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Progress in backreacted holographic QCD

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Holographics Methods and Applications

Reykjavik – 18 August 2014

Outline

1. Brief intro to holographic V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

2. V-QCD at finite quark mass

[MJ, arXiv:14nn.xxxx]

   - Scaling at finite quark mass
   - S-parameter
   - Gell-Mann-Oakes-Renner relation

3. CP-odd Lagrangian

[Arean, Iatrakis, MJ, Kiritsis]

   - η' mass and Witten-Veneziano relation

Finite T and μ $\rightarrow$ Timo Alho's talk this morning

Motivation

QCD: $SU(N_c)$ gauge theory with N_f quark flavors (fundamental)

- Often useful: "quenched" or "probe" approximation, $N_f \ll N_c$, 't Hooft limit
- Here Veneziano limit: large N_f, N_c with $x = N_f/N_c$ fixed $\Rightarrow$ backreaction

Backreaction $\Rightarrow$ better modeling of (ordinary) QCD?

Important new features can be captured in the Veneziano limit:

- Phase diagram of QCD (at zero temperature, baryon density, and quark mass), varying $x = N_f/N_c$
- The QCD thermodynamics as a function of x
- Phase diagram as a function of baryon density
- Effect of turning on a finite quark mass at finite x (this talk)

Holographic V-QCD: the fusion

The fusion:

1. IHQCD: model for glue by using dilaton gravity

   [Gursoy, Kiritsis, Nitti; Gubser, Nellore]

2. Adding flavor and chiral symmetry breaking via tachyon brane actions

   [Klebanov,Maldacena; Bigazzi,Casero,Cotrone,Iatrakis,Kiritsis,Paredes]

Consider 1. + 2. in the Veneziano limit with full backreaction
$\Rightarrow$ V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

Defining V-QCD

Degrees of freedom ($T = \tau\mathbb{I}$):

- The tachyon $\tau \leftrightarrow \bar{q}q$,   and the dilaton $\lambda \leftrightarrow \mathrm{Tr}F^2$
- $\lambda = e^{\phi}$ is identified as the 't Hooft coupling $g^2 N_c$

Terms relevant in the classical vacuum:

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x V_f(\lambda,\tau) \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau)}$$

$$V_f(\lambda,\tau) = V_{f0}(\lambda)\exp(-a(\lambda)\tau^2) \; ; \qquad ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}x^\mu x^\nu)$$

Need to choose V_{f0}, a, and κ ... (V_g chosen as before)

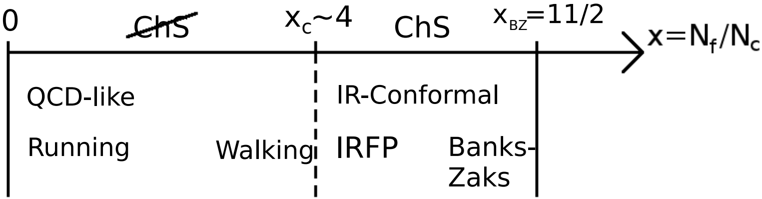
The simplest and most reasonable choices do the job!

(See Timo Alho's talk for more details on choosing the potentials)

Phase diagram of V-QCD

Different phases $\leftrightarrow$ different IR geometries

With reasonable potentials, at zero quark mass and temperature, constructing numerically all vacua, QCD phase diagram reproduced:



- Conformal transition (BKT) at $x = x_c \simeq 4$ [Kaplan,Son,Stephanov;Kutasov,Lin,Parnachev]
- Miransky scaling, $\langle \bar{q}q \rangle \sim \exp\left[-\frac{2K}{\sqrt{x_c - x}}\right]$, in walking regime
- For $x < x_c$, "good" IR singularity + tachyon
- For $x > x_c$, IR AdS_5, zero tachyon

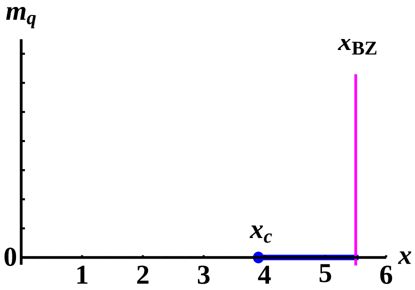
Turning on finite m_q

Quark mass defined through the tachyon boundary conditions in the UV:

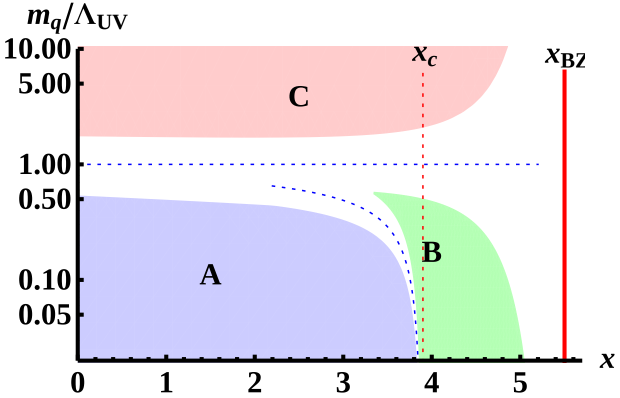
$$\tau(r) \simeq m_q(-\log r)^{-\gamma_0/\beta_0} r + \sigma(-\log r)^{\gamma_0/\beta_0} r^3$$

with the 5th coordinate $r \sim 1/\Lambda \to 0$ and $\sigma \sim \langle \bar{q} q \rangle$

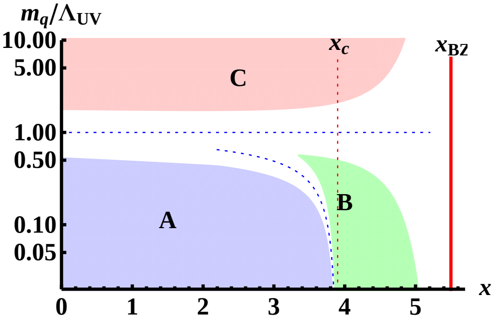
- Logarithmic running obtained by fitting the potentials (κ, a and V_f)
- Implies nonzero tachyon and chiral symmetry breaking
- Conformal transition becomes a crossover
- Discontinuous change of IR geometry in the conformal window



Analysis of the tachyon solution $\Rightarrow$ separate different regimes:



Border between A and B $\sim \exp\left[-\frac{2K}{\sqrt{x_c - x}}\right] \sim \langle\bar{q}q\rangle$



