The thermoelectric properties of inhomogeneous holographic lattices

Workshop on "Holographic Methods and Applications"

Aristomenis Donos[1] Jerome P. Gauntlett[2]

[1]DAMTP, U. of Cambridge

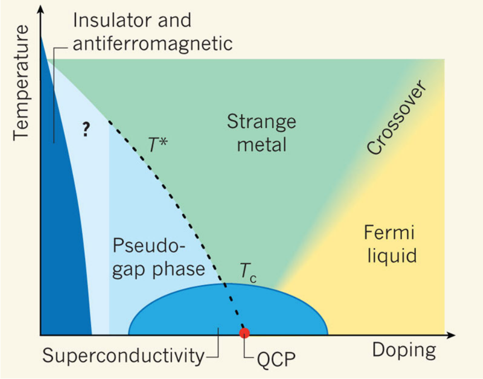
[2]Imperial College London

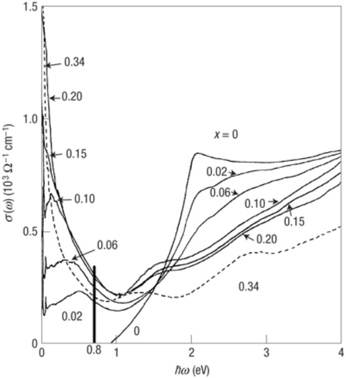
August 21, 2014

Outline

Outline

The Cuprates





Strong coupling in the Cuprates leads to:

- Interesting phase diagram
- Peculiar transport properties
- Use holographic methods!

AdS/CMT

The recipe says:

Field Theory	**Bulk**
Start with CFT$_d$	AdS_{d+1} Asymptotics
Chemical potential μ	$U(1)$ electric charge
Finite T	Killing horizon
2-point function $G_{JJ}(\omega)$	Bulk perturbation $\delta A_x, \ldots$

Use Kubo's formula

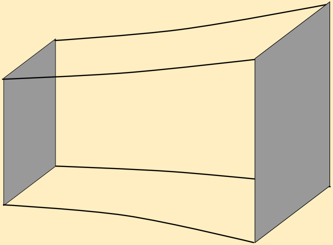
$$\sigma(\omega) = \frac{G_{JJ}(\omega)}{\imath\omega}$$

Perfect Holographic Conductor

Do it in $D = 4$ Einstein-Maxwell-Dilaton with AdS asymptotics:

$$\mathcal{L}_{EMD} = R - \frac{Z(\varphi)}{4}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\,(\partial\varphi)^2 - W\,(\varphi)$$
$$ds_4^2 = -U\,dt^2 + U^{-1}\,dr^2 + e^{2\,V}\,\left(dx_1^2 + dx_2^2\right)$$
$$A = a\,dt, \quad \varphi = \varphi(r)$$

$r = r_+$



$r = \infty$

$a = \mu - q\,r^{-1} + \cdots$

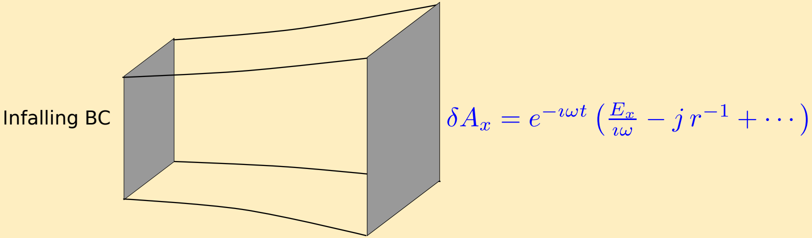
$\varphi = \varphi_+$

$\varphi = c_\varphi\,r^{3-\Delta} + \cdots$

Background black hole has temperature T , energy E, pressure P, entropy s and charge q.

Perfect Holographic Conductor

- To calculate conductivity need to source $\delta A_x = -e^{-\imath\omega t}\frac{E_x}{\imath\omega}$ on the boundary
- Momentum (δg_{tx}) couples because of background charge



$$\omega << \mu \Rightarrow \sigma = j/E_x = \frac{\imath}{\omega}\frac{q^2}{E+P} + \frac{(T\,s)^2}{(E+P)^2}\,Z_+$$

Generalisation from RN [Hartnoll, Herzog]

Minimal Gauged SUGRAs

There is two interesting gauged SUGRAs with bosonic sectors coming from dim. reductions of string/M-theory [Gauntlett, Varela]

