# Pairing induced superconductivity in holography

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Based on 1403.3699

(see also Y. Liu, K. Schalm, Y. Sun, J. Zaanen; 1404.0571)

Holographic methods and applications
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# Motivation

We would like to use the power of AdS/CFT to analyze **realistic** quantum field theories at strong coupling and finite density.

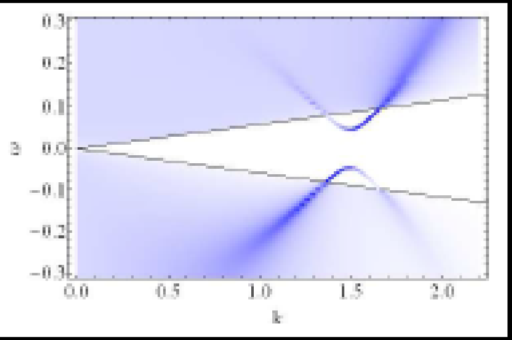
In real solid state systems the “strong coupling” is rather the coupling between electrons than the gauge coupling.

# Motivation

- The main goal: to formulate a complete dynamical holographic model of unconventional SC with an explicit strong fermionic pairing mechanism.
- A step on this way: to reformulate the standard BCS theory in holographic terms

# Fermionic pairing in holography

- Fermionic probes of a holographic superconductor (Faulkner, Horowitz, McGreevy, Roberts, Vegh; 0911.3402)




- Cooper pairing near a black hole (Hartman&Hartnoll; 1003.1918)

- Top-down models of fermionic bound states – N=2 SYM, meson superfluid (Ammon, Erdmenger, Kaminski, Kerner; 0903.1864)

# Backreacting fermions

Quantum electron star – fluid limit:

- Fermions backreacting on the U(1) gauge field (Allais, McGreevy, Suh; 1202.5308)
- Fermions backreacting both on the gauge field and the bulk metric (Allais, McGreevy; 1306.6075)

Holographic Fermi liquid – QM limit:

- Fermions backreacting on the gauge field in a hard wall model (Sachdev; 1107.5321)

# Our strategy

- Consider the pure AdS4 hard wall holographic model of Fermi liquid
- Introduce scalar field (order parameter) and Yukawa-like coupling in the bulk
- Numerically solve the system of interacting scalar, fermionic, and electromagnetic fields in the bulk (metric is fixed)
- Read off spectrum of fermions and vev’s in the boundary field theory

# Holographic Fermi liquid

- Well-defined long living quasiparticles
- Certain transport properties

$$\mathcal{G}(\omega, k) \sim \frac{1}{i\omega - \epsilon_{\mathbf{k}} - \Sigma(k, \omega)}$$

$$\Sigma(k, \omega) \sim m_*^3 \frac{(\pi T)^2 + \omega^2}{1 + e^{-\beta\omega}}$$

# Holographic Fermi liquid

- Fermions in pure AdS4 spacetime + finite charge density, i.e. U(1) gauge field

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left( R - \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\Psi} \Gamma^\mu D_\mu \Psi - m_\Psi \overline{\Psi} \Psi \right)$$

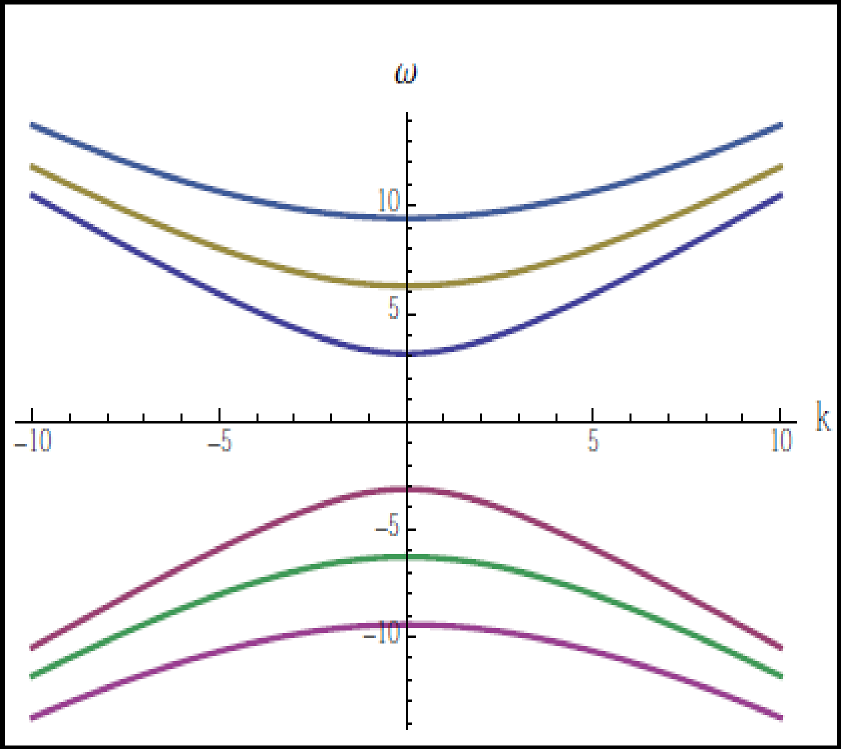
- Gapless IR degrees of freedom are to be cut off