A : m_q is a small perturbation

$$m_n = m_n(m_q = 0) + C_n m_q + \cdots$$

B : “Scaling” regime: amount of walking determined by m_q

$m_n \sim m_q^{1/\Delta_*}$ “model independent, standard hyperscaling”

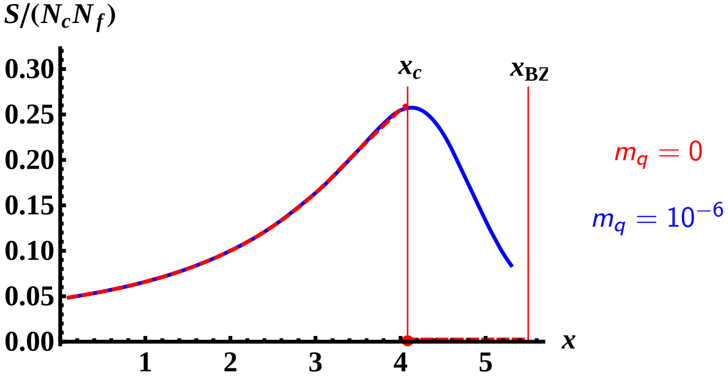
[Evans, Scott]

C : Large quark mass: model dependent,
in V-QCD large mass gap

S-parameter

After adding gauge fields dual to vector operators in the DBI action

$$S = 4\pi \frac{\partial}{\partial q^2} \left[q^2 \Pi_V(q^2) - q^2 \Pi_A(q^2) \right]_{q^2=0}$$



- Discontinuity at $m_q = 0$ in the conformal window
- Qualitative agreement with field theory expectations

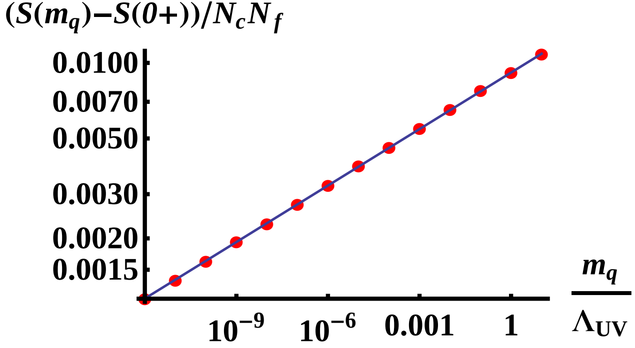
[Sannino]

Scaling of the S-parameter

As $m_q \to 0$ in the conformal window,

$$S(m_q) \simeq S(0+) + c\left(\frac{m_q}{\Lambda_{\rm UV}}\right)^{\frac{\Delta_{FF}-4}{\Delta_*}}$$

- Limiting value $S(0+) = \lim_{m_q \to 0+} S(m_q)$ is finite and positive (while $S(0) = 0$)
- Δ_{FF} is the dimension of $\mathrm{tr} F^2$ at the fixed point



Gell-Mann-Oakes-Renner relation

Combination of two computations:

1. Pion mass at small m_q (analyzing the fluctuation equations)
2. Chiral condensate as $\frac{d}{dm_q} S_{\text{on-shell}}$, when $m_q \to 0$

$$f_\pi^2 m_\pi^2 \simeq m_q \langle \bar{q} q \rangle \ , \qquad m_q \to 0$$

Numerical proportionality constant ($= 1$) is

- sensitive to the backreaction of the flavor to the glue
- correct when backreaction taken into account

The CP-odd term

Bulk axion a

- dual to $\mathrm{tr} F \wedge F$
- background value identified as θ/N_c, where θ is the theta angle of QCD

Tachyon Ansatz $T = \tau e^{i\xi}\, \mathbb{I}$

String motivated CP-odd term added in the action

$$S_a = -\frac{M^3 N_c^2}{2} \int d^5x \sqrt{-\det g}\, Z(\lambda) \times \left[da - x\left(2V_a(\lambda,\tau)\, A - \xi\, dV_a(\lambda,\tau)\right)\right]^2$$

[Casero, Kiritsis, Paredes]

Symmetry

$$A_\mu \to A_\mu + \partial_\mu \epsilon\ , \qquad \xi \to \xi - 2\epsilon\ , \qquad a \to a + 2x\, V_a \epsilon$$

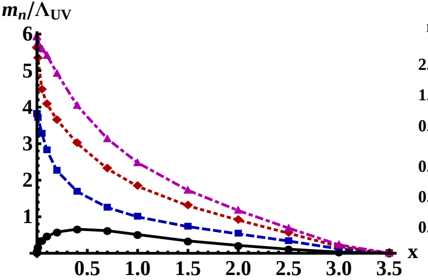
reflects the axial anomaly in QCD (with $\epsilon = \epsilon(x_\mu)$)

The mass of η'

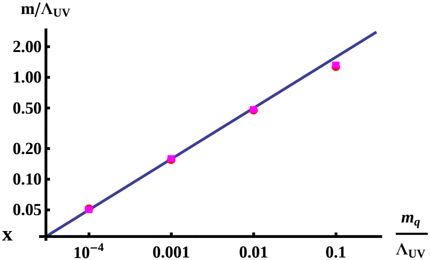
Perturbative analysis of the coupled flavor singlet (pseudoscalar meson+glueball) fluctuation equations $\Rightarrow$
The Witten-Veneziano relation: η' becomes light as $x \to 0$

$$m_{\eta'}^2 \simeq m_\pi^2 + x \frac{N_f N_c \chi}{f_\pi^2}$$

$m_q = 0$



$x = 0.0001$



Conclusions

- Backreaction is important
- Finite (flavor independent) quark mass and axial anomaly can be implemented in V-QCD
- Dependence of mass spectra on m_q matches with QCD at qualitative level
- Next step: fitting the potentials of the model quantitatively to QCD data

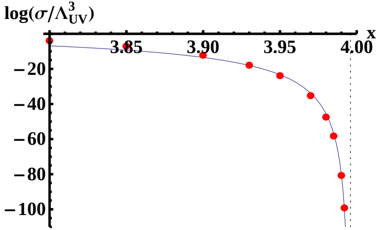
Extra slides

Energy scales at zero quark mass

V-QCD reproduces the expected picture:

1. QCD regime: single energy scale Λ
2. Walking regime ($x_c - x \ll 1$): two scales related by Miransky/BKT scaling law

$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\kappa}{\sqrt{x_c - x}}\right)$$



