Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

FROGS
FRont Of pro-Galician Scientists

Disordered Holographic Superconductors

In collaboration with

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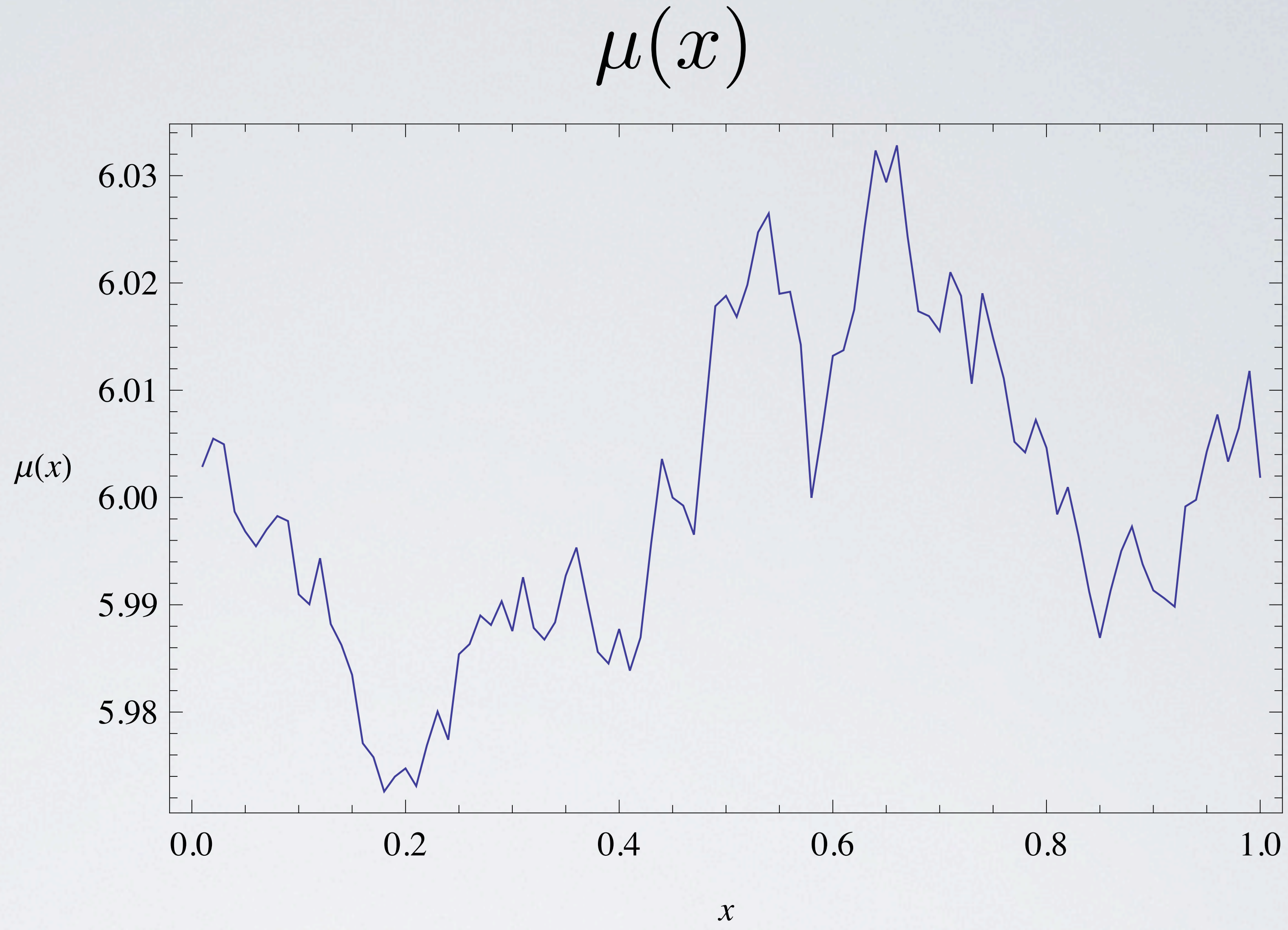
I. Salazar Landea (La Plata, Argentina)
A. Scardicchio (ICTP, Italy)

[1308.1920, 1407.7526, ...]

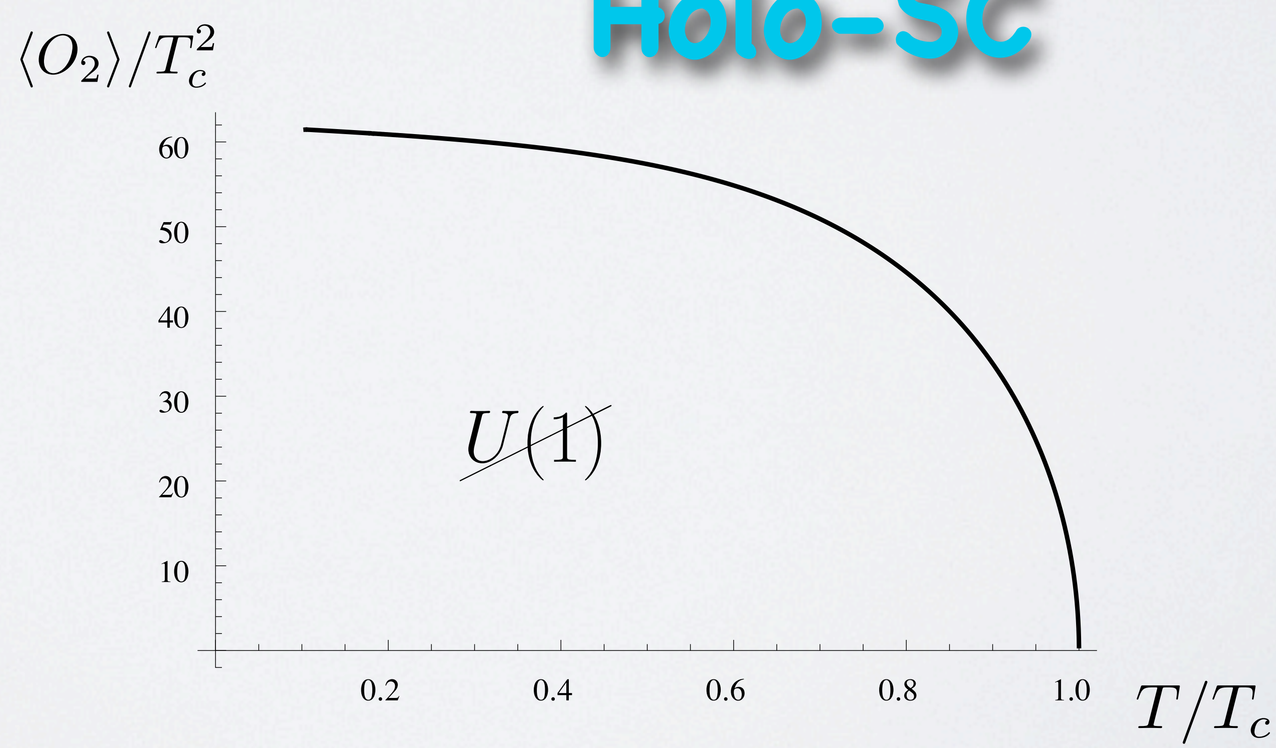
Daniel Areán
Reykjavík, August 2014

Noise

[charged impurities]

