s_{34}} \tilde{f}^{abc} T^b_{i\bar{\jmath}} T^c_{k\bar{l}} \Big(\langle 1|\, \varepsilon_5 |2] \, \langle 3|\, 5|4] - \langle 1|\, 5|2] \, \langle 3|\, \varepsilon_5|4] - 2\, \langle 13\rangle \, [24]((k_1{+}k_2)\cdot\varepsilon_5) \Big) = \frac{c_5 n_5}{D_5}$$

Not gauge invariant, but accidentally satisfy color-kinematics duality

$$c_1 - c_2 = -c_5 \qquad \Leftrightarrow \qquad n_1 - n_2 = -n_5$$

Double copy = gravity amplitudes

Indistinguishable matter:

$$A(1_\gamma^-, 2_\gamma^+, 3_\gamma^-, 4_\gamma^+, 5_h^{++}) = \sum_{i=1}^{10} \frac{n_i^2}{D_i}$$

$$A(1_\phi^{-+}, 2_\phi^{+-}, 3_\phi^{-+}, 4_\phi^{+-}, 5_h^{++}) = \sum_{i=1}^{10} \frac{n_i \bar{n}_i}{D_i}$$

distinguishable matter

$$A(1_\gamma^-, 2_\gamma^+, 3_{\gamma'}^-, 4_{\gamma'}^+, 5_h^{++}) = \sum_{i=1}^{5} \frac{n_i^2}{D_i}$$

$$A(1_\phi^{-+}, 2_\phi^{+-}, 3_{\phi'}^{-+}, 4_{\phi'}^{+-}, 5_h^{++}) = \sum_{i=1}^{5} \frac{n_i \bar{n}_i}{D_i} \qquad (\varepsilon_5 = \varepsilon_5^+)$$

4 and 5pts are well behaved for accidental reasons.
What kinematic algebra should be imposed on numerators in general?

‘Adjoint’ Color-Kinematics Duality

pure Yang-Mills theories are controlled by an ‘adjoint’ kinematic algebra

- Amplitude in cubic graph expansion:

numerators

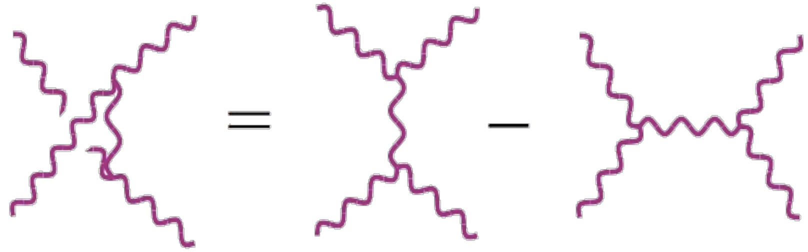
color factors

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

propagators

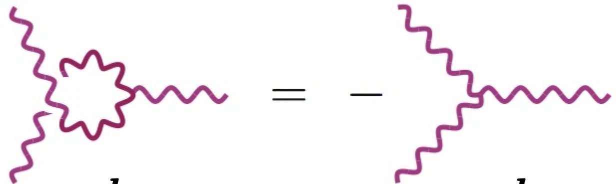
Color & kinematic numerators satisfy same relations:

Bern, Carrasco, HJ



$$f^{adc} f^{ceb} = f^{eac} f^{cbd} - f^{abc} f^{cde}$$

Jacobi identity



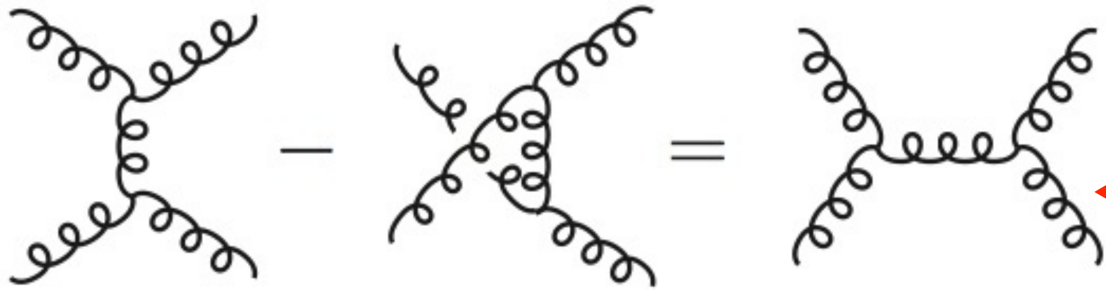
$$f^{bac} = - f^{abc}$$

antisymmetry

The ‘adjoint’ & ‘fundamental’ algebra

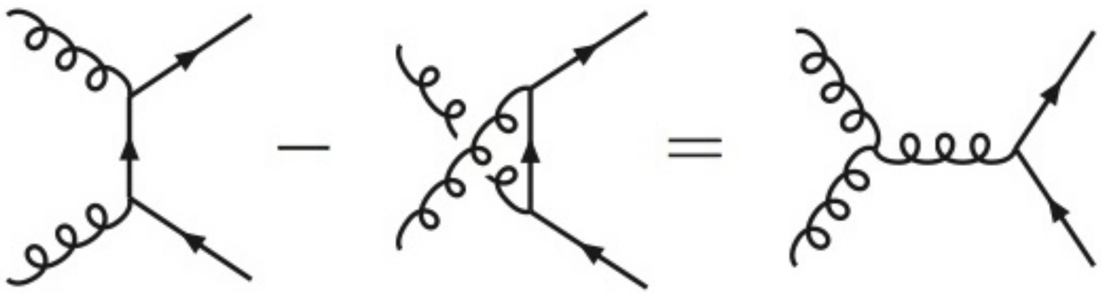
adjoint repr.
or gluon, or
vector multipl.

