Flavored ABJM with flux and Hall states

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[Work in progress with N. Jokela, M. Lippert, A. V. Ramallo, D. Zoakos]

General idea

- Aim: construct an ABJM-gravity dual of a Hall system
- Previous examples in the literature: D3-D7' D2-D8'

General idea:

- Introduce flavor branes $\rightarrow$ D6-branes in AdS_4 and $\mathbb{RP}^3 \subset \mathbb{CP}^3$
- Electromagnetic set up $\rightarrow E, B, d, j_y, j_x$.
- Turn on the proper WZ term, by turning on an internal flux $\rightarrow$ flux on $\mathbb{RP}^3$:

$$A = L^2 a(r)(d\psi + \cos\alpha d\beta) \quad \Rightarrow \quad C_1 \wedge F \wedge F \wedge F$$

- Regularity at the tip $\rightarrow \sigma_{xx} = 0$, $\sigma_{xy} \neq 0$
- Cycle at the tip $\rightarrow \sigma_{xy} = \frac{\nu}{2\pi}$ quantized $(\mathbb{S}^2_*)$

ABJM theory

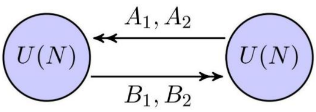
"N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals", arXiv:0806.1218

[Aharony-Bergman-Jafferis-Maldacena]

CS-matter theory in 2+1 $\Leftrightarrow$ M2 branes placed at $\mathbb{C}^4/\mathbb{Z}_k$

Field Theory side:

- $\mathcal{N} = 6$ Chern-Simons-matter theory in 2+1 dimensions with gauge group $U(N)_k \times U(N)_{-k}$



- Two gauge fields $A_\mu, \hat{A}_\mu$
- Four complex scalar fields: $C^{I=1,2,3,4}$ bifundamentals $(N, \bar{N})$

- Action:

$$S = k\mathrm{CS}[A] - k\mathrm{CS}[\hat{A}] - kD_\mu C^{I\dagger} D^\mu C^I - V_{\mathrm{pot}}(C)$$

Gravity side:

- Associated with M2-branes in $\mathbb{C}^4/\mathbb{Z}_k$ in M-theory.
- Large N limit $\rightarrow$ M-theory in $AdS_4 \times \mathbb{S}^7/\mathbb{Z}_k$.
- $N^{1/5} \ll k \ll N \rightarrow$ type IIA description:

$$AdS_4 \times \mathbb{CP}^3 + \text{fluxes}$$

$$ds_{10}^2 = L^2 \left(r^2 dx_{1,2}^2 + \frac{dr^2}{r^2} \right) + L^2 \; ds_{\mathbb{CP}^3}^2 \qquad e^{\phi} = \frac{2L}{k}$$

$$F_2 = 2kJ \quad , \quad F_4 = \frac{3k}{2} L^2 \Omega_{AdS_4}$$

Unquenched massless flavored ABJM

- "On the gravity dual of Chern-Simons-matter theories with unquenched flavor", arXiv:1105.6045

[Conde-Ramallo]

- Type II A regime $\rightarrow$ flavor D6-branes in AdS_4 and $\mathbb{RP}^3 \subset \mathbb{CP}^3$
- Action: $S_{SUGRA} + S_{D6}$. Smearing technique.
  Squashed $\mathbb{CP}^3$, base $\mathbb{S}^4$ fibre $\mathbb{S}^2$
- $\mathcal{N} = 1$ BPS equations. Solution:

$$\eta = 1 + \frac{3N_f}{4k} \qquad q = \frac{3}{2}(\eta+1) - 2\sqrt{\eta + \frac{9}{16}(\eta-1)} \qquad b = \frac{2q}{1+q}$$

$$ds^2_{10} = L^2\left(r^2 dx^2_{1,2} + \frac{dr^2}{r^2}\right) + \frac{L^2}{b}\left(q ds^2_{\mathbb{S}^4} + E_1^2 + E_2^2\right) \qquad e^{\phi} = \frac{4}{b}\frac{2-q}{\eta+q}\frac{L}{k}$$

$$F_2 = \frac{k}{2}\left(E_1 \wedge E_2 - \eta\left(\mathcal{S}_\xi \wedge \mathcal{S}_3 + \mathcal{S}_1 \wedge \mathcal{S}_2\right)\right) \qquad F_4 = \frac{3k}{4}\frac{(\eta+q)b}{2-q}L^2\Omega_{AdS_4}$$