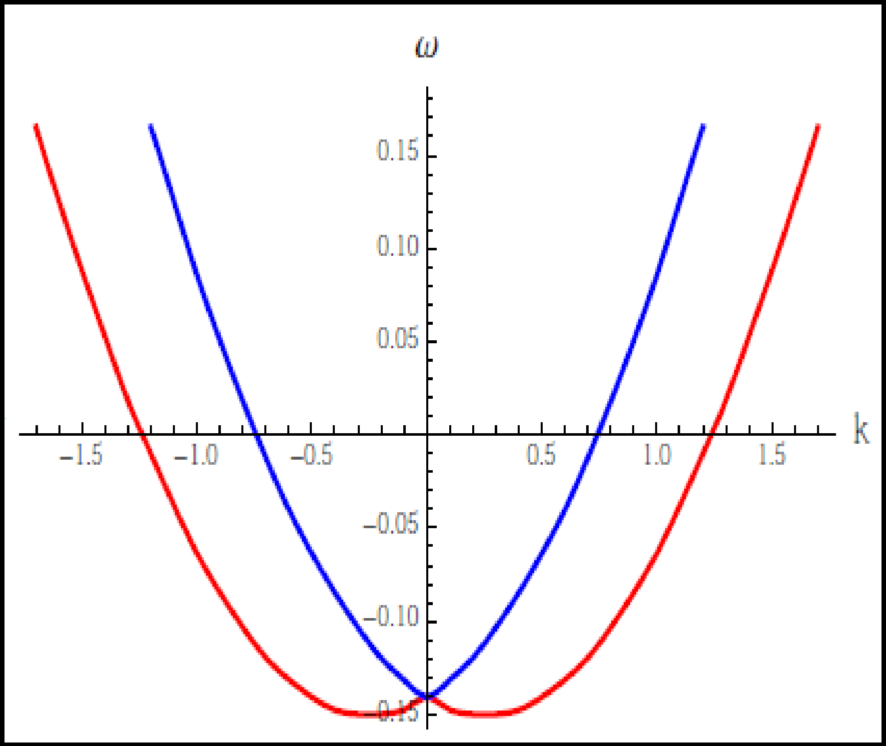
$$ds^2 = \frac{1}{z^2} \left( -dt^2 + dz^2 + dx^2 + dy^2 \right), \quad z \in [0, z_w]$$

- All relevant information is contained in the spectrum of bulk fermions

# Spin splitting

- Zero vs. non-zero chemical potential



# Majorana interaction

- The most general Lagrangian of relativistic BCS theory (D. Bertrand, PhD thesis, Leuven U.)

$$\mathcal{L}_{int} = g_1(\bar{\psi}\psi)^2 + g_2(\bar{\psi}\gamma_5\psi)^2 + g_3(\bar{\psi}\gamma^\mu\psi)^2 + g_4(\bar{\psi}\gamma^\mu\gamma_5\psi)^2 + g_5(\bar{\psi}\sigma^{\mu\nu}\psi)^2$$

- The only important coupling

$$\mathcal{L}_{int} = g\left(\bar{\psi}\gamma_5\psi\right)\left(\bar{\psi}\gamma_5\psi\right)^\dagger$$

$$\mathcal{L} = \bar{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma^\mu\psi - m\bar{\psi}\psi - eA_\mu\bar{\psi}\gamma^\mu\psi + g\left(\bar{\psi}\gamma_5\psi\right)\left(\bar{\psi}\gamma_5\psi\right)^\dagger$$

# Nambu-Gorkov transformation

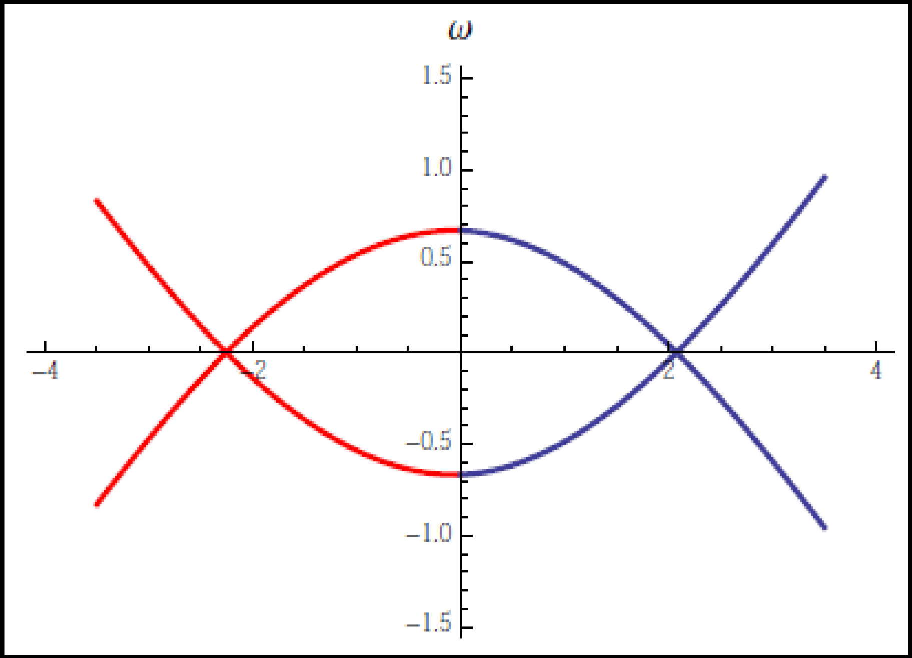
$$S_D + S_M = \int d^4x \overline{\chi} K \chi$$