3. Conformal window ($x_c \leq x < 11/2$): again one scale Λ, but slow RG flow

Vector correlators and S-parameter

1. Introduce bulk gauge fields dual to vector operators
$$A_\mu^{L/R} \leftrightarrow \bar{q}\gamma_\mu(1 \pm \gamma_5)q$$

2. Fluctuate full flavor action of V-QCD
$$S_f = -\frac{1}{2} M^3 N_c \mathbb{T}r \int d^4x\, dr \left(V_f(\lambda, T^\dagger T)\sqrt{-\det \mathbf{A}_L} + (L \to R) \right)$$
$$\mathbf{A}_{L/R\,MN} = g_{MN} + w(\lambda, T) F_{MN}^{(L/R)} +$$
$$+ \frac{\kappa(\lambda, T)}{2}\left[(D_M T)^\dagger (D_N T) + (D_N T)^\dagger (D_M T) \right]$$
Here T and $A^{(L/R)}$ matrices in flavor space

3. Compute vector-vector correlators using standard recipes
$$-i\langle J_\mu^{a\,(V)} J_\nu^{b\,(V)} \rangle \propto \delta^{ab} \left(q^2 \eta_{\mu\nu} - q_\mu q_\nu \right) \Pi_V(q^2)$$
$$-i\langle J_\mu^{a\,(A)} J_\nu^{b\,(A)} \rangle \propto \delta^{ab} \left[\left(q^2 \eta_{\mu\nu} - q_\mu q_\nu \right) \Pi_A(q^2) + q_\mu q_\nu \Pi_L(q^2) \right]$$

Matching to QCD

In the UV ($\lambda \to 0$):

- UV expansions of potentials matched with perturbative QCD beta functions $\Rightarrow$

$$\lambda(r) \simeq -\frac{1}{\beta_0 \log r}, \quad \tau(r) \simeq m(-\log r)^{-\gamma_0/\beta_0} r + \sigma(-\log r)^{\gamma_0/\beta_0} r^3$$

  with the 5th coordinate $r \sim 1/\Lambda \to 0$

In the IR ($\lambda \to \infty$):

- $V_{f0}(\lambda)$, $a(\lambda)$, and $\kappa(\lambda)$ chosen to produce tachyon divergence: several possibilities ($\to$ Potentials I and II)
- Extra constraints from the asymptotics of the meson spectra
- Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (i.e, first guesses usually work!)

How does the phase structure arise?

Turning on a tiny tachyon in the conformal window

$$\tau(r) \sim m_q r^{\Delta_*} + \sigma r^{4-\Delta_*}$$

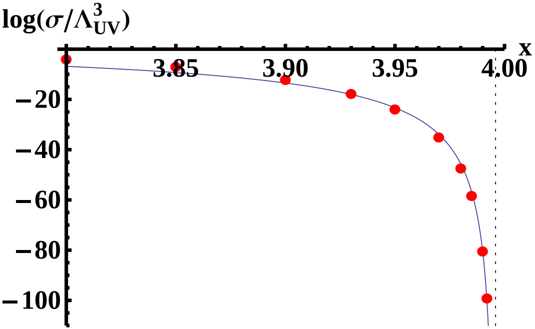
Breitenlohner-Freedman (BF) bound

$$\Delta_*(4-\Delta_*) = -m_\tau^2 \ell_*^2 \leq 4$$

Violation of BF bound $\Rightarrow$ instability

- $\Rightarrow$ bound saturated at the conformal phase transition ($x = x_c$)
- BF bound violation leads to a BKT transition quite in general

Consequences of the BKT transition



$$\langle \bar{q} q \rangle \sim \sigma \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

1. Miransky/BKT scaling as $x \to x_c$ from below
   - E.g., The chiral condensate $\langle \bar{q} q \rangle \propto \sigma$
2. Unstable Efimov vacua observed for $x < x_c$
3. Turning on the quark mass possible

Finite T and μ – definitions

Add gauge field

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right] - N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \times \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda) F_{ab})}$$

$$F_{r0} = \partial_r \Phi \qquad \Phi = \mu - n r^2 + \cdots$$

A more general metric (A and f solved from EoMs)

$$ds^2 = e^{2A(r)} \left(\frac{dr^2}{f(r)} - f(r) dt^2 + d\mathbf{x}^2 \right)$$

Nontrivial blackening factor f: black hole solutions possible

Various solutions

Two classes of IR geometries:

1. Black hole solutions $\rightarrow$ temperature and entropy through BH thermodynamics
   - $f'(r_h) = -4\pi T$; $\quad s = 4\pi M^3 N_c^2 e^{3A(r_h)}$
2. Thermal gas solutions ($f \equiv 1$)
   - Any T and μ, zero s

Two types of tachyon behavior ($\tau \leftrightarrow \bar{q}q$, quark mass and condensate from UV boundary behavior):

1. Vanishing tachyon – chirally symmetric
2. Nontrivial tachyon – chirally broken

$\Rightarrow$ four possible types of background solutions

Computation of pressure

Three phases turn out to be relevant (at small x)

- Tachyonic Thermal gas (chirally broken)
- Tachyonic BH (chirally broken)
- Tachyonless BH (chirally symmetric)

Nontrivial numerical analysis:

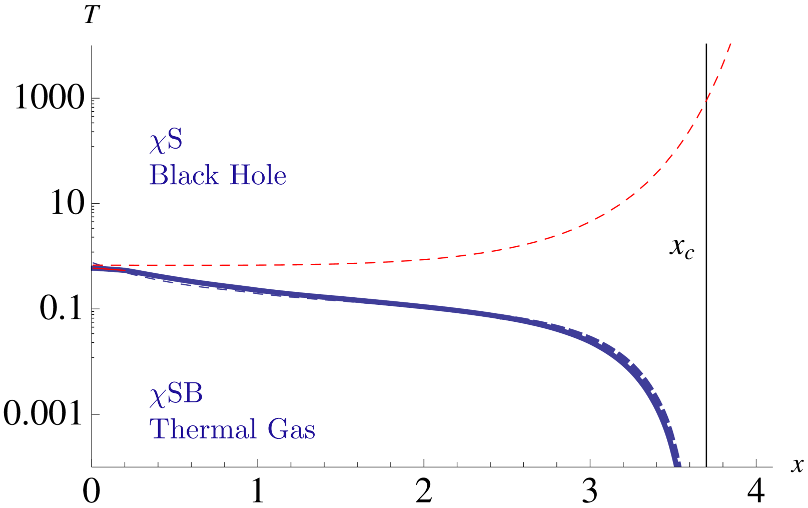
1. T, μ not input parameters, they need to be calculated first
2. Integrate numerically for each phase