ABJ theory

"Fractional M2-branes", arXiv:0807.4924

[Aharony-Bergman-Jafferis]

Generalization:

$$U(N)_k \times U(N)_{-k} \;\rightarrow\; U(N)_k \times U(N+M)_{-k}$$

Parity is broken!

- (Type IIA) New ingredient: NS-NS B_2 field, $dB_2 = 0$
- Same SUGRA solution with B_2 field (same SUSY)
- B_2 couples to brane action!
- Page Charge is quantized $\rightarrow$ B_2 flux quantized

ABJ theory + flavor probe brane

- Consider a flavor probe D6-brane in AdS_4 and $\mathbb{RP}^3 \subset \mathbb{CP}^3$
  Brane profile $\theta(r)$ ($\mathbb{CP}^3$ angle). κ symmetry $\rightarrow \cos\theta(r) = \frac{r_*}{r}$
- massive brane: pullback of B_2

$$\hat{B}_2 = -\pi \frac{M}{k} \left(\theta'(r) \sin\theta(r) dr \wedge (d\psi + \cos\alpha d\beta) + \cos\theta(r) \cos\alpha d\alpha \wedge d\beta\right)$$

Still kappa symmetric

- Non trivial $\mathbb{S}^2_*$ cycle at the tip $\rightarrow$ Quantization condition:

$$\frac{-1}{2\pi\alpha'} \int_{\mathbb{S}^2_*} \hat{B}_2 = \frac{2\pi M}{k} \qquad M \in \mathbb{Z}$$

ABJ theory + flavor probe brane

- But...alternatively, we can think it as a brane gauge field

$$S = -T_{D6}\int d\xi^7 e^{-\phi}\sqrt{g + F - \hat{B}_2} + T_{D6}\int \sum_i \hat{C}_i \wedge e^{F - \hat{B}_2}$$

$$\hat{B}_2 \quad \to \quad -F$$

$$F = \pi \frac{M}{k}\left(\theta'(r)\sin\theta(r) dr \wedge (d\psi + \cos\alpha d\beta) + \cos\theta(r) d\alpha \wedge d\beta\right)$$

Whose gauge potential is:

$$F = dA \qquad\qquad A = -\pi \frac{M}{k}\cos\theta(r)(d\psi + \cos\alpha d\beta)$$

- Generalize:

$$A = L^2 a(r)(d\psi + \cos\alpha d\beta)$$

- Determine $a(r)$ from EOMs obtained from the DBI+WZ action (or kappa symmetry).

Flavored ABJM with internal flux

- Apply idea to unquenched massless flavored ABJM.
- D6 flavor probe brane in AdS_4 and $\overline{\mathbb{RP}^3} \subset \overline{\mathbb{CP}^3}$ with flux:

$$A = L^2 a(r)(d\psi + \cos\alpha \, d\beta)$$

- Imposing kappa symmetry $\rightarrow$ flux function

$$\cos\theta(r) = \left(\frac{r_*}{r}\right)^b \qquad a(r) = -Q\left(\frac{r_*}{r}\right)^{\frac{b}{q}} = -Q\left(\cos\theta(r)\right)^{\frac{1}{q}}$$

- Quantization condition due to the $\mathbb{S}^2_*$ at the tip:

$$\frac{1}{2\pi\alpha'}\int_{\mathbb{S}^2_*} F = \frac{2\pi M}{k} \qquad M \in \mathbb{Z} \qquad \rightarrow \qquad Q = \frac{\pi M}{kL^2}$$

- Which observables change? $\rightarrow$ the meson mass spectrum.