- $N = 2$, $D = 4$ just Einstein-Maxwell

$$\mathcal{L} = \sqrt{-g}\left(R + 6 - \frac{1}{4}F^2\right)$$

- $N = 1$, $D = 5$ just Einstein-Maxwell with CS coupling

$$\mathcal{L} = *R + *12 - \frac{1}{2}F \wedge *F - \frac{1}{3^{\frac{3}{2}}}A \wedge F \wedge F$$

- Chemical potential from $U(1)_R$ charge

Stick with $D = 4$ Einstein-Maxwell for now. CS term will make a difference [AD, Hartnoll]

AdS-RN black hole

AdS RN black hole

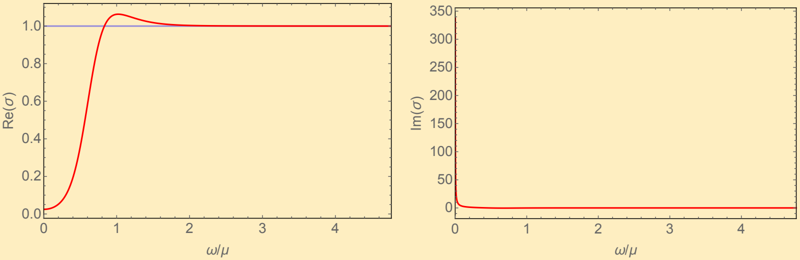
$$\mathcal{L} = \sqrt{-g}\,\left(R + 6 - \frac{1}{4}\,F^2\right)$$

reads

$$\begin{aligned} ds_4^2 &= r^2\,\left(-f\,dt^2 + f^{-1}\,dr^2 + dx^2 + dy^2\right) \\ A &= \mu\,\left(1 - \frac{r_+}{r}\right)\,dt \\ f &= (r - r_+)\,\left(\frac{1}{r} + \frac{r_+}{r^2} + \frac{r_+}{r^3} - \frac{\mu^2\,r_+}{4\,r^4}\right) \end{aligned}$$

- AdS_4 boundary at $r = \infty$, BH horizon at $r = r_+$
- Finite entropy density at $T = 0$, $\frac{s}{\mu^2} = \frac{\pi}{3}$
- Also semilocal

AdS-RN black hole



[Hartnoll, Herzog]

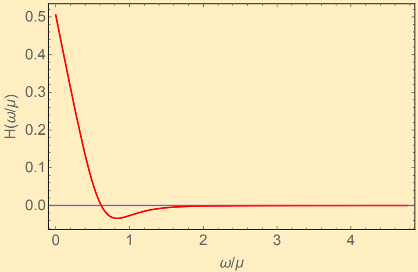
- Recover analytic expression for $\omega << \mu$
- For $\omega >> \mu$ asymptotes to $\sigma_{CFT} = 1$

Sum Rules

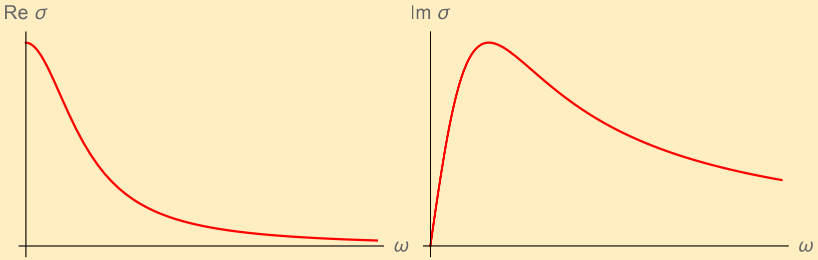
- Conductivity is $\sigma = \frac{G(\omega)}{\imath\omega}$
- Causality implies G analytic for $\mathrm{Im}\,\omega \geq 0$
- Charge redistributes spectral weight

$$\mathrm{Re}\,G\,(0) - \mathrm{Re}\,G_{CFT}\,(0) = -\frac{2\,\mu}{\pi}\int_{0^+}^{\infty}\mathrm{Re}\,(\sigma - \sigma_{CFT})$$

$$H(y) = \int_0^y \left[\frac{\pi}{\mu}\,\frac{q^2}{E+T^{xx}}\,\delta\left(\frac{\omega}{\mu}\right) + \mathrm{Re}\sigma - \mathrm{Re}\sigma_{CFT}\right]$$



Drude peaks



- Drude peaks are guaranteed
  [Hartnoll, Hofman]

$$\sigma = \frac{nq^2}{m}\frac{\tau}{1 - \imath\omega\tau}$$

- When the momentum relaxation rate τ is parametrically the largest scale
- Without collisions $\tau \to \infty \Rightarrow \sigma = \frac{nq^2}{m}\left(\delta(\omega) + \frac{\imath}{\omega}\right)$

