Charge transport in holography

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Based on *Charge transport in holography with momentum dissipation*, arxiv:1401.5436
and *Holographic metals and insulators with helical symmetry*, arxiv:1406.6351, with A. Donos and E. Kiritsis,

1 Transport in high T_c superconductors

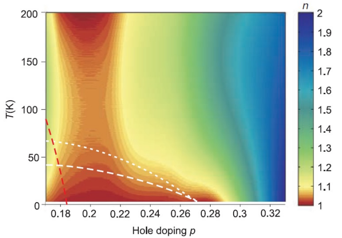
2 Computing finite DC conductivities

3 A holographic landscape of metals and insulators

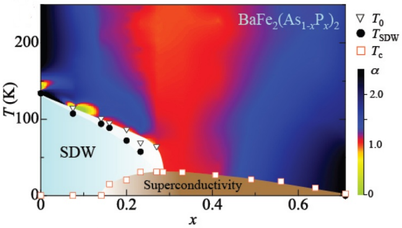
Scaling laws in high T_c superconductors

$$\sigma_{DC}(T) = \lim_{\omega\to 0} \Re[\sigma(\omega, T)]\,, \qquad \rho(T) = (\sigma_{DC})^{-1} \sim T^{\alpha}$$

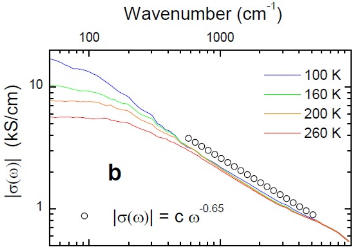
$La_{2-x}Sr_xCuO_4$



[Cooper et al, Science'09]



[Kasahara et al, PRB'10]



Data for an optimally doped high Tc superconductor $Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$,
[van der Marel et al, Nature'03]

$$\sigma_{AC} \sim \omega^{-2/3} \quad (T \lesssim \omega \ll \mu)$$

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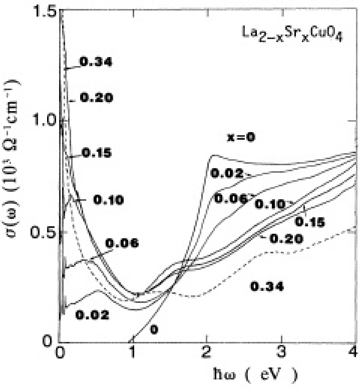
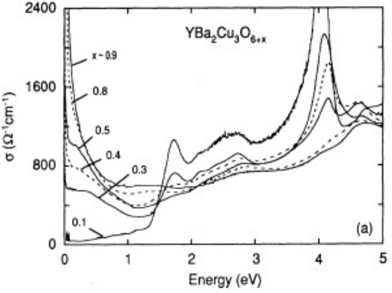
In-plane metal/insulator transitions





[Cooper et al, PRB'91]

[Uchida et al, PRB'91]

- **Drude-like feature** at low frequencies for high doping
- **Incoherent transport** at intermediate doping
- **Metal/insulator transition** at low doping
- Reorganisation of degrees of freedom assumed to be driven by strong correlations

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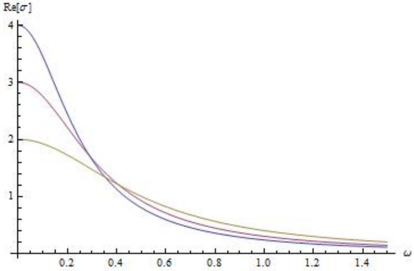
Drude model of conduction

Drude model: postulates the **existence (quasi)-particles** whose momentum relaxes on a typical scale τ

$$\sigma(\omega, T) = \frac{\sigma_{DC}}{1 - \imath\omega\tau}$$

$\sigma_{DC} \to +\infty$ when $\tau \to +\infty$ (no momentum relaxation)



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Models of conduction

- What happens for strong breaking of translation invariance (metals without Drude peaks, insulators)?