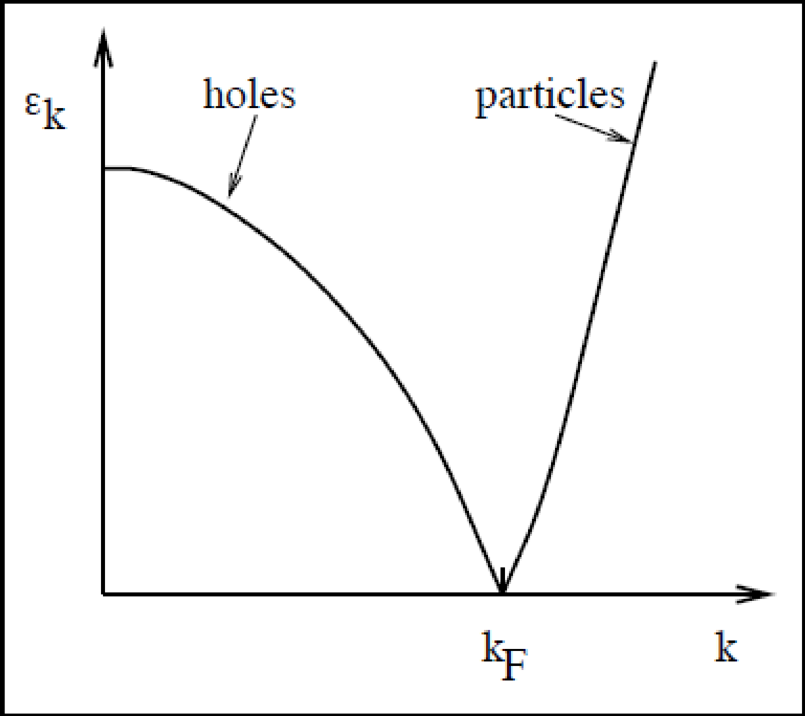
$$\chi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3^* \\ \psi_4^* \end{pmatrix} \qquad K = \begin{pmatrix} D_{11} & 2\eta_5 \frac{\phi}{z}\sigma_3 \\ -2\eta_5^* \frac{\phi^*}{z}\sigma_3 & D_{22} \end{pmatrix}$$

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (\chi_1, i\chi_2, \chi_3, i\chi_4)$$

# Nambu-Gorkov transformation

- Flipped spectrum




Schulz et. al., Fermi liquids and Luttinger liquids

# Backreacting fermions

- Fermionic bilinears backreact on the scalar order parameter and on the gauge field

$$\langle \psi^{+} \psi \rangle = \frac{1}{2\pi} \sum_{n} \int dk|k| \left(\alpha_{k,n,1}^{2} + \alpha_{k,n,2}^{2}\right) \Theta\left(-\omega_{k,n}\right)$$

$$\langle \bar{\psi^{C}} \Gamma^{5} \psi \rangle = \frac{i}{2\pi} \sum_{n} \int_{-\Lambda(\omega_D)}^{\Lambda(\omega_D)} dk|k| \left[\Theta\left(\omega_{k,n}\right) \left(\alpha_{k,n,1}\alpha_{k,n,4} - \alpha_{k,n,2}\alpha_{k,n,3}\right)\right]$$

# Equations of motion

$$z^2\phi'' - 2z\phi' + 4q^2z^2A_0^2\phi - m_\phi^2\phi = \frac{\eta_5 z^3}{2\pi}\sum_n \int_{-\Lambda(\omega_D)}^{\Lambda(\omega_D)} dk|k|\Theta\left(\omega_{k,n}\right)\left(\alpha_{k,n,1}\alpha_{k,n,4} - \alpha_{k,n,2}\alpha_{k,n,3}\right),$$

$$z^2A_0'' - 8q^2A_0 = \frac{qz^2}{2\pi}\sum_n \int dk|k|\left(\alpha_{k,n,1}^2 + \alpha_{k,n,2}^2\right)\Theta\left(-\omega_{k,n}\right),$$

$$\begin{pmatrix} \partial_z - \frac{m_\Psi}{z} & -(\omega - k) - qA_0 & 2\eta_5\frac{\phi}{z} & 0 \\ (\omega + k) + qA_0 & \partial_z + \frac{m_\Psi}{z} & 0 & 2\eta_5\frac{\phi}{z} \\ 2\eta_5\frac{\phi}{z} & 0 & \partial_z + \frac{m_\Psi}{z} & (\omega - k) - qA_0 \\ 0 & 2\eta_5\frac{\phi}{z} & -(\omega + k) + qA_0 & \partial_z - \frac{m_\Psi}{z} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = 0$$

# Boundary conditions

- At the holographic boundary – all fields should be normalizable
- At the hard wall