$$dp = s\,dT + n\,d\mu$$

3. Phase with highest p dominates

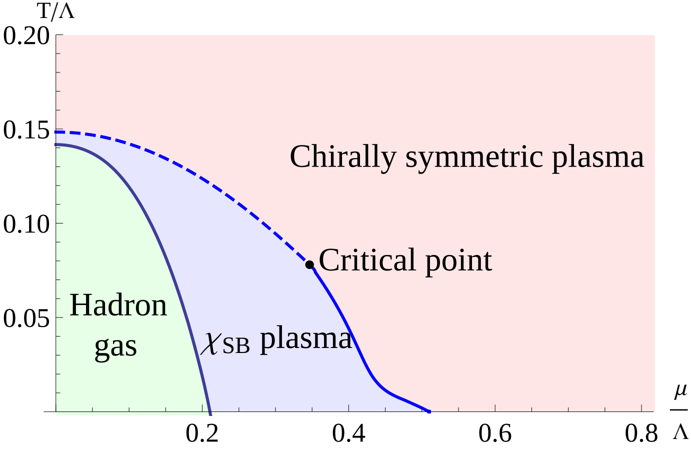
Phase diagram: example at zero μ

Phases on the (x, T)-plane – as expected from QCD



Phase diagram at finite μ (example at fixed x)

First attempt: $x = N_f/N_c = 1$, Veneziano limit, zero quark mass



- $\mathrm{AdS}_2 \times \mathbb{R}^3$ IR geometry as $T \to 0$
- Finite entropy at zero temperature $\Rightarrow$ instability?

Fluctuation analysis

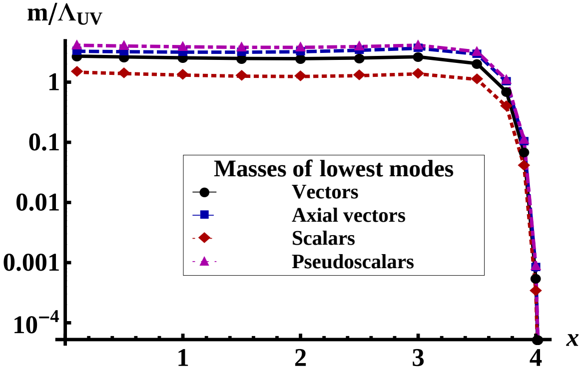
1. Meson spectra (at zero temperature and quark mass)
   - Implement (left and right handed) gauge fields in $\mathcal{S}_{\mathrm{V-QCD}}$
   - Four towers: scalars, pseudoscalars, vectors, and axial vectors
   - Flavor singlet and nonsinglet ($SU(N_f)$) states

In the region relevant for "walking" technicolor ($x \to x_c$ from below):

- Possibly a light "dilaton" (flavor singlet scalar): Goldstone mode due to almost unbroken conformal symmetry.
  Could the dilaton be the 125 GeV Higgs?

Meson masses

Flavor nonsinglet masses (Example: PotI)



- Miransky scaling:

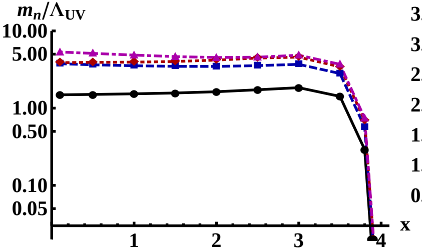
$$m_n \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

- Radial trajectories $m_n^2 \sim n$ or $m_n^2 \sim n^2$ depending on potentials

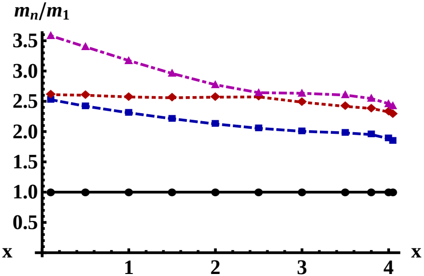
Scalar singlet masses

Scalar singlet (0^{++}) spectrum (PotI):

In log scale



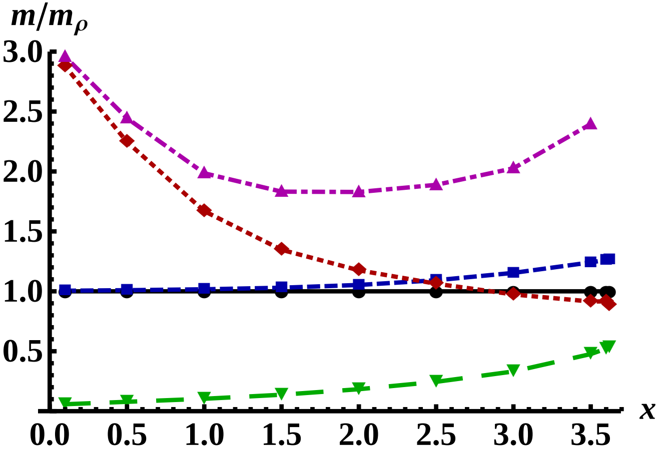
Normalized to the lowest state



- No light dilaton state as $x \to x_c$?

Meson mass ratios

Mass ratios (PotII): Lowest states normalized to ρ



All ratios tend to constants as $x \to x_c$: indeed no dilaton

S-parameter

$$S \sim \frac{d}{dq^2} q^2 \left[\Pi_V(q^2) - \Pi_A(q^2)\right]_{q^2=0}$$

where (at zero quark mass)

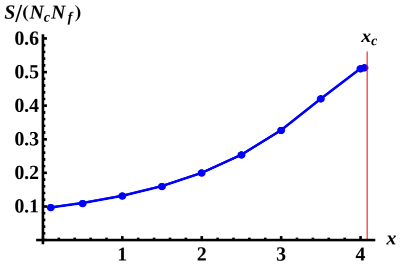
$$\Pi_{V/A}(q^2) \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right) \delta^{ab} \propto \langle J^{\mu\, a}_{V/A} J^{\nu\, b}_{V/A} \rangle$$

in terms of the vector-vector and axial-axial correlators

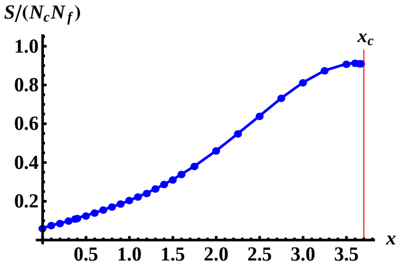
- The S-parameter might be reduced in the walking regime