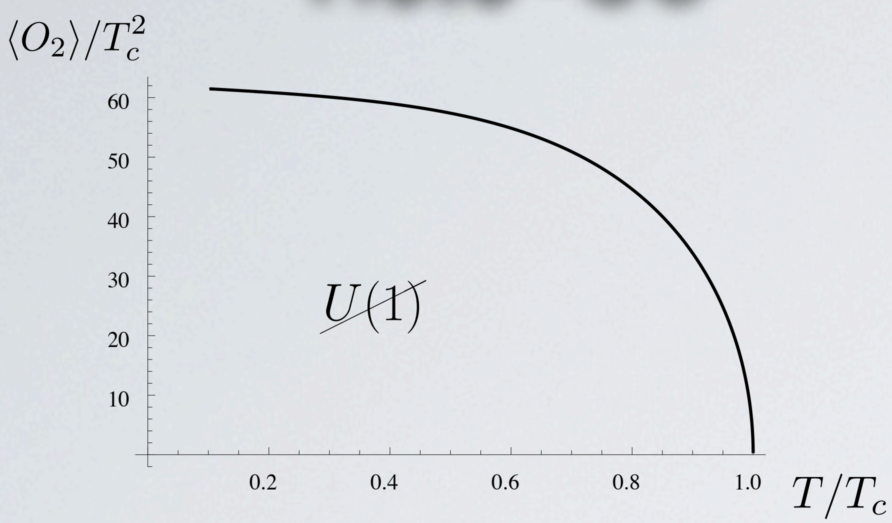


Holo-SC



in holography

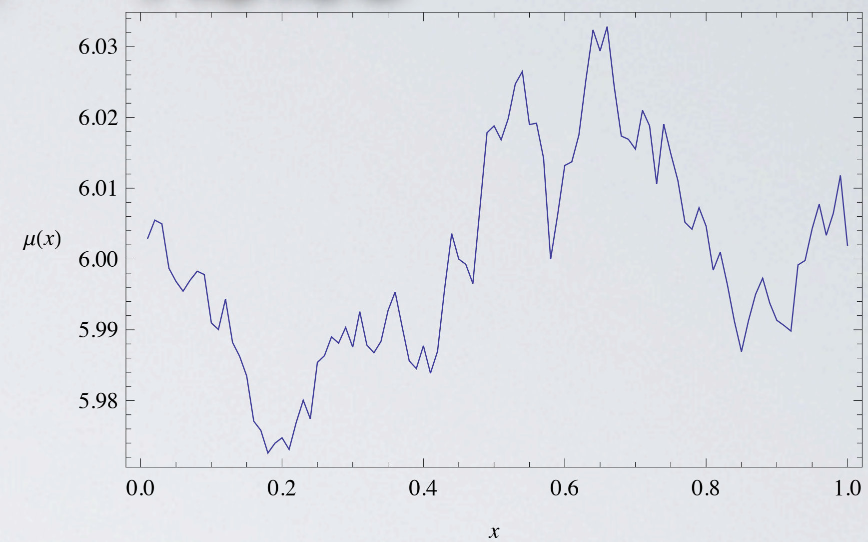
Holo-SC



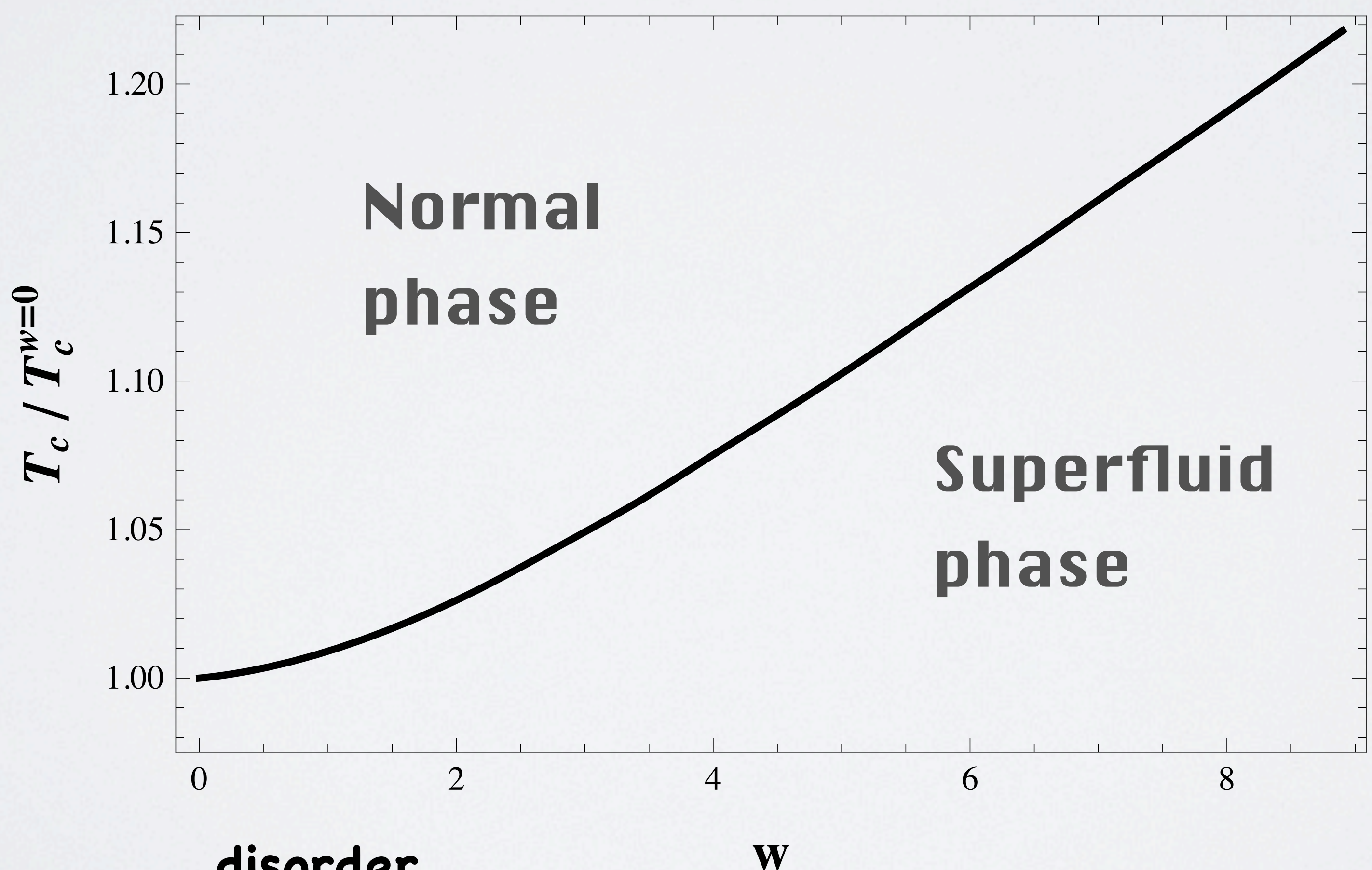
+

Noise

$\mu(x)$

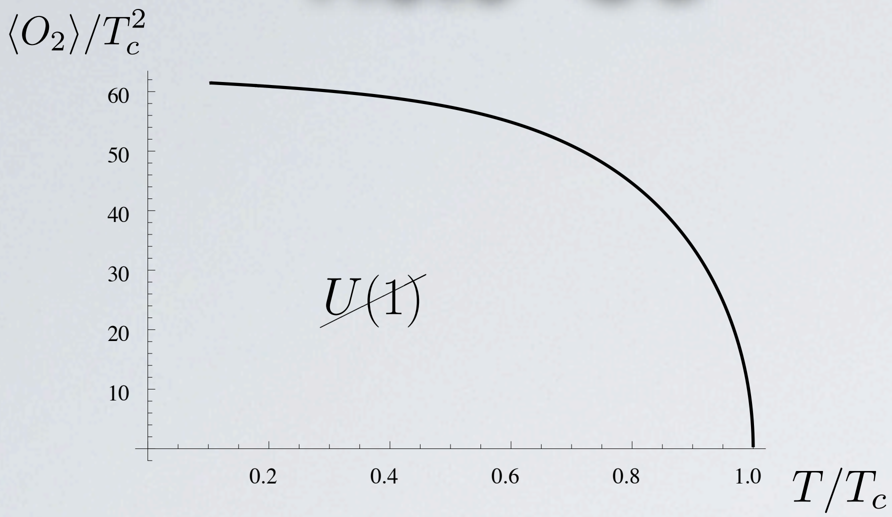


= ★ **Enhancement of SC**



disorder

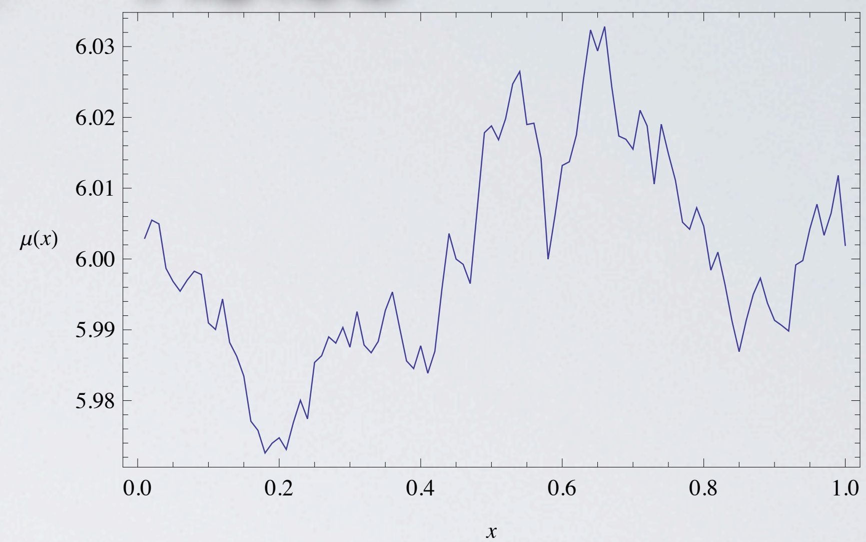
Holo-SC



+

Noise

$\mu(x)$

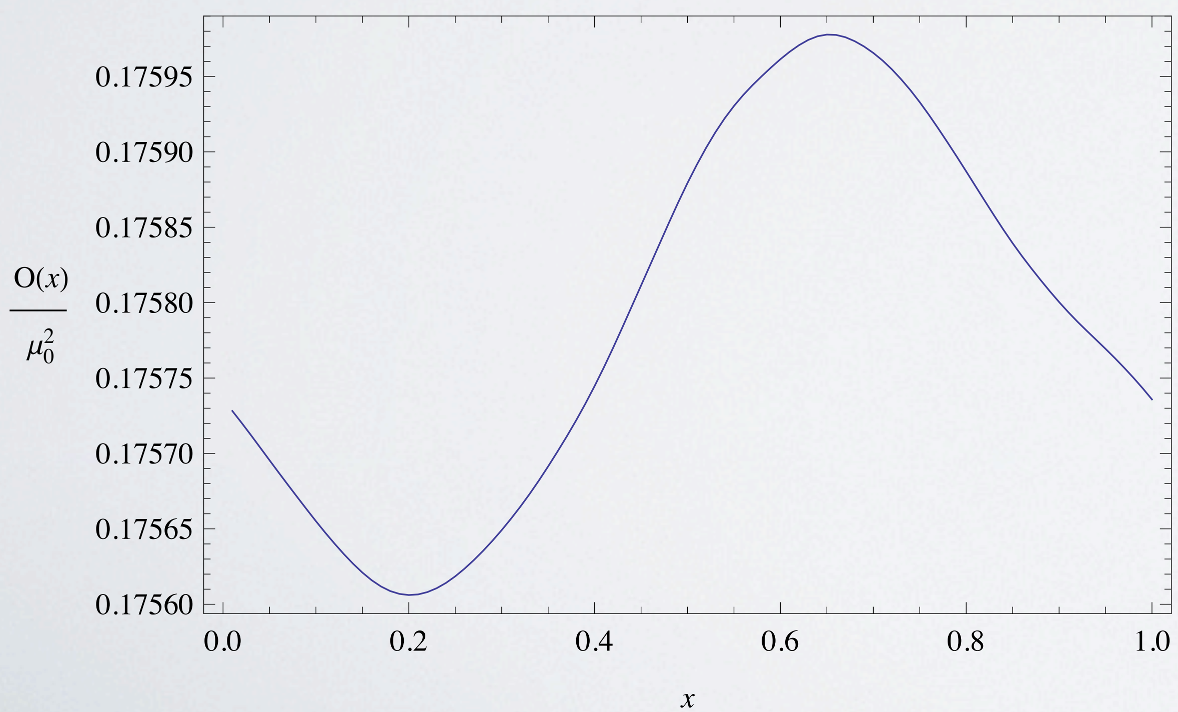


$$S_k = \frac{1}{k^{2\alpha}}$$

= ★ **Spectrum 'renormalization'**

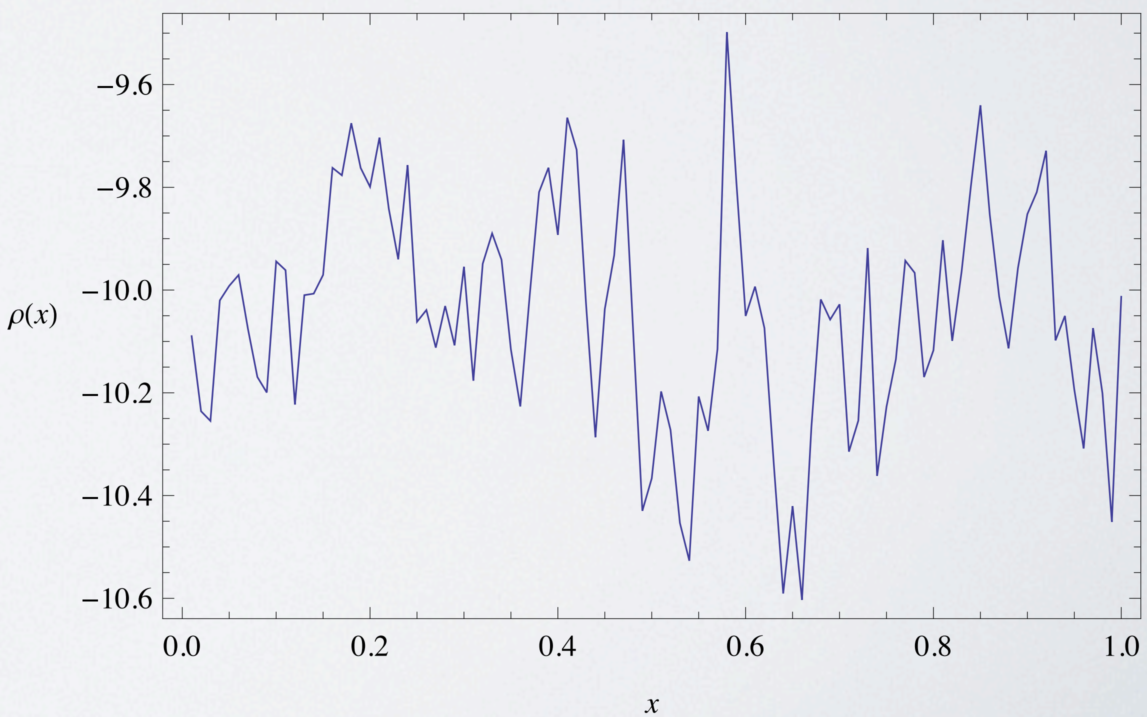
Condensate

$$S_k = \frac{1}{k^{2\alpha+4}}$$



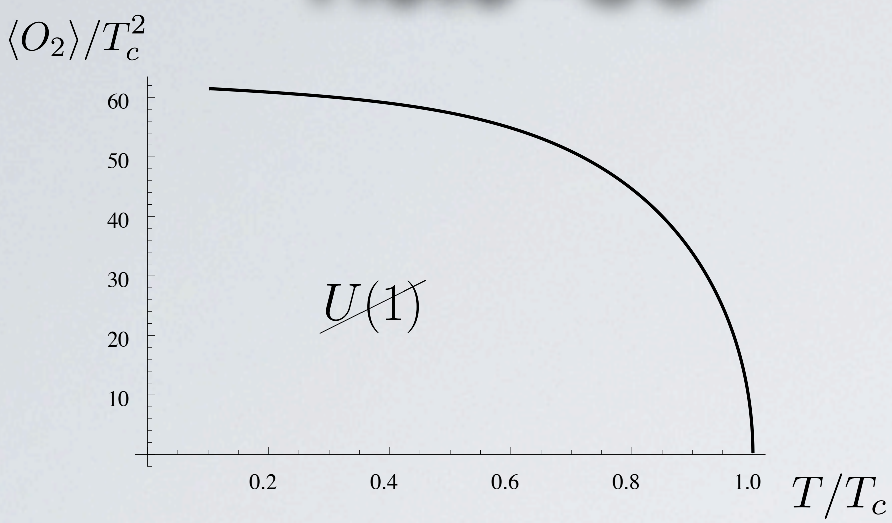
Charge density

$$S_k = \frac{1}{k^{2\alpha-2}}$$



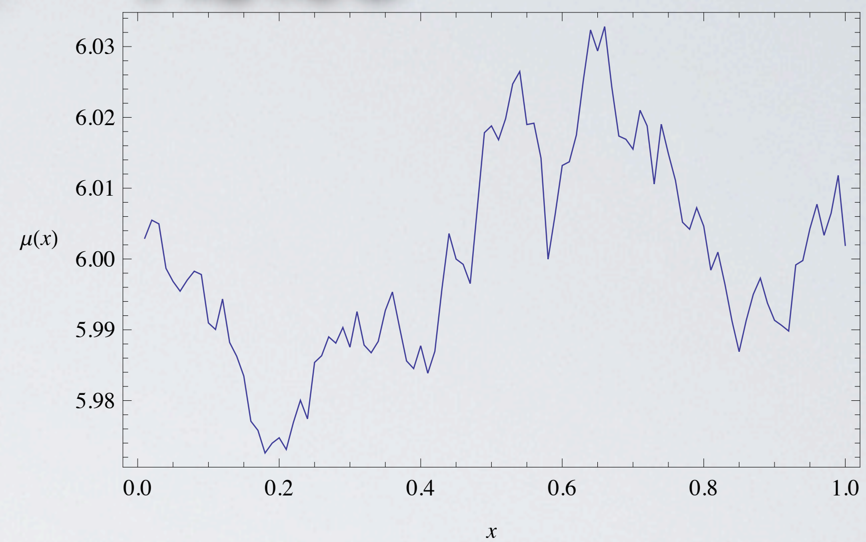
[also in brane intersections]

Holo-SC



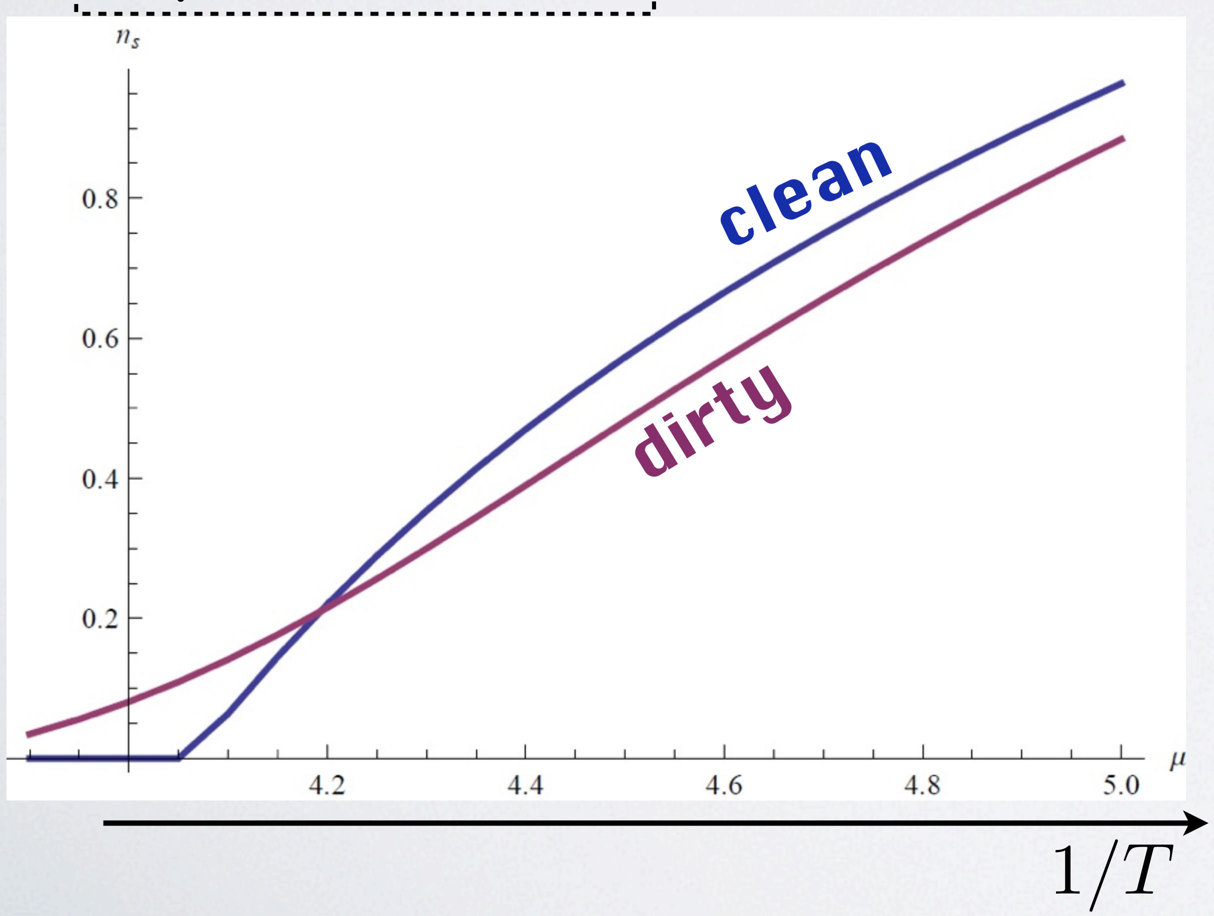
+

Noise $\mu(x)$



= ⋆ **Conductivities of disordered systems** [for branes too]

Superfluid density



'noise lowers the conductivity'

OUTLINE

> **Motivation:** Strong coupling, Disorder, Superconductors

> **Review:** Holographic Superconductors

> Dirty Holographic (p-wave) Superconductors

> **Results:** Phase diagram, spectrum, (some) noisy σ

> **Future:** Dirty Thin Films (islands of SC?), noisy σ, . . .

> Challenges in Condensed Matter:

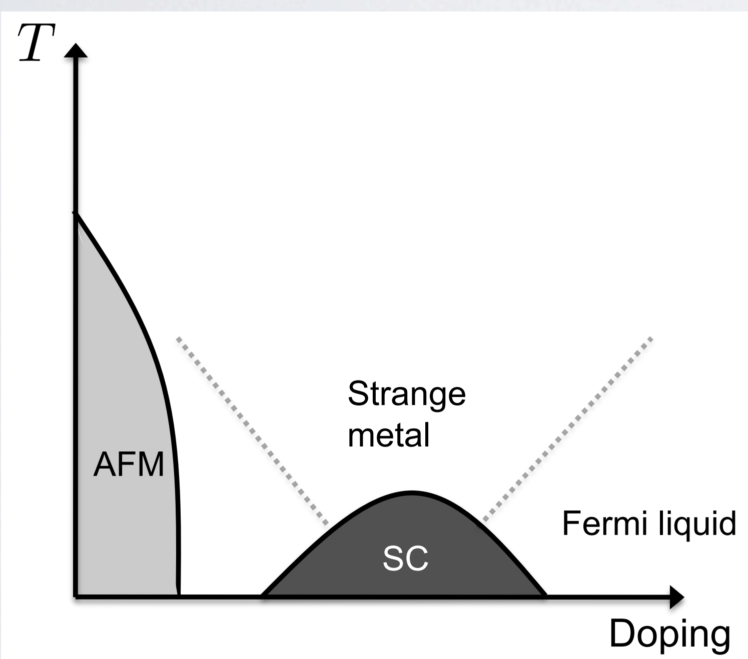
> Strong Coupling: High Tc Superconductors (strange metals), heavy fermions, ...