Jacobi Id.



$$\tilde{f}^{dac}\,\tilde{f}^{cbe} - \tilde{f}^{dbc}\,\tilde{f}^{cae} = \tilde{f}^{abc}\,\tilde{f}^{dce}$$

Fundamental
algebra



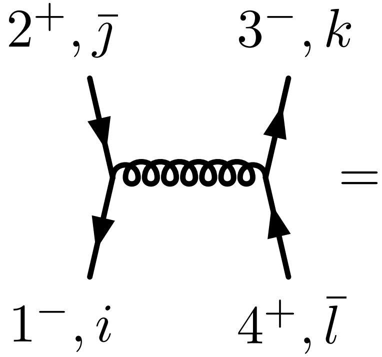
fund. repr.
or fermion, or
complex scalar,
or matter multipl.

$$T^{a}_{i\bar{k}}\,T^{b}_{k\bar{\jmath}} - T^{b}_{i\bar{k}}\,T^{a}_{k\bar{\jmath}} = \tilde{f}^{abc}\,T^{c}_{i\bar{\jmath}}\,.$$

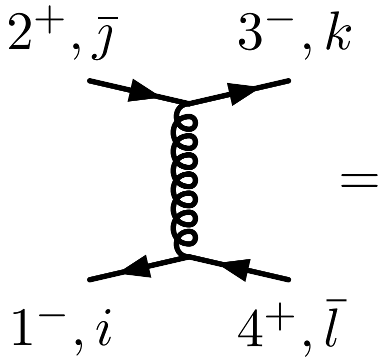
Are there additional algebraic relations?

Recall the 4pt example

The four-quark tree amplitude has an obvious numerator identity



$$= -i\, T^a_{i\bar{\jmath}} T^a_{k\bar{l}} \frac{\langle 13\rangle\,[24]}{s} = \frac{c_1 n_1}{D_1}$$



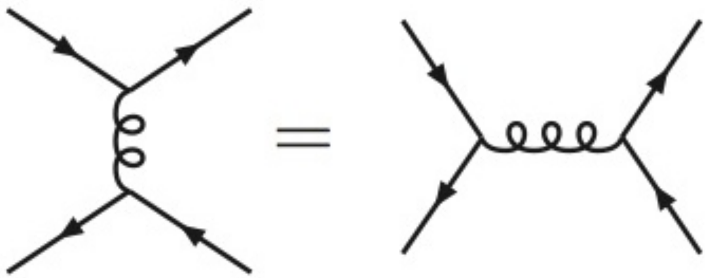
$$= -i\, T^a_{i\bar{l}} T^a_{k\bar{\jmath}} \frac{\langle 13\rangle\,[24]}{t} = \frac{c_2 n_2}{D_2}$$

$$\rightarrow \quad n_s = n_t$$

Is the identity part of the kinematic algebra?

→ Optional kinematical identity

Two-term Id.



Possible to enforce for single scalar or fermion in *D*=3,4,6,10 (Chiodaroli, Jin, Roiban)

$$\Longleftrightarrow$$

Color identity? Not for fundamental matter, but holds for certain complex representations of U(*N*)

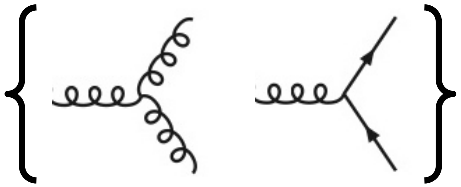
$$T^a_{i\bar{\jmath}} T^a_{k\bar{l}} = T^a_{i\bar{l}} T^a_{k\bar{\jmath}} \,, \qquad U(1) : T^a_{i\bar{\jmath}} = 1$$

Amplitude representation for non-pure SYM

super-Yang-Mills amplitude with one fundamental matter multiplet:

$$\mathcal{A}_m^{(L)} = \sum_i \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{D_i}$$

sum is over all cubic vector–matter graphs with vertices



Color factors c_i are built out of $f^{abc}, \ T^a_{i\bar{\jmath}}$

If n_i satisfy the kinematic algebra, and external legs are vector multiplets, then consistent sugra ampl's are generated by replacing the color factors:

e.g. $c_i \to (N_V)^{|i|} n_i$ or $c_i \to (N_X)^{|i|} \bar{n}_i$

$|i|$ counts number of closed matter loops

$\overline{n}_i$ conjugation denotes reversal of all matter arrows

$N_X + 1$ = number of complex matter multiplets in theory

N_V = number of abelian vector multiplets in theory

Factorizable non-pure gravities

archetype: $(\mathcal{N}=0)\times(\mathcal{N}=0) \;\rightarrow\;$ Einstein gravity + axion+ dilaton