Results:

PotI



PotII

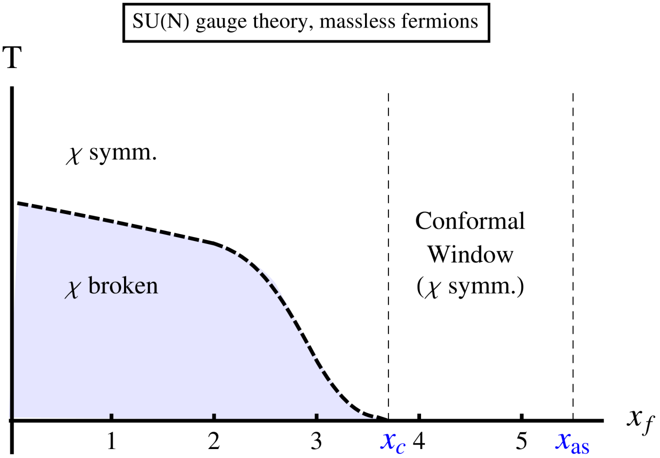


The S-parameter increases with x: expected suppression absent

Jumps discontinuously to zero at $x = x_c$

QCD at finite T (and x)

Expected phase structure at finite temperature (and x)



Potentials I

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x) \\
\kappa(\lambda) &= \frac{1}{\left(1+\frac{115-16x}{288\pi^2}\lambda\right)^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges exponentially:

$$
\tau(r) \sim \tau_0 \exp\left[\frac{81\ 3^{5/6}(115-16x)^{4/3}(11-x)}{812944\ 2^{1/6}}\frac{r}{R}\right]
$$

Potentials II

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x)\frac{1+\frac{115-16x}{216\pi^2}\lambda + \lambda^2/(8\pi^2)^2}{(1+\lambda/(8\pi^2))^{4/3}} \\
\kappa(\lambda) &= \frac{1}{(1+\lambda/(8\pi^2))^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges as

$$
\tau(r) \sim \frac{27\ 2^{3/4}3^{1/4}}{\sqrt{4619}}\sqrt{\frac{r-r_1}{R}}
$$

Effective potential

For solutions with $\tau = \tau_* = \text{const}$

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - xV_f(\lambda, \tau_*) \right]$$

IHQCD with an effective potential

$$V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_f(\lambda, \tau_*) = V_g(\lambda) - xV_{f0}(\lambda) \exp(-a(\lambda)\tau_*^2)$$

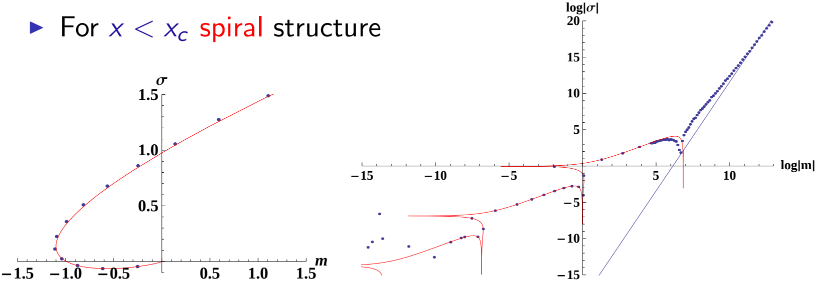
Minimizing for τ_* we obtain $\tau_* = 0$ and $\tau_* = \infty$

- $\tau_* = 0$: $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$;
  fixed point with $V'_{\text{eff}}(\lambda_*) = 0$
- $\tau_* \to \infty$: $V_{\text{eff}}(\lambda) = V_g(\lambda)$ (like YM, no fixed points)

Efimov spiral

Ongoing work: the dependence $\sigma(m)$ of the chiral condensate on the quark mass

- For $x < x_c$ spiral structure

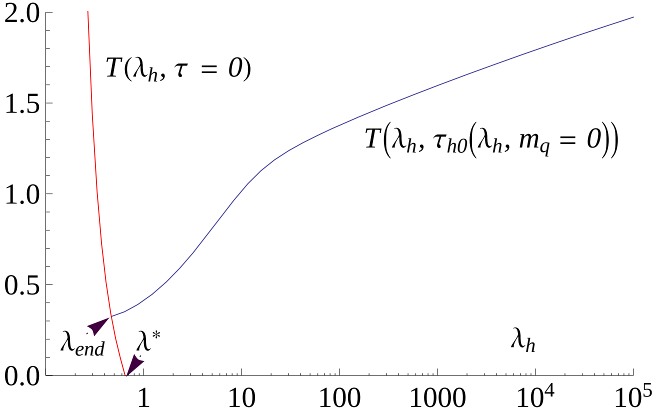


- Dots: numerical data
- Continuous line: (semi-)analytic prediction

Allows to study the effect of double-trace deformations

Black hole branches

Example: PotII at $x = 3$, $W_0 = 12/11$

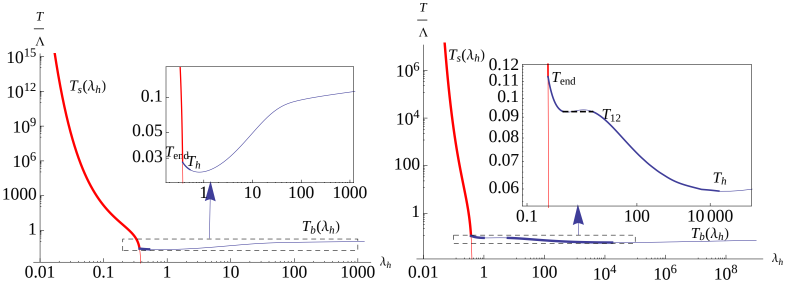


Simple phase structure: 1st order transition at $T = T_h$ from thermal gas to (chirally symmetric) BH

More complicated cases:

PotII at $x = 3$, W_0 SB

PotI at $x = 3.5$, $W_0 = 12/11$

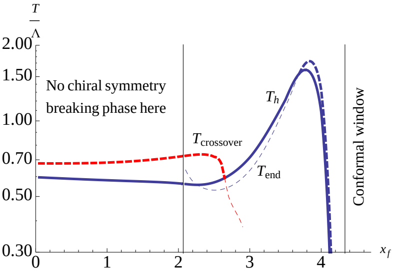


- Left: chiral symmetry restored at 2nd order transition with $T = T_{\text{end}} > T_h$
- Right: Additional first order transition between BH phases with broken chiral symmetry

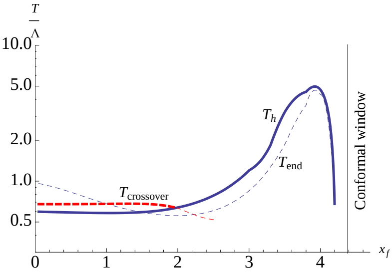
Also other cases . . .

