Holographic Pomeron and Odderon at Strong Coupling

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Outline

1. Introduction, Pomeron and Odderon
2. Intercept & $\Delta(j)$ Curve
3. Pomeron and Odderon at Strong Coupling
4. Applications
5. Beyond Leading Order

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- Recently, there has been a renewed interest in both the Pomeron and the Odderon. These are the leading Regge trajectories with the quantum numbers of the vacuum.
- We can study these objects at both weak (using pQCD) and strong coupling (using gauge/string duality).
- The recent focus has been mostly on the question of what the intercept of these objects is. By calculating the intercepts to higher order from both the weak and the strong coupling side we can try to interpolate to the non-perturbative region from both sides. This can be a very important test of the gauge/string duality.
- Using both methods, the Odderon has two solutions, one fixed at 1 and one slightly below 1. An additional question for the Odderon is whether the first solution is exactly equal to one to all order.
- We will give an introduction to what the Pomeron and Odderon are, and show how they arise in string theory on AdS, focusing on the Odderon, and discuss possible applications and extensions.

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- Consider $2 \rightarrow 2$ scattering.
- Work in the Regge limit

$$s \gg t$$

- We can expand the amplitude into partial waves

$$A(s,t) = 16\pi \sum_{j=0}^{\infty} (2j+1) A_j(t) P_j(\cos\theta_t),$$

- In the Regge limit,

$$P_j\left(1+\frac{2s}{t}\right) \rightarrow \frac{\Gamma(2j+1)}{\Gamma^2(j+1)}\left(\frac{s}{2t}\right)^j \sim f(t)s^j.$$

- If exchanged particle has spin j

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- Optical theorem:

$$\sigma_{tot} = \frac{1}{s}\Im A(s,0)$$

- Experimentally

$$\sigma_{tot} \sim s^{0.08}$$

- The amplitude will depend on an infinite number of exchanged particles.
- We can continue the amplitude into the complex plane

$$A^{\pm}(j,t) = \begin{cases} A_j^+(t) & j\,even \\ A_j^-(t) & j\,odd \end{cases}$$

- $A(j,t)$ will have as singularities poles at integer j for fixed t. As we change t, the position of the pole will change, leading to a trajectory

$$j = \alpha(t)$$

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- After performing the Sommerfeld-Watson transform, we can show that the leading term at high energy behaves as

$$A^{\pm}(s,t) \sim (1 \pm e^{-i\pi\alpha^{\pm}(t)})\beta(t)(\frac{s}{s_0})^{\alpha^{\pm}(t)}.$$

- $\alpha(t)$ is the term with the largest value of $\Re\alpha_i(t)$
- Amplitude corresponds to an exchange of a whole trajectory of particles $\alpha^{\pm}(t)$.
- Equivalently, we are exchanging a 'Regge trajectory' - object with spin $\alpha^{\pm}(t)$.

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- This will give us a sum in powers of s. At high energy, we can keep just the leading term

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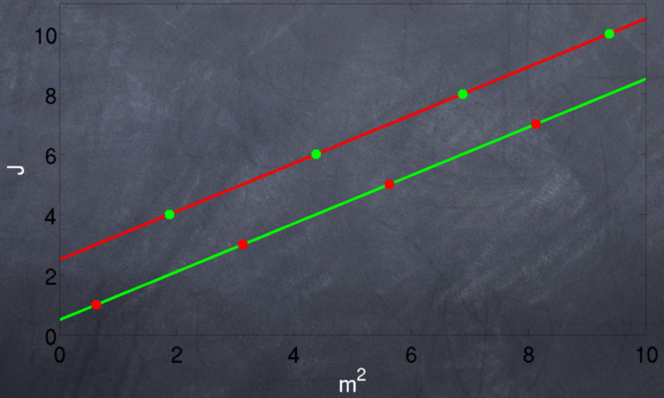

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Figure: Regge trajectories.