$(\mathcal{N}=1)\times(\mathcal{N}=0) \;\rightarrow\; \mathcal{N}=1$ sugra + $\mathcal{N}=2$ matter

$(\mathcal{N}=1)\times(\mathcal{N}=1) \;\rightarrow\; \mathcal{N}=2$ sugra + two $\mathcal{N}=2$ matter

$(\mathcal{N}=2)\times(\mathcal{N}=0) \;\rightarrow\; \mathcal{N}=2$ sugra + $\mathcal{N}=2$ vector

$(\mathcal{N}=2)\times(\mathcal{N}=1) \;\rightarrow\; \mathcal{N}=3$ sugra + $\mathcal{N}=4$ vector

$(\mathcal{N}=2)\times(\mathcal{N}=2) \;\rightarrow\; \mathcal{N}=4$ sugra + two $\mathcal{N}=4$ vectors

Definition: $\mathcal{N}=2$ matter: $(\lambda, 2\phi, \overline{\lambda})$

In terms of on-shell superspace multiplets this can be summarized as:

$$\mathcal{H}_{\mathcal{N}+\mathcal{M}} \equiv \mathcal{V}_{\mathcal{N}} \otimes \mathcal{V}'_{\mathcal{M}} = H_{\mathcal{N}+\mathcal{M}} \oplus X_{\mathcal{N}+\mathcal{M}} \oplus \overline{X}_{\mathcal{N}+\mathcal{M}}$$

$$\begin{aligned} \text{factorizable graviton multiplet}: \quad & \mathcal{H}_{\mathcal{N}+\mathcal{M}} \equiv \mathcal{V}_{\mathcal{N}} \otimes \mathcal{V}'_{\mathcal{M}} \\ \text{gravity matter}: \quad & X_{\mathcal{N}+\mathcal{M}} \equiv \Phi_{\mathcal{N}} \otimes \overline{\Phi}'_{\mathcal{M}} \\ \text{gravity antimatter}: \quad & \overline{X}_{\mathcal{N}+\mathcal{M}} \equiv \overline{\Phi}_{\mathcal{N}} \otimes \Phi'_{\mathcal{M}} \end{aligned}$$

Obtaining Pure Supergravities

recall:

$N_\phi + 1$ = number of complex scalars in theory

$N_X + 1$ = number of complex matter multiplets in theory

if we want no extra matter in the (super)gravity theory
we are forced to pick $N_\phi = -1$ and $N_X = -1$

What does it mean?

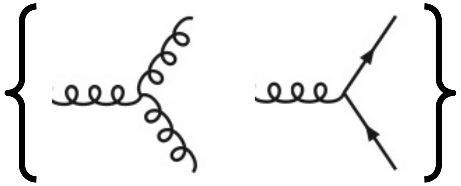
The double-copied matter has the wrong-sign statistics;
that is, those matter fields are ghosts!

This is a welcome feature of the construction, not a bug
– Removes the unwanted states in the vector double copy
– Preserves the double-copy factorization of states
– Preserves Lorentz invariance

Pure $\mathcal{N}$=0,1,2,3 supergravity amplitudes

(super-)Yang-Mills amplitude with one fundamental matter multiplet

$$\mathcal{A}_m^{(L)} = \sum_i \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{D_i}$$

sum is over all cubic vector–matter graphs with vertices 

Color factors c_i are built out of $f^{abc}, \; T^a_{i\bar{\jmath}}$

If n_i, n_i' satisfy the kinematic algebra → pure (super-)gravity

$$\mathcal{M}_m^{(L)} = \sum_i \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{(-1)^{|i|}}{S_i} \frac{n_i \overline{n}_i'}{D_i}$$

$|i|$ counts number of closed matter loops (i.e. ghost loops)