Phase diagrams on the (x, T)-plane

PotI$_*$ W_0 SB

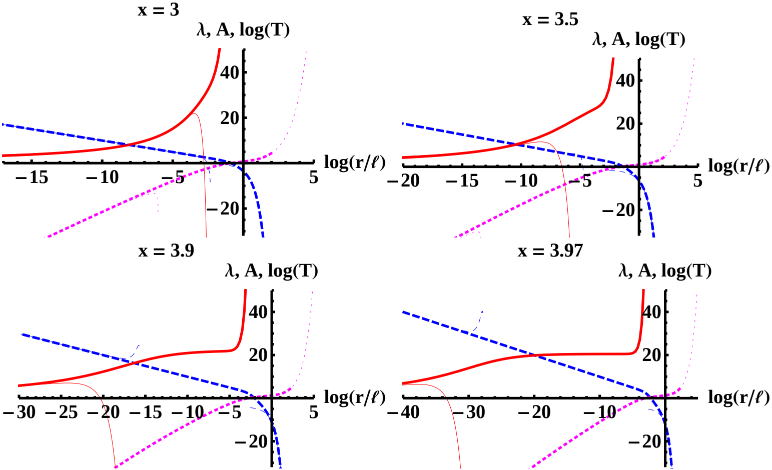


PotII$_*$ W_0 SB

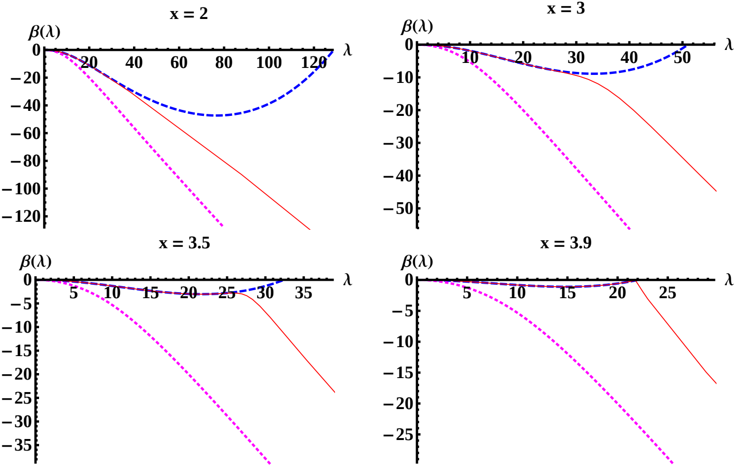


Backgrounds in the walking region

Backgrounds with zero quark mass, $x < x_c \simeq 3.9959$ (λ, A, τ)



Beta functions along the RG flow (evaluated on the background), zero tachyon, YM $x_c \simeq 3.9959$



Holographic beta functions

Generalization of the holographic RG flow of IHQCD

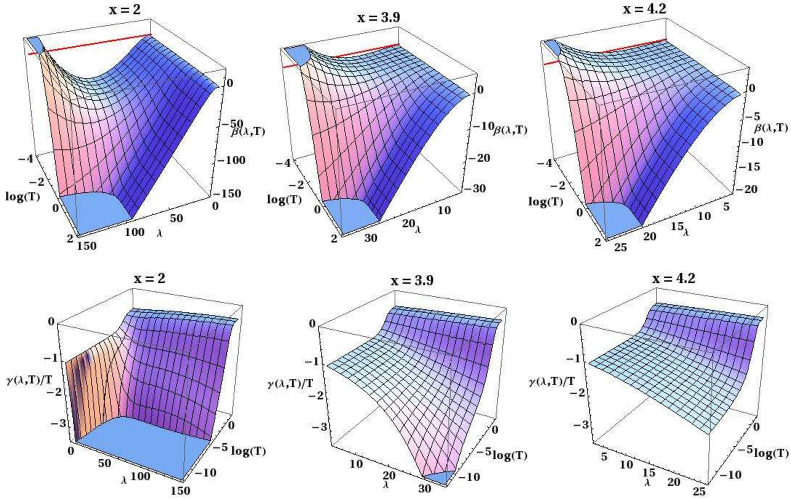
$$\beta(\lambda,\tau) \equiv \frac{d\lambda}{dA}\ ; \qquad \gamma(\lambda,\tau) \equiv \frac{d\tau}{dA}$$

linked to

$$\frac{dg_{\text{QCD}}}{d\log\mu}\ ; \qquad \frac{dm}{d\log\mu}$$

The full equations of motion boil down to two first order partial non-linear differential equations for β and γ

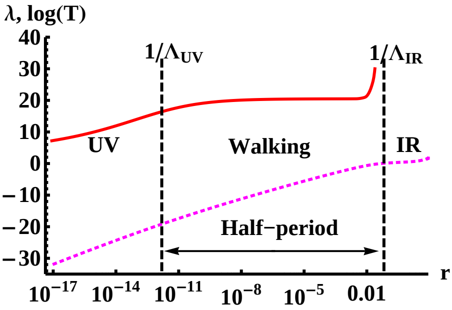
“Good” solutions numerically (unique)



Miransky/BKT scaling

As $x \to x_c$ from below: walking, dominant solution

- BF-bound for the tachyon violated at the IRFP
- x_c fixed by the BF bound:
  $\Delta = 2$ & $\gamma_* = 1$
  at the edge of the conformal window

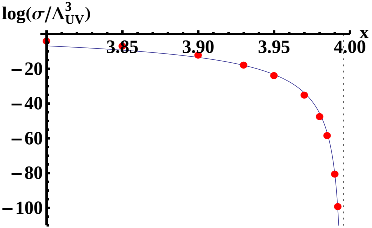


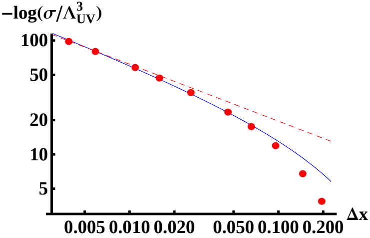
- $\tau(r) \sim r^2 \sin(\kappa\sqrt{x_c - x} \log r + \phi)$ in the walking region
- "0.5 oscillations" $\Rightarrow$ Miransky/BKT scaling,
  amount of walking $\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$