Pomeron

- Look again at the factor

$$1 \pm e^{-i\pi\alpha^{\pm}(t)}$$

- When $\alpha^{+}(t)$ is odd, $1 + e^{-i\pi\alpha^{+}(t)} = 0$, and similarly when $\alpha^{-}(t)$ is even, $1 - e^{-i\pi\alpha^{-}(t)} = 0$.
- Two sets of trajectories, one with only particles with even non-negative spin, and one with particles with odd positive spin.
- For trajectories with that don't have the quantum numbers of the vacuum, $\alpha(0) < 1$, leading to vanishing σ_{tot}
- The leading Reggeon which has the quantum numbers of the vacuum, $C = +1$ and $I = 0$, is known as the Pomeron.
- The intercept $\alpha(0) > 1$ leading to non-vanishing

$$\sigma_{tot} \sim s^{\alpha(0)-1}$$

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Odderon

- Similarly, the Odderon is the leading negative signature trajectory
- Its quantum numbers are

$$I = 0\,, \quad C = -1$$

- Its intercept is either

$$\alpha(0) = 1 \;\; or \;\; \alpha(0) < 1$$

- It is more elusive experimentally.
- We will revisit the Pomeron and Odderon from string theory.
- The Pomeron is very important - the leading exchange in total cross sections.

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Let us now examine the Regge limit in the context of conformal field theories [Cornalba, Costa, Penedones, 2007].

- Consider the connected component for a four point correlation function of primary operators $\mathcal{O}_i$ of dimension Δ_i.

$$A(x_i) = \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle_c = \frac{1}{(x_{12}^2)^{\Delta_1}(x_{34}^2)^{\Delta_3}} F(u,v)\,,$$

  where

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}\,, \qquad x_{ij} = x_i - x_j$$

  are the cross ratios and for simplicity we have assumed $\Delta_1 = \Delta_2$ and $\Delta_3 = \Delta_4$.

- The Regge limit corresponds to $u \to 0$ and $v \to 1$, with $(1-v)/\sqrt{u}$ fixed. Alternatively, defining $u = z\bar{z}$ and $v = (1-z)(1-\bar{z})$ with $z = \sigma e^{\rho}$ and $\bar{z} = \sigma e^{-\rho}$, the Regge limit can be specified by $\sigma \to 0$ for fixed ρ.

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- We can expand $F(u,v)$ in a conformal partial wave expansion:

$$F(u,v) = \sum_j \sum_\alpha C_{\alpha,j}^{(12),(34)} G(j,\Delta_\alpha(j);u,v)\,.$$

- We can express this in a double Mellin representation

$$F(u,v) = -\int_{-i\infty}^{i\infty} \frac{dj}{2\pi i} \frac{1 \pm e^{-i\pi j}}{\sin \pi j} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi}\, a(j,\nu)\, \mathcal{G}(j,\nu;u,v)\,.$$

- To recover the standard expansion, the contour in j is to stay to the right of singularities of $a(j,\nu)$, and the ν integral is done by closing the contour in the lower-half ν-plane picking up poles in $a(j,\nu)$ at $\nu(j) = -i(\Delta(j) - 2)$.
- The conformal harmonics, $\mathcal{G}(j,\nu;u,v)$, are eigen-functions of the quadratic Casimir operator of $SO(4,2)$, and in the Regge limit

$$\mathcal{G}(j,\nu;u,v) \sim \sigma^{1-j}\Omega_{i\nu}(\rho)\,, \qquad \Omega_{i\nu}(\rho) = \frac{1}{4\pi^2} \frac{\nu \sin(\nu\rho)}{\sinh \rho}\,.$$

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$$F(u,v) = \sum_j \sum_\alpha C^{(12),(34)}_{\alpha,j}\, G(j, \Delta_\alpha(j); u, v)\,.$$

- We can express this in a double Mellin representation

$$F(u,v) = -\int_{-i\infty}^{i\infty} \frac{dj}{2\pi i}\, \frac{1 \pm e^{-i\pi j}}{\sin \pi j} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi}\, a(j,\nu)\, \mathcal{G}(j,\nu; u,v)\,.$$