Holographic Lattice

To add momentum dissipation introduce a UV benign lattice:

- UV relevant deformation $\mathcal{O}(x)$ with period L
- To be "Drude" at low T the IR operator should be irrelevant
- Solve the PDEs and show that the IR remained the same
- Charge density is a universal relevant operator in our constructions $\Rightarrow$ Impose $A_t = \mu(x) - J^t(x)\, r^{-1} + \cdots$
  [Hartnoll, Hofman][Horowitz, Santos, Tong]

$$\mu(x) = \mu_0 + A(x), \quad \langle A \rangle_L = 0$$

- $\mu_0 \Rightarrow$ chemical potential, $A'(x) \Rightarrow$ periodic electric field

Inhomogeneity

Why bother with inhomogeneous lattices/impurities?:

- Deform the theory by a single mode $\phi\,(x) \sim \lambda\, A\,(r)\, \cos\,(k\, x)$ of a relevant operator
- Higher harmonics sourced due to non-linearities but suppressed
- Q-lattices (also helical) fine-tuned situation where higher modes consistently drop out

  $\rightarrow$ Momentum dissipation

  $\rightarrow$ Metal - Insulator transitions
  [AD, Hartnoll], [AD,Gauntlett], [Andrade, Withers], [Gouteraux], [AD, Gouteraux, Kiritsis], [AD, Blake]

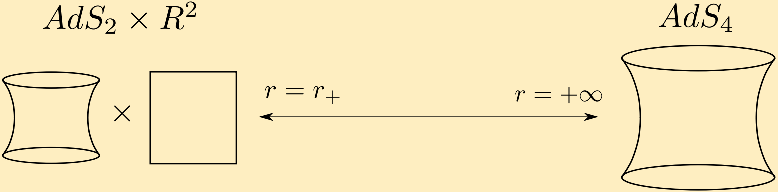
- Sourcing higher modes does have further impact on mid-IR physics. Perhaps interesting

Outline

1. Introduction/Motivation
2. Inhomogeneous Lattice in Einstein-Maxwell
   - Background black holes
   - AC/DC Transport
3. Summary

The (perturbative) ionic lattice

At $T = 0$ the solution is a domain wall



Add a lattice perturbation on top:

$$\delta g^{\mu}{}_{\nu}\,(r,x) = \varepsilon\,\delta h^{\mu}{}_{\nu}\,(r)\,\cos{(k\,x)}\,, \quad (\mu,\nu) = \{tt,\, xx,\, yy\}$$
$$\delta A_t\,(r,x) = \varepsilon\,\delta a_t\,(r)\,\cos{(k\,x)}\,, \quad \varepsilon << \mu$$

- Horizon + Infinity are singular points for the ODEs

$\Rightarrow$ Find converging modes close to the singular points

The (perturbative) ionic lattice

Three modes close to AdS_2

$$\delta h^{\mu}{}_{\nu} = c^{\mu}{}_{\nu} \left(r - \mu/\sqrt{12}\right)^{\delta} + \cdots$$

$$\delta a_t = c_t \left(r - \mu/\sqrt{12}\right)^{1+\delta} + \cdots$$

$$\delta_{\pm}(k) = \frac{1}{2}\left(-1 + \sqrt{5 + 8\,\frac{k^2}{\mu^2} \pm \sqrt{1 + 4\,\frac{k^2}{\mu^2}}}\right), \quad \delta_g = 0$$

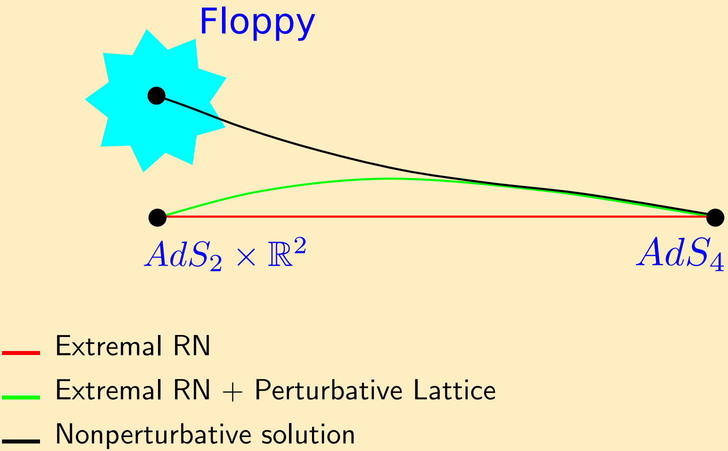
- Use it to find horizon profiles at small $T << \varepsilon << \mu$.
- Leading mode that breaks translations on the horizon has e.g.

$$\delta g^{\mu}{}_{\nu} \sim T^{\delta_-(k)} \cos(k\,x), \quad \delta A_t \sim T^{\delta_-(k)} \cos(k\,x)$$

- Notice $\delta_-(0) = 0 \Rightarrow$ Large periods lead to tiny exponents

“Floppy” Horizons?