> Disorder + Interactions: Anderson localization in many body int. systems

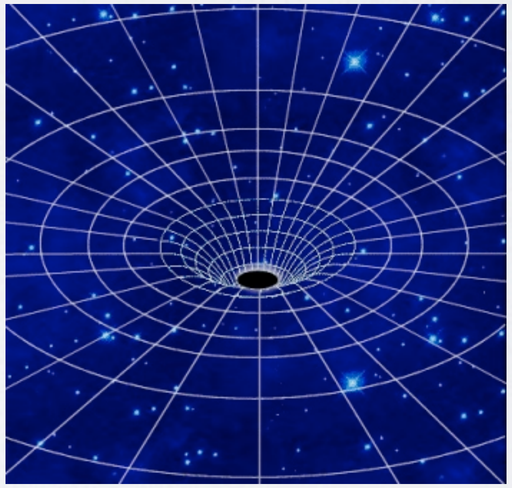
✦ **AdS/CFT** weak / strong coupling duality

> High Tc Superconductors

['gravities' + matter in ~ AdS]



$\sim$

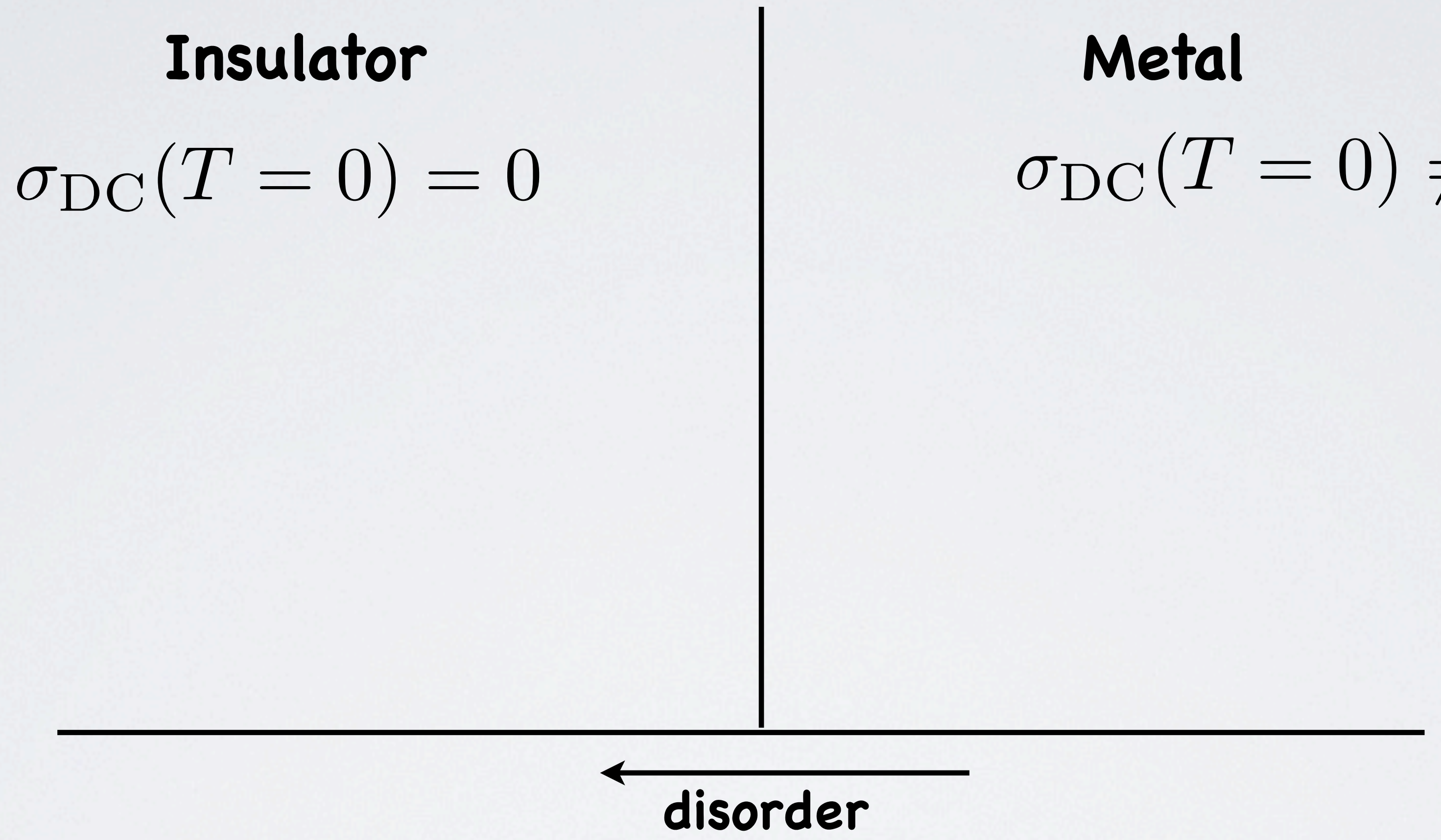


- Superconducting phase → ~~BCS~~
- Strange metal → Non-Fermi liquid

> Black holes with hair, domain wall geometries, electron stars . . .

> Disorder and interactions

> Anderson Localization '58: disorder suppresses conductivity

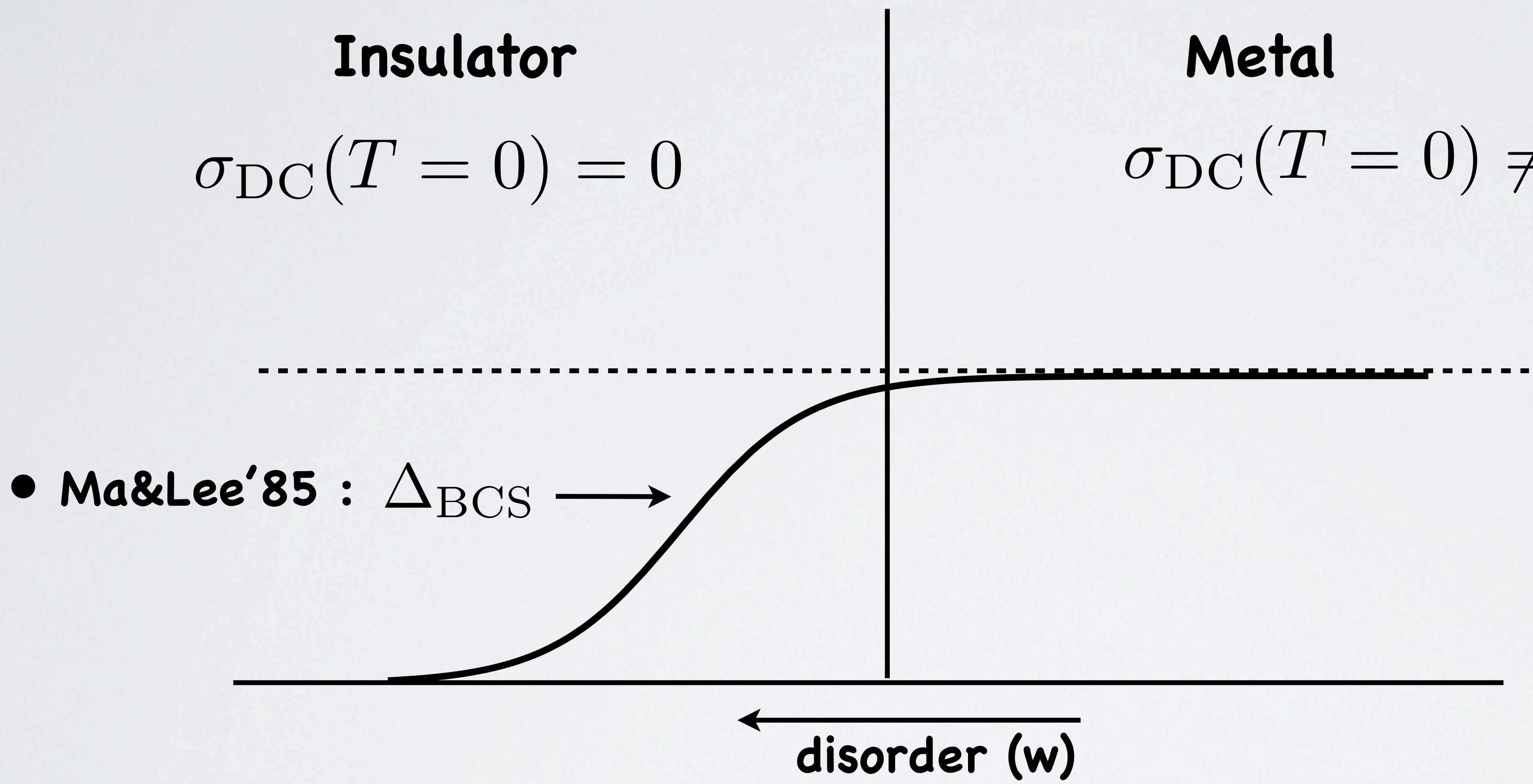


Disorder and superconductors?

> Disorder and interactions

> Anderson Localization '58: disorder suppresses conductivity

> Disorder and superconductors?



- **Anderson Theorem**

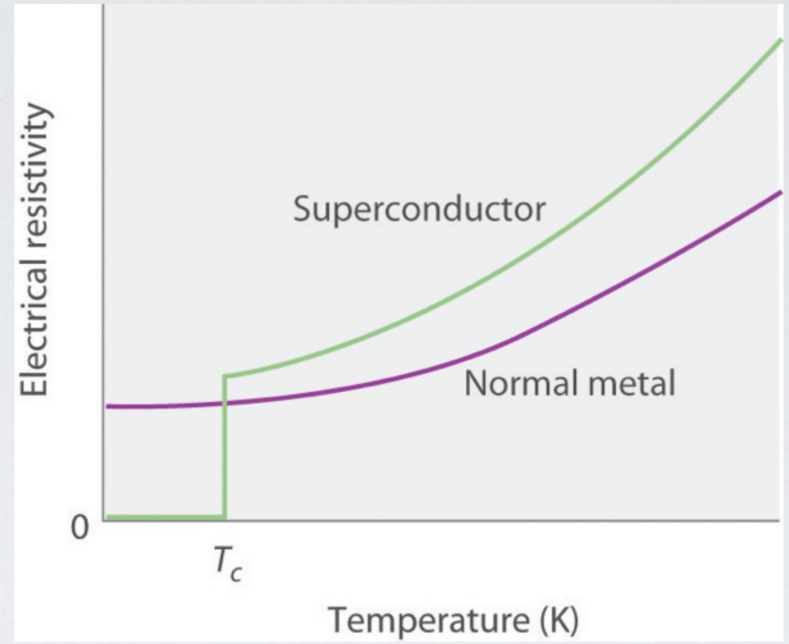
  $\Delta \neq \Delta(r,w)$

> Disorder + many body interacting system -> difficult! (see cond-mat/0506617)

→ **AdS/CFT ?** ⟶ **Dirty Holographic Superconductors !**

★ Holographic Superconductors (Hartnoll, Herzog, Horowitz,'08)

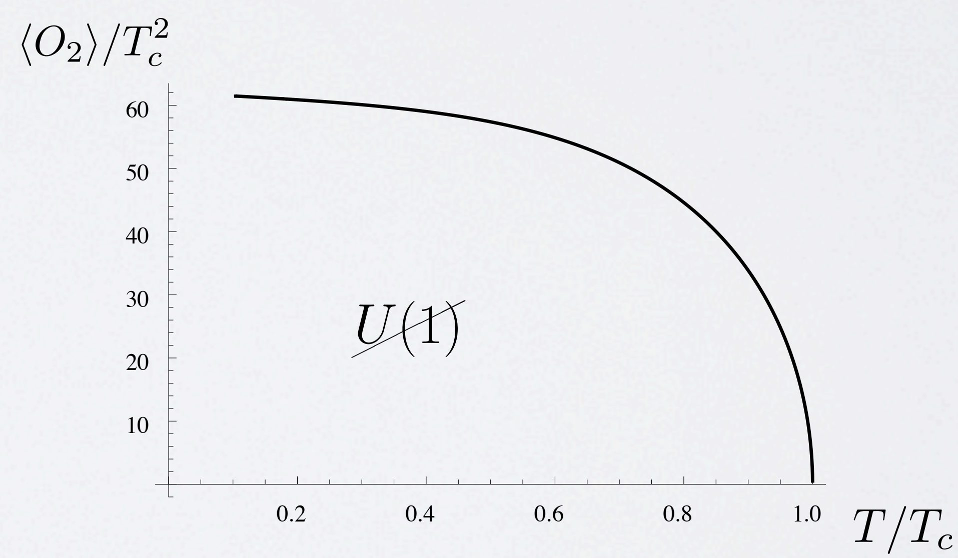
- **Black Hole gets hair** $\sim$



$\langle \cancel{U(1)} \rangle$

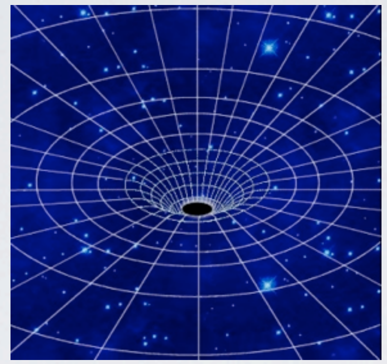
◆ **Charged BH unstable against scalar condensation (Gubser'08)**

$\sim$



> Holographic p-wave Superconductor (Gubser'08)

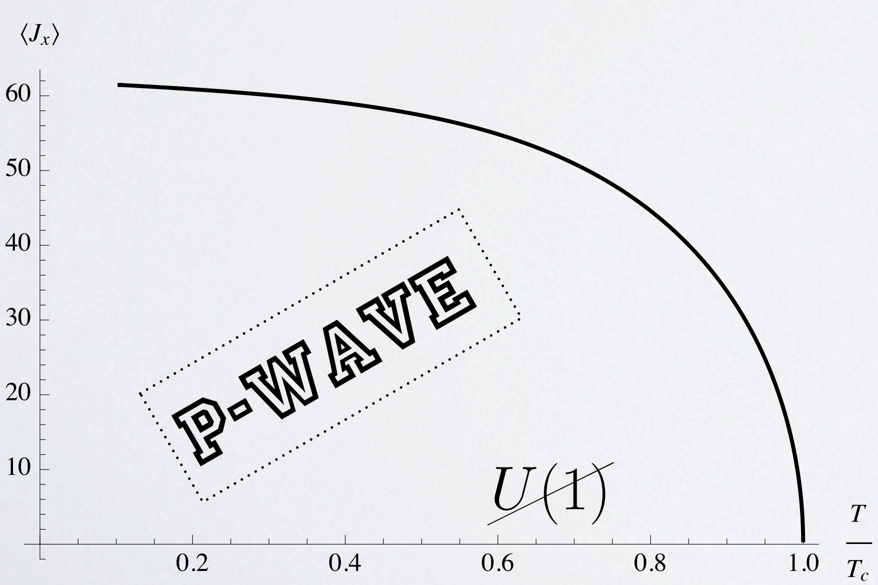
AdS$_4$

SU(2) $F_{ab} F^{ab}$ in  $\sim 2+1\ \mathrm{CFT}\ (T \neq 0)$

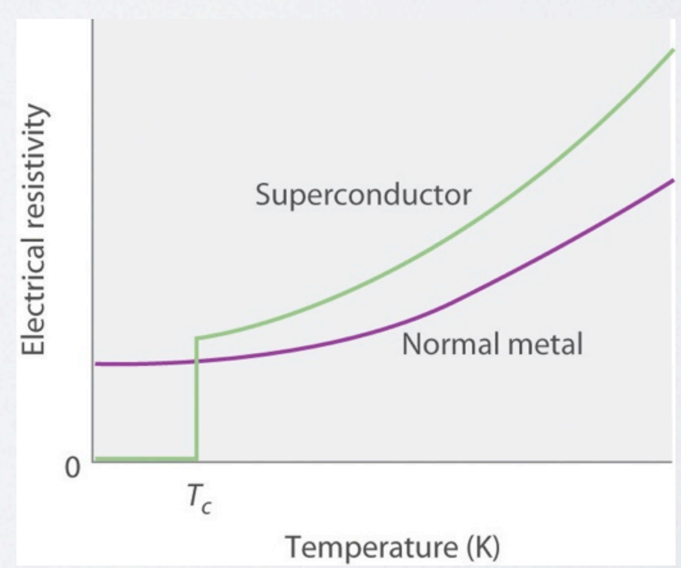
with

chemical potential // U(1) $\subset$ SU(2) $A^3_t(z) \sim \mu \quad [SU(2) \longrightarrow U(1)]$

p-wave condensate $A^1_x(z) \sim \langle \mathcal{J}^1_x \rangle \quad [\langle \cancel{U(1)} \rangle]$ **[➥ ~~rotational invariance~~]**



$\sim$

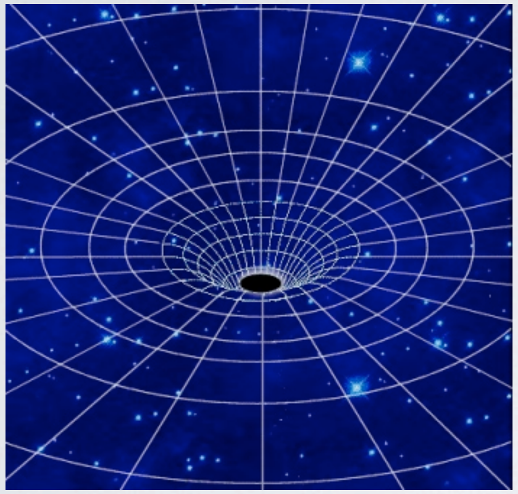


Adding Impurities!