- To recover the standard expansion, the contour in j is to stay to the right of singularities of $a(j,\nu)$, and the ν integral is done by closing the contour in the lower-half ν-plane picking up poles in $a(j,\nu)$ at $\nu(j) = -i(\Delta(j) - 2)$.
- The conformal harmonics, $\mathcal{G}(j,\nu; u,v)$, are eigen-functions of the quadratic Casimir operator of $SO(4,2)$, and in the Regge limit

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- As an illustration, let us consider the $C = +1$ case and focus on the contribution from a single conformal Pomeron pole in ν^2,

$$a(j,\nu) = \frac{r(j)}{\nu^2 + (\Delta(j) - 2)^2},$$

characterized by a spectral curve $\Delta(j)$

- Closing the contour first in the lower-half ν-plane, leads to a single-Mellin representation (in the Regge limit)

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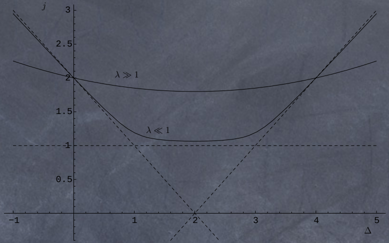

- We can expand j

$$j \approx j_0 + \frac{1}{2\sqrt{\lambda}}(\Delta - 2)^2 + \cdots$$

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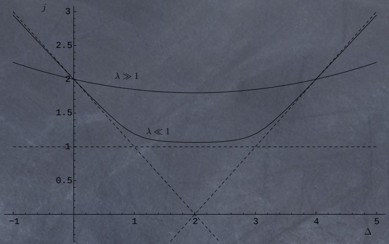

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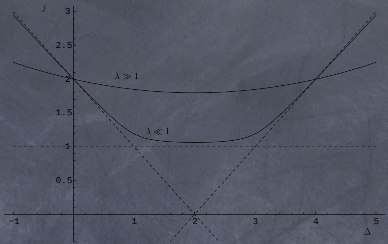

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- Note that by doing the analysis in momentum space we can see that

$$\sigma = \sqrt{u} \longleftrightarrow (zz's)^{-1}\,.$$

- Take home message of the first two sections: There is an analogy between $\Delta(j)$ and $\alpha(t)$ from the traditional Regge theory. In fact the intercept $\alpha(0) = j_0$, and this gives the leading behavior in cross sections. j_0 in the minimum of the $j(\Delta)$ curve and can be calculated by studying the $\Delta(j)$, and specifically by setting $\Delta = 2$.

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BPST Pomeron

At strong coupling the Pomeron was first introduced by Brower, Polchinski, Strassler and Tan, 2006.

- They show that the Pomeron emerges as the Regge trajectory of the graviton. We can introduce a vertex operator

$$\mathcal{V}_P(j,\pm) = (\partial X^{\pm}\bar{\partial}X^{\pm})^{\frac{j}{2}} e^{\mp ik\cdot X}\phi_{\pm j}(r).$$

- This operator must satisfy the on-shell condition.

$$[\frac{j-2}{2} - \frac{\alpha'}{4}\Delta_j]e^{\mp ik\cdot X}\phi_{\pm j}(r) = 0$$

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- They show how we can expand the differential operator to order $1/\sqrt{\lambda}$ around $j=2$

$$[j-2-\frac{\alpha' t}{2}e^{-2u}-\frac{1}{2\sqrt{\lambda}}(\partial_u^2-4)]\phi_\pm(u)=0$$

- From here we can obtain the intercept

$$j_0=2-\frac{2}{\sqrt{\lambda}}$$

- We can also use the above equation to calculate the propagator for Pomeron exchange

$$\chi(\tau,L)=(\cot(\frac{\pi\rho}{2})+i)g_0^2e^{(1-\rho)\tau}\frac{L}{\sinh L}\frac{\exp(\frac{-L^2}{\rho\tau})}{(\rho\tau)^{3/2}}$$

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  Weak coupling:

$$\mathcal{K}(k_\perp, k'_\perp, s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\log s}} e^{-(\log k_\perp - \log k'_\perp)^2/4\mathcal{D}\log s}$$