- Nice setup but recent numerical study suggested this is not the whole story [Hartnoll, Santos]



- Shouldn't see AdS_2 in the full solution

The (non-perturbative) ionic lattice

Non-linear ansatz:

$$ds_4^2 = -U\,H_{tt} + \frac{H_{rr}}{U}\,dr^2 + \Sigma\,\left[e^{B}\,(dx + W\,dr)^2 + e^{-B}\,dy^2\right]$$

$$A = a_t\,dt$$

- Plan is to use DeTurck's method
  [Headrick, Kitchen, Wiseman][Figueras, Lucietti, Wiseman]
- A Smarr-type relation has to hold since ∂_y is Killing and non-deganarate
  [AD, Gauntlett]

$$\int\,\left[T^{tt}\,(x) + T^{yy}\,(x) - \mu\,(x)\,J^t\,(x)\right] = T\,S$$

- Reduces to $E + P - \mu\,q - T S = 0$

The (non-perturbative) ionic lattice

$$ds_4^2 = -U\,H_{tt}\,dt^2 + \frac{H_{rr}}{U}\,dr^2 + \Sigma\left[e^B\,(dx + W\,dr)^2 + e^{-B}\,dy^2\right]$$
$$A = a_t\,dt$$

Say we have our horizon at $r = 0$. Analyticity + Killing implies

$$U\,(r) = 4\pi\,T\,r + \ldots,\quad a_t(r,x) = a_t^{(0)}\,(x)\;r + \ldots$$
$$H_{tt}(r,x) = H_{tt}^{(0)}\,(x) + \ldots,\quad H_{rr}(r,x) = H_{tt}^{(0)}\,(x) + \ldots$$
$$\Sigma(r,x) = \Sigma^{(0)}\,(x) + \ldots,\quad B(r,x) = B^{(0)}\,(x) + \ldots$$

The (non-perturbative) ionic lattice

Horizon invariants to check "Floppiness"

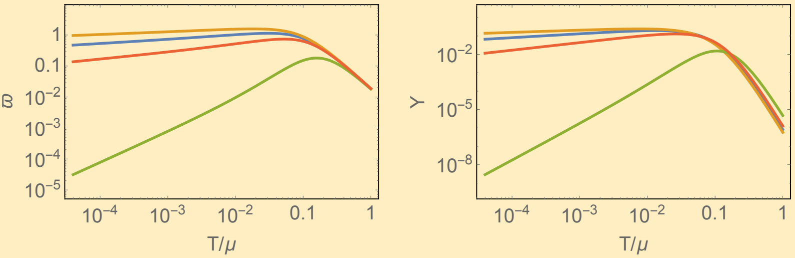
- $\varpi = \frac{\max \|\partial_y\|^2}{\min \|\partial_y\|^2} - 1$
  [Santos, Hartnoll]
- $\Upsilon = \int \frac{1}{\Sigma^{(0)}} \left[\partial_x \ln \frac{e^{B^{(0)}}}{\Sigma^{(0)}}\right]^2$

If AdS_2 is the IR geometry then at $T << \mu$

$$\varpi \sim T^{\delta_-(k/\lambda)}, \quad \Upsilon \sim T^{2\,\delta_-(k/\lambda)}$$

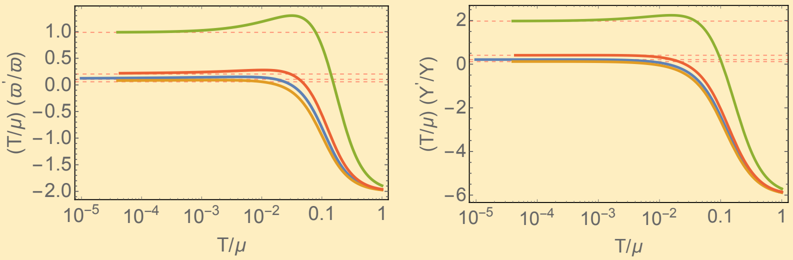
- λ would be fixed from by a low T limit
- σ_{DC} and $\bar{\kappa}_{DC}$ also suitable