- Holographic methods are applicable for strongly-coupled systems both with weak and strong breaking of translation invariance.

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Outline

1. Transport in high T_c superconductors
2. Computing finite DC conductivities
3. A holographic landscape of metals and insulators

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1 Transport in high T_c superconductors

2 Computing finite DC conductivities

3 A holographic landscape of metals and insulators

Effective holographic actions for momentum relaxation

Option 1: relax momentum with spatially dependent scalars

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - \partial\phi^2 - Z_e(\phi){F_e}^2 + V(\phi) - Y(\phi)\sum_{i=1}^{p-1}(\partial\psi_i)^2\right]$$

- Contains gravity, a gauge field (finite density), a neutral scalar [Charmousis, **B.G.**, Kim, Kiritsis & Meyer '10].

- Assume that the massless scalars ('axions')

  $$\psi_i = kx^i$$

  (preserves homogeneity of the eoms: ODEs) [Andrade & Withers'13, **B.G.**'14, Donos&Blake'14, Taylor & Woodhead'14]. (similar mechanism to Q-lattices [Donos & Gauntlett'13,'14] – see J. Gauntlett's talk)

- The metric will be isotropic:

  $$\mathrm{d}s^2 = -B\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B} + C\mathrm{d}\vec{x}^2$$

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Effective holographic actions for momentum relaxation

Option 2: relax momentum by breaking translations to a helical Bianchi VII subgroup:

$$S = \int \mathrm{d}^5 x \sqrt{-g} \left[R - \partial\phi^2 - Z_e(\phi) {F_e}^2 - Z_m(\phi) {F_m}^2 + V(\phi) \right]$$

- Contains gravity, an electric (finite density) and a magnetic field, a neutral scalar [Charmousis, **B.G.**, Kim, Kiritsis & Meyer '10].
- In the metric Ansatz, replace the $\mathbf{R}^d$ spatial factor by a helical, Bianchi VII symmetry in a 5D bulk [Kachru et al'12, Donos & Gauntlett'12, Donos & Hartnoll'12], [Donos, **B.G.** & Kiritsis'14].

$$\mathrm{d}s^2 = -B \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B} + \sum_{i=1}^{3} C_i {\omega_i}^2$$

$$\omega_1 = \mathrm{d}x\,, \quad \omega_2 = \cos(kx)\mathrm{d}y + \sin(kx)\mathrm{d}z\,, \quad \omega_3 = \sin(kx)\mathrm{d}y - \cos(kx)\mathrm{d}z$$

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Other options

- Break diffeomorphism invariance in the bulk: massive gravity [Vegh'13, Davison'13, Blake & Tong'13, Davison, Schalm & Zaanen'13, Amoretti et al.'14]. Very similar to axions.
- Inhomogeneous lattices, [Horowitz, Santos & Tong'12, Donos & Hartnoll'12, Horowitz & Santos'13, Chesler, Lucas & Sachdev'13, Blake, Tong & Vegh'13]. A. Donos' talk
- Random-field disorder, [Hartnoll & Herzog'08, Davison, Schalm & Zaanen'13, Lucas, Sachdev & Schalm'14]

The electric perturbation problem

The conductivity gives the response of the current to a small applied electric field

$$\sigma(\omega, T) = \frac{\delta J_x}{\delta E_x} = \frac{G^R_{J_x J_x}(\omega, \mathbf{k}=0)}{\imath\omega}$$

Turn on a small electric field along x on the boundary:

$$\delta A_x(r, \omega, \vec{k}=0) \sim \delta A_{x(0)} + r\, \delta A_{x(1)} + \ldots, \quad r \to 0$$