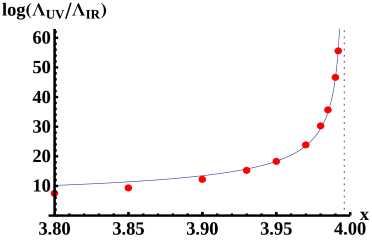
As $x \to x_c$ with known κ

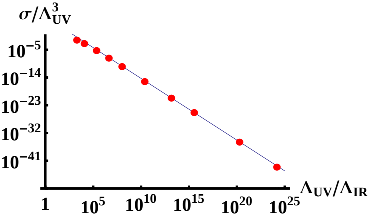
$$\langle \bar{q} q \rangle \sim \sigma \sim \exp(-2\pi/(\kappa\sqrt{x_c - x}))$$

$$\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$$







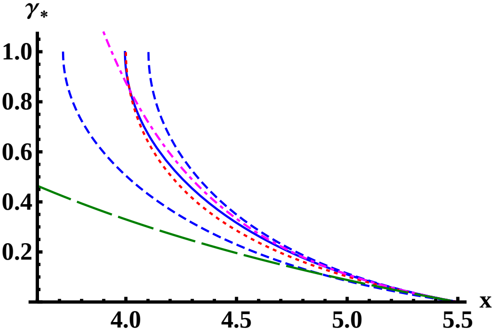


γ_* in the conformal window

Comparison to other guesses

V-QCD (dashed: variation due to W_0)
Dyson-Schwinger
2-loop PQCD
All-orders β

[Pica, Sannino arXiv:1011.3832]



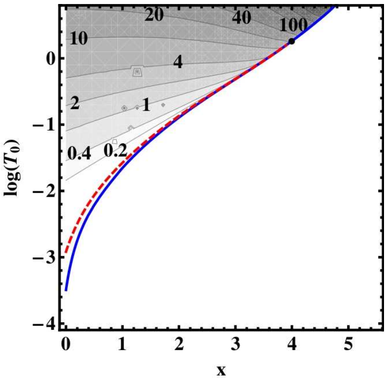
Parameters

Understanding the solutions for generic quark masses requires discussing parameters

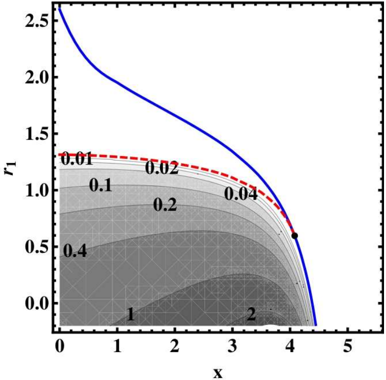
- YM or QCD with massless quarks: no parameters
- QCD with flavor-independent mass m: a single (dimensionless) parameter m/Λ_{QCD}
- In this model, after rescalings, this parameter can be mapped to a parameter (τ_0 or r_1) that controls the diverging tachyon in the IR
- x has become continuous in the Veneziano limit

Map of all solutions

All "good" solutions ($\tau \neq 0$) obtained varying x and τ_0 or r_1
Contouring: quark mass (zero mass is the red contour)

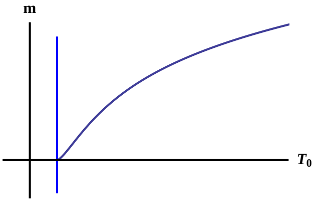


"Potentials I" $\leftrightarrow$ T_0



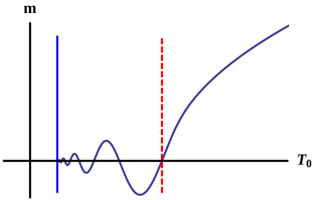
"Potentials II" $\leftrightarrow$ r_1

Mass dependence and Efimov vacua



Conformal window ($x > x_c$)

- For $m = 0$, unique solution with $\tau \equiv 0$
- For $m > 0$, unique "standard" solution with $\tau \neq 0$

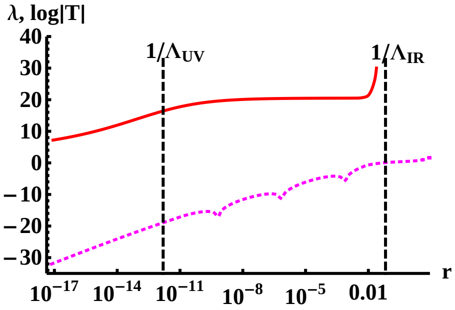


Low $0 < x < x_c$: Efimov vacua

- Unstable solution with $\tau \equiv 0$ and $m = 0$
- "Standard" stable solution, with $\tau \neq 0$, for all $m \geq 0$
- Tower of unstable Efimov vacua (small $|m|$)

Efimov solutions

- Tachyon oscillates over the walking regime
- $\Lambda_{UV}/\Lambda_{IR}$ increased wrt. "standard" solution

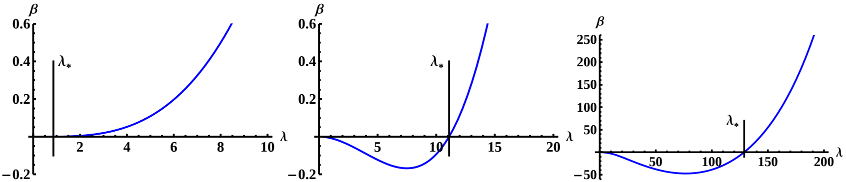


Effective potential: zero tachyon

Start from Banks-Zaks region, $\tau_* = 0$, chiral symmetry conserved $(\tau \leftrightarrow \bar{q}q)$, $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$

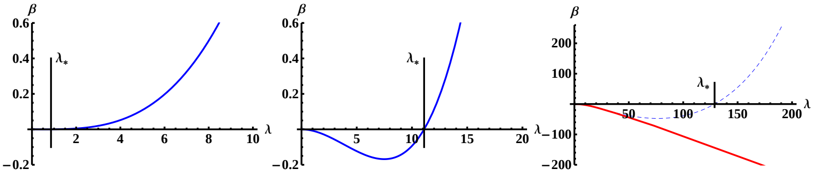
- V_{eff} defines a β-function as in IHQCD – Fixed point guaranteed in the BZ region, moves to higher λ with decreasing x
- Fixed point λ_* runs to ∞ either at finite $x (< x_c)$ or as $x \to 0$

Banks-Zaks	Conformal Window	
$x \to 11/2$	$x > x_c$	$x < x_c$??