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- In AdS/CFT, the Odderon was identified as the Regge trajectory of the Kalb-Ramond $\tilde{B}_{\mu\nu}$ field [Brower, MD, Tan, 2008]

$$\mathcal{V}_O(j,\pm) = (\partial X^{\pm}\bar{\partial} X^{\perp} - \partial X^{\perp}\bar{\partial} X^{\pm})(\partial X^{\pm}\bar{\partial} X^{\pm})^{\frac{j-1}{2}} e^{\mp ik\cdot X}\phi_{\pm j\perp}(r).$$

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$$(\Box_{Maxwell} - (k+4)^2)\tilde{B}_{IJ}^{(1)} = 0\,, \quad (\Box_{Maxwell} - k^2)\tilde{B}_{IJ}^{(2)} = 0$$

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  where m_{AdS}^2 is either 16 or 0, depending on which of the two solutions we are considering.
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Summary

We can summarize both the strong and weak coupling results.

	Weak Coupling	Strong Coupling
$C=+1$	$j_0^{(+)} = 1 + (\ln 2)\,\lambda/\pi^2 + O(\lambda^2)$	$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$
$C=-1$	$j_{0,(1)}^{(-)} \simeq 1 - 0.24717\,\lambda/\pi + O(\lambda^2)$ $j_{0,(2)}^{(-)} = 1 + O(\lambda^3)$	$j_{0,(1)}^{(-)} = 1 - 8/\sqrt{\lambda} + O(1/\lambda)$ $j_{0,(2)}^{(-)} = 1 + O(1/\lambda)$

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We can apply these methods to calculate the amplitude for any process where Pomeron or Odderon exchange dominates.

- Eikonal approximation in AdS space (Brower, Strassler, Tan; Cornalba,Costa,Penedones)

$$A(s,t) = 2is \int d^2 l e^{-i\mathbf{l}_\perp \cdot \mathbf{q}_\perp} \int dz d\bar{z} \, P_{13}(z) P_{24}(\bar{z}) (1 - e^{i\chi(s,b,z,\bar{z})})$$

- Single Pomeron exchange would correspond to expanding the above to first order in χ.
- To study different processes, we just provide different wavefunction for the external states.
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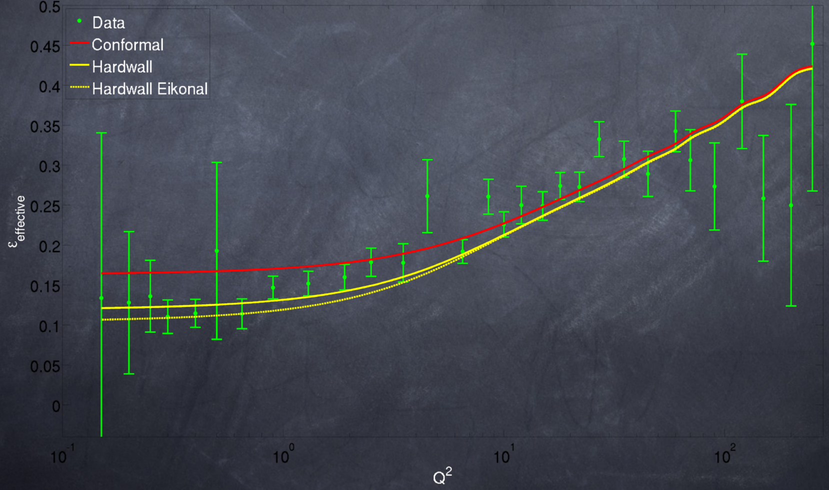
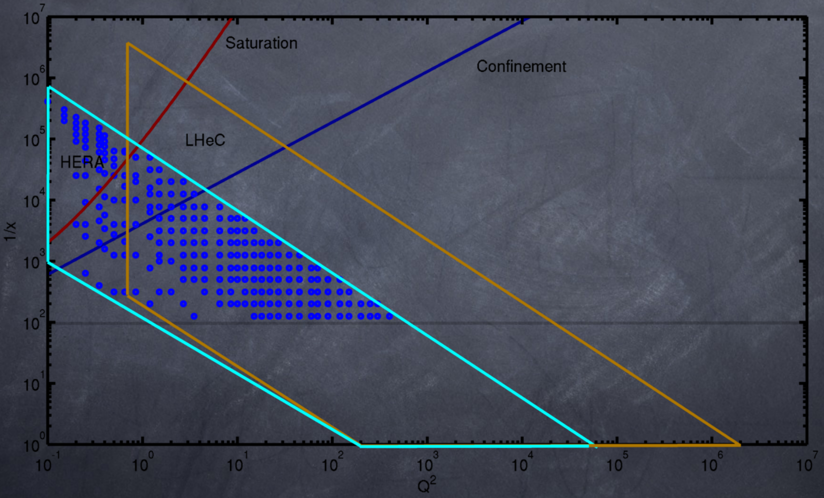
- Already applied to DIS [Brower, MD, Sarcevic, Tan], DVCS [Costa, MD] and vector meson production [Costa, MD, Evans].