This couples to other (vector) perturbations: g_{tx}, $\delta\psi\ldots$

The holographic dictionary tells us that the 2-point function (the conductivity) is the variation of the vev wrt the source:

$$\sigma(\omega, T) = \frac{\delta A_{x(1)}}{\imath\omega\, \delta A_{x(0)}}$$

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The DC conductivity

- At $\omega = 0$, there is a **radially conserved quantity**:

$$\partial_r [\sigma(r)] = 0$$

- It can be evaluated at the horizon (IR)!

$$\sigma_{DC} = \lim_{r \to 0} \sigma(r) = \lim_{r \to r_h} \sigma(r) = \sigma_{pc} + \sigma_{mr}$$

[Iqbal & Liu'08, Blake & Tong'13, Andrade & Withers'13, **B.G.**'14, Donos & Gauntlett'14]

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The DC conductivity: momentum relaxation term

$$\sigma_{DC} = \sigma_{pc} + \sigma_{mr}$$

Model-dependent, proportional to the density, dominates in the clean limit.

- Massive gravity: [Blake & Tong'13]

$$\sigma_{mr} = \frac{{Q_e}^2}{s\, m_h^2}$$

- axions: [Andrade & Withers'13, **B.G.**'14, Donos & Gauntlett'14]

$$\sigma_{mr} = \frac{{Q_e}^2}{k^2 s\, Y_h}$$

- Bianchi VII_0: [Donos, **B.G.** & Kiritsis'14]

$$\sigma_{mr} = \frac{{Q_e}^2 \sqrt{C_2 C_3 / C_1}}{k^2 \left((C_2 - C_3)^2 + C_2 Z_m A_m^2 \right)}$$

- Similar result for random, perturbative disorder

[Lucas, Schalm & Sachdev'14]

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The DC conductivity: pair creation term

$$\sigma_{DC} = \sigma_{pc} + \sigma_{mr}$$

$$\sigma_{pc} = \frac{Z_e(\phi)}{C_x}\left(\prod_{i=1}^{d} C_i\right)^{1/2}\Bigg|_{r=r_h} \qquad \mathrm{d}s^2 = -B\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B} + \sum_{i=1}^{d} C_i \mathrm{d}{x_i}^2$$

- Independent of the relaxation mechanism, dominates in the dirty limit;
- production of charged particles in an electric field without heat flow [Donos & Gauntlett '14].
- Unless there is no overlap between the current and momentum operator (CFT at zero density), does not give rise to finite conductivities by itself: $\sigma(\omega) \sim \sigma_{pc}\left(\delta(\omega) + \frac{i}{\omega}\right)$

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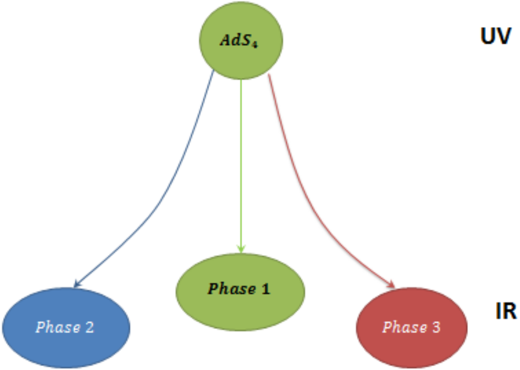
Philosophy

Assume the UV is Anti-de Sitter.
However, we wish to study the IR : **different IR phases can compete**.



How do we characterise these phases?

Effective holographic theories at low temperatures

Write down **effective holographic theories** describing the desired dynamics and characterize the possible IR phases by their symmetries and their behaviour under scaling transformations:

- Phases with unbroken U(1) symmetry (fractionalized phases), [Charmousis, **B.G.**, Kim, Kiritsis & Meyer '10, **B.G.** & Kiritsis'11]

- Phases with broken U(1) symmetry (e.g. cohesive phases like superfluids or electron stars), [**B.G.** & Kiritsis'12, **B.G.**'13];

- This talk: holographic metals and insulators with broken translation symmetry, [**B.G.**'14, Donos, **B.G.** & Kiritsis'14]

