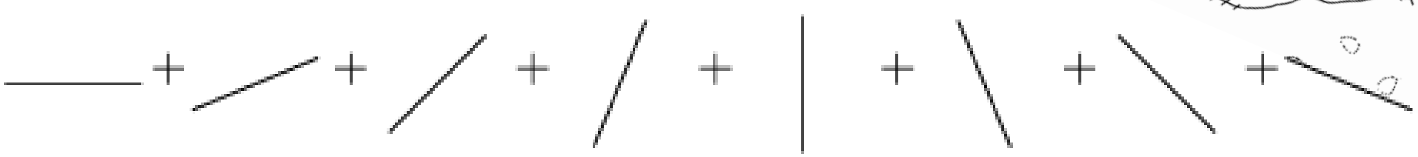
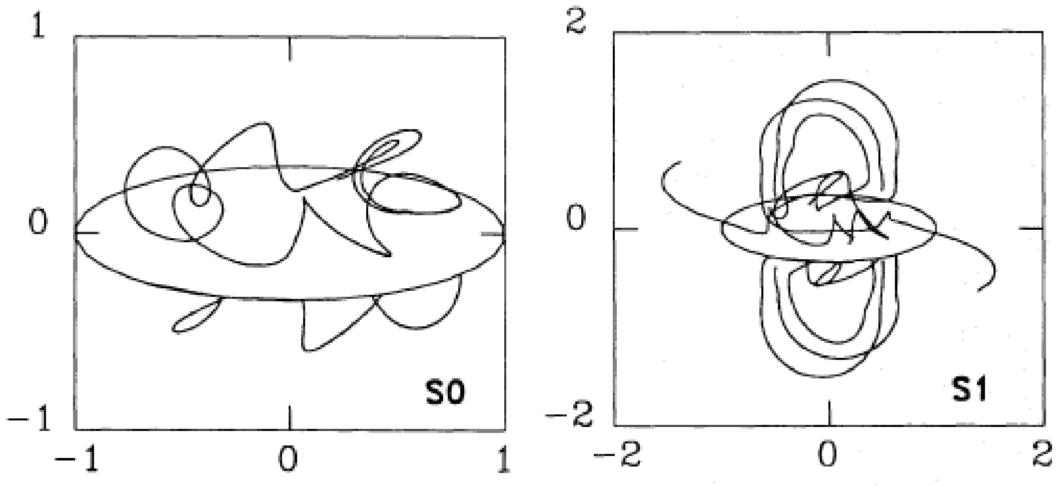
Faraday signature of helical and bi-helical magnetic fields

Axel Brandenburg (Nordita) & Rodion Stepanov (Perm)

- Compensating Faraday depolarization
  - By a helical field (single sense of twist)
  - By a bi-helical field & RM dependence
- Application to edge-on galaxies

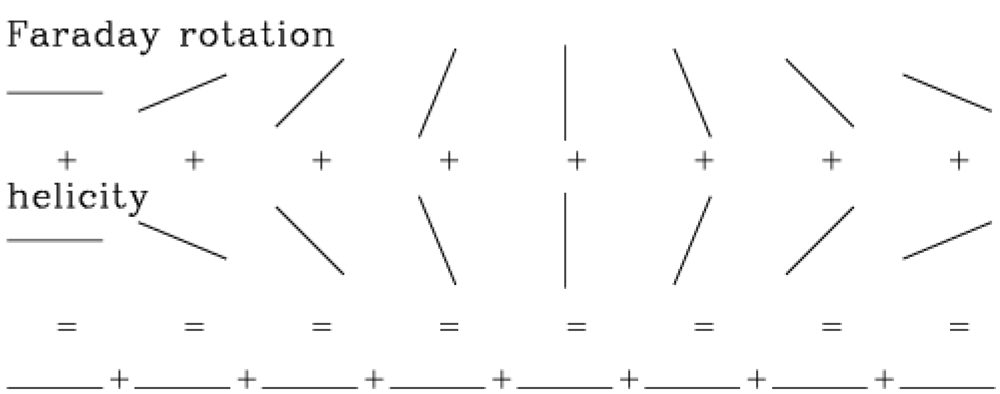


Helical (swirling) magnetic fields



Brandenburg & Donner (1990)