Effective potential: what actually happens

Banks-Zaks $x \to 11/2$ | Conformal Window $x > x_c$ | $x < x_c$



$\tau \equiv 0$ | $\tau \equiv 0$ | $\tau \neq 0$

- For $x < x_c$ vacuum has nonzero tachyon (checked by calculating free energies)
- The tachyon screens the fixed point
- In the deep IR τ diverges, $V_{\mathrm{eff}} \to V_g \Rightarrow$ dynamics is YM-like

Where is x_c?

How is the edge of the conformal window stabilized?
Tachyon IR mass at $\lambda = \lambda_*$ $\leftrightarrow$ quark mass dimension

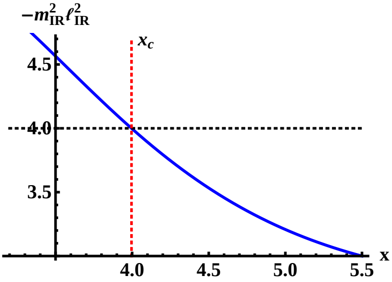
$$-m_{\text{IR}}^2 \ell_{\text{IR}}^2 = \Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) = \frac{24 a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - x V_0(\lambda_*))}$$

$$\gamma_* = \Delta_{\text{IR}} - 1$$

Breitenlohner-Freedman (BF) bound (horizontal line)

$$-m_{\text{IR}}^2 \ell_{\text{IR}}^2 = 4 \;\Leftrightarrow\; \gamma_* = 1$$

defines x_c



Why $\gamma_* = 1$ at $x = x_c$?

No time to go into details ... the question boils down to the linearized tachyon solution at the fixed point

- For $\Delta_{\rm IR}(4 - \Delta_{\rm IR}) < 4 \qquad (x > x_c)$:

$$\tau(r) \sim m_q r^{\Delta_{\rm IR}} + \sigma r^{4-\Delta_{\rm IR}}$$

- For $\Delta_{\rm IR}(4 - \Delta_{\rm IR}) > 4 \qquad (x < x_c)$:

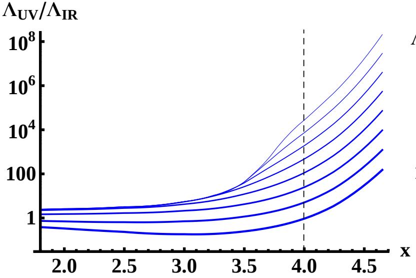
$$\tau(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\rm IR}) \log r + \phi\right]$$

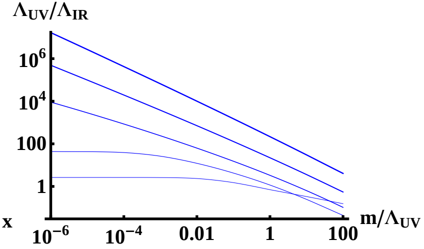
Rough analogy:
Tachyon EoM $\leftrightarrow$ Gap equation in Dyson-Schwinger approach
Similar observations have been made in other holographic frameworks

[Kutasov, Lin, Parnachev arXiv:1107.2324, 1201.4123]

Mass dependence

For $m > 0$ the conformal transition disappears
The ratio of typical UV/IR scales $\Lambda_{UV}/\Lambda_{IR}$ varies in a natural way
$m/\Lambda_{UV} = 10^{-6}, 10^{-5}, \ldots, 10$ $\quad x = 2, 3.5, 3.9, 4.25, 4.5$





sQCD phases

The case of $\mathcal{N} = 1$ $SU(N_c)$ superQCD with N_f quark multiplets is known and provides an interesting (and more complex) example for the nonsupersymmetric case. From Seiberg we have learned that:

- $x = 0$ the theory has confinement, a mass gap and N_c distinct vacua associated with a spontaneous breaking of the leftover R symmetry Z_{N_c}.
- At $0 < x < 1$, the theory has a runaway ground state.
- At $x = 1$, the theory has a quantum moduli space with no singularity. This reflects confinement with ChSB.
- At $x = 1 + 1/N_c$, the moduli space is classical (and singular). The theory confines, but there is no ChSB.
- At $1 + 2/N_c < x < 3/2$ the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f - N_c)$ IR free.
- At $3/2 < x < 3$, the theory flows to a CFT in the IR. Near $x = 3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows. However near $x = 3/2$ the dual magnetic $SU(N_f - N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.
- At $x > 3$, the theory is IR free.

Saturating the BF bound (sketch)

Why is the BF bound saturated at the phase transition (massless quarks)??

$$\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

- For $\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) < 4$:
  $\tau(r) \sim m_q r^{4-\Delta_{\mathrm{IR}}} + \sigma r^{\Delta_{\mathrm{IR}}}$
- For $\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) > 4$:
  $\tau(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\mathrm{IR}}) \log r + \phi\right]$
- Saturating the BF bound, the tachyon solutions will engtangle $\rightarrow$ required to satisfy boundary conditions
- Nodes in the solution appear trough UV $\rightarrow$ massless solution

Saturating the BF bound (sketch)

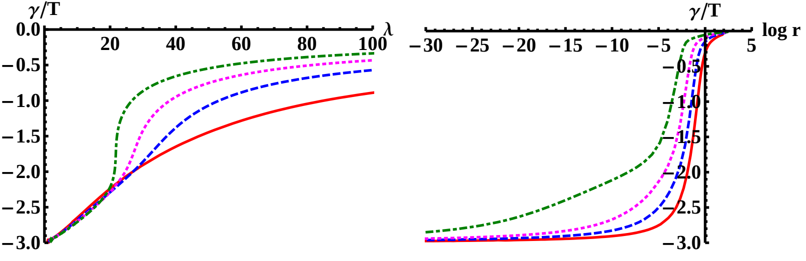
Does the nontrivial (ChSB) massless tachyon solution exist?
Two possibilities:

- $x > x_c$: BF bound satisfied at the fixed point $\Rightarrow$ only trivial massless solution ($\tau \equiv 0$, ChS intact, fixed point hit)
- $x < x_c$: BF bound violated at the fixed point $\Rightarrow$ a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: phase transition at $x = x_c$
As $x \to x_c$ from below, need to approach the fixed point to satisfy the boundary conditions $\Rightarrow$ nearly conformal, "walking" dynamics

Gamma functions

Massless backgrounds: gamma functions $\frac{\gamma}{\tau} = \frac{d \log \tau}{dA}$



$x = 2, 3, 3.5, 3.9$