Effective holographic actions for momentum relaxation

Option 1: momentum relaxation by **massless scalars** $\psi_i = kx^i$

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - \partial\phi^2 - Z_e(\phi){F_e}^2 + V(\phi) - Y(\phi)\sum_{i=1}^{p-1}(\partial\psi_i)^2\right]$$

Option 2: momentum relaxation by **helical Bianchi VII lattices**

$$S = \int \mathrm{d}^5x\sqrt{-g}\left[R - \partial\phi^2 - Z_e(\phi){F_e}^2 - Z_m(\phi){F_m}^2 + V(\phi)\right]$$

In the IR, we can assume that the scalar ϕ

- either settles in an **extremum** of its effective potential, $\phi = \phi_\star$;
- or has a **runaway behaviour** $\phi \to \pm\infty$. Then:

$$Z_{e,m}(\phi) \sim e^{\gamma_{e,m}\phi}, \qquad V(\phi) \sim V_0 e^{-\delta\phi}, \quad Y(\phi) \sim e^{\lambda\phi}, \quad \phi \sim \kappa \ln r$$

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Ansatz for holographic quantum critical points

Assume that translation symmetry is broken in the IR by **irrelevant** deformations:

$$ds^2 = r^{\frac{2\theta}{d}} \left(-\frac{dt^2}{r^{2z}} + L^2 \frac{dr^2}{r^2} + \frac{d\vec{x}_{(d)}^{\,2}}{r^2} \right), \quad \phi = \kappa \ln r$$

- Captures **deviation** from symmetry under

$$t \to \lambda^z t\,, \qquad (r, x^i) \to \lambda (r, x^i)$$

- $\theta = 0 \Rightarrow$ constant scalar, hyperscaling solutions

$$S \sim T^{\frac{d}{z}}$$

AdS_{p+1} Lifshitz $AdS_2 \times \mathbf{R}^{p-1}$

The conduction (or vector hyperscaling violation) exponent

$$A_t \sim Q_e r^{\zeta - z}\,\mathrm{d}t$$

A **new exponent** parameterizes the scaling of the electric potential, [**B.G.** & Kiritsis'12, **B.G.**'13]. It can behave in two ways:

- The Maxwell stress-tensor is subleading compared to logarithmic derivatives from the Einstein tensor. Then the **charge density** Q_e sources an **irrelevant** mode of the solution:

$$\mathrm{d}s^2 = \mathrm{d}s_0{}^2\left(1 + Q_e{}^2 r^{2\beta} + \cdots\right), \qquad \beta = \frac{\zeta + d - \theta}{2}$$

  β is the (anomalous) scaling dimension of the charge density [**B.G.**'13,'14, Karch'14]. $\zeta = d - \theta$ gives back the usual scaling in $d - \theta$ dimensions.

- The Maxwell stress-tensor appears at the same order in r as logarithmic derivatives from the Einstein tensor. Then the **charge density** Q_e sources a **marginal** mode of the solution and $\zeta = \theta - d$.

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Clean limit: irrelevant translation-breaking deformations for helical phases [Donos, **B.G.** and Kiritsis'14]

Two translation-breaking modes from the metric and the magnetic field. The **momentum-relaxing** term dominates the DC conductivity

$$\sigma_{diss} = \frac{{Q_e}^2 \sqrt{C_2 C_3 / C_1}}{k^2 \left((C_2 - C_3)^2 + C_2 Z_m A_m^2 \right)}$$

- $0 < z < +\infty$: the modes are **exponentially suppressed**, as expected from the dispersion relation $\omega \sim k^z$ – No dofs at finite momentum if $z < +\infty$ [Hartnoll&Shaghoulian'12, Anantua et al.'12]

$$\sigma_{DC} \sim \frac{{Q_e}^2}{k^2} T^{\frac{z-1}{z}} \exp\left(2kT^{-\frac{1}{z}} \right)$$

- **Semi-locally critical limit** $z \to +\infty$, $\theta = \eta z$: the exponent $\beta(k)$ of the modes **depends on momentum** k [Hartnoll&Hofman'12]

$$\sigma_{DC} \sim \frac{{Q_e}^2}{k^2} T^{\eta + \beta_m}$$

  where β_m is the irrelevant mode turning on the magnetic field.