Rotation from swirl compensates Faraday rotation



See also Sokoloff et al. (1998)

Singly helical field

Stokes Q and U parameters $P = Q + iU$

$$Q = p_0 \int_{-\infty}^{\infty} \varepsilon \cos 2\left(\psi + \phi\lambda^2\right) dz$$

$$U = p_0 \int_{-\infty}^{\infty} \varepsilon \sin 2\left(\psi + \phi\lambda^2\right) dz$$

Intrinsic polarized emission from B

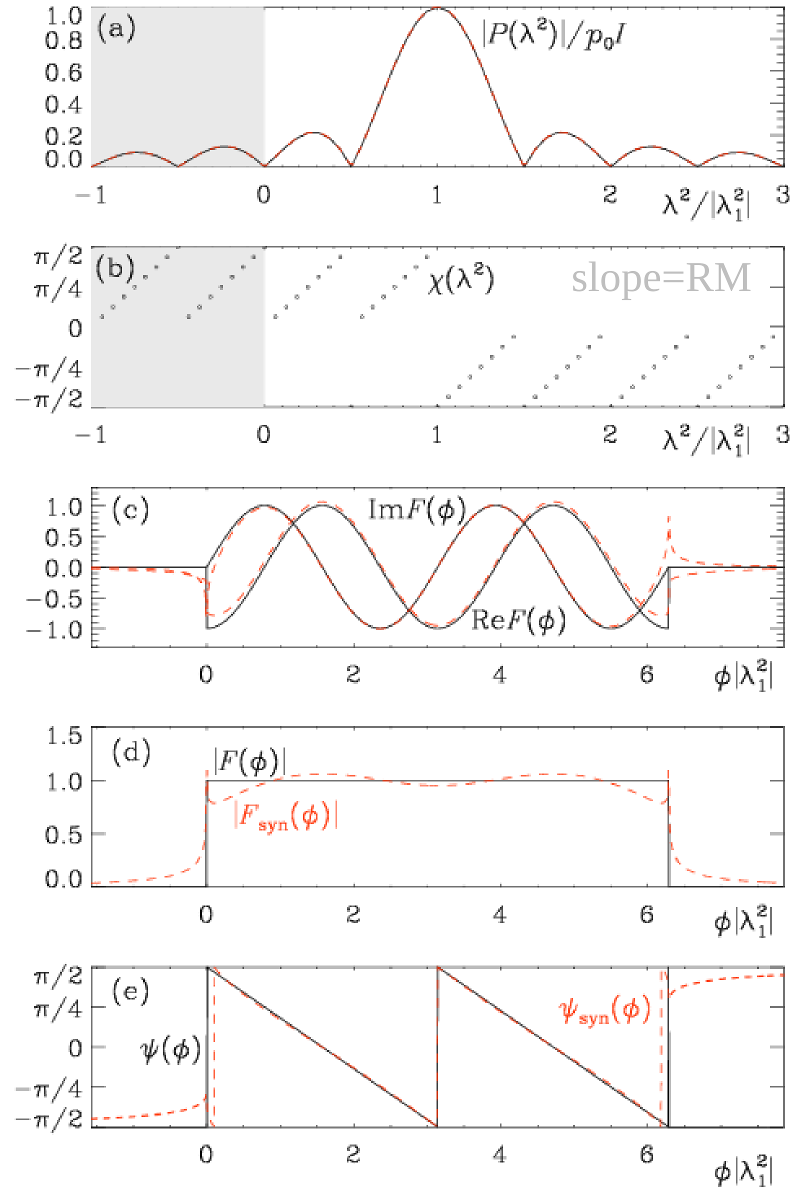
$$B_\perp = \left|B_\perp\right| e^{i\psi_B}, \quad \psi = \psi_B + \tfrac{1}{2}\pi$$