Effective Pomeron Intercept

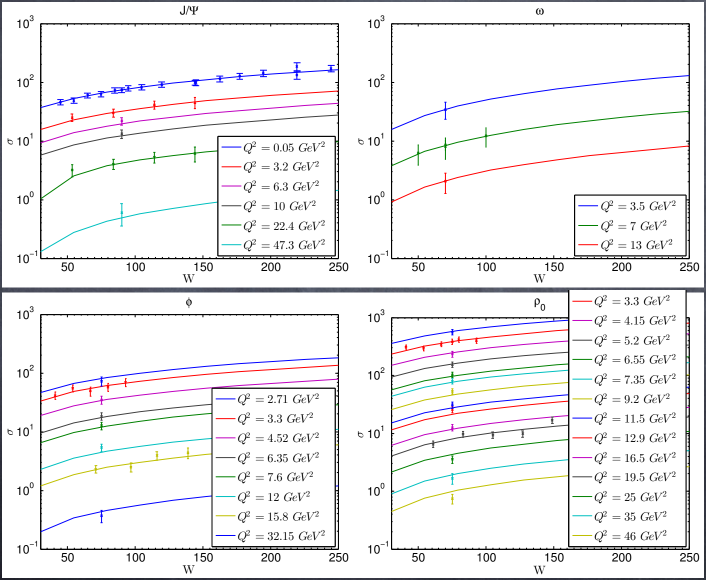
$$F_2 \sim \left(\frac{1}{x}\right)^{\epsilon_{eff}}$$



Confinement vs. Saturation



VM Production Cross Sections



Outline

1. Introduction, Pomeron and Odderon
2. Intercept & $\Delta(j)$ Curve
3. Pomeron and Odderon at Strong Coupling
4. Applications
5. Beyond Leading Order

We now want to calculate j_0 by expandin $\Delta(j)$ into a power series, and setting $\Delta = 2$.

- The $\mathcal{N} = 4$ SYM operators corresponding to the Pomeron spectral curve are

$$Tr[F_{\pm\perp} D_{\pm} \cdots D_{\pm} F_{\perp\pm} Z^{\tau-2}] + \cdots$$

  and for the type-a Odderon

$$Tr(F_{\perp\pm} F^2 Z^k) + \cdots , .$$

- By supersymmetry both of these can be related to the spectral curve of $Tr[D_{\pm}^{S} Z^{\tau}]$, for which a lot of study has been done using spin chains. In particular in an expansion

$$\Delta_Z(S,\tau) = \tau + \alpha_1(\tau,\lambda) S + \alpha_2(\tau,\lambda) S^2 + \alpha_3(\tau,\lambda) S^3 + \cdots$$

  the coefficients α_1 and α_2 have been recently calculated.
- The Pomeron and Odderon spectral curves can then be related to this, e.g. for the Pomeron

$$\Delta_P(j) = 2 + \Delta_Z(j-2, 2) .$$

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$$\alpha_1(\lambda, \tau) = \frac{\sqrt{\lambda}}{\tau} Y_\tau(\sqrt{\lambda}) .$$