Including E and B fields, density and currents

- Motivation: make contact with CMT
- Idea: turn on E, B, d, j_y, j_x on the probe brane ~~SUSY~~

$$A = L^2\left(a_0(r)dt + (Et + a_x(r))dx + (Bx + a_y(r))dy + a(r)(d\psi + \cos\alpha d\beta)\right)$$

- (consistent) EOMs:

$$\Delta := b^4 r^2 a'^2 + \sin^2\theta\left[b^2 + r^2\theta'^2 + b^2\frac{(Ba_0' + Ea_y')^2 + r^4(a_0'^2 - a_x'^2 - a_y'^2)}{E^2 - B^2 - r^4}\right]$$

$$r^4\sin^2\theta\frac{\sqrt{q^2 + b^4a^2}\,a_x'}{\sqrt{\Delta}\sqrt{B^2 - E^2 + r^4}} = constant$$

$$\frac{q+\eta}{2b(2-q)}\partial_r\left[\frac{\sqrt{q^2+b^4a^2}}{\sqrt{\Delta}\sqrt{B^2-E^2+r^4}}\sin^2\theta\left[B(Ba_0' + Ea_y') + r^4a_0'\right]\right] - B\left(\eta\cos\theta a' - a\sin\theta\theta'\right) = 0$$

$$\frac{q+\eta}{2b(2-q)}\partial_r\left[\frac{\sqrt{q^2+b^4a^2}}{\sqrt{\Delta}\sqrt{B^2-E^2+r^4}}\sin^2\theta\left[E(Ba_0' + Ea_y') - r^4a_y'\right]\right] - E\left(\eta\cos\theta a' - a\sin\theta\theta'\right) = 0$$

$$\partial_r\left[\frac{r^2}{\sqrt{\Delta}}\sqrt{q^2+b^4a^2}\sqrt{B^2-E^2+r^4}\,a'\right] - \frac{\sqrt{\Delta}\sqrt{B^2-E^2+r^4}}{\sqrt{q^2+b^4a^2}}a + 3r^2a - \frac{2(2-q)\eta}{b(q+\eta)}(Ba_0' + Ea_y')\cos\theta = 0$$

$$\partial_r\left[\frac{r^2}{\sqrt{\Delta}}\sin^2\theta\sqrt{q^2+b^4a^2}\sqrt{B^2-E^2+r^4}\,\theta'\right] - \frac{\sqrt{q^2+b^4a^2}\sqrt{B^2-E^2+r^4}}{\sqrt{\Delta}}\left[\Delta - b^4r^2a'^2\right]\cot\theta -$$

$$-(3-2b)qr^2\sin\theta\cos\theta + \frac{2b^3(2-q)}{q+\eta}a\sin\theta(Ba_0' + Ea_y') = 0$$

Including E and B fields, density and currents

- But,... still a SUSY solution!! $\mathcal{N} = 1/2$

$$E = B \qquad a_0'(r) = -a_y'(r) \qquad j_x = 0$$

Analytical solution:

$$\cos\theta(r) = \left(\frac{r_*}{r}\right)^b \qquad a(r) = -Q\left(\frac{r_*}{r}\right)^{\frac{b}{q}} \qquad a_0'(r) = (4-3b)\frac{b^2}{q}QB\frac{r^{2b}}{r^4}\frac{r^2 - r_*^2}{r^{2b} - r_*^{2b}}$$

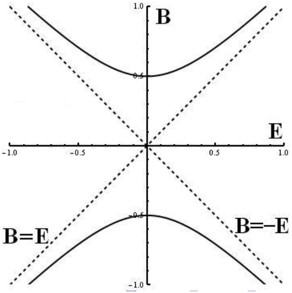
- Non-SUSY: looks ugly ...But, $O(1,1)$ symmetry!!

$$E^2 - B^2 \qquad a_0'^2 - a_y^2 \qquad Ba_0' + Ea_y'$$

$$\begin{pmatrix} B' \\ E' \end{pmatrix} = \begin{pmatrix} \cosh\gamma & \sinh\gamma \\ \sinh\gamma & \cosh\gamma \end{pmatrix} \begin{pmatrix} B \\ E \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Reduction:magnetic, electric, SUSY.