$$\phi'(z_w) = 0 \ , \quad A_0'(z_w) = 0$$

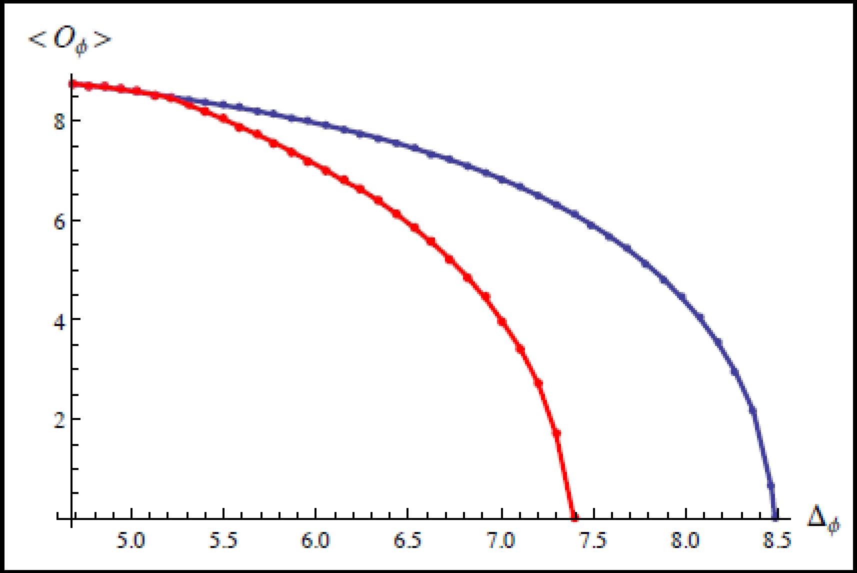
$$\alpha_1(z_w) = \alpha_4(z_w) = 0$$

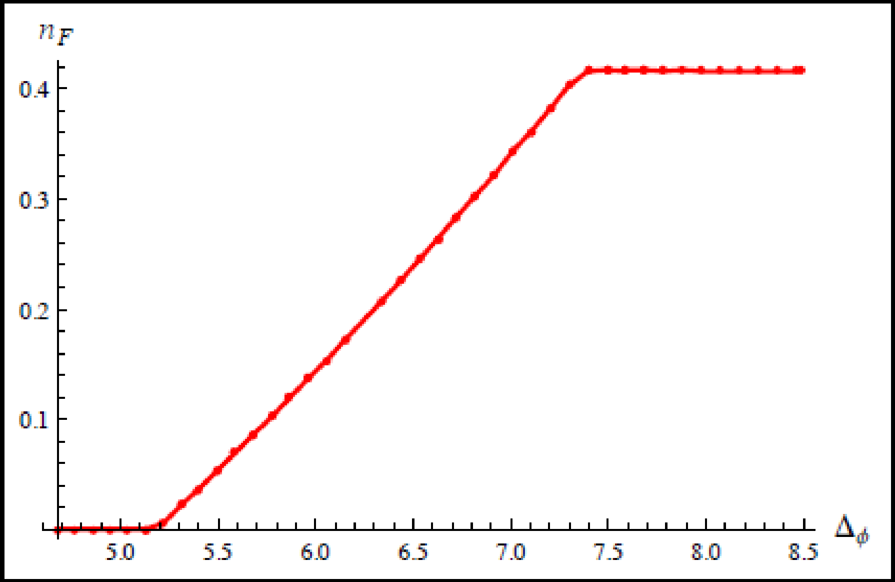
# Numerical scheme

- We deal with a system of coupled integro-differential equations
- We split it in two parts: bosonic (scalar + gauge fields) and fermionic
- Fermionic subsystem - standard shooting scheme;
  Bosonic subsystem – Newton relaxation scheme

# Bose-Fermi competition

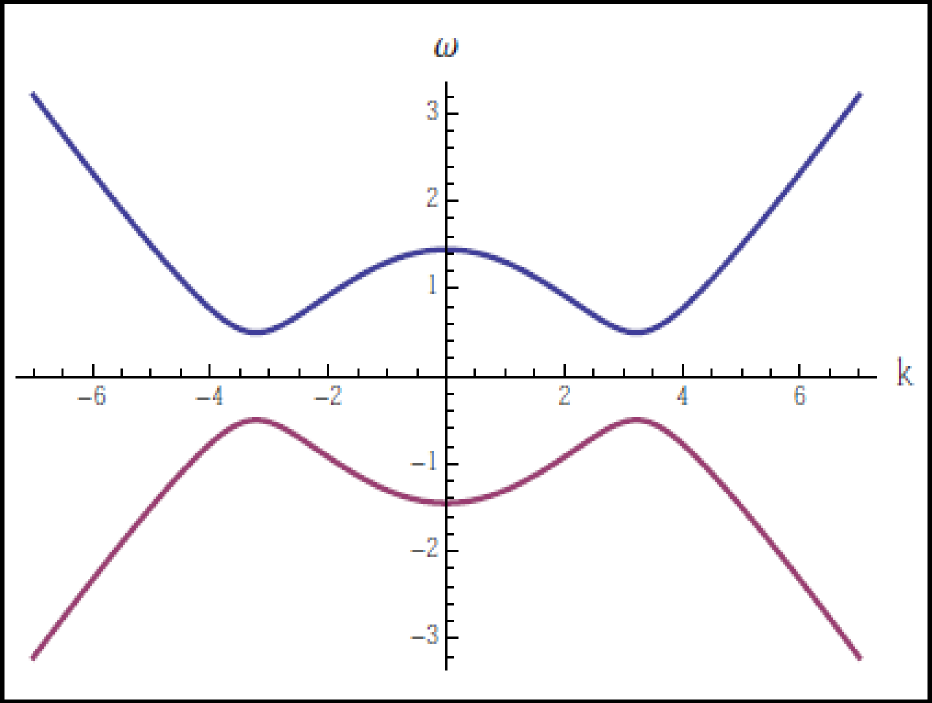
- Scalar order parameter and fermionic charge density at zero Majorana coupling