- Always metals.

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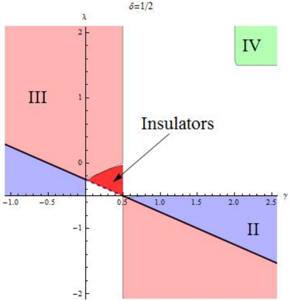
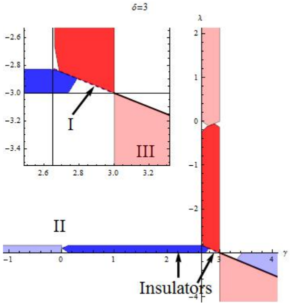
Clean limit: irrelevant translation-breaking deformations for axions [B.G.'14]

- The modes are not exponentially suppressed, no momentum scale.
- The **momentum-relaxing** term dominates the DC conductivity with a **non-universal temperature dependence**.

$$\sigma_{DC} \sim \frac{{Q_e}^2}{k^2} T^{\frac{\kappa\lambda - d + \theta}{z}}$$

- Can give rise both to metals (class III and IV) and insulators (class III, $z < 0$) (see also [Donos & Gauntlett'14])





Dirty limit for the helical phases [Donos, B.G. and Kiritsis'14]

Anisotropic saddle points, with leading behaviour

$$\mathrm{d}s^2 = r^{2\theta/3}\left(-\frac{\mathrm{d}t^2}{r^{2z_1}} + \frac{L^2\mathrm{d}r^2 + {\omega_1}^2}{r^2} + \frac{{\omega_2}^2 + \lambda r^{-2}{\omega_3}^2}{r^{2z_2}}\right)\left(1 + O(k^2 r^2)\right)$$

$$\phi = \kappa \ln r + \cdots\,, \quad A_e = Q_e r^{\zeta - z_1} + \cdots\,, \quad A_m = Q_m \omega_2 + \cdots$$

$$z_1 < 0$$

The conductivity is **pair creation dominated**, and gives rise to **insulators** ($\zeta < 2$) or **metals** ($\zeta > 2$ + irrelevant density mode)

$$\sigma_{DC} \sim T^{\frac{\zeta-2}{z_1}}$$

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Dirty limit for the phases with axions [B.G.'14]

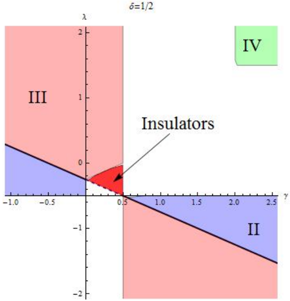
The metric is **isotropic** (see [Donos&Gauntlett'14] for anisotropic metrics)

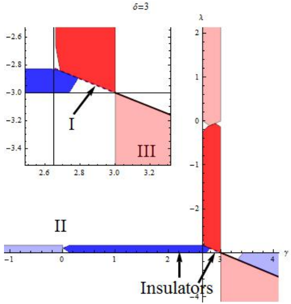
$$\mathrm{d}s^2 = r^{\frac{2\theta}{d}}\left(-\frac{\mathrm{d}t^2}{r^{2z}} + L^2\frac{\mathrm{d}r^2}{r^2} + \frac{\mathrm{d}\vec{x}_{(d)}^{\,2}}{r^2}\right), \quad \phi = \kappa \ln r$$

The conductivity is **pair creation dominated**, and gives rise both to **insulators** (and then $z < 0$) and **metals** – classes I and II

$$\sigma_{DC} \sim T^{\frac{\zeta-2}{z}}$$

Same scaling with T as for helical phases – Universal?