$\overline{n}_i$ conjugation denotes reversal of all matter arrows

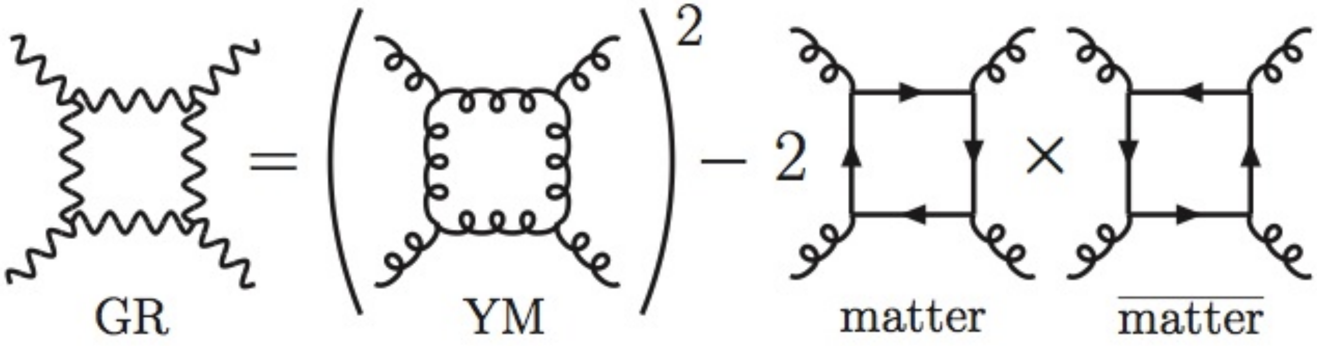
Example: one-loop 4pt

Collect all diagrams with the same denominators

→ i = {Box, triangle, bubble}

$$\mathcal{M}_4^{(1)} = \sum_{\mathcal{S}_4} \sum_{i=\{B,t,b\}} \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{S_i} \frac{n_i^V {n_i}^{V'} - \overline{n}_i^m {n_i}^{m'} - n_i^m {\overline{n}_i}^{m'}}{D_i}$$

If left and right states are the same→ effective gravity numerator is



...and similarly for triangle and bubble

Bosonic YM+scalar theory → YM-Einstein SUGRA

YM + scalar theory

Consider the bosonic YM + scalar theory

Chiodaroli, Gunaydin, HJ, Roiban

$$
\begin{aligned}
L_{\mathcal{N}=0} \;=\; & -\frac{1}{4}F^{\hat{a}}_{\mu\nu}F^{\mu\nu}_{\hat{a}} \\
& +\frac{1}{2}(D_\mu\phi^a)^{\hat{a}}(D^\mu\phi^b)_{\hat{a}}\delta_{ab}+\frac{\mathrm{g}^2}{4}(i\mathrm{f}_{\hat{a}\hat{b}\hat{c}}\phi^{\hat{b}b}\phi^{\hat{c}c})(i\mathrm{f}^{\hat{a}}{}_{\hat{b}'\hat{c}'}\phi^{\hat{b}'b'}\phi^{\hat{c}'c'})\delta_{bb'}\delta_{cc'} \\
& +\frac{\mathrm{g}g'}{3!}(i\mathrm{f}_{\hat{a}\hat{b}\hat{c}})F_{abc}\phi^{\hat{a}a}\phi^{\hat{b}b}\phi^{\hat{c}c}
\end{aligned}
$$

(initially studied in 1999 by Bern, De Freitas, Wong)

It has two coupling constants, and
two copies of Lie algebras: color & flavor

Non-trivially satisfies color-kinematics duality !

(checked at tree level up to 6pts, and at 1-loop 4pts.)

Multiple gauged supergravities obtained after double copying:
e.g. certain *N*=0,1,2,4 Yang-Mills-Einstein supergravities

E.g. *N*=2 Yang-Mills-Einstein SUGRA

Consider the tensor product of the spectrum

($\mathcal{N}$=0 YM + scalars) × ($\mathcal{N}$=2 SYM)

$$\{A_+, \phi^a, A_-\} \otimes \{A_+, \lambda_+, \varphi, \bar{\varphi}, \lambda_-, A_-\}$$

This gives the spectrum of the generic Jordan family of *N* = 2 Maxwell-Einstein and Yang-Mills-Einstein supergravities

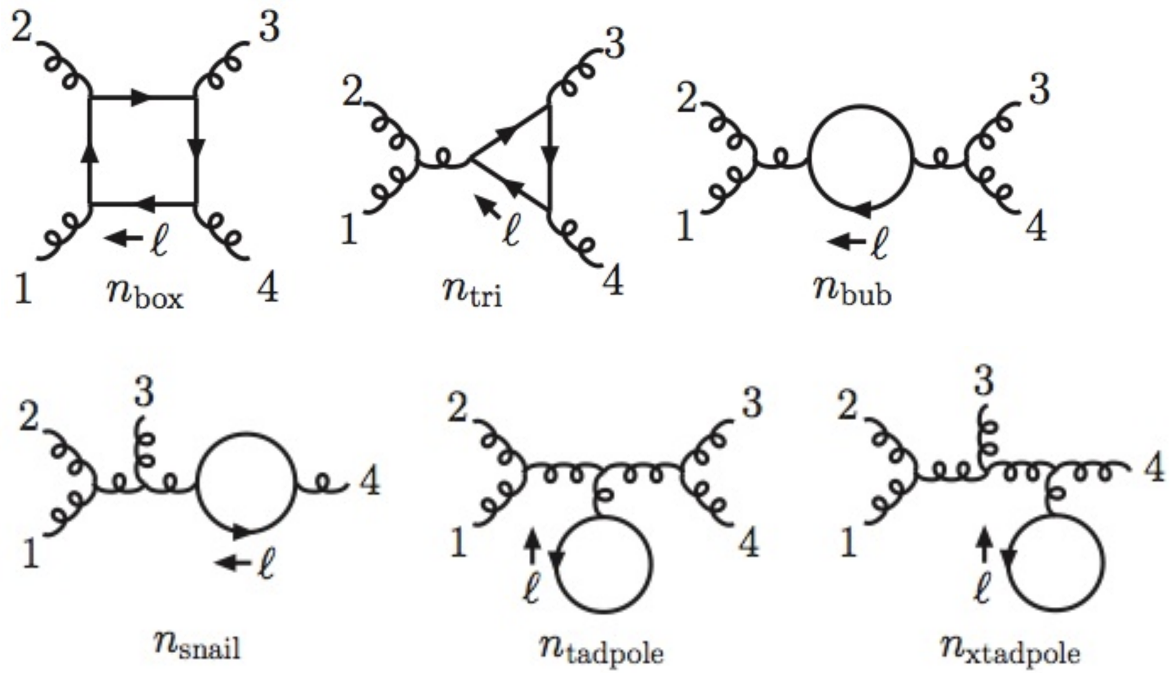
Gunaydin, Sierra, Townsend;
de Wit, Lauwers, Philippe, Su, Van Proeyen;
de Wit, Lauwers, Van Proeyen