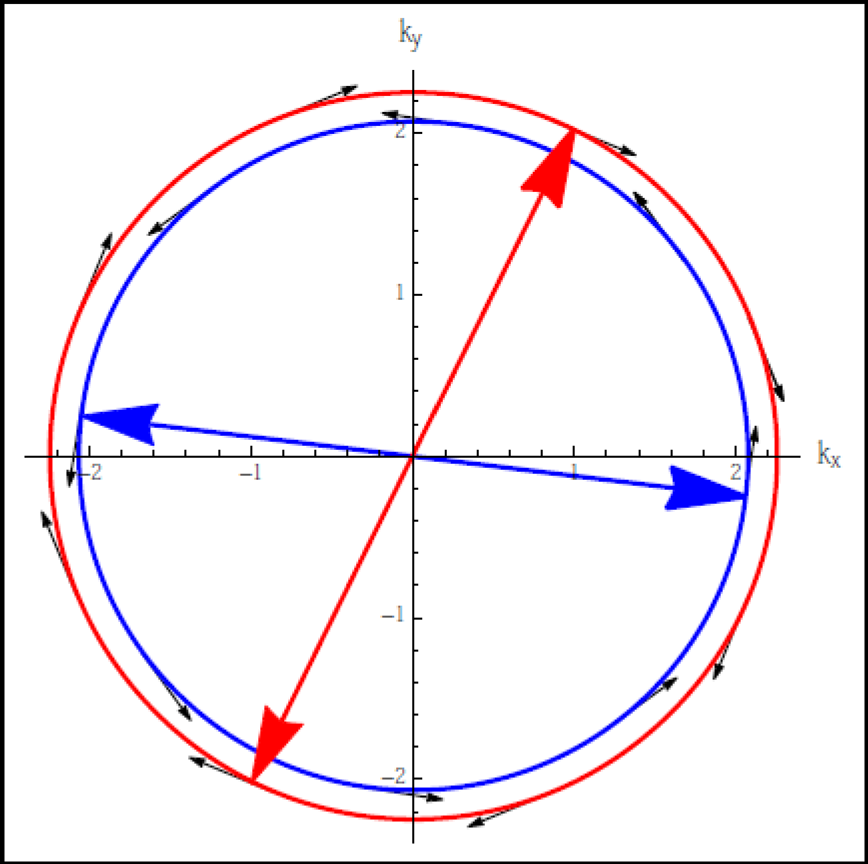



# Superconducting gap

- Gap opening

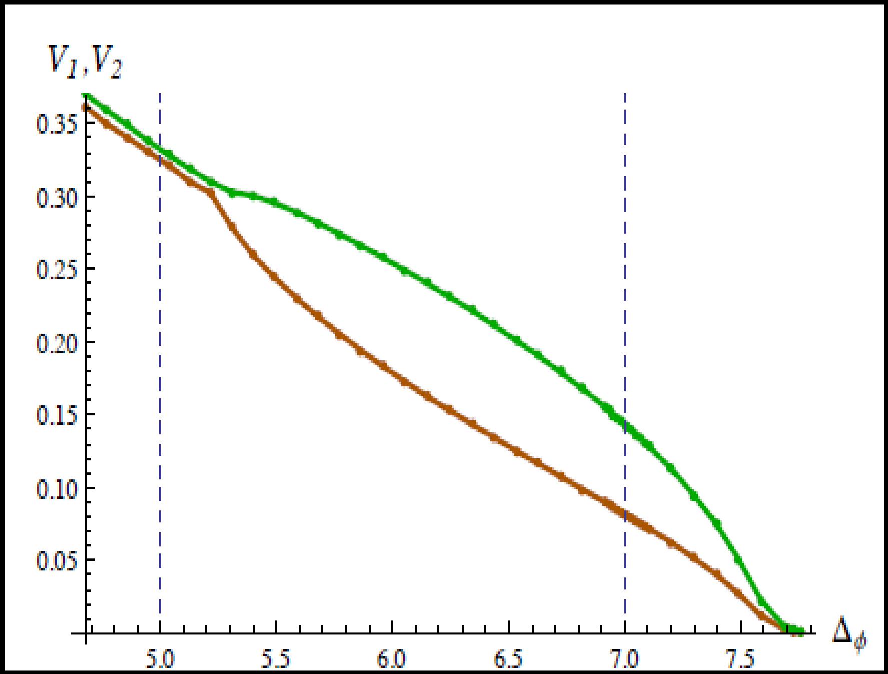


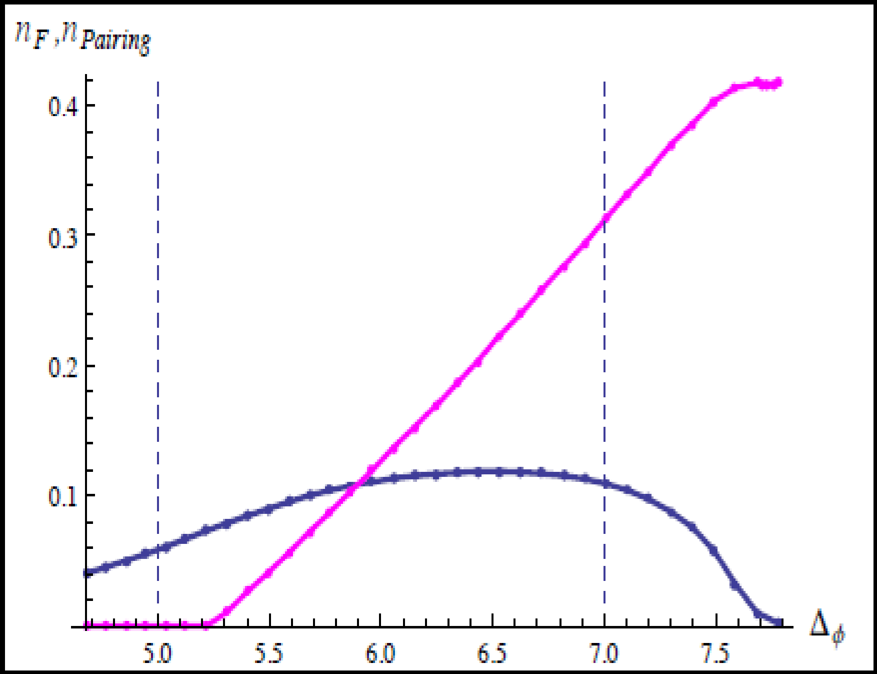

- Spin splitting



# Fully interacting system

- Two gaps
- Charge densities



# How to read off vacuum expectation values?

- In absence of fermions:

$$\phi(z) = A z^{3-\Delta_\phi} \cdot (1 + a_1 z + a_2 z^2 + \ldots) + B z^{\Delta_\phi} \cdot (1 + b_1 z + b_2 z^2 + \ldots) \,,$$

$$\Delta_\phi = \frac{3}{2} + \frac{1}{2}\sqrt{9 + 4m_\phi^2},$$

- In presence of fermions:

$$z^2 \phi''(z) - 2z\phi'(z) + q_\phi^2 z^2 A_0^2(z)\phi(z) - m_\phi^2 \phi(z) = -i\eta_5 z^3 \langle \overline{\psi^c} \Gamma^5 \psi \rangle$$

$$\phi(z) \;=\; \phi_{hom}(z) + \phi_{part}(z)$$

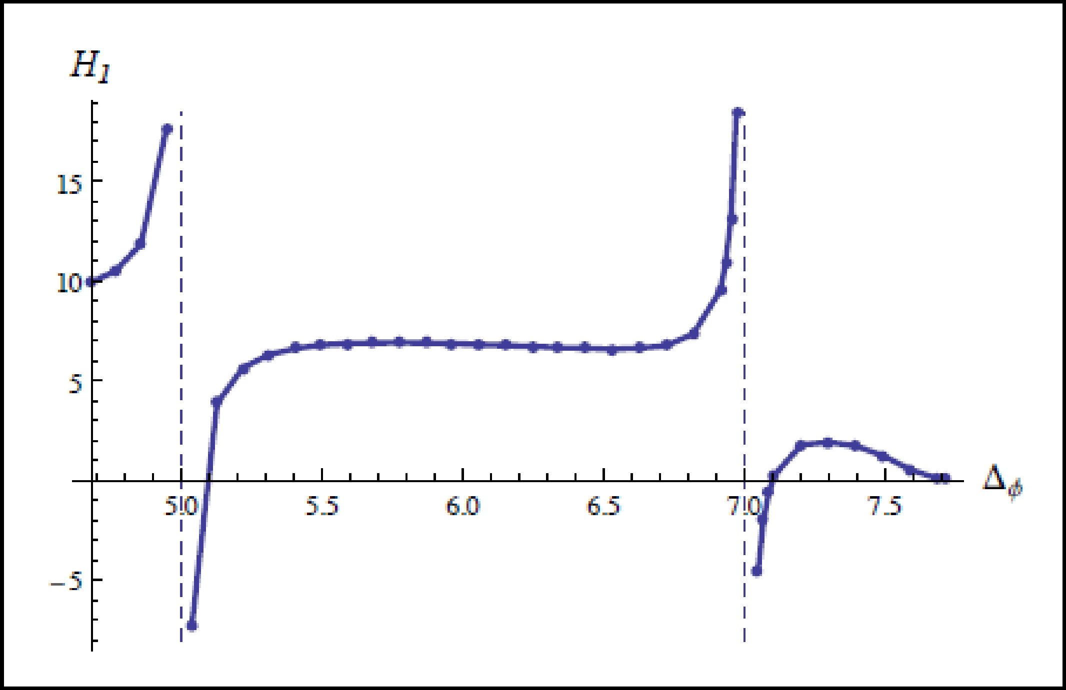
$$\phi_{part}(z) \;=\; \mathcal{P}_1 z^{2\Delta_\Psi} + \mathcal{P}_2 z^{2\Delta_\Psi + 1} + \mathcal{P}_3 z^{2\Delta_\Psi + 2} + \ldots$$

# How to read off vacuum expectation values?

- Does the standard AdS/CFT prescription work?

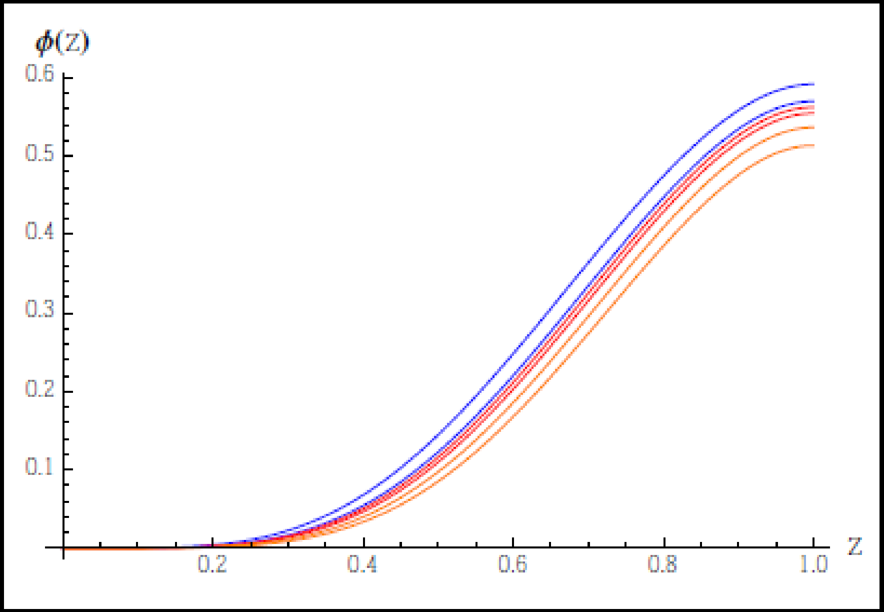
$$\langle \mathcal{O}_\phi \rangle = \lim_{z\to 0} z^{-d+1} \partial_z \left( z^{d-\Delta_\phi} \phi(z) \right)$$