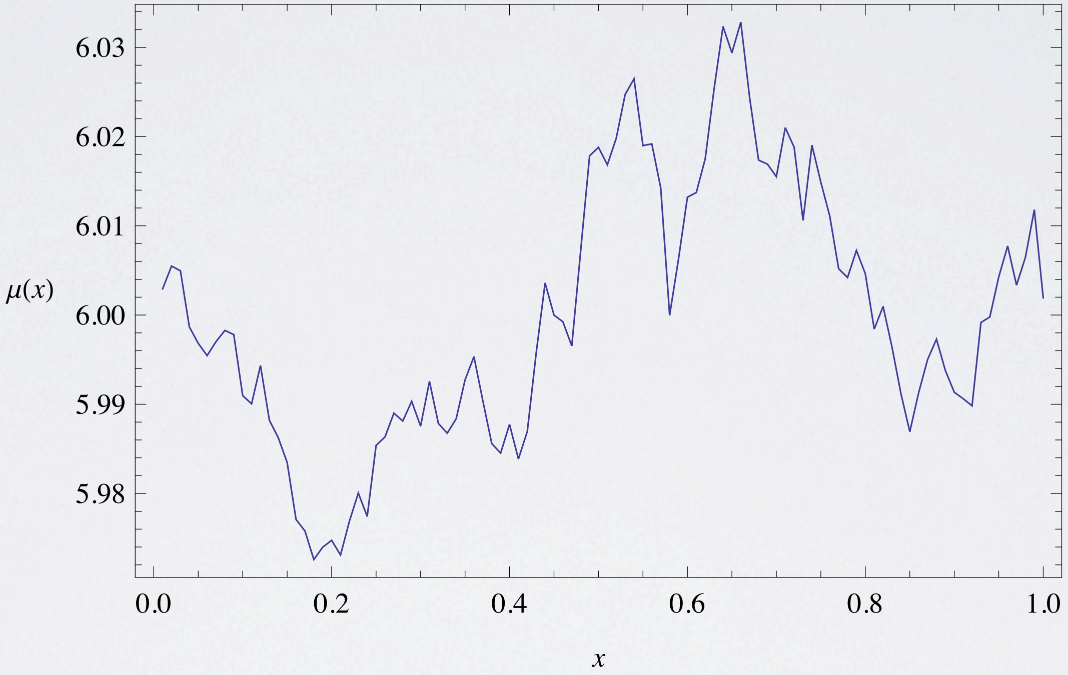
> 2+1 Holographic Superconductors + Noisy chemical potential $\mu = \mu(x)$

Disorder $\mu(x)$

p-wave

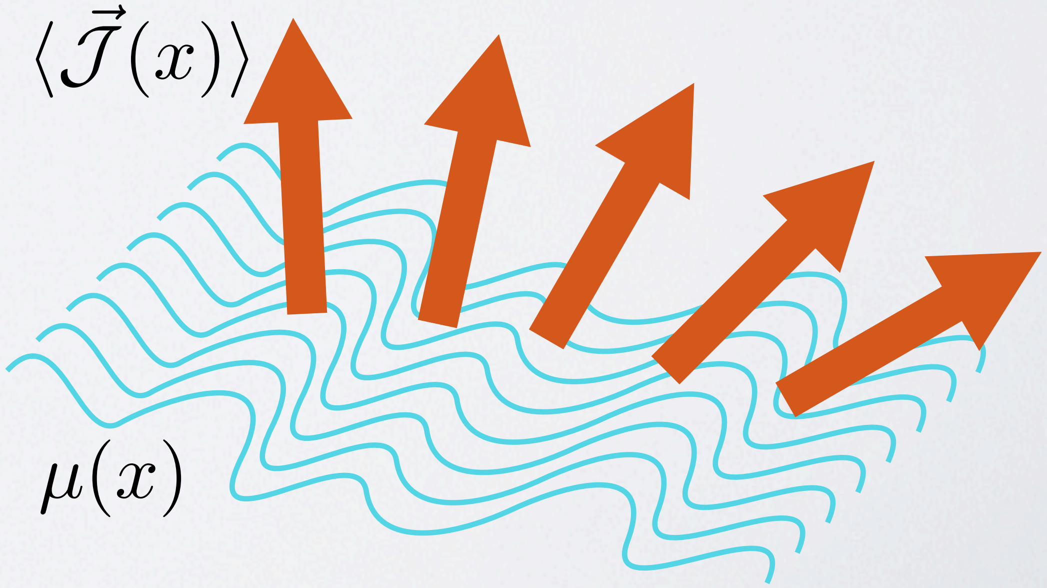


+



? Condensate
? Phase diagram
? Spectrum

➥ *p-wave condensate picks a direction?*



[./Tech Specs/pwave_1]

- **Action**

Probe Limit

$$S = \int d^4x \sqrt{-g} \left(\boxed{-\frac{1}{4} F^c_{\mu\nu} F_c^{\mu\nu}} + \frac{R}{\mathcal{K}} + \frac{6}{\mathcal{K} L^2} \right)$$

- **Field content**

$$\boxed{A_t^3(x,z) \sim \mu(x)} \qquad \boxed{\left(A_x^1(x,z), A_y^1(x,z)\right) \sim \left(\langle \mathcal{J}_x^1(x) \rangle, \langle \mathcal{J}_y^1(x) \rangle\right)}$$

$$\boxed{A_t^2(x,z)}$$

2nd 'charge density'

- **UV boundary conditions (z=0)**

$$A_t^3(x,z) = \mu(x) + \ldots$$
$$A_i^1(x,z) = \cancel{w_i^{(0)}}(x) + \langle \mathcal{J}_i^{(1)}(x) \rangle\, z + \ldots$$
$$A_t^2(x,z) = \cancel{\mu_2}(x) - \rho_2(x)\, z + \ldots$$

4 Coupled PDEs → Numerics

s-wave: 2 PDEs

Charged impurities >>> Noisy chemical potential

- Noise through random phases

Random phases

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k} \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha} \cos(k\,x + \delta_k)$$

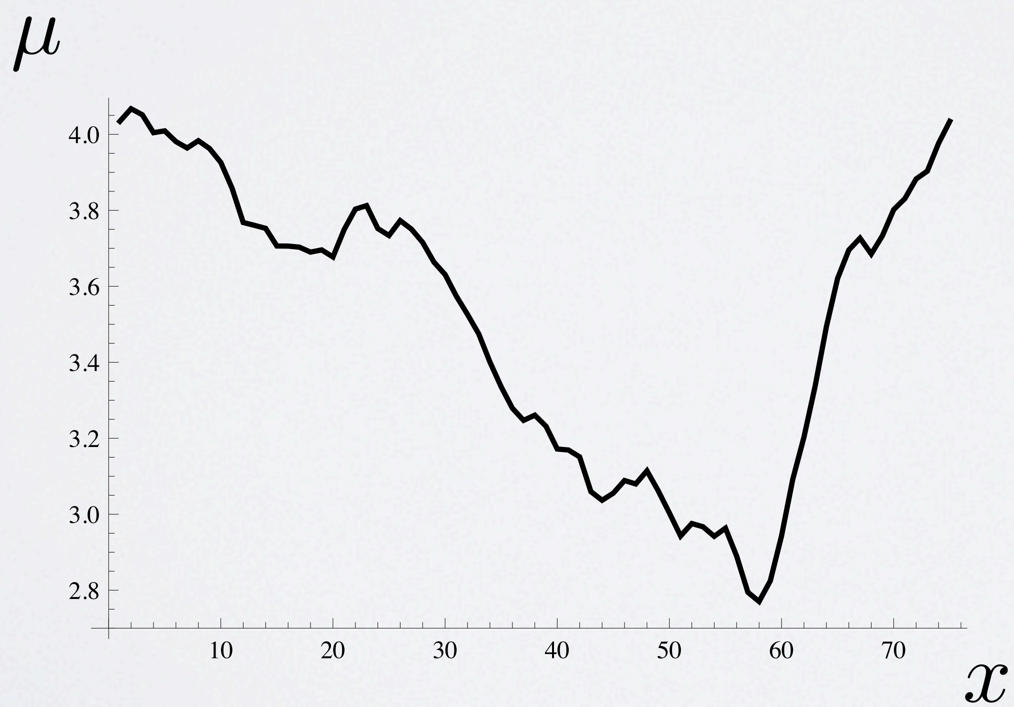
Power spectrum

Scales: $k_0\,,\ k_*\,,\ (\epsilon)$

Strength of noise $w = 25\epsilon/\mu_0$

[see also Scardicchio cond-mat/0505050]

- $\mu_0 = 3.50\,,\ \alpha = 1.50\,,\ w = 3.50 \quad [\mu_0 < \mu_c = 3.66]$



$L_x = 2\pi \rightarrow K_0 = 1$

$N_x = 75$

Charged impurities >>> Noisy chemical potential

- **Noise through random phases**

Random phases

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{\boxed{S_k}}\cos(k\,x + \boxed{\delta_k}) = \mu_0 + \boxed{\epsilon} \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\cos(k\,x + \delta_k)$$

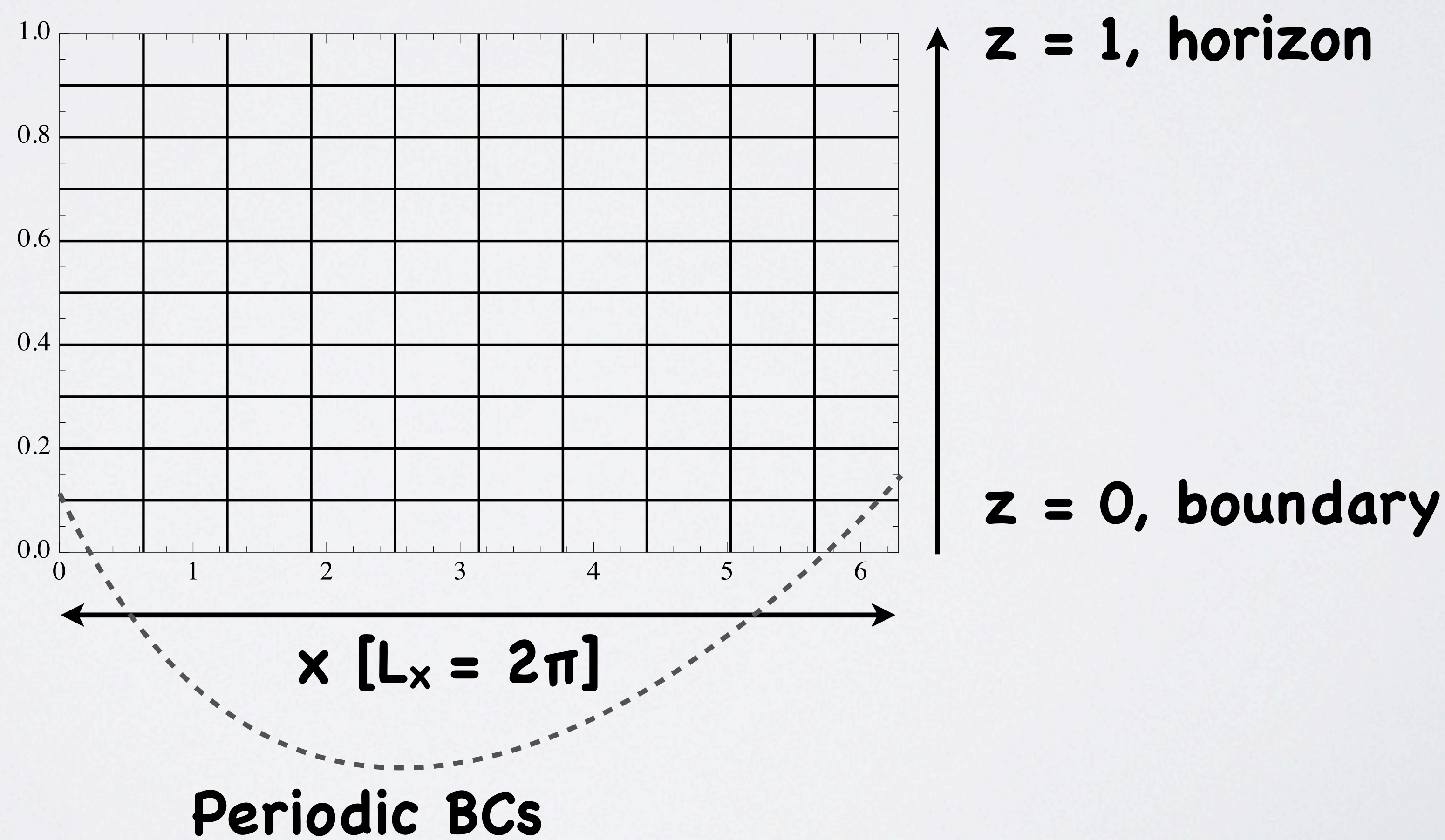
Power spectrum

Strength of noise $w = 25\epsilon/\mu_0$

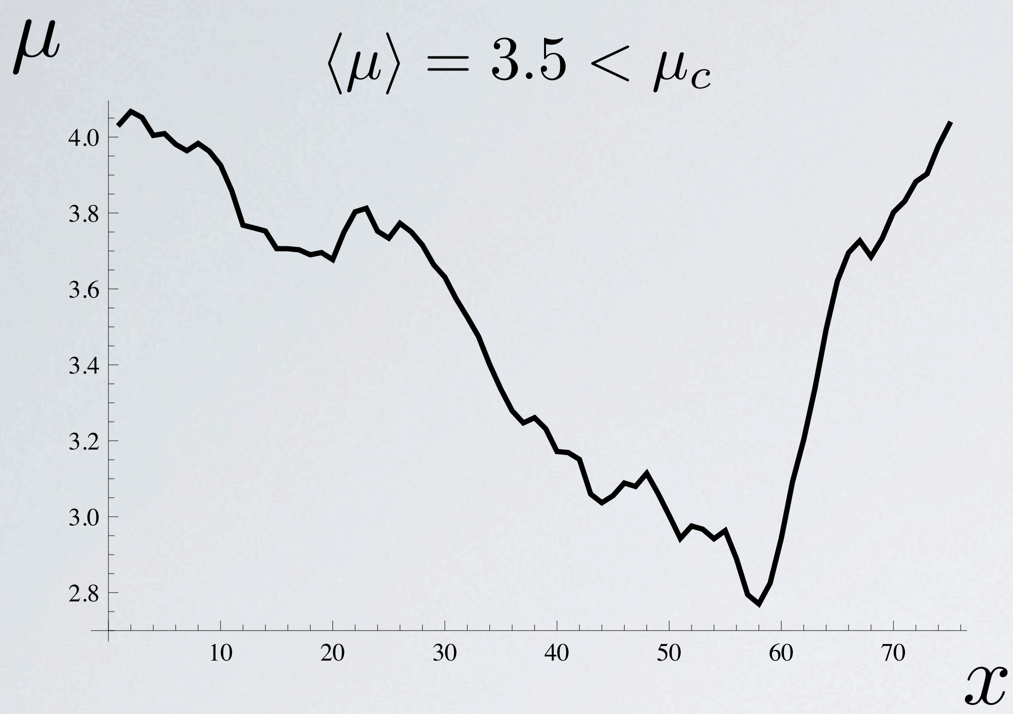
[see also Scardicchio cond-mat/0505050]