- Knowledge of this coefficient is enough to calculate the Pomeron intercept up to $O(\lambda^{-2})$ [Costa, Goncalves, Penedones, 2012].
- More recently, using numerical analysis the coefficient α_2 has been calculated as well [Gromov, Levkovich-Maslyuk, Sizov, Valatka 2014]

$$\begin{aligned}\alpha_2(\lambda, \tau) &= -\frac{8\pi^2\lambda}{\tau^3} + \frac{2\pi\lambda^{1/2}}{\tau^3} + \frac{1}{4\tau} + \frac{1-\tau^2(24\zeta(3)+1)}{64\pi\tau^3\lambda^{1/2}} \\ &\quad - \frac{8\tau^4+\tau^2(72\zeta(3)+11)}{512\pi^2\tau^3\lambda} \\ &\quad + \frac{24\tau^4(16\zeta(3)+20\zeta(5)-7)-48\tau^2(31\zeta(3)+20\zeta(5)+7)+75}{16384\pi^3\tau^3\lambda^{3/2}} + O(\lambda^{-2}) .\end{aligned}$$

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$$(\Delta_P(j,\tau)-2)^2 = \tau^2+\beta_1(\lambda,\tau)(j-2)+\beta_2(\lambda,\tau)(j-2)^2+\beta_3(\lambda,\tau)(j-2)^3+\cdots$$

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- The coefficients β_n themselves allow an expansion

$$\beta_n=\lambda^{\frac{2-n}{2}}\left(b_{n0}+\frac{b_{n1}}{\lambda^{1/2}}+\frac{b_{n2}}{\lambda}+\frac{b_{n3}}{\lambda^{3/2}}+\cdots\right)$$

With this expansion, one has

$$(\Delta_P(j,\tau)-2)^2 = \tau^2+\sum_{n=1}\sum_{m=0} b_{nm}\lambda^{\frac{3-m-n}{2}}(j-2)^n+O((j-2)^{n+1})\,.$$

- The intercept can now be found iteratively by expanding

$$j_0=2+\frac{c_1}{\lambda^{1/2}}+\frac{c_2}{\lambda}+\frac{c_3}{\lambda^{3/2}}+\frac{c_4}{\lambda^2}+\cdots\,.$$

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- To illustrate how this goes, the first few coefficients are

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\begin{aligned}
c_1 &= -b_{10}^{-1}\tau^2 \\
c_2 &= -b_{10}^{-1}b_{11}c_1 \\
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$$

  and so on (up to c_6). The coefficients b_{ij} can be extracted from α_1 and α_2.
- This gives for the Pomeron intercept

$$
\begin{aligned}
\alpha_P(\lambda,\tau) = j_0(\lambda,\tau) &= 2 - \frac{\tau^2}{2\lambda^{1/2}} - \frac{\tau^2}{4\lambda} + \frac{\tau^2(\tau^2-3)}{16\lambda^{3/2}} - \frac{\tau^2[\tau^2(24\zeta(3)+11)-12]}{64\lambda^2} \\
&+ \frac{-2\tau^6+6\tau^4(48\zeta(3)+19)-63\tau^2}{256\lambda^{3/2}} \\
&+ \frac{\tau^2[-216-2\tau^4(17+36\zeta(3)+60\zeta(5))+\tau^2(637+1536\zeta(3)+480\zeta(5))}{512\lambda^3} \\
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The same steps can be repeated for the type-a Odderon, expanding around $j = 1$ instead of $j = 2$, and having $\tau = 4$ instead of 2.