The (non-perturbative) ionic lattice



- Fix $\mu(x) = \mu_0 \left(1 + \frac{1}{2} \cos(k\,x)\right)$ for $\frac{k}{\sqrt{2}\mu_0} = \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, 1$
- LogLog plots suggest we have power laws with k-dependent exponents at low T
- k-independent power laws for hight $T \to$ related to UV scaling dimensions

The (non-perturbative) ionic lattice



- Log derivatives with T give scaling exponents
- Estimate exponents based on AdS_2 expansion

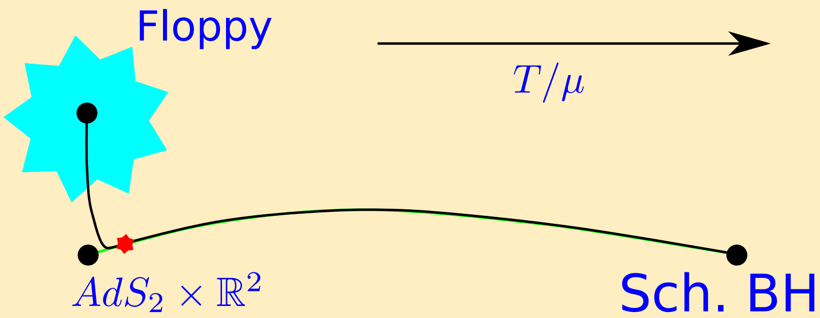
$$\delta_- = 0.059,\, 0.105,\, 0.204,\, 0.987$$

- Good agreement with low T Log derivatives

The (non-perturbative) ionic lattice

Logical possibilities:

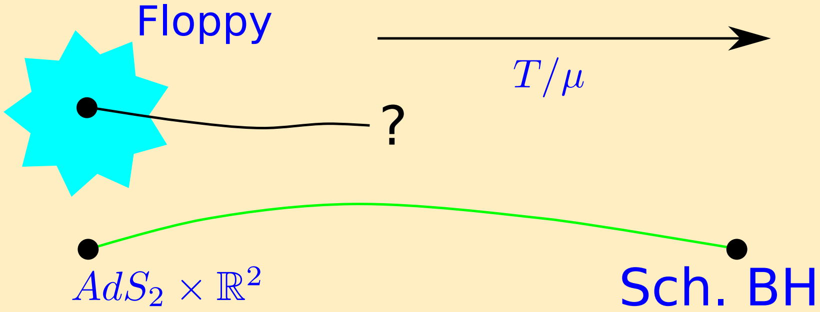
- We are not low enough in T. Something happens at lower T
- There is a disconnected branch of black holes
- The continuum limit of the domain walls with Floppy horizons is just $AdS_2 \times \mathbb{R}^2$



The (non-perturbative) ionic lattice

Logical possibilities:

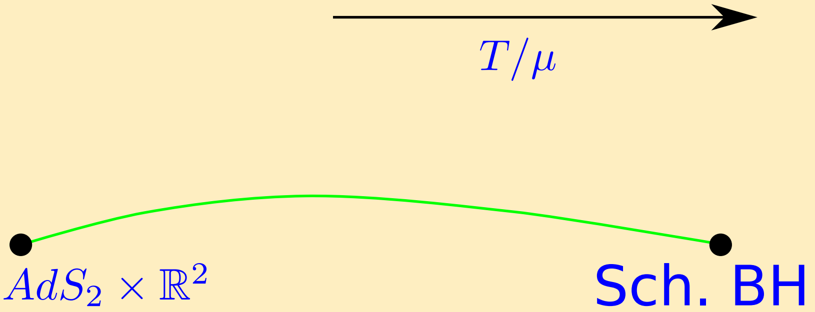
- We are not low enough in T. Something happens at lower T
- There is a disconnected branch of black holes
- The continuum limit of the domain walls with Floppy horizons is just $AdS_2 \times \mathbb{R}^2$



The (non-perturbative) ionic lattice

Logical possibilities:

- We are not low enough in T. Something happens at lower T
- There is a disconnected branch of black holes
- The continuum limit of the domain walls with Floppy horizons is just $AdS_2 \times \mathbb{R}^2$



AC/DC Transport properties

Need to introduce sources to study transport

- AC transport:
- Electric Field: $\delta A_x \sim -\imath\, E\, \omega^{-1}\, e^{-\imath\omega\, t}$
  [Horowitz, Santos, Tong]

$$\delta g_{tt},\, \delta g_{tx},\, \delta g_{xx},\, \delta g_{yy},\, \delta A_t,\, \delta A_x$$
$$\sigma\,(\omega) = (\imath\,\omega)^{-1}\; G_{x,x}\,(\omega)$$