Indeed, using the explicit Feynman rules of this *N*=2 gravity theory we have confirmed that the double-copy construction reproduces amplitudes in this theory in *D*=4 and *D*=5 dimensions.

One-loop 4pt amplitudes

Algebra for one-loop 4pt calculations

diagrams:



vanish after integration

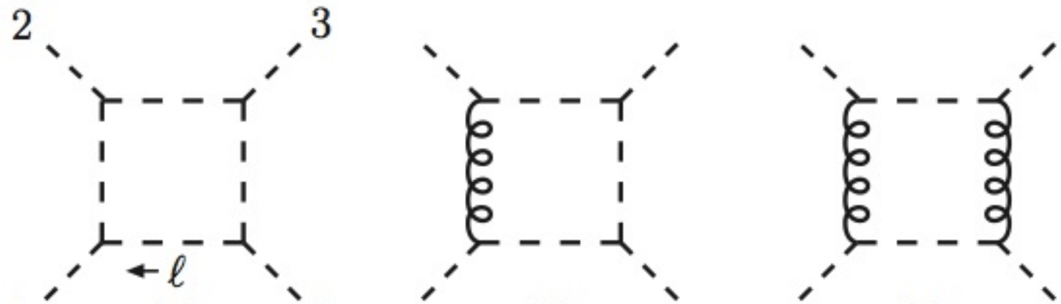
kinematic algebra:

$$\begin{aligned}
n_{\rm tri}(1,2,3,4,\ell) &= n_{\rm box}([1,2],3,4,\ell)\,, \\
n_{\rm bub}(1,2,3,4,\ell) &= n_{\rm box}([1,2],[3,4],\ell)\,, \\
n_{\rm snail}(1,2,3,4,\ell) &= n_{\rm box}([[1,2],3],4,\ell)\,, \\
n_{\rm tadpole}(1,2,3,4,\ell) &= n_{\rm box}([[1,2],[3,4]],\ell)\,, \\
n_{\rm xtadpole}(1,2,3,4,\ell) &= n_{\rm box}([[[1,2],3],4],\ell)\,.
\end{aligned}$$

Amplitude in YM + scalar theory

Consider the 4pt amplitude with external scalars in the YM+ scalar theory

Three types of contributions:



(a) (b) (c)

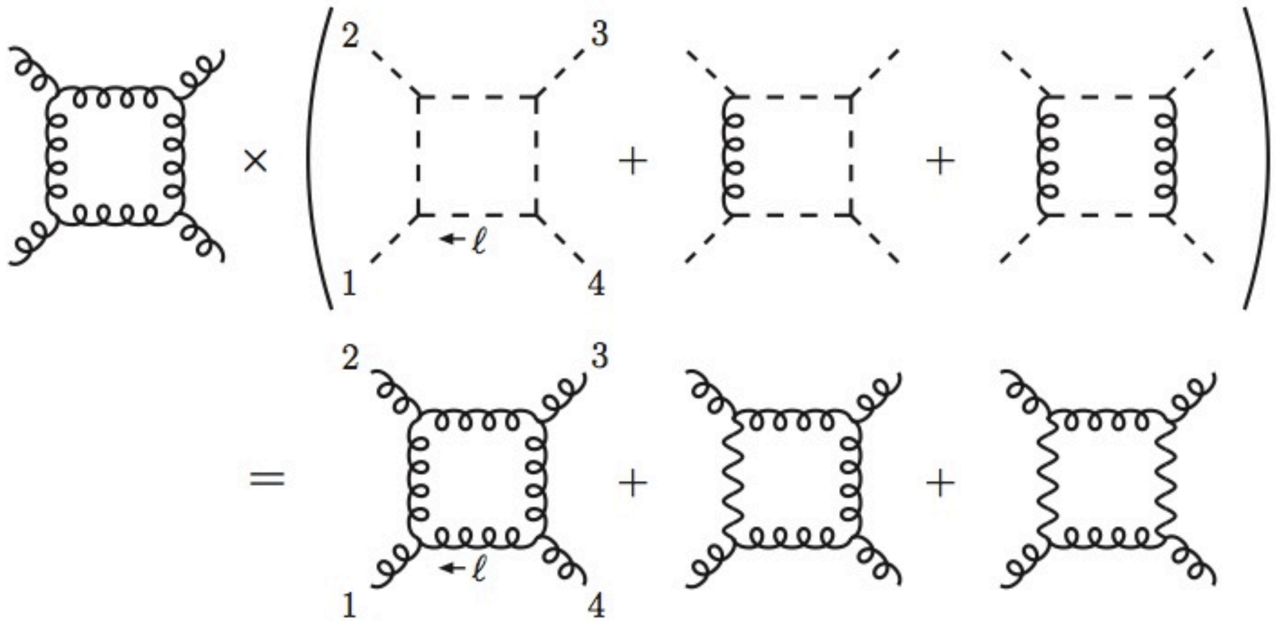
$$n^{(\mathrm{a})}_{\mathrm{box}}(1,2,3,4) = -ig'^4 F^{ba_1c}F^{ca_2d}F^{da_3e}F^{ea_4b}$$