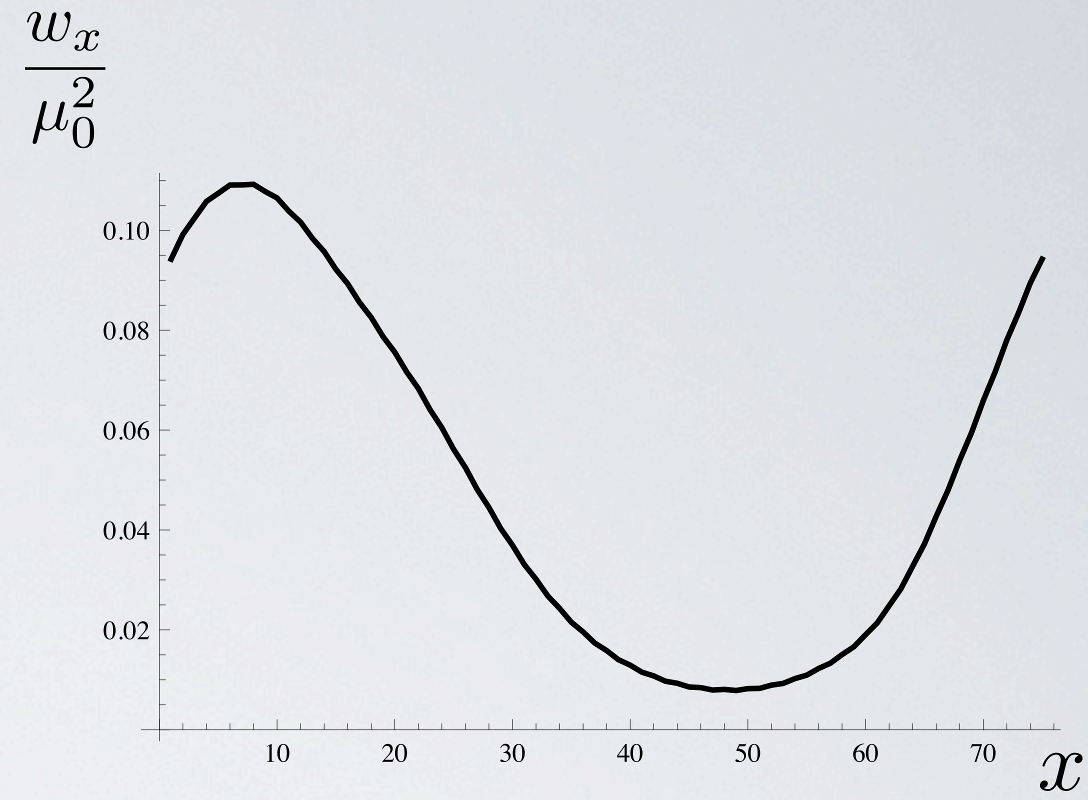
Scales: $k_0\,,\ k_*\,,\ (\epsilon)$

- **System on a grid**

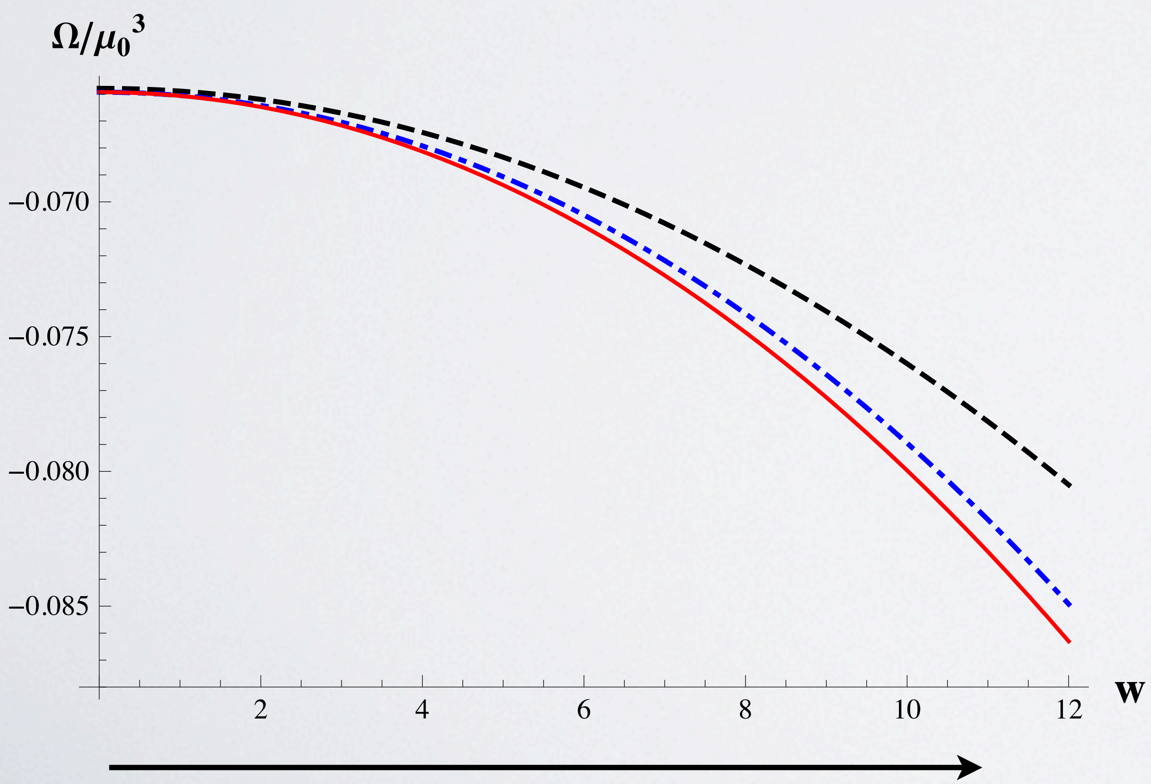


Condensate likes noise





> **Free energy of competing solutions**



disorder

➥ p-wave picks x:

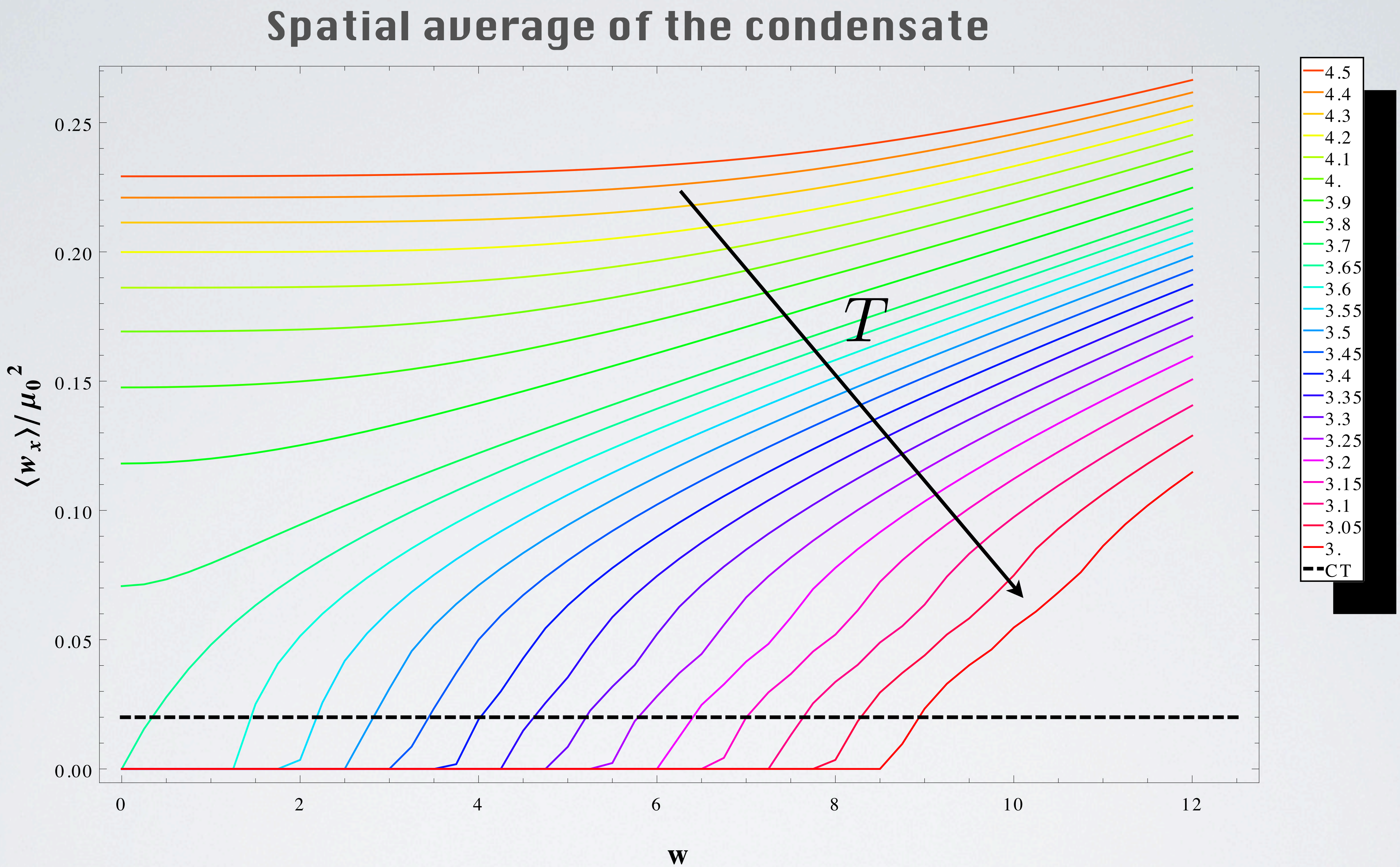
- - - - Normal phase

• • • • • • • Condensate ⊥ Noise

——— Condensate // Noise

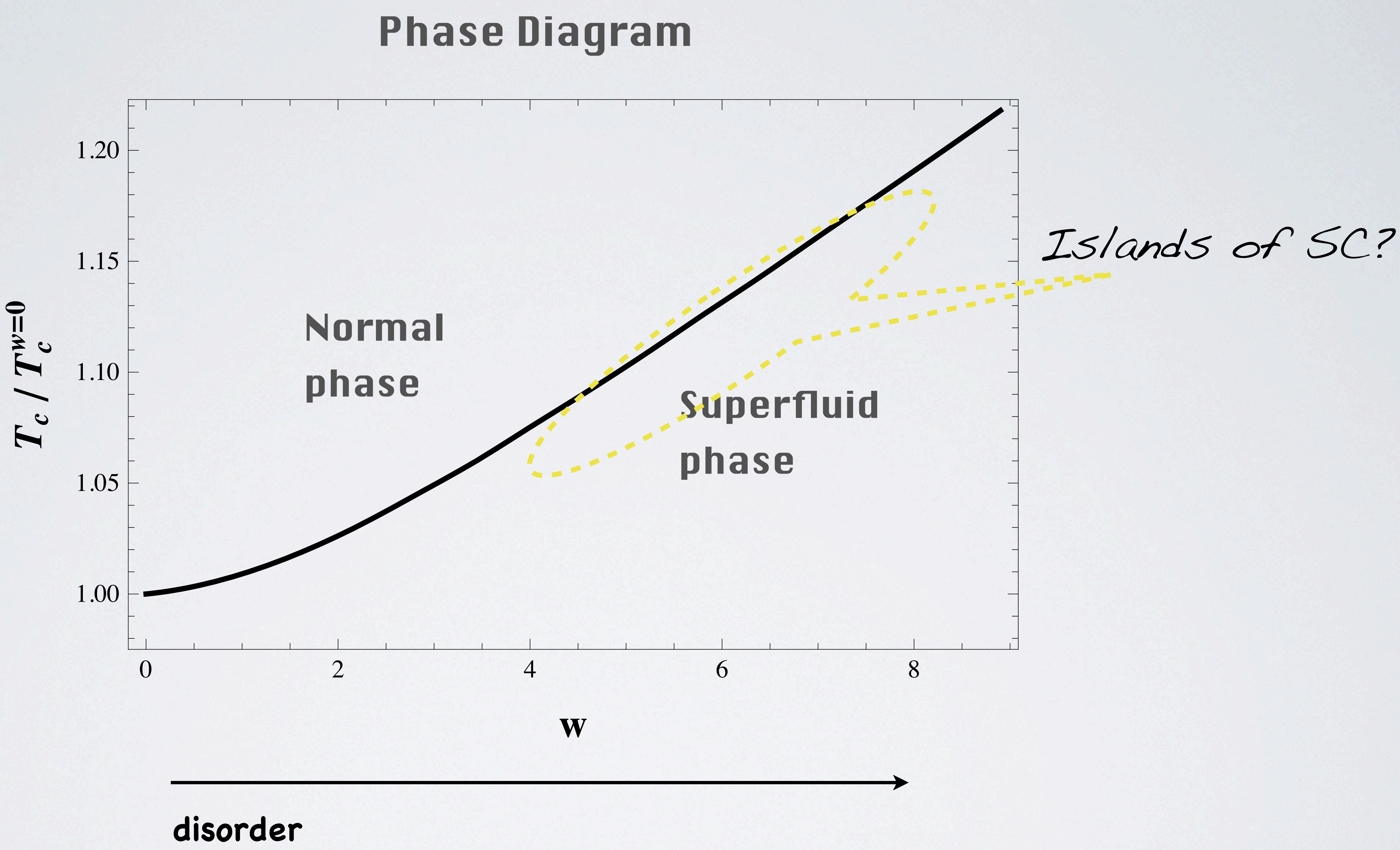
always wins!

★ Enhancement of SC



disorder

⋆ Enhancement of SC



Seen before in CM (hard-core bosons)

- 'Disorder-induced superfluidity', Dang et al, Phys. Rev. B 79, 214529

⋆ Spectrum 'renormalization'

>>> Noisy chemical potential

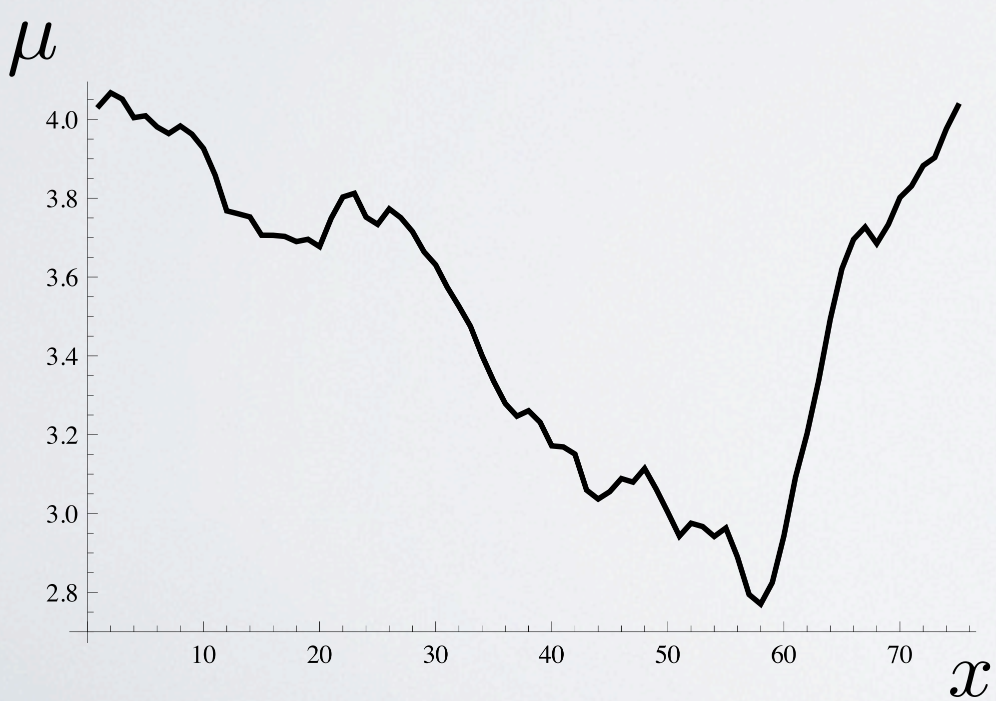
$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\cos(k\,x+\delta_k)$$