- DC transport:
  [Iqbal, Liu][Iqbal, Liu][Davison][Blake, Tong, Vegh][Andrade, Withers]
- Electric field: $\delta A_x \sim -E\, t$
- Temperature gradient: $\delta g_{tx} \sim -r^2\, \zeta\, t$, $\delta A_x \sim \mu\,(x)\;\zeta\, t$ where $\zeta = \nabla_x T/T$

DC Transport

Introduce time dependent perturbation

$$\delta ds^2 = \delta g_{\mu\nu}\,(r,x)\;dx^\mu\,dx^\nu$$
$$\delta A = \delta a_\mu\,(r,x)\;dx^\mu - Et\,dx$$

Once again there is a Killing vector $k = \partial_t$

$$\partial_\mu\left(\sqrt{-g}\,F^{\mu\nu}\right) = 0$$
$$\partial_\mu\left(\sqrt{-g}\,G^{\mu\nu}\right) = 3\,k^\nu$$

for

$$G^{\mu\nu} = \nabla^\mu k^\nu + \frac{1}{2}k^{[\mu}F^{\nu]\sigma}A_\sigma + \frac{1}{4}(\psi - 2\theta)F^{\mu\nu}$$
$$L_k A = d\psi, \quad i_k F = d\theta$$

DC Transport

Define

$$J = \sqrt{-g}\, F^{rx}, \quad Q = 2\sqrt{-g}\, G^{rx}$$

- Use previous to show $\partial_\mu J = 0$, $\partial_\mu Q = 0$
- Fall offs at infinity give J is the electric current and

  $$Q = T^{tx} - \mu\, J$$

  is the heat current.
- Constant in x is just boundary Ward identities
- Constant in r is used to relate J, Q to E

The same quantities constant when introducing a T gradient on the boundary

DC Transport

In the end:

$$\sigma = \frac{1}{C} + \frac{M^2}{X\,C}, \quad \bar{\alpha} = \alpha = 4\pi\frac{M}{X},$$
$$\bar{\kappa} = \frac{(4\pi)^2 T\,C}{X}, \quad \kappa = \frac{(4\pi)^2 T\,C}{X + M^2}$$

where

$$X = \left(\int e^{B^{(0)}}\right)\left(\int e^{B^{(0)}}\left(\frac{a_t^{(0)}}{H_{tt}^{(0)}}\right)^2\right) - \left(\int e^{B^{(0)}}\frac{a_t^{(0)}}{H_{tt}^{(0)}}\right)^2$$
$$+ C\Upsilon$$
$$C = \int e^{B^{(0)}}, \quad M = \int e^{B^{(0)}}\frac{a_t^{(0)}}{H_{tt}^{(0)}}$$

Similar structure with homogeneous lattices but not quite the same

DC Transport

- At $T >> \mu$

$$\sigma = 1 + \frac{(\int \mu)^2}{\int \mu^2 - (\int \mu)^2}\,, \qquad \alpha = \frac{(4\pi)^2}{3}\frac{\int \mu}{\int \mu^2 - (\int \mu)^2}\, T$$

$$\bar{\kappa} = \frac{(4\pi)^4}{9}\frac{1}{\int \mu^2 - (\int \mu)^2}\, T^3, \qquad \kappa = \frac{(4\pi)^4}{9}\frac{1}{\int \mu^2}\, T^3$$

- At $T << \mu$ and assuming AdS_2

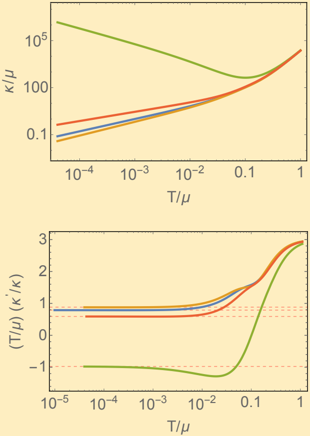
$$\sigma \sim T^{-2\delta_-(k/\lambda)}, \quad \alpha \sim T^{-2\delta_-(k/\lambda)}, \quad \bar{\kappa} \sim T^{1-2\delta_-(k/\lambda)}$$

$$\kappa \sim T$$

Agrees with memory matrix formalism

[Hartnoll, Hofman]