$$\begin{aligned}
n^{(\mathrm{b})}_{\mathrm{box}}(1,2,3,4,\ell) \;=\; & -\frac{i}{12}g'^2\Big\{(N_V+2)\big(F^{a_1a_4b}F^{ba_3a_2}(\ell_2^2+\ell_4^2)+F^{a_1a_2b}F^{ba_3a_4}(\ell_1^2+\ell_3^2)\big)\\
&+24\big(sF^{a_1a_4b}F^{ba_3a_2}+tF^{a_1a_2b}F^{ba_3a_4}\big)+\delta^{a_3a_4}\mathrm{Tr}_{12}(6\ell_3^2-\ell_2^2-\ell_4^2)\\
&+\delta^{a_2a_3}\mathrm{Tr}_{14}(6\ell_2^2-\ell_1^2-\ell_3^2)+\delta^{a_1a_4}\mathrm{Tr}_{23}(6\ell_4^2-\ell_1^2-\ell_3^2)\\
&+\delta^{a_1a_2}\mathrm{Tr}_{34}(6\ell_1^2-\ell_2^2-\ell_4^2)+(\ell_1^2+\ell_2^2+\ell_3^2+\ell_4^2)(\delta^{a_2a_4}\mathrm{Tr}_{13}+\delta^{a_1a_3}\mathrm{Tr}_{24})\Big\}
\end{aligned}$$

Amplitude in YM-Einstein SUGRA

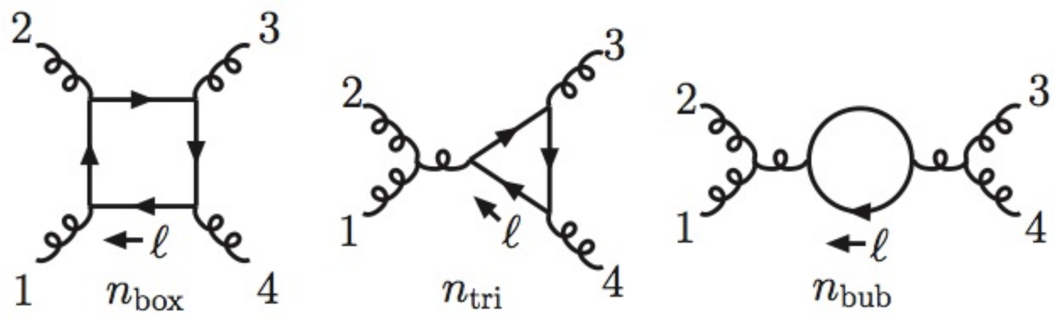
Consider the 4pt amplitude with external vectors in the YM-Einstein theory

The three types of contributions are obtained as double copies



$$
\begin{aligned}
\tilde{n}^{\mathcal{N}=2,\text{mat.}}_{\text{box}}(1,2,3,4,\ell) \;=\; & (\kappa_{12}+\kappa_{34})\frac{(s-\ell_s)^2}{2s^2} + (\kappa_{23}+\kappa_{14})\frac{\ell_t^2}{2t^2} + (\kappa_{13}+\kappa_{24})\frac{st+(s+\ell_u)^2}{2u^2} \\
& + \mu^2\Big(\frac{\kappa_{12}+\kappa_{34}}{s} + \frac{\kappa_{23}+\kappa_{14}}{t} + \frac{\kappa_{13}+\kappa_{24}}{u}\Big) \\
& - 2i\epsilon(1,2,3,\ell)\frac{\kappa_{13}-\kappa_{24}}{u^2}\;,
\end{aligned}
$$