Random phases

Power spectrum

Strength of noise

>input spectrum $S_k = \frac{1}{k^{2\alpha}}$



>output spectra

Condensate

Charge density

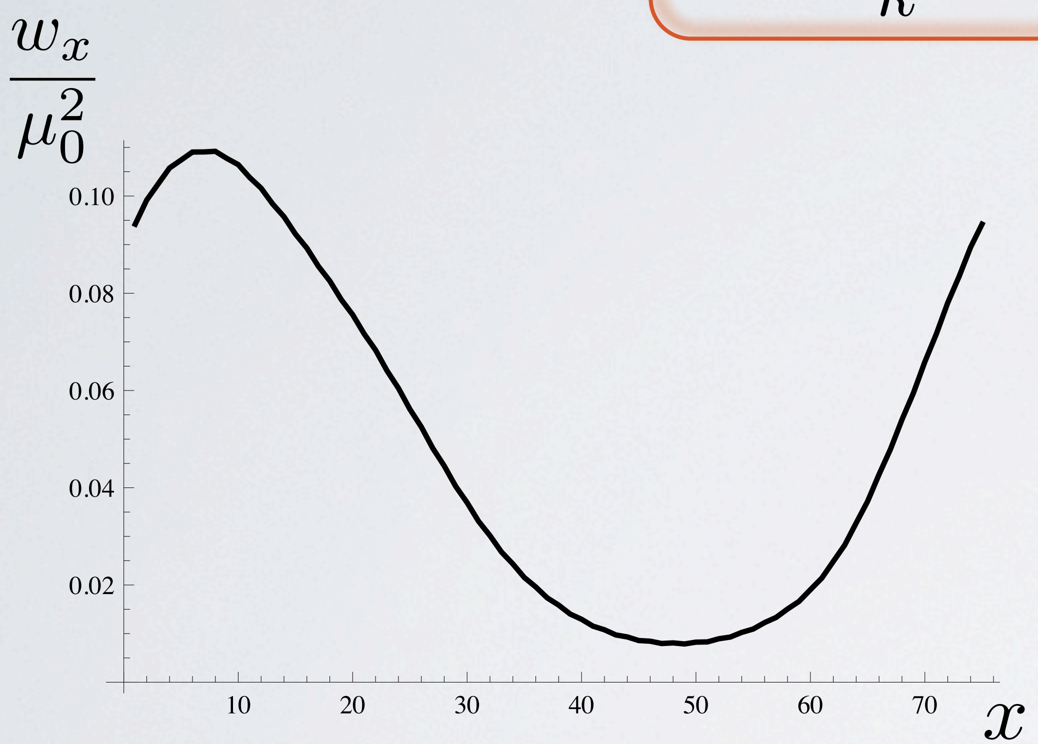
$$S_k = \frac{1}{k^\Gamma}$$?

⋆ Spectrum 'renormalization'

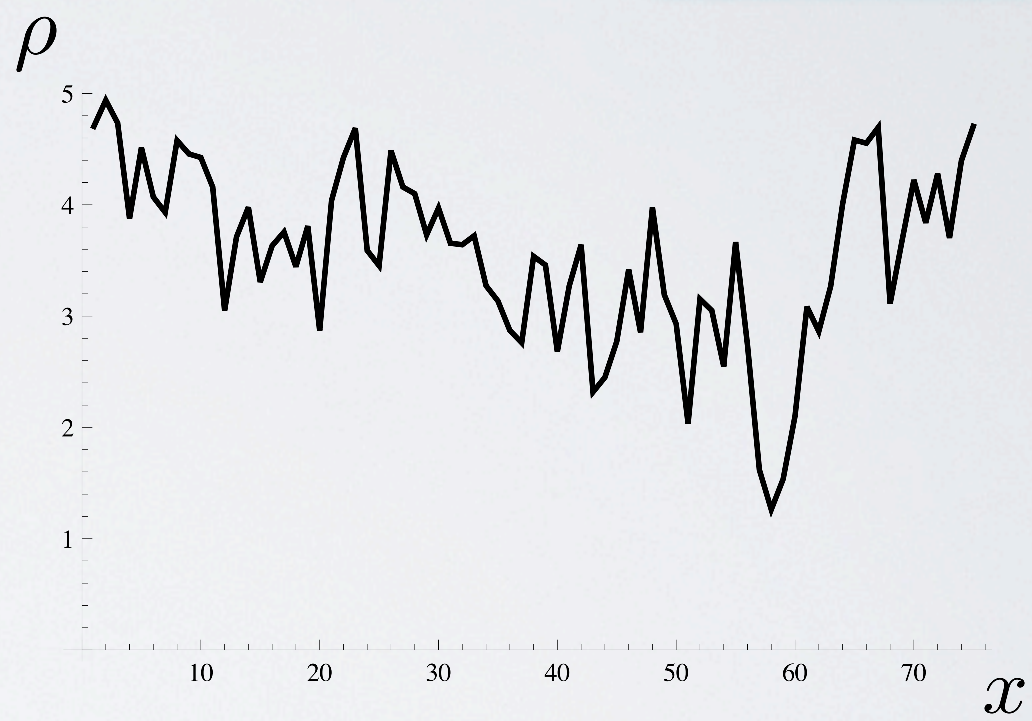
>input spectrum $\boxed{S_k = \frac{1}{k^{2\alpha}}}$

> OUTPUT

Condensate $\boxed{S_k = \frac{1}{k^{2\alpha+4}}}$



Charge density $\boxed{S_k = \frac{1}{k^{2\alpha-2}}}$



Hints of universality

- **S-wave** [1308.1920]
- [Hartnoll&Santos 1402.0872]
- **Fundamental matter (D3-D5)** [w/ M. Araújo, J. Lizana, I.S. Landea]
- **FT: noisy U(1) @ finite T** [D. Musso, I.S. Landea]

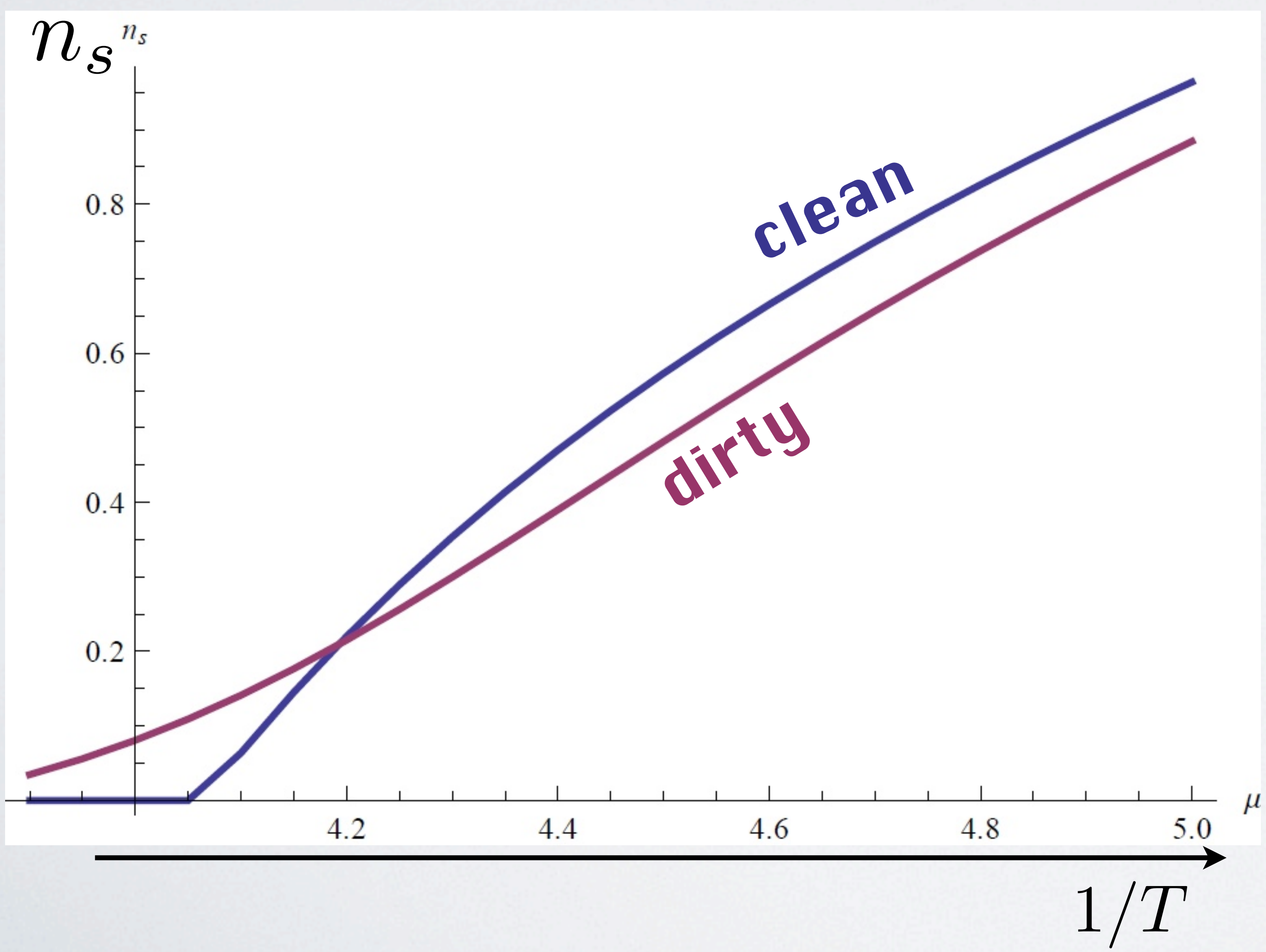
A taste of 'disordered conductivities'

[WORK IN PROGRESS!]
[see also Ryu et al 1103.6068]

- STUDY FLUCTUATIONS ($a_x \sim j_x$) [in the SC phase they'll see the noise, even in the probe limit]

- AVERAGED CONDUCTIVITY $\sigma_x(\omega) = \dfrac{\langle j_x(x,\omega)\rangle}{E_x(\omega)}$

- SC PHASE: $\sigma_{DC} \to \infty$. SUPERFLUID DENSITY n_s : $\sigma_x \approx n_s \left(\pi\,\delta(\omega) + \dfrac{i}{\omega}\right)$



>s-wave holo SC
>fixed noise strength

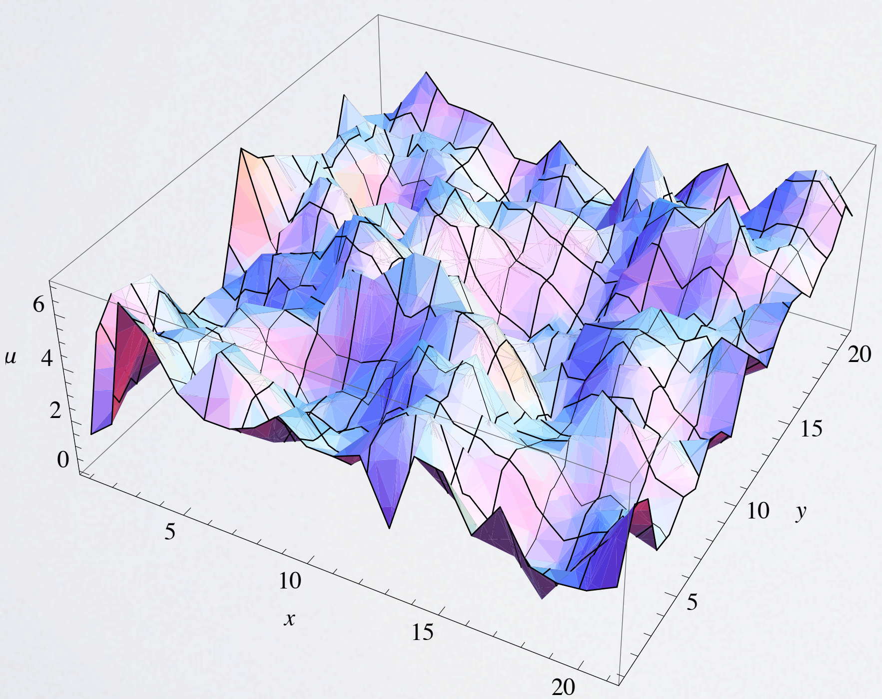
> FUTURE & ONGOING

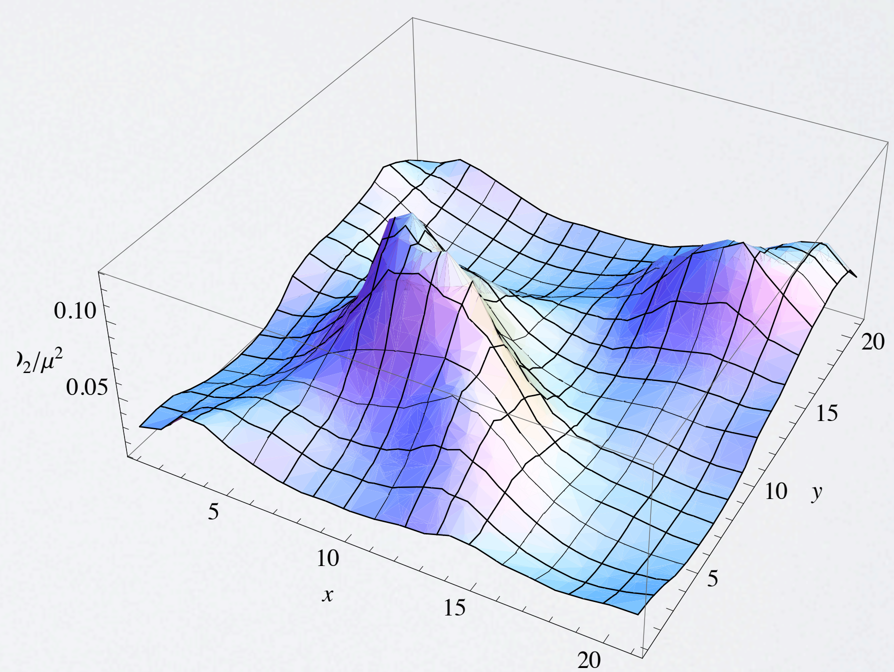
>Disordered holo SCs: both s- and p-wave ✔

>Enhancement of SC & ‘spectrum renormalization’ ✔
(thermo limit OK)

>Conductivity of disordered strongly coupled systems [....%%%%]

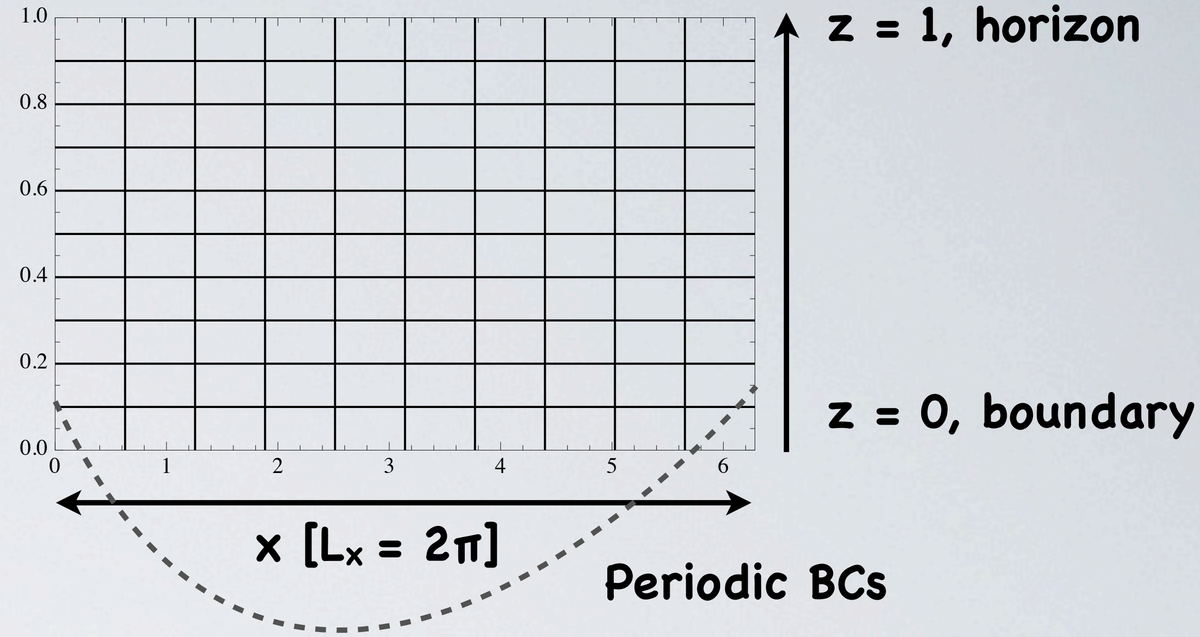
> Dirty Thin Films (islands of superfluidity?) [....%%%%]