Cancellation condition

$$\psi = -kz, \quad \phi = -Kn_{\rm th} B_0 z$$

Helical field w/ positive helicity

$$\mathbf{B} = \begin{pmatrix} B_1 \cos kz \\ -B_1 \sin kz \\ B_0 \end{pmatrix}$$



Only works if RM > 0 and k > 0

Peak determined by single parameter

$$\lambda_1^2 = -k / K n_{\rm th} B_0 \propto k / {\rm RM}$$

But difficult/impossible to recover F(ϕ)

(Burn 1966)

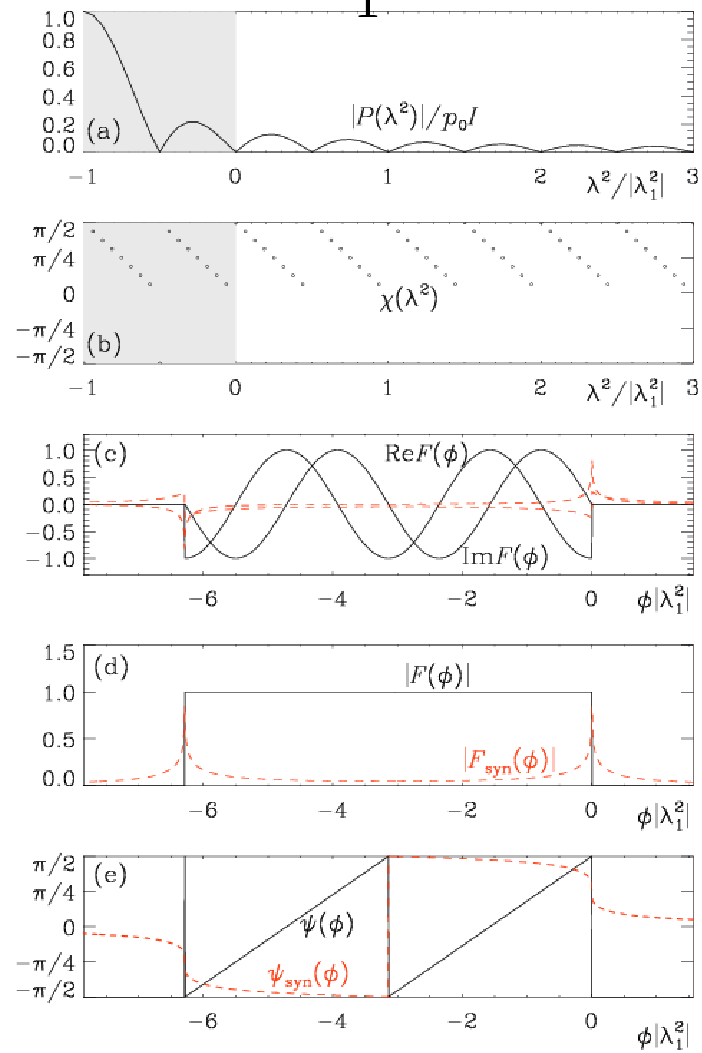
$$P(\lambda^2) = \int_{-\infty}^{\infty} F(\phi) e^{2i\phi\lambda^2} \, d\phi$$

$$F(\phi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\lambda^2) e^{-2i\phi\lambda^2} \, d(2\lambda^2)$$

Positivity:

$$F_{\rm syn}(\phi) = \frac{1}{2\pi} \int_{0}^{\infty} P(\lambda^2) e^{-2i\phi\lambda^2} \, d(2\lambda^2)$$