Ampl's for fundamental *N*=1 SYM and pure *N*=2 SG



***N*=1 matter parity odd:**

$$
\begin{aligned}
n_{\text{box}}^{\mathcal{N}=1,\text{odd}} = &-(\kappa_{12}-\kappa_{34})\frac{(s+\tau_{35}+\tau_{45})^3}{2s^3} - (\kappa_{14}-\kappa_{23})\frac{(\tau_{25}+\tau_{35})^3}{2t^3} \\
&+ (\kappa_{13}-\kappa_{24})\Big[s\Big(\frac{1}{2u} - \frac{3(\tau_{15}+\tau_{35})}{2u^2} + \frac{3(\tau_{15}+\tau_{35})^2}{2u^3}\Big) \\
&+ \frac{(\tau_{15}+\tau_{35})^3}{2u^3}\Big] - 2i(\kappa_{13}+\kappa_{24})\frac{2\tau_{15}+2\tau_{35}-u}{u^3}\epsilon(1,2,3,5)
\end{aligned}
$$

HJ, Ochirov

$$\tau_{i5} = 2k_i\cdot\ell$$

parity even: Carrasco, Chiodaroli, Gunaydin, Roiban; Nohle; Ochirov, Tourkine

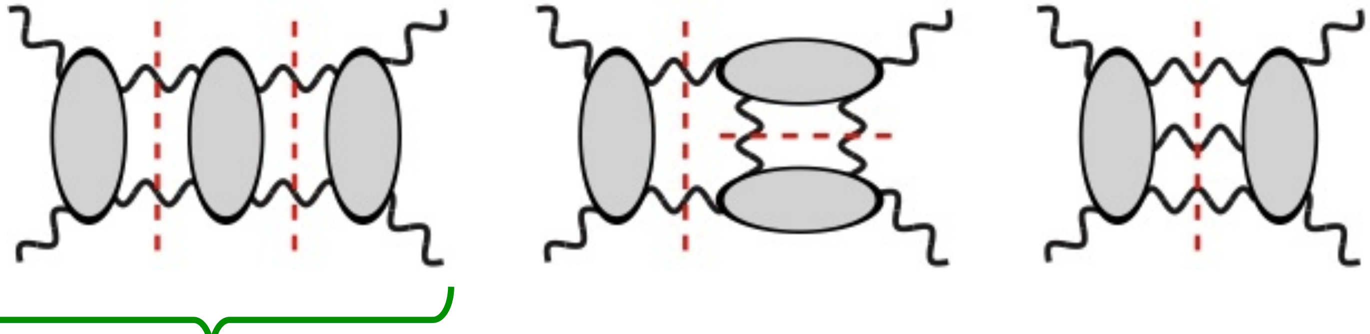
E.g. pure *N*=2 gravity numerator:

$$
\begin{aligned}
n_{\text{box}}^{\mathcal{N}=2\ \text{SG}} &= (n_{\text{box}}^{\mathcal{N}=1\ \text{SYM}})^2 - 2(n_{\text{box}}^{\mathcal{N}=1,\text{even}} + n_{\text{box}}^{\mathcal{N}=1,\text{odd}})(n_{\text{box}}^{\mathcal{N}=1,\text{even}} - n_{\text{box}}^{\mathcal{N}=1,\text{odd}}) \\
&= (n_{\text{box}}^{\mathcal{N}=1\ \text{SYM}})^2 - 2(n_{\text{box}}^{\mathcal{N}=1,\text{even}})^2 + 2(n_{\text{box}}^{\mathcal{N}=1,\text{odd}})^2
\end{aligned}
$$

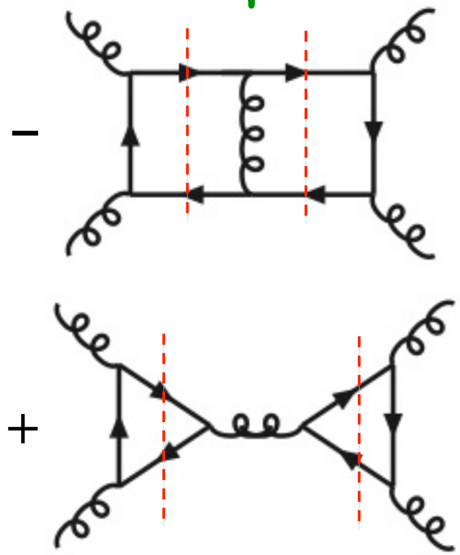
After integration: agreement with Dunbar & Norridge ('94)

Two-loop check

We have checked that the ghost prescription removes all dilaton and axion contributions in the physical unitarity cuts in 2-loop 4pt Einstein gravity



D=4



The double s-channel cut is the most nontrivial
It mixes two diagrams of different statistics:

The cancellation between the ghosts, dilaton and axion is highly intricate in this case

→ works well for $(\lambda^+ \otimes \lambda^-) \oplus (\lambda^- \otimes \lambda^+) \to \phi, a$

→ glitch/feature: $(\phi^+ \otimes \phi^-) \oplus (\phi^- \otimes \phi^+) \to \phi, \phi'$

Summary

- In the past color-kinematics duality could not be used for pure N<4 (super)gravity theories – impeded studies of gravity UV behavior

- Similarly, how to properly "double-copy construct" supergravity coupled to abelian or non-abelian vectors was an open problem