> Noisy chemical potential

- GRID

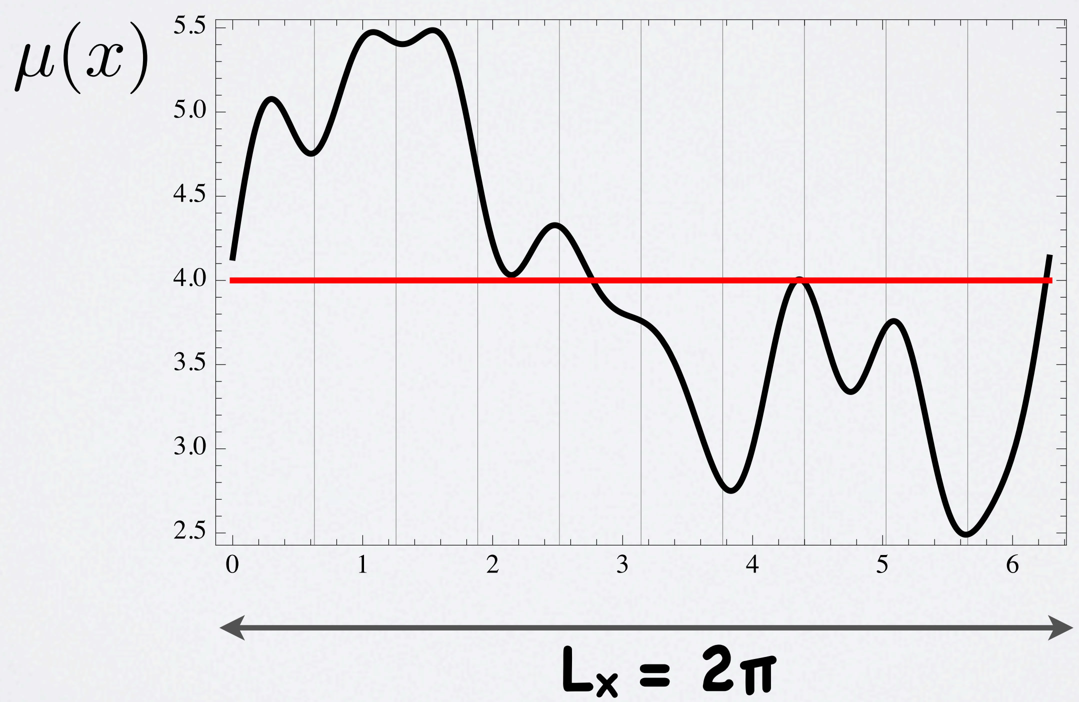


Random phases

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^{\alpha}}\cos(k\,x + \delta_k)$$

Power spectrum

Strength of noise



> Noisy chemical potential

- **Noise through random phases**

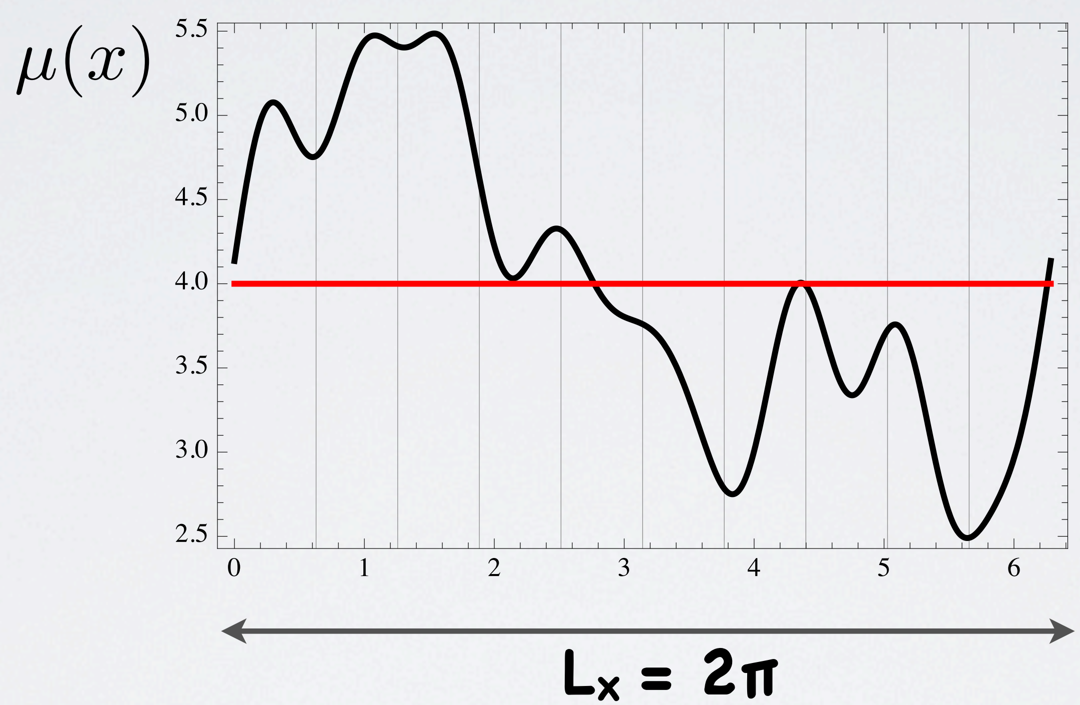
Random phases

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\cos(k\,x + \delta_k)$$

Power spectrum

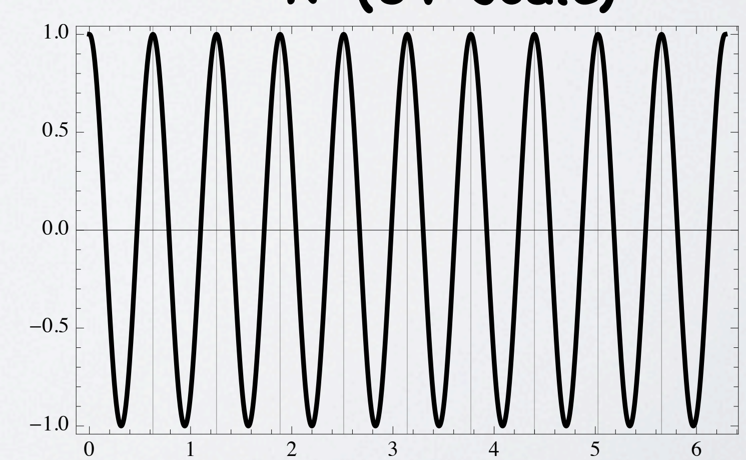
Strength of noise

- **System on a grid**



K_0 (IR scale)

K_* (UV scale)

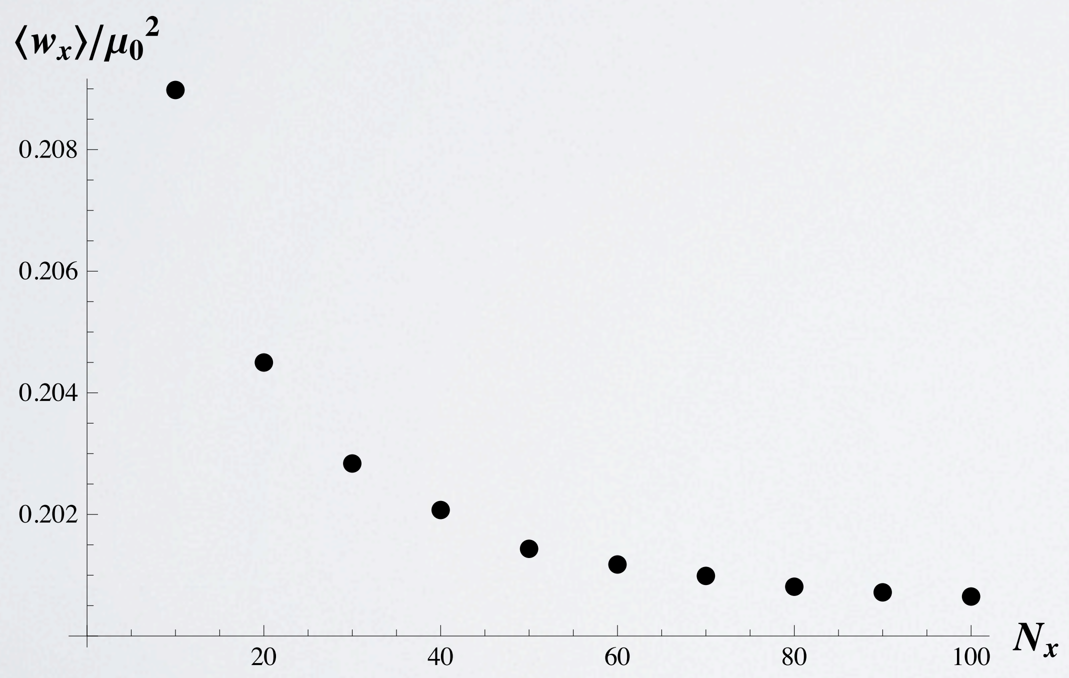


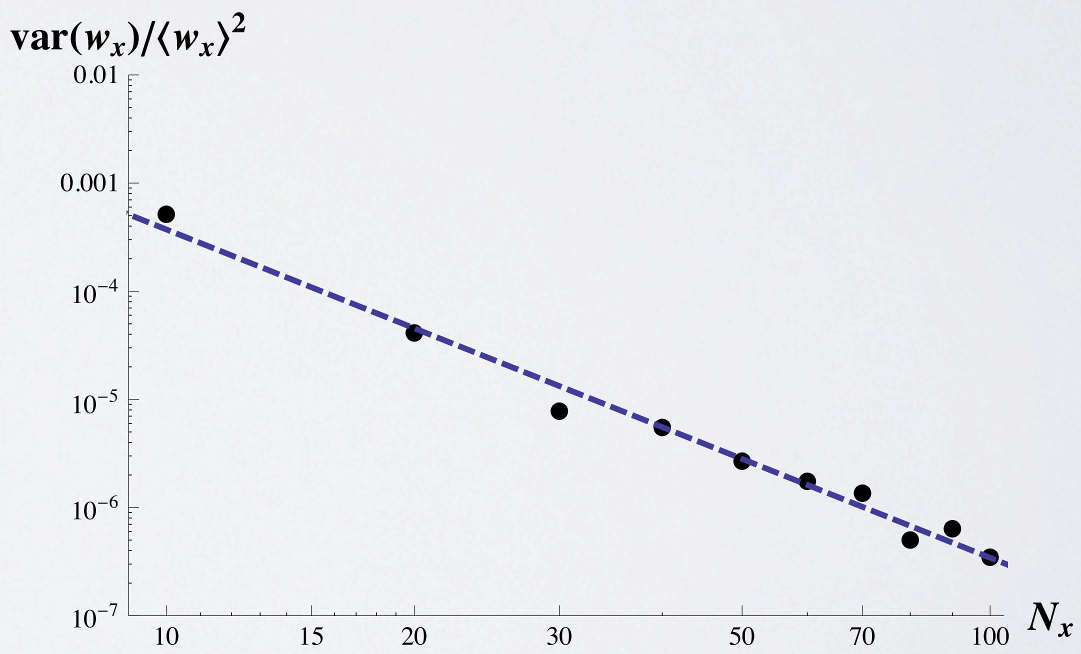
Thermodynamic limit

- **Thermo limit: Noise correlation length << System length**
  - Flat spectrum noise: correlation length $\propto$ 1 / (grid size)

- **Condensate and Charge density are self-averaging in the thermo limit:**
  - X_n is self-averaging when $$\frac{< X_n^2 > - < X_n >^2}{< X_n >^2} \to 0$$

Condensate





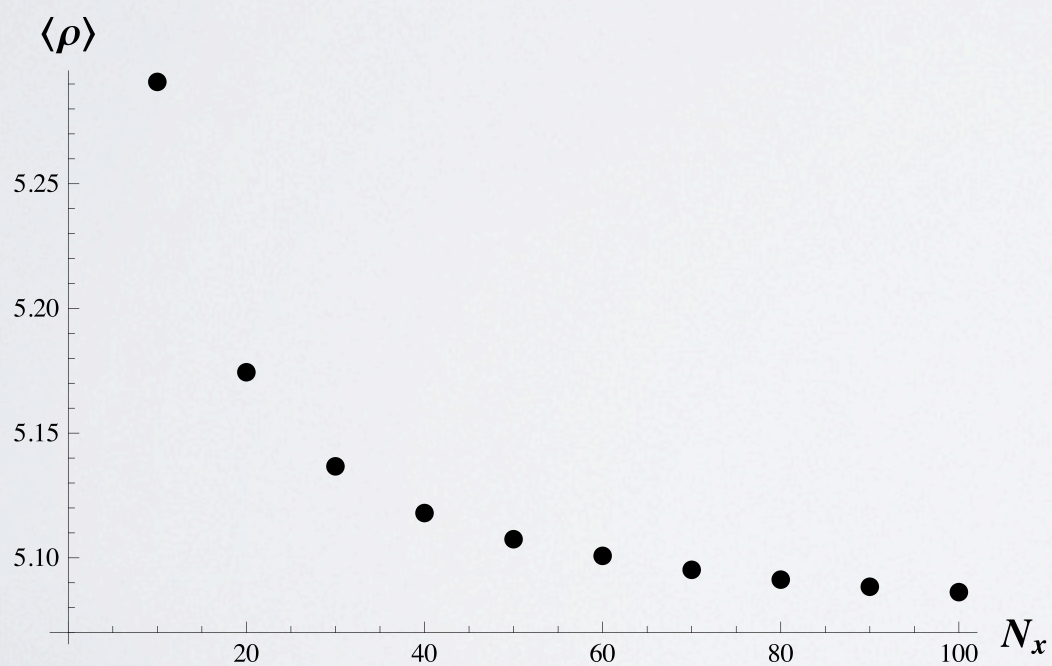
$$\boxed{\log(\mathrm{var}(w_x)/\langle w_x \rangle^2) = -0.90 - 3.03\,\log(N_x)}$$

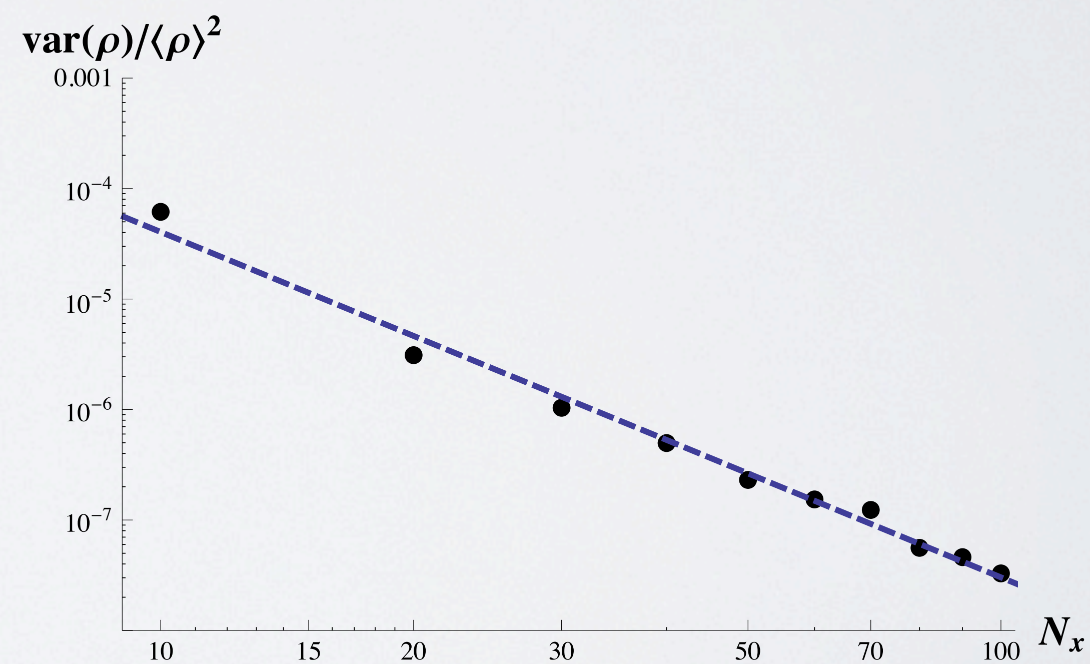
> Thermodynamic limit

- **Thermo limit: Noise correlation length << System length**
  - > Flat spectrum noise: correlation length $\propto$ 1 / (grid size)