DC Transport



- Fixed $\mu(x)/\mu = 1 + \frac{1}{2}\cos(k\,x)$
- Different $k/\left(\sqrt{2}\,\mu\right) = 1/3,\ 2/5,\ 1/2, 1$

DC Transport

Examine Wiedemann-Franz type of laws

- Form the Lorentz ratio $\bar{L} = \frac{\bar{\kappa}}{\sigma T}$. At low temps or small lattices

  $$\bar{L} \to \frac{s^2}{q^2}$$

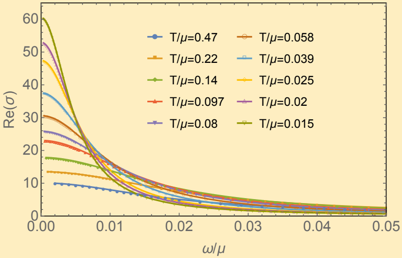
  Agrees with memory matrix formalism

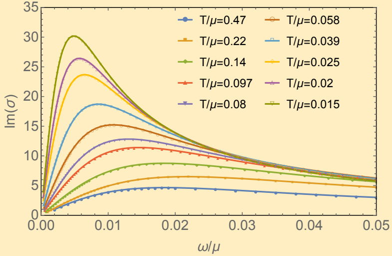
- Interesting to also examine

  $$\frac{\bar{\kappa}}{\alpha} = \frac{4\pi T \int e^{B^{(0)}}}{\int e^{B^{(0)}} \frac{a_t^{(0)}}{H_{tt}^{(0)}}}$$

- Different from homogeneous lattices but the same for small temperature or lattice $\frac{Ts}{q}$

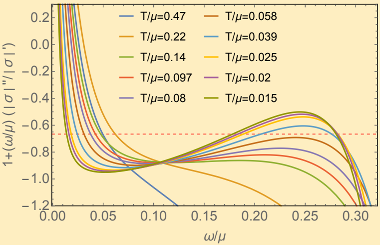
AC Transport

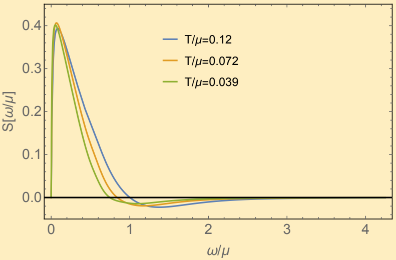




- Fix $\mu(x)/\mu = 1 + \frac{1}{2}\cos(k\,x)$ with $k/\mu = 2^{-1/2}$
- At low frequencies $\omega < T$ there is now a "Drude" peak [Horowitz, Santos, Tong]

AC Transport





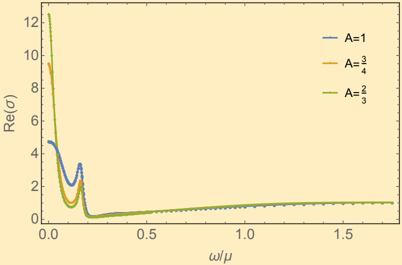
- Plot $1 + \frac{\omega}{\mu}\frac{\sigma''}{\sigma'}$ but no clear sign of scaling laws
- Sum rule seems to work

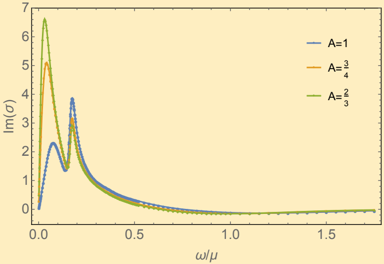
$$S\left(y\right) = \int_0^y \left(\mathrm{Re}\sigma - \mathrm{Re}\sigma_{CFT}\right)$$

Lattice resolves the delta function at the origin

AC Transport

- Deform by $\mu(x)/\mu = 1 + A\,\cos(k\,x)$ for fixed $k/\mu = \left(3\sqrt{2}\right)^{-1}$





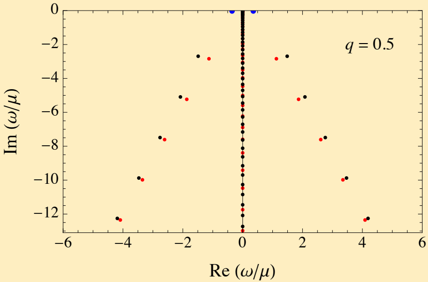
- CFT result for $\omega >> k, T$
- Large lattices shift more spectral weight to mid-infrared
- Mid-infrared peak