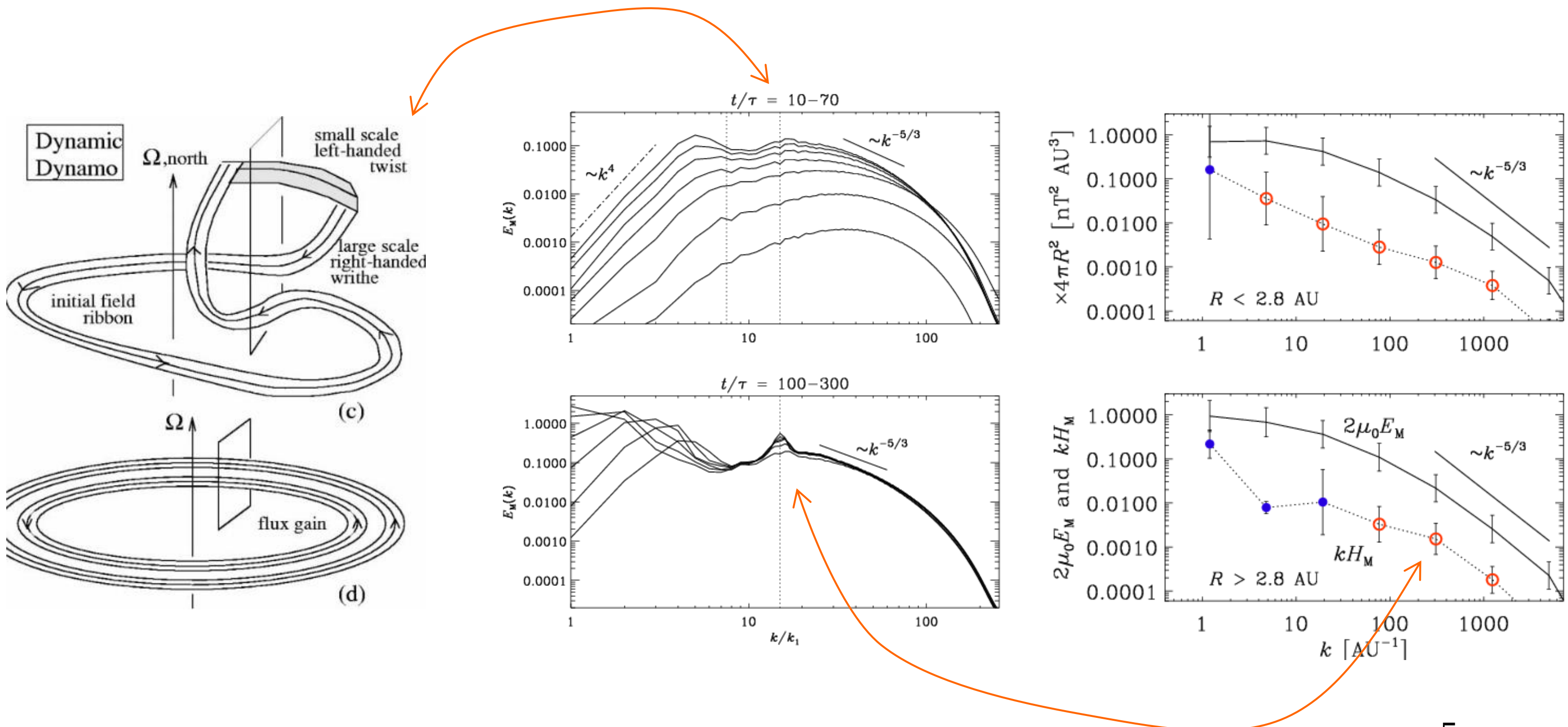
$$\lambda_1^2 < 0$$



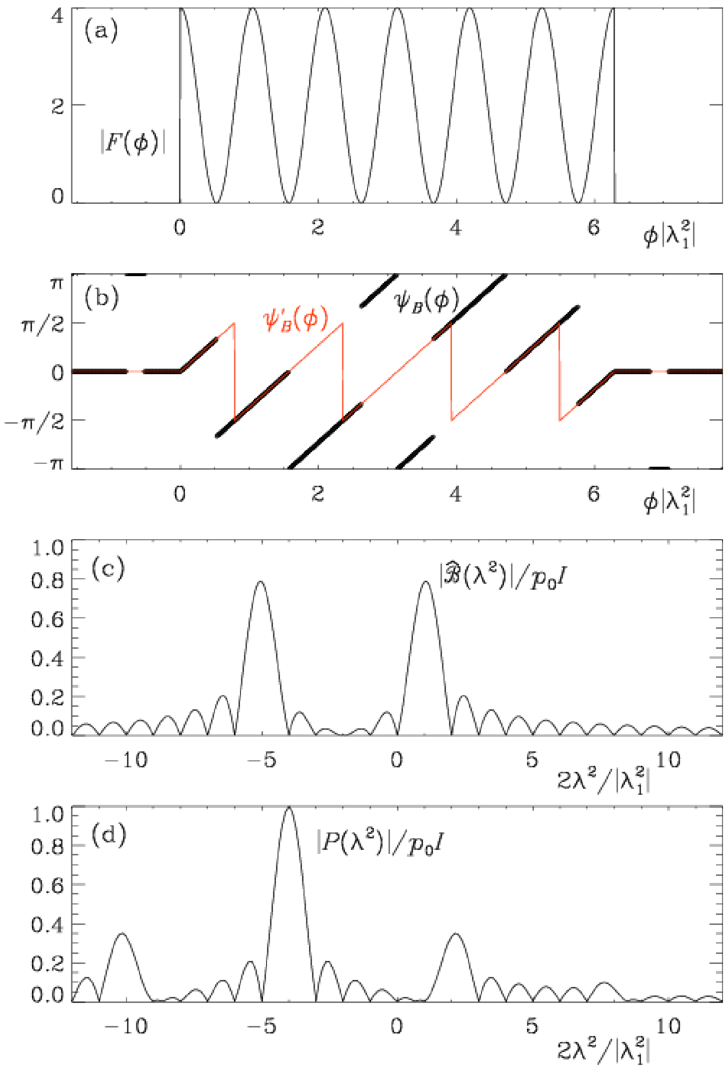
Expect bi-helical fields

- Magnetic helicity conserved
- Inverse cascade produces small-scale waste!
- Opposite sign of helicity (or k)

Blackman & Brandenburg (2003)



$\prec$ ambiguity lead to "line splitting"