- No…



# How to read off vacuum expectation values?

- Everything is regular in the bulk:

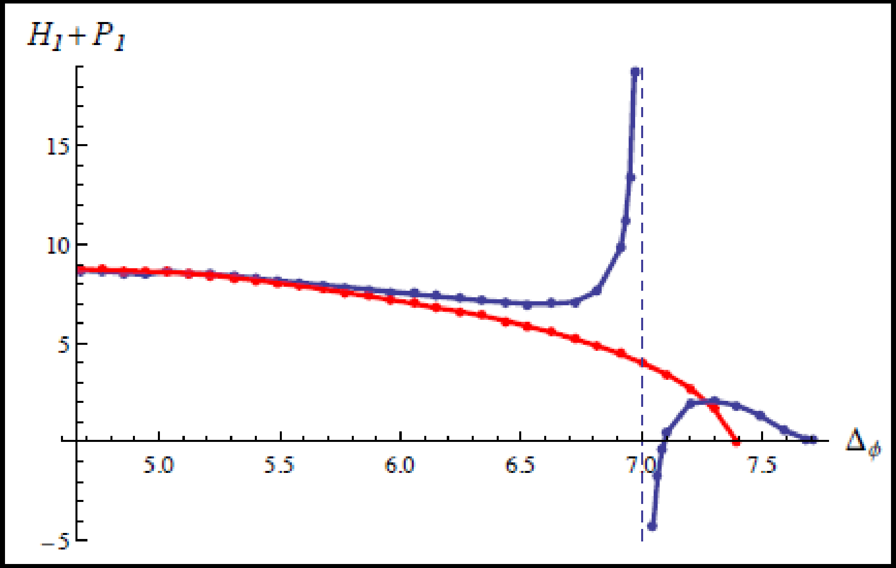


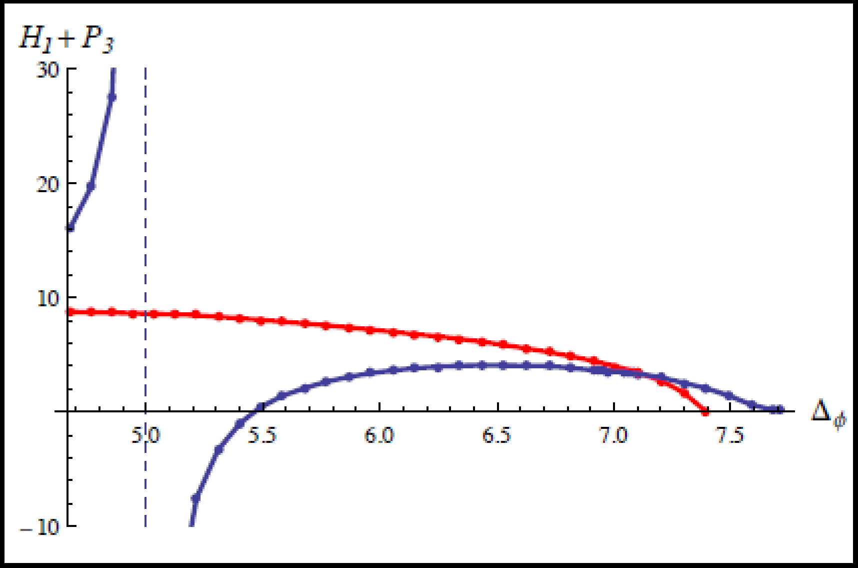

- At the critical point:

$$\phi(z) = \mathcal{H}_1 z^{2\Delta_\Psi + n} + \ldots + \mathcal{P}_1 z^{2\Delta_\Psi} + \ldots + \mathcal{P}_{n+1} z^{2\Delta_\Psi + n} \ln(z) + \ldots$$