AC Transport

- At linearised level, background lattice couples current to longitudinal modes of fluctuations of undeformed RN bh

$$\delta g_{tt},\ \delta g_{tx},\ \delta g_{xx},\ \delta g_{yy},\ \delta A_t,\ \delta A_x$$

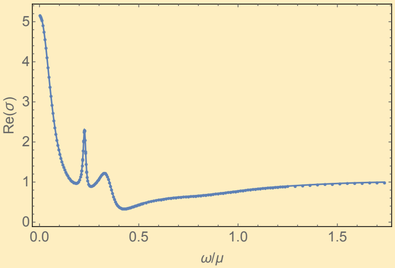
- Sound mode of RN bh lies in this sector
  [Policastro, Son, Starinets][Edalati, Jottar, Leigh]

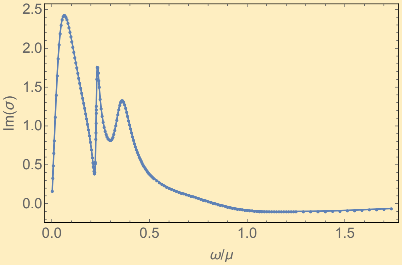


- Sound mode has $\mathrm{Re}(\omega) \sim \frac{1}{\sqrt{2}}\,k \to$ Peak in σ at $\omega \sim k/\sqrt{2}$

AC Transport

- Deform by higher harmonics
  $\mu(x)/\mu = 1 + A\,\cos(k\,x) + B\,\cos(2\,k\,x)$

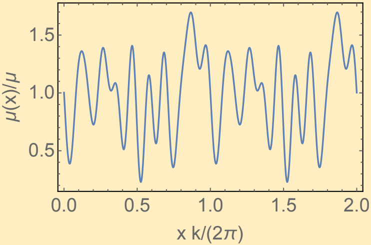


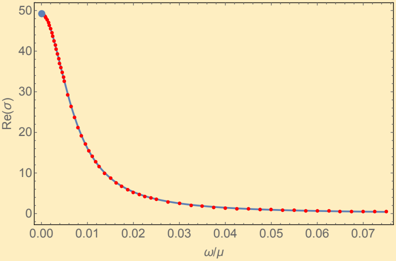


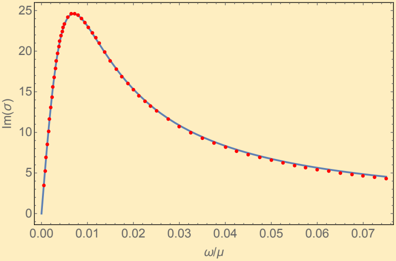
- Second peak appears

$\Rightarrow$ Sourcing higher modes affects the mid-infrared. How?

AC Transport

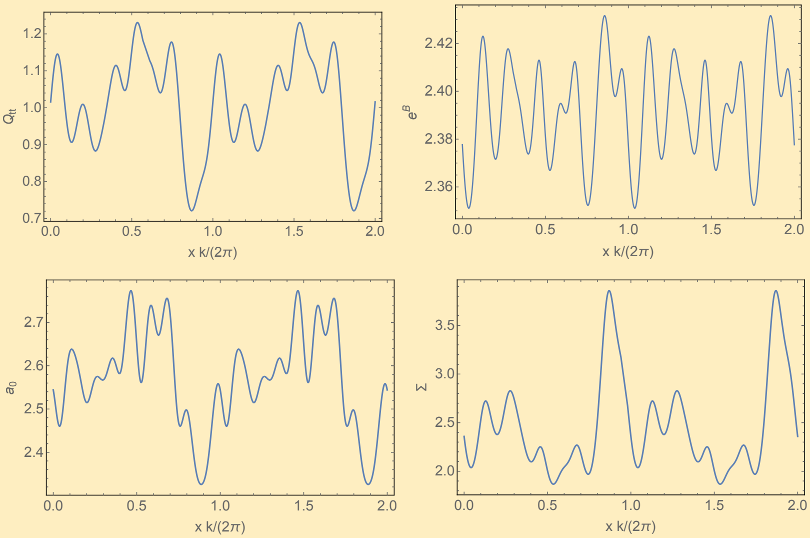






- Consider a "dirty lattice"
- Small frequency regime $\omega < T < k, A$ gives a "Drude" peak

AC Transport



- Used horizon data to find σ_{DC}

Outline

Summary

- Examined low temperature behaviour of lattice deformed RN black hole
- Showed analytic expressions of transport coefficients in terms of bh horizon data
- Revisited intermediate scaling laws in the optical conductivity
- Connection between sourced lattice modes and optical conductivity peaks