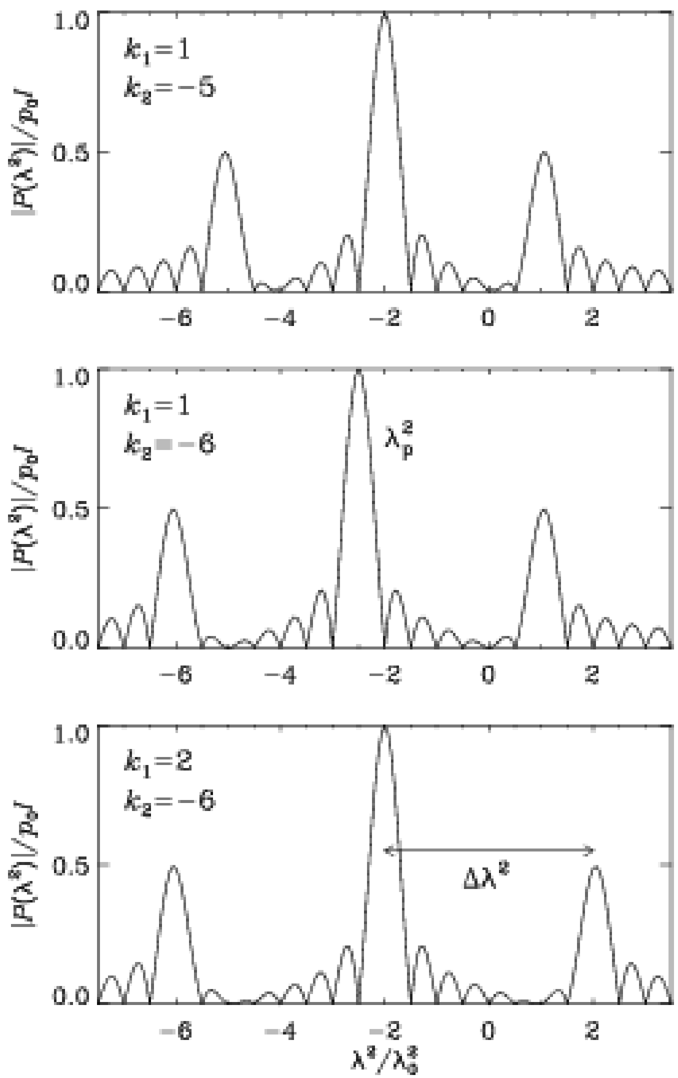
Peaks at k_1=1 and k_2=-5

translate to k_1+k_2 = -4
and to k_1-k_2 = 6

(i) peak in P at -4
peak separation 6

(ii) in Faraday dispersion:
frequency 6
-2x phase gradient -4

≺ ambiguity: other examples



Peaks at k_1=1 and k_2=-5
1 and -6, or, 2 and -6

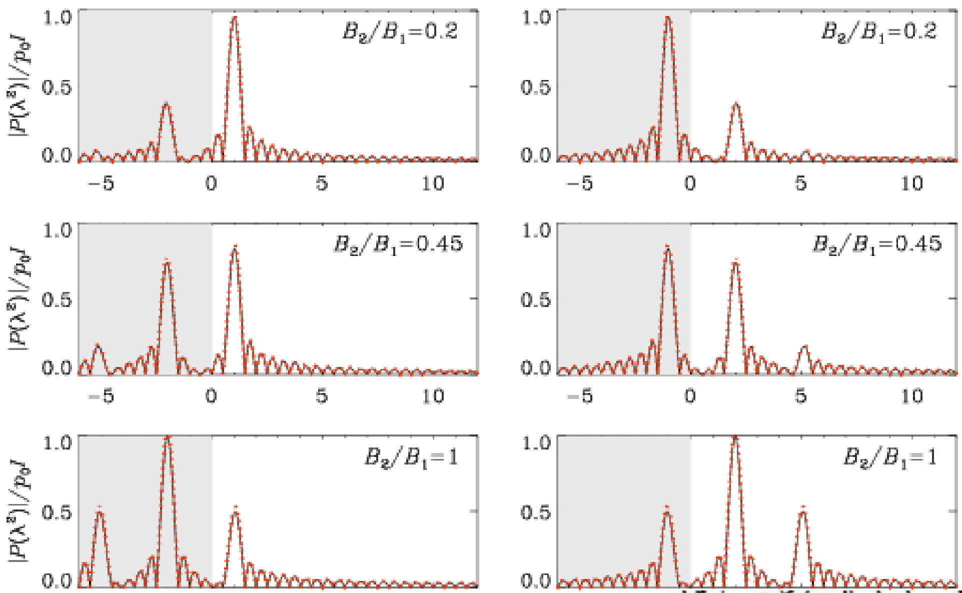
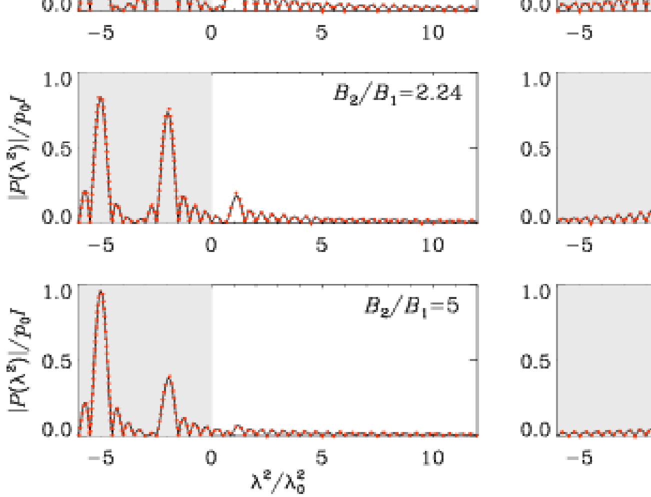
translate to k_1+k_2 =-4
and k_1-k_2 =6