- **Condensate and Charge density are self-averaging in the thermo limit:**
  - > X_n is self-averaging when $$\frac{< X_n^2 > - < X_n >^2}{< X_n >^2} \to 0$$

Charge density





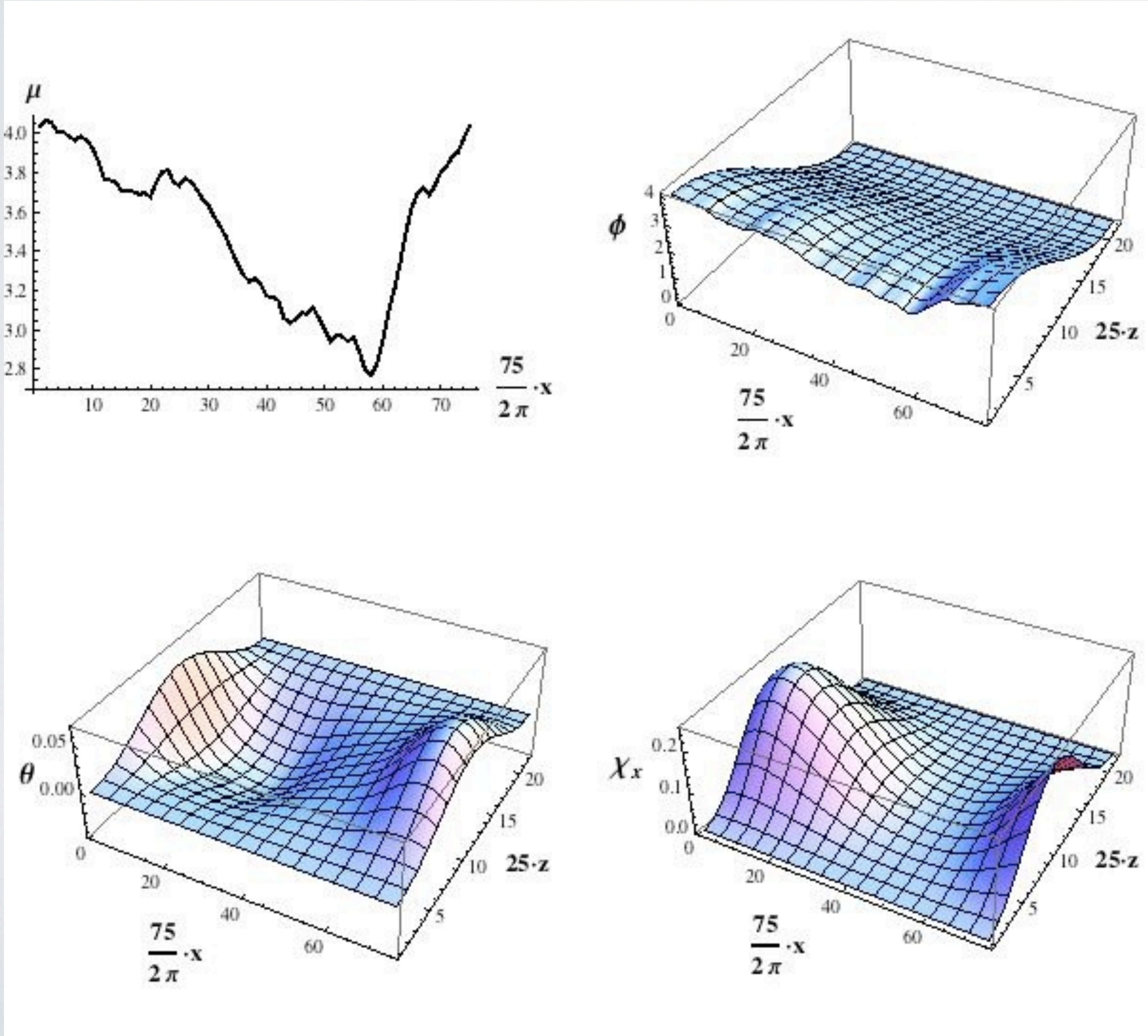
$$\boxed{\log(\mathrm{var}(\rho)/\langle\rho\rangle^2) = -2.92 - 3.13\ \log(N_x)}$$

> **Simulation #1**

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\,\cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\,\cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

- $\mu_0 = 3.50\,,\ \alpha = 1.50\,,\ w = 3.50 \quad [\mu_0 < \mu_c = 3.66]$



$L_x = 2\pi \rightarrow K_0 = 1$

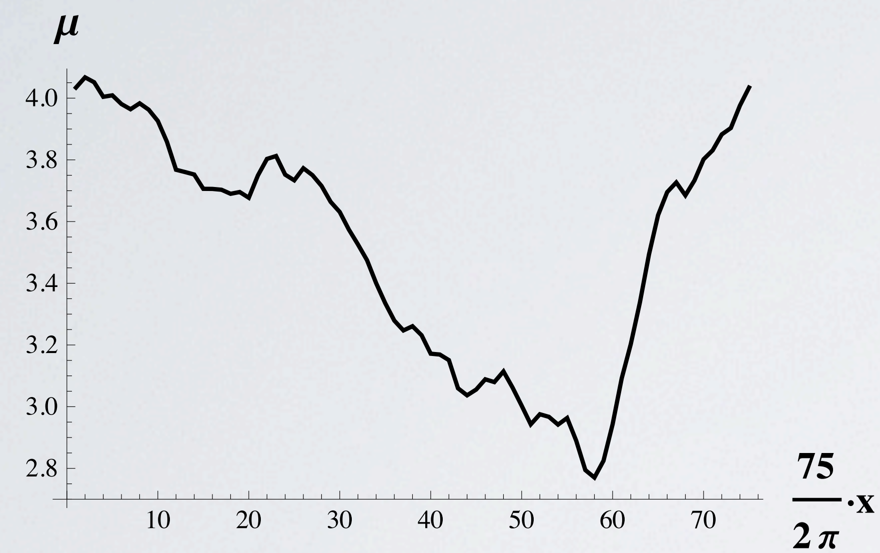
$N_z \times N_x = 25 \times 75$

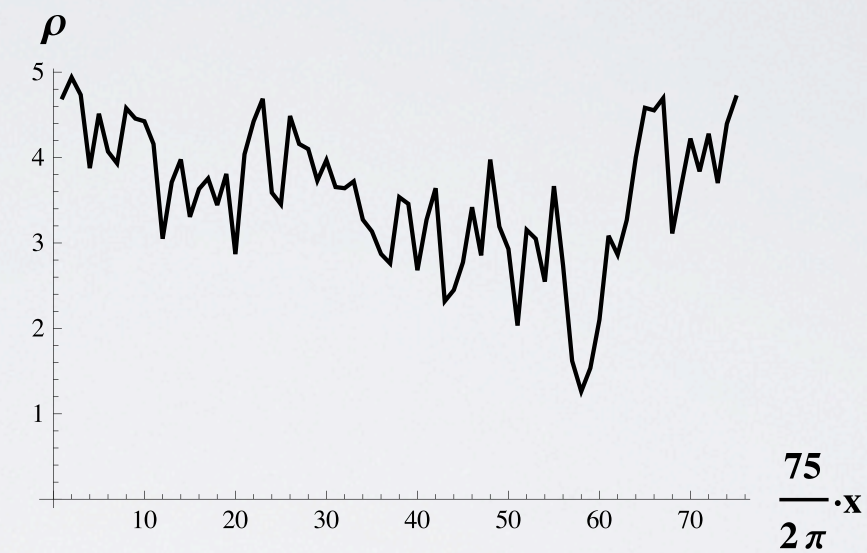
> Simulation #1

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

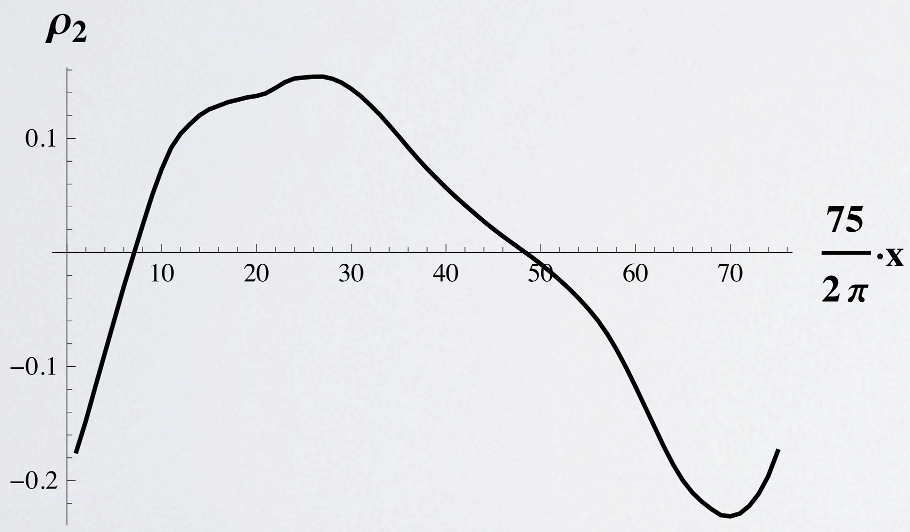
- $\mu_0 = 3.50\,,\ \alpha = 1.50\,,\ w = 3.50 \quad [\mu_0 < \mu_c = 3.66]$

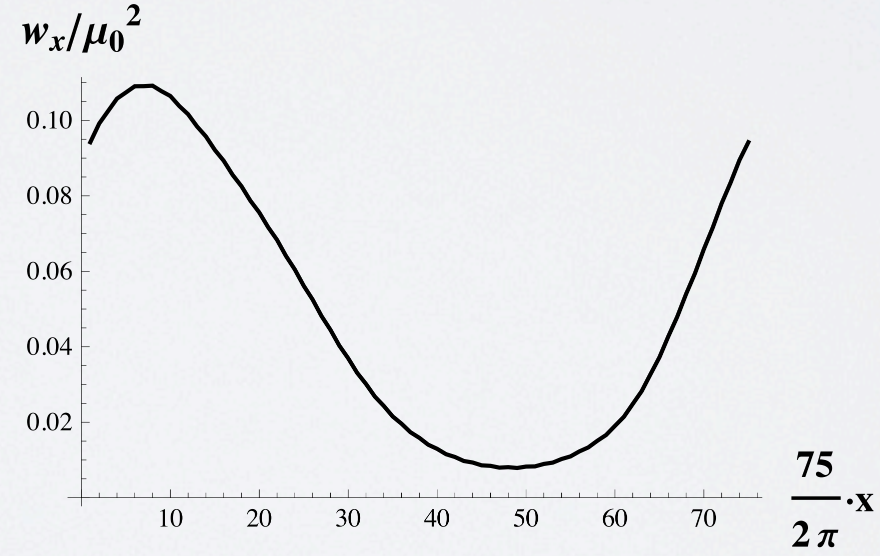




$L_x = 2\pi \rightarrow K_0 = 1$

$N_z \times N_x = 25 \times 75$





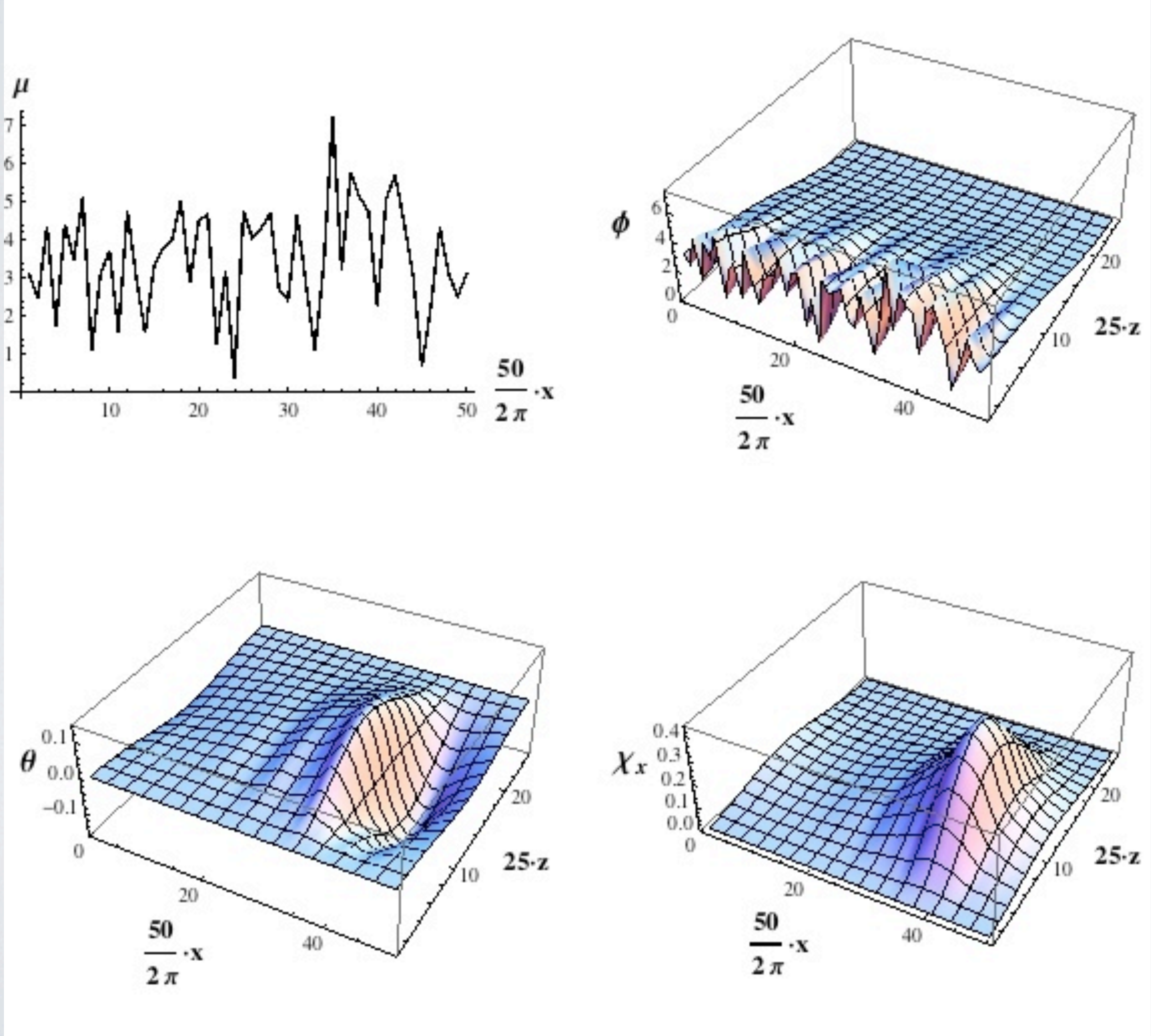
> **Simulation #2**

Flat Noise

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

- $\mu_0 = 3.50\,,\ \alpha = 0\,,\ w = 3.50 \quad [\mu_0 < \mu_c = 3.66]$



$L_x = 2\pi \rightarrow K_0 = 1$

$N_z \times N_x = 25 \times 75$

> Simulation #2

Flat Noise

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

- $\mu_0 = 3.50\,,\ \alpha = 0\,,\ w = 3.50\quad [\mu_0 < \mu_c = 3.66]$

$L_x = 2\pi \rightarrow K_0 = 1$

$N_z \times N_x = 25 \times 75$