AC conductivity

At small frequency and zero temperature, there is a matched asymptotic argument relating IR and UV retarded Green's functions [Donos&Hartnoll'12]:

$$\Im\left[G^R_{J_x J_x}(\omega, T)\right] = \sum_I d^I \Im\left[\mathcal{G}^R_{\mathcal{O}_I \mathcal{O}_I}(\omega, T)\right]$$

The scaling of the AC conductivity is then given by the scaling of the least irrelevant operator.

For the Bianchi VII solution: 3 propagating modes.

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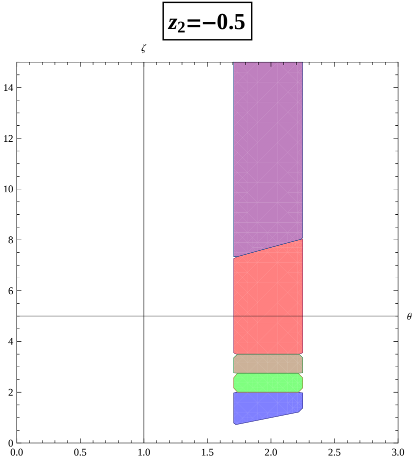
$T = 0$ IR asymptotics of the optical conductivity

Bianchi VII solution with irrelevant charge density mode



$$\sigma_{DC} \sim T^{n_{DC}}, \quad \sigma_{AC}(T=0) \sim \omega^{n_{AC}}$$

$$n_{DC} = \frac{\zeta - 2}{z_1}$$

- Blue: insulators with $n_{DC} = n_{AC} > 0$
- Green: metals with $n_{DC} = n_{AC} < 0$
- Brown: metals with $n_{DC} \neq n_{AC} < 0$, $n_{AC} < 0$: σ is not scale covariant!
- Red and purple: metals with $n_{DC} \neq n_{AC}$, $n_{AC} > 0$: reappearance of a delta function? – see also [Donos&Gauntlett'14]

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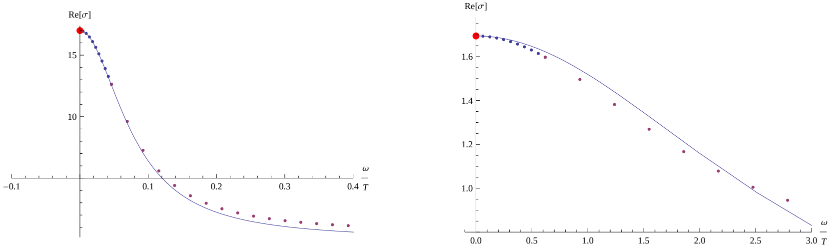
Clean/dirty vs coherent/incoherent limits

- What happens to the optical conductivity when either term dominates the DC conductivity? Is there always a Drude peak?
- Charged black hole analogous to the Reissner-Nordström black hole with IR $AdS_2 \times \mathbf{R}^2$ [Bardoux, Caldarelli & Charmousis'12, Andrade & Withers '13]
- $\sigma_{DC} = 1 + \mu^2/k^2$.



- When the pair creation term is negligible (left), the conductivity exhibits a Drude peak. Clean/Coherent limit – particle-like physics.
- When it is not negligible (right), the conductivity deviates from the Drude model. Dirty/Incoherent limit – unparticle physics.

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Conclusions

- Recipe to compute the DC conductivity analytically.

$$\sigma_{DC} = \sigma_{pc} + \sigma_{mr}$$

- Either term can dominate: clean vs dirty limit.

- Do they always coincide with the interplay between coherent/incoherent behaviour, e.g. Drude peaks vs other behaviour – particle vs unparticle-like?

- **Insulators** seem to require $\mathbf{z} < 0$ – field theory interpretation? Clearly the non-relativistic scaling $\omega \sim k^z$ does not make sense. No apparent mass gap.