Table 1. Summary of the three examples shown in Fig. 4 for RM $>$ 0 with bi-helical magnetic fields of wavenumbers k_1 and k_2, the corresponding values of $k_{\pm} = k_1 \pm k_2$, the peak wavenumber k_p, the peak separation Δk, the phase gradient (ϕ derivative, indicated by ∇ for brevity), and corresponding values for λ_p^2 and $\Delta\lambda^2$. All values of k are normalized by k_0 and all values of λ^2 are normalized by λ_0^2.

k_1	k_2	k_+	k_-	k_p	Δk	$\nabla\psi'_B$	λ_p^2	$\Delta\lambda^2$	$\nabla\psi$
1	−5	−4	6	−4	6	2	−2	3	2
1	−6	−5	7	−5	7	2.5	−2.5	3.5	2.5
2	−6	−4	8	−4	8	2	−2	4	2

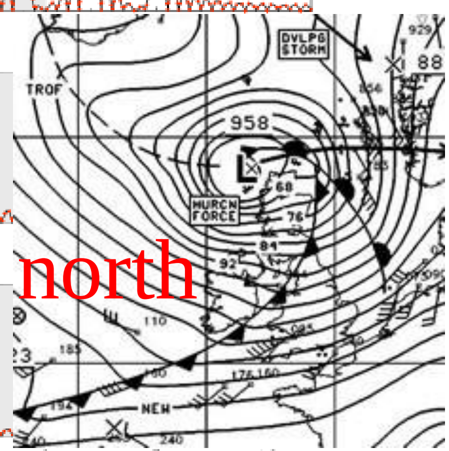
For RM>0, only +ve helicity obs

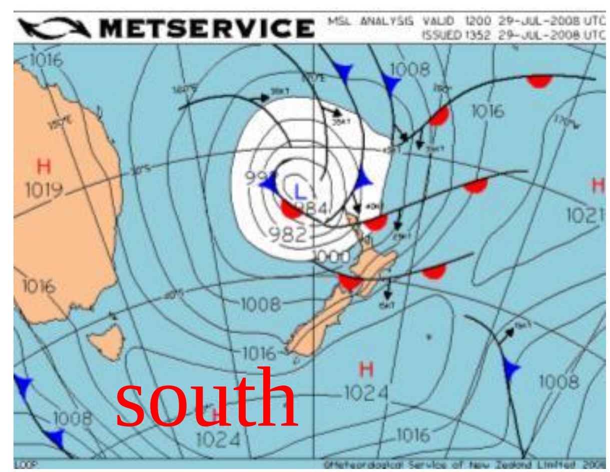
RM > 0 RM < 0

For RM > 0,
Left: north
Right: south

For RM < 0,
Left: south
Right: north





Scales and applications

$$z \leftrightarrow \phi$$

$$k \leftrightarrow \lambda^2$$

- L = 1 kpc → k = 6 kpc^{-1} → ●=30 cm
- L<0.1 kpc → k > 60 kpc^{-1} → ●=1 m
- Assuming B = 3 ❍G, n_e=0.03 cm^{-3}

λ coverage only possible with SKA: 2 cm – 6m

Conclusions

- RM synthesis: measure magnetic helicity
- Need line of sight component: edge-on galaxy
- Expect polarized intensity only in 2 quadrants
- 2 characteristic peaks