# How to read off vacuum expectation values?

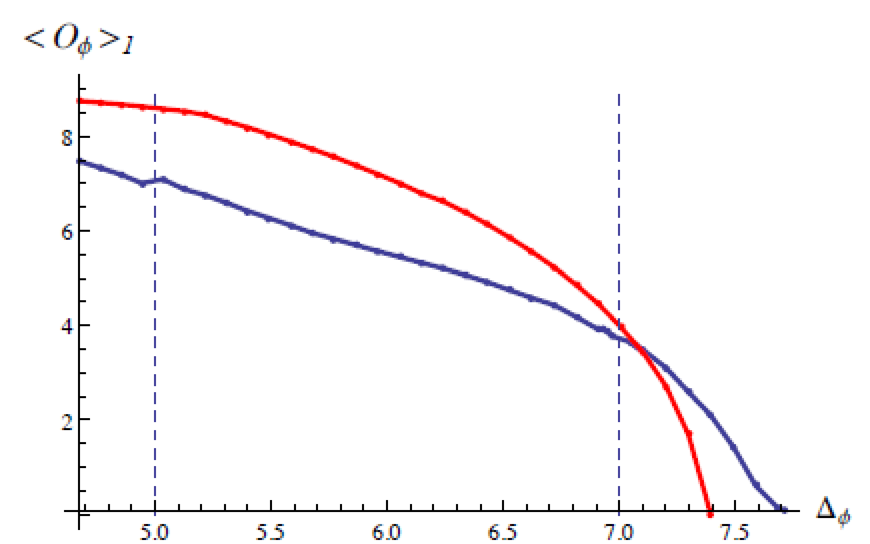
- Is it possible to regulate these resonances?
- Yes, if we take into account higher normalizable terms

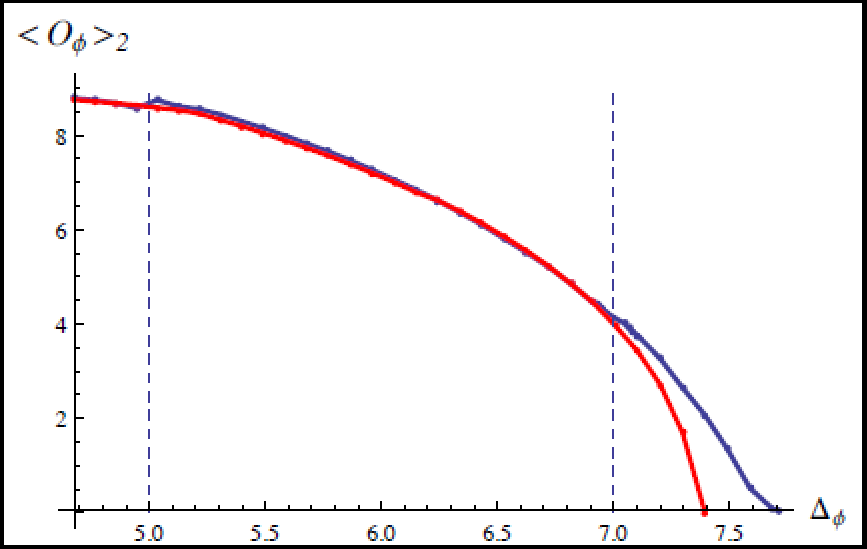


# How to read off vacuum expectation values?

- Two possible linear combinations




$$\langle \mathcal{O}_\phi \rangle_1 = \mathcal{H}_1 + \tfrac{1}{2}((2\Delta_\Psi + 2) - \Delta_\phi)\mathcal{P}_1 + \tfrac{1}{2}(\Delta_\phi - 2\Delta_\Psi)\mathcal{P}_3$$

$$\langle \mathcal{O}_\phi \rangle_2 = \mathcal{H}_1 + \frac{1}{2} e^{-(2\Delta_\Psi - \Delta_\phi)}((2\Delta_\Psi + 2) - \Delta_\phi)\mathcal{P}_1 + \frac{1}{2} e^{-(2\Delta_\Psi + 2 - \Delta_\phi)}(\Delta_\phi - 2\Delta_\Psi)\mathcal{P}_3$$

# Mixing of double trace operators

- Infinite tower of operators

$$\mathcal{O}_{(0)} = \mathcal{O}_{\overline{\Psi^C}} \mathcal{O}_{\Psi}$$

$$\mathcal{O}_{(1)} = \mathcal{O}_{\overline{\Psi^C}} (\overleftarrow{\partial}_\mu - \overrightarrow{\partial}_\mu)(\overleftarrow{\partial}^\mu - \overrightarrow{\partial}^\mu) \mathcal{O}_{\Psi}$$

$$\mathcal{O}_{(2)} = \mathcal{O}_{\overline{\Psi^C}} (\overleftarrow{\partial}_\mu - \overrightarrow{\partial}_\mu)(\overleftarrow{\partial}^\mu - \overrightarrow{\partial}^\mu)(\overleftarrow{\partial}_\nu - \overrightarrow{\partial}_\nu)(\overleftarrow{\partial}^\nu - \overrightarrow{\partial}^\nu) \mathcal{O}_{\Psi}$$

$$\vdots$$

- Extra boundary terms

$$S_{counter} \sim \int_{z=\epsilon} d^3x \left( -\phi^2 - \phi \bar{\Psi}^C_+ \Psi_- - \phi \bar{\Psi}^C_+ \overleftrightarrow{\partial}_\mu \overleftrightarrow{\partial}^\mu \Psi_- - \ldots \right)$$

# BCS/BEC crossover

- At small scalar conformal dimensions we have the Klein-Gordon equation for bosons
- At higher conformal dimensions its kinetic term can be neglected, and we enter the BCS regime
- Instead of a sharp $2^{nd}$ order phase transition we have an exponential tail

# Conclusions

- Holographic fermions compete with bosons and suppress the scalar condensation
- Yukawa-like interaction enhances condensation of the order parameter and opens a superconducting gap
- Infinite tower of double trace operators mixes in, so modification of the standard AdS/CFT dictionary for vev's is required

# Outlook

- Formulate a proper prescription for the boundary condensates in the case of operator mixing (in progress)
- Release the hard wall and consider a deconfined state
- Include backreaction of fermions on the metric