Hall States

- Internal flux + electromagnetic set up $\rightarrow C_1 \wedge F \wedge F \wedge F$

  Regularity at the tip $\rightarrow j_x = 0 \;\; d = B \;\; I(r_*) \;\; j_y = E \;\; I(r_*)$

$$\sigma_{xx} := \frac{j_{x\ phys}}{E_{phys}} = 0 \qquad \sigma_{xy} := \frac{j_{y\ phys}}{E_{phys}} = \frac{\nu}{2\pi}$$

- $C_1 \wedge F \wedge F \wedge F \rightarrow \sigma_{xy} \neq 0$
- $\mathbb{S}^2_*$ cycle at the tip $\rightarrow \sigma_{xy}$ quantized

QHE!

Hall States

- Previous examples in the literature: D3-D7' D2-D8'
- This new solution has much more analytical control!

Filling fraction:

- Non-SUSY general case:

$$\nu = \frac{M}{2}\left(1 + \frac{3N_f}{4k}\int_{r_*}^{\infty} \cos\theta(r)\frac{a'(r)}{Q}dr\right) \qquad M \in \mathbb{Z}$$

  Flavor correction depends on $\theta(r)$ and $a(r)$ along the brane.

- SUSY case $E = B$, $d = -j_y$:

$$\nu = \frac{M}{2}\left[1 + \frac{3N_f}{8k}(2-b)\right] \qquad M \in \mathbb{Z}$$

Generalizations

- Unquenched massive flavored ABJM background
  [Bea,Conde,Jokela,Ramallo]

  IR $\rightarrow$ ABJM
  UV $\rightarrow$ unquenched massless flavored ABJM

  Interpolating filling fraction:

$$\nu_{IR} = \frac{M}{2} \qquad \rightarrow \qquad \nu_{UV} = \frac{M}{2}\left(1 + \frac{3N_f}{4k}\int_{r_*}^{\infty} \cos\theta(r)\frac{a'(r)}{Q}dr\right)$$

- Temperature in unquenched massless flavored ABJM
  [Jokela,Mas,Ramallo,Zoakos]

  First order phase transition:

  - High T $\rightarrow$ BH embeddings (Karch-O'Bannon) $\rightarrow$ metal phase
  - Low T $\rightarrow$ MN embeddings (regularity at the tip) $\rightarrow$ Hall phase

Summary

- Starting point: unquenched massless flavored ABJM
- D6 flavor probe brane + flux: κ symmetric!

$$A = a(r)(d\psi + \cos\alpha d\beta) \qquad a(r) = -Q\left(\frac{r_*}{r}\right)^{\frac{b}{q}}$$

- E, B, d, j_y, j_x: SUSY broken
  - O(1,1) $\rightarrow$ reduction of solutions
  - $\mathcal{N} = \frac{1}{2}$ SUSY solution
- $C_1 \wedge F \wedge F \wedge F \rightarrow \sigma_{xx} = 0 \ \sigma_{xy} = \frac{\nu}{2\pi} \neq 0$
- Gravity dual of a Hall system!

$$\nu = \frac{M}{2}\left(1 + \frac{3N_f}{4k}\int_{r_*}^{\infty} \cos\theta(r)\frac{a'(r)}{Q}dr\right) \qquad M \in \mathbb{Z}$$

- Generalizations: temperature, massive background...

Thank you for your attention