- The problems are solved by the introduction of fundamental-matter color-kinematics duality, and a bosonic YM+scalar theory, respectively

- Double copies give amplitudes in a wide range of matter-coupled (super)gravity theories

- A ghost prescription is proposed to give pure gravities.
  Ghosts obtained from double copies of matter-antimatter pairs

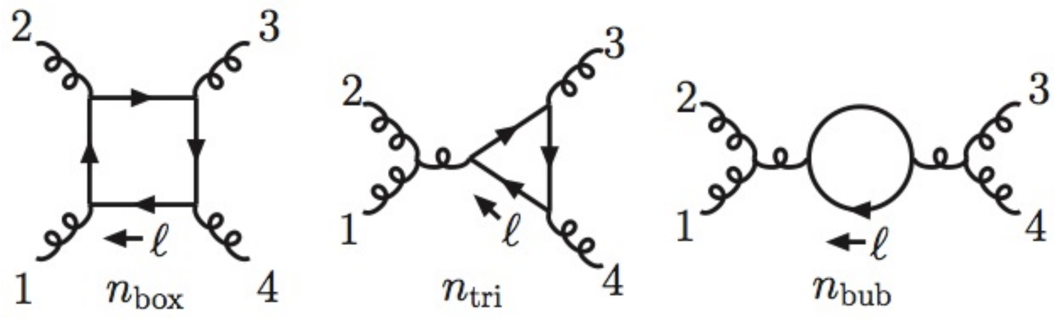
- Checks: At tree-level, one and two loops.

- Opens a new window into the study of gravity UV properties

Extra Slides

Notation for one-loop calculations

diagrams:



ansatz for 4pt MHV amplitude with internal matter, in any SYM theory: HJ, Ochirov

$$n_{\text{box}}(1,2,3,4,\ell) = \sum_{1\le i<j\le 4} \frac{\kappa_{ij}}{s_{ij}^N}\Big(\sum_k a_{ij;k} M_k^{(N)} + \epsilon(1,2,3,\ell)\sum_k \tilde{a}_{ij;k} M_k^{(N-2)}\Big)$$

power-counting factor: $N = 4 - \mathcal{N}$ ← SUSY

momentum monomials: $M^{(N)} = \Big\{ \prod_{i=1}^{N} m_i \;\Big|\; m_i \in \{s,\, t,\, \ell\cdot k_j,\, \ell^2,\, \mu^2\} \Big\}$

state dependence: $\kappa_{ij} = \dfrac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\delta^{(2\mathcal{N})}(Q)\,\langle i\,j\rangle^{4-\mathcal{N}}\,\theta_i\theta_j$

(**vector multiplet:** $\mathcal{V}_{\mathcal{N}} = V_{\mathcal{N}} + \overline{V}_{\mathcal{N}}\,\theta$)

Other Spacetime Dimensions

The prescription for adding matter-antimatter ghosts is similar in other dimensions, however, starting in *D*=6 an unwanted 4-form show up.

Dim	$A^\mu \otimes A^\nu \to \phi \oplus h^{\mu\nu} \oplus B^{\mu\nu}$	tensoring matter	$(\text{matter})^2 \to \phi \oplus B^{\mu\nu} \oplus D^{\mu\nu\rho\sigma}$	resulting states
$D=3$	$1 \otimes 1 \to 1 \oplus 0 \oplus 0$	$\lambda \otimes \overline{\lambda}$	$1 \otimes 1 \to 1 \oplus 0 \oplus 0$	topological
$D=4$	$2 \otimes 2 \to 1 \oplus 2 \oplus 1$	$(\lambda^+ \otimes \lambda^-) \oplus (\lambda^- \otimes \lambda^+)$	$(1 \otimes 1) \oplus (1 \otimes 1) \to 1 \oplus 1 \oplus 0$	$h_{\mu\nu}$
$D=5$	$3 \otimes 3 \to 1 \oplus 5 \oplus 3$	$\lambda^\alpha \otimes \overline{\lambda}^\beta$	$2 \otimes 2 \to 1 \oplus 3 \oplus 0$	$h_{\mu\nu}$
$D=6$	$4 \otimes 4 \to 1 \oplus 9 \oplus 6$	$(\lambda^\alpha \otimes \lambda^\beta) \oplus (\widetilde{\lambda}^{\dot\alpha} \otimes \widetilde{\lambda}^{\dot\beta})$	$(2 \otimes 2) \oplus (2 \otimes 2) \to 1 \oplus 6 \oplus 1$	$h_{\mu\nu}, D^{\mu\nu\rho\sigma}$ (ghost)
$D=10$	$8 \otimes 8 \to 1 \oplus 35 \oplus 28$	$\lambda^A \otimes \lambda^B$	$8 \otimes 8 \to 1 \oplus 28 \oplus 35$	$h_{\mu\nu}, D^{\mu\nu\rho\sigma}$ (ghost)

possibly cured cured by:

- Projecting out states in the matter-antimatter double copy ?
- Adding back a 4-form (axion in *D*=6) ?
- Other way ?