- One finds

$$\alpha_{O,a} = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \frac{96\zeta(3)+41}{\lambda^2} + \frac{288\zeta(3)+\frac{1249}{16}}{\lambda^{5/2}} + \frac{-720\zeta(5)+192\zeta(3)+\frac{159}{4}}{\lambda^3} + \cdots .$$

- For type-b odderons we don't expect the expansions of α_1 and α_2 to hold, so we will not use them.
- We proceed by making the expansion for β_n, as before, and arriving at

$$
\begin{aligned}
(\Delta_{O,b}(j,\tau_b)-2)^2 = \quad & \tau_b^2 \quad +\lambda^{1/2}\Big(2-\frac{b_{11}}{\lambda^{1/2}}+\frac{b_{12}}{\lambda^{3/2}}+\frac{b_{13}}{\lambda^2}+\cdots\Big)(j-1) \\
+ \quad & \Big(b_{20}+\frac{b_{21}}{\sqrt{\lambda}}+\frac{b_{22}}{\lambda}+\frac{b_{23}}{\lambda^{3/2}}+\cdots\Big)(j-1)^2 \\
+ \quad & \lambda^{-1/2}\Big(b_{30}+\frac{b_{31}}{\lambda^{1/2}}+\frac{b_{32}}{\lambda}+\frac{b_{33}}{\lambda^{3/2}}+\cdots\Big)(j-1)^3 \\
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- As before we expand everything in λ and we can relate the coefficients iteratively

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\begin{aligned}
c_1(\tau_b) &= \left(\frac{-1}{2}\right)\tau_b^2\,, \\
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and so on.

- For $k=0$ the twist $\tau_b=0$, which if we plug in above we can see

$$\alpha_{O,b}=1\,,$$

without higher order correction in an $1/\sqrt{\lambda}$ expansion. To state it more graphically, in the limit $\tau_b \to 0$, higher order corrections can change the shape of the spectral curve without changing its minimum at $\Delta=2$. For $\tau_b \neq 0$, more information is required in order to determine the higher order expansion for their intercepts.

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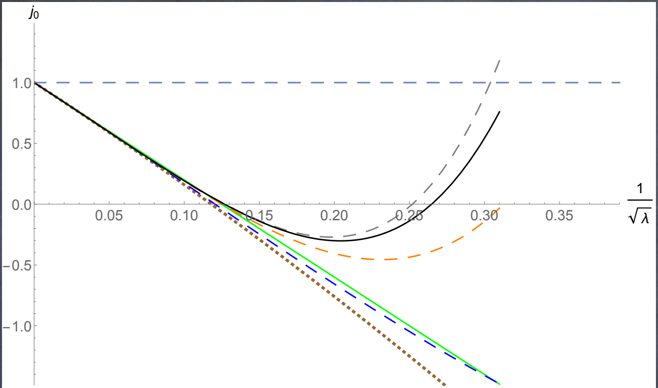

Figure: Odderon-a intercept at strong coupling. The solid green-line is to first order in $1/\sqrt{\lambda}$, the dotted brown-line is to second order, the dashed blue-, orange- and grey-line are to third order, fourth order and fifth order respectively. Finally the solid black-line is the intercept up to sixth order.

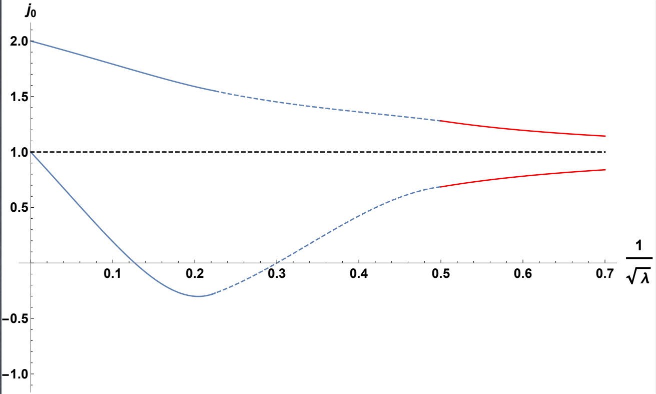

Figure: The Pomeron and Odderon-(a) intercepts from strong to weak coupling. The dark blue curves are the calculated strong coupling results, the red curves are the known weak coupling intercepts, and the dashed line is an interpolation. It is interesting to note that up to their current orders, both the Pomeron and Odderon intercepts appear consistent with weak coupling results in the transition region. Black dashed line is for the Odderon-(b) solution where $\alpha_{O,b} = 1